Question 1

$f = 10\ MHz$
$\sigma = 10^{-4}\ S/m$
$\varepsilon = 2.25\,\varepsilon_0$
$\mu = \mu_0$

champ $\vec{E}$ suivant $+\hat{e}_x$
propagation suivant $+\hat{e}_z$
$P(z=0) = 1\ mW/m^2$

a) $$\frac{\sigma}{\omega\varepsilon} = \frac{10^{-4}}{2\pi \times 10 \times 10^{6} \times 2.25 \times 8.85 \times 10^{-12}} = 0{,}08$$

$\sigma \neq 0$ mais $\frac{\sigma}{\omega\varepsilon} \ll 1$, diélectrique imparfait.

b) $$v_p = \frac{c}{\sqrt{\varepsilon_r}} = \frac{3\times10^{8}}{\sqrt{2.25}} = 2\times10^{8}\ m/s$$

ou $$v_p = \frac{c}{\sqrt{\frac{\varepsilon_r}{2}\left(\left(1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2\right)^{1/2}+1\right)^{1/2}}} = 1{,}998\times10^{8}\ m/s$$

c) $$\eta = \sqrt{\frac{\mu}{\varepsilon}}\left(1 + j\frac{\sigma}{2\omega\varepsilon}\right) = \frac{376{,}8}{\sqrt{2.25}}\left(1 + j\,0{,}04\right)$$

$$\eta = 251\sqrt{1+(0.04)^2}\ \exp\left(j\,tg^{-1}\,0.04\right)$$

$$\eta = 252\ e^{j0.04}$$

d) $$\alpha = \frac{1}{2}\sigma\sqrt{\frac{\mu}{\varepsilon}} = \frac{1}{2}\sigma\frac{\eta_0}{\sqrt{\varepsilon}} = \frac{10^{-4}\times376{,}8}{2\times1.5} = 0{,}0126\ m^{-1}$$

$(1{,}256\times10^{-2}\ m^{-1})$

$$\beta = \frac{\omega}{v_p} = \frac{2\pi\times10\times10^{6}}{2\times10^{8}} = 0{,}314\ m$$

e) $\lambda = \frac{2\pi}{\beta} = 20\ m$

f) $P = \frac{1}{2|\eta|} |E|^2 \cos(\tau) = 1\ \frac{mW}{m^2} = 1 \times 10^{-3}\ \frac{W}{m^2}$

$|E| = 0.71\ V/m$

g) suivant $+\ \hat{a}_y$

h) $\vec{E} = E_0\, e^{-\alpha z} \cos(\omega t - \beta z)\, \hat{a}_x$

$\vec{H} = \frac{E_0}{|\eta|}\, e^{-\alpha z} \cos(\omega t - \beta z - \tau)\, \hat{a}_y$

$\vec{E} = 0.71\ e^{-0{,}0126 z} \cos(20\pi \times 10^6 t - 0{,}314 z)\, \hat{a}_x$ $\frac{V}{m}$

$\vec{H} = 0{,}0028\ e^{-0{,}0126 z} \cos(20\pi \times 10^6 t - 0{,}314 z - \tau)\, \hat{a}_y$ A/m

i) $\frac{P(z)}{P(0)} = 0{,}5 = e^{-2\alpha z} \Rightarrow z = -\frac{\ln(0.5)}{2 \times 0.0126}$

$\underline{z = 27.5\ m}$

j) $\tilde{\vec{E}} = 0{,}71\ e^{-0{,}0126 z}\ e^{-j\,0{,}314 z}\ \hat{a}_x$

$\tilde{\vec{E}} = 0{,}0028\ e^{-0{,}0126 z}\ e^{-j\,0{,}314 z}\ e^{-j\,0{,}04}\ \hat{a}_y$

Questions 2 :

a) Etant donné qu'il s'agit de conducteurs parfaits, les charges peuvent se déplacer et elles se retrouveront en surface, donc

à $r=a$ $\quad \rho_s = \dfrac{Q}{4\pi a^2}$

à $r=b$ $\quad \rho_s = \dfrac{-Q}{4\pi b^2}$ (pour annuler le champ ds le conducteur).

b) En utilisant la loi de Gauss, on trouve

$r<a \quad \vec{E}=0$

$a<r<b \quad \vec{E} = \dfrac{Q}{4\pi\varepsilon_0 r^2}\,\hat{a}_r$

$r>b \quad \vec{E}=0$

c) $U_e = \dfrac{1}{2}\iiint \varepsilon_0 |E|^2 \, dv$ $\Rightarrow$ entre $r=a$ et $r=b$ car le champ est nul partout ailleurs.

$$U_e = \frac{1}{2}\cancel{4\pi}\int_a^b \frac{\cancel{\varepsilon_0}\, Q^2}{\cancel{16}^{\,4}\pi^2 \varepsilon_0^{\cancel{2}}\, r^{\cancel{4}}}\, \cancel{r^2}\, dr$$

$$U_e = \frac{1}{8\pi\varepsilon_0}\, Q^2 \left.\frac{-1}{r}\right|_a^b = \frac{Q^2}{8\pi\varepsilon_0}\left(\frac{1}{a}-\frac{1}{b}\right)$$

Question 4 :

a) $$V(r,\theta,\varphi) = \frac{1}{4\pi\varepsilon_0}\left(\frac{-q}{r_{+a}} + \frac{2q}{r} + \frac{-q}{r_{-a}}\right)$$

$$V(r,\theta,\varphi) = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}\Bigg(-\left(1 + \frac{a}{r}\cos\theta - \frac{1}{2}\left(\frac{a}{r}\right)^2 + \frac{3}{2}\left(\frac{a}{r}\right)^2\cos^2\theta\right)$$
$$+ 2 - \left(1 - \frac{a}{r}\cos\theta - \frac{1}{2}\left(\frac{a}{r}\right)^2 + \frac{3}{2}\left(\frac{a}{r}\right)^2\cos^2\theta\right)\Bigg)$$

$$V(r,\theta,\varphi) = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}\left(\left(\frac{a}{r}\right)^2 - 3\left(\frac{a}{r}\right)^2\cos^2\theta\right)$$

$$V(r,\theta,\varphi) = \frac{1}{4\pi\varepsilon_0}\frac{a^2q}{r^3}\left(1 - 3\cos^2\theta\right)$$

$$V(r,\theta,\varphi) = \frac{1}{4\pi\varepsilon_0}\frac{a^2q}{r^3}\left(1-3\cos^2\theta\right)$$

b) $$\vec{E} = -\nabla V = -\frac{\partial V}{\partial r}\,\hat{\imath}_r - \frac{1}{r}\frac{\partial V}{\partial \theta}\,\hat{\imath}_\theta$$

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{a^2q}{r^4}\left[(1-3\cos^2\theta)\,\hat{\imath}_r - 6\cos\theta\sin\theta\,\hat{\imath}_\theta\right]$$

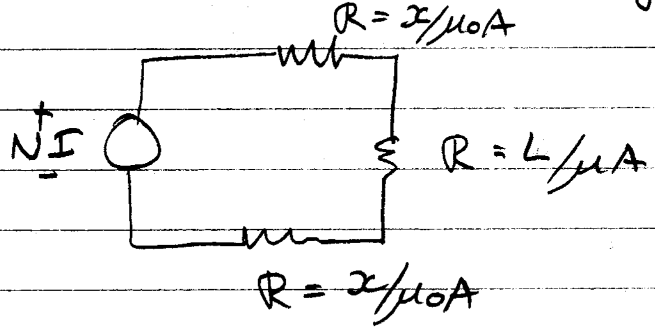
Question boni

$$\vec{E} = E_0\left(\cos(\omega t - \beta z)\,\hat{\imath}_x + \cos(\omega t - \beta z + \pi/2)\,\hat{\imath}_y\right)$$

$$\left.\begin{array}{l}\text{Amplitude égale}\\ \text{Différence de phase de } \pi/2\end{array}\right\}\ \text{circulaire}$$

Question 4 :

a) On utilise l'analogie électrique



$$\mathcal{R}_{tot} = \frac{2x}{\mu_0 A} + \frac{L}{\mu A}$$

$$\mathcal{R}_{tot} = \frac{1}{A}\left(\frac{2x}{\mu_0} + \frac{L}{\mu}\right)$$

$$\phi = \frac{NI}{\mathcal{R}_{tot}} = \frac{NIA}{\left(\frac{2x}{\mu_0} + \frac{L}{\mu}\right)}$$

b) $$L = N\frac{d\phi}{dI} = \frac{N^2 A}{\left(\frac{2x}{\mu_0} + \frac{L}{\mu}\right)}$$

c) $$U_m = \frac{1}{2}\iiint \mu |H|^2 dv$$

$$U_m = \frac{1}{2}\iiint \frac{1}{\mu}|B|^2 dv \qquad \text{avec} \qquad B = \frac{\phi}{A}$$

$$U_m = \frac{1}{2}\iiint \frac{1}{\mu}\frac{\phi^2}{A^2} dv$$

$$U_m = \frac{1}{2}\frac{\phi^2}{A^{\cancel{2}}}\left(\frac{1}{\mu_0}2x\cancel{A} + \frac{L\cancel{A}}{\mu}\right)$$

$$U_m = \frac{1}{2}\frac{\phi^2}{A}\left(\frac{2x}{\mu_0} + \frac{L}{\mu}\right)$$

d) $$U_m = \frac{1}{2}\frac{N^2 I^2 A^{\cancel{2}}}{\cancel{A}}\frac{1}{\left(\frac{2x}{\mu_0} + \frac{L}{\mu}\right)^{\cancel{2}}}\cancel{\left(\frac{2x}{\mu_0} + \frac{L}{\mu}\right)}$$

$$\boxed{U_m = \frac{1}{2}LI^2}$$