Assignment 1

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1 Problem 1

- Perhaps you had a typo, the crystal is $CaAl_2$
- See file in the folder

2 Problem 2

Using the notation that $\mathbf{G} = \sum_{i=1}^{3} n_i \vec{a}_i$ and $\mathbf{R} = \sum_{i=1}^{3} m_i \vec{b}_i$, the periodic condition for lattice is

$$e^{i\mathbf{G}\cdot\mathbf{R}} = 1 \iff |\vec{a}_i \cdot \vec{b}_j| = 2\pi\delta_{ij}$$

Using the geometrical fact that $\vec{v}_{ij} \equiv \vec{a}_i \times \vec{a}_j$ satisfies $\vec{v}_{ij} \perp \vec{a}_{i,j}$. And $\vec{x} \cdot \vec{y} = \vec{0}$ if $\vec{x} \perp \vec{y}$. Therefore $\vec{a} \cdot (\vec{a} \times \vec{b}) = \vec{a} \cdot (\vec{b} \times \vec{a}) = 0$ We have

$$\begin{aligned}
\vec{a}_i \cdot \vec{b}_j &= \vec{a}_i \cdot 2\pi \frac{\vec{a}_k \times \vec{a}_l}{\vec{a}_j \cdot \vec{a}_k \times \vec{a}_l} && (1) \\
&= \begin{cases} 0, & \text{if } i = k \text{ or } i = l. \\ 2\pi, & \text{if } i = j. \end{cases} && (2) \\
&= 2\pi\delta_{ij} \quad \square && (3)
\end{aligned}$$