Problem 6

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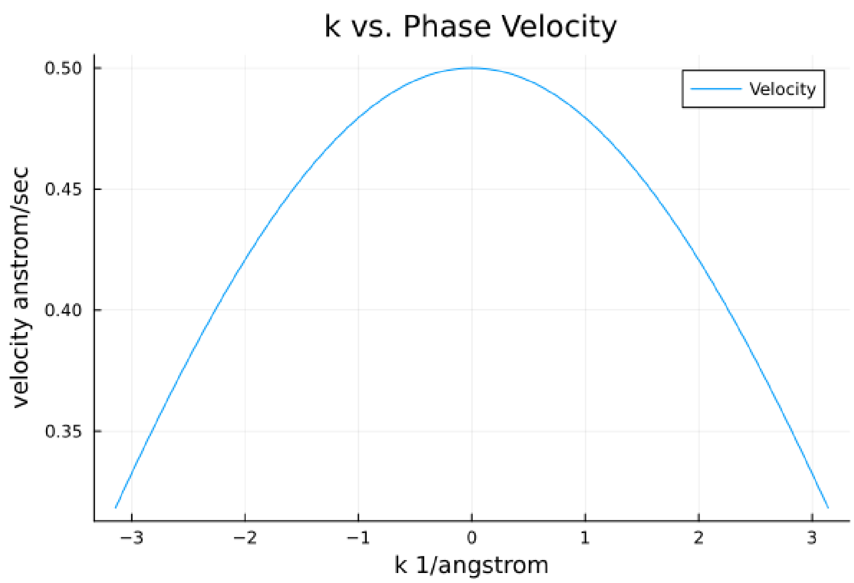
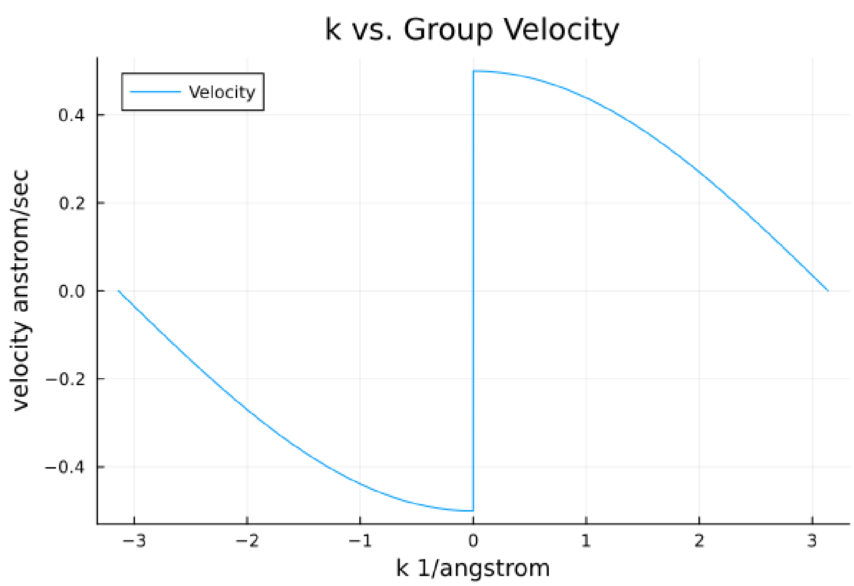
Contents

1 Problem 1

- Phase velocity is $v_p = \frac{\omega}{k} = \sqrt{\frac{4C}{Mk^2}} |\sin(\frac{1}{2}ka)|$
- Group velocity is $v_g = \frac{\partial \omega}{\partial k} = \pm\sqrt{\frac{4C}{M}} \frac{a}{2} \cos(\frac{1}{2}ka)$
- In the following plot, I made $a = 1$ and $4C/M = 1$

```
using Plots
vp(k) = abs(sin(k/2))/abs(k)
vg(k) = k >= 0 ? cos(k/2)/2 : - cos(k/2)/2
k = -:0.001:
vps = vp.(k)
vgs = vg.(k)
plot(k,vps;title="k vs. Phase Velocity",label="Velocity")
xlabel!("k 1/angstrom")
ylabel!("velocity anstrom/sec")
savefig("phase.png")
plot(k,vgs;title="k vs. Group Velocity",label="Velocity")
xlabel!("k 1/angstrom")
ylabel!("velocity anstrom/sec")
savefig("group.png")
```





2 Problem 2

- At small k we assume linear relation between phonon momentum and ω. Therefore $v_s = v_p|_{k \ll 1} = \frac{\omega}{k} = \sqrt{\frac{4C}{Mk^2}}|\sin(\frac{1}{2}ka)| \approx \sqrt{\frac{C}{M}}a$.
- Since the chain density is $\rho = M/a$, for $v_s \equiv \frac{1}{\sqrt{\beta\rho}}$, $\beta = \frac{1}{aC}$