Wednesday 24 April 2015; Duration: 11:30 to 13:20
No documentation allowed; calculator allowed

Problem1 (25 points over 100)

Supposw we have a second order PLL whose loop filter response is

$$F(\omega) = \frac{1}{j\omega + 1}$$

and whose VCO gain is either K_0=1 or K_0=10. For each of these two values of K_0, respond to questions A and B.

A. (10 points) Give the PLL estimate of the phase, $\hat{\theta}(t)$, when there is a unitary phase jump at t=0.

B. (10 points) What is the asymptotic error?

   a. when there is a unitary phase jump at t=0.

   b. when there is a linear phase variation with slope 1

C. (5 points) How and under what circumstances can we exploit the VOC gain, K_0, to improve PLL performance?

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$g(t)$	$G(j\omega)$
$u(t) = \begin{cases} 1 & t > 0 \\ 0 & t < 0 \end{cases}$	$\frac{1}{j\omega}$
$\frac{1}{\omega_0} u(t)\left[1 - e^{-\omega_0 t}\right]$	$\frac{1}{j\omega}\frac{1}{j\omega + \omega_0}$
$\frac{1}{\omega_0} u(t)\left[t - \frac{1 - e^{-\omega_0 t}}{\omega_0}\right]$	$\frac{1}{(j\omega)^2}\frac{1}{j\omega + \omega_0}$
$1 - \frac{e^{-\varsigma\omega_n t}}{\sqrt{1-\varsigma^2}} \sin\left(\omega_n t\sqrt{1-\varsigma^2} + \cos^{-1}\varsigma\right)$	$\frac{1}{j\omega}\frac{\omega_n^2}{(j\omega)^2 + j\omega 2\varsigma\omega_n + \omega_n^2}$

Problem2 (20 points over 100)

The following is the control matrix for a block code:

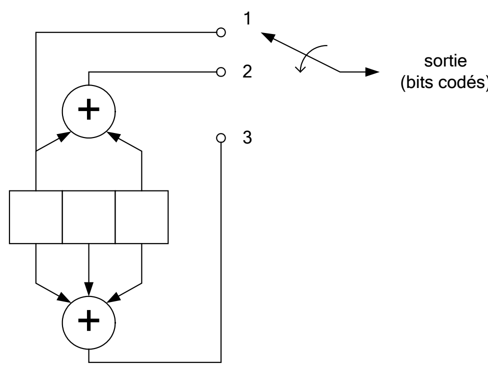
$$H^T = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$$

A. (10 points) Sketch a shift-register implementation of this code.

B. (10 points) Give the syndrome table for this code..

Problem3 (25 points over 100)

Here is the shift register implementation of convolutional code:



Use the sheets provided for parts A and B. Do not forget to write your name and ID number on each sheet before turning in your exam.

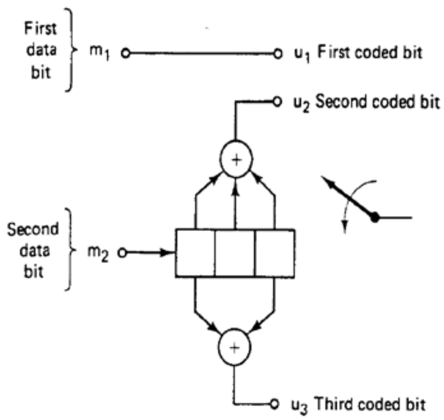
A. (10 points) Sketch the state diagram for this code.

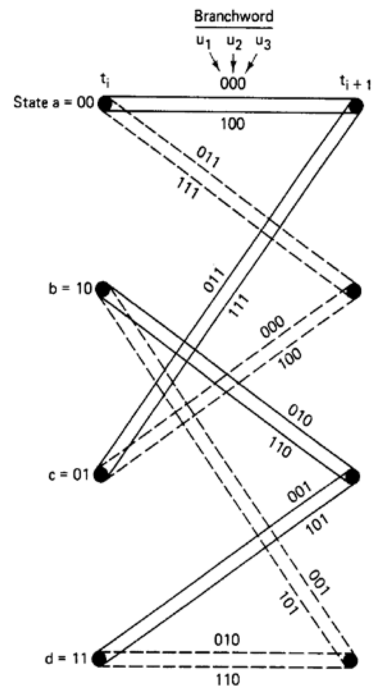
B. (5 points) Give the trellis encoder for this code.

C. (10 points) Find the minimal distance for the convolutional code using Hamming distance as a metric.

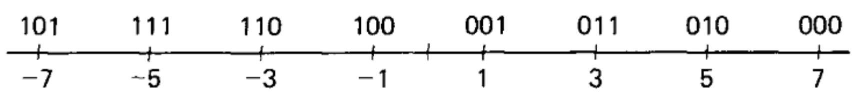
Problem4 (30 points over 100)

The following is a TCM encoder and trellis.





The 8QAM constellation and corresponding bit assignments are the following.

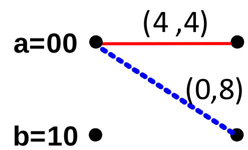


Suppose that the received signal (soft decisions) from a Gaussian channel are (3,-3,-5,7). You must find the most probably path through the trellis using ***Euclidian distance.***

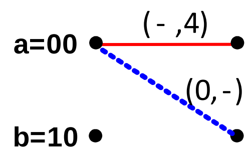
A. (5 points) Complete the trellis (sheet provided) with the Euclidean distances between the code words and the received signal for each transition.

For example, in the first interval we have:

- From state **a** to state **a** :
  - o distance between 3 and 000 (7) is 4,
  - o distance between 3 and 100 (-1) is 4
- From state **a** to state **b** :
  - o distance between 3 and 011 (3) is 0,
  - o distance between 3 and 111 (-5) is 8
- The diagram is



B. (5 points) Simplify the trellis using distances from part A, by selecting the shortest distance for the first data bits (m_1). Use the sheet provided. For example, in the first interval we have

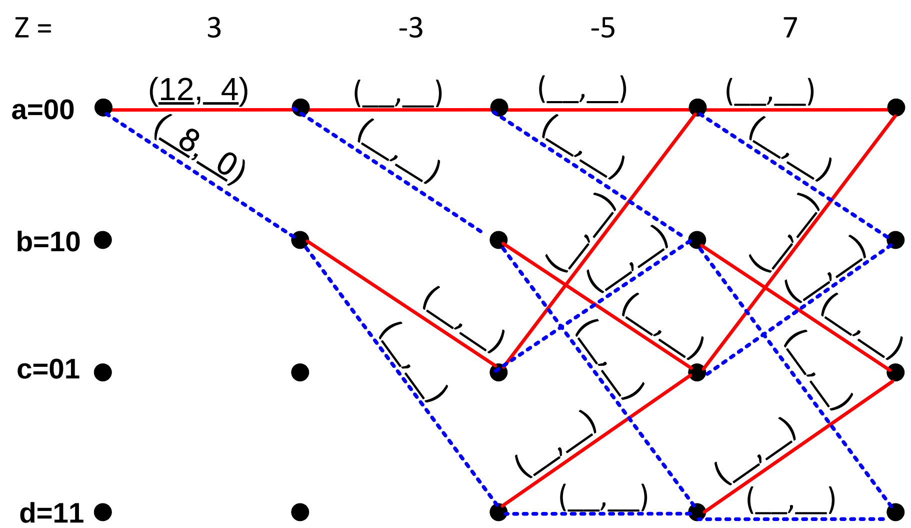


C. (10 points) For time intervals 2, 3 and 4, calculate the metrics (distances) for the two paths between each unique state (sheet proivded). Indicate the winning paths for each state at each time interval. The Euclidian distance is calculated from

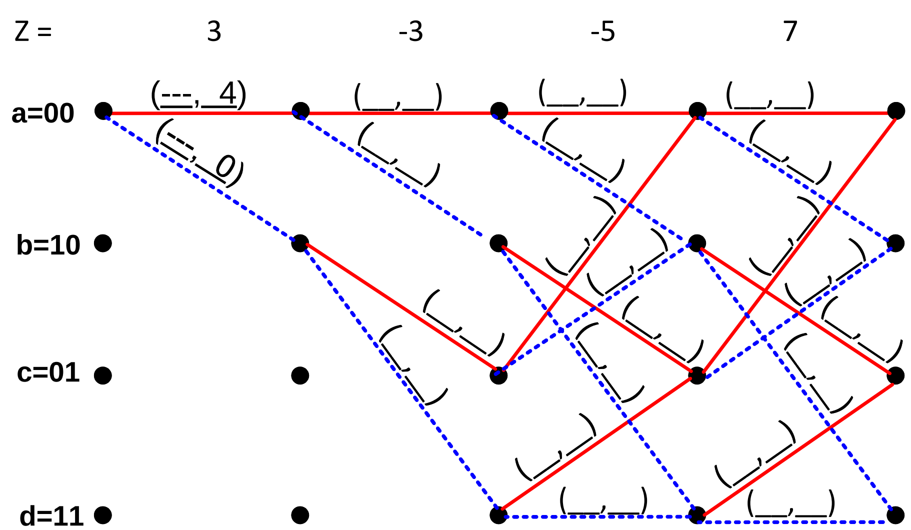
$$d_{\text{chemin}} = \sqrt{d_1^2 + d_2^2 + \cdots + d_n^2}$$

D. (10 points) Indicate the most probably path after the fourth time interval. Indicate the decoded data (m_1 and m_2 for the four time intervals) for each segment of the winning path.

Partie A – calculate the distances

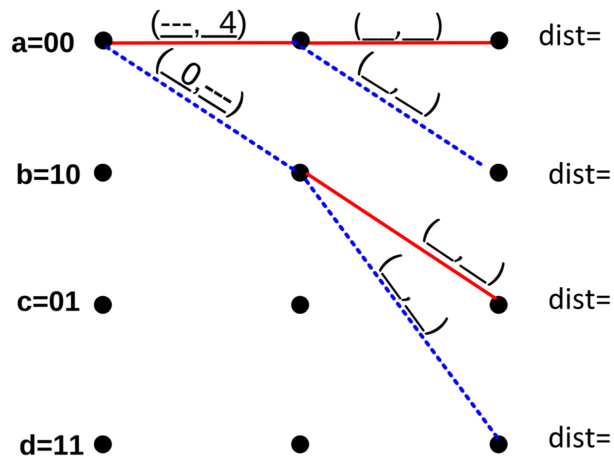


Partie B – eliminate the larger distances

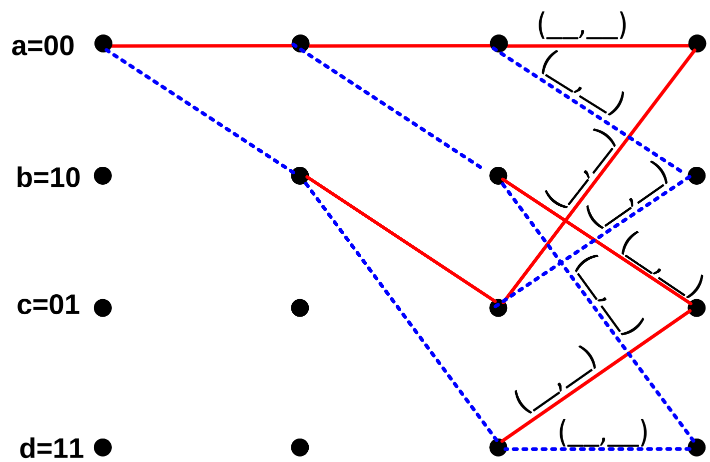


Nom :____________________ Matricule :__________________

Partie C – calculate the metrics (distances)

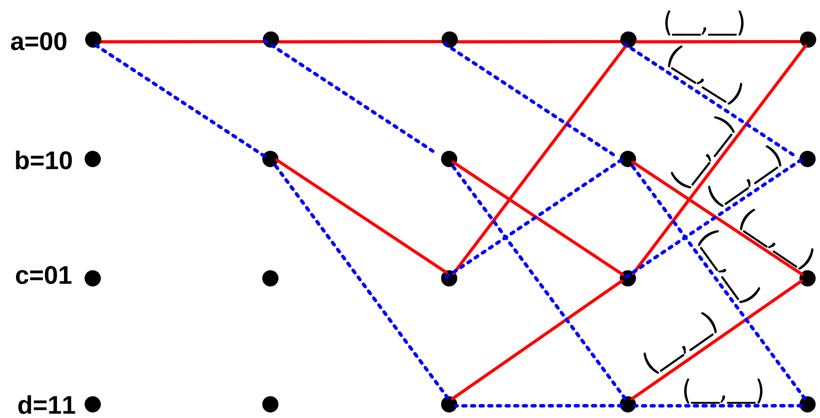


	distance	
upper <u>path</u>	lower <u>path</u>	winning <u>path</u>

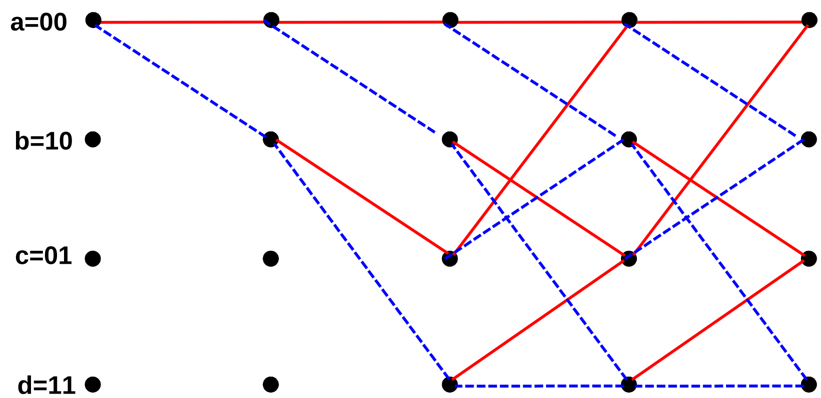


Nom :____________________ Matricule :__________________

	distance	
upper path	lower path	winning path



Partie D – indiquate the most probable path (winning)



Data

____ ____ ____ ____ ____ ____ ____ ____

m_1 m_2 m_1 m_2 m_1 m_2 m_1 m_2

Nom :____________________ Matricule :__________________