The parallelogram inequality $\|v+w\|^2 + \|v-w\|^2 = 2(\|v\|^2 + \|w\|^2)$ guarantees a closest point to a convex closed set in Hilbert space $\mathcal{H}$. This is how we define orthogonal projection.

The decomposition $\ker\phi + (\ker\phi)^\perp$ is used to construct Reisz Representation.