For vector space V, the following are equivalant:

- The topology on V is the initial topology from a collection of semi-norms on V.
- The topology on V is translation invariant and has a subbase of convex sets.

When the above scernaios occur, we say the topology on V is **locally convex**.

Alaoglu's theorem. If V is a normed vector space, then the closed unit ball B in V' is compact for the weak-* topology.

Point of confusion. Two different topologies are used in the statement of the theorem. The closed unit ball is defined by the norm-topology, but the actual topology on V' is weak-*.

Proof gist. Use Tynchoff's theorem to construct a compact space and establish a homeomorphism between that compact space and B with the weak-* topology.

More detail. Define $D_v = \{t \in \mathbb{R} : |t| < ||v||\}$. Then consider the map

$$J : B \to \prod_{v \in V}^{\infty} D_v = P$$

given by the (informal) expression

$$J(\phi) \mapsto \prod_{v \in V}^{\infty} \phi(v).$$

By comparing the subbases of B and $J(B)$, we conclude they are homeomorphic. To show $J(B)$ is closed (hence compact), we note each element of P determines a function f. If f is in the closure of $J(B)$, then by approximating f by elements of $J(B)$, we show f is linear and thus an element of $J(B)$.