

Floquet Code

From Honeycomb Code to Toric Code

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Outline

Motivations

Static Code

Floquetification

Conclusion

Appendix

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Benefit of Floquet Code

Low-weight Measurement

- As a result, good threshold?

Low Connectivity Requirement

- Give visualization

Floquet Code's Stats

Floquet Code has good qualities

- **Threshold** of 0.2% – 0.3% without native **weight-2 measurement** [1]¹
- Threshold of 1.5% – 2.0% with native weight-measurements [1]
- Photon loss threshold: 6.4% on photonic platform [2]
- Code Overhead: $\lim_{n \rightarrow \infty} \frac{k}{n} \rightarrow \frac{1}{2}$ on qudit codes [3]
- $5.6 \times$ fewer physical qubits are needed to implement Floquet code at depolarizing noise of 0.1% compare to surface code [4]

¹0.5% – 0.7% for surface code (which surface code was this? which error model?)

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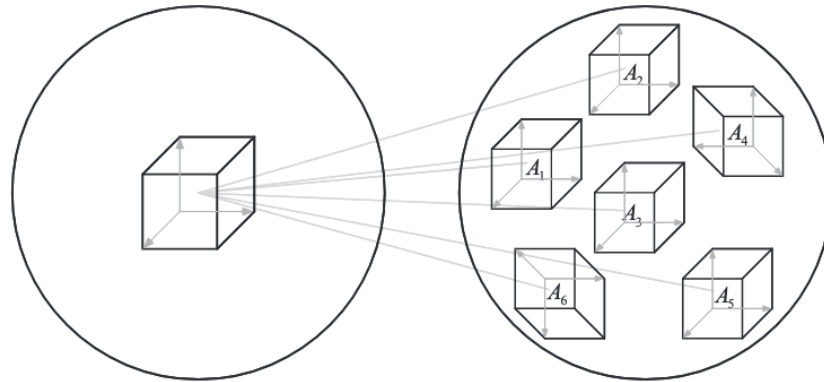
Conclusion

Appendix

Stabilizer Code

Example: $[[4,2,2]]$ Code

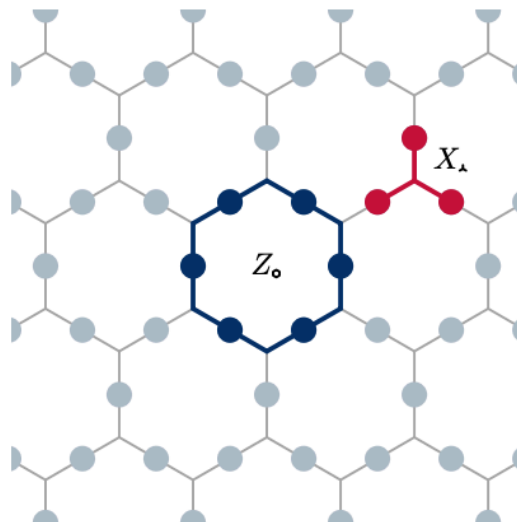
- Stabilizers are product of Pauli operators on qubits: $X_1 X_2 X_3 X_4$ and $Z_1 Z_2 Z_3 Z_4$
- Measurement result of stabilizers signals whether you have an error



- Logical Operators commutes with stabilizers but cannot be generated by them: $\tilde{X}_1 = X_1 X_2$, $\tilde{X}_2 = X_1 X_3$, $\tilde{Z}_1 = Z_1 Z_3$ and $\tilde{Z}_2 = Z_1 Z_2$

Stabilizer Code

Example: Toric Code



1

¹Kott, Viktor, et al. "Quantum robustness of the toric code in a parallel field on the honeycomb and triangular lattice." arXiv preprint arXiv:2402.15389 (2024).

Subsystem Code

Example: $[[4,1,2]]$ Code

- **Checks:** $X_1 X_3$, $X_2 X_4$, $Z_1 Z_2$, $Z_3 Z_4$
- **Not necessarily commute** with each other
- Generated group has center $X_1 X_2 X_3 X_4$ and $Z_1 Z_2 Z_3 Z_4$.
- Logical operators commutes with all **checks**: $X_1 X_2$, and $Z_1 Z_3$

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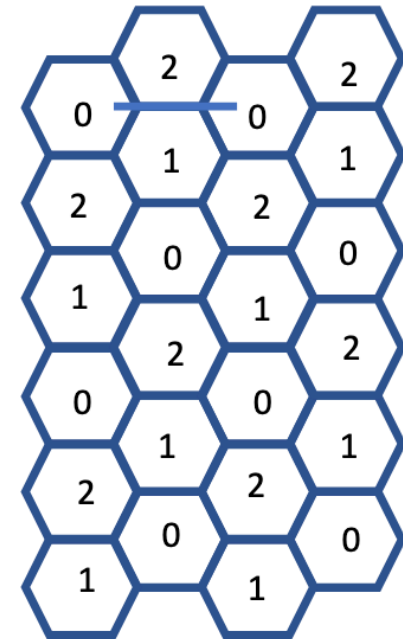
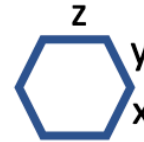
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Appendix

Honeycomb Code:

Definition:

1. Qubits on vertices of lattice
2. Each edge associated with a check
3. Each plaquette associated with a type 0,1,2
4. Each edge associated with a type 0,1,2
5. Measurement sequence according to edge type

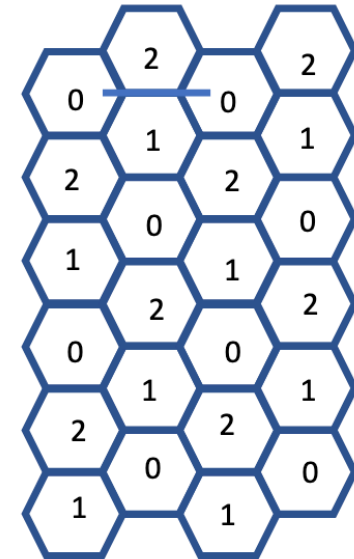
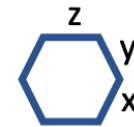


Honeycomb Code:

Check Measurement gives rise to instantaneous stabilizer groups

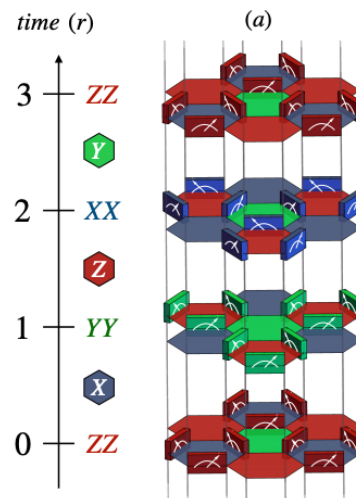
Given a state stabilized by \mathcal{S} : a group generated by Pauli String operators, projective measurement of Pauli String operators P modifies the stabilizer group of the state as

1. if $P \in \mathcal{S}$ or $-P \in \mathcal{S}$, ISG remains
2. if $P \notin \mathcal{S}$ and $-P \notin \mathcal{S}$
 1. P commutes with all of \mathcal{S} , include $\pm P$ in ISG depending on the measurement result
 2. P commutes with all of $\mathcal{S}_0 \subset \mathcal{S}$, ISG is $\mathcal{S}_0 \cup \pm P$



Honeycomb Code:

Measurement Visualized



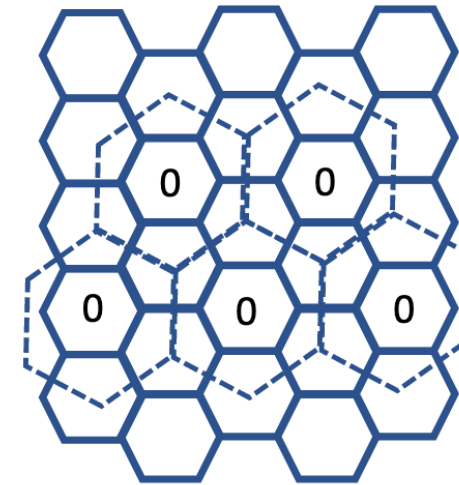
1

¹Zhu, Guo-Yi, and Simon Trebst. “Qubit fractionalization and emergent Majorana liquid in the honeycomb Floquet code induced by coherent errors and weak measurements.” arXiv preprint arXiv:2311.08450 (2023).

Honeycomb Code:

Embedded Toric Code

- ISG at step $r \geq 3$ contains all hexagons and $r \bmod 3$ edge checks.
- Each edge check halves the degree of freedom of qubits on the edge.
- Effectively, the code is a **lattice of hexagons** with **embedded toric code** on each hexagon.



[5]

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Appendix

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- Honeycomb Code on a hexagonal lattice is “equivalent” to Toric Code on a hexagonal superlattice
- Floquet code has comparable quality as surface code but requires lower connectivity on hardware

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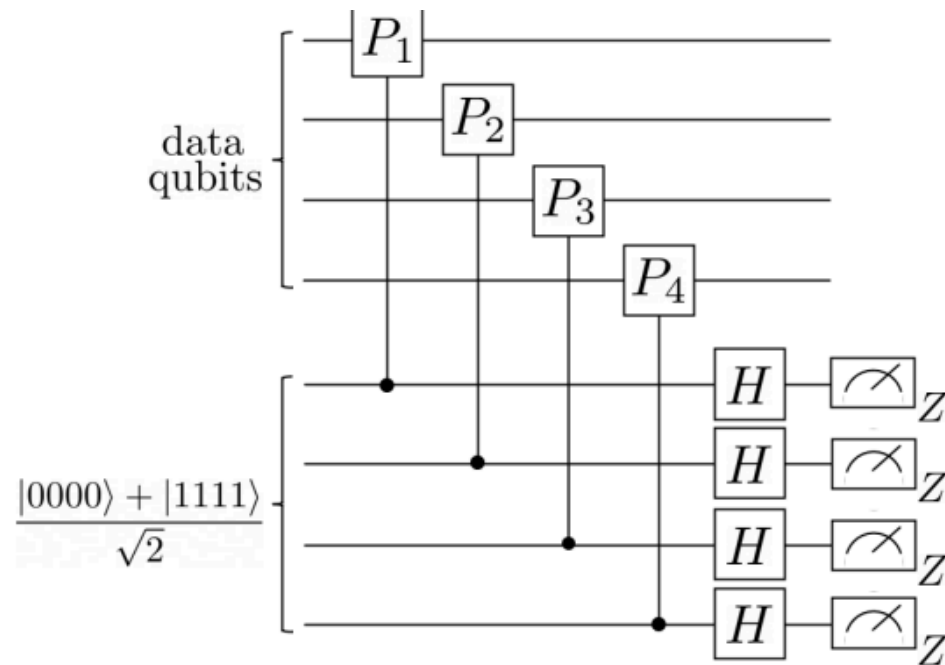
Appendix

Terms

- “The **teraquop footprint** is the number of physical qubits required to create a logical qubit reliable enough to survive one trillion operations.”

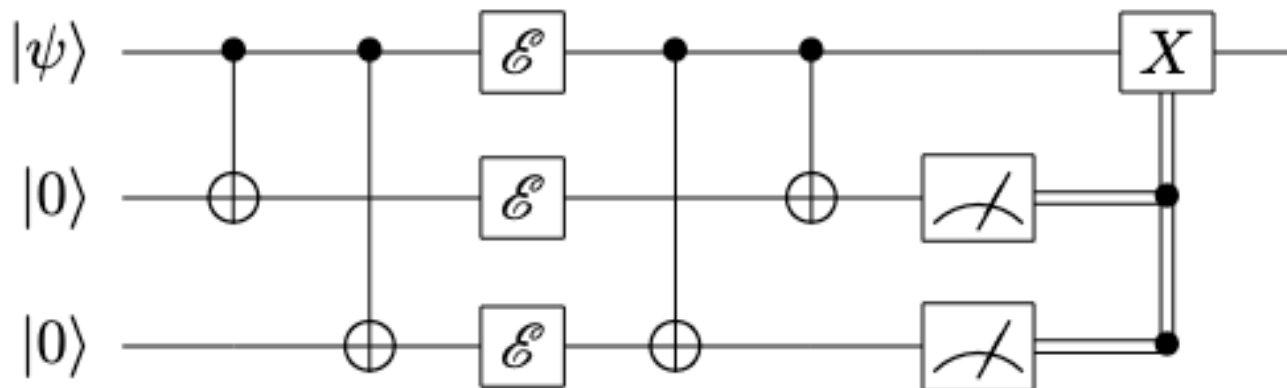
Terms

Static Code: Shor-style Measurement



Terms

Repetition Code: Encoding, Syndrome Extraction, and Error Correction



References

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1. Gidney, C., Newman, M., Fowler, A., Broughton, M.: A Fault-Tolerant Honeycomb Memory. Quantum. 5, 605–606 (2021). <https://doi.org/10.22331/q-2021-12-20-605>
2. Hilaire, P., Dessertaine, T., Bourdoncle, B., Denys, A., Gliniasty, G. de, Valentí-Rojas, G., Mansfield, S.: Enhanced Fault-tolerance in Photonic Quantum Computing: Floquet Code Outperforms Surface Code in Tailored Architecture, <http://arxiv.org/abs/2410.07065>
3. Tanggara, A., Gu, M., Bharti, K.: Simple Construction of Qudit Floquet Codes on a Family of Lattices, <http://arxiv.org/abs/2410.02022>
4. Higgott, O., Breuckmann, N. P.: Constructions and Performance of Hyperbolic and Semi-Hyperbolic Floquet Codes, <http://arxiv.org/abs/2308.03750>

References

5. Hastings, M. B., Haah, J.: Dynamically Generated Logical Qubits. *Quantum*. 5, 564–565 (2021). <https://doi.org/10.22331/q-2021-10-19-564>