ME 3870 Lab 4: Temperature Measurement with Thermocouples

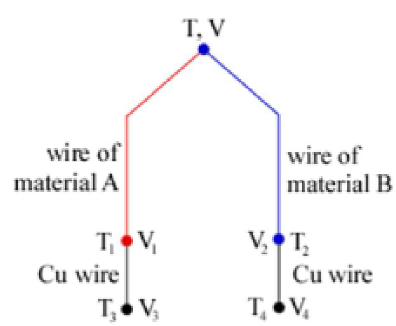
by Walker Blevins, Eshwar Pamula, and Hiromu Yamamoto

Executive Summary: The aim of this investigation was to utilize the theory of the Seebeck effect in conductive materials to construct functional thermocouple circuits capable of accurately yet indirectly measuring temperatures over specified input temperature ranges and output multimeter voltage ranges. Data for both the static calibration of each thermocouple and the measurement of each thermocouple's dynamic step response to a common input temperature shift were captured using appropriately tailored virtual instruments in LabView. The subsequent data analyses focus on the derivation of linear regression models in MATLAB for both of the static and dynamic calibration datasets, which were in turn used to draw conclusions regarding measurement errors, uncertainties, and correlation factors associated with the static calibration data as well as time constants associated with the dynamic responses of each thermocouple. Ultimately this investigation served to validate the assumptions that thermocouples make for accurate temperature measurement tools due to their high accuracy and fast dynamic responses.

1. Introduction

The general motivation for the investigation of thermocouples lies in the necessity for robust, precise temperature measurement systems throughout a wide range of industrial applications. Thermocouples are cost-efficient and easily integrable system components and can be used to measure an extremely wide range of temperatures, while also exhibiting very quick responses to dynamic inputs. The critical aspect which makes them a beneficial tool in the development of controlled-temperature systems is their basis in the theory of the Seebeck effect for conductive materials; i.e. they exhibit an approximately linear relationship between input temperature differences and output voltages. The scale factor for this ideally linear relationship is known as the Seebeck coefficient, which is unique for different materials as it depends on electron mobility/conductivity. Materials with high Seebeck coefficients have structures that hinder the flow of electrons from one end to another, generating a larger electric potential difference than a material with a low Seebeck coefficient would for the same applied temperature gradient. This property of semiconductors provides the foundation for the choice of thermocouple configuration utilized in this investigation.

The goal in constructing the circuit for this lab was to connect the multimeter in a way that would not affect the true measured temperature reading. For any thermocouple circuit, the electric potential across a segment of conductive material is proportional to the temperature difference across its respective ends. Hence, the summation of these individual sensitivity relations can yield the critical linear relation between the measured voltage across the free ends of the thermocouples and the temperature at any particular junction with respect to that at a reference junction held at constant temperature. For a measurement junction formed between wires of two different materials, it was thus necessary to set the free ends at a common reference temperature, with copper leads attached to the multimeter in order to avoid creating additional junctions between the thermocouple materials and the copper-wire circuitry of the multimeter. The summation of individual sensitivity relations under the described conditions results in a theoretically accurate system for which the multimeter voltage varies linearly with the temperature at the measurement junction.



$$V_4 - V_3 = (\alpha_A - \alpha_B)(T - T_{ref})$$

Figure A.1: Thermocouple System with Copper Leads

The central objective of this investigation was the analysis of the datasets containing measurements for both static and dynamic thermocouple calibration, and the subsequent interpretation of key statistics to draw conclusions regarding the true accuracy of the measurement system. In order to accomplish this, regression models were used to derive polynomial curves of minimized error for both the static calibration data and the natural logarithms of the normalized responses to a common step input. Subsequent statistical analyses for the static behavior of the thermocouple, as explained in the experimental procedure, thus allowed for a comparison between the measured thermocouple data and the expected results based on the Seebeck effect. As for the dynamic behavior of the thermocouple, the data analysis and regression function allowed for the calculation of the time constant for each thermocouple.

II. Experimental Procedure

To begin the experimental thermocouple investigation, a total of four thermocouples were constructed by cutting approximately equal lengths of iron and constant wire. Measurement junctions were then established; this was accomplished for one set of wires by twisting their ends tightly together, and for the remaining wires by creating a weldment between the two materials. Finally, in order to ensure voltage measurements accurately reflected the temperature behavior at the measurement junction exclusively, copper leads were attached to the free ends of each thermocouple to match the material of the internal wiring of the voltmeter. This served to preserve the measurement system's accuracy, preventing the generation of extraneous thermoelectric voltages which would distort the input signal to the voltmeter.

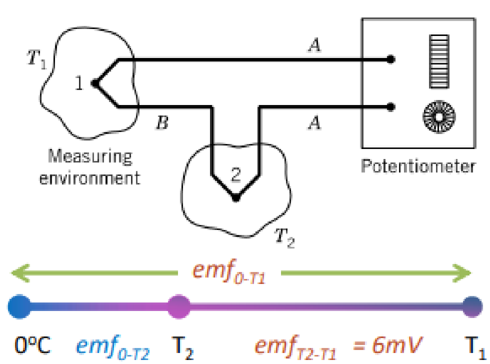


Figure A.1: Experimental Setup

In order to ensure the production of proper regression curves, it was first necessary to conduct an initial investigation of the thermocouples. The lead on the end of the iron wire was connected to the positive terminal of the multimeter to ensure positive values were read by the multimeter; this configuration was consistent with the material's relatively higher Seebeck coefficient (sensitivity to temperature differences). A thermometer was used to measure the temperatures of the laboratory room and a prepared cold-water bath, while a multimeter was used to measure the thermocouple circuit output voltage under various combinations of measurement and reference junction location conditions. The goal of this initial experimentation was to confirm that the thermocouple functioned in a reasonably similar manner as would be expected based on theory and the tabulated data can be found in Tables A.1 and A.2.

The collection of calibration data required a slightly adjusted setup, utilizing an AD594 thermocouple amplifier chip with a single power supply. For the static calibration, a banana-to-BNC adapter was used to connect the amplifier output and ground to the NI PCle-6321 data acquisition board, while the thermocouple wires were connected directly to the corresponding amplifier input terminals. The AD594 chip effectively amplified the thermocouple signal for analysis purposes while also eliminating the necessity of the copper leads and ice bath, as it electronically compensated for the reference junction. The LabVIEW VI for static and dynamic calibration was then updated to accurately document the data for the given objectives.

Using the thermocouple in conjunction with the amplifier chip allowed for the acquisition of static calibration data for each of the three welded thermocouples as the measurement junction was submerged in a water bath of increasing temperature, with measurements taken every 5 ° C based on thermometer readings. Collecting data for the step response of each thermocouple required a similar setup, with the only major difference being the connection of the AD594 chip output to the analog input of the NI PCle 6321 DAQ board. In order to capture the step responses of all four thermocouples, the measurement junctions were individually plunged into a bath of boiling water, and both the amplified static calibration and step response data were saved in LabVIEW for analysis.

With all experimental data stored in appropriate files, the focus of the investigation shifted to data analysis in MATLAB. A linear least squares regression function was created in MATLAB in order to generate coefficients for a best-fit line, the expected standard deviation of the measurements, and the coefficient of determination for the regression model. These statistics of interest, as well as the calculated voltage resolution, quantization error, and experimental uncertainty, were tabulated and can be found in Tables B.3 and B.4, while the measured data and calculated regression line were overlaid in Figure B.2 for clarity. In order to analyze the step response of the thermocouples, the natural logarithm of the normalized response of each system was determined. These values were plotted along with their corresponding regression lines. The time constants associated with each thermocouple were then determined and tabulated in Table C.1.

III. Results and Discussion

The results of the initial investigation for one of the welded thermocouples were consistent with theoretical expectations and served to confirm that the device was reliable enough for more in-depth analyses. Output voltage values from the multimeter were converted to appropriate temperature values using sensitivity information tabulated in Table A.1 for an array of junction location combinations. As expected, the multimeter read output voltage values were extremely close to 0 mV for the cases where both the measurement and reference junctions were kept at the same temperature. Furthermore, when the welded junction was held at ice water temperature, the behavior of the output voltage was predictable, following an approximately linear growth as the reference junction was moved from air temperature to a warmer temperature in a group member's hands. The below table helped establish a general sense of the measurement device's sensitivity and behavior prior to obtaining discrete data for analysis.

Table A.1: Multimeter Output Voltages for Initial Investigation

Reference junction in: →			Air	Hands	Ice Water
Welded junction location	Thermometer reading (°C)	Thermocouple voltage, table (mV)	Thermocouple reading (mV)	Thermocouple reading (mV)	Thermocouple reading (mV)
Ambient air	25	1.277	0.028	-0.412	0.724
Ice water	8	0.405	0.500	0.848	0.012

For each reference junction condition, the theoretical air temperature was calculated based on the tabulated thermocouple reading and the corresponding temperature in the Type-J Thermocouple Reference Table found in Figure A.2. It was determined that placing the reference junction in ambient air produced the most theoretically accurate results based on the measured air temperature of 25°C, which is consistent with the fact that this configuration also corresponded to the minimal temperature gradient, which reduces the influence of variation presented by the thermocouple. Placing the reference junction in ice water also predictably led to the most inaccurate measurement, as the reference temperature was much different than that at the measurement junction and thus the temperature gradient was larger. Furthermore, it was confirmed in this stage of the investigation that the sensitivities for both reference junction locations with non-negligible voltage readings. While the accuracy for the reference-in-air measurement was expected, the similarity of the other two ensures an approximately common sensitivity value which would, in theory, be unique to that particular device. Relevant calculations are provided in the Appendix.

Table A.2: Theoretical Air Temperatures and Static Sensitivities for Initial Investigation

Junction Conditions	Theoretical Air Temp. (°C)	Static Sensitivity K (mV/°C)
Welded: Air; Reference: Air	25.1	0.28
Welded: Air; Reference: Hands	27.0	0.0412
Welded: Air; Reference: Ice Water	22.0	0.04259

$(T_{air})_{approx} = T_{ref} + \Delta T_{table}\quad T$; $(T_{ref})_{hands} \cong 35$ °C

As for the static calibration of the welded thermocouples, the data analysis was mostly consistent with theoretical expectations but had some noticeable deviations, providing insight into the accuracy of the thermocouples for static measurements. The experimental calibration data gathered by taking static measurements over a range of 0 °C to 100 °C at every 5 °C interval was read into MATLAB in order to perform some initial calculations of input and output ranges associated with each thermocouple; this data is tabulated in Table B.1 and the raw data is plotted in Figure B.1 for clarity.

Table B.1: Dataset Specifications for Static Analysis:

Dataset	Temperature Range (°C)	Voltage Range (V)	Mean Voltage (V)
Static 1	25 : 100	0.5234 : 0.7615	0.6487
Static 2	25 : 100	0.5213 : 0.7609	0.6401
Static 3	25 : 100	0.5095 : 0.7615	0.6377

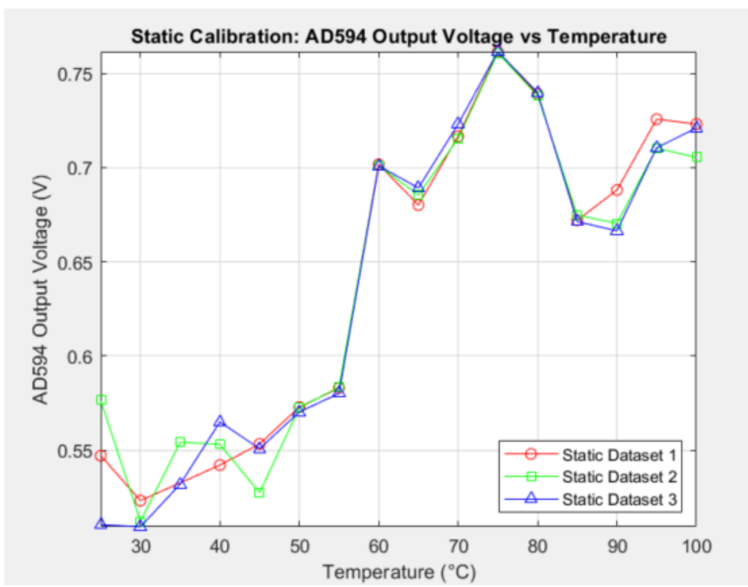


Figure B.1: Experimental Thermocouple Data for Static Analysis

Upon application of the user-defined linear least squares regression function LSR.m (Appendix B) to each dataset, the appropriate slopes and intercepts were determined such that the overall error for each set would be minimized. The equations used for these calculations can be found in Appendix B. Parameters corresponding to the output of the LSR.m function are found in Table

B.2, and an analysis of the returned values suggests that the regression function accurately estimated the lines of minimal error for each dataset, allowing for a subsequent error analysis for the static data. Upon analyzing the plot, it is worthy of note that the data collected during static analysis exhibited a fair degree of variability, with this further supported by the relatively low coefficients of determination, which provide insight into the accuracy of the thermocouples with respect to their regression models. This is best explained by the fact that measurements were taken at a relatively low sampling frequency, increasing the likelihood that random noise caused unexpectedly high deviations from the theoretically linear behavior of the thermocouples.

Table B.2: Linear Regression Outputs for Static Analysis:

Dataset	**Sensitivity (V/°C)**	**Intercept (V)**	**Std. Deviation, Individual (V)**	**Std. Deviation, Mean (V)**	**Coefficient of Determination**
Static 1	0.003085	0.4502	0.05012	0.022394	0.7370
Static 2	0.002846	0.4623	0.05450	0.023460	0.6703
Static 3	0.003206	0.4373	0.05211	0.022452	0.7384
Average	0.003046	0.4450	0.05224	0.02277	0.7152

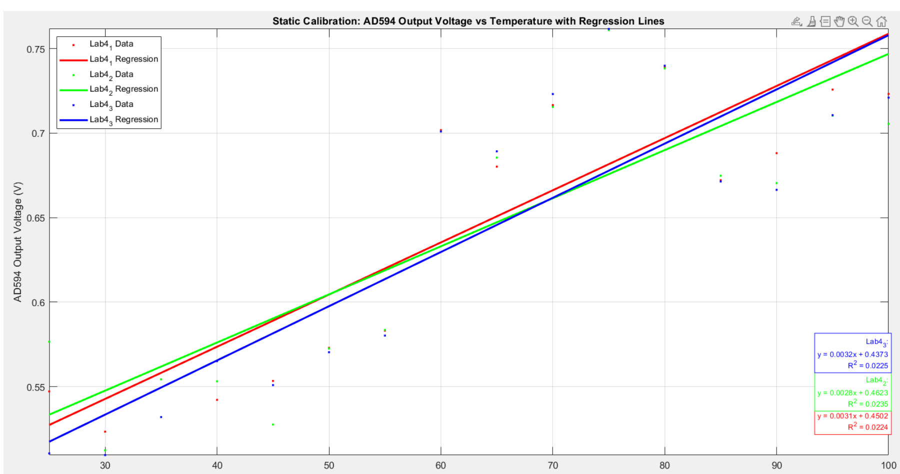


Figure B.2: Regression Lines for Static Analysis

The derivation of critical statistics for the static analysis supports the conclusion that the thermocouples have an approximately constant sensitivity and provide reasonably accurate temperature measurements at their welded junctions. All three thermocouple datasets exhibited relatively consistent values for measurement uncertainty, with an average value of 0.1052 mV among the samples. According to the Type-J Thermocouple Reference Table (Appendix A.1), an uncertainty of this magnitude would correspond to a temperature uncertainty of approximately 3°C, implying that these particular measurement systems are accurate within 3°C of the true temperature value. Relevant statistics for this portion of the analysis, including voltage resolution based on the precision of the NI-PCLe-6321 connector box and the quantization error, are tabulated in Tables B.3 and B.4.

Table B.3: Voltage Resolution and Quantization Error for NI-PCLe-6321:

Voltage Resolution (mV)	0.30518
Quantization Error (°C)	0.09986

Table B.4: Static Measurement Uncertainties for Welded Thermocouples:

Dataset	Measurement Uncertainty (mV)
Static 1	0.100941
Static 2	0.109765
Static 3	0.104957

The remainder of the experimental analysis focused on the dynamic response of the thermocouples in order to validate the theoretical assumption that they provide quick and effective dynamic responses for measurement systems. The dynamic data captured from quickly submerging each thermocouple's measurement junction in boiling water was loaded and plotted in MATLAB in order to generate both time-domain step response plots and their corresponding linearized semi-log plots, for each thermocouple sample. The logarithmic response z was calculated using Equation* in order to accurately perform the transformation. Figures B.1 through B.4 show the plots of the step responses and logarithmic responses for each dataset.

The linear regression employed for the step responses of the thermocouples utilized the same function as for the static calibration analysis, with the only difference being that the z-plots needed to have restricted domains in order to analyze only the approximately linear portions of the signal. All of the step response signals contained a substantial amount of noise, which made it important that the linear regressions were performed across appropriate boundaries because the derived sensitivity from the regression models, in theory, is equivalent to the negative reciprocal of the system time constant as seen in Equation 19. The time constants for each of the welded thermocouples, as well as that of the twisted thermocouple, are tabulated in Table*. It is worthy of note that the twisted thermocouple took a considerably longer time to respond to the step input than the welded thermocouples did, which supports the use of welded junctions to measure dynamic temperature information more accurately. The difference can be explained by the fact that weldments provide a more continuous connection, allowing for better thermal conductivity between the iron and constantan wires. Ultimately, the step response data gathered supports the initial assumption that thermocouples respond to dynamic inputs very quickly, thus making them effective for applications requiring precise temperatures to be controlled relatively quickly.

Table C.1: Time Constants for Dynamic Calibration

Thermocouple	**Time Constant τ (s)**
Welded 1	0.0868
Welded 2	0.0906
Welded 3	0.007
Twisted	0.6047

IV. Conclusion

The importance we learned from the lab was observing the behavior of thermocouples given the parameters. In our situation, we discovered as far as what kind of thermocouple, twisted or welded, that welded thermocouples have a quicker time response due to a better connection. As far as the environments go, we discovered there is a sweet spot for the temperature to produce the highest voltage output. In our boiling water experiment, we found that around 60°C between 80°C had the highest voltage output.

Appendix A: Tables, Figures, and Equations for Initial Investigation

Figure A.1: Thermocouple System with Copper Leads

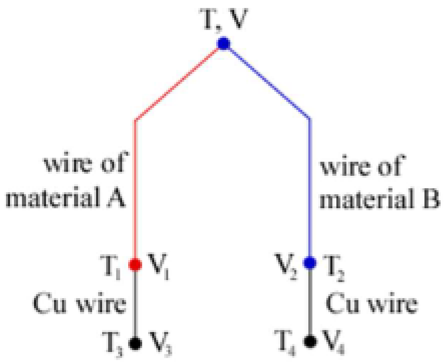


Figure A.2: Thermocouple Reference Table for Type-J Thermocouple

Table 8.6 Thermocouple Reference Table for Type-J Thermocouple[a]

Temperature (°C)	Thermocouple emf (mV)									
	0	−1	−2	−3	−4	−5	−6	−7	−8	−9
−210	−8.095									
−200	−7.890	−7.912	−7.934	−7.955	−7.976	−7.996	−8.017	−8.037	−8.057	−8.076
−190	−7.659	−7.683	−7.707	−7.731	−7.755	−7.778	−7.801	−7.824	−7.846	−7.868
−180	−7.403	−7.429	−7.456	−7.482	−7.508	−7.534	−7.559	−7.585	−7.610	−7.634
−170	−7.123	−7.152	−7.181	−7.209	−7.237	−7.265	−7.293	−7.321	−7.348	−7.376
−160	−6.821	−6.853	−6.883	−6.914	−6.944	−6.975	−7.005	−7.035	−7.064	−7.094
−150	−6.500	−6.533	−6.566	−6.598	−6.631	−6.663	−6.695	−6.727	−6.759	−6.790
−140	−6.159	−6.194	−6.229	−6.263	−6.298	−6.332	−6.366	−6.400	−6.433	−6.467
−130	−5.801	−5.838	−5.874	−5.910	−5.946	−5.982	−6.018	−6.054	−6.089	−6.124
−120	−5.426	−5.465	−5.503	−5.541	−5.578	−5.616	−5.653	−5.690	−5.727	−5.764
−110	−5.037	−5.076	−5.116	−5.155	−5.194	−5.233	−5.272	−5.311	−5.350	−5.388
−100	−4.633	−4.674	−4.714	−4.755	−4.796	−4.836	−4.877	−4.917	−4.957	−4.997
−90	−4.215	−4.257	−4.300	−4.342	−4.384	−4.425	−4.467	−4.509	−4.550	−4.591
−80	−3.786	−3.829	−3.872	−3.916	−3.959	−4.002	−4.045	−4.088	−4.130	−4.173
−70	−3.344	−3.389	−3.434	−3.478	−3.522	−3.566	−3.610	−3.654	−3.698	−3.742
−60	−2.893	−2.938	−2.984	−3.029	−3.075	−3.120	−3.165	−3.210	−3.255	−3.300
−50	−2.431	−2.478	−2.524	−2.571	−2.617	−2.663	−2.709	−2.755	−2.801	−2.847
−40	−1.961	−2.008	−2.055	−2.103	−2.150	−2.197	−2.244	−2.291	−2.338	−2.385
−30	−1.482	−1.530	−1.578	−1.626	−1.674	−1.722	−1.770	−1.818	−1.865	−1.913
−20	−0.995	−1.044	−1.093	−1.142	−1.190	−1.239	−1.288	−1.336	−1.385	−1.433
−10	−0.501	−0.550	−0.600	−0.650	−0.699	−0.749	−0.798	−0.847	−0.896	−0.946
0	0.000	−0.050	−0.101	−0.151	−0.201	−0.251	−0.301	−0.351	−0.401	−0.451

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Equations:

1. $\Delta V = \alpha(\Delta T)$
2. $\Delta V = (\alpha_B - \alpha_A)\Delta T$
3. $K = \Delta V / \Delta T$

Appendix B: Tables, Figures, Equations, and Codes for Static Calibration and Analysis

Figure B.1: Experimental Thermocouple Data for Static Analysis

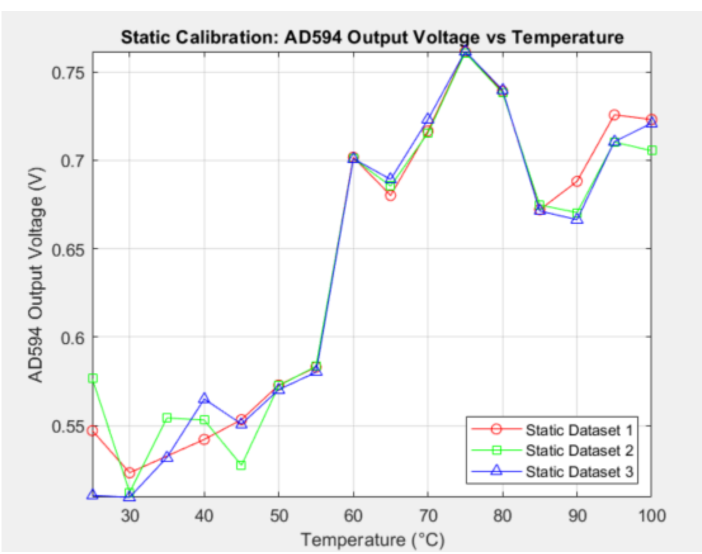


Figure B.2: Regression Lines for Static Analysis

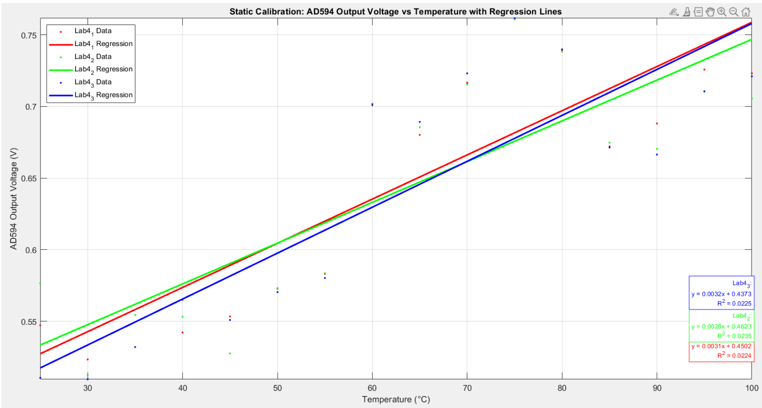
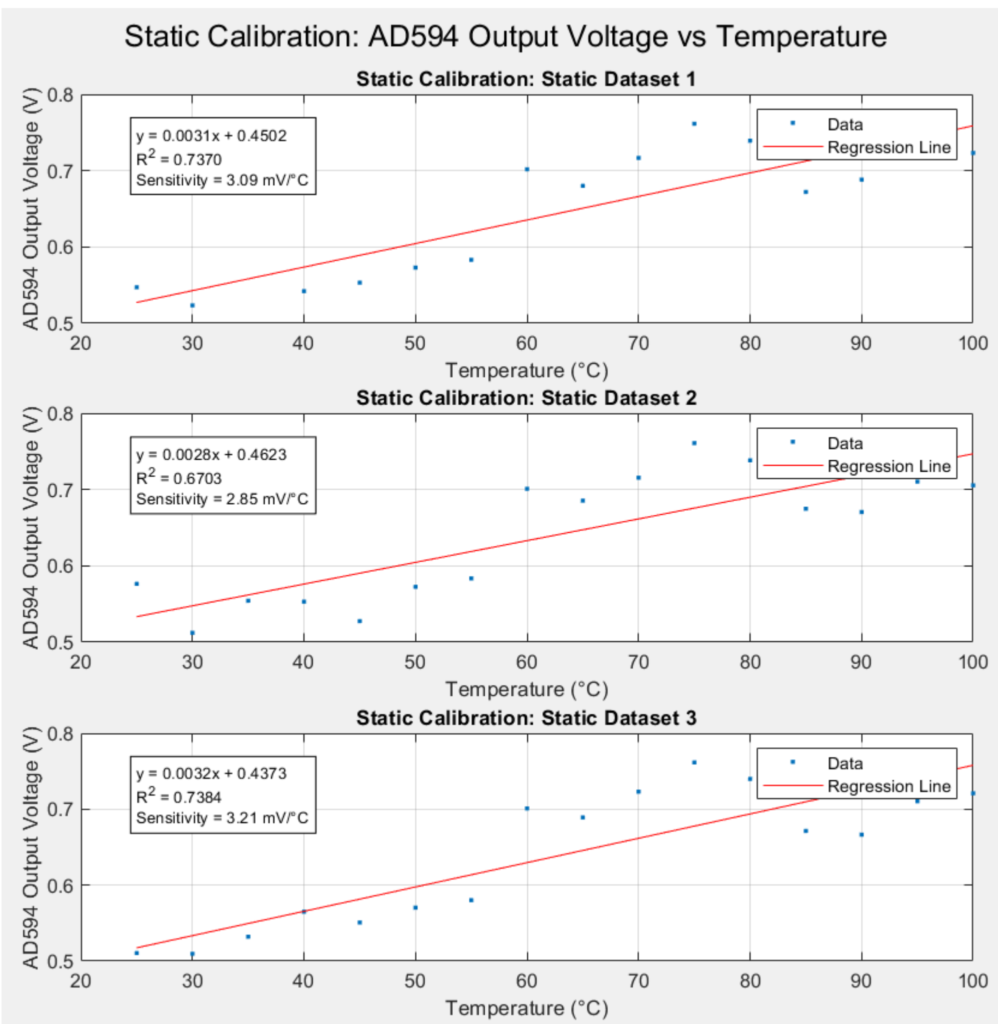


Figure B.3: Individual Regression Lines and Parameters for Static Calibration Data



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Appendix C: Tables, Figures, Equations, and Codes for Static Calibration and Analysis

Figure C.1: Step Response and Z-Plot for Welded Thermocouple 1

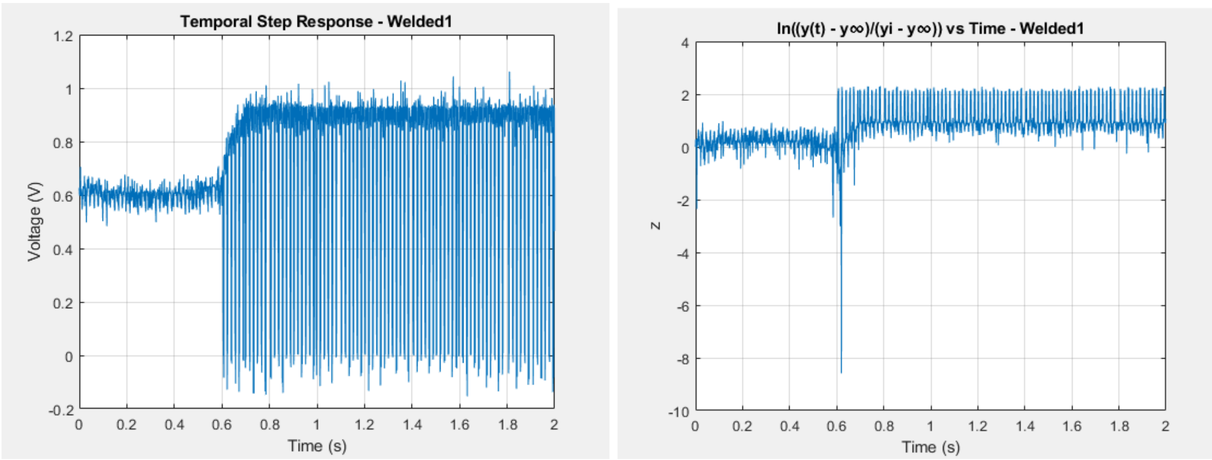


Figure C.2: Step Response and Z-Plot for Welded Thermocouple 2

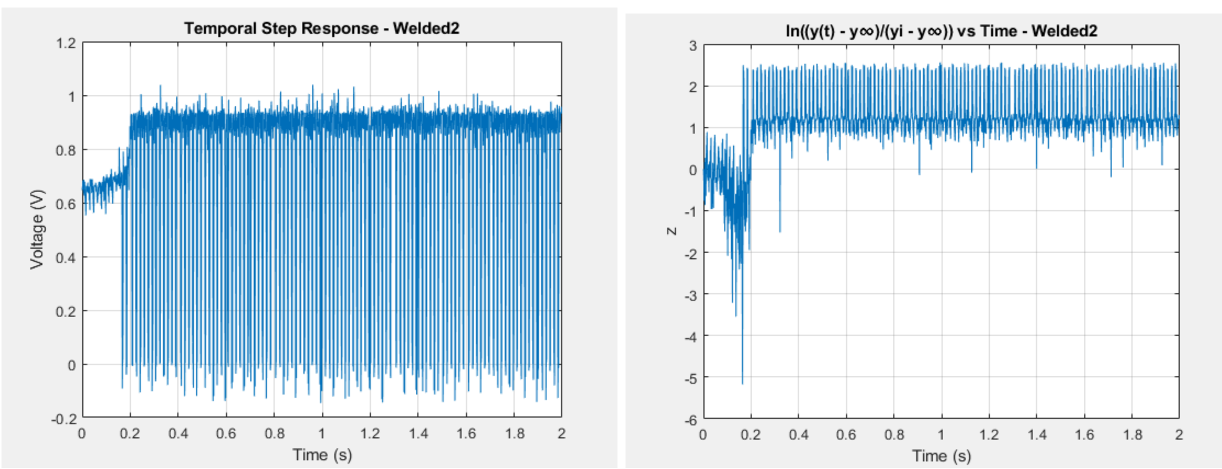


Figure C.3: Step Response and Z-Plot for Welded Thermocouple 3

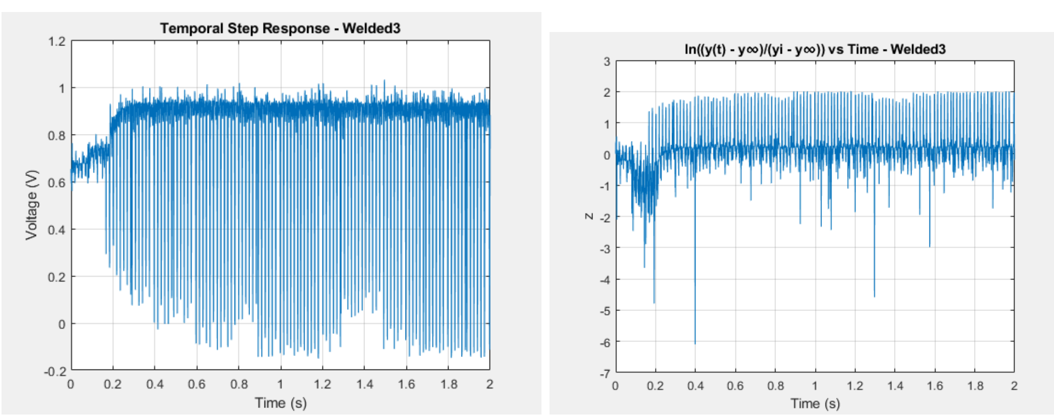


Figure C.4: Step Response and Z-Plot for Twisted Thermocouple

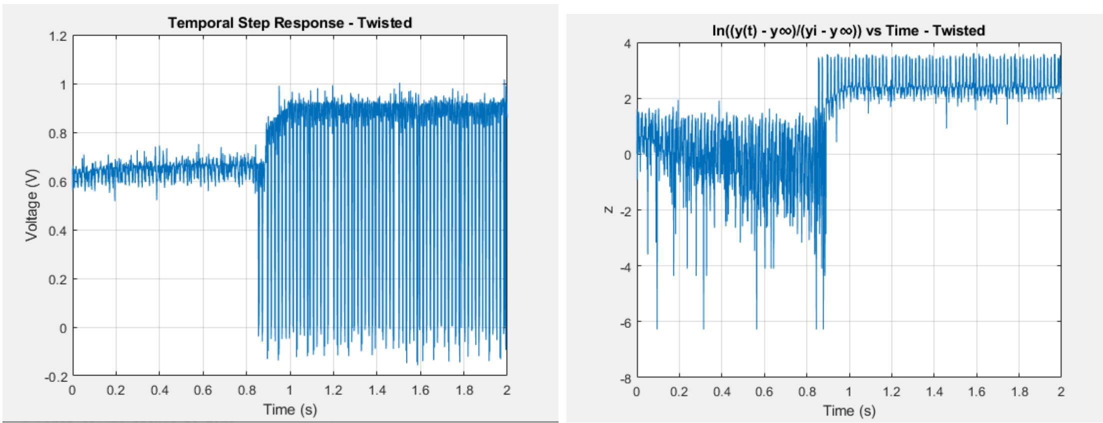


Figure C.5: Linear Regression for Welded Thermocouple 1

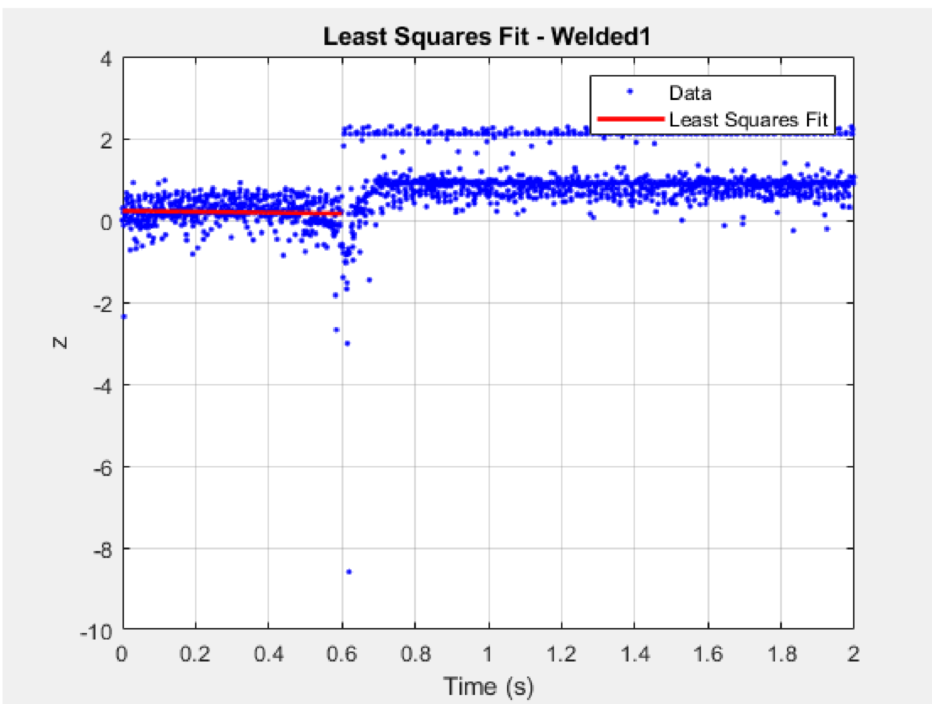


Figure C.6: Linear Regression for Welded Thermocouple 2

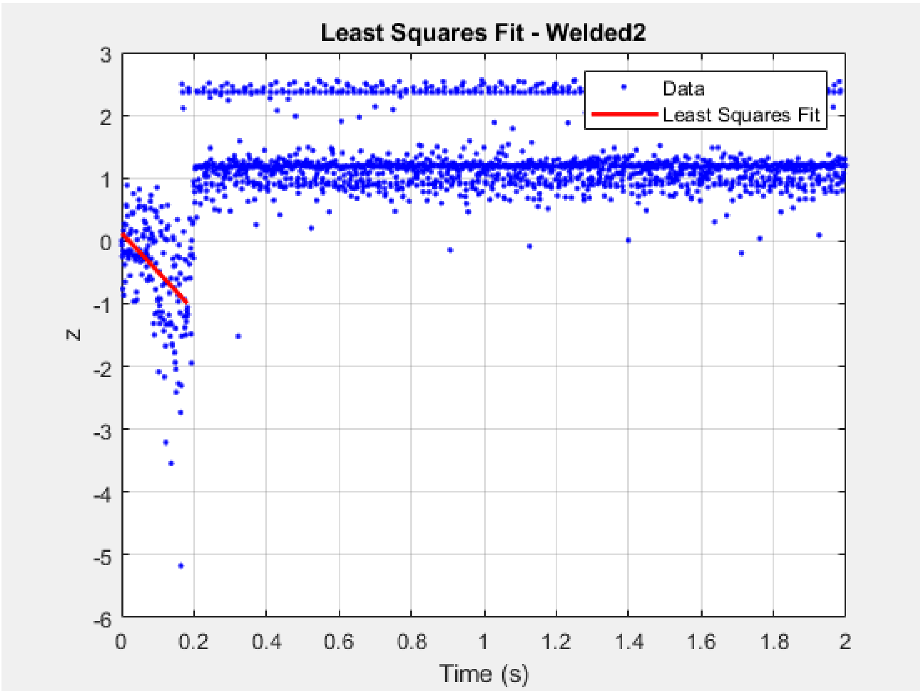


Figure C.7: Linear Regression for Welded Thermocouple 3

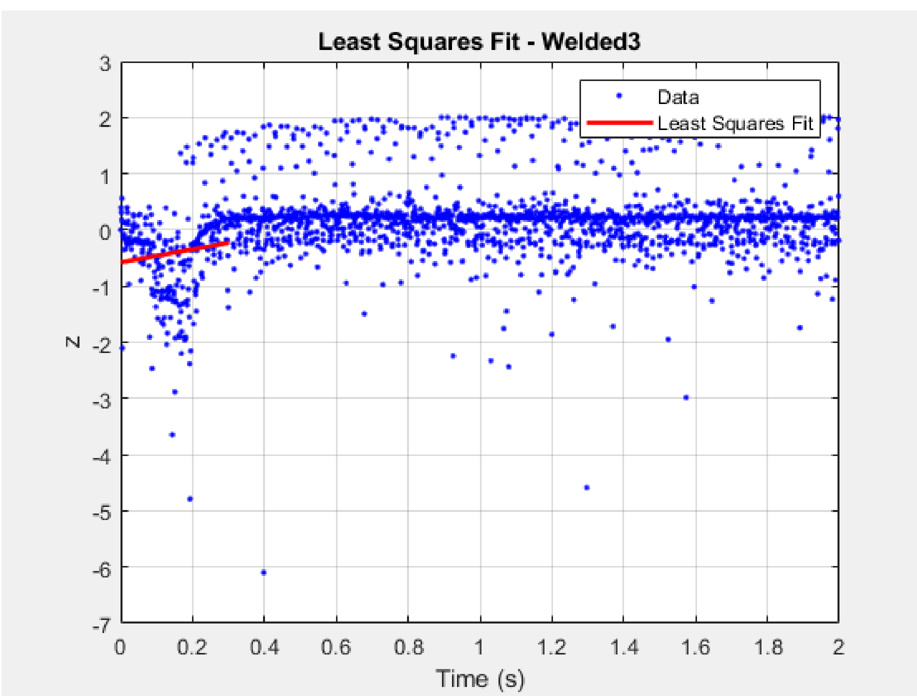
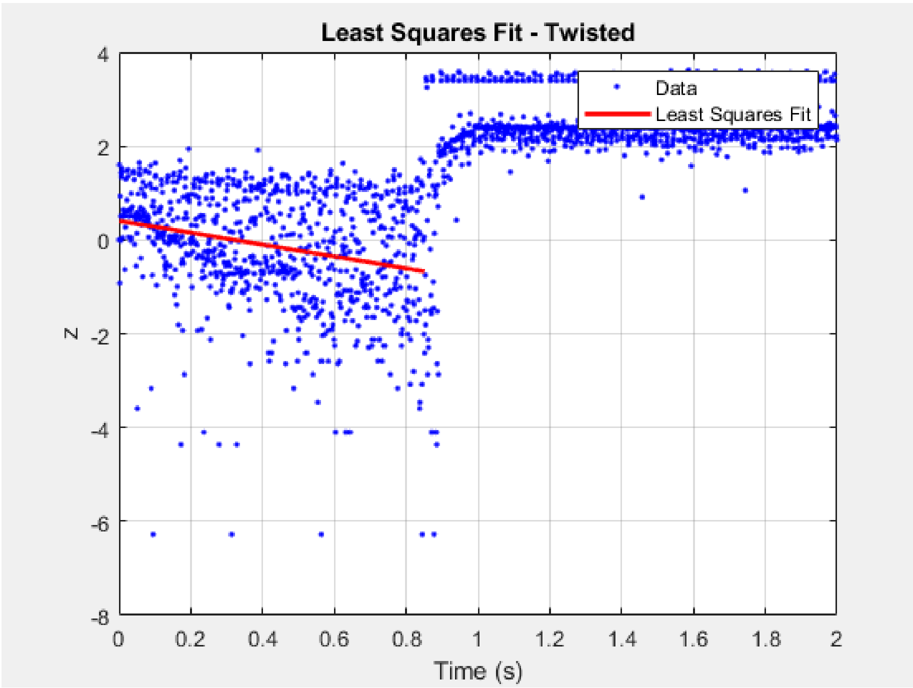


Figure C.8: Linear Regression for Twisted Thermocouple



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Analysis A: Initial Investigation

1. Table in appendix
2.

	0	1	2	3	4	5	6	7	8	9	10	
°C	**-10**	**-9**	**-8**	**-7**	**-6**	**-5**	**-4**	**-3**	**-2**	**-1**	**0**	**°C**
200	-8.095	-8.076	-8.057	-8.037	-8.017	-7.996	-7.976	-7.955	-7.934	-7.912	-7.890	-200
190	-7.890	-7.868	-7.846	-7.824	-7.801	-7.778	-7.755	-7.731	-7.707	-7.683	-7.659	-190
180	-7.659	-7.634	-7.610	-7.585	-7.559	-7.534	-7.508	-7.482	-7.456	-7.429	-7.403	-180
170	-7.403	-7.376	-7.348	-7.321	-7.293	-7.265	-7.237	-7.209	-7.181	-7.152	-7.123	-170
160	-7.123	-7.094	-7.064	-7.035	-7.005	-6.975	-6.944	-6.914	-6.883	-6.853	-6.821	-160
150	-6.821	-6.790	-6.759	-6.727	-6.695	-6.663	-6.631	-6.598	-6.566	-6.533	-6.500	-150
140	-6.500	-6.467	-6.433	-6.400	-6.366	-6.332	-6.298	-6.263	-6.229	-6.194	-6.159	-140
130	-6.159	-6.124	-6.089	-6.054	-6.018	-5.982	-5.946	-5.910	-5.874	-5.838	-5.801	-130
120	-5.801	-5.764	-5.727	-5.690	-5.653	-5.616	-5.578	-5.541	-5.503	-5.465	-5.426	-120
110	-5.426	-5.388	-5.350	-5.311	-5.272	-5.233	-5.194	-5.155	-5.116	-5.076	-5.037	-110
100	-5.037	-4.997	-4.957	-4.917	-4.877	-4.836	-4.796	-4.755	-4.714	-4.674	-4.633	-100
-90	-4.633	-4.591	-4.550	-4.509	-4.467	-4.425	-4.384	-4.342	-4.300	-4.257	-4.215	-90
-80	-4.215	-4.173	-4.130	-4.088	-4.045	-4.002	-3.959	-3.916	-3.872	-3.829	-3.786	-80
-70	-3.786	-3.742	-3.698	-3.654	-3.610	-3.566	-3.522	-3.478	-3.434	-3.389	-3.344	-70
-60	-3.344	-3.300	-3.255	-3.210	-3.165	-3.120	-3.075	-3.029	-2.984	-2.938	-2.893	-60
-50	-2.893	-2.847	-2.801	-2.755	-2.709	-2.663	-2.617	-2.571	-2.524	-2.478	-2.431	-50
-40	-2.431	-2.385	-2.338	-2.291	-2.244	-2.197	-2.150	-2.103	-2.055	-2.008	-1.961	-40
-30	-1.961	-1.913	-1.865	-1.818	-1.770	-1.722	-1.674	-1.626	-1.578	-1.530	-1.482	-30
-20	-1.482	-1.433	-1.385	-1.336	-1.288	-1.239	-1.190	-1.142	-1.093	-1.044	-0.995	-20
-10	-0.995	-0.946	-0.896	-0.847	-0.798	-0.749	-0.699	-0.650	-0.600	-0.550	-0.501	-10
0	-0.501	-0.451	-0.401	-0.351	-0.301	-0.251	-0.201	-0.151	-0.101	-0.050	0.000	0
0	0.000	0.050	0.101	0.151	0.202	0.253	0.303	0.354	0.405	0.456	0.507	0
10	0.507	0.558	0.609	0.660	0.711	0.762	0.814	0.865	0.916	0.968	1.019	10
20	1.019	1.071	1.122	1.174	1.226	1.277	1.329	1.381	1.433	1.485	1.537	20

Figure 2: j-table used to find our voltages at our measured temperatures

3. Calculate Sensitivity

For theoretical, we found the expected values from the thermocouple table in the previous question above (Analysis A, part 2). Then we used equation 3 from the appendix.

Theoretical Sensitivity = (1.277 - 0.405) V / (25 - 8) °C = 0.05129 V/°C

The following three were actual measured values for voltage. The thermocouple was placed in different environments as labeled below. We once again used equation 3 in the appendix.

Sensitivity for Reference Junction in Air = |(0.028 - 0.500) V| / (25 - 8) °C = 0.02776 V/°C

Sensitivity for Reference Junction in Hands = |(0.212 - 0.848) V| / (25 - 8) °C = 0.0374 V/°C

Sensitivity for Reference Junction in Ice Water = (0.645 - 0.012) V / (25 - 8) °C = 0.0372 V/°C

Analysis B: Static Calibration

1. Reading static calibration data into MATLAB
2. Plotting static calibration data in MATLAB
3. Linear Least Squares Regression Function
4. Fitting Linear Regression line to static calibration data
5. Tabulating Statistics

	R-Squared	Slope	Y-Intercept
Thermocouple 1	0.0071	0.0031	0.4502
Thermocouple 2	0.0068	0.0028	0.4623
Thermocouple 3	0.0079	0.0032	0.4373

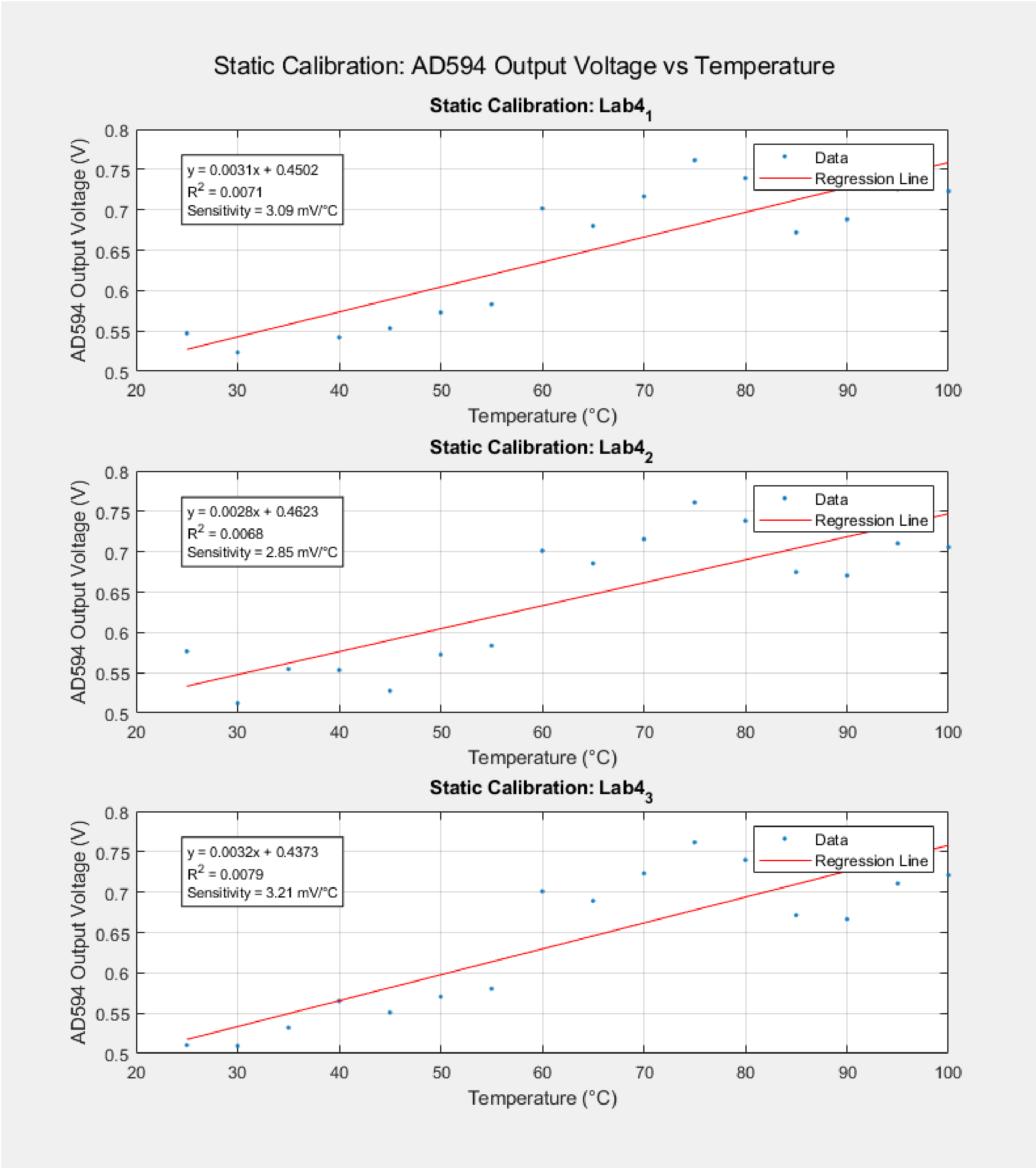

Figure 4: Plots of the 3 trials

6. Plot overlaying static calibration data and regression line

Analysis C: Uncertainty Analysis for Static Calibration

Refer to equations 12 & 13 in the appendix for error and uncertainty calculations.

Analysis D: Dynamic Calibration

Plots for twisted and all 3 welded tests are in appendix. To calculate the time constant, equation 20 was used in the appendix. Below are the time constants we got for each experiment respectively.

```
Thermocouple        TimeConstant_s
____________    ____________________

{'Welded2'}      0.090606-0.080548i
{'Twisted'}        0.6047-0.33502i
{'Welded1'}        8.6814+0i
{'Welded3'}     -0.0070382-0.077658i
```

Appendix

Reference junction in: →			Air	Hands	Ice Water
Welded junction location	Thermometer reading (C)	Thermocouple voltage, table (mV)	Thermocouple reading (mV)	Thermocouple reading (mV)	Thermocouple reading (mV)
Ambient air	25	1.277	0.028	0.212	0.645
Ice water	8	0.405	0.500	0.848	0.012

Equations:

4. $\Delta V = \alpha(\Delta T)$
5. $\Delta V = (\alpha_B - \alpha_A)\Delta T$
6. $K = \Delta V / \Delta T$
7. $m = \frac{n(\sum_{i=1}^{n} x_i y_i) - (\sum_{i=1}^{n} x_i)(\sum_{i=1}^{n} y_i}{n(\sum_{i=1}^{n} x_i^2) - (\sum_{i=1}^{n} x_i)^2}$

8. $m = \frac{n(\sum_{i=1}^{m} x_i^2)(\sum_{i=1}^{n} y_i) - (\sum_{i=1}^{n} x_i)(\sum_{i=1}^{n} x_i y_i)}{n(\sum_{i=1}^{n} x_i^2) - (\sum_{i=1}^{n} x_i)^2}$

9. $s_e^2 = \frac{1}{n-2} \sum_{i=1}^{n} (mx_i + b - y_i)^2$

10. $s_{ye}^2 = \frac{1}{n-1} \sum_{i=1}^{n} (y_i - \bar{y})^2$

11. $s_y^2 = s_e^2 \left[1 + \frac{1}{n} + \frac{(r_i/2)^2}{\sum_{i=1}^{n} (x_i - \bar{x})^2} \right]$

12. $\bar{y} = \frac{1}{n} \sum_{i=1}^{n} y_i$

13. $s_y^2 = s_e^2 \left[\frac{1}{n} + \frac{(r_i/2)^2}{\sum_{i=1}^{n} (x_i - \bar{x})^2} \right]$

14. $r^2 = 1 - \frac{(n-2)s_e^2}{(n-1)s_{ye}^2}$

15. $U_d^2 = e_q^2 + e_y^2 + e_T^2$

16. $e_y = \frac{ts_y}{m}$

17. $t = tinv(P_m, v)$

18. $v = n - 1,\ (P\%)$

19. $y(t) = y_\infty + (y_o - y_\infty)e^{-t/\tau}$

20. $\frac{y(t) - y_\infty}{y_o - y_\infty} = e^{-t/\tau}$

21. $Z \equiv ln\left(\frac{y(t) - y_\infty}{y_o - y_\infty}\right) = -\frac{1}{\tau}t$

22. $(T_{air})_{approx} = T_{ref} + \Delta T_{table}\, T; (T_{ref})_{hands}$

23. $\tau = \frac{-1}{m}$

Tasks:

Executive Summary - Walker

Introduction - Walker

Experimental Procedure - Walker

Results and Discussion - Walker, Hiromu

Conclusion - Hiromu

Appendix - Hiromu

Nomenclature - Hiromu

Coding - Eshwar, Hiromu, Walker