Notes on Curves and Surfaces

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Notation I

Suppose $x(u, v)$ is a local parameterization of a surface. Then we write

$$x_u = \frac{\partial x}{\partial u}, \quad x_v = \frac{\partial x}{\partial v}.$$

Observe that x_u and x_v are the column vectors of the derivative dx, i.e.

$$x_u = dx(u), \quad x_v = dx(v).$$

As parameterizations are diffeomorphisms, dx is invertible, and thus x_u and x_v are linearly independent.

First fundamental form

The first fundamental form at a point $p \in S \subset \mathbb{R}^3$ is the inner product on $T_p(S)$ inherited from $\mathbb{R}^3$. It is often expressed as a quadratic form using the basis x_u, x_v as follows:

$$\langle ax_u + bx_v, ax_u + bx_v \rangle = \langle x_u, x_u \rangle a^2 + 2\langle x_u, x_v \rangle ab + \langle x_v, x_v \rangle b^2.$$

The three coefficents are often denoted E, F, G.