HW 7

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Contents

1 Problem 1

Following Hook's Law, we have the next system of equation.

$$\begin{cases} M_A \frac{\partial u_i^2}{\partial t^2} = \lambda(w_{i-1} - u_i) + \lambda(v_i - u_i) \\ M_B \frac{\partial v_i^2}{\partial t^2} = \lambda(w_i - v_i) + \lambda(u_i - v_i) \\ M_C \frac{\partial w_i^2}{\partial t^2} = \lambda(v_i - w_i) + \lambda(u_{i+1} - w_i) \end{cases} \tag{1}$$

Due to periodicity of our system, we solve with ansatz $u_i = Ae^{i\omega t}e^{ika}$, $v_i = Be^{i\omega t}e^{ika}$ and, $w_i = Ce^{i\omega t}e^{ika}$. we have

$$\begin{cases} -M_A\omega^2 u_i = \lambda(w_{i-1} - u_i) + \lambda(v_i - u_i) \\ -M_B\omega^2 v_i = \lambda(w_i - v_i) + \lambda(u_i - v_i) \\ -M_C\omega^2 w_i = \lambda(v_i - w_i) + \lambda(u_{i+1} - w_i) \end{cases} \tag{2}$$

$$\begin{cases} -M_A\omega^2 A = \lambda(Ce^{-ika} - A) + \lambda(B - A) \\ -M_B\omega^2 B = \lambda(C - B) + \lambda(A - B) \\ -M_C\omega^2 C = \lambda(B - C) + \lambda(Ae^{ika} - C) \end{cases} \tag{3}$$

$$\begin{cases} -\frac{M_A}{\lambda}\omega^2 A = (Ce^{-ika} - A) + (B - A) \\ -\frac{M_B}{\lambda}\omega^2 B = (C - B) + (A - B) \\ -\frac{M_C}{\lambda}\omega^2 C = (B - C) + (Ae^{ika} - C) \end{cases} \tag{4}$$

$$\begin{pmatrix} \frac{M_A}{\lambda}\omega^2 - 2 & 1 & e^{-ika} \\ 1 & \frac{M_B}{\lambda}\omega^2 - 2 & 1 \\ e^{ika} & 1 & -\frac{M_C}{\lambda}\omega^2 - 2 \end{pmatrix} \begin{pmatrix} A \\ B \\ C \end{pmatrix} = \vec{0} \tag{5}$$