Final Exam Problem Bank

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Contents

1 Purely Linear Algebra

1.1 MAT 310 II

1.1.1 P2: How to take determinant

(15pts) Let

$$A = \begin{pmatrix} 4 & 3 & 1 & 2 \\ 1 & 9 & 0 & 2 \\ 8 & 3 & 2 & -2 \\ 4 & 3 & 1 & 1 \end{pmatrix}$$

1. Calculate the determinant of A using any method that you know.
2. What is the determinant of $-2A$?

1.1.2 P3: Determinant and Invertibility

(13pts) Let A and B be $n \times n$ matrices such that $AB = -BA$. Prove that if n is odd, then A or B is not invertible.

1.1.3 P4: Diagonalizing a matrix

(13pts) Determine if the following matrix is diagonalizable and justify your answer. If so, find an invertible matrix Q and a diagonal matrix D such that $A = QDQ^{-1}$.

$$A = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

1.2 MAT 310 I

1.2.1 P3: Basis

(15 pts) Determine whether or not $\{(1,1,0),(2,0,-1),(-3,1,1)\}$ is a basis for $\mathbb{R}^3$.

1.2.2 P4: Rank Nullity Theorem

(15pts) Let $T:\mathbb{R}^4 \to \mathbb{R}^3$ denote a linear transformation such that $T((1,0,0,0)) = (3,-1,0), T((1,1,1,1)) = (-2,1,3)$, and $T((0,0,1,1)) = (0,1,1)$. Compute the dimension of the null space $\dim(N(T))$. Hint:$dim(R^4) = rank(T) + null(T)$

1.3 NYU Final

1.3.1 P5: Rank Nullity Theorem

1. Suppose A is an 8×7 matrix in which $\dim(\operatorname{Nul} A) = 6$. Then rank $A =$

(a) 1 (b) 2 (c) 6 (d) 7 (e) 8

1.3.2 P7: Invertibility

Consider the 3×3 matrix $A = \begin{bmatrix} 0 & 1 & k \\ 2 & k & -6 \\ 2 & 7 & 4 \end{bmatrix}$. For what values of k is matrix A invertible?

(a) $k \in \mathbb{R}$ (b) all real k except 2 and 5 (c) $k \geq 0$ (d) $k = 0$ (e) no value of k makes A invertible

1.3.3 P8: Definition of Eigenvector

$\begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}$ is an eigenvector of $\begin{bmatrix} 4 & -2 & 1 \\ 2 & 0 & 1 \\ 2 & -2 & 3 \end{bmatrix}$. What is the corresponding eigenvalue? (a) 0 (b) 2 (c) 3 (d) 7 (e) need more information to determine eigenvalue associated with eigenvector $\begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}$

1.3.4 P13: Exponentiating a matrix

(10 points) Compute A^{2017} where $A = \begin{bmatrix} 1 & 0 \\ 2 & -1 \end{bmatrix}$. (Hint: Diagonalize A.)

2 General

2.1 Sakurai

2.2 Likharev

- 2.1
- 4.4
- 4.9
- 4.19

3 Quantum Information

3.1 Bacon Final

- P1
- P2
- P3
- P4

3.2 MIT P3

- P1
- P4
- P5

3.3 ETH P3

- Shor's code

4 Atomic Physics

4.1 The Quantum Mechanics Solver

5 Condensed Matter Physics

- Provided by Prof. Liu