Floquet Code

Establishing Connection between Floquet Code and Toric Code

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Outline

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Floquet Code needs more attention

- Study of Fault-tolerant quantum computation: **Quantum Memory** and logical operations
- Design of quantum memory concerns the following properties of a **Quantum Error Correction Code** [1]

  a) Code distance
  b) Ease of implementing logical gates
  c) Tradeoffs between the number of logical qubits and distance

- Surface code is not optimal by standard a) and c) [2] but has higher threshold in practice [3] due to **low-weight measurement** (Figure 7) and **lower connectivity** hardware requirements compare to many families of qLDPC codes [4, 5]
- Floquet code is a family of codes that pushes these strength of surface code even further [6]

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Floquet Code has good qualities

- **Threshold** of $0.2\% - 0.3\%$ without native **weight-2 measurement** [6][1]
- Thershold of $1.5\% - 2.0\%$ with native weight-measurements [6]
- Photon loss threshold: 6.4% on photonic platform [7]
- Code Overhead: $\lim_{n\to\infty} \frac{k}{n} \to \frac{1}{2}$ on qudit codes [8]
- $5.6\times$ fewer physical qubits are needed to implement Floquet code at depolarizing noise of 0.1% compare to surface code [4]

[1] $0.5\% - 0.7\%$ for surface code

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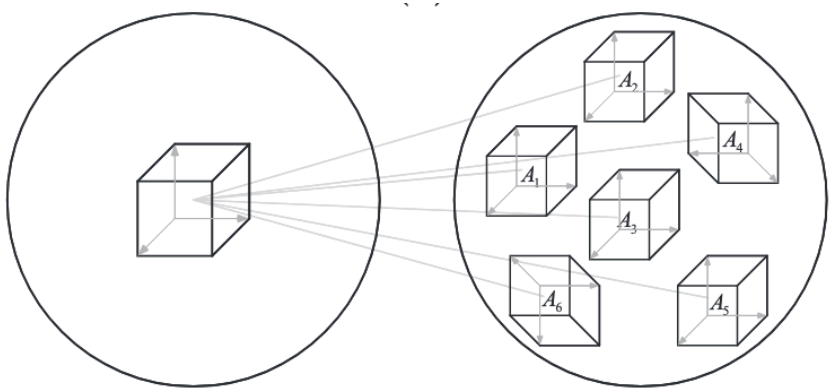
Stabilizer Code

Example: [[4,2,2]] Code

- Stabilizers are product of Pauli operators on qubits: $X_1X_2X_3X_4$ and $Z_1Z_2Z_3Z_4$
- Measurement result of stabilizers signals whether you have an error



- Logical Operators commutes with stabilizers but cannot be generated by them: $\widetilde{X}_1 = X_1X_2$, $\widetilde{X}_2 = X_1X_3$, $\widetilde{Z}_1 = Z_1Z_3$ and $\widetilde{Z}_2 = Z_1Z_2$

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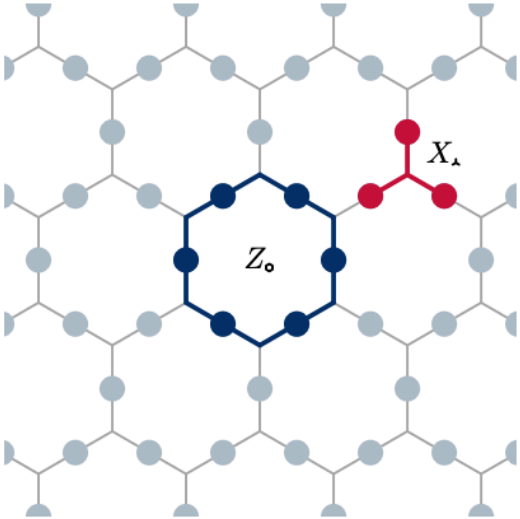
Stabilizer Code

Example: Toric Code



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[1]Kott, Viktor, et al. "Quantum robustness of the toric code in a parallel field on the honeycomb and triangular lattice." arXiv preprint arXiv:2402.15389 (2024).

Subsystem Code

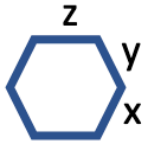
Example: [[4,1,2]] Code

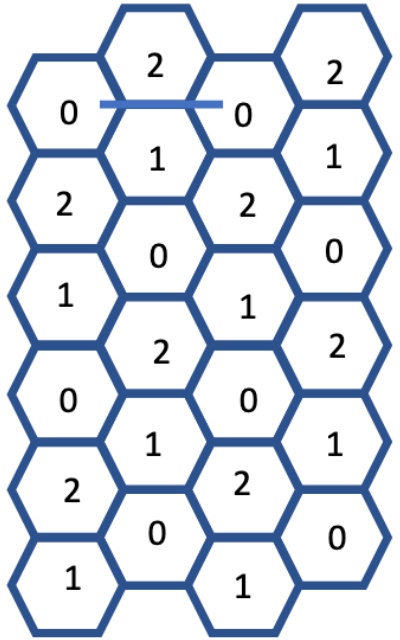
- **Checks**: X_1X_3, X_2X_4, Z_1Z_2, Z_3Z_4
- **Not necessarily commute** with each other
- Generated group has center $X_1X_2X_3X_4$ and $Z_1Z_2Z_3Z_4$.
- Logical operators commutes with all **checks**: X_1X_2, and Z_1Z_3

Honeycomb Code:

Definition:

1. Qubits on vertices of lattice
2. Each edge associated with a check
3. Each plaquette associated with a type 0,1,2
4. Each edge associated with a type 0,1,2
5. Measurement sequence according to edge type





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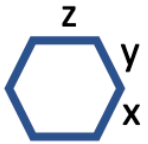
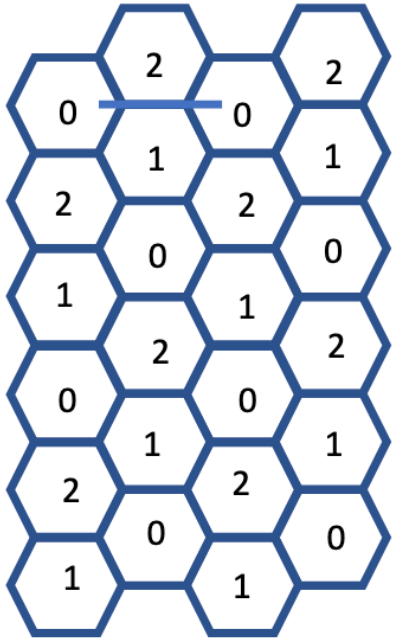
Honeycomb Code:

Check Measurement gives rise to instantaneous stabilizer groups

Given a state stabilized by $\mathcal{S}$: a group generated by Pauli String operators, projective measurement of Pauli String operators P modifies the stabilizer group of the state as

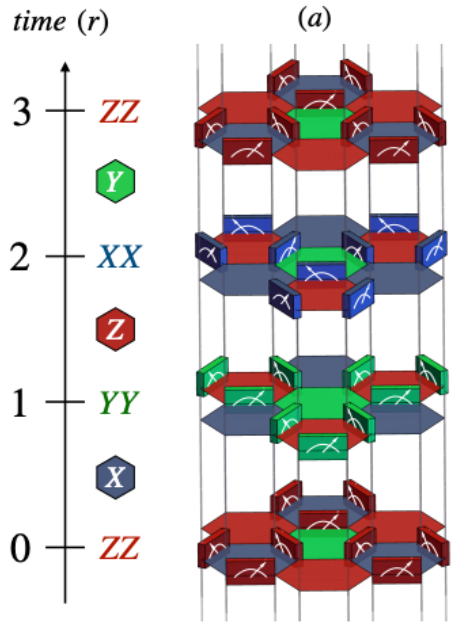
1. if $P \in \mathcal{S}$ or $-P \in \mathcal{S}$, ISG remains
2. if $P \notin \mathcal{S}$ and $-P \notin \mathcal{S}$
   1. P commutes with all of $\mathcal{S}$, include $\pm P$ in ISG depending on the measurement result
   2. P commutes with all of $\mathcal{S}_0 \subset \mathcal{S}$, ISG is $\mathcal{S}_0 \cup \pm P$





Honeycomb Code:

Measurement Visualized



Honeycomb Code:

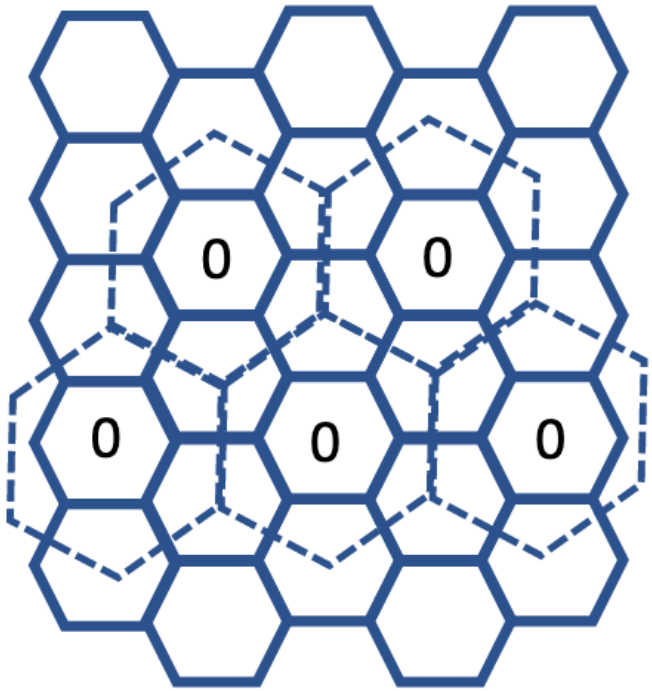
1

Embeded Toric Code

[1]Zhu, Guo-Yi, and Simon Trebst. "Qubit fractionalization and emergent Majorana liquid in the honeycomb Floquet code induced by coherent errors and weak measurements." arXiv preprint arXiv:2311.08450 (2023).

Honeycomb Code:

- ISG at step $r \geq 3$ contains all hexagons and $r \bmod 3$ edge checks.
- Each edge check halfs the degree of freedom of qubits on the edge.
- Effectively, the code is a **lattice of hexagons** with **embedded toric code** on each hexagon.



Honeycomb Code:

[9]

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Conclusion

- Honeycomb Code on a hexagonal lattice is “equivalent” to Toric Code on a hexagonal superlattice
- Floquet code has comparable quality as surface code but requires lower connectivity on hardware

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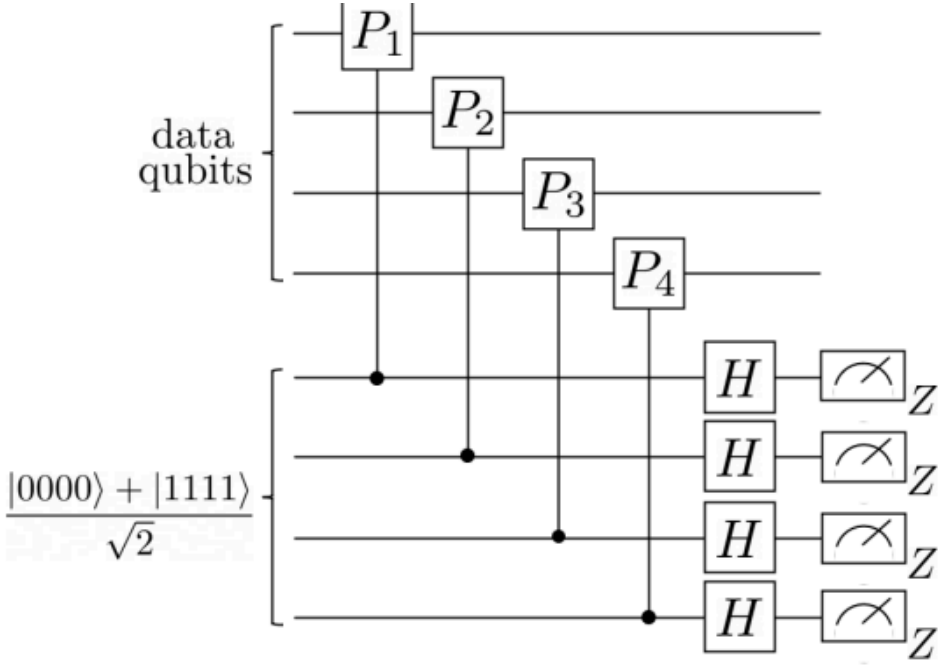
Terms

- “The **teraquop footprint** is the number of physical qubits required to create a logical qubit reliable enough to survive one trillion operations."

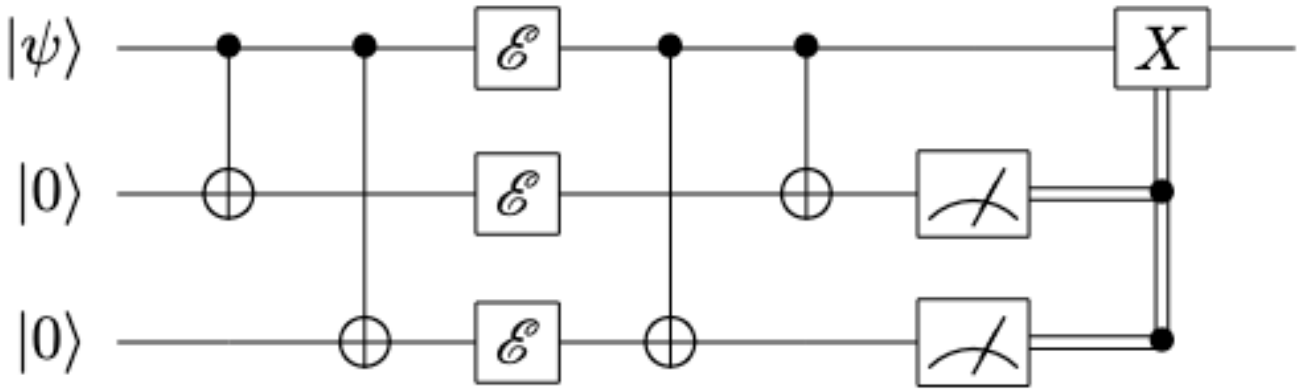
Terms

Static Code: Shor-style Measurement



Terms

Repetition Code: Encoding, Syndrome Extraction, and Error Correction



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Bibliography

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