The weak topology is tight. Suppose V is a vector space, and W is a collection of linear functionals on V. Then if ϕ is a continuous linear functional for the W-weak topology, $\phi \in W$.

Key trick. If $\phi_1, \ldots, \phi_n$ are a collection of linear functionals, then

$$\bigcap \ker \phi_i \subset \ker \phi \implies \phi \in \operatorname{span}\{\phi_i\}.$$

To show the LHS, we compare the topology generated by ϕ with the subbase of the weak topology.

Krein-Milman. Let C be a closed convex subset of a locally convex topological vector space V (assumed Hausdorff). Then the convex hull of extreme points of C is C.

Main idea. Consider the poset P of all the faces of C. Then the minimal points of P are the extreme points (need Zorn for existence). Then use this property to prove this theorem.

Key technique. If D is compact convex set, we consider a continuous linear functional ϕ not constant on D (use H-B or H-B separation). Then

$$\{v \in D : \phi(v) \text{ achieves its maximum.}\}$$

is a proper (compact convex) face.