Physics and Computation

Topologically protected qubits

Yusheng Zhao

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Outline

My interest: Quantum Computer

- Motivation: quantum advantage on some problems
- How does it differ from classical one?
  1. Information encoded on a qubit: $|\psi\rangle = \alpha |0\rangle + e^{i\theta}\beta |1\rangle$
  2. $|0\rangle$ and $|1\rangle$ are usually degrees of freedom of underlying physical system

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Note

- Let me first introduce quantum computer for those who are not familiar
- Qubit vs Bit
- Quantum Computers could solve some problems more efficiently compared to classical computers. For example, Shor's algorithm is able to factor large integers N while providing an exponential speed up compare to classical one.

Noise in Quantum Computer

- Noise is the archenemy
- Example: Imp and abucus
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- However, this technology is plagued by noise. Roughly speaking, noise is like a little daemon who flips the abucus that you use to do calculation. The source of those noise come from unwanted physical interaction. or even badly calibrated actions.
- Loss of phase: $a_j^\dagger a_j = (1 + i\gamma_{2j-1}\gamma)/2$

Observation and Solution

- Observation: Physical interactions are local
- Proposition: Design a qubit to store information non-locally
- "If a physical system were to have quantum topological (necessarily nonlocal) degrees of freedom, which were insensitive to local probes, then information contained in them would be automatically protected against errors caused by local interactions with the environment." - A. Kitaev

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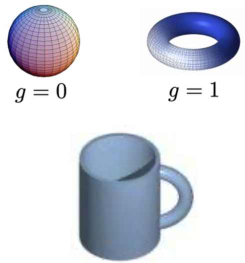
Note

- This gives us an idea. Since all known physical interactions are local, could be store our information non-locally to alleviate the effect of noise?
- The following quote of Alexi Kitaev is too good to be missed

Topological Invariant



Gauss-Bonnet theorem

$$\int K dS = 2\pi(2 - 2g)$$

$K = \kappa_1 \kappa_2$ is the Gaussian curvature

[Prof. Li Slides]

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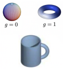

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Note

- This is very abstract, can we get a better intuition from what we have learned in class?
- Consider Topological Invariant
- Recall our definition of a topological invariant, as previously defined in class. Both the Gauss-Bonnet theorem and the calculation of the Berry phase require an integral over the entire system. Local information alone is insufficient to determine the topological invariants of a system. Similarly, if one has the ability to alter a system locally, they cannot alter the topological invariant number of that system.

Topological Degeneracy

- Defined on low temperature gapped quantum many body system
- Degenerate ground state emerges after phase transition
- Arises from topologically non-trivial phase
- Protected from local perturbation
- Encode qubit onto these d.o.f

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- Using this as a guide, we turn our attention to topological systems. In the limit of large system size, a topologically non-trivial phase can emerge from a gapped quantum many-body system. One key feature of the non-trivial phase is that it possesses topologically degenerate ground states that are not degenerate in the topologically trivial phase. Therefore, the degeneracy of the ground states is a direct consequence of the topology of the system's phase. Applying the logic from the previous paragraph, we expect these ground states to be protected from local noise, meaning decoherence should not affect them.

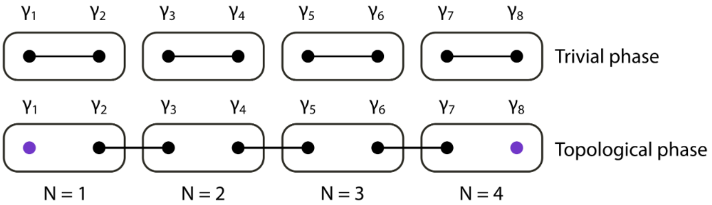
Hamiltonian [Kitaev,]

- $H = \sum_{n=1}^{N}[-\mu(a_n^\dagger a_n - \frac{1}{2}) - w(a_n^\dagger a_{n+1} + a_{n+1}^\dagger a_n) + \Delta a_n a_{n+1} + \Delta^* a_{n+1}^\dagger a_n^\dagger)]$
- 1D chain of fermions
- μ: chemical potential
- w: hopping strength
- Δ: induced superconducting gap

Emergence of Non-trivial Phase [Huang,]

- $|\Delta| = w > 0$
- $a_n = \frac{1}{2}(e^{-i\theta/2}\gamma_{2n} + e^{i\theta/2}\gamma_{2n-1})$
- γ is the Majorana creation/anhilation operator
- $\tilde{a}_n = (\gamma_{2n} - i\gamma_{2n+1})/2$
- Diagonalize: $H = 2w\sum_{n=1}^{N-1}(\tilde{a}_n^\dagger \tilde{a}_n - 1/2)$

A picture is worth a thousand words



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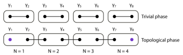

Note

- Note, γ_1 and γ_{2N} are not in Hamiltonian
- Have zero energy.
- Combine to make fermonic mode $\tilde{a}_0 = (\gamma_1 + i\gamma_{2N})/2$
- $|0\rangle$ and $|1\rangle$ of above creation operator have degenerate energy.
- Also protected by topology. Can be made into protected qubits!

Physics and Computation

- "Information is Physical" [Landauer,]
- Topologically degenerate degree of freedom sees not local perturbation

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Note

- Information is physical, meaning that the effecacy of the computation relies very much so on the system that realizes it. Computation is not merely something on the paper. It's very much so related to the physical world.
- Topological degree of freedom is calculated from the system-wide point of view. Therefore, it could not be probed locally hence it's immune to local error.

References I

Huang, S.
Introduction to Majorana Zero Modes in a Kitaev Chain.

Kitaev, A.
Unpaired Majorana fermions in quantum wires.
44:131–136.

Landauer, R.
There are no unavoidable energy consumption requirements per step in a computer. Related analysis has provided insights into the measurement process and the communications channel, and has prompted speculations about the nature of physical laws.