Topological Order and Noise

Emergence of Majorana Zero Mode

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Outline

Noise in Quantum Computer

- Quantum Computing has advantage over Classical
- Noise is the archenemy
- Solution: store information non-locally
- "If a physical system were to have quantum topological (necessarily nonlocal) degrees of freedom, which were insensitive to local probes, then information contained in them would be automatically protected against errors caused by local interactions with the environment." - A. Kitaev

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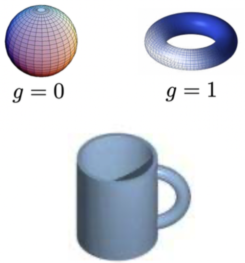
Note

- Quantum Computers could solve some problems more efficiently compared to classical computers. For example, Shor's algorithm is able to factor large integers N, in $\mathcal{O}(log(N)^2 loglog(N)$ time. Meanwhile, the best known classical algorithm is $\mathcal{O}(e^{1.9 log(N)^{1/3} loglog(N)^{2/3}}$.
- However, this technology is plagued by noise. Roughly speaking, noise is like a little daemon who flips the abucus that you use to do calculation. The source of those noise come from unwanted physical interaction. or even badly calibrated actions.
- For the purpose of this talk, we focus on unwanted physical interaciton.
- This gives us an idea. Since all known physical interactions are local, could be store our information non-locally to alleviate the effect of noise?

Topological Invariant

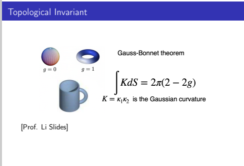


Gauss-Bonnet theorem

$$\int K dS = 2\pi(2 - 2g)$$

$K = \kappa_1 \kappa_2$ is the Gaussian curvature

[Prof. Li Slides]



Note

- Recall our definition of a topological invariant, as previously defined in class. Both the Gauss-Bonnet theorem and the calculation of the Berry phase require an integral over the entire system. Local information alone is insufficient to determine the topological invariants of a system. Similarly, if one has the ability to alter a system locally, they cannot alter the topological invariant number of that system.

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- Protected from local perturbation

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- Using this as a guide, we turn our attention to topological systems. In the limit of large system size, a topologically non-trivial phase can emerge from a gapped quantum many-body system. One key feature of the non-trivial phase is that it possesses topologically degenerate ground states that are not degenerate in the topologically trivial phase. Therefore, the degeneracy of the ground states is a direct consequence of the topology of the system's phase. Applying the logic from the previous paragraph, we expect these ground states to be protected from local noise, meaning decoherence should not affect them.

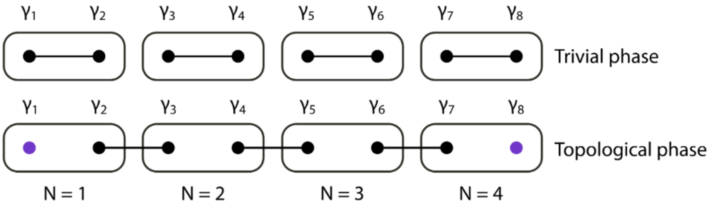
Hamiltonian [Kitaev,]

- $H = \sum_{n=1}^{N} [-\mu(a_n^\dagger a_n - \frac{1}{2}) - w(a_n^\dagger a_{n+1} + a_{n+1}^\dagger a_n) + \Delta a_n a_{n+1} + \Delta^* a_{n+1}^\dagger a_n^\dagger)]$

Emergence of Non-trivial Phase [Huang,]

- $|\Delta| = w > 0$
- $a_n = \frac{1}{2}(e^{-i\theta/2}\gamma_{2n} + e^{i\theta/2}\gamma_{2n-1})$, γ is the Majorana creation/anhilation operator
- $a_n^\dagger = \frac{1}{2}(e^{i\theta/2}\gamma_{2n} - e^{-i\theta/2}\gamma_{2n-1})$
- $\tilde{a}_n = (\gamma_{2n} - i\gamma_{2n+1})/2$
- $H = 2w\sum_{n=1}^{N-1}(\tilde{a}_n^\dagger \tilde{a}_n - 1/2)$

A picture is worth a thousand words



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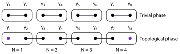

Note

- Note, γ_1 and γ_{2N} are not in Hamiltonian
- Have zero energy.
- Combine to make fermonic mode $\tilde{a}_0 = (\gamma_1 + i\gamma_{2N})/2$
- $|0\rangle$ and $|1\rangle$ of above creation operator have degenerate energy.
- Also protected by topology. Can be made into protected qubits!

Physics and Computation

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Note

- Information is physical, meaning that the effecacy of the computation relies very much so on the system that realizes it. Computation is not merely something on the paper. It's very much so related to the physical world.
- Topological degree of freedom is calculated from the system-wide point of view. Therefore, it could not be probed locally hence it's immune to local error.

References I

Huang, S.
Introduction to Majorana Zero Modes in a Kitaev Chain.

Kitaev, A.
Unpaired Majorana fermions in quantum wires.
44:131–136.

Landauer, R.
There are no unavoidable energy consumption requirements per step in a computer. Related analysis has provided insights into the measurement process and the communications channel, and has prompted speculations about the nature of physical laws.