Quotient norm. If W is a closed subspace of Banach space V, then equip V/W with the norm

$$\|\pi(v)\| = \inf\{\|v - w\| : w \in W\}.$$

Note that if W is not closed, then we only get a semi-norm.

Tweak technique. There are a few miscellaneous statements about quotient norms and spaces, and they often use the following technique:

- Fix $\pi(v)$. Tweak $v \to v + w$ until you get a specific property.

As an exercise, prove the following statement: For any $v \in V$, there is a $\phi \in V'$ such that $\phi(W) = 0$, $\phi(v) = \|\pi(v)\|$, and $\|\phi\| = 1$.

Theorem. V/W is a Banach space.

Proof gist. "Pull back" a rapidly Cauchy sequence from V/W to V. Tweak technique is needed.

Theorem. Space of continuous linear operators $\mathcal{B}(V, W)$ is a Banach space.

Proof gist. Let T be the point of convergence of a Cauchy sequence. Show T is bounded and linear (very routine).