Random Resistor Networks

Physics 514

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Introduction

- Random resistor networks (RRNs): lattices where sites are either conducting or insulating
- Useful for studying electrical conductivity of composite substances, so long as the distribution of its constituents can be modelled as random
- Capable of predicting potential failure points in electrical networks

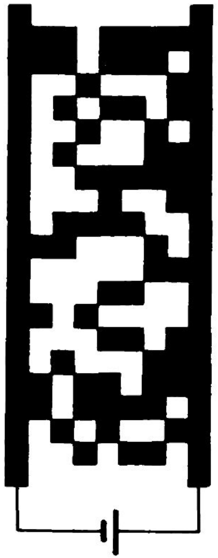


Fig. 1 Image of an RRN, taken from [1].

Rules and Definitions

- Conductance: the current that flows through the lattice if a unit voltage is applied across it
  - In 2D: Proportional to N (width), inversely proportional L (height)
- Conductivity, Σ: the factor of proportionality relating conductance and geometry
  - For a 2D square lattice, they are the same! Call it L.
- Mass of the lattice: ratio of the number of sites in the percolating cluster to the total amount of sites
  - Related to Σ, or not?

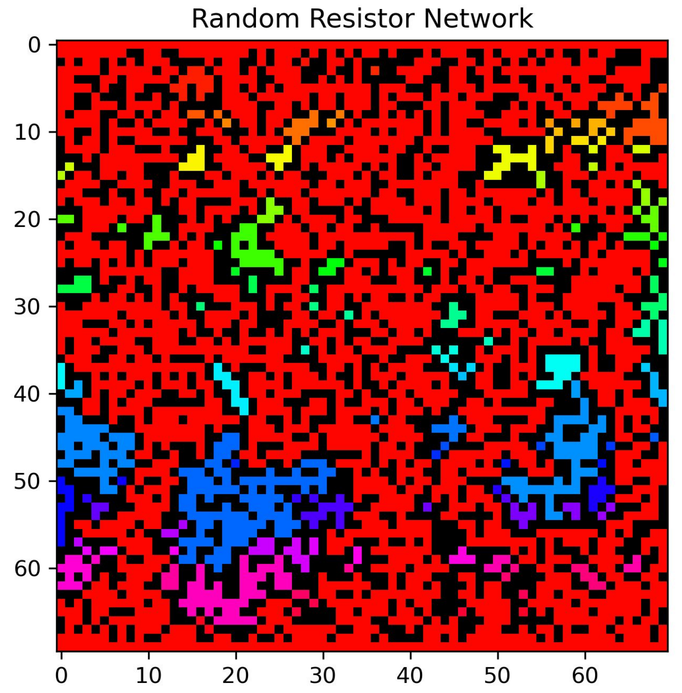


Fig. 2 Sample L=70 RRN.

Quick Check: Finding p_C and Scaling of Δ

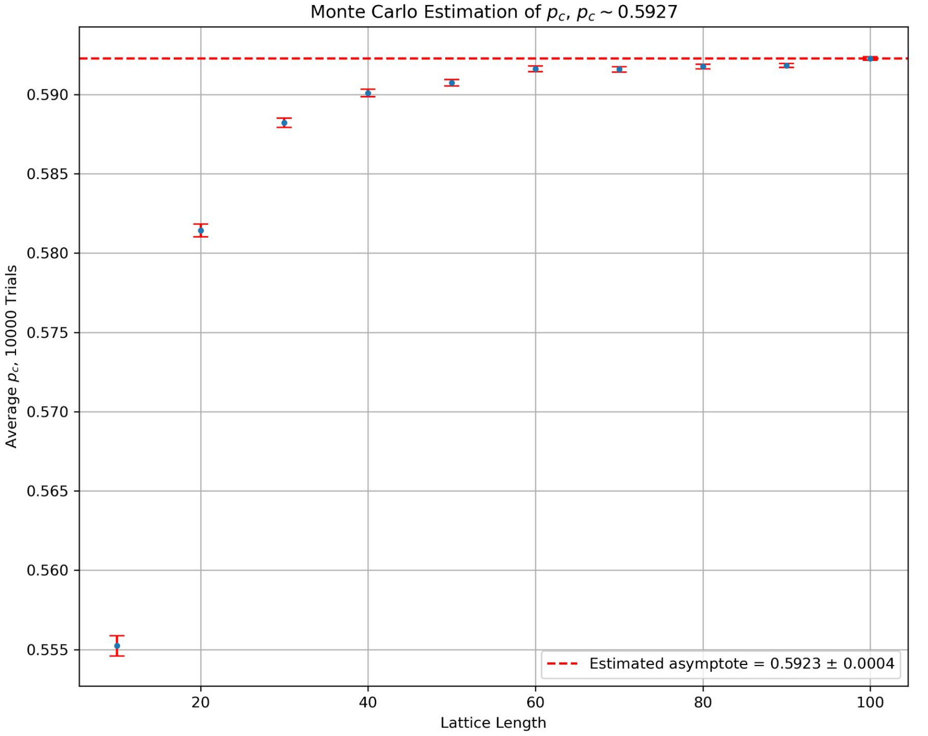


Fig. 3 Estimation of critical fraction at various lattice sizes.

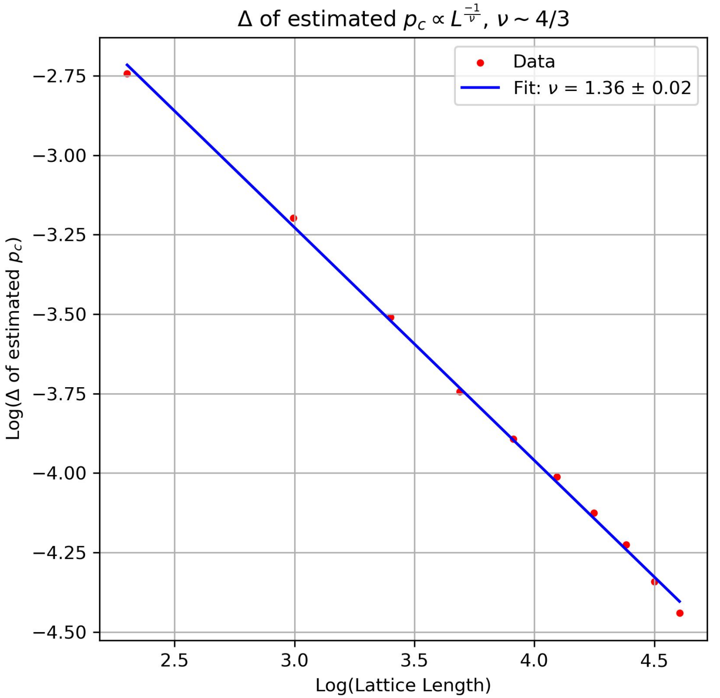


Fig. 4 Dependence of the standard deviation of the estimated critical fraction, Δ, on lattice size.

Calculating Conductivity

- Pick a site in the network, determine conductivity of its neighbors, replace the block with a single equivalent conductor, do this for the entire network … [2]
- Use current conservation and the microscopic form of Ohm's law to apply relaxation methods [3,4]:

$$\boxed{\begin{gathered}\vec{j} = \sigma\vec{E} = \sigma\vec{\nabla}V \\ \vec{\nabla}\cdot\vec{j} = 0\end{gathered}}$$

Calculating Conductivity (cont.)

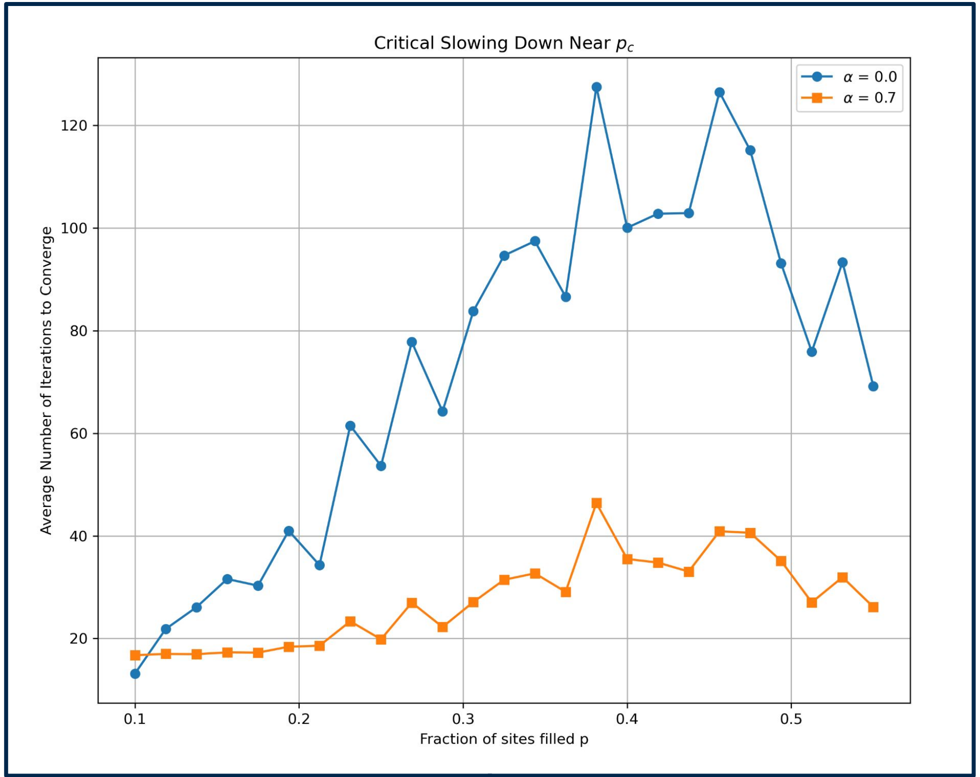
- Write down gradient operator on the lattice, do some algebra, get a nice(?) formula for relaxation:

$$\mathrm{V}_{k+1}(x_i, y_j) = \frac{\sigma(x_i+1,y_j)V_k(x_i+1,y_j)+\sigma(x_i,y_j)V_k(x_i-1,y_j)+\sigma(x_i,y_j+1)V_k(x_i,y_j+1)+\sigma(x_i,y_j)V_k(x_i,y_j-1)}{\sigma(x_i+1,y_j)+\sigma(x_i,y_j+1)+2\sigma(x_i,y_j)}$$

- Sigma is 1 if site is occupied, zero otherwise
- This is really just an average of the potential at nearby points weighted by sigma!
- Calculate the potential over the lattice, then compute the local current density according to Ohm's law, then sum the current densities in top and bottom row to find conductivity

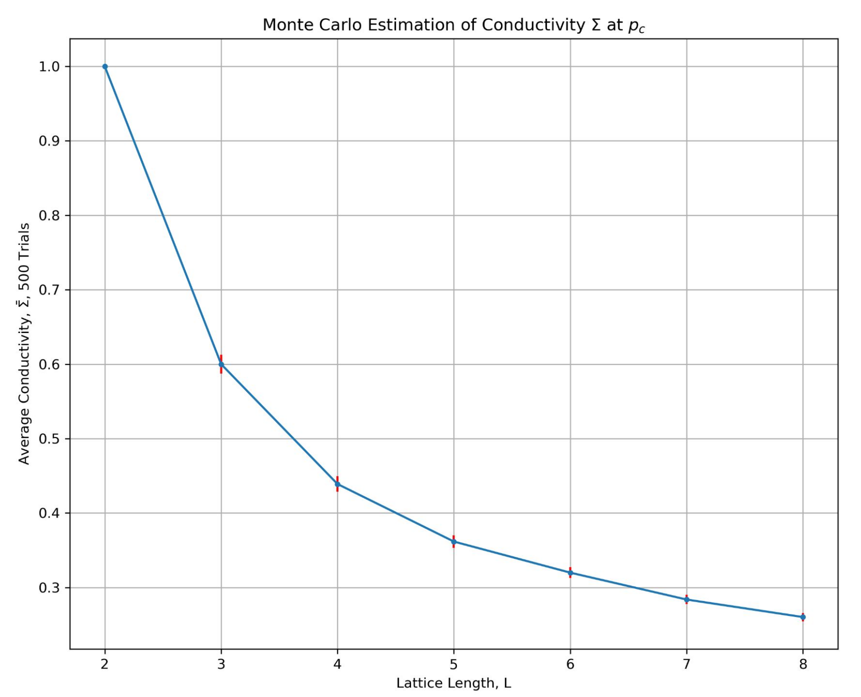
Overrelax To Avoid Critical Slowdown

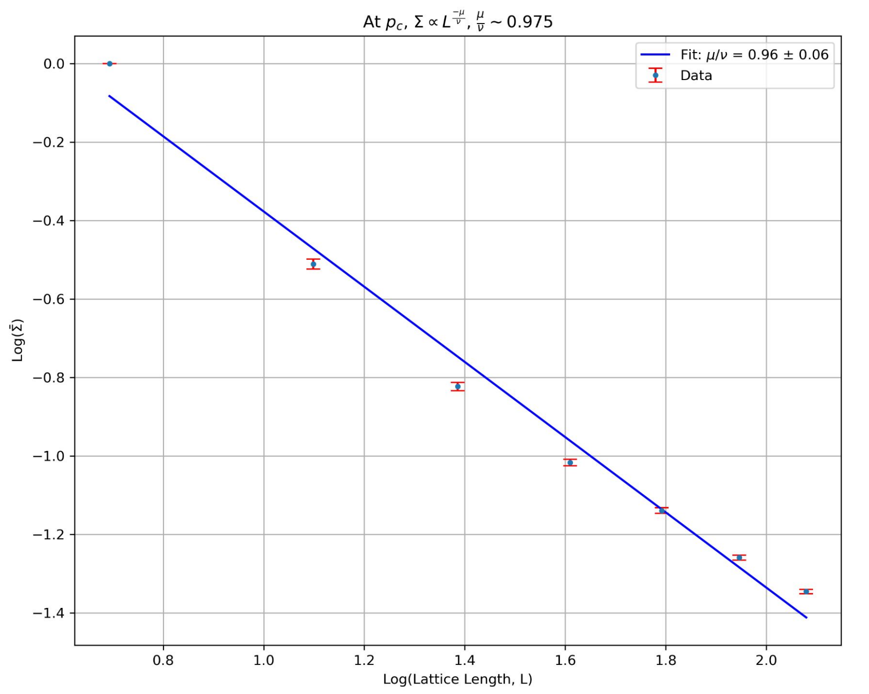
$$V^{-}_{k+1} = (1+\alpha)V_{k+1} - \alpha V_k$$



Figs. 5 Number of iterations for potential to converge with L=10 as a fraction of sites filled, using relaxation and overrelaxation.

How Does Conductivity Scale at p_C?





Figs. 6,7 Estimation of the L dependence of Σ at p_C.

Conductivity and Mass

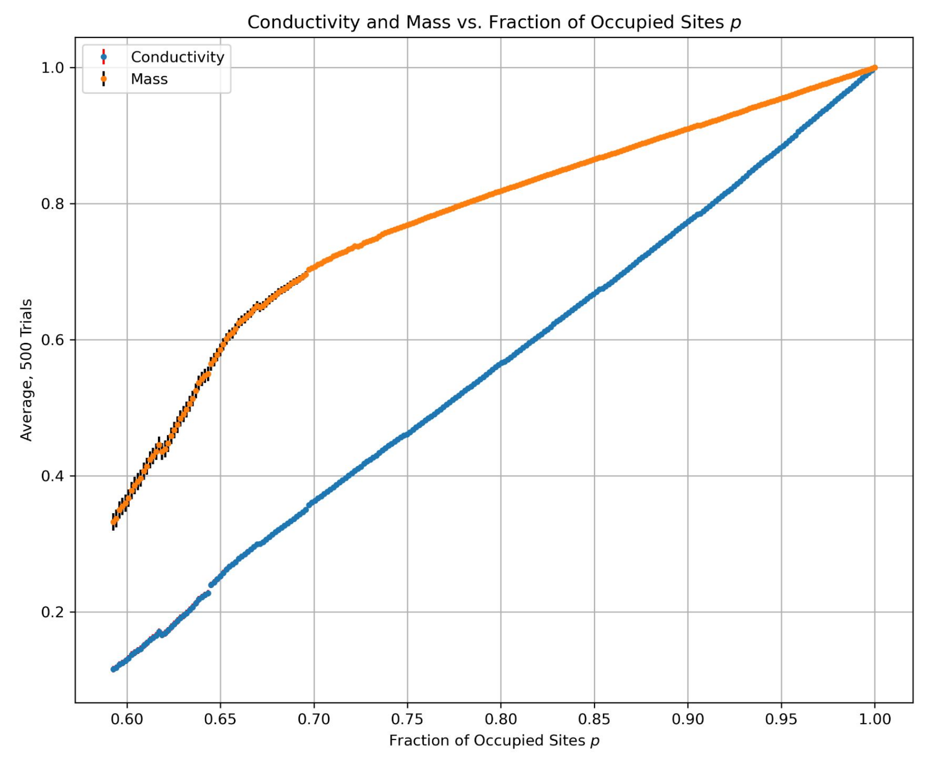


Fig. 8 Dependence of conductivity and mass on fraction of occupied sites, as determined by simulation of L=20.

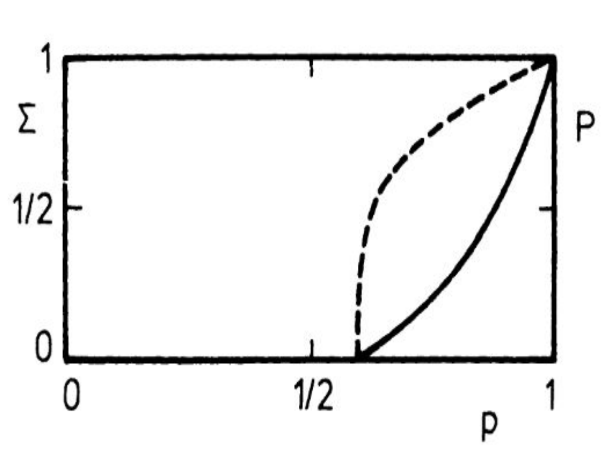


Fig. 9 Σ (solid) and mass (dashed) vs. p, as determined by experiment, taken from [5].

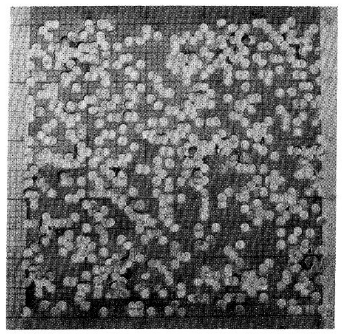
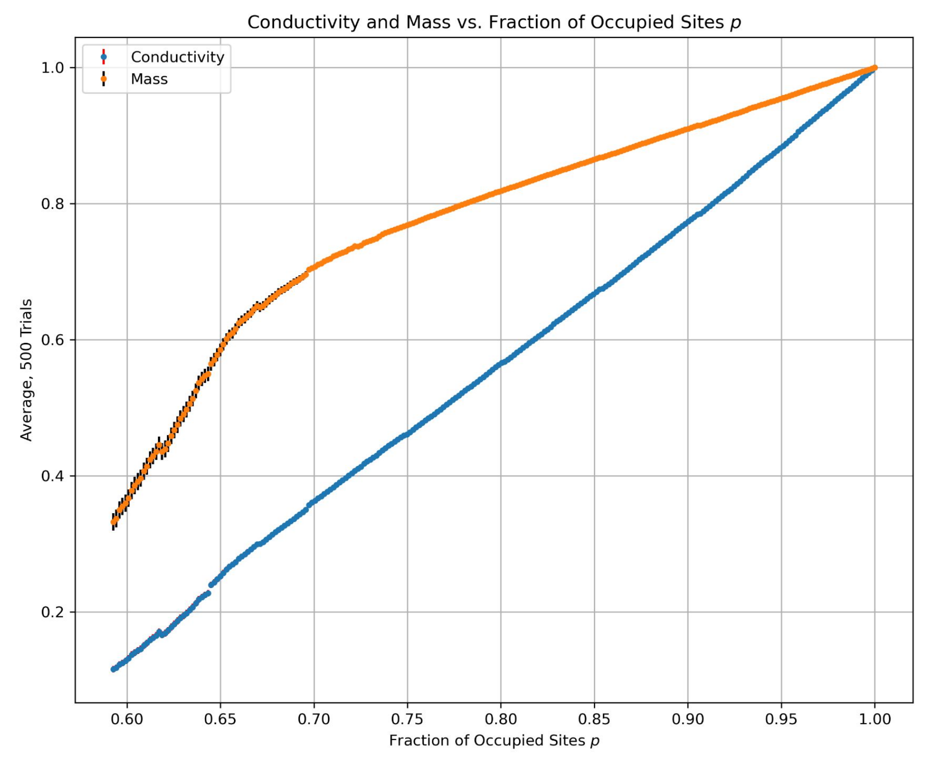


Fig. 10 Conducting paper with holes randomly punched into it, used to produce Fig. 8, taken from [5].

Conductivity and Mass (cont.)



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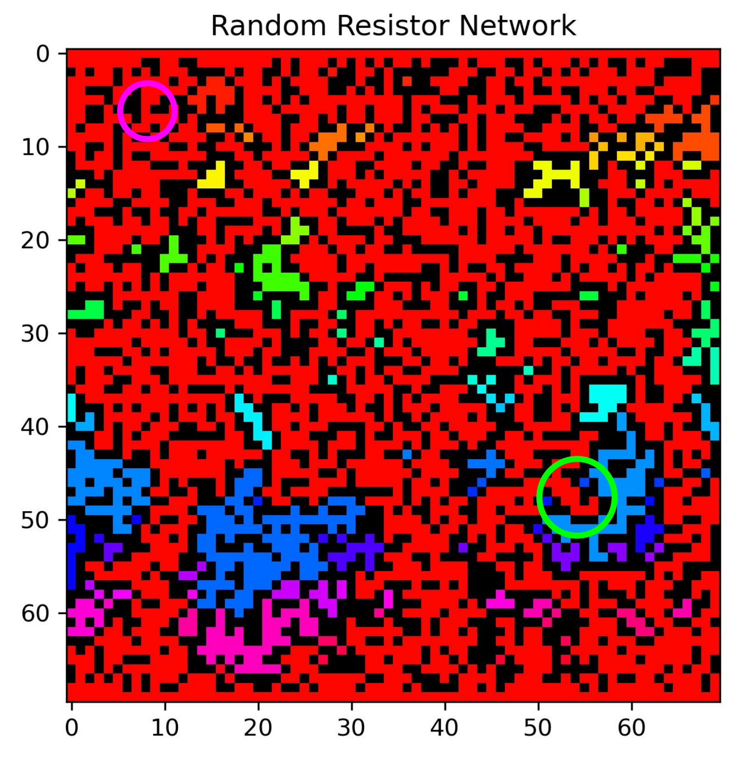

Fig. 11 Sample L=70 RRN with features identified.

References

[1] D. Stauffer and A. Aharony, *An Introduction to Percolation Theory*, 2nd ed. London, UK: Taylor & Francis, 1992.

[2] R. Fogelholm, "Percolation and Conduction," *J. Phys. C: Solid State Phys.*, vol. 13, no. 15, pp. L571-L578, 1980.

[3] S. Kirkpatrick, "Percolation and Conduction," *Rev. Mod. Phys.*, vol. 45, no. 4, pp. 574-588, Oct. 1973.

[4] E. Dagotto, *Nanoscale Phase Separation and Colossal Magnetoresistance: The Physics of Manganites and Related Compounds*. New York, NY, USA: Springer, 2002, ch. 16.

[5] B. J. Last and D. J. Thouless, "Percolation Theory and Electrical Conductivity," *Phys. Rev. Lett.*, vol. 27, no. 25, pp. 1719-1721, Dec. 1971, doi: 10.1103/PhysRevLett.27.1719.