1

a) $$H(\omega) = \frac{K_o F(\omega)}{j\omega + K_o F(\omega)} \qquad K_o = 1 \qquad F(\omega) = \frac{1}{j\omega + \sqrt{2}}$$

$$\frac{1/j\omega+\sqrt{2}}{j\omega + \frac{1}{j\omega+\sqrt{2}}} = \frac{1}{j\omega(j\omega+\sqrt{2})+1} = \frac{1}{1+\sqrt{2}j\omega - \omega^2}$$

b) $$E(\omega) = \frac{j\omega\, \Theta(\omega)}{j\omega + K_o F(\omega)} = j\omega \frac{\cdot 1/j\omega}{j\omega + \frac{1}{j\omega+\sqrt{2}}} = \frac{j\omega+\sqrt{2}}{1+\sqrt{2}j\omega - \omega^2}$$

$$\lim_{j\omega\to 0} E(\omega) = \lim_{j\omega \to 0} j\omega \frac{j\omega+\sqrt{2}}{1+\sqrt{2}j\omega-\omega^2} = \sqrt{2}\cdot 0$$

c) $$\Theta(\omega) = \cancel{K}(j\omega)^2 \Rightarrow \lim_{j\omega\to 0} E(\omega) = \frac{\cancel{j\omega}}{j\omega} \cdot \frac{j\omega+\sqrt{2}}{1+\sqrt{2}j\omega-\omega^2}$$

$$= \sqrt{2}$$

2. $U = mG \quad G = [P \; I_k] \quad P = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix}$

vérification :

$$[m_1 \; m_2 \; m_3] \begin{bmatrix} 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} m_1 + m_2 \\ m_2 + m_3 \\ m_1 + m_2 + m_3 \\ m_1 + m_3 \end{bmatrix} = \begin{bmatrix} p_1 \\ p_2 \\ p_3 \\ p_4 \end{bmatrix} \checkmark$$

a) $G = \begin{bmatrix} 1 & 0 & 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 & 1 \end{bmatrix}$

2. b) $m \in \{000 \;\; 001 \;\; 010 \;\; 011 \;\; 100 \;\; 101 \;\; 110 \;\; 111\}$

$mG = m \begin{bmatrix} 1 & 0 & 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 & 1 \end{bmatrix}$

m	u	poids
000	0000000	0
001	0111001	4
010	1110010	4
011	1001011	4
100	1011100	4
101	1100101	4
110	0101110	4
111	0010111	4

poids minimale = 4 = distance minimale

c) $H = I_4 \; P^T = \begin{bmatrix} 1000 & 110 \\ 0100 & 011 \\ 0010 & 111 \\ 0001 & 101 \end{bmatrix}$ $\qquad H^T = \begin{bmatrix} 1000 \\ 0100 \\ 0010 \\ 0001 \\ 1011 \\ 1110 \\ 0111 \end{bmatrix}$

d) $S = eH^T \Rightarrow$

$\underline{e}$	$\underline{S}$
0000001	0111
0000010	1110
0000100	1011
0001000	0001
0010000	0010
0100000	0100
1000000	1000

e) $[1101011] \begin{bmatrix} 1000 \\ 0100 \\ 0010 \\ 0001 \\ 1011 \\ 1110 \\ 0111 \end{bmatrix} = [0100]$

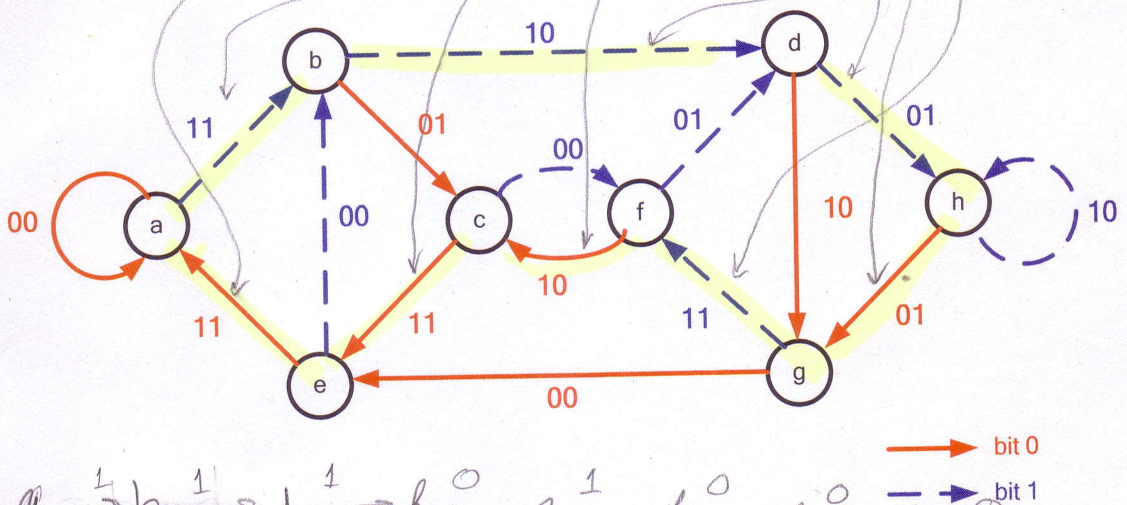
$m = r + e = 1101011 + [0100000]$

$= \underline{\underline{1001011}}$

Problème 3 (10 points sur 100)

Voici le diagramme de l'état d'un code convolutif avec longueur de contrainte quatre. Les mots de codes sont indiqués à côté de chaque transition possible. L'état initial est « a », l'état où tous les registres contiennent zéro.

Donnez la séquence encodée quand les données sont 1 1 1 0 1 0 0 0.



$a \xrightarrow{1} b \xrightarrow{1} d \xrightarrow{1} h \xrightarrow{0} g \xrightarrow{1} f \xrightarrow{0} c \xrightarrow{0} e \xrightarrow{0} a$

11 10 01 01 11 10 11 11

4. délai 0 : $\quad -1 \quad 1 \quad 1 \quad 1 \quad -1 \quad -1 \quad 1$ donnée

$$
\begin{array}{rrrrrrrr}
 & -1 & 1 & 1 & 1 & -1 & -1 & 1 \\
\times & -1 & -1 & 1 & -1 & 1 & 1 & 1 \\
\hline
 & 1 & -1 & 1 & -1 & -1 & -1 & 1
\end{array}
\quad \Sigma = -1
$$

délai 1
(rotation)

$$
\begin{array}{rrrrrrrr}
 & -1 & 1 & 1 & 1 & -1 & -1 & 1 \\
\times & 1 & -1 & -1 & 1 & -1 & 1 & 1 \\
\hline
 & -1 & -1 & -1 & 1 & +1 & -1 & 1
\end{array}
\quad \Sigma = -1
$$

délai 2

$$
\begin{array}{rrrrrrrr}
 & -1 & 1 & 1 & 1 & -1 & -1 & 1 \\
\times & 1 & 1 & -1 & +1 & -1 & -1 & 1 \\
\hline
 & -1 & +1 & -1 & -1 & -1 & +1 & +1
\end{array}
\quad \Sigma = -1
$$

délai 3

$$
\begin{array}{rrrrrrrr}
 & -1 & 1 & 1 & 1 & -1 & -1 & 1 \\
\times & 1 & 1 & 1 & -1 & -1 & 1 & -1 \\
\hline
 & -1 & +1 & +1 & -1 & +1 & -1 & -1
\end{array}
\quad \Sigma = -1
$$

délai 4

$$
\begin{array}{rrrrrrrr}
 & -1 & 1 & 1 & 1 & -1 & -1 & 1 \\
\times & -1 & 1 & 1 & 1 & -1 & -1 & 1 \\
\hline
 & 1 & 1 & 1 & 1 & 1 & 1 & 1
\end{array}
\quad \Sigma = 7 \ \checkmark
$$

pic d'autocorrélation

Bonne rotation

$-1 \quad 1 \quad 1 \quad 1 \quad -1 \quad -1 \quad 1$

ou on peut le lire directement des données!!!

4, b) bit 1 : -1 1 1 1 -1 -1 1

$\times$ -1 1 1 1 -1 -1 1

1 1 1 1 1 1 1 $\Sigma = 7 \Rightarrow$ bit1 = 1

bit 2 -1 1 1 1 -1 -1 1

$\times$ -1 1 1 1 -1 -1 1

1 1 1 1 1 1 1 $\Sigma = 7 \Rightarrow$ bit2 = 1

bit 3 1 -1 -1 -1 1 1 -1

-1 1 1 1 -1 -1 1

-1 -1 -1 -1 -1 -1 -1 $\Sigma = -7 \Rightarrow$ bit3 = -1

c) délai = 2 chips $\Rightarrow$ ISI rejeté par $1/G$

G = largeur de code
= expansion spectrale
= gain d'étalement
= gain de traitement

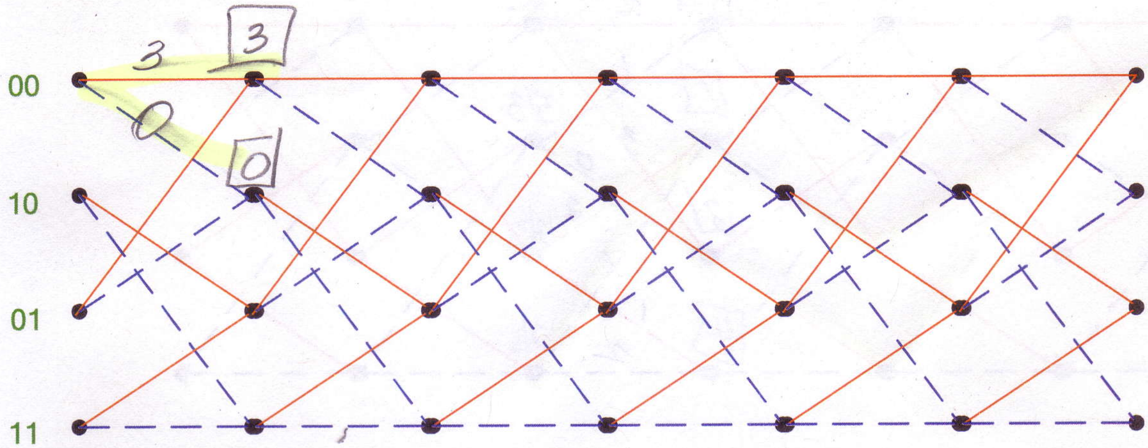
$1/G = 1/7 \Rightarrow 10 \log_{10} 1/7 = -8{,}45\,dB$

Nous voulons <u>10dB</u> donc encore 1,55dB

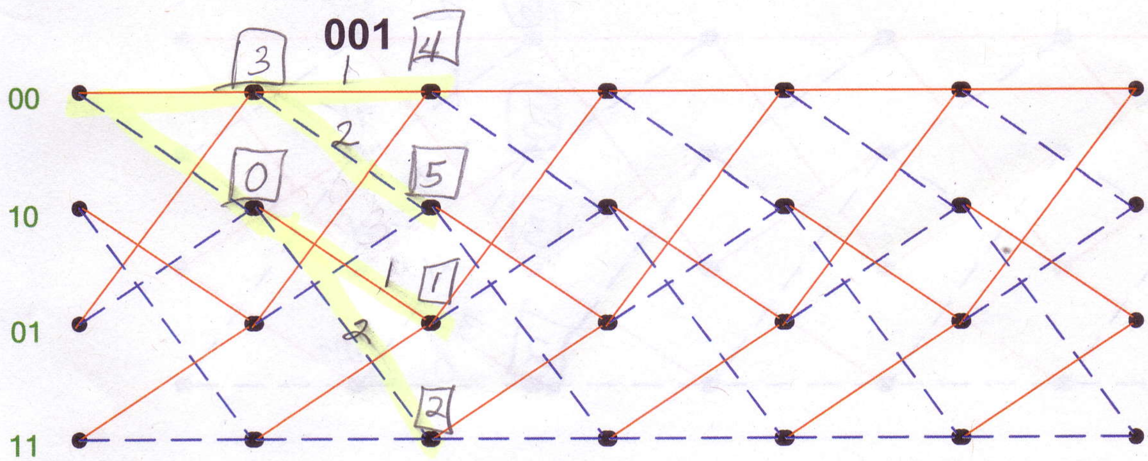
$-1.55\,dB \Rightarrow 10^{-.155} = .7$

$\boxed{\alpha \leq .7} \Rightarrow$ ISI rejection $\geq 10\,dB$

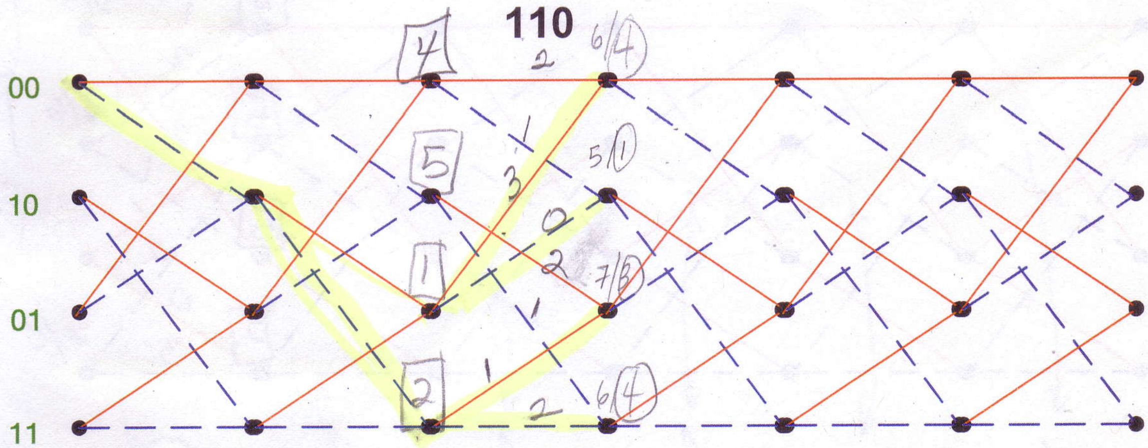
111

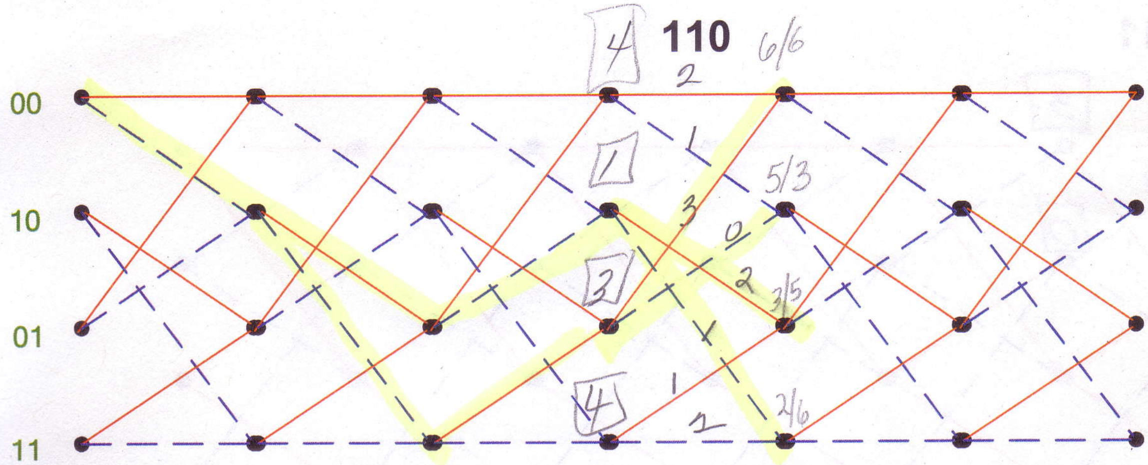

00
10
01
11
3
3
0
0

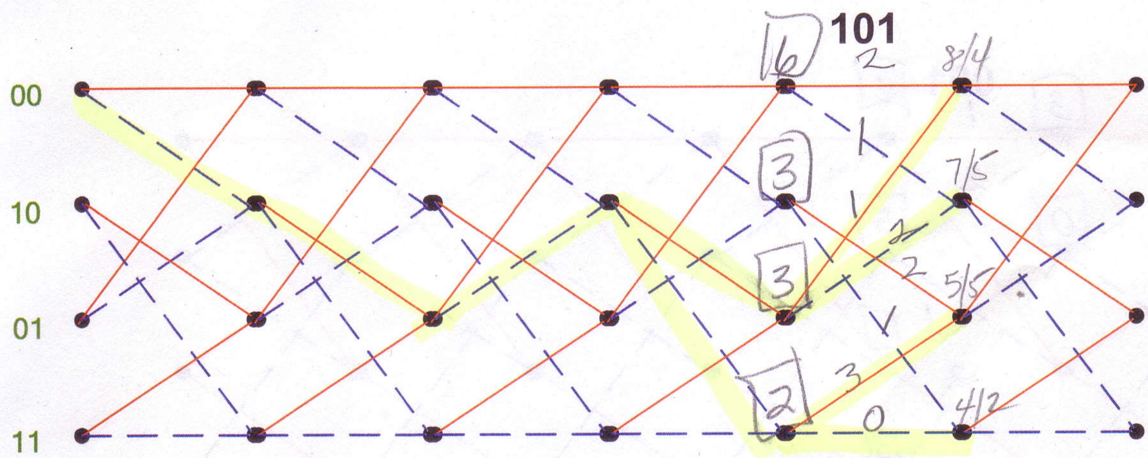
001

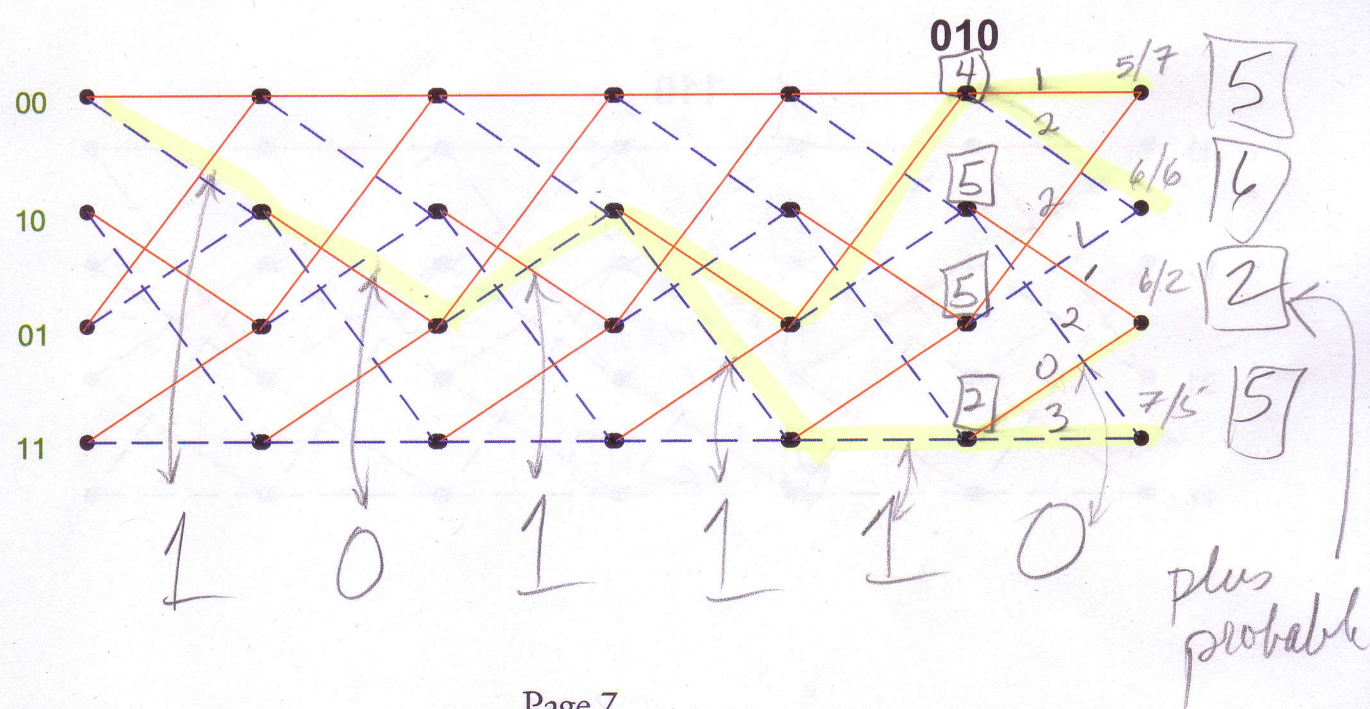

00
10
01
11
3
4
1
2
5
0
1
1
2
2

110


00
10
01
11
4
2
6/4
5
1
3
5/1
1
0
2
7/3
2
1
1
2
6/4


110
00
10
01
11
6/6
5/3
3/5
2/6


101
00
10
01
11
8/4
7/5
5/5
4/2


010
00
10
01
11
5/7
6/6
6/2
7/5
1 0 1 1 1 0
plus probable