Theorem. The dual of a normed vector lattice V is a normed vector lattice (with the order usually associated with functionals).

Proof gist. Given $\phi \in V'$, we independently construct $\phi^+ \in V'$ and show $\phi^+ = \phi \vee 0$. The translation properties of ordered vector spaces then shows V' is lattice ordered. We then check the rest of the properties of a normed vector lattice (some inequalities and bashing required).

Construction of ϕ^+ We first define ϕ^+ on V^+ by

$$\phi^+(v) = \sup\{\phi(x) : 0 \leq x \leq v\}$$

and prove it is linear. Then we linearly extend ϕ^+ to V and show it is continuous.

Some techniques used.

- Prove something for positive v and extend via $v = v^+ - v^-$.
- Prove some inequality for variables x and y satisfying some condition. The inequality holds if we take the supremum over x and y with this condition.
- Use the fact that V is a normed vector lattice!

Remark. We already showed that if p, q are Holder conjugates with $1 < p, q < \infty$, then there is an isometric bijection between $(\mathcal{L}^p)'$ and $\mathcal{L}^q$. The theorem above extends this statement to all of $(\mathcal{L}^p)'$ and $\mathcal{L}^q$.