3:00 – 3:16

2014 Exam1, P1

October 30, 2014 4:48 PM

$$f(t) = e^{-\pi t^2} \Longleftrightarrow F(\omega) = e^{-\omega^2/4\pi}$$

a) $f(t+2) = ?$

$f(t+a)$	$e^{ja\omega}F(\omega)$

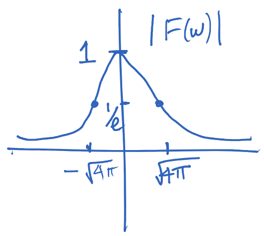
$a = 2$

$$\boxed{\underbrace{e^{j2\omega}}_{\text{phase seulement}} \; e^{-\underbrace{\omega^2/4\pi}_{}}}$$

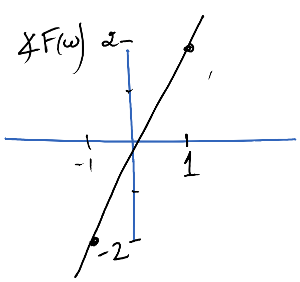
purement réel et positive

$\Rightarrow$ contribution à la phase $= 0$

$|F(\omega)| = e^{-\omega^2/2\cdot 4\pi}$



$$\angle F(\omega) = j2\omega + \text{Arg } e^{\omega^2/4\pi} = j2\omega + 0 = j2\omega$$



b) $\cos \omega_0 t \, f(t) = \frac{1}{2} e^{j\omega_0 t} f(t) + \frac{1}{2} e^{-j\omega_0 t} f(t)$

$e^{jbt} f(t)$	$F(\omega - b)$

$$F(\omega) = \frac{1}{2} e^{-(\omega^2/4\pi)}\Big|_{\omega = \omega - \omega_0} + \frac{1}{2} e^{-\omega^2/4\pi}\Big|_{\omega = \omega + \omega_0}$$

$(\omega - \omega_0)^2/ \quad (\omega+\omega_0)^2/_{4\pi}$

$'w = w - w_0$ $\qquad\qquad$ $'w = w + \omega_0$

$$\boxed{= \frac{1}{2} e^{-(w-w_0)^2/4\pi} + \frac{1}{2} e^{-(w+w_0)^2/4\pi}}$$

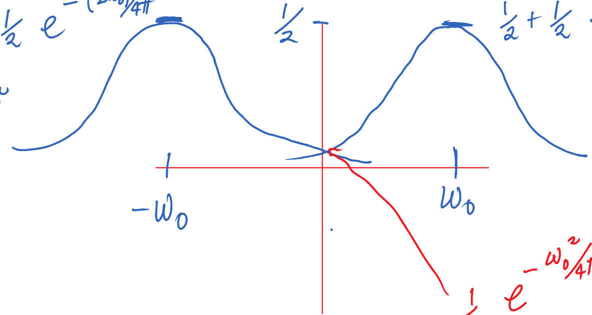
purement réel et positive ⇒ phase est nul

$= \frac{1}{2} + \frac{1}{2} e^{-(2w_0)^2/4\pi}$

$\frac{1}{2} + \frac{1}{2} e^{-w_0^2/\pi}$

$\approx 1/2$

$\frac{1}{2} + \frac{1}{2} e^{-(-2w_0)^2/4\pi} = \frac{1}{2} + \frac{1}{2} e^{-w_0^2/\pi} \approx 1/2$



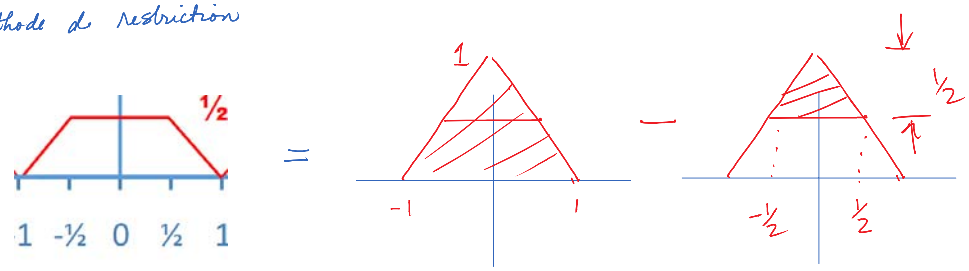
$\frac{1}{2} e^{-w_0^2/4\pi} + \frac{1}{2} e^{-w_0^2/4\pi} = e^{-w_0^2/4\pi} \approx 0$

c) $t \cdot e^{-\pi t^2}$

$t^n f(t)$	$(j)^n \frac{d^n}{d\omega^n} F(\omega)$

$$j \frac{d}{dw} e^{-w^2/4\pi} = j\left(\frac{-2w}{4\pi}\right) \cdot e^{-w^2/4\pi} = -\frac{jw}{2\pi} e^{-w^2/4\pi}$$

Méthode de restriction



$\text{Tri}\left(\frac{t}{1}\right)$ $\quad\quad$ $\frac{1}{2}\text{Tri}\left(\frac{t}{1/2}\right)$

$$f(t) = \text{Tri}\left(\frac{t}{c}\right)\Big|_{c=1} - \frac{1}{2}\text{Tri}\left(\frac{t}{c}\right)\Big|_{c=1/2}$$

$\text{Tri}\left(\dfrac{t-t_0}{\tau}\right)$ triangle de hauteur un, centré sur $t=t_0$, avec un base de longueur 2τ.

$$\Rightarrow F(\omega) = TF\left\{\text{Tri}\left(\frac{t}{1}\right)\right\} - \frac{1}{2}TF\left\{\text{Tri}\,\frac{t}{1/2}\right\}$$

$\text{Tri}(t/\tau)$ (2) $\quad\quad$ $\tau\,\text{Sa}^2(\omega\tau/2)$

$$F_r(\omega) = Sa^2\frac{\omega}{2} - \frac{1}{2}\cdot\frac{1}{2}Sa^2\frac{\omega\cdot\frac{1}{2}}{2}$$

$$= Sa^2\frac{\omega}{2} - \frac{1}{4}Sa^2\frac{\omega}{4}$$

$$F(n) = \frac{F_r(\omega)}{T_0}\Big|_{n\omega_0} \qquad T_0 = 2 \qquad \omega_0 = \pi$$

$$F(n) = \frac{Sa^2\frac{n\pi}{2}}{2} - \frac{1}{4}\frac{Sa^2 n\pi/4}{2}$$

$$= \frac{1}{2}Sa^2\frac{n\pi}{2} - \frac{1}{8}Sa^2\frac{n\pi}{4}$$

$$F(\omega) = \pi\sum_{-\infty}^{\infty}\left[Sa^2\frac{n\pi}{2} - \frac{1}{4}Sa^2\frac{n\pi}{4}\right]\delta(\omega - n\pi)$$

b) $-2.5\pi < \omega < 2.5\pi \Rightarrow n=0,\ n=1$ et $n=2$ tombent dans cet interval

$$F(0) = \pi\left(1 - \tfrac{1}{4}\right) = \pi \cdot \tfrac{3}{4} \approx \tfrac{9}{4} \approx 2.25$$

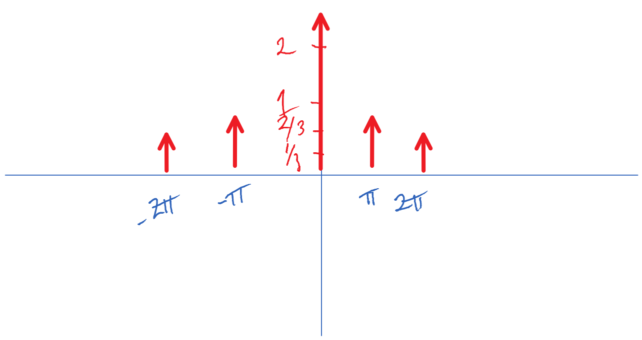
$$F(-1) = F(1) = \pi\left[Sa^2\tfrac{\pi}{2} - \tfrac{1}{4}Sa^2\tfrac{\pi}{4}\right] = \pi\left[\left(\frac{1}{\pi/2}\right)^2 - \frac{1}{4}\left(\frac{1/\sqrt{2}}{\pi/4}\right)^2\right]$$

$$\approx \pi\left[\frac{4}{\pi^2} - \frac{1}{4}\cdot\frac{1}{2}\cdot\frac{16}{\pi^2}\right] = \pi\left[\frac{4}{\pi^2} - \frac{2}{\pi^2}\right] = \frac{2}{\pi} \approx \frac{2}{3}$$

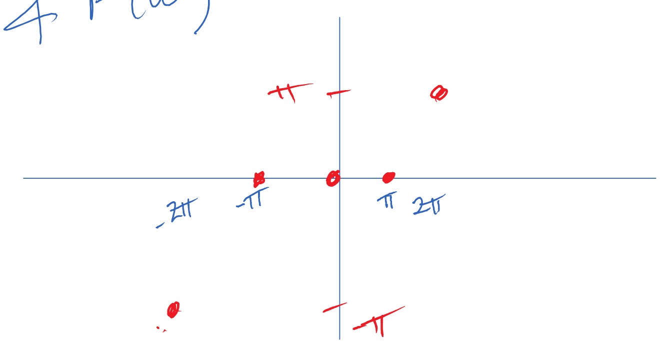
$$F(-2) = F(2) = \pi\left[Sa^2\pi - \tfrac{1}{4}Sa^2\tfrac{\pi}{2}\right] = \pi\left[-\frac{1}{4}\left(\frac{1}{\pi/2}\right)^2\right]$$

$$= \pi\left[-\frac{1}{4}\,\frac{4}{\pi^2}\right] \approx -\frac{1}{\pi} \approx -\frac{1}{3}$$

$|F(\omega)|$



$\measuredangle F(\omega)$



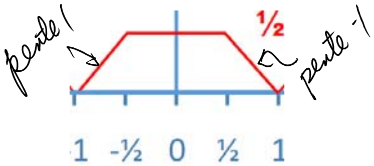
c) périodique $\Rightarrow$ signal de puissance

d) $$P(0) = |F(n)|^2_{n=0} = \left(\frac{1}{2} - \frac{1}{8}\right)^2 = \left(\frac{3}{8}\right)^2 = \frac{9}{64}$$

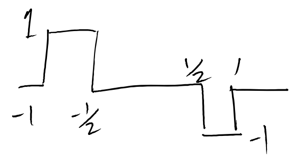
$$P(0) = \left[\frac{1}{T_0}\int_{-T_0/2}^{T_0/2} f(t)\,dt\right]^2 = \left\{\frac{1}{2}\left[\frac{1}{8} + \frac{1}{2} + \frac{1}{8}\right]\right\}^2$$

$$= \left\{\frac{1}{2}\;\frac{3}{4}\right\}^2 = \left(\frac{3}{8}\right)^2 = \frac{9}{64}$$

méthode de dérivés



$f_r'(t)$



$$\mathrm{Rect}\,\frac{t+3/4}{1/2} - \mathrm{Rect}\,\frac{t-3/4}{1/2}$$

$$\Updownarrow$$

$$j\omega F_r(\omega) = e^{+\frac{3}{4}\omega j}\, Sa\,\frac{\omega\cdot\frac{1}{2}}{2}\cdot\frac{1}{2} - e^{-j\frac{3}{4}\omega}\,\frac{1}{2}\,Sa\,\frac{\omega\frac{1}{2}}{2}$$

$$= \frac{1}{2}\,Sa\,\frac{\omega}{4}\left[e^{\frac{3}{4}j\omega} - e^{-j\frac{3}{4}\omega}\right]$$

$$= \frac{2j}{2}\,Sa\frac{\omega}{4}\cdot\sin\frac{3}{4}\omega$$

$$F_r(\omega) = \frac{\cancel{j}}{j}\,Sa\frac{\omega}{4}\cdot\frac{\sin\frac{3}{4}\omega}{\omega\cdot 3/4}\cdot 3/4$$

$$= \frac{3}{4}\,Sa\frac{\omega}{4}\,Sa\frac{3}{4}\omega$$

$$F(n) = \frac{F_r(n\omega_0)}{T_0} = \frac{1}{2}\cdot\frac{3}{4}\,Sa\,\frac{n\pi}{4}\,Sa\,\frac{3}{4}n\pi = \frac{3}{8}\,Sa\frac{n\pi}{4}\,Sa\,\frac{3\pi n}{4}$$

$$F(0) = 2\pi\cdot\frac{3}{8} = \pi\,\frac{3}{4}$$

$$F(1) = 2\pi\cdot\frac{3}{8}\,Sa\,\frac{\pi}{4}\,Sa\,\frac{3\pi}{4} = 2\pi\,\frac{3}{8}\,\frac{1/\sqrt{2}}{\pi/4}\cdot\frac{1/\sqrt{2}}{3\pi/4}$$

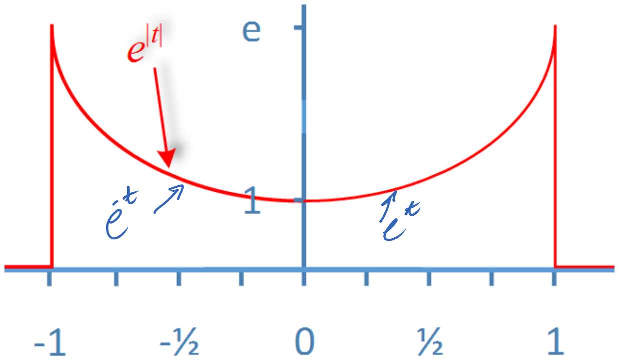
$$\frac{6\pi}{8}\ \frac{14}{3\pi^2}\ \frac{1}{2} = \frac{\sim}{\pi}$$

$$F(2) = 2\pi\ \frac{3}{8}\ Sa\frac{\pi}{2}\ Sa\frac{\pi 3}{2} = 2\pi\ \frac{3}{8}\ \frac{1}{\pi/2}\cdot\frac{(-1)}{3\pi/2}$$

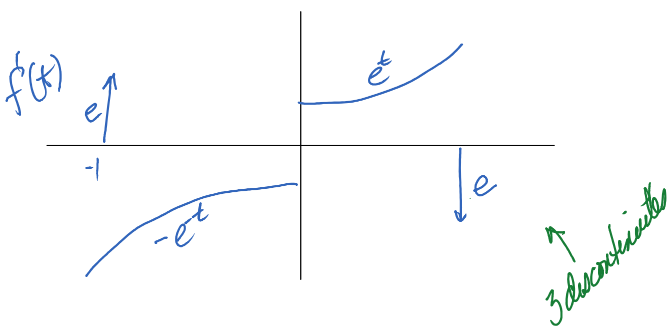
$$= \frac{6\pi}{8}\cdot\frac{2}{\pi}\cdot\frac{-2}{3\pi} = \frac{-1}{\pi}$$

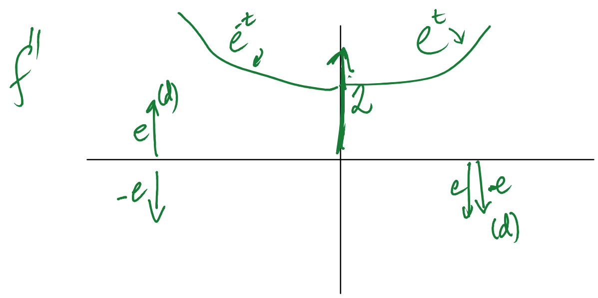
2014 Exam1, P3

November 1, 2014 12:03 AM



méthode de dérivés





$$f''(t) = e\delta'(t+1) - e\delta(t+1) + 2\delta(t) - e\delta'(t-1) - e\delta(t-1) + f(t)$$

$$(j\omega)^2 F(\omega) = e\,TF\{\delta'(t+1) - \delta'(t-1)\} - e\,TF\{\delta(t+1) + \delta(t-1)\} + f(t) + 2\delta(t)$$

$\delta(t)$	1	$\frac{d^n}{dt^n} f(t)$	$(j\omega)^n F(\omega)$

$\delta'(t) \Leftrightarrow j\omega$

$f(t+a)$	$e^{ja\omega} F(\omega)$

$$(j\omega)^2 F(\omega) = e\,\underbrace{TF\{\delta'(t+1) - \delta'(t-1)\}}_{j\omega\left[e^{j\omega} - e^{-j\omega}\right]} - e\,TF\underbrace{\{\delta(t+1) + \delta(t-1)\}}_{\left[e^{-j\omega} + e^{j\omega}\right]} + F(\omega) + 2$$

$$j\omega\left[e^{j\omega} - e^{-j\omega}\right] = j\omega \sin\omega \cdot 2j = -2\omega \sin\omega$$

$$\left[e^{-j\omega} + e^{j\omega}\right] = 2\cos\omega$$

$$(-w^2-1)F(w) = 2 - 2e\cos w - 2w\sin w$$

$$F(\omega) = \frac{2e\cos w + 2w\sin w - 2}{1+w^2}$$

Méthode d'intégration

$$F(w) = \int_{-\infty}^{\infty} f(t)\, e^{-jwt}\, dt = \int_{-1}^{0} e^{-t} e^{-jwt}\, dt + \int_{0}^{1} e^{t} e^{-jwt}\, dt$$

$$= -\frac{e^{-t(1+jw)}}{1+jw}\Bigg|_{-1}^{0} + \frac{e^{t(1-jw)}}{1-jw}\Bigg|_{0}^{1}$$

$$= -\frac{1-e^{1+jw}}{1+jw} + \frac{e^{1-jw}-1}{1-jw}$$

$$= \frac{-1+jw+e^{1+jw}-jwe^{1+jw}+e^{1-jw}+jwe^{1-jw}-1-jw}{1+w^2}$$

$$= \frac{-2+e\left(e^{jw}+e^{-jw}\right)+jwe\left[e^{-jw}-e^{jw}\right]}{1+w^2}$$

$$= \frac{-2+2e\cos w + jw\cdot 2j\cdot e\cdot(-1)\sin w}{1+w^2}$$

$$= \frac{-2+2e\cos w + 2we\sin w}{1+w^2}$$

b) 2 discontinuités $\Rightarrow$ f discontinue $\Rightarrow F(\omega) \propto 1/\omega$

$$F(\omega) = \frac{-2 + 2e\cos\omega + 2\omega e\sin\omega}{1+\omega^2}$$

comme $1/\omega$