Reisz-Markov

Here, we wish to provide a intuitive presentation of Reisz-Markov and a motivation of its proof.

Suppose ϕ is a positive linear functional on $C_c(\mathbb{R})$, i.e. a positive Radon measure. It is not far-fetched to hope for a representation like

$$\phi(f) = \int f d\mu_\phi.$$

After all, integration plays a large part in the dual spaces of L^q. So we ask ourselves: if such a representation existed, what would μ_ϕ look like?

Let f be the following function:

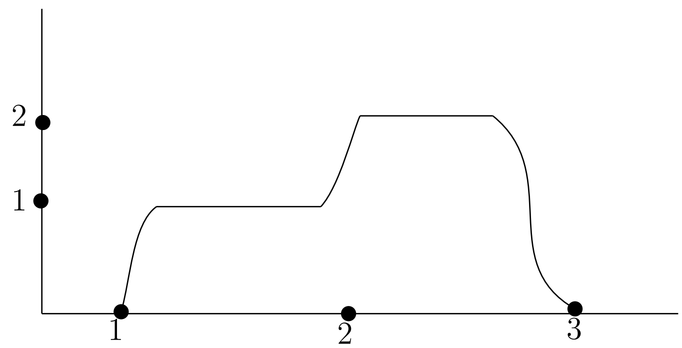

Figure 1: A Simple Example

By linearity, we can decompose $\phi(f) \approx 1 \cdot \phi(g_{(1,2)}) + 2 \cdot \phi(g_{(2,3)})$ where g_E is the continuous analog to the charateristic function χ_E, i.e.

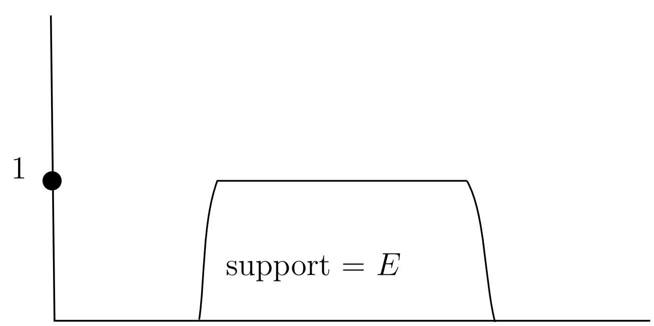

Figure 2: Decomposition

This suggests that we should set $\mu_\phi(E) = \phi(g_E)$. If you think about it (intuitively), this indeed turns ϕ into an integral. Hence, we start our construction of μ_ϕ by defining

$$\mu_\phi(U) = \sup\{\phi(f) : 0 \leq f \leq \chi_U,\ f \in C_c(X),\ \mathrm{supp}\{f\} \subset U\}$$

for open sets U. (Note as ϕ is positive, this supremum is really just approximating χ_U by continuous functions.)