CMM3 Group Design Project

Project Title: "Hot Stuff, Cool Savings: The 400mm Wall Optimization"

Group Number: Group 13

GitHub Repository URL: [exactxacto/cmm3-house-heating: Computational Methods and Modelling 3 group project]

Group Member Contributions

NAME	STUDENT ID	MAIN CONTRIBUTIONS
Christina Fotou	S2329017	- Organised weekly meetings. - Researched and formulated thermal modelling and solar irradiance data - Refined project scope - Edited and formatted the final report and cover page - Completed documentation
Carmen Alonso Lidon	S2513717	- Conducted cost–performance analysis - Integrated team code into final model - Developed root-finding algorithm - Completed the report cover page
Manas Rayadurg	S2484606	- Developed solar irradiance simulation and ODE model - Implemented effective temperature calculations and ODE solver - Created and maintained GitHub repository
Cailean Johnson	S2532763	- Processed temperature data - Implemented interpolation and R-value variation modelling - Contributed to debugging - LaTeX report outline
Haziq Wan Huzaini	S2478600	- General debugging - Assisted in developing initial Fourier-series interpolation code.

Summary of Group Work

Our group primarily used Microsoft Teams for communication and GitHub for version control and tracking code progress. Our weekly Thursday morning meetings were organised and scheduled by Christina. Coding responsibilities were divided based on each member's assigned tasks, while most team members contributed to the project's modelling work.

Tasks were distributed organically according to the solutions required within the project scope. Challenges that arose during development were discussed in our meetings, where we collaboratively addressed issues, refined our approaches, and adjusted the project model when necessary. Collaboration was ongoing, with members supporting each other on individual tasks as needed.

No. of commits:

Christina:8	Carmen: 21	Manas: 48	Cailean: 15	Haziq: 5

Tools used for planning or editing:

- Microsoft Teams
- Spyder
- VS Code

Specific pairings /review examples:

- Code reviews**:** Peer reviewed each other's functions and/or scripts for errors, clarity, or efficiency during the weekly meetings as part of our agenda.
- Model validation: Members compared their independent results to check for consistency.
- Data verification: Cross-checking datasets, interpolations, or numerical outputs for accuracy.
- Report review: Team members reviewed each other's sections for clarity, formatting, and alignment with project objectives.

Notes on how workflow evolved during the project:

The workflow evolved from initial independent exploration of the project scope and tools to a more coordinated process supported by GitHub version control. As the project progressed, task allocation became clearly defined, and collaboration increased through joint debugging, code reviews, and validation of model components. Weekly meetings shifted from broad planning to focused integration activities, culminating in a final phase dedicated to merging contributions, resolving conflicts, refining outputs, and conducting iterative reviews to ensure consistency across all models and documentation.

Generative AI Use Declaration

AI Statement - "Academic integrity is an underlying principle of research and academic practice. All submitted work is expected to be your own. AI tools (e.g., ELM) should not be used to generate content for this assessment. However, you are allowed to use these tools to identify ideas, key themes, and plan your assessment. You may also use it to improve the clarity of your writing. If you use AI software, you must acknowledge its use in your submission."

AI models used:

- ChatGPT (plus)
- Google Gemini

Please state the purpose of use:

- Brainstorming/outlining
- Concept clarification
- Code debugging
- Report structure and formatting

Contents

1 Introduction

Ask any student in Edinburgh about the negatives of studying in the city, and cold flats in winter will almost always be mentioned. With many tenements being over a hundred years old, poor insulation and limited winter sunlight often cause indoor temperatures to drop well below comfortable levels. This creates a clear engineering problem: understanding how modern insulation affects indoor conditions over a full year.

This project aims to model the internal temperature of a typical Edinburgh student room to determine how different insulation material thicknesses influence comfort. With different insulation materials having different levels of performance at varying costs, we aim to find the optimal combination of materials and thicknesses that maximise hours between the 12–18 °C band at the lowest possible cost

1.1 Project Overview

Indoor temperatures of 12–18°C are generally considered comfortable, yet many Edinburgh flats fall far below this range in winter. The model predicts the room's annual temperature profile and evaluates how increasing insulation thickness (up to 0.4 m) improves comfort relative to cost. The outcome identifies the most cost-effective insulation configuration for a representative tenement room.

2 System Overview

2.1 Physical System Model

The representative room used in this report is modelled as shown in Figure 1, with dimensions based on UK national minimum standards for a one-bedroom flat [1].

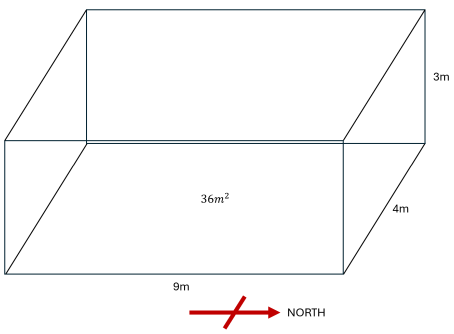

Figure 1: Model of house

For simplicity, the furniture and room contents are treated as a uniformly distributed thermal mass with a total heat capacity of 1500 kJ/K [2], while the air heat capacity is considered separately.

Typical Edinburgh tenements have a narrow frontage and deeper plan, and their north–south orientation limits sunlight on the north face. The model assumes this orientation when calculating solar gains.

The external wall insulation thickness is varied up to a maximum of 0.4 m, consistent with the upper limit of what the Scottish Government defines as a "well-insulated" home [3].

2.2 Key Inputs, Outputs, and Physical Principles

Inputs

- Annual ambient temperature
- Solar irradiance and heat gains
- Wall layers and insulation thickness
- Material thermal properties

Outputs

- Indoor temperature profile
- Annual comfort hours (12–18°C)
- Insulation cost and comfort–cost trade-off
- Optimal insulation thickness

Physical principles

- Heat conduction through multilayer walls
- Solar heat gain on the external surface
- Energy balance on room air and thermal mass
- Transient temperature response via an ODE model

3 Mathematical and Numerical Methods

This section outlines the thermal model governing the room system and the numerical methods used for temperature reconstruction, heat-flow simulation, and optimisation of insulation performance.

3.1 Simplified Model of the System

To keep the analysis tractable, the room is modelled as a single, well-mixed thermal zone with a uniform indoor temperature $T_{\text{interior}}(t)$. The total thermal mass is represented by a lumped heat capacity combining the air within the room and the furniture. The air heat capacity is calculated using

$$C_{\text{air}} = \rho V_{\text{room}} c_{\text{air}}, \tag{1}$$

and combined with the furniture heat capacity ($C_{\text{room}} = 1500\,\text{kJ/K}$) to obtain

$$C_{\text{tot}} = C_{\text{air}} + C_{\text{room}}. \tag{2}$$

Heat transfer with the outdoors is assumed to occur only through one representative external wall. The conductive resistance of this wall varies depending on the insulation materials and thicknesses, and is given by

$$R_{\text{wall}} = \frac{R_{\text{val}}}{A_{\text{wall}}}, \tag{3}$$

where $A_{\text{wall}} = 12\,\text{m}^2$. This resistance forms the dominant thermal pathway to the outside environment.

Modelling Assumptions

- Indoor air is well-mixed, resulting in a uniform interior temperature.
- Heat transfer through wall is one-dimensional.
- Ventilation, infiltration, and internal gains are neglected.
- Solar radiation affects the system only through its contribution to the external wall temperature.
- A single wall is used to represent heat exchange with the exterior.

These assumptions maintain computational efficiency while preserving key physical behaviour needed for comparing different insulation designs.

3.2 Solar Irradiance and Effective Outdoor Temperature

Solar irradiance warms the exterior wall surface and is incorporated into the model as an equivalent heat gain. The solar heat flux incident on the wall is described by

$$Q_{\text{solar}}(t) = \eta\, \alpha\, A_{\text{wall}}\, I_{\text{global}}(t), \tag{4}$$

where $\eta = 0.07$ represents the fraction of absorbed heat that effectively contributes to the indoor system within the time-step [4], $\alpha = 0.6$ is the absorptance of sandstone [5], and $I_{\text{global}}(t)$ is the hourly global irradiance. This formulation captures the fact that only a portion of the absorbed solar energy meaningfully influences indoor temperatures, while the remainder is dissipated through outdoor convection and re-radiation.

The absorbed solar flux raises the exterior wall surface temperature above ambient. A convective boundary condition relates this flux to the outer surface temperature:

$$q'' = h_o \left(T_{s,\text{outer}} - T_\infty\right), \tag{5}$$

where h_o is the outside convective heat-transfer coefficient and T_∞ is the ambient air temperature. Writing $Q = q''A$ and solving for the surface temperature yields

$$T_{s,\text{outer}} = T_\infty + \frac{Q_{\text{solar}}}{h_o A}. \tag{6}$$

The term $\frac{1}{h_o A}$ represents the *outside convective resistance*, which is the resistance to heat flow between the moving outdoor air and the solid wall surface. This surface temperature rise is what ultimately drives conductive heat flow across the insulation layer toward the interior.

To avoid explicitly modelling transient heat diffusion through the outermost wall layers, this effect is incorporated through an *effective outdoor temperature*. Introducing the outside convective resistance $R_{\text{conv,out}} = 1/(h_o A)$ and similarly defining an inside convective resistance $R_{\text{conv,in}}$, the effective boundary condition becomes

$$T_{\text{out,eff}} = T_\infty + Q_{\text{solar}}\, R_{\text{conv,out}}. \tag{7}$$

This effective temperature captures the combined influence of solar heating and outdoor convection on the external wall surface. It enables the model to include radiative heat gains without requiring a full multilayer transient conduction model, significantly reducing computational cost while retaining the dominant physics governing solar-driven temperature fluctuations.

3.3 Interpolation of Ambient Temperature Data

Ambient temperature data from the MIDAS dataset is provided in discrete hourly measurements and must be converted into a continuous function for the ODE solver [6]. A Discrete Fourier Transform (DFT) is used to decompose the signal into sinusoidal components:

$$X[k] = \sum_{n=0}^{N-1} x[n]\, e^{-i2\pi kn/N}. \tag{8}$$

A truncated Fourier series reconstructs a smooth annual temperature profile. This approach preserves the main seasonal and daily cycles while suppressing high-frequency noise, producing a continuous function $T(t)$ evaluated during ODE integration.

3.4 Numerical Solution of the Heat-Flow ODE

Heat transfer from the outside to the inside is modelled using a first-order energy balance of the form

$$\frac{dT_{\text{interior}}}{dt} = \frac{T_{\text{out,eff}} - T_{\text{interior}}}{R_{\text{tot}}\, C_{\text{tot}}}, \tag{9}$$

where the total thermal resistance is

$$R_{\text{tot}} = R_{\text{wall}} + R_{\text{conv,in}} + R_{\text{conv,out}}. \tag{10}$$

As this is a first-order ODE, a single initial condition is required. In this model, the simulation is started at an initial indoor temperature T_0.

The equation is solved numerically using the RK45 implementation of the Runge–Kutta family in `scipy.solve_ivp`. RK45 combines fourth- and fifth-order estimates to achieve good accuracy with adaptive timestepping. Since the governing ODE is smooth and non-stiff, RK45 provides an effective balance of stability, speed, and computational efficiency.

3.5 Insulation Performance Interpolation and Root-Finding

After solving the heat-flow ODE for each insulation configuration, the resulting comfort-hour data forms a discrete set of points that must be converted into a continuous relationship with thermal resistance. To achieve this, a one-dimensional interpolation function,

$$\text{Comfort} = f(R), \tag{11}$$

was created using the `scipy.interpolate.interp1d` library. A linear interpolation scheme was selected for numerical stability, as cubic interpolation proved unsuitable due to the non-smooth nature of the simulation data.

With this continuous representation, the required insulation level for a specified comfort target can be determined by solving

$$f(R) - \text{TargetComfort} = 0, \tag{12}$$

where the Brentq algorithm from `scipy.optimize.b` is used to locate the root. Brentq is a robust bracketing method that guarantees convergence provided the function crosses the target value within the chosen interval, allowing the model to directly compute the R-value needed to satisfy a specified design criterion (e.g. 2460 comfortable hours).

4 Design Analysis

4.1 Model Overview

Diagram 2 gives an overview of the model and code architecture, showing how data flows from weather inputs and material properties through to indoor temperature and comfort-hour analysis.

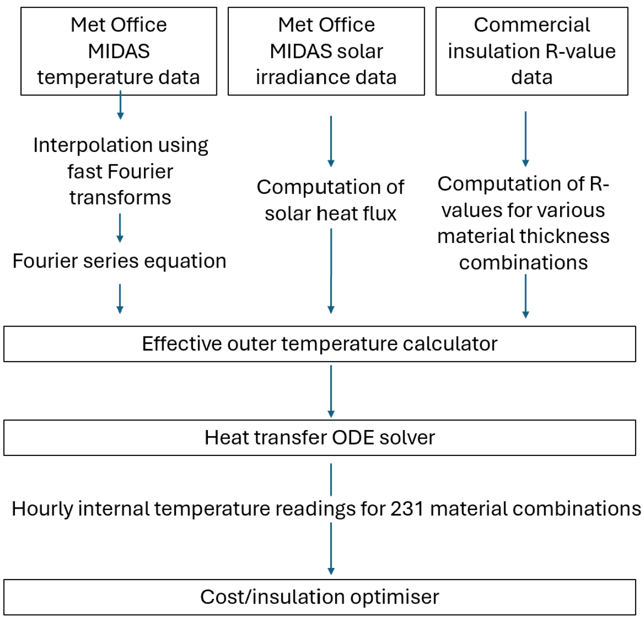

Figure 2: Overview of model and code architecture.

4.2 Temperature Data

Meteorological data were obtained from the UK Met Office Integrated Data Archive System (MIDAS) [6] for the Edinburgh Gogarbank weather station. The dataset provided hourly measurements of ambient temperature and global solar irradiance for the year 2023.

Raw CSV files were downloaded from the MIDAS datasheet and parsed into a clean format. The code automatically extracted temperature (°C) and solar irradiance (W/m^2) columns, resolved incomplete entries, and converted timestamps to a consistent format using `pandas`. Following data preparation, two summary plots were produced to visualise trends throughout the year.

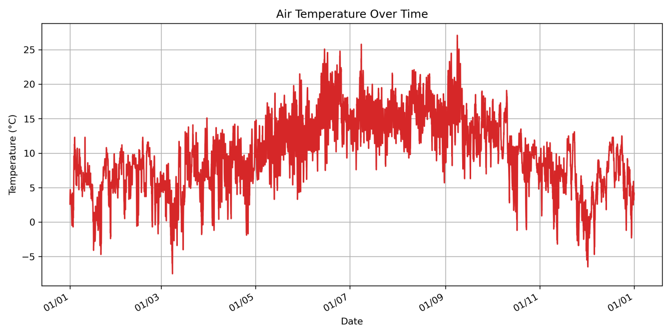

Figure 3: Hourly ambient temperature throughout 2023 from the MIDAS Edinburgh dataset.

4.3 Insulation – Variation in R

A Python-based model was developed to evaluate how the thermal resistance R varies with different insulation materials, layer combinations, and thicknesses. Material properties and conductivities k were loaded from input files and stored in a structured database for easy comparison.

For each material or layer assembly, resistance is calculated using

$$R = \frac{t}{k}, \tag{13}$$

where t is the thickness in metres and k is the thermal conductivity in W m^{-1} K^{-1}. For multilayer walls, resistances are summed in series. The program then performs automated sweeps across all valid combinations, generating the full dataset of R-values and associated outputs.

4.4 Interpolation – Temperature

This section outlines the process used to transform the Edinburgh MIDAS temperature dataset into a symbolic equation suitable for analytical modelling and integration with the project's ODE solvers. The aim is to replace discrete hourly measurements with a continuous temperature function so that the simulation can use realistic climatic inputs.

4.4.1 Frequency Analysis using DFT

The cleaned temperature data are converted from the time domain to the frequency domain using a Discrete Fourier Transform (DFT) implemented via the built-in `numpy` function. This identifies all cyclic frequencies present in the temperature dataset; the resulting amplitude–frequency spectrum provides a view of how strongly each frequency contributes to the overall temperature variation.

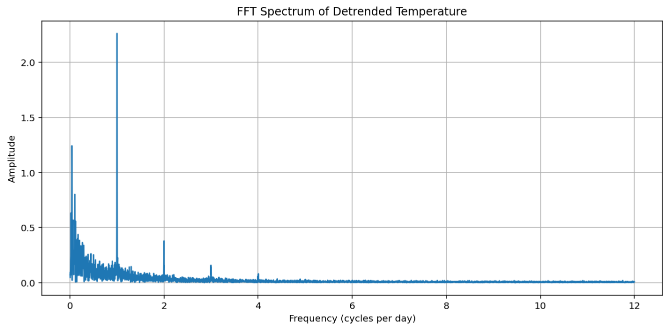

Figure 4: Frequency spectrum of the temperature signal obtained using DFT.

4.4.2 Fourier Reconstruction of Temperature Signal

Using a selected number of frequencies identified in the DFT, the program reconstructs a simplified temperature curve by summing cosine and sine terms with the corresponding amplitudes, phases, and angular frequencies. This captures the dominant behaviour of the measured data while reducing noise and data volume. The symbolic approximation is given by:

$$\begin{aligned} f(t) \approx A_0 + \sum_{k=1}^{k/2-1} A_k \cos(k\omega_0 t) \\ + \sum_{k=1}^{k/2-1} B_k \sin(k\omega_0 t) + A_{k/2} \cos\left(\tfrac{k}{2}\omega_0 t\right), \end{aligned} \tag{14}$$

where A_0 is the mean value (taken to be zero), and A_k and B_k are the DFT-derived coefficients. Including more terms improves accuracy but increases computation time; therefore, a reduced set of 150–300 terms was used to balance efficiency and precision.

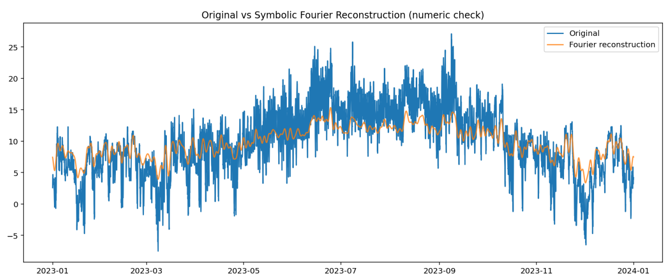

Figure 5: Original and reconstructed temperature signal using a truncated Fourier series.

4.4.3 Computational Cost vs Accuracy

An analysis of computational cost versus accuracy was performed by varying the number of terms used in the Fourier reconstruction. The Root Mean Square Error (RMSE) between the reconstructed series and the original temperature data is defined as

$$\mathrm{RMSE} = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(y_i - \hat{y}_i)^2}, \tag{15}$$

where y_i are the original temperature values and $\hat{y}_i$ are the reconstructed values.

The resulting RMSE values for different k_{max} are shown in Table 1.

Table 1: Root Mean Square Error (RMSE) comparison for Fourier approximation.

$\mathbf{k}_{\mathbf{max}}$	**RMSE (°C)**
50	3.55763
100	3.39197
300	3.23747
500	2.76434
1000	2.70171

It was determined that a k_{max} of 300 provided the best balance between computational cost and accuracy, and was therefore used for the final analysis in this report.

4.5 Heat Flow Results

Two Python scripts, `v2_simulate_solar_irradiance` and `solve_ODE`, are used to solve the heat transfer ODE outlined in Section 3.4 and obtain T_{interior} for each configuration.

4.5.1 Effective External Temperature Calculator

Thermal properties such as the heat capacity of air and values for $R_{\mathrm{conv,out}}$ and $R_{\mathrm{conv,in}}$ are calculated as outlined in Section 3.2. The values for h_o and h_i were taken to be 35 and 8 $\mathrm{W\,m^{-2}\,K^{-1}}$, respectively [7], accounting for the difference between typical outdoor and indoor air velocities. These elements are combined in the function `calculate_t_out_eff`, which applies the heat addition method described previously to compute an hourly effective external temperature for each of the 231 material combinations over the course of a year.

4.5.2 ODE Solver

The second script calculates C_{tot} as described in Section 3.1. This, along with the 231 values of R_{tot}, is then used as input to the ODE defined in Section 3.4. The function `simulate` solves the ODE by sampling the Fourier series temperature function hourly for 8760 data points (the number of hours in a year) and integrating to obtain `t_hourly`, the timestamps for the data points, and `T_hourly`, the corresponding indoor temperatures across all 231 material combinations.

Figure 6 shows the outdoor and indoor temperatures for three representative material combinations over the course of a year.

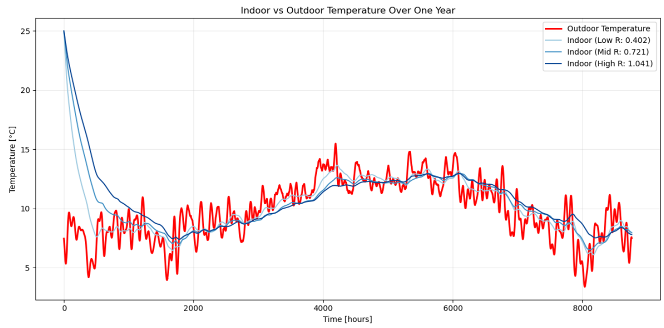

Figure 6: Indoor vs outdoor temperature for three representative insulation configurations with $T_0 = 25°C$.

5 Insulation Optimisation and Design Recommendation

This chapter identifies the most cost-effective insulation design based on thermal, comfort, and cost results.

5.1 Cost–Performance Optimisation

To identify the optimal insulation design, the total cost of each averaged combination was plotted against its resulting R-value, as shown in Figure 7. This plot illustrates the diminishing returns associated with increasing thermal resistance: although higher R-values provide more comfortable hours, the marginal gains decrease significantly at the upper end of the range.

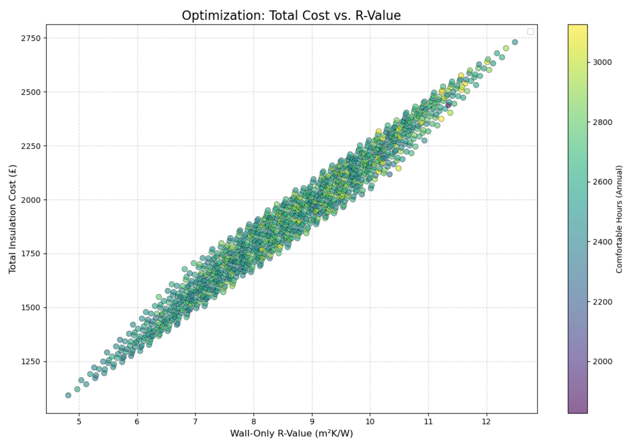

Figure 7: Cost vs. R-value, with the optimal trade-off point marked by a red star.

To select a single "best" design, a knee-finding algorithm was applied to the cost–performance curve. This provides an objective means of identifying the point at which further investment yields disproportionately small gains in comfort hours. The optimal trade-off point was found to be:

- **Optimal R-value:** 8.909 m^2K/W
- **Total cost:** £1843.92
- **Comfortable hours:** 2802 hours/year
- **Material composition (0.4 m total):**
  - 15.0 cm Expanded Polystyrene (EPS)
  - 9.4 cm Mineral Wool
  - 5.0 cm Polyurethane (PUR)

This configuration is approximately 40% cheaper than the most expensive combination (£4680), yet achieves the highest comfort-hour count. It therefore represents the best value among all simulated material designs.

5.2 Simulation Results Comparison

To illustrate the practical impact of insulation performance, indoor temperatures for the best, median, and worst configurations were plotted over the same 72-hour period. Figure 8 demonstrates that the "Best Comfort" configuration (R = 16.0, 100% PUR) maintains substantially more stable indoor temperatures than the "Worst Comfort" configuration (R = 11.1, 100% Mineral Wool), which shows pronounced oscillations. The median case behaves as expected between the two extremes.

These comparisons reinforce the physical effect of increasing thermal resistance: thicker or higher-performance insulation leads to reduced temperature volatility and improved indoor comfort.

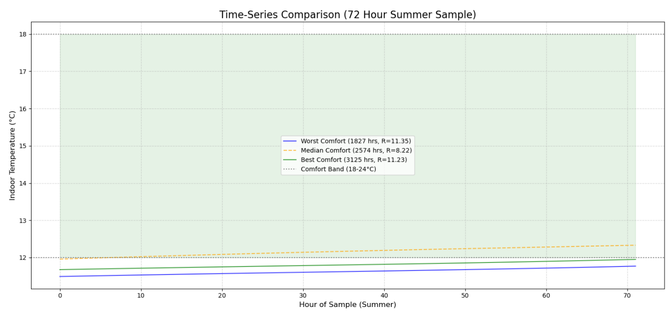

Figure 8: Simulated indoor temperatures for the best, median, and worst R-value combinations across a 72-hour summer period.

5.3 Design Target Solution (Root-Finding)

In addition to identifying an optimal configuration, the model can solve for the insulation level required to meet a specified performance target. Using the interpolated comfort-hour function and the `brentq` root-finding algorithm described in Section 3.5, the design target of 2460 comfortable hours was evaluated.

The calculated solution is:

- **Target R-value:** $5.444\,\mathrm{m^2K/W}$
- **Interpolated cost:** £1206.69

Figure 9 validates this result, showing the interpolated comfort-hour curve intersecting the 2460-hour target at the expected R-value. This demonstrates that the model can be used not only to identify optimal designs but also to support user-defined performance specifications.

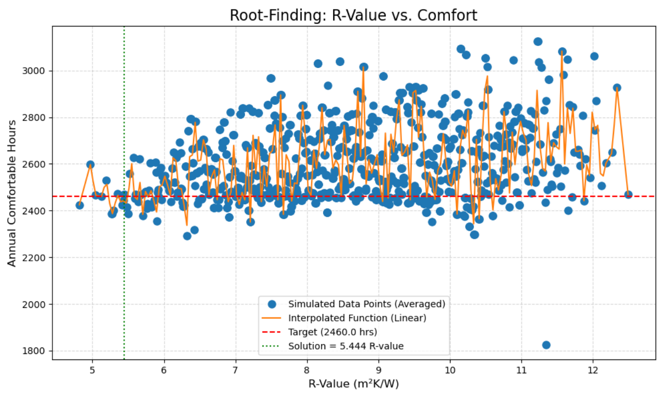

Figure 9: Root-finding validation plot showing the solution for 2460 comfortable hours.

5.4 Validation

To verify the optimisation results, several validation checks were performed.

5.4.1 Trend Validation (Thermal Stability)

Physical principles dictate that increasing thermal resistance reduces heat transfer. This is confirmed in Figure 8, where the highest-R configuration exhibits the smallest indoor temperature swings. This agreement with expected behaviour validates the model's thermal physics.

5.4.2 Cost Model Sanity Check

The relatively high cost of the optimal configuration aligns with real-world pricing for high-performance insulation materials. Standard domestic insulation is thinner (100–270 mm) and typically uses low-cost mineral wool, whereas the present model specifies a 400 mm fixed depth and includes premium polyurethane foam. The resulting unit cost (approximately £42/m^2) is therefore consistent with market expectations.

5.4.3 Numerical Validation

The accuracy of the root-finding solution is further supported by the clear intersection shown in Figure 9, confirming the consistency between the simulation data, interpolation function, and numerical solver.

6 Conclusion

This project developed a computational model to assess the thermal performance and cost of 1000 insulation combinations for a 0.4 m wall. Using real weather data, Fourier-based temperature reconstruction, and an ODE heat-flow model, indoor temperatures and annual comfort hours were computed for every design.

The analysis identified an optimal configuration with an R-value of $8.909\,\mathrm{m^2K/W}$ at a cost of £1843.92, delivering 2802 comfort hours and offering the best comfort–cost trade-off. Interpolation and root-finding methods also showed that the model can determine the insulation level required to meet any chosen performance target.

Key findings show that increasing R-value improves thermal stability but with diminishing returns relative to cost. With additional time or data, the model could be extended by incorporating ventilation, internal gains, more detailed solar modelling, and real indoor temperature measurements to further strengthen validation and design accuracy.

References

[1] Ministry of Housing, Communities and Local Government, "Technical housing standards – nationally described space standard." https://www.gov.uk/government/publications/technical-housing-standards-nationally-described-space-standard/technical-housing-standards-nationally-described-space-standard, aug 24 2021. [Online; accessed 2025-11-13].

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[7] "Understanding convective heat transfer: Coefficients, formulas examples," *Engineering Toolbox*, aug 28 2003. [Online; accessed 2025-11-17].