Sorting (V): Merge Sort

CSCD 300 – Data Structures

Eastern Washington University

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Goal

We will learn the mechanism of the Merge Sort algorithm and then analyze its time complexity in the best as well as in the worst case.

Outline

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Merge sort

Basic idea

- Divide the n-number sequence into two halves. Each subsequence has $n/2$ numbers.
- Recursively sort each n/2-number subsequence.
- Merge two sorted subsequences into one sequence.

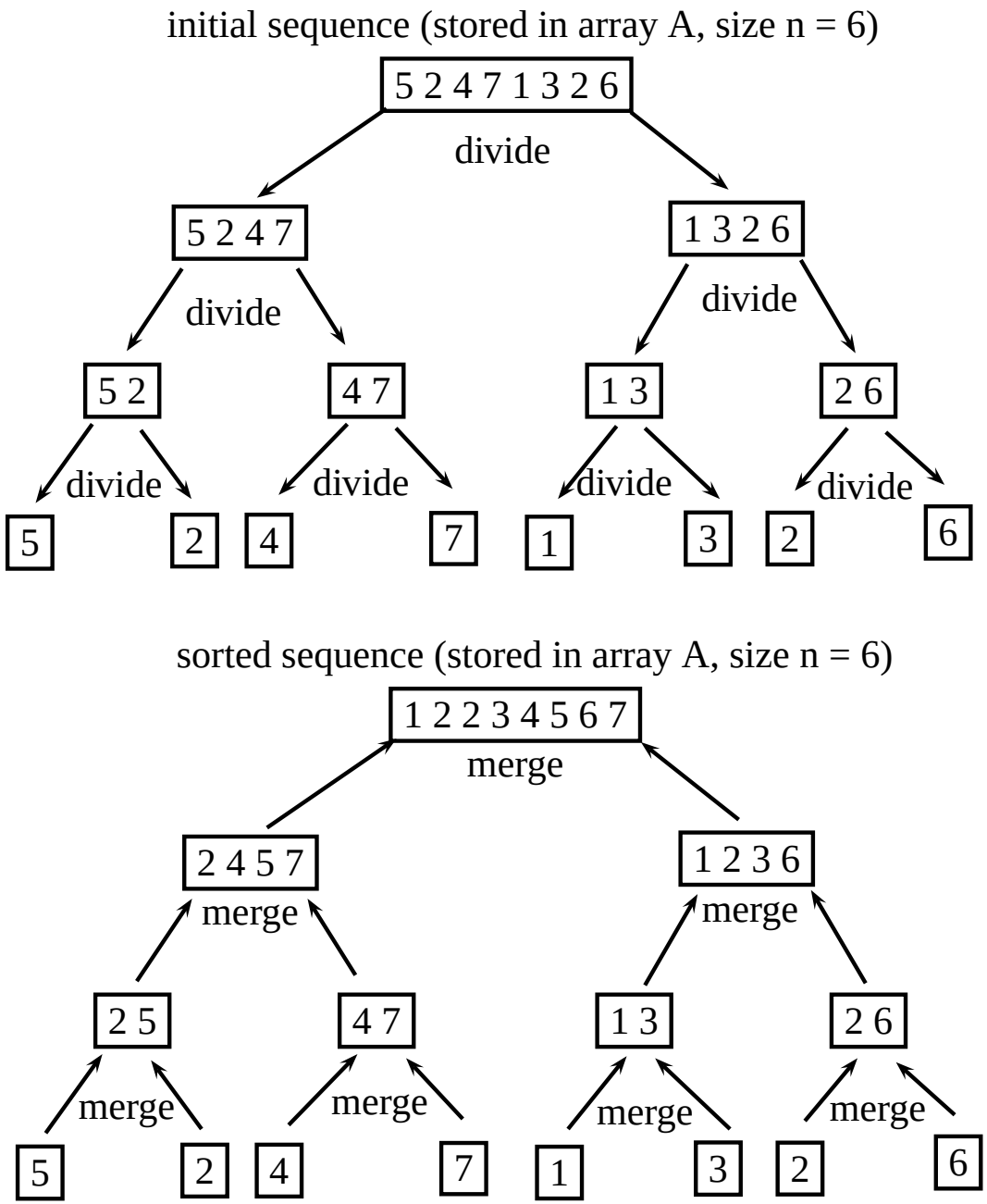
Psuedocode

```
MERGE_SORT(A, p, r)
{
   if(p<r)
      q = floor((p+r)/2)

      MERGE_SORT(A, p, q)
      MERGE_SORT(A, q+1, r)

      MERGE(A, p, q, r)
}
```

```
MERGE(A, p, q, r)
{
   copy A[p ... q] into L[0 ... q-p]
   copy A[q+1 ... r] into R[0 ... r-q-1]

   i = 0; j = 0;
   for k = p to r
      if(i == q-p+1)
         A[k] = R[j]; j ++;
      else if(j == r-q)
         A[k] = L[i]; i ++;
      else if L[i] <= R[j]
         A[k] = L[i]; i ++;
      else
         A[k] = R[j]; j ++;
}
```

Call "MERGE_SORT(A, 0, n-1)" will sort the sequence of n numbers that are stored in array A. Let's look at an example ...

An example


initial sequence (stored in array A, size n = 6)
5 2 4 7 1 3 2 6
divide
5 2 4 7
1 3 2 6
divide
divide
5 2
4 7
1 3
2 6
divide
divide
divide
divide
5
2
4
7
1
3
2
6
sorted sequence (stored in array A, size n = 6)
1 2 2 3 4 5 6 7
merge
2 4 5 7
1 2 3 6
merge
merge
2 5
4 7
1 3
2 6
merge
merge
merge
merge
5
2
4
7
1
3
2
6

Time complexity of Merge Sort: recurrence equation

```
MERGE_SORT(A, p, r)
{
   if(p<r)
      q = floor((p+r)/2)

      MERGE_SORT(A, p, q)
      MERGE_SORT(A, q+1, r)

      MERGE(A, p, q, r)
}
```

Initial call: `Merge_Sort(A,0,n-1)`

Let $T(n)$ =time cost for sorting the n numbers in $A[0...n-1]$, then we have:

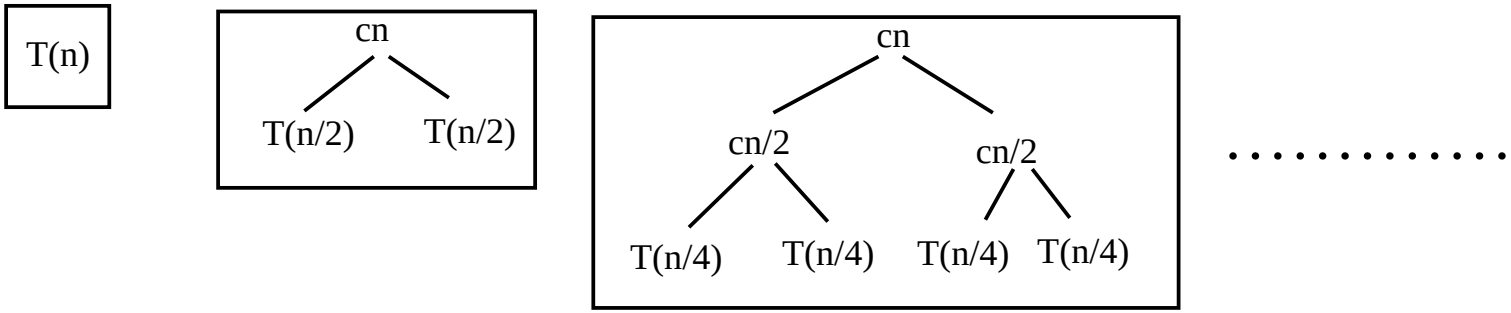
$$T(n) = 2T(n/2) + O(n)$$

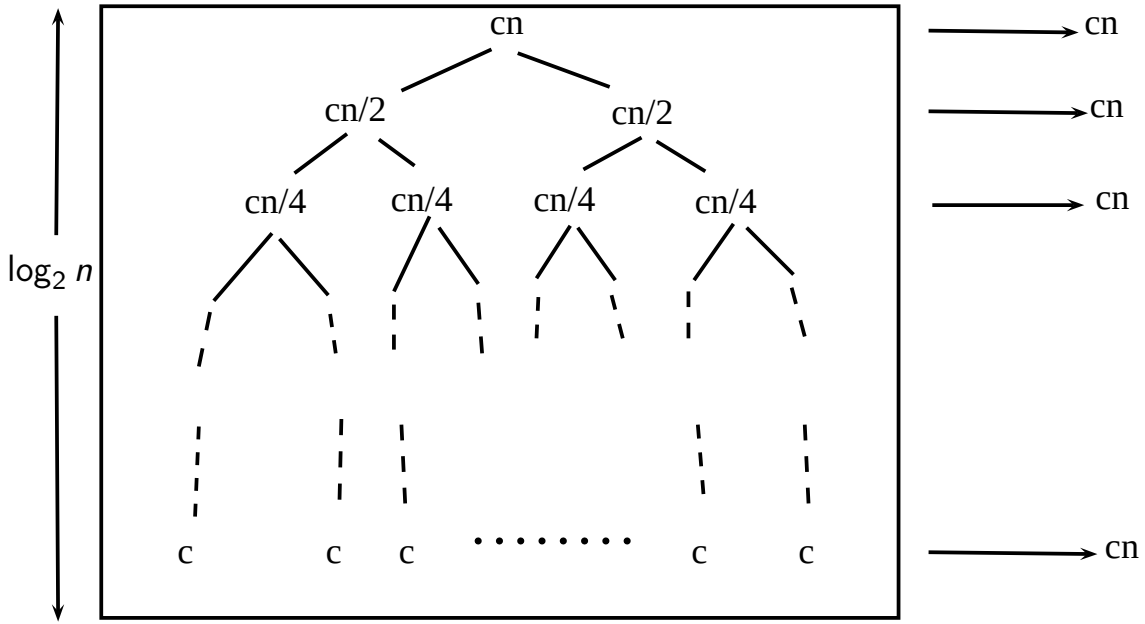
because

- sorting A[0...n-1] is divided into sorting of two subsequences. Sorting each subsequence takes time $T(n/2)$ as each subsequence has size $n/2$.
- merging the two sorted subsequences of each sized $n/2$ takes n constant-time operations.

Solve the recurrence

Note: c is some positive constant.





Total: $T(n) = cn \log_2 n = O(n \log n)$

Question

How do you use the Merge sort if the data sequence is saved in a singly linked list ?