Practical - 4

DFT

4x4 gray Scale.

$$f(x,y) \xrightarrow{2DFT} F(u,v)$$

$$F(u,v) = \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} f(x,y)\, e^{-j2\pi\left(\frac{ux}{M} + \frac{vy}{N}\right)}$$

$$F(u,v) \xrightarrow{2IDFT} f(x,y)$$

$$f(x,y) = \frac{1}{MN} \sum_{u=0}^{M-1} \sum_{v=0}^{N-1} F(u,v)\, e^{j2\pi\left(\frac{ux}{M} + \frac{vy}{N}\right)}$$

$$T = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix}$$

$$f = \begin{bmatrix} 0 & 1 & 2 & 1 \\ 1 & 2 & 3 & 2 \\ 2 & 3 & 4 & 3 \\ 1 & 2 & 3 & 2 \end{bmatrix}$$

$F = TfT$

$$= \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 0 & 1 & 2 & 1 \\ 1 & 2 & 3 & 2 \\ 2 & 3 & 4 & 3 \\ 1 & 2 & 3 & 2 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix}$$

$0+1+2+1$
$= 4$

$$= \begin{bmatrix} 4 & 8 & 12 & 8 \\ -2 & -2 & -2 & -2 \\ 0 & 0 & 0 & 0 \\ -2 & -2 & -2 & -2 \end{bmatrix} \times \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix}$$

$$= \begin{bmatrix} 32 & -8 & 0 & -8 \\ -8 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -8 & 0 & 0 & 0 \end{bmatrix} \Rightarrow \underline{\text{DFT}} \text{ matrix}$$

Circular Convolution

$$x[m,n] = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}_{2\times 2} \qquad h(m,n) = \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix}_{2\times 2}$$

form block matrices H_0 & H_1

$$H_0 = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$$

$$H_1 = \begin{bmatrix} 3 & 4 \\ 4 & 3 \end{bmatrix}$$

Toeplitz matrix

$$\underline{A} = \begin{bmatrix} H_0 & H_1 \\ H_1 & H_0 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & 4 & 3 \\ 3 & 4 & 1 & 2 \\ 4 & 3 & 2 & 1 \end{bmatrix}$$

$$y(m \times n) = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & 4 & 3 \\ 3 & 4 & 1 & 2 \\ 4 & 3 & 2 & 1 \end{bmatrix} \times \begin{bmatrix} 5 \\ 6 \\ 7 \\ 8 \end{bmatrix}$$

$$y(m,n) = \begin{bmatrix} 70 \\ 68 \\ 62 \\ 60 \end{bmatrix}$$

$$y(m,n) = \begin{bmatrix} 70 & 68 \\ 62 & 60 \end{bmatrix}$$

Practical - 2

$$x(m,n) = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}_{2\times 2} \begin{matrix} \rightarrow H_0 \\ \rightarrow H_1 \end{matrix} \quad h(m,n) = \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix}$$

Linear Convolution

$$\therefore H_0 = \begin{bmatrix} \cancel{1} & \cancel{2} \\ 2 & 1 \end{bmatrix} \quad \begin{bmatrix} 1 & 0 \\ 2 & 1 \\ 0 & 2 \end{bmatrix}$$

$$H_1 = \begin{bmatrix} 3 & 0 \\ 4 & 3 \\ 0 & 4 \end{bmatrix}$$

$$A = \begin{bmatrix} H_0 & 0 \\ H_1 & H_0 \\ 0 & H_1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 2 & 1 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 3 & 0 & 1 & 0 \\ 4 & 3 & 2 & 1 \\ 0 & 4 & 0 & 2 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 4 & 3 \\ 0 & 0 & 0 & 4 \end{bmatrix} \times \begin{bmatrix} 5 \\ 6 \\ 7 \\ 8 \end{bmatrix}$$

$$A = \begin{bmatrix} 5 \\ 16 \\ 12 \\ 22 \\ 60 \\ 40 \\ 21 \\ 52 \\ 32 \end{bmatrix}$$

$$\therefore y(m,n) = \begin{array}{c} \begin{matrix} C_1 & C_2 & C_3 \end{matrix} \\ \begin{bmatrix} 5 & 16 & 12 \\ 22 & 60 & 40 \\ 21 & 52 & 32 \end{bmatrix} \end{array}$$

Step (1) $C_3 + C_1$ bcz C_3 is alias of C_1

$$= \begin{array}{cc} \begin{matrix} C_1 & C_2 \end{matrix} & \\ \begin{bmatrix} 17 & 16 \\ 62 & 60 \\ 53 & 52 \end{bmatrix} & \begin{matrix} \rightarrow R_1 \\ \rightarrow R_2 \\ \rightarrow R_3 \end{matrix} \end{array}$$

Step (2) $R_3 + R_1$ bcz R_3 is alias of R_1

$$y(m,n) = \begin{bmatrix} 70 & 68 \\ 62 & 60 \end{bmatrix}$$

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(1) 2D Linear Cross Correlation:

$$x_1[m,n] = \begin{bmatrix} 3 & 1 \\ 2 & 4 \end{bmatrix}$$

$$x_2[m,n] = \begin{bmatrix} 1 & 5 \\ 2 & 3 \end{bmatrix}$$

Step (1):

→ From the given $x_2[m,n]$ determine $[\cancel{x_1}, -m, -n]$

(1.a)

→ To get fold version of $x_2[m,n]$ first fold columnwise to get $x_2[-m,-n]$

$$= \begin{bmatrix} 5 & 1 \\ 3 & 2 \end{bmatrix}$$

1(b)

→ Then fold $x_2[m,-n]$ along row-wise to get $x_2[-m,-n]$

$$x[-m,-n] = \begin{bmatrix} 3 & 2 \\ 5 & 1 \end{bmatrix}$$

Step 2.

→ Now to perform linear convolution.

2a → To form block matrix i.e. H_0 & H_1

$$H_0 = \begin{bmatrix} 3 & 0 \\ 1 & 3 \\ 0 & 1 \end{bmatrix} \qquad H_1 = \begin{bmatrix} 2 & 0 \\ 4 & 2 \\ 0 & 4 \end{bmatrix}$$

$$A = \begin{bmatrix} H_0 & 0 \\ H_1 & H_0 \\ 0 & H_1 \end{bmatrix} = \begin{bmatrix} 3 & 0 & 0 & 0 \\ 1 & 3 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 2 & 0 & 3 & 0 \\ 4 & 2 & 1 & 3 \\ 0 & 4 & 0 & 1 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 4 & 2 \\ 0 & 0 & 0 & 4 \end{bmatrix} \times \begin{bmatrix} 3 \\ 2 \\ 5 \\ 1 \end{bmatrix} = \begin{bmatrix} 9 \\ 9 \\ 2 \\ 21 \\ 24 \\ 9 \\ 10 \\ 22 \\ 4 \end{bmatrix}$$

$$\cancel{A} = y(m,n) = \begin{bmatrix} 9 & 9 & 2 \\ 21 & 24 & 9 \\ 10 & 22 & 4 \end{bmatrix}$$

② Circular Correlation:

$x_1(m,n) = \begin{bmatrix} 1 & 5 \\ 2 & 4 \end{bmatrix}$ and $x_2(m,n) = \begin{bmatrix} 3 & 2 \\ 4 & 1 \end{bmatrix}$

Step ① Form^ of block matrix

H_0 & H_1

$H_0 = \begin{bmatrix} 1 & 5 \\ 5 & 1 \end{bmatrix}$ $H_1 = \begin{bmatrix} 2 & 4 \\ 4 & 2 \end{bmatrix}$

$A = \begin{bmatrix} H_0 & H_1 \\ H_1 & H_0 \end{bmatrix} = \begin{bmatrix} 1 & 5 & 2 & 4 \\ 5 & 1 & 4 & 2 \\ 2 & 4 & 1 & 5 \\ 4 & 2 & 5 & 1 \end{bmatrix}$

$x_2[-m,-n]$

first Column-wise $x_2[-m,-n] = \begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix}$

then, row-wise :. $x_2[-m,-n] = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$

$y(m,n) = A \times x_2(-m,-n)$

20
+16
36

$= \begin{bmatrix} 1 & 5 & 2 & 4 \\ 5 & 1 & 4 & 2 \\ 2 & 4 & 1 & 5 \\ 4 & 2 & 5 & 1 \end{bmatrix} \times \begin{bmatrix} 1 \\ 4 \\ 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 1+20+4+12 \\ 5+4+8+6 \\ 2+16+2+15 \\ 4+8+10+3 \end{bmatrix} = \begin{bmatrix} 37 \\ 23 \\ 35 \\ 25 \end{bmatrix}$

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$\therefore$ The matrix $y(m,n) = \begin{bmatrix} 37 & 23 \\ 35 & 25 \end{bmatrix}$

3 Auto Correlation   Ok Clear

$$x[m,n] = \begin{bmatrix} 3 & 2 \\ 1 & 5 \end{bmatrix}$$

Formn of ~~of~~ block matrix

$$H_0 = \begin{bmatrix} 3 & 2 \\ 2 & 3 \end{bmatrix}$$

$$H_1 = \begin{bmatrix} 1 & 5 \\ 5 & 1 \end{bmatrix}$$

$$A = \begin{bmatrix} H_0 & H_1 \\ H_1 & H_0 \end{bmatrix} = \begin{bmatrix} 3 & 2 & 1 & 5 \\ 2 & 3 & 5 & 1 \\ 1 & 5 & 3 & 2 \\ 5 & 1 & 2 & 3 \end{bmatrix}$$

Now, multiplying the block Circular matrix A with $x_2[-m,-n]$, we get

$$y[m,n] = \begin{bmatrix} 3 & 2 & 1 & 5 \\ 2 & 3 & 5 & 1 \\ 1 & 5 & 3 & 2 \\ 5 & 1 & 2 & 3 \end{bmatrix} \times \begin{bmatrix} 5 \\ 1 \\ 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 34 \\ 26 \\ 22 \\ 39 \end{bmatrix}$$

$$y[m,n] = \begin{bmatrix} 34 & 26 \\ 22 & 39 \end{bmatrix}$$

For rest of the row.

$$C(u,v) = \begin{array}{c|cccc} & 0 & 1 & 2 & 3 \\ \hline 0 & 0.5 & 0.5 & 0.5 & 0.5 \\ 1 & 0.653 & 0.2705 & -0.2705 & -0.653 \\ 2 & 0.5 & -0.5 & -0.5 & +0.5 \\ 3 & 0.2705 & -0.653 & 0.653 & -0.2705 \end{array}$$

① $C \neq C^T$ → Not Symmetric Matrix
Real terms & orthogonal

② JPEG Compression

$$C^T = \begin{array}{c|cccc} & 0 & 1 & 2 & 3 \\ \hline 0 & 0.5 & 0.653 & 0.5 & 0.2705 \\ 1 & 0.5 & 0.2705 & -0.5 & 0.653 \\ 2 & 0.5 & -0.2705 & -0.5 & 0.653 \\ 3 & 0.5 & +0.653 & 0.5 & -0.2705 \end{array}$$

4x4.

$C(u,v) = C \cdot f \cdot C^T$

$= \cdot$

0 → black
1 → white:

KL transform

~~Expression $y = A(x - \mu_x)$~~

x_2				
3	0	0	0	0
2	0	1	1	0
1	0	0	1	1
0	0	0	0	0
	0	1	2	3 x_1

(1) Find (x)

$$x = \left\{ \begin{pmatrix} 1 \\ 2 \end{pmatrix} \begin{pmatrix} 2 \\ 1 \end{pmatrix} \begin{pmatrix} 2 \\ 2 \end{pmatrix} \begin{pmatrix} 3 \\ 1 \end{pmatrix} \right\}$$

(2) Find mean (μ_x)

$$(\mu_x) = \left\{ \begin{pmatrix} 2 \\ 1.5 \end{pmatrix} \right\}$$

(3) $C_x = E[(x_i - \mu_x)(x_i - \mu_x)^T]$

$1-2$

$$(x_i - \mu_x) = \begin{pmatrix} -1 \\ 0.5 \end{pmatrix}, \quad (x_i - \mu_x)^T = \begin{pmatrix} -1 & 0.5 \end{pmatrix}$$

$$C_1 = \begin{pmatrix} -1 \\ 0.5 \end{pmatrix} \begin{pmatrix} -1 & 0.5 \end{pmatrix} = \begin{bmatrix} 1 & -0.5 \\ -0.5 & 0.25 \end{bmatrix}_{2\times 2}$$

$$C_2 = \begin{bmatrix} 0 & 0 \\ 0 & 0.25 \end{bmatrix} \quad C3 = \begin{bmatrix} 0 & 0 \\ 0 & 0.25 \end{bmatrix} \; \& \; C_4 = \begin{bmatrix} 1 & -0.5 \\ -0.5 & 0.25 \end{bmatrix}$$

= Mean of 1^{st} term

ie: $1 + 0 + 0 + 1 = \frac{2}{4} = \frac{1}{2} = 0.5$

Mean of 2^{nd} term

$$-0.5 + 0 + 0 + -0.5 = -\frac{1}{4} = -0.25 = -0.25$$

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$$C_x = \begin{bmatrix} 0.5 & -0.25 \\ -0.25 & 0.25 \end{bmatrix}$$

4) Eigen Values

$$|C_x - \lambda I| = 0 \quad \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\left| \begin{bmatrix} 0.5 & -0.25 \\ 0.25 & 0.25 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} \right| = 0$$

$$\begin{vmatrix} 0.5 - \lambda & -0.25 \\ 0.25 & 0.25 - \lambda \end{vmatrix}_{2\times 2} = 0$$

$0.5 - \lambda$

$$\lambda^2 - 0.75\lambda + 0.625 = 0$$

$\therefore \lambda_1 = 0.6545$ ← highest EV

$\lambda_2 = 0.09549$ $\quad \lambda_1 > \lambda_2$

rule: $\lambda_1 > \lambda_2 > \lambda_3 > \lambda_4$

5) Eigen Vector

for $\lambda_1 = 0.6545$

Eigen vector

$$[C_x - \lambda I]\,[X] = 0$$

$$\begin{bmatrix} 0.5 - 0.6545 & -0.25 \\ -0.25 & 0.25 - 0.6545 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$-0.1545\,x_1 - 0.25\,x_2 = 0$$

$$x_1 = \frac{-0.25}{0.1545} x_2$$

$$\therefore x_1 = -1.6181\, x_2$$

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$\cancel{-1.6181}\ x_1 = \cancel{-1.6181}\ x_2$

$\therefore$ Eigen Vector

$$= \begin{bmatrix} -1.6181\ x_1 \\ 1\ x_2 \end{bmatrix} = e'$$

For $\lambda_2 = 0.09549.$

$[Cx - \lambda I][x] = [0]$

$$\begin{bmatrix} 0.4045 & -0.25 \\ -0.25 & 0.1545 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

0.5000
$-$ 0.6545
545

0.6545
0.5000
1545

$$\therefore x_1 = \frac{0.25}{0.4045} x_2$$

$$x_1 = 0.618\ x_2$$

$\therefore$ Eigen Veetor

$$= \begin{bmatrix} 0.618 \\ 1 \end{bmatrix} = e_1$$

Transformation matrix

$\lambda_1 \to e_0 \to (\text{highest } \lambda)$
$\lambda_2 \to e_1 \to (\text{lowest } \lambda)$

$$A = \begin{bmatrix} e_0^T \\ e_1^T \end{bmatrix} = \begin{bmatrix} -1.61 & 1 \\ 0.61 & 1 \end{bmatrix}$$

$[e_2^2]$ $[\ \ 1]$

$$y = A(x - \mu_x) =$$
$$= \begin{bmatrix} -1.61 & 1 \\ 0.61 & 1 \end{bmatrix} \begin{bmatrix} \\ \end{bmatrix}$$

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(1) Linear Convolution

$$x(m,n) = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}_{3\times3} \quad h = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}_{3\times1}$$

→ ~~Bas~~ Forming the Block matrix

$$H_0 = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \quad H_1 = \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix} \quad H_2 = \begin{bmatrix} 7 \\ 8 \\ 9 \end{bmatrix}$$

$$\boxed{\text{Toeplitz matrix}} = \begin{bmatrix} H_0 & 0 & 0 \\ H_1 & H_0 & 0 \\ H_2 & H_1 & H_0 \\ 0 & H_2 & H_1 \\ 0 & 0 & H_2 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 0 & 0 \\ 3 & 0 & 0 \\ 4 & 1 & 0 \\ 5 & 2 & 0 \\ 6 & 3 & 0 \\ 7 & 4 & 1 \\ 8 & 5 & 2 \\ 9 & 6 & 3 \\ 0 & 7 & 4 \\ 0 & 8 & 5 \\ 0 & 9 & 6 \\ 0 & 0 & 7 \\ 0 & 0 & 8 \\ 0 & 0 & 9 \end{bmatrix} \times \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 5 \\ 7 \\ 9 \\ 12 \\ 15 \\ 18 \\ 11 \\ 13 \\ 15 \\ 7 \\ 8 \\ 9 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 3 \\ 5 & 7 & 9 \\ 12 & 15 & 18 \end{bmatrix}$$

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