

Assessing Extreme Droughts: An Extreme Value Theory Analysis of Common Drought Indices



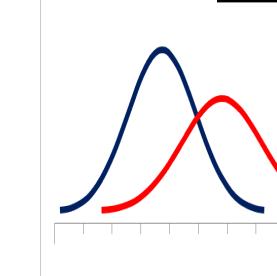
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EXALT

Why this work?

As part of the EXALT Project, this work focuses on **attributing extreme droughts** using **Extreme Value Theory**, leveraging its tools and limit theorems to extrapolate beyond observed data for rare, extreme events.

Here is shown the starting point of this research which has been the implementation of **Peaks-over-Threshold method** or **Pareto-tails modelling** to analyse the distribution and behaviour of extreme droughts events over Europe.

Data & Indices – What is a drought?

- E-OBS data (0.25° resolution) from **1950 to 2024**. From raw data we defined **P3** as **mean precipitation [mm/day] accumulated over 3 months** to capture drought duration. For each location l and month m we define:
- **Index D3**: by subtracting the climatological mean of P3 and dividing by its climatological standard deviation, mapping **P3** to its **standard version**.

$$D3(m, l) \stackrel{\text{def}}{=} (P3(m, l) - \mu^{clim}[P3](m, l)) / \sigma^{clim}[P3](m, l)$$

- **Index SPI3**: obtained by **fitting a Gamma distribution** to non-zero precipitation, adjusting for the probability of zeros, and **applying a Gaussian quantile transformation**.

$$SPI3(m, l) \stackrel{\text{def}}{=} \Phi^{-1}\{p_0 + (1 - p_0)\Gamma(P3(m, l))\}$$

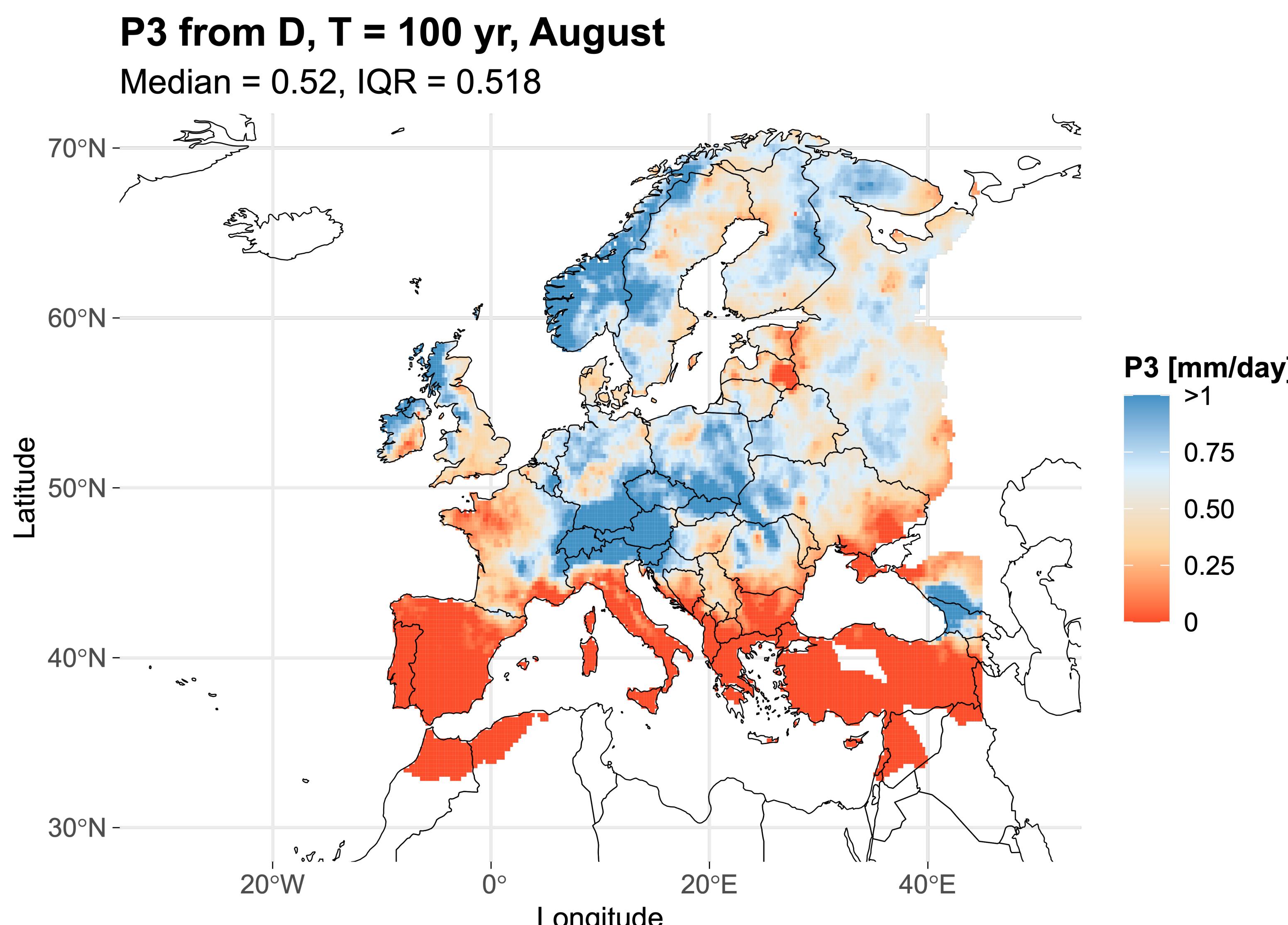


Figure 1. Return level of mean precipitation [mm/day] obtained inverting $D3$ to $P3$. Fitting of the **GPD at 5%**.

Methods – Peaks-over-threshold

- GPD fitted to the **negated $D3$ series (minima)**, with **5% threshold** and MLE per grid point.
- **Declustering**: contiguous negative-valued months (or those separated by **up to 2 over-threshold months**) are treated as single events.
- We compute **fitted quantiles** from models (GPD for $-D3$, Gaussian for $SPI3$) and **reverse the normalization process** to express them in physical precipitation units, using **monthly mean and standard deviation for $D3$** (Figure 1), and the **inverse Gamma mapping for $SPI3$** .

Difference $P3_D - P3_{SPI}$, $T = 100$ yr, August

Median = -0.047, IQR = 0.132

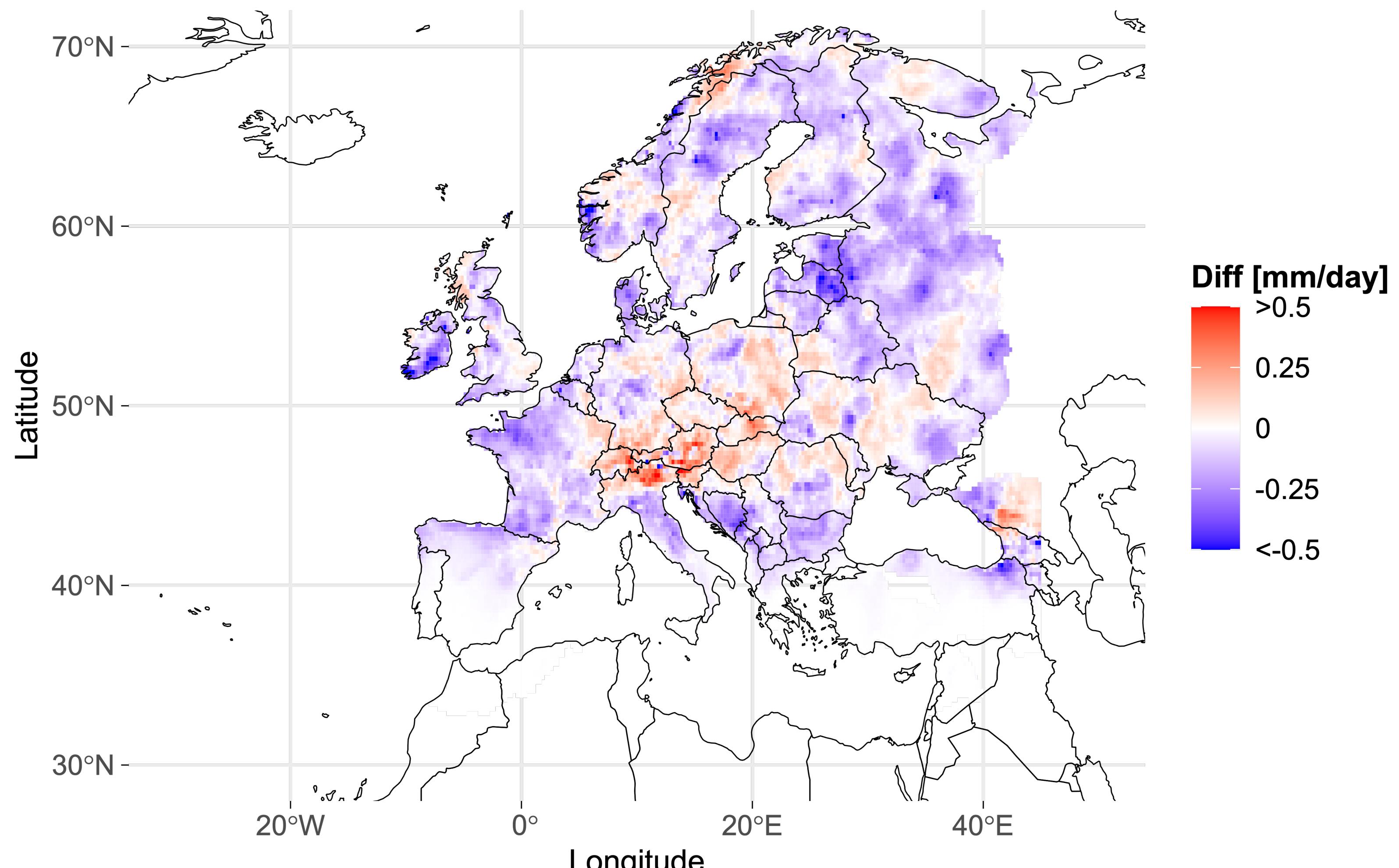


Figure 2. Difference between the return levels obtained inverting $D3$ to $P3$ and the ones obtained inverting $SPI3$ to $P3$.

Results – What is the difference?

- Because of its mathematical definition, SPI fixes the tails and is incompatible with EVT.
- Divergence between $D3$ and $SPI3$ return levels (Figure 3), due to different tail models (GPD vs Gaussian). **SPI underestimates** drought severity (lack of precipitation) **compared to D**. A similar behaviour can be seen over the all Europe as the map of Figure 2 is mostly blue → $P3(D3) < P3(SPI3)$.
- **WHAT IS NEXT?**
 - Use more advanced model to consider **spatial correlations and clustering**.
 - More data...** we need longer time span to test the long perspective differences.

References

- Burke, E. J., Perry, R. H., & Brown, S. J. (2010). An extreme value analysis of UK drought and projections of change in the future. *Journal of Hydrology*, 388 (1-2), 131–143.
- Coles, S. (2001). An introduction to statistical modeling of extreme values. Springer-Verlag London.
- Cornes, R., van der Schrier, G., van den Besselaar, E.J.M., Jones, P., (2018): An Ensemble Version of the E-OBS Temperature and Precipitation Datasets, *J. Geophys. Res. Atmos.*, 123.

Return Level Curves, $D3$ vs $SPI3$ – August

P3 return levels from D and SPI (inverse-transformed)

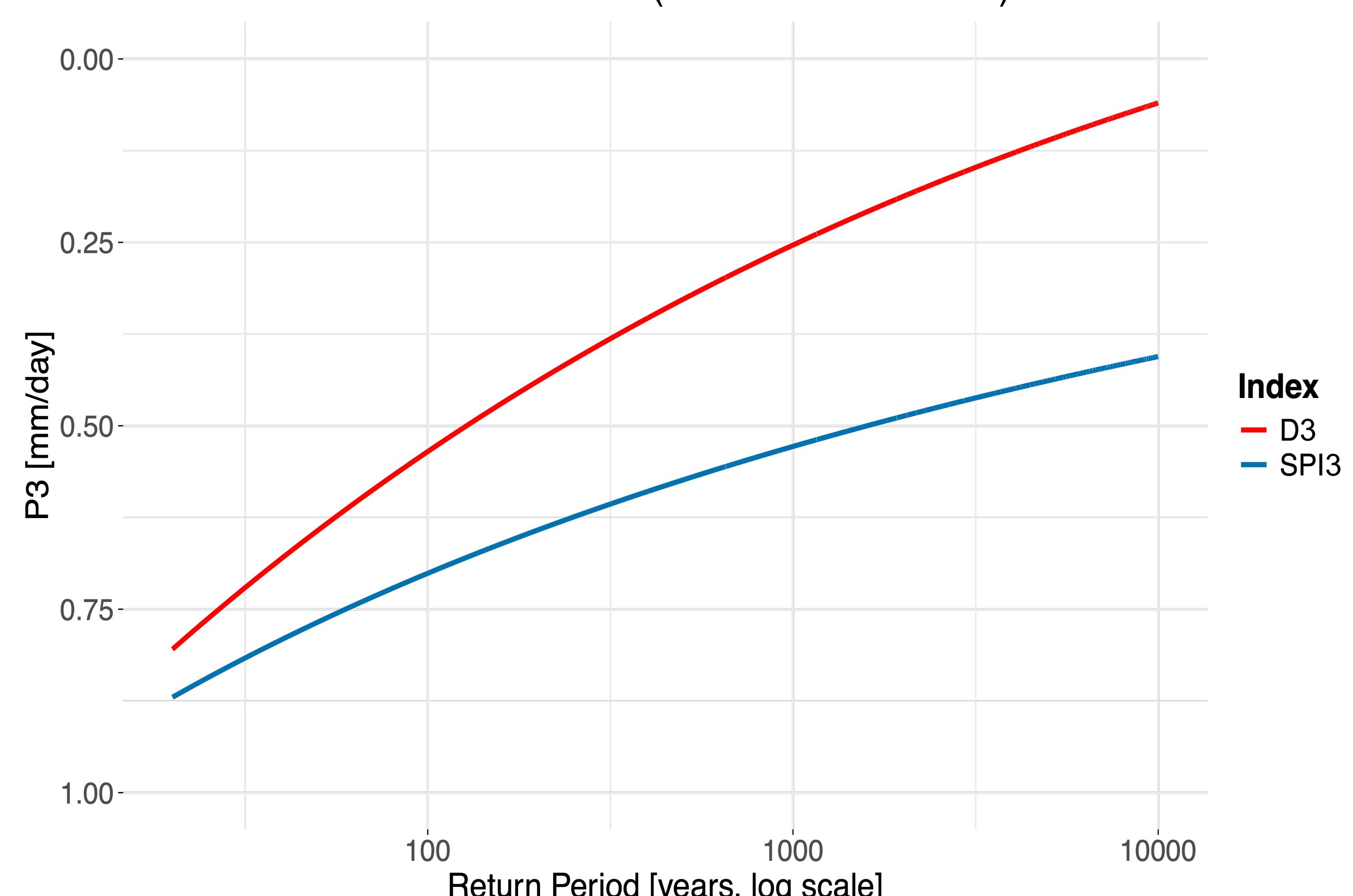


Figure 3. Direct comparison of the return levels of the $D3$ and $SPI3$. Example location of Brussels (50.85°N, 4.35°E). The indices have been inverted back into precipitation and August as been chosen as the most representative month.