Homework 3 Answer

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1 Problem 1

Recall, Born-von Karman boundary condition, $\psi(\vec{r}) = \psi(\vec{r} + n_i\vec{a}_i)$. For momentum eigenstates, we need $e^{i\vec{k}_i \cdot \delta\vec{r}_i} = 1$. Where we define $\delta\vec{r}_i = n_i\vec{a}_i$ and $n_i \in [1, N_i]$.

To satisfy the condition, we need $(\vec{k}_i \cdot n_i\vec{a}_i) = 2\pi * m, \forall i \in \{x, y, z\}$, where $\vec{k}_i = h_i\vec{b}_i$. And $\vec{b}_i$, $\vec{a}_i$ are basis vector in the momentum and real space respectively and m is an integer.

Within the first Brillouin Zone, $m = 1$. We then have, $\vec{k}_i \cdot \vec{a}_i = 2\pi/n_i$. Recall $\vec{k}_i$ and $\vec{a}_i$ are parallel. We see $||\vec{k}_i|| = 2\pi/(||\vec{a}_i||n_i)$ where $n_i \in [1, N_i]$. We see now, $\vec{k}_i$ could take N_i different values.

The arguement above should be repeated in all three dimensions, in result $\sum_i \vec{k}_i$ could take $\prod_i N_i$ different results which is also the number of lattice points in real space.

2 Problem 2