GEL-16120

Examen de mi-session

Automne 2006

Question 1 :

$$m(t) = \text{sinc}^2(t)$$

et $x(t) = m(t)\cos(2\pi f_c t) + m_h(t)\sin(2\pi f_c t)$

où $m_h(t) \triangleq \mathcal{H}[m(t)]$

a) pré-enveloppe positive $x_+(t)$

$$x_+(t) = x(t) + j\,x_h(t)$$

$$x_+(t) = [m(t)\cos(2\pi f_c t) + m_h(t)\sin(2\pi f_c t)] + j\,\mathcal{H}[m(t)\cos(2\pi f_c t) + m_h(t)\sin(2\pi f_c t)]$$

$$\boxed{x_+(t) = m(t)\cos(2\pi f_c t) + m_h(t)\sin(2\pi f_c t) + j\,m(t)\sin(2\pi f_c t) - j\,m_h(t)\cos(2\pi f_c t)}$$

ou encore

$$x_+(t) = [m(t) - j\,m_h(t)]\cos(2\pi f_c t) + [m_h(t) + j\,m(t)]\sin(2\pi f_c t)$$

ou encore

$$x_+(t) = m(t)\cos(2\pi f_c t) + j\,m(t)\sin(2\pi f_c t) + m_h(t)\sin(2\pi f_c t) - j\,m_h(t)\cos(2\pi f_c t)$$

$$\boxed{x_+(t) = m(t)\,e^{j2\pi f_c t} - j\,m_h(t)\,e^{j2\pi f_c t}}$$

b) représentation complexe en bande de base :

$$x_+(t) = \tilde{x}(t)\, e^{j2\pi f_c t}$$

$$\rightarrow \tilde{x}(t) = x_+(t)\, e^{-j2\pi f_c t}$$

$$\tilde{x}(t) = \left[m(t)\, e^{j2\pi f_c t} - j\, m_h(t)\, e^{j2\pi f_c t} \right] e^{-j2\pi f_c t}$$

$$\boxed{\tilde{x}(t) = m(t) - j\, m_h(t)}$$

c) spectre d'amplitude de $x(t)$: $X(f)$

$$X(f) = \mathcal{F}[x(t)]$$

$$X(f) = \mathcal{F}[m(t)\cos(2\pi f_c t) + m_h(t)\sin(2\pi f_c t)]$$

$$X(f) = \left[\mathcal{F}[m(t)] * \mathcal{F}[\cos(2\pi f_c t)]\right] + \left[\mathcal{F}[m_h(t)] * \mathcal{F}[\sin(2\pi f_c t)]\right]$$

$$\boxed{X(f) = M(f) * \frac{1}{2}\left[\delta(f-f_c) + \delta(f+f_c)\right] - j\,\mathrm{sgn}(f)\, M(f) * \frac{1}{2j}\left[\delta(f-f_c) - \delta(f+f_c)\right]}$$

où $M(f) = \mathcal{F}[m(t)] = \mathcal{F}[\mathrm{sinc}^2(t)] = \Lambda(f)$

(fonction triangulaire)

3

$$X(f) = \Lambda(f) * \frac{1}{2}[\delta(f-f_c) + \delta(f+f_c)]$$
$$- j\, \mathrm{sgn}(f)\Lambda(f) * \frac{1}{2j}[\delta(f-f_c) - \delta(f+f_c)]$$

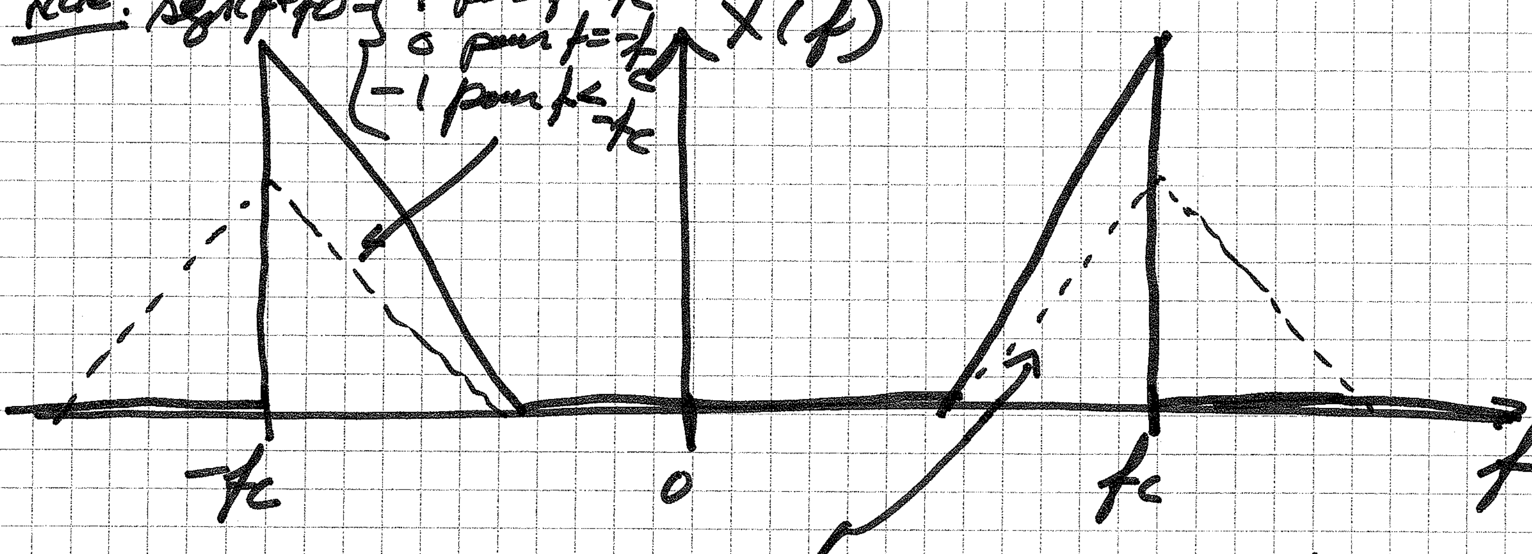
$$X(f) = \frac{\Lambda(f-f_c) + \Lambda(f+f_c)}{2}$$
$$\frac{-j\, \mathrm{sgn}(f-f_c)\Lambda(f-f_c) + j\, \mathrm{sgn}(f+f_c)\Lambda(f+f_c)}{2j}$$

$$\boxed{X(f) = \frac{\Lambda(f-f_c)[1 - \mathrm{sgn}(f-f_c)]}{2} + \frac{\Lambda(f+f_c)[1 + \mathrm{sgn}(f+f_c)]}{2}}$$

autour de $+f_c$

autour de $-f_c$

note: $\mathrm{sgn}(f+f_c) = \begin{cases} 1 & \text{pour } f > -f_c \\ 0 & \text{pour } f = -f_c \\ -1 & \text{pour } f < -f_c \end{cases}$



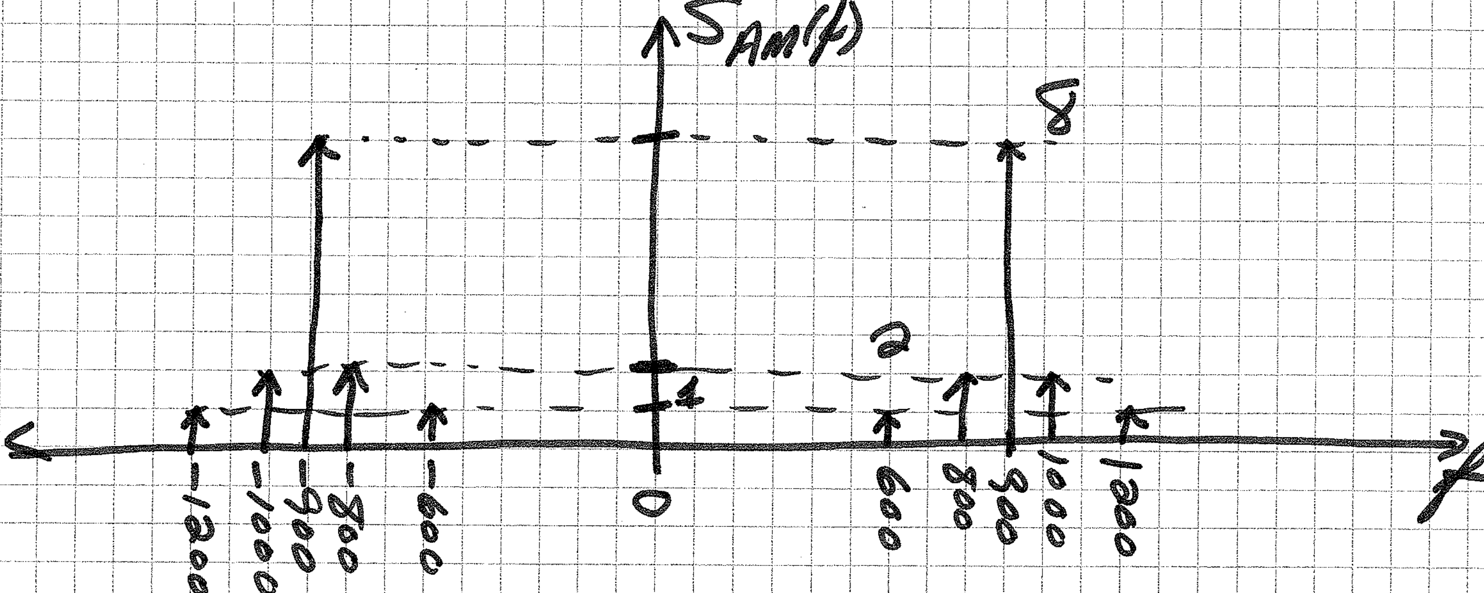
note: $\mathrm{sgn}(f-f_c) = \begin{cases} +1 & \text{pour } f > f_c \\ -1 & \text{pour } f < f_c \\ 0 & \text{pour } f = f_c \end{cases}$

Question 2

$$s_{AM}(t) = 2\cos(1200\pi t) + 4\cos(1600\pi t) + 16\cos(1800\pi t)$$
$$+ 4\cos(2000\pi t) + 2\cos(2400\pi t)$$

a) $S_{AM}(f) = \mathcal{F}[s_{AM}(t)]$

$$S_{AM}(f) = \delta(f-600) + \delta(f+600) + 2[\delta(f-800) + \delta(f+800)]$$
$$+ 8[\delta(f-900) + \delta(f+900)] + 2[\delta(f-1000) + \delta(f+1000)]$$
$$+ \delta(f-1200) + \delta(f+1200)$$



b) La porteuse est à $f_c = 900\,[Hz]$ et $k_a = 1/4$

$$s_{AM}(t) = A_c[1 + k_a m(t)]\cos(2\pi f_c t)$$

$$S_{AM}(f) = \frac{A_c}{2}[\delta(f-f_c) + \delta(f+f_c)]$$
$$+ \frac{A_c k_a}{2}[M(f+f_c) + M(f-f_c)]$$

$$\Rightarrow \frac{A_c}{2}\left[\delta(f-f_c)+\delta(f+f_c)\right] = 8\left[\delta(f-900)+\delta(f+900)\right]$$

$$\Rightarrow f_c = 900\ Hz \text{ et } \frac{A_c}{2} = 8 \Rightarrow A_c = 16$$

$$\Rightarrow \boxed{c(t) = A_c\cos(2\pi f_c t) = 16\cos(1800\pi t)}$$

$$\Rightarrow \frac{A_c k_a}{2}\left[M(f+f_c)+M(f-f_c)\right] = \frac{16\times 1/4}{2}\left[M(f+900)+M(f-900)\right]$$

$$= 2\left[M(f+900)+M(f-900)\right] = 2M(f) * \left[\delta(f+900)+\delta(f-900)\right]$$

$$= \delta(f-600)+\delta(f+600)+\delta(f-1200)+\delta(f+1200)$$
$$+2\left[\delta(f-800)+\delta(f+800)+\delta(f-1000)+\delta(f+1000)\right]$$

$$= \left[\delta(f-300)+2\delta(f-100)+2\delta(f+100)+\delta(f+300)\right]$$
$$* \left[\delta(f+900)+\delta(f-900)\right]$$

$$\Rightarrow M(f) = \frac{1}{2}\left[\delta(f-300)+\delta(f+300)\right] + \left[\delta(f-100)+\delta(f+100)\right]$$

$$\Rightarrow m(t) = \mathcal{F}^{-1}\left[M(f)\right]$$

$$m(t) = \cos(2\pi\times 300\times t) + 2\cos(2\pi\times 100\times t)$$

$$\boxed{m(t) = \cos(600\pi t) + 2\cos(200\pi t)}$$

vérification: ③

$$s_{Am}(t) = A_c\,[1 + k_a\, m(t)]\cos(2\pi f_c t)$$

$$s_{Am}(t) = 16\left[1 + \frac{1}{4}\left[\cos(600\pi t) + 2\cos(200\pi t)\right]\right]\cos(1800\pi t)$$

$$s_{Am}(t) = 16\cos(1800\pi t) + 4\cos(600\pi t)\cos(1800\pi t) + 8\cos(200\pi t)\cos(1800\pi t)$$

$$s_{Am}(t) = 16\cos(1800\pi t) + \frac{4}{2}\left[\cos(1200\pi t) + \cos(2400\pi t)\right] + \frac{8}{2}\left[\cos(1600\pi t) + \cos(2000\pi t)\right]$$

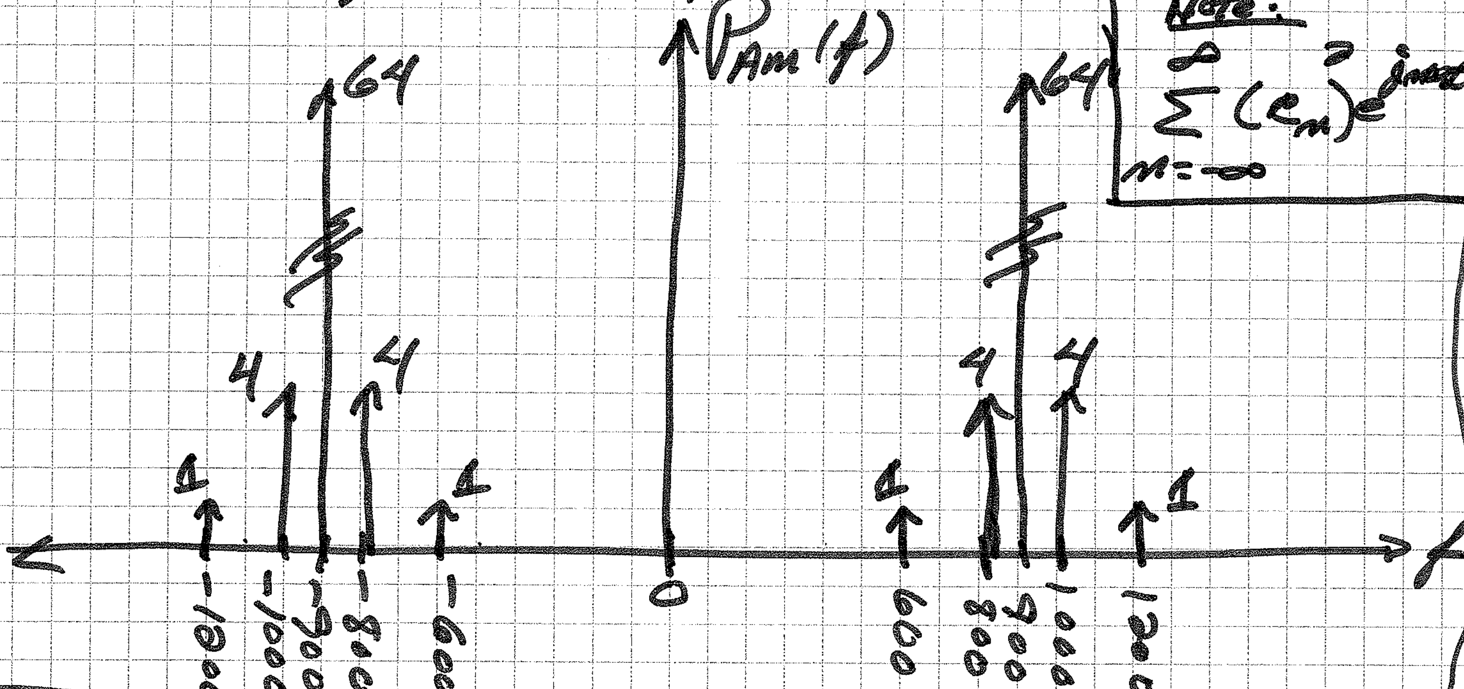
$$\boxed{s_{Am}(t) = 16\cos(1800\pi t) + 2\cos(1200\pi t) + 2\cos(2400\pi t) + 4\cos(1600\pi t) + 4\cos(2000\pi t)}$$

c) Densité spectrale de puissance $P_{AM}(f)$ 4

$$P_{AM}(f) = \frac{16^2}{4}\left[\delta(f-900)+\delta(f+900)\right]$$
$$+\ \frac{4^2}{4}\left[\delta(f+800)+\delta(f+800)+\delta(f-1000)+\delta(f+1000)\right]$$
$$+\ \frac{2^2}{4}\left[\delta(f-600)+\delta(f+600)+\delta(f-1200)+\delta(f+1200)\right]$$

$$\boxed{\begin{aligned}P_{AM}(f) = {} & \delta(f-600)+\delta(f+600)+4\delta(f-800)+4\delta(f+800)\\ & +64\delta(f-900)+64\delta(f+900)+4\delta(f-1000)+4\delta(f+1000)\\ & +\delta(f-1200)+\delta(f+1200)\end{aligned}}$$

Note : $\sum_{n=-\infty}^{\infty} (c_n)^2 e^{jn\omega t}$



d) $\eta_{AM} = \dfrac{P_{\text{bandes latérales}}}{P_{\text{TOTALE}}} = \dfrac{(4\times1)+(4\times4)}{(4\times1)+(4\times4)+(2\times64)}$

$$\boxed{\eta_{AM} = \frac{20}{148} = 13.51\,\%}$$

Question 3 : ①

$m(t) = 25 \text{ sinc } (4000t)$

$\beta_f = 8$ et $c(t) = 400 \cos(2 \times 10^7 \pi t)$

a) Sensibilité de la modulation FM D_f :

$\beta_f = \dfrac{\Delta f_{max}}{B}$ où $\Delta f_{max} = \dfrac{D_f}{2\pi} \max[|m(t)|]$

or $\max[|25 \text{ sinc}(4000t)|] = 25$

et $M(f) = \mathcal{F}[m(t)] = \left(\dfrac{25}{4000}\right) \Pi \left(\dfrac{f}{4000}\right)$

$\Rightarrow B = 2000 \ [Hz]$

$\Rightarrow \beta_f = \dfrac{D_f}{2\pi B} m[|m(t)|]$

$\Rightarrow D_f = \dfrac{2\pi \beta_f B}{m[|m(t)|]} = \dfrac{2\pi \times 8 \times 2000}{25}$

$$\boxed{D_f = 1280\pi = 4021 \ [rad/s\text{-}V]}$$

b) $s_{FM}(t) = A_c \cos\left[2\pi f_c t + D_f \int_0^t m(\tau)\, d\tau\right]$

$$\boxed{s_{FM}(t) = 400 \cos\left[(2 \times 10^7 \pi t) + 1280\pi \int_0^t 25 \text{ sinc}(4000\tau)\, d\tau\right]}$$

$s_{FM}(t) = 400 \cos\left[(2 \times 10^7 \pi t) + 32000\pi \int_0^t \text{sinc}(4000\tau)\, d\tau\right]$

$s_{FM}(t) = 400 \cos\left[(2 \times 10^7 \pi t) + 100531 \int_0^t \text{sinc}(4000\tau)\, d\tau\right]$

c) déviation maximale de fréquence Δf_{max}:

$$\Delta f_{max} = \beta_f B = 8 \times 2000$$

$$\boxed{\Delta f_{max} = 16\,000 \ [Hz]}$$

d) Largeur de bande effective B_T:

$$B_T = 2(\beta_f + 1)B$$ (règle de Carson)

$$B_T = 2(8+1)\,2000 = 18 \times 2000$$

$$\boxed{B_T = 36\,000 \ [Hz]}$$

Question 4 :

$$s(t) = 200\cos\left[(2\times10^{7}\pi t) + 12\sin(2\times10^{3}\pi t)\right]$$

a) Puissance moyenne :
modulation d'angle

$$\boxed{P = \frac{A_c^2}{2} = \frac{(200)^2}{2} = 20\,000\ [W]}$$

b) $\Delta\theta$ max (déviation maximale de phase)

$$\max\left[\,|\theta(t)|\,\right]$$

$$\max\left[\,|12\sin(2\times10^{3}\pi t)|\,\right]$$

$$\boxed{\Delta\theta_{max} = 12\ [\text{radians}]}$$

c) Δf_{max} (déviation maximale de fréquence)

$$\Delta f_{max} = \frac{1}{2\pi}\max\left\{\left|\frac{d}{dt}\left[\theta(t)\right]\right|\right\}$$

$$= \frac{1}{2\pi}\max\left\{\left|\frac{d}{dt}\,12\sin(2\times10^{3}\pi t)\right|\right\}$$

$$= \frac{12}{2\pi}\max\left\{\left|\frac{d}{dt}(2\times10^{3}\pi t)\times\cos(2\times10^{3}\pi t)\right|\right\}$$

$$= \frac{12}{2\pi}\times(2\times10^{3}\pi)\max\left\{|\cos(2\times10^{3}\pi t)|\right\}$$

$$\boxed{\Delta f_{max} = 12000\ Hz}$$

2

d) $$\boxed{\beta_F = \frac{\Delta f_{MAX}}{B} = \frac{12\,000\,Hz}{1000\,Hz} = 12}$$

$$B_T = 2(\beta_f + 1)B$$

$$B_T = 2(12+1)\,1000$$

$$\boxed{B_T = 26\,000\,Hz}$$

e) $$\beta_p = \Delta\theta_{MAX} = 12\,[\text{radians}]$$

$$B_T = 2(\beta_p + 1)B$$

$$B_T = 2(12+1)\,1000$$

$$\boxed{B_T = 26000\,Hz}$$