Physics and Computation

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1 Disclaimer

Yes, chatgpt is good at spitting out nonsense. But I used it to proof-read and polish my essay.

2 Quantum Computation: A plagued technology

Quantum computers possess the capability to solve certain problems with greater efficiency than classical computers. Shor's algorithm, for instance, can factor large integers N in $\mathcal{O}(log(N)^2 loglog(N))$ time, whereas the most efficient classical algorithm is $\mathcal{O}(e^(1.9log(N)^(1/3)loglog(N)^(2/3)))$[Sho,]. Consequently, quantum computers have the potential to provide exponential speedup for factoring large integers. However, modern encryption techniques that rely on the difficulty of factoring integers are unlikely to be compromised anytime soon.

One of the most significant challenges in quantum computing is noise. In essence, noise refers to a disruptive force that perturbs the qubits used for computation. It is analogous to a mischievous imp that upsets the abacus one uses for arithmetic operations. In other words, computation results from a noisy quantum computer are no longer precise. These noises arise from unwanted physical interactions between the qubits undergoing coherent evolution and their environment or poorly calibrated operations used for computation.

Although techniques such as quantum error correction code can protect data in a fault-tolerant quantum memory, they rely on relatively good physical operation[Girvin,]. Presently, our technology is still struggling to keep up[Kitaev,]. Consequently, there is a need to develop a naturally robust computing approach that is immune to unwanted interactions causing noise.

3 Topology: A Global Picture

As Rolf Landauer famously stated, "Information is Physical." [Landauer,] When performing computations, we must first encode information onto physical systems and evolve those systems to carry out the necessary computations. If we could identify a physical system that is inherently resistant to noise and encode information onto it, we would be able to construct a naturally fault-tolerant quantum computer. Furthremore, current knowledge suggests that all physical interactions are local. Therefore, if we could identify a system that is immune to local interactions, we could construct a decoherence immune qubit.

Recall our definition of a topological invariant, as previously defined in class. Both the Gauss-Bonnet theorem and the calculation of the Berry phase require an integral over the entire system. Local information alone is insufficient to determine the topological invariants of a system. Similarly, if one has the ability to alter a system locally, they cannot alter the topological invariant number of that system.

Using this as a guide, we turn our attention to topological systems. In the limit of large system size, a topologically non-trivial phase can emerge from a gapped quantum many-body system. One key feature of the non-trivial phase is that it possesses topologically degenerate ground states that are not degenerate in the topologically trivial phase. Therefore, the degeneracy of the ground states is a direct consequence of the topology of the system's phase. Applying the logic from the previous paragraph, we expect these ground states to be protected from local noise, meaning decoherence should not affect them.

In conclusion, we can encode information onto these ground states and expect this encoding to be immune to decoherence. This insight offers a promising avenue for constructing a fault-tolerant quantum computer by leveraging the properties of topological systems.

4 Majorana Zero Mode: Physical Realization

As an illustration, Alexi Kitaev proposed a one-dimensional chain of fermions in which the spin is disregarded [Kitaev,]. The Hamiltonian of the chain is given by $H = \sum_{n=1}^{N}[-\mu(a_n^\dagger a_n - \frac{1}{2}) - w(a_n^\dagger a_{n+1} + a_{n+1}^\dagger a_n) + \Delta a_n a_{n+1} + \Delta^* a_{n+1}^\dagger a_n^\dagger)]$, where Kitaev discovered that when $|\Delta| = w > 0$, the system is in a topologically non-trivial phase [Alicea,]. The degenerate ground state can be obtained through the following transformation, where θ is the superconducting phase and γ is the Majorana operator: $a_n = \frac{1}{2}(e^{-i\theta/2}\gamma_{2n} + e^{i\theta/2}\gamma_{2n-1})$ and $a_n^\dagger = \frac{1}{2}(e^{i\theta/2}\gamma_{2n} - e^{-i\theta/2}\gamma_{2n-1})$. This transformation leads to the Hamiltonian $H = -iw\sum_{n=1}^{N-1}\gamma_{2n}\gamma_{2n+1}$, which can be easily diagonalized to obtain its eigenstates by introducing new fermionic operators $\tilde{a}_n = (\gamma_{2n} - i\gamma_{2n+1})/2$. It can be observed that $H = 2w\sum_{n=1}^{N-1}(\tilde{a}_n^\dagger\tilde{a}_n - 1/2)$ [Huang,]. As γ_1 and γ_{2N} do not exist in the Hamiltonian, they commute and have zero energy. Excitation modes of these creation operators correspond to Majorana modes with zero energy, hence the name "Majorana zero mode". The two Majorana modes can be combined into one fermionic mode by $\tilde{a}_0 = (\gamma_1 + i\gamma_{2N})/2$. The $|0\rangle$ and $|1\rangle$ states with respect to such fermionic excitation have degenerate energy but are topologically protected. Therefore, they can be used to encode decoherence-free qubits.

5 Bibliography

5.1 References

References

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