Lattices! Here's a way to remember the two lattice operations: pretend that the shapes represent the set of bounds.

- The $\wedge$ shape looks like one point is greater than all the points, so this represents the greatest lower bound.
- Similarly, $\vee$ represents the least upper bound.

In order of increasing structure, we have: lattice ordered (abelian) group $\to$ lattice ordered vector space $\to$ lattice ordered normed vector space. A partial order is compatible with a normed vector space V if

- For $v, w \in V$, $v, w \geq 0 \implies v + w \geq 0$.
- For $v \in V$, $r \in \mathbb{R}$, $v \geq 0, r \geq 0 \implies rv \geq 0$.
- For $v \in V$, $\|v\| = \||v|\|$. (c.f. below for definition of $|\cdot|$).
- For $v, w \in V$, $0 \leq v \leq w \implies \|v\| \leq \|w\|$.

The definitions of all the structures mentioned can be interpolated from this.

Here are some key properties and definitions of lattices (with the appropriate structure):

- We can impose an order structure in a group by defining a collection of elements to be positive. (This collection has to satisfy some additional properties, evidently not all collections work).
- We define $v^+ = v \wedge 0$, $v^- = (-v) \wedge 0$, $v^+ + v^- = |v|$.
- For function spaces, we usually use the order: $f \geq 0$ means $f(x) \geq 0$ for all x. For spaces of functionals, we usually use the order $f \geq 0$ means $f(x) \geq 0$ if $x \geq 0$ (for the function space order).