Ch1 - Estimation

(*Point and Interval Estimation)*

ECO 204
Statistics For Business and Economics - II

Shaikh Tanvir Hossain

East **W**est **U**niversity, Dhaka
Last updated: June 26, 2025

Outline

Outline

Descriptive vs Inferential Statistics

Motivating Picture

...a picture may be worth a thousand words.. - [Djikstra]

Consider following picture,

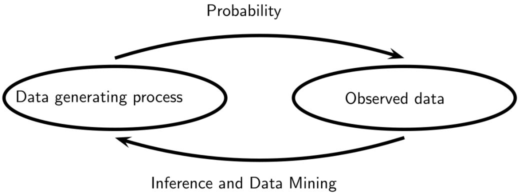


Figure 1: The figure is directly taken from Wasserman (2013), clearly explains what you did in ECO 104 (indicated by the arrow at the top going to right direction) and what you will do in ECO 204 (indicated by the arrow at the bottom going to left).

The work of *Probability Theory* is to describe - how the data / sample has been generated from a population, and the work of *Statistics* (or in particular Statistical Inference) is to make conclusions about the population using a sample.

Introduction to Statistical Inference

- Welcome to ECO 204!
- ECO 204 is about *Inferential Statistics* (as opposed to *Descriptive Statistics* which you did in ECO 104, as a side note - ECO 104 was about two things - i) Descriptive Statistics and ii) Probability Theory)
- Any idea about *Inferential Statistics*...?
- Roughly, It's is a branch of Statistics that helps us to *make conclusions about the population using a sample*
- Now couple of questions from ECO 104
  - What is *the population* for any particular study,
  - What is *a sample*, and
  - How do you make *Inference* using example?
- Let's see one example and answer these questions systematically, suppose we have the following data from 5 students studying currently at EWU (this is a hypothetical data, perhaps *randomly* collected!).

	Gender	Monthly Family Income (tk)
1.	Male	70,150
2.	Female	20,755
3.	Male	44,758
4.	Female	38,790
5.	Male	20,579

- You should already know that the columns are called *variables* and the rows are called *observations*. Let me stat with some questions.

Important Questions

Suppose our goal is to understand the Income and Gender of the current students at EWU, then consider following questions,

- Q1: What is the population of the study?
- Q2: Currently what proportion of students are female at EWU? Is it possible to calculate this?
- Q3: Currently what is the average income of all students at EWU? Is it possible to calculate this?
- Q4: Can we **estimate** proportion of students who are female at EWU? If yes then how?
- Q5: Can we **estimate** average monthly family income all students at EWU? If yes then how?
- Q6: What's the difference between sample and population quantities?
- Q7: If I have a population data, then *do I need a sample*? Why would I go for a sample?
- Q8: What are the possible issues in a sample?
- Q9: Are the sample quantities fixed or random?
- Q10: Do our predictions improve if we collect more samples?

Here are some answers,

- Ans 1: The population is the set of all units in the study, so in this case the population is *all currently enrolled EWU students*.

- Ans 2: We don't know this proportion exactly unless we have the data from all the students who are currently studying at EWU. In other words, we need data from the full population. Usually getting full population is very hard or time consuming. Although this is the quantity we are after, in other words this is one of our targets, but still we cannot calculate this exactly!

- Ans 3: Again the reasoning is same as Ans 2. Since we don't have the full population it's almost impossible to get the average income of all the current students.

- Ans 4: Yes, by using sample we can calculate **sample proportion** and then this sample proportion can work as an **estimate** for the **population proportion** of female students. In this sample proportion of female is 40%. So we can say, roughly, given that our sample represents the population, it is possible that the population proportion is close to 40%. Again, always remember here *population proportion* is for the entire population, and usually this is impossible to calculate. This is the first example of Statistical Inference, in particular we call this Statistical Estimation (more on this later)

- Ans 5: The answer is similar to the last answer. Yes we can estimate the **population mean** using *sample mean* of the students in the sample, in this case the sample mean is $39,006.4$ taka and we can take this **sample mean** as an **estimate** of the *population mean*. Again to make it clear here *population mean* is the average of the income of all the current students. This is the second example of Statistical Inference and in particular Statistical Estimation.

- Ans 6: We already answered this, but let's make it concrete, **sample quantities are estimates of the population quantities**. Population quantities are always our targets. Definitely chances are very low that they will be exactly same, however with a "good" sample we might be close to our target.

- Ans 7: NO, I have all information I need, so no need for sampling. I go for sampling because usually I don't have access to the population.

- Ans 8: Obvious issue - *biased sample* (sample doesn't represent population properly). Another issue - *small sample size*. From now on, we will avoid issues related to "biased sample", and assume our sample is a fairly good representative of the population....but ...?

- Ans 9: Definitely random since if we have a different sample then sample proportion or sample mean both will change.

- Ans 10: Of course ....

Introduction to Statistical Inference

- In the last example, we used a sample to calculate some sample quantities, in fact we calculated, *sample proportion* of female students in the sample, and *sample mean of family income*, and then we argued that we can use these objects to *"predict"*, or or *"conclude"* or *"infer"* about the unknown population quantities.
- This was an example of *Inferential Statistics* or *Statistical Inference*, in particular this is what we call *Statistical Estimation!*. The formal definition of *Statistical Inference* would require us to carefully define many things, but perhaps informally we can say -

Definition: Statistical Inference

Statistical Inference is a procedure where we have a **target parameter** θ defined for the population, and then we use a sample to make conclusions regarding the target parameter. There are two key branches of Statistical Inference,

- *Statistical Estimation (in short Estimation)*
- *Statistical Hypothesis Testing (in short Testing)*

We can also categorize estimation as point and interval estimation. Typical examples of Statistical Inference include making conclusion about the population mean, population proportion, population variance, etc., using sample mean, sample proportion, sample variance.

We will **generally use** the Greek letter θ to represent the target parameter. In the examples above,

- for the first one θ is the population proportion. If we write population proportion with p, then in this case $\theta = p$
- for the second one θ is the population mean. If we write population mean with μ, then in this case $\theta = \mu$

From the next section, first we will focus on **Estimation** and then in the next chapter we will move to **Testing**. In both cases, we will always start with the some *numerical techniques* that you have already learned before, for example, sample mean $\overline{x}$, sample proportion $\bar{p}$, etc. But our goal is one step more - that is making *inference* about the population.

Introduction to Statistical Inference

- The practice of Statistics falls broadly into two categories *Descriptive and Inferential*
- Descriptive Statistics is about describing the data using both numerical and graphical techniques / methods,
  - **Numerical Methods:** Calculating Sample Mean, Median, Mode, Variance, Standard Deviation, etc.
  - **Graphical Methods:** Looking at Bar Charts, Pie Charts, Histograms, etc
- The goal in this case is to describe the data, not to make any inference (or predict) about the population. This is what you did in ECO 104. You will do some recap exercises in the first problem set (which I will have to prepare, sorry not done yet!)

- However Inferential Statistics goes one step more, we try to make *"good" conclusions about the population using a sample*.
- There are two major themes of Inferential Statistics,
  - Statistical Estimation, in short we say *Estimation*
  - Hypothesis Testing, in short we say *Testing*
- You have already seen two examples of Estimation, we will see more. First we will focus on Estimation and then we will move to Hypothesis Testing.
- In both cases we will almost always start with the same *numerical techniques* that you have already learned in ECO104, that is we will use
  - sample mean,
  - sample median,
  - sample mode,
  - sample variance or sample standard deviation
  - sample quantiles or percentiles,
  - sample covariance, or sample correlation etc,

  but our goal is in this course will be one step more - that is making *inference* about the population.

  In the next section we will talk about *Estimation*.

Inferential Statistics - Part I, Estimation

Estimation, Estimate and Estimator

Estimator

We have already seen two examples of *Statistical Estimation*, in particular we saw

- we can use *sample proportion* $\bar{p}$ to estimate the unknown population proportion p.
- also, we can use *sample mean* $\bar{x}$ to estimate the unknown population mean μ.

Below we clearly define Statistical Estimation, in particular we will define what is *the target parameter, an estimate and an estimator, point estimation and interval estimation*,

Estimator

Definition (Statistical Estimation)

Statistical estimation is a statistical inference procedure which assigns numerical values to the unknown population parameters θ (e.g. mean μ, proportion p, variance σ^2) using data from a **random sample** $X_1, X_2, \ldots, X_n$. There are two types of Statistical Estimation,

- **Point estimation:** Provide a single best-guess number $\hat{\theta}$ for θ.
- **Interval estimation:** Provide an interval of possible values that, in repeated sampling, contains θ with a specified coverage probability (e.g. a 95 % confidence interval).

Any function of the random sample $(X_1, \ldots, X_n)$ is called a *statistic*. When a statistic is used to infer a parameter it is called an *estimator*. Both Statistic and Estimator are random variables, and their values change from sample to sample. For a fixed sample the value of the estimator is called an *estimate*. The probability distribution of a Statistic (or an estimator) is called *sampling distribution*.

There are several things to understand in the definition,

Estimator

Key Terms in the Definition

- Q1. What is a **Target Parameter** θ?
- Q2. What is a **Random Sample** $(X_1, \ldots, X_n)$?
- Q3. What is a **Statistic**?
- Q4. What is an **Estimator**?
- Q5. What is an **Estimate**?
- Q6. What is a **Sampling distribution of a Statistic / Estimator**?
- Q7. What is **Point Estimation** and **Interval Estimation**.

Again we will answer these questions and also understand the definition using an example. Consider the following sample, this is the same sample as above but just with one variable that is - Monthly family Income

sl.	Monthly Family Income (tk)	R.V.
1.	70,150	$X_1 =?$
2.	20,755	$X_2 =?$
3.	44,758	$X_3 =?$
4.	38,790	$X_4 =?$
5.	20,579	$X_5 =?$

Estimator

Remarks on Notation: Usually for fixed numbers we will use lower case letters $x_1, x_2, x_3, \ldots, x_n$, rather than numbers, just to make it more general...and for random variables we use upper case letters $X_1, X_2, X_3, \ldots, X_n$. Also generally when we think about n random variables, we write $X_1, X_2, X_3, \ldots, X_n$, and similarly for n fixed numbers we write $x_1, x_2, x_3, \ldots, x_n$.

Estimator

- **Ans 1 :** In this case the target parameter θ can be population mean of all current students at EWU. Since it's a mean in this case we often use μ for the target parameter.

- **Ans 2 :** The idea of the random sample is when we think each row as a *random data point*. How this is random? The idea is before sampling it is possible to have different values in each row. So we can think we have a random variable $X_1 =?$ at first row, X_2 at the second row and so on... so in principle each row can take different values from the population, and each value before sampling is a random variable. Also note after the sampling, when we have **observed the sample**, a random data becomes a fixed number and in this case the random variable X_1 has already taken a value, so we get $X_1 = 70,150$, and similarly for $X_2 = 20,755$. Here we don't have any randomness, we call it *observed data or realized data*. Important, *there is no randomness after we have observed the sample!*. So in the sample we have 5 random variables, they are X_1, X_2, X_3, X_4, X_5, together we call it a *random sample*. And also we have 5 fixed numbers, and they are $70,150$; $20,755$; $44,758$; $38,790$ and $20,579$, together we call it an *observed sample* or a *realized sample*.

Estimator

- **Ans 3:** A statistic is simply a function of the random sample, for example following are examples of statistic,
  - $\sum_{i=1}^{n} X_i$
  - $\overline{X} = \frac{1}{n}(X_1 + X_2 + \ldots, X_n) = \frac{1}{n}\sum_{i=1}^{n} X_i$
  - $\frac{\overline{X}-\mu}{\sigma/\sqrt{n}}$, where μ and σ are simply constants, μ is population mean and σ is population standard deviation
  - $\frac{\overline{X}-\mu}{S/\sqrt{n}}$ where S is the sample standard deviation and $S^2 = \frac{1}{n-1}\sum_{i=1}^{n}(X_i - \overline{X})^2$ and $S = \sqrt{S^2}$

  In a nutshell a statistic is any quantity made from the random sample $X_1, X_2, \ldots, X_n$
- **Ans 4:** When a Statistic is used for Estimation, we call it an **Estimator**. If Population mean μ is the target quantity, we can use $\overline{X}$ to calculate random sample mean. In this case $\overline{X}$ is a statistic and also it's an estimator.
- **Ans 5:** If we have a fixed sample then the value of the estimator becomes fixed, and on that case the numerical value of the estimator is called an **estimate**. For example if μ (or the population mean) is the target parameter then $\overline{X}$ is the estimator (this is the random sample mean) and for fixed sample we will get a value of $\overline{X}$ which will be an **estimate**. For example for this particular sample the estimate is $\bar{X} = 39,006$ taka. Again an estimate is a fixed number for a specific sample and an estimator is a random variable.

Estimator

- **Ans 6:** Now since **a statistic** or **an estimator** are random variables and their values will be different from sample to sample, it will have many possible values. The probability distribution over these values is called **sampling distribution**. For a concrete example, think about $\overline{X}$, this is an estimator when the target parameter is μ, and this will have many possible values for different samples. So we can think about a probability distribution over the values of $\overline{X}$, and this is the sampling distribution of $\overline{X}$. We will talk about sampling distribution detail in the next section, but there is a theorem which says if population is normally distributed with mean μ and variane σ^2, then the distribution of $\overline{X}$ is also normal with mean μ and variance σ^2/n. Using notation we write this as $\overline{X} \sim \mathcal{N}(\mu, \sigma^2/n)$. Again sampling distribution of sample mean $\overline{X}$ means distribution of different sample means, you should keep the shown picture (on the left) in your mind.

- **Ans 7:** When the parameter is μ, point estimation means we propose one best value, for example in this case sample mean $\bar{x}$ for a fixed sample. On the other hand interval estimation means we propose an interval of possible values with certain probability, such that in repeated sampling the parameter μ will be inside the interval. We will talk about interval estimation in the coming sections. But before that we need to discuss about sampling distribution of sample means.

Inferential Statistics - Part I, Estimation

Sampling Distribution of Sample Means Assuming Normality

Sampling Distribution of Sample Means With Normality Assumption

- Now when it comes to random variable $\overline{X}$, one of the most important question is,

  What is the probability distribution of $\overline{X}$ in repeated sampling?

- The answer to this question is not straightforward and it depends on probability distribution of the population. In particular we can think about two cases,
  - **Case 1: Population is Normally distributed** This means we are assuming the population data is normally distributed/
  - **Case 2: Population has some unknown probability distribution** This means we don't know the distribution of the population data (may or may not be normal).
- **I.I.D. Assumption:** In both cases we will always assume we have i.i.d. random sample. This means two things
  - The random variables in the random sample are all distributed as same distribution, which means $X_1, X_2, \ldots, X_n$ *are all identically distributed*, i.e., all are normally distributed with same mean μ and variance σ^2.
  - And $X_1, X_2, \ldots, X_n$ *are independent random variables*, this means in our sample each row is independently sampled from other.
- This i.i.d. assumption is often a standard assumption in cross section data. However in time series data the independence assumption breaks down!

Sampling Distribution of Sample Means With Normality Assumption

- Below we write the results of the two cases separately.
- The *first case* is actually quite restrictive since in practice we never know whether the probability distribution of the population is normal or not. But we will see that there are some strong features of the results of this case is, that is the results of the sampling distribution can be applied to any sample size. The *second case* is more practical, since in practice we never the probability distribution of the population. However there is an issue, we can only use the results from this case when we have a large sample size.
- Here is the result for Case - 1

Theorem (Sampling Distribution of $\overline{X}$ with Normality and I.I.D. Assumption)

If the population data is Normally distributed with mean μ and variance σ^2. Note that this further means the random variables $X_1, X_2, \ldots, X_n$ are Normal with same mean μ and variance σ^2 (this means for all i we have $X_i = \mathcal{N}(\mu, \sigma^2)$), and moreover they are all independent, then we can prove that,

$$\overline{X} \sim \mathcal{N}(\mu, \frac{\sigma^2}{n}) \tag{1}$$

Sampling Distribution of Sample Means With Normality Assumption

Proof.

When it comes to theorems we need to prove it. But for this one I will skip the proof, since this requires materials which are beyond the scope of this course! So you can also skip it. But at some point I will give the proof in the Appendix. □

In Words: This means *if the population data is Normally distributed and the random sample have i.i.d. random variables, then the sampling distribution of $\overline{X}$ is also normal with mean μ and variance σ^2/n, where n is the sample size.*

Standard Error: As a side note, the square root of this variance which is $\sqrt{\mathbb{V}(\overline{X})} = \sigma/\sqrt{n}$ is called the **Standard Error of the Sampling Distribution**. In general the square root of the variance for any distribution is called **Standard deviation**, but in this case when we are talking about sampling distribution we have a special name.

So **Standard Error** of the sampling distribution is, $\sigma/\sqrt{n}$, If we replace σ with S, then we call it *estimate of the Standard Error*, which is, $S/\sqrt{n}$

- This theorem has some immediate consequences, in *Math the consequence are called corollaries*, in particular we can write following corollary from the theorem.

Corollary (More Results Assuming Normality)

Following the last theorem, we can also get following results if the assumptions of the theorem holds,

$$i) \quad \mathbb{E}\left(\overline{X}\right) = \mu \tag{2}$$

$$ii) \quad \mathbb{V}\left(\overline{X}\right) = \frac{\sigma^2}{n} \tag{3}$$

$$iii) \quad Z \sim \mathcal{N}(0,1) \text{ where } Z = \frac{\overline{X} - \mu}{\frac{\sigma}{\sqrt{n}}} \tag{4}$$

$$iv) \quad T \sim t_{n-1} \tag{5}$$

$$\text{where } T = \frac{\overline{X} - \mu}{\frac{S}{\sqrt{n}}} \text{ and } S^2 = \frac{1}{n-1}\sum_{i=1}^{n}(X_i - \overline{X})^2 \tag{6}$$

Sampling Distribution of Sample Means With Normality Assumption

Proof.

Again we will skip the proof for now □

- *In Words:* Number i) and ii) says that *the expectation of the sample mean $\overline{X}$ (i.e., the average of the averages) will be equal to the population mean μ, i.e., $\mathbb{E}(\overline{X}) = \mu$, and the variance of the sample mean is $\mathbb{V}(\overline{X}) = \sigma^2 / n$.*

- Number (iii) is a direct consequence of the relationship between standard normal and normal distributions in general. Note that here

$$Z = \frac{\overline{X} - \mu}{\frac{\sigma}{\sqrt{n}}}$$

is actually *a Statistic*, so in this sense this distribution of Z is also sampling distribution.

Sampling Distribution of Sample Means With Normality Assumption

- The last one is a result for T statistic and *t distribution*, here $n-1$ is the parameter of the distribution, we call it *degrees of freedom*. Let's talk about the T statistic. First of all, note that T is a Statistic and a random variable, since it's a function of $\overline{X}$, in particular we have,

$$T = \frac{\overline{X} - \mu}{\frac{S}{\sqrt{n}}}$$

  number iv) says the distribution of this Statistic is a new distribution which is called t distribution. We write t_{n-1}, to write the parameter of the distribution which is in this case is $n-1$ and the name of this parameter is degrees of freedom.
- Interestingly there is a common pattern for these two statistics, that is

$$Z = \frac{\overline{X} - \mathbb{E}(\overline{X})}{\sqrt{\mathbb{V}(\overline{X})}} \text{ and } T = \frac{\overline{X} - \mathbb{E}(\overline{X})}{\sqrt{\widehat{\mathbb{V}}(\overline{X})}}$$

Inferential Statistics - Part I, Estimation

Interval Estimation Assuming Normality

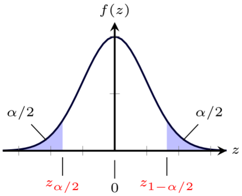
Interval Estimation With Normality Assumption

- Now we have the sampling distribution of sample means $\overline{X}$, and we can use this to construct *confidence intervals* for the population mean μ.
- Let's see how do we get the formula for the confidence interval,
- Again recall,

$$\overline{X} \sim \mathcal{N}(\mu, \frac{\sigma^2}{n}) \quad \text{equivalently} \quad Z \sim \mathcal{N}(0,1) \text{ where } Z = \frac{\overline{X} - \mu}{\frac{\sigma}{\sqrt{n}}}$$

- Now we start from the Z statistic, and write

$$\mathbb{P}(z_{\alpha/2} \leq Z \leq z_{1-\alpha/2}) = 1 - \alpha$$



- Where α is any probability $0 < \alpha < 1$, and $z_{\alpha/2}$ and $z_{1-\alpha/2}$ are values such that $\alpha/2$ and $1-\alpha/2$ are the probabilities to the left of $z_{\alpha/2}$ and $z_{1-\alpha/2}$ respectively.

Interval Estimation With Normality Assumption

- We now start with Z statistic and do some algebra such that μ comes in between

$$
\begin{aligned}
z_{\alpha/2} \leq Z \leq z_{1-\alpha/2} &= -z_{1-\alpha/2} \leq Z \leq z_{1-\alpha/2} \text{ [using symmetry of the normal]} \\
&= -z_{1-\alpha/2} \leq \frac{\overline{X} - \mu}{\frac{\sigma}{\sqrt{n}}} \leq z_{1-\alpha/2} \\
&= -\frac{\sigma}{\sqrt{n}}\, z_{1-\alpha/2} \leq \overline{X} - \mu \leq \frac{\sigma}{\sqrt{n}}\, z_{1-\alpha/2} \text{ [multiplying all sides by } \sigma/\sqrt{n}\text{]} \\
&= \frac{\sigma}{\sqrt{n}}\, z_{1-\alpha/2} \geq -\overline{X} + \mu \geq -\frac{\sigma}{\sqrt{n}}\, z_{1-\alpha/2} \text{ [multiplying all sides by } -1\text{]} \\
&= -\frac{\sigma}{\sqrt{n}}\, z_{1-\alpha/2} \leq -\overline{X} + \mu \leq \frac{\sigma}{\sqrt{n}}\, z_{1-\alpha/2} \text{ [rewriting the inequalities]} \\
&= \overline{X} - \frac{\sigma}{\sqrt{n}}\, z_{1-\alpha/2} \leq \mu \leq \overline{X} + \frac{\sigma}{\sqrt{n}}\, z_{1-\alpha/2} \text{ [adding } \overline{X} \text{ to all sides]}
\end{aligned}
$$

- This gives us following probability

$$
\mathbb{P}\left(\overline{X} - \frac{\sigma}{\sqrt{n}}\, z_{1-\alpha/2} \leq \mu \leq \overline{X} + \frac{\sigma}{\sqrt{n}}\, z_{1-\alpha/2}\right) = 1 - \alpha
$$

Interval Estimation With Normality Assumption

- And this is the formula for the interval estimation, in particular,

$$\text{lower limit} = \overline{X} - \frac{\sigma}{\sqrt{n}} z_{1-\alpha/2} \text{ and upper limit} = \overline{X} + \frac{\sigma}{\sqrt{n}}\ z_{1-\alpha/2}$$

  So the interval estimator is

$$[\overline{X} - \frac{\sigma}{\sqrt{n}} z_{1-\alpha/2} \quad , \quad \overline{X} + \frac{\sigma}{\sqrt{n}}\ z_{1-\alpha/2}]$$

  Now if we calculate this for a fixed sample we will call it an *interval estimate* which will be

$$[\overline{x} - \frac{\sigma}{\sqrt{n}} z_{1-\alpha/2} \quad , \quad \overline{x} + \frac{\sigma}{\sqrt{n}}\ z_{1-\alpha/2}]$$

  For an interval estimate, there is no probabilistic interpretation.

  But for the interval estimator, can think about the frequency interpretation of probability, that is, *if we do repeated sampling* 100 *times, then* 95 *out* 100 *times the intervals that we constructed will contain the true parameter* μ

Let's do an example first where we will calculate interval estimate for a fixed sample.

Example 1.1: (Interval Estimator and Interval Estimate/Confidence Interval)
Suppose we have $\overline{x} = 82$, population standard deviation $\sigma = 20$, sample size $n = 100$, and we are asked to compute the 95% *confidence interval or interval estimate of the population mean* μ, then since $z_{1-\alpha/2} = z_{0.975} = 1.96$ (this is $1 - \alpha/2$ quantile of the standard normal distribution and you can calculate this using Excel, `NORM.INV(.975, 0, 1)` and R function `qnorm(.975)`), the *interval estimator* is

$$[\overline{X} - 1.96\frac{20}{\sqrt{100}}, \quad \overline{X} + 1.96\frac{20}{\sqrt{100}}] \tag{7}$$

The *interval estimate or confidence interval* is

$$\begin{aligned} & [82 - 1.96\frac{20}{\sqrt{100}}, \quad 82 + 1.96\frac{20}{\sqrt{100}}] \\ & = [82 - 3.92, \quad 82 + 3.92] \\ & = [78, \quad 85.92] \end{aligned} \tag{8}$$

Now note, the first one at (7) is a *random interval* since $\overline{X}$ is random but the second one (8) is a deterministic interval (there is no randomness here!), this is the interval estimate, another name is *confidence interval*.

Interval Estimation With Normality Assumption

- So in the second one either our population mean μ is there or it is not there. If you say that there is a 95% probability that true parameter μ will fall inside $[78, \quad 85.92]$, this is a *wrong interpretation*. We can say "for this particular sample, *the interval estimate* is $[78, 85.92]$".
- Where does the word confidence comes? You can say *if we construct these kinds of intervals* 100 *times, then roughly* 95 *times our true parameter will fall inside*.
- So now we have a probabilistic interpretation.
- *A Side Note:* Note that when we constructed the interval estimate, we added and subtracted the following same number with $\overline{x}$

$$\frac{\sigma}{\sqrt{n}} \times z_{1-\alpha/2}$$

  Here $\sigma/\sqrt{n}$ is the standard error and the whole term is called the *margin of error* of the point estimate.

References

Abraham, B. and Ledolter, J. (2006), *Introduction to Regression Modeling*, Duxbury applied series, Thomson Brooks/Cole, Belmont, CA.

Anderson, D. R., Sweeney, D. J., Williams, T. A., Camm, J. D., Cochran, J. J., Fry, M. J. and Ohlmann, J. W. (2020), *Statistics for Business & Economics*, 14th edn, Cengage, Boston, MA.

Bertsekas, D. and Tsitsiklis, J. N. (2008), *Introduction to probability*, 2nd edn, Athena Scientific.

Blitzstein, J. K. and Hwang, J. (2015), *Introduction to Probability*.

Casella, G. and Berger, R. L. (2002), *Statistical Inference*, 2nd edn, Thomson Learning, Australia ; Pacific Grove, CA.

DeGroot, M. H. and Schervish, M. J. (2012), *Probability and Statistics*, 4th edn, Addison-Wesley, Boston.

Hansen, B. (2022), *Econometrics*, Princeton University Press, Princeton.

James, G., Witten, D., Hastie, T. and Tibshirani, R. (2023), *An introduction to statistical learning*, Vol. 112, Springer.

Newbold, P., Carlson, W. L. and Thorne, B. M. (2020), *Statistics for Business and Economics*, 9th, global edn, Pearson, Harlow, England.

Pishro-Nik, H. (2016), *Introduction to probability, statistics, and random processes*.

Ramachandran, K. M. and Tsokos, C. P. (2020), *Mathematical Statistics with Applications in R*, 3rd edn, Elsevier, Philadelphia.

References

Rice, J. A. (2007), *Mathematical Statistics and Data Analysis*, Duxbury advanced series, 3rd edn, Thomson/Brooks/Cole, Belmont, CA.

Wasserman, L. (2013), *All of statistics: a concise course in statistical inference*, Springer Science & Business Media.

Wooldridge, J. M. (2009), *Introductory Econometrics: A Modern Approach*, 4th edn, South Western, Cengage Learning, Mason, OH.