Recursive token distribution in continuous time

Objective:

Reward programs typically rely on either a fixed emission schedule over time or on a predetermined distribution function rule. However, these approaches are usually ineffective in attracting a diverse user base and often leads to a concentration of rewards among a few select wallets. To address this issue, we are experimenting with the idea of an adaptive model at Exactly. The model implements an incentive function based on how closely or far the program's objectives are being met during the rewards period.

Scheme:

The proposed model is comprised of two distinct modules: the distribution module and the allocation module. A) The distribution module is responsible for determining the quantity of tokens to be allocated during each time frame (as determined by two consecutive protocol transactions). B) The allocation module is responsible for determining the proportion of tokens to be allocated to each class of protocol user in each specific period.

A) Distribution module

Definition:

$$
\begin{aligned}
N_{tg} : &\quad \rightarrow \quad \text{target volume for loans} \\
TotRew : &\quad \rightarrow \quad \text{total available rewards for distribution} \\
TauRew : &\quad \rightarrow \quad \text{reward distribution period}
\end{aligned}
$$

Reward natural minting rate per unit time:

$$
\mu = \begin{cases} \frac{TotRew}{TauRew} & , \quad t \leq TauRew \\ 0 & , \quad t > TauRew \end{cases}
$$

Definition:

$$
\begin{aligned}
X_t : &\quad \rightarrow \quad \text{cumulative distributed rewards up to time } t/ \\
Y_t : &\quad \rightarrow \quad \text{cumulative undistributed rewards up to time } t/
\end{aligned}
$$

A) Distribution module

Loans to target ratio at any time:

$$\gamma(t) = \min\left(\frac{totLoans(t)}{N_{tg}}, 1\right)$$

Note: For every time interval (t_a,t_b) between two transactions, we can consider γ to be approximately constant

Moving law (evolution over time):

$$\begin{aligned} dX_t &= \gamma\left(\mu + \omega\frac{Y}{\tau_{Rew}}\right)dt \\ dY_t &= \mu dt - dX_t \end{aligned}$$

At any given interval (t_a,t_b), the system does not distribute the fixed amount $\mu(t_b-t_a)$, but only the fraction $\gamma(t_a)$ of it. Furthermore, a specific proportion of rewards not being previously distributed is also allocated.

A) Distribution module

Solving for Y_t

$$\int_{Y_a}^{Y_b} \frac{dY_t}{\left(1 - \frac{\gamma\omega}{(1-\gamma)\mu\tau_{Rew}} Y\right)} = \int_{t_a}^{t_b} (1-\gamma)\,\mu dt$$

$$Y_b = \frac{(1-\gamma)\,\mu\tau_{Rew}}{\gamma\omega} - \left[\frac{(1-\gamma)\,\mu\tau_{Rew}}{\gamma\omega} - Y_a\right] \exp\left[-\frac{\omega\gamma}{\tau_{Rew}}(t_b - t_a)\right]$$

$$\Delta Y_{ba} = Y_b - Y_a = \left[\frac{(1-\gamma)\,\mu\tau_{Rew}}{\gamma\omega} - Y_a\right]\left(1 - \exp\left[-\frac{\omega\gamma}{\tau_{Rew}}(t_b - t_a)\right]\right)$$

Solving for X_t

$$\int_{X_a}^{X_b} dX_t = \int_{t_a}^{t_b} \mu dt - \int_{Y_a}^{Y_b} dY_t$$

$$\Delta X_{ba} = X_b - X_a = \mu\,(t_b - t_a) - \left[\frac{(1-\gamma)\,\mu\tau_{Rew}}{\gamma\omega} - Y_a\right]\left(1 - \exp\left[-\frac{\omega\gamma}{\tau_{Rew}}(t_b - t_a)\right]\right)$$

A) Distribution module - Calculation Flow

- Set the initial value $$Y_a = 0$$

- Compute

$$\Delta Y_{ba} = \frac{(1-\gamma)\,\mu\tau_{Rew}}{\gamma\omega}\left(1-\exp\left[-\frac{\omega\gamma}{\tau_{Rew}}(t_b-t_a)\right]\right) - Y_a\left(1-\exp\left[-\frac{\omega\gamma}{\tau_{Rew}}(t_b-t_a)\right]\right)$$

$$Y_b = Y_a + \Delta Y_{ba}$$

- Period distributed rewards per unit asset:

$$\boxed{\frac{\Delta X_{ba}}{totSupply} = \frac{\mu\,(t_b-t_a) - Y_b + Y_a}{totSupply}}$$

where: *totSupply = totLoans + totDeposits*

- Prepare for next iteration $$Y_a = Y_b$$

B) Allocation module

Objective:

Once we know the incentives to be distributed during a certain period, it is necessary to define how they will be allocated among the different users. The allocation scheme aims to direct these incentives according to the needs of the protocol, based on the following criteria:

- To Reward borrowers (key players in any lending/deposit model) by reducing their effective cost of debt
- To offer attractive yields to yield farmers.
- In cases where utilization levels are very high, to concentrate rewards on depositors as a means of restoring equilibrium.

- Borrower incentive rule:

$$BI(U) = \left[\varpi_B R(U)\left(1-(1-\delta)U\right)+\mu_1\right]\left(1-sigmoid(U)\right)$$

- Depositor incentive rule:

$$DI(U) = \mu_2\left(1-sigmoid(U)\right)+\mu_3 sigmoid(U)$$

where $\mu 1$, $\mu 2$ and $\mu 3$ are free parameters

B) Allocation module

- Sigmoid function in terms of U:

$$sigmoid\left(U\right) = \frac{1}{1 + \left[\frac{(1-U)\alpha}{U(1-\alpha)}\right]^{k}}$$

$$\begin{aligned} x &= \ln\left(\frac{U}{1-U}\right), \\ x_0 &= \ln\left(\frac{\alpha}{1-\alpha}\right) \end{aligned}$$

$$sigmoid\left(x\right) = \frac{1}{1 + \exp\left[-k\left(x - x_0\right)\right]}$$

- Weight definition:

for borrowers:

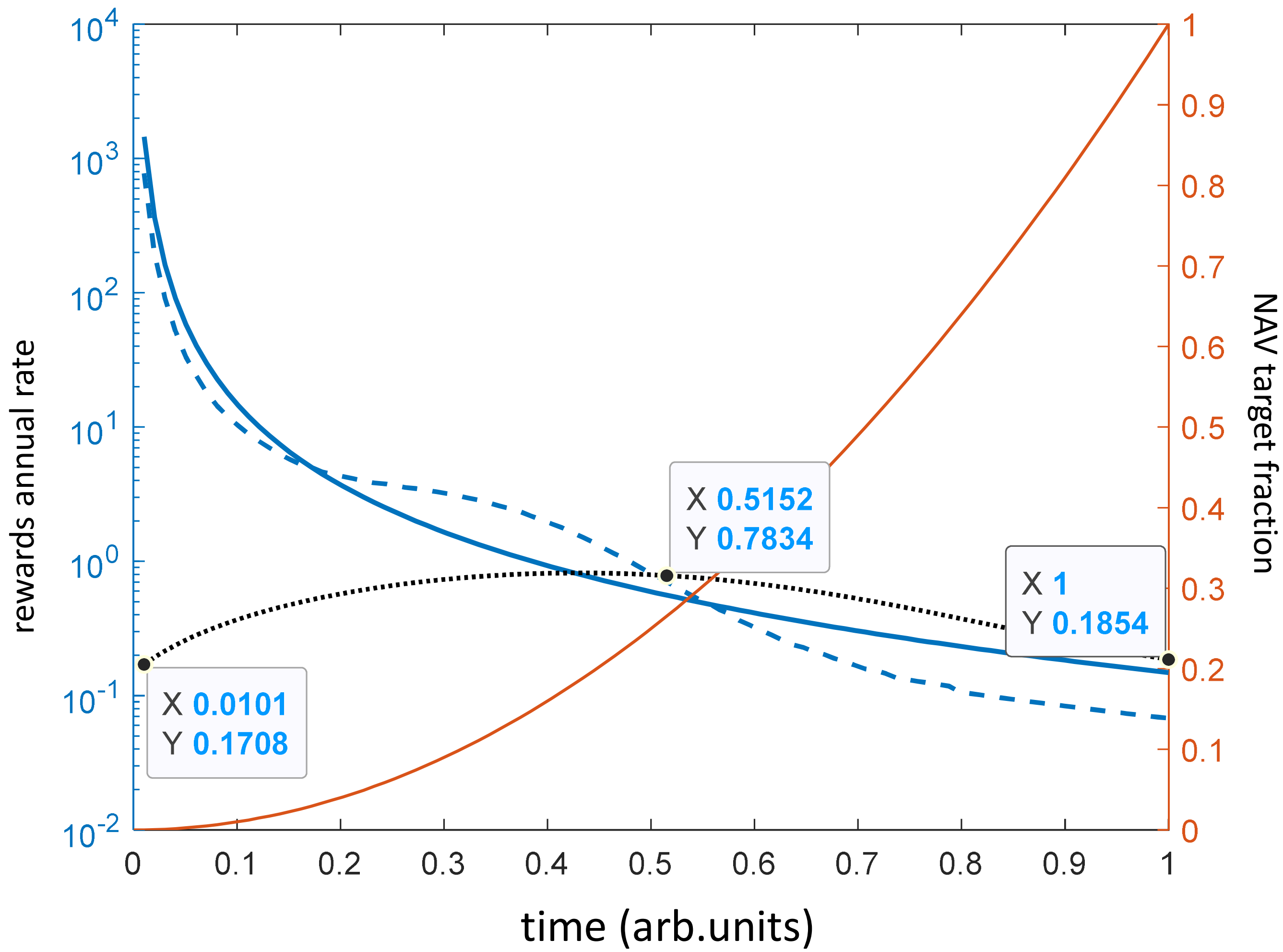
$$\theta_B\left(U\right) = \frac{BI\left(U\right)}{BI\left(U\right) + DI\left(U\right)}$$

for depositors:

$$\theta_D\left(U\right) = \frac{DI\left(U\right)}{BI\left(U\right) + DI\left(U\right)} = 1 - \theta_B\left(U\right)$$

Exapmple 1: Reward effective rate under a quadratic growth adoption

Annualized rate (left axis) a function of the target fraction for a) uniform dist. (solid blue), b) Weibull dist. (dashed blue) and c) adaptive scheme (dotted black). Left axis in log scale.



Exapmple 2: Allocation effect

Rewards rate (G1), net return rate for depositors and yield farmers (G2) and net return rate for borrowers (G3)