

Introduction

- **Research question:**
To what extent does anthropogenic forcing increase the frequency, intensity, and duration of extreme heat events?
- **Method:** Compare CMIP6 climate simulations with and without anthropogenic forcing using space-time extreme value theory.
- **What do we focus on?**
Quantifying changes in heat wave characteristics over Europe.

Datasets

- **Observational data:** Daily maximum temperature from the Berkeley Earth dataset [2] ($1^\circ \times 1^\circ$ resolution, 1880–present), combining over 30 000 weather stations with spatial reconstruction for exploratory analysis and clustering.
- **Model simulations:** CMIP6 climate models under **historical** (with human forcing, Factual) and **natural** (without human forcing, Counterfactual) runs (1850–2020).

Clustering region

- How: Partitioning around medoids.
- Distance based on pairwise tail dependence coefficient [3,4]:

$$d(\mathbf{s}_l, \mathbf{s}_k | u) = \frac{1 - \chi_{\mathbf{s}_l, \mathbf{s}_k, t_q, t_r}(u)}{2(3 - \chi_{\mathbf{s}_l, \mathbf{s}_k, t_q, t_r}(u))}$$

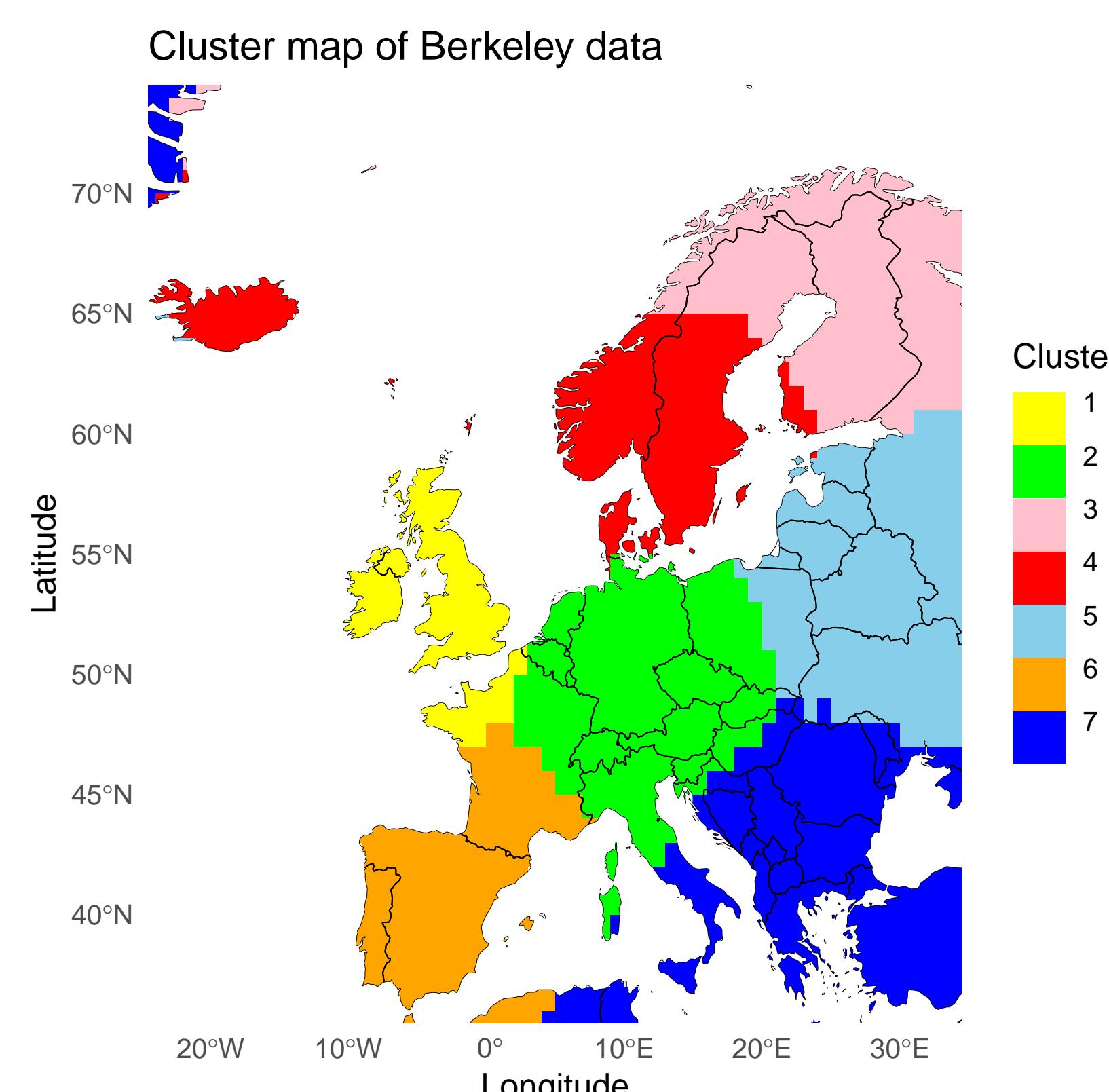


Figure 1. Cluster map of summer (JJA) temperature anomalies from the Berkeley dataset. The seven spatial clusters were identified using the partitioning around medoids (PAM) algorithm applied to a distance based on pairwise tail dependence.

Marginal model

The marginal distribution of each location $i \in \{1, \dots, n_j\}$ in a given cluster $j \in \{1, \dots, 7\}$ is modelled with a nonstationary generalized extreme value distribution:

$$G_{i,j}(z) = \exp \left\{ - \left[1 + \xi_{i,j} \left(\frac{z - \mu_{i,j}(t)}{\sigma_{i,j}} \right) \right]_+^{-1/\xi_{i,j}} \right\}$$

$$\mu_{i,j}(t) = \mu_{0,i,j} + \mu_{1,i,j}\text{GMST}(t) + \mu_{2,i,j}\text{ENSO3.4}(t) + \mu_{3,i,j}\text{NAO}(t)$$

where GMST(t) is the annual Global Mean Surface Temperature anomaly, ENSO3.4(t) is the July-centered 5-month running mean Niño 3.4 anomaly, and NAO(t) is the standardized summer (JJA) North Atlantic Oscillation index.

We define the parameter vector for location i in cluster j as follows:

$$\boldsymbol{\beta}_{i,j} := (\mu_{0,i,j}, \mu_{1,i,j}, \mu_{2,i,j}, \mu_{3,i,j}, \sigma_{i,j}, \xi_{i,j})^\top \in \mathbb{R}^6$$

We penalize variation across neighboring locations. Let $\boldsymbol{\gamma}_{p,j} \in \mathbb{R}^{n_j}$ be the vector of the p -th parameter across all n_j locations in cluster j .

We estimate the parameters by minimizing a penalized negative log-likelihood [5,6,7]:

$$\hat{\boldsymbol{\beta}}_j = \arg \min_{\boldsymbol{\beta}_j} \left[-\log \mathcal{L}(\boldsymbol{\beta}_j) + \sum_{p=1}^4 \lambda_{j,p} \boldsymbol{\gamma}_{j,p}^\top \mathbf{S}_j \boldsymbol{\gamma}_{j,p} \right]$$

where $\mathbf{S}_j \in \mathbb{R}^{n_j \times n_j}$ is an adjacency matrix:

$$s_{lk} = \begin{cases} |\mathcal{N}_l| & \text{if } l = k \\ -1 & \text{if } l \text{ and } k \text{ are neighbors} \\ 0 & \text{otherwise} \end{cases}$$

and \mathcal{N}_l denotes the set of neighbors of location l .

Spatio-temporal modeling

We model extremal dependence via the copula of a latent process [8,9] that is applied on all daily temperatures:

$$Z(\mathbf{s}, t) = R(t)^\delta W(\mathbf{s}, t)^{1-\delta}, \quad \delta \in [0, 1]$$

- $R(t)$: IID standard unit-Pareto
- $W(\mathbf{s}, t) = \frac{1}{1-\Phi(W^*(\mathbf{s}, t))}$: space-time process
- $W^*(\mathbf{s}, t)$: zero-mean Gaussian field with separable covariance

$$C((\mathbf{s}_l, t_q), (\mathbf{s}_k, t_r)) = C_s(\mathbf{s}_l, \mathbf{s}_k)C_t(t_q, t_r)$$

Spatial covariance (Spatially nonstationary, anisotropic) [10]:

$$C_s(\mathbf{s}_l, \mathbf{s}_k) = \frac{\left(\sqrt{2}\theta_l^{1/4}\theta_k^{1/4}/(\theta_l + \theta_k)^{1/2} \right)^2}{1 + \left(\frac{\|A(\mathbf{s}_l - \mathbf{s}_k)\|}{\theta_l + \theta_k} \right)^2}$$

- $\theta_i \sim \text{Gamma}(\alpha, \tau)$: random local range
- A : anisotropy matrix

$$A = \begin{pmatrix} \cos \omega & -\sin \omega \\ \sin \omega & \cos \omega \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & \rho \end{pmatrix}$$

Temporal covariance [11]:

$$C_t(t_q, t_r) = \frac{2^{1-\nu}}{\Gamma(\nu)} \left(\frac{\sqrt{2\nu}|t_q - t_r|}{\kappa} \right)^\nu K_\nu \left(\frac{\sqrt{2\nu}|t_q - t_r|}{\kappa} \right)$$

- Likelihood-free parameter estimation with neural Bayes estimators [12].

Preliminary results on CanESM5 model [13] cluster 1

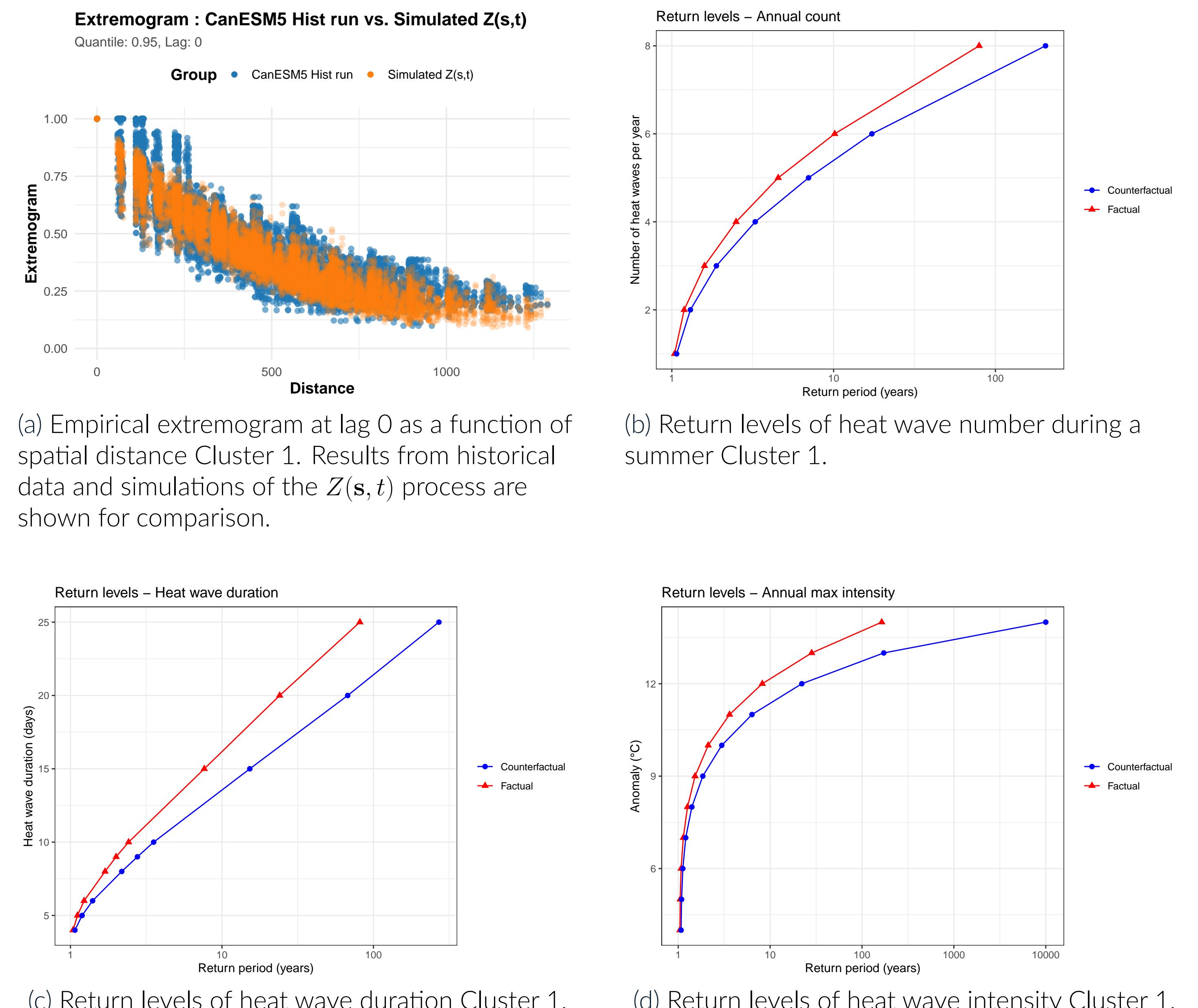


Figure 2. Preliminary results on CanESM5 model cluster 1. Historical (Factual) and natural (Counterfactual) scenarios are shown for comparison.

References

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