EWSNET DOCUMENTATION

June 9, 2021

1 Introduction

Transition from one steady state to another occur in many complex systems, such as financial markets, human societies, climate systems and various other domains. Transitions may be abrupt and irreversible (i.e., catastrophic), or smooth and reversible (i.e., non-catastrophic), and can occur due to gradual external forcing or random fluctuations in the system. In such scenarios, on crossing a threshold (known as a tipping or bifurcation point), structural changes occur in the underlying system. This is often termed a critical transition, prior to which the system's return to an equilibrium slows down - a phenomenon known as critical slowing down (CSD) [**?**]. The phenomenon of CSD is related to the fact that the real part of the dominant eigenvalue of the system goes to zero at the bifurcation point [**?**, **?**]. In all such cases, where the dominant eigenvalue approaches zero close to the tipping point irrespective of catastrophic or non-catastrophic transitions, the phenomenon of CSD persists, and there exist statistical indicators that forewarn the vicinity of a tipping point [**?**]. Understanding the causes of sudden transitions and forecasting them using statistical indicators have recently emerged as an important area of research due to the management implications of preventing catastrophes in natural systems

The traditional approach of forecasting a critical transition relies on CSD based indicators such as variance, autocorrelation, and skewness showing an increasing trend before a transition.

2 EWSNet

EWSNet is a parameterized function that employs an LSTM to capture long-term dependencies in the sequential time series data, in addition to a fully convolutional submodule that helps automatically extract complex non-linearities from the data, requisite to learn the characteristics indicative of a future transition. EWSNet can classify catastrophic and non-catastrophic and no transitions in raw time series data learning fundamental properties that characterize transitions alongwith critical slowing down which helps it to distinguish a no transition from the other two variants. EWSNet makes none but one pressumtion that the test data must belong to a similar distributionn as that of training data. By a similar distribution, it is meant that test data have critical behavior (bifurcations) similar to underlying bifurcations of time series data that are deployed for training. The present EWSNet is not specific to models on which it is trained but critical behavior specific. There is always a choice to update and retrain EWSNet continuously to cover an exhaustive space of such critical behaviors under each transition label increase robustness of its prediction and get rid of this assumption.

3 Data Generation

We generate stochastic time series that serve as training data pertaining to the 3 labels (catastrophic, non-catastrophic and no transition). Time series are obtained by simulating 9 most well established models that depict the above transitions in diverse systems such as ecological, climatic and systems biology. We consider data only prior to tipping to train the EWSNET. The normal form theory confirms that at the vicinity of a tipping, dynamics associated with a specific bifurcation is similar and not model dependent. Yet we train with diverse non-linear models for variance in the training set that adds to the robustness of the EWSNET. In the process, we first obtain the deterministic tipping by plotting the bifurcation diagram in XPPAUT. We also vary parameters apart from the control (bifurcation parameter) wherever possible. It is to be noted that an extra parameter is varied in models [1],[3],[6],[7],[9] only after reconfirming that the afforementioned bifurcation persist within the range by means of a two parameter bifurcation diagram plotted in XPPAUT. This aids to more variability across timeseries. On varying an extra parameters in the above models , the tipping points may be altered, we note the minimum value of the deterministic tipping for the range of parameters. Further, to ensure the tipping is never crossed on interaction of noise and nonlinearity especially for highly correlated noise, we choose the control parameter value quite prior to tipping and solve the system using Euler Muryama method for 400 time steps within this range. For each time step, we solve the system until a stationary state is reached (t=1000, dt=0.01). As we move forward in time $t = 1, 2....400$, the control parameter is gradually changed, while all other parameters are generally fixed. If any other parameter is varying as in models [1],[3],[6],[7],[9], the parameter value is picked up randomly for each simulation which remains fixed for all time steps and varies only across time series. 0.1 million time series, each for Dataset-W and Dataset-C are generated with the training data majorly common in both sets. However, in Dataset-C, we train with an additional number of time series by simulating models for $\kappa = -0.2$ and 0.2, while the corresponding test data contain time series generated from the underlying models for high amplitude auto-correlation coefficient values [$\kappa = -0.64, 0.64, -0.8, 0.8$]. Time series for both training and testing are generated in equal proportions for all the 3 labels. Model [1]-[5] correspond to the label critical (catastrophic) transition, [6]-[8] replicate smooth or gradual (non-catastrophic) transition and model 9 represent no transitions. All the chosen parameter values for model [1]-[9] are taken from published literature which are cited in the main text and supplementary. Variation in parameters in order to add variability to the datasets are done only after plotting both one-parameter and two-parameter bifurcations. Fig **??**

We explicitly explain the data generation process for model1. The deterministic dynamics of model1 can be depicted by the differential equation:

$$\frac{dN}{dt} = rN(1 - \frac{N}{K}) - \frac{cN^2}{b^2 + N^2}$$

Here $r = 1$ represents the maximum growth rate, $k \in [8, 10]$ is the carrying capacity and $b = 1$ is the half saturation constant. c is the bifurcation parameter which is varied in the range 1-3 (Fig. 1). Time series generated by simulating model 1 correspond to the label catastrophic transition. In generating the stochastic time series, the parameters r,b are fixed while k takes a uniform random number in the range [8,10] in order to add variability across time series. This is done by confirming that the saddle node bifurcation persists with the help of a two parameter bifurcation (Fig. 2) in the above range for c for the given k value. The deterministic bifurcation point is c=2.13 for k=8 and c=2.6 for k=10. For generating time series, we simulate the model for 400 time steps for c=1-1.9 prior to the minimum tipping. This also allows for variance in dataset and reduce bias if any, as pre-transition time series that are used to train the EWSNET are at varying distances from the tipping. Stochastic time series are generated by adding the multiplicative noise component to the deterministic skeleton ($\kappa = 0$ for Dataset-W and $\kappa = 0$ and 0.2 for training data in Dataset-C and $\kappa = 0.64, -0.64, 0.8, -0.8$ for test set). Only 20% of the training data for Dataset-C are generated by setting $\kappa = -0.2$ and 0.2.

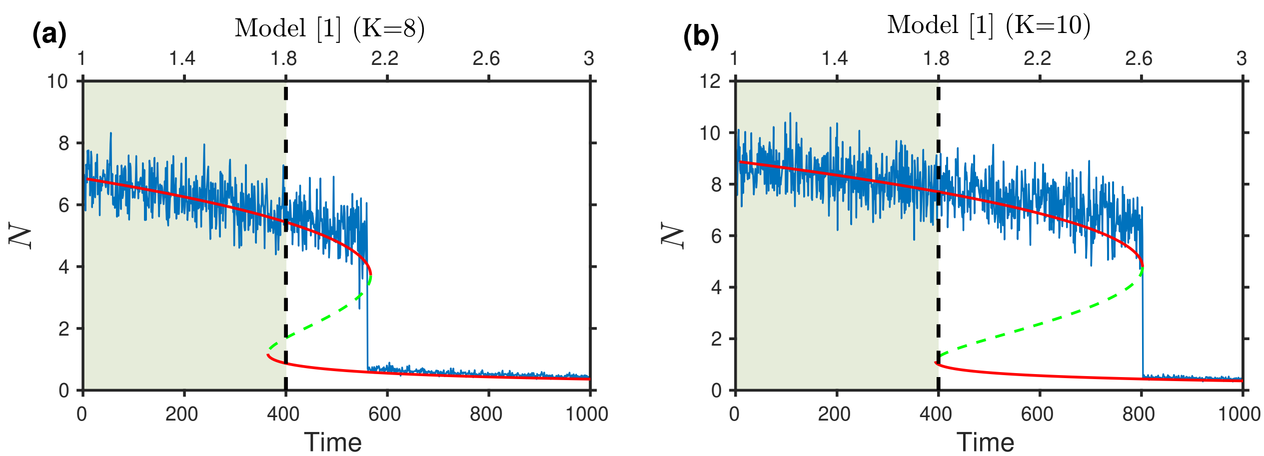

Figure 1: **Stochastic time series embedded over bifurcation diagram**: One parameter bifurcation diagram for model 1 for K=8 and K=10 respectively for the bifurcation parameter c (maximum grazing rate) varying in the range 1-3. Other parameters are r=1 and b=1. The shaded region mark the pre-transition time series used for training the EWSNET.

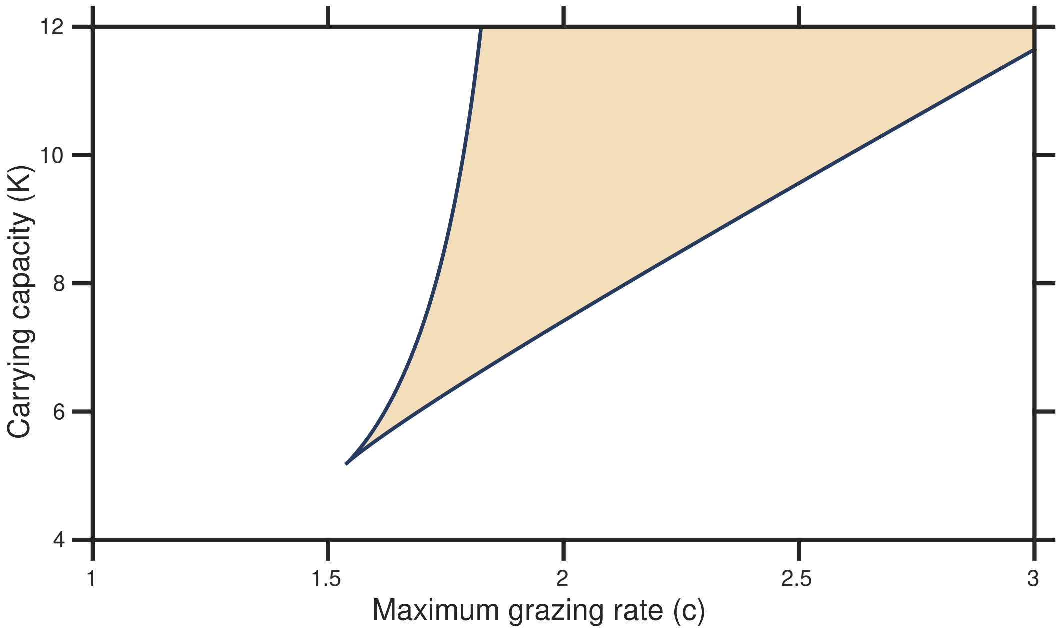

Figure 2: **Two parameter bifurcation diagram**: The colored region show the region of bistability and the fold bifurcation point corresponding to the k values.

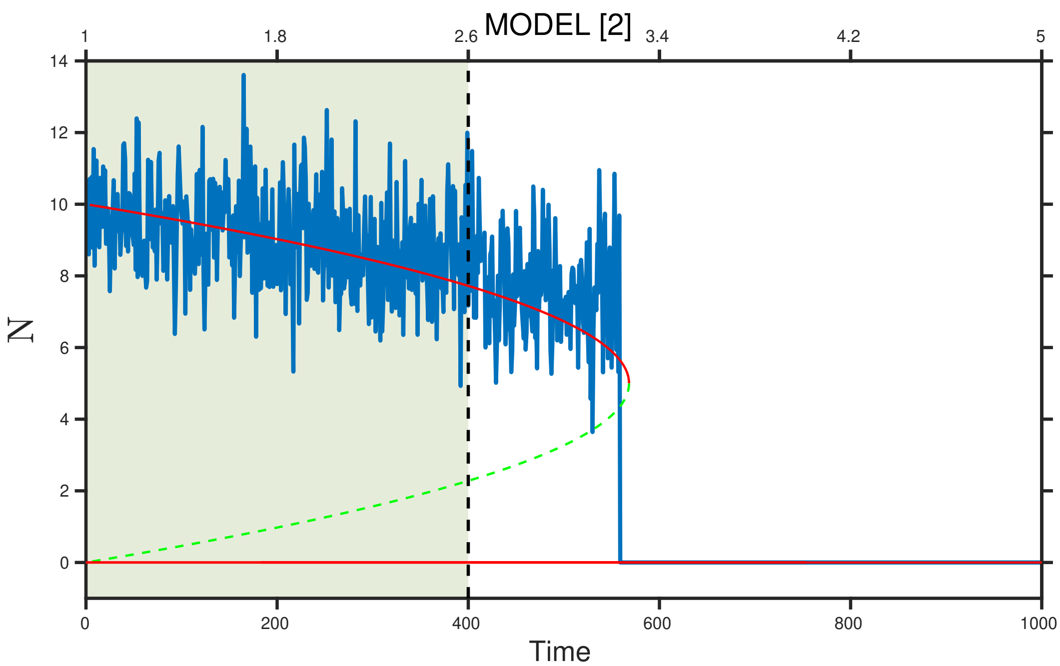

Figure 3: **Stochastic time series embedded over bifurcation diagram**:One parameter bifurcation diagram for model 2 for the bifurcation parameter c (maximum grazing rate) varying in the range 1-5. Other parameter is K=11. The shaded region mark the pre-transition time series used for training the EWSNET

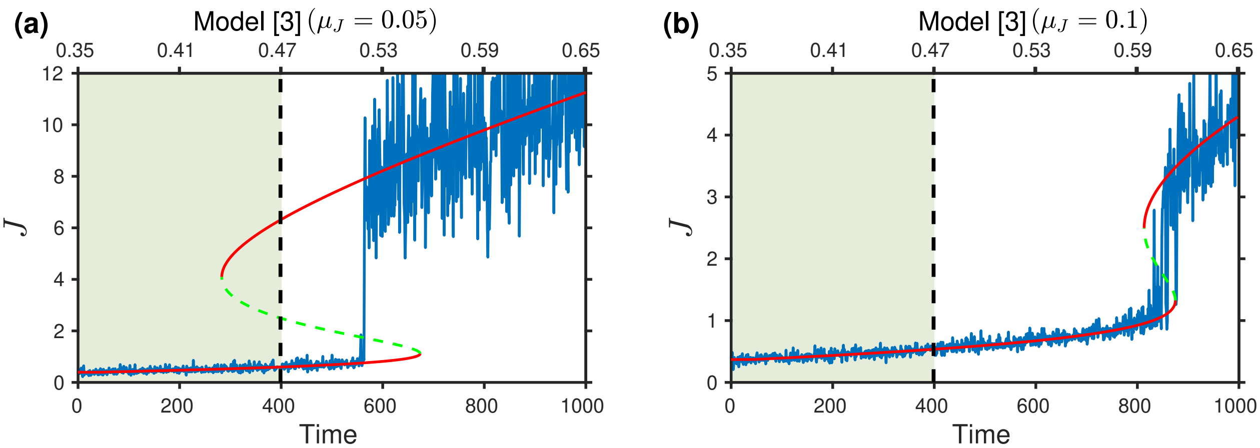

Figure 4: **Stochastic time series embedded over bifurcation diagram**: One parameter bifurcation diagram for model 1 for $\mu_J = 0.05$ and $\mu_J = 0.1$ respectively for the bifurcation parameter μ_P (predator mortality rate) varying in the range 0.35-0.65. Other parameters are b=1, c=1, $\mu_A = 0.01$. The shaded region mark the pre-transition time series used for training the EWSNET.

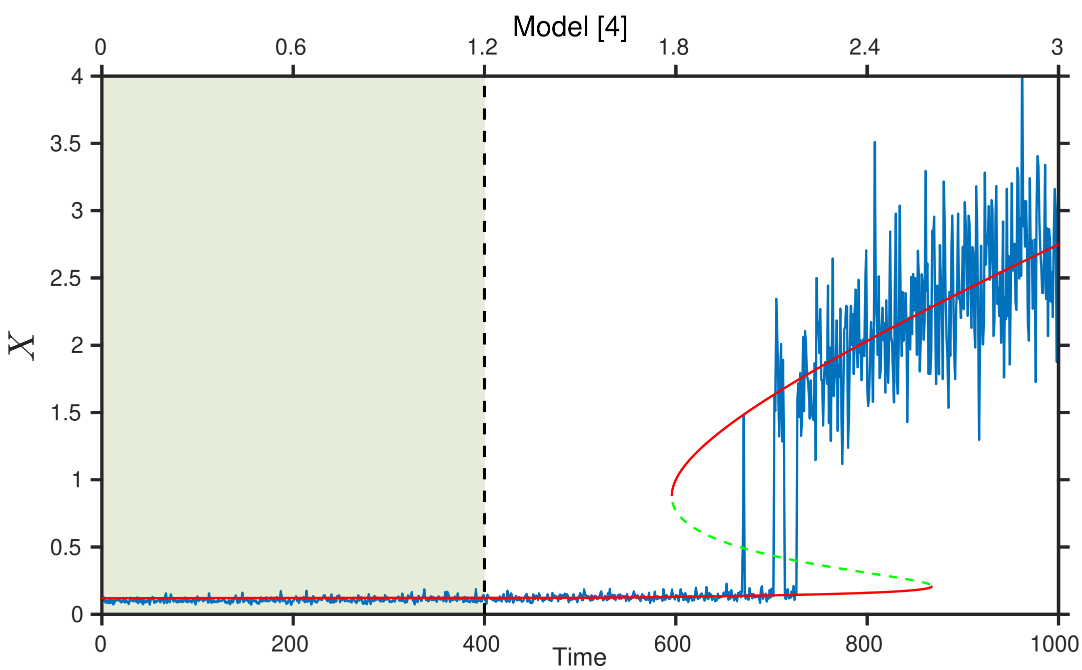

Figure 5: **Stochastic time series embedded over bifurcation diagram**:One parameter bifurcation diagram for model 4 for the bifurcation parameter c (maximum transcription rate) varying in the range 0-3. Other parameter is r=0.1. The shaded region mark the pre-transition time series used for training the EWSNET

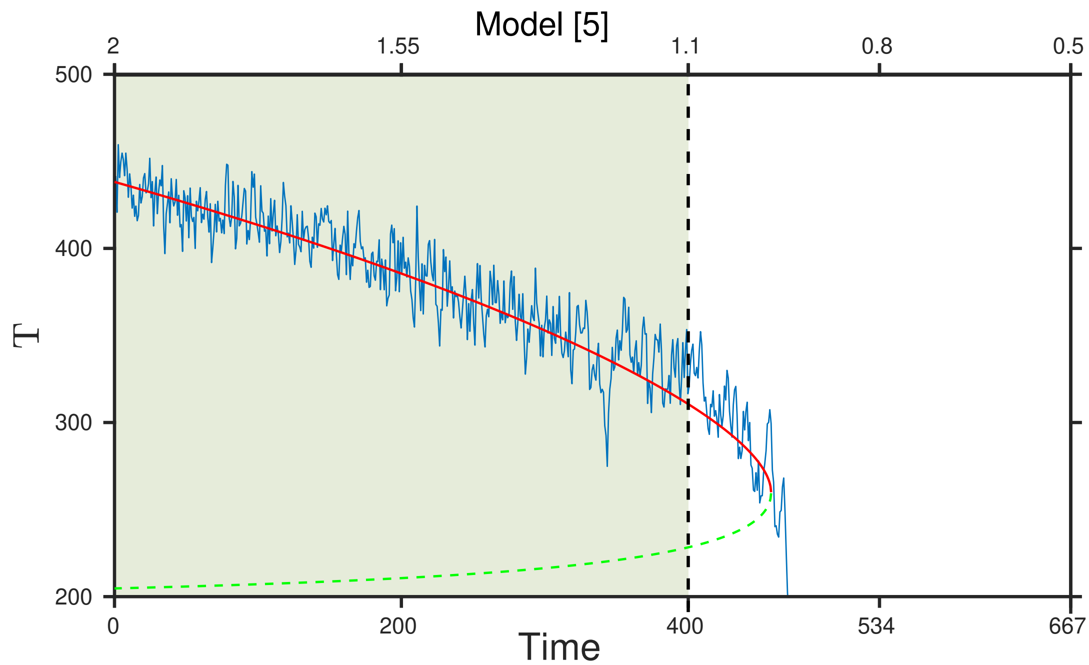

Figure 6: **Stochastic time series embedded over bifurcation diagram**:One parameter bifurcation diagram for model 5 for the bifurcation parameter μ (relative intensity of solar radiation) varying in the range 0.5-2. Other parameters are $a = 2.81$, $b = 0.009$, $c = 1.5*10^8$, $\epsilon = 0.69$,$I_0 = 0.03$,$\sigma = 1.251 * 10^{-}12$. The density T represent temperature in Kelvin. The shaded region mark the pre-transition time series used for training the EWSNET.

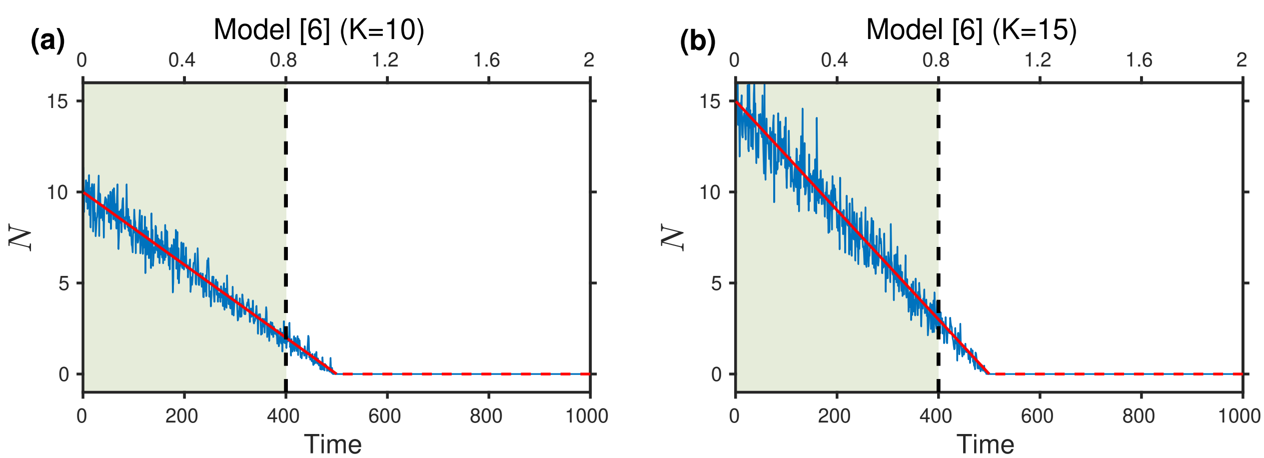

Figure 7: **Stochastic time series embedded over bifurcation diagram**: One parameter bifurcation diagram for model 6 for K=10 and K=15 respectively for the bifurcation parameter c (maximum grazing rate) varying in the range 0-2. Other parameter is r=1. The shaded region mark the pre-transition time series used for training the EWSNET.

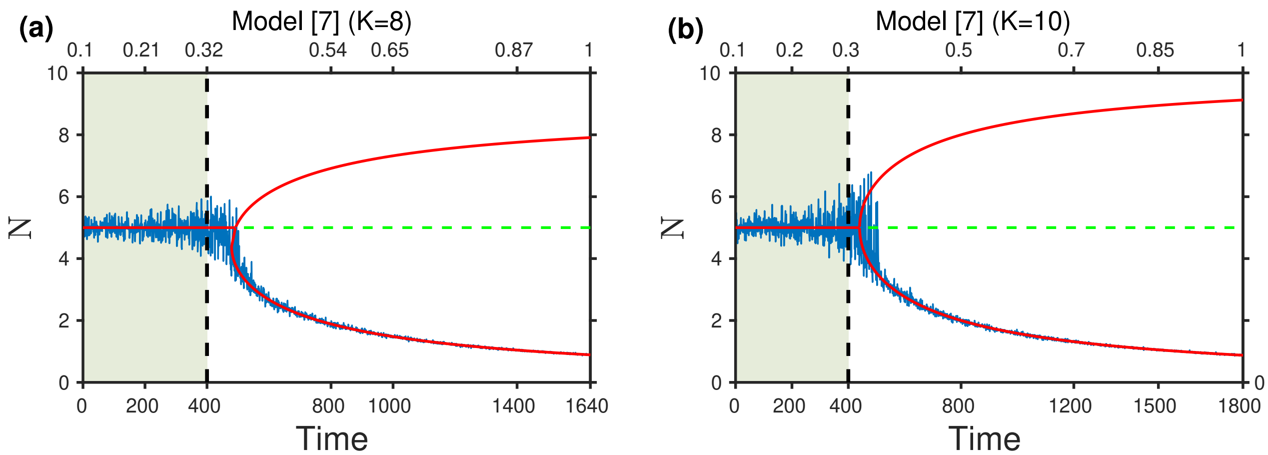

Figure 8: **Stochastic time series embedded over bifurcation diagram**: One parameter bifurcation diagram for model 7 for K=8 and K=10 respectively for the bifurcation parameter c (maximum grazing rate) varying in the range 0.1-1. Other parameters are c=0.8, $N_c = 5$, I=4. The shaded region mark the pre-transition time series used for training the EWSNET.

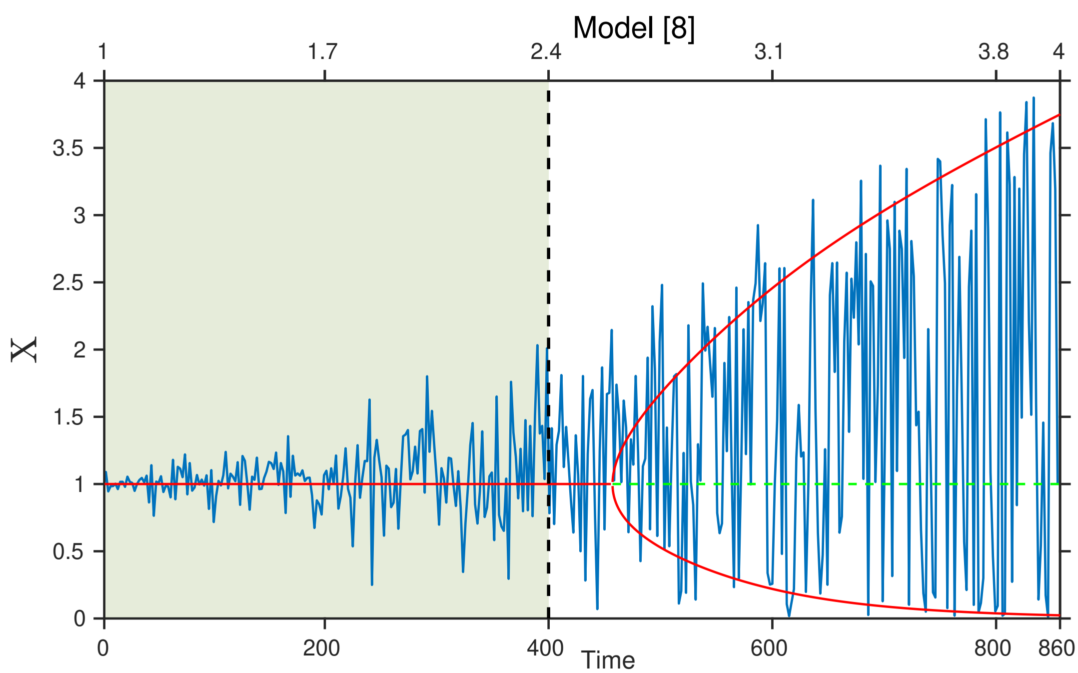

Figure 9: **Stochastic time series embedded over bifurcation diagram**:One parameter bifurcation diagram for model 8 for the bifurcation parameter K (carrying capacity of resource) varying in the range 1-4. Other parameters are r=0.5, a=0.4, b=0.6, e1=0.6, d1=0.15. The shaded region mark the pre-transition time series used for training the EWS-NET

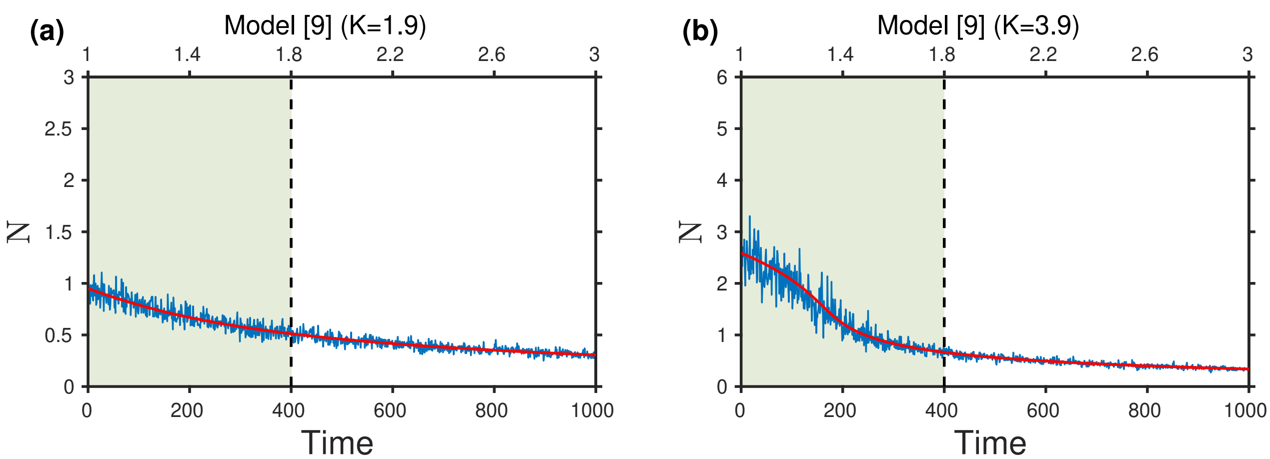

Figure 10: **Stochastic time series embedded over bifurcation diagram**: One parameter bifurcation diagram for model 9 for K=1.9 and K=3.9 respectively for the bifurcation parameter c (maximum grazing rate) varying in the range 1-3. Other parameters are r=1, b=1. The shaded region mark the pre-transition time series used for training the EWSNET.