Question 1

a) $\oint \vec{D}\cdot d\vec{s} = \int \rho \, dV$

$2\pi r h \, D(r) = \int \rho \, dV$

pour $r<a$ $\quad 2\pi r h \, D(r) = \frac{\rho_L}{\pi a^2} \pi r^2 h$

$D(r) = \frac{\rho_L}{2\pi a^2} r$

$\Rightarrow \quad \vec{E}(r) = \frac{\rho_L r}{2\pi \varepsilon_0 a^2} \hat{e}_r$

$a<r<b$ $\quad 2\pi r h \, D(r) = \frac{\rho_L \pi a^2 h}{\pi a^2}$

$D(r) = \frac{\rho_L}{2\pi r}$

$\Rightarrow \quad \vec{E}(r) = \frac{\rho_L}{4\pi \varepsilon_0 r} \hat{e}_r$

$r>b$ $\quad 2\pi r h \, D(r) = \rho_L h - \rho_L h = 0$

$\Rightarrow \quad \vec{E}(r) = 0$

b) $w_e = \frac{1}{2} \varepsilon_0 E^2$

$r<a$ $\quad w_e = \frac{1}{2} \varepsilon_0 \frac{\rho_L^2 r^2}{4\pi^2 \varepsilon_0^2 a^4} = \frac{1}{8\pi^2} \frac{\rho_L^2}{\varepsilon_0 a^4} r^2$

$a<r<b$ $\quad w_e = \frac{1}{2} \frac{2\varepsilon_0 \, \rho_L^2}{16\pi^2 \varepsilon_0^2 r^2} = \frac{1}{16\pi^2} \frac{\rho_L^2}{\varepsilon_0} \frac{1}{r^2}$

$W_e = \iiint w_e \, dv = \iiint w_e \, r \, d\varphi \, dr \, dz$

$\frac{W_e}{h} = 2\pi \int w_e \, r \, dr$

$$\frac{W_e}{h} = \frac{1}{4\pi}\frac{\rho_L^2}{\varepsilon_0 a^4}\int_0^a r^3\,dr + \frac{1}{8\pi}\frac{\rho_L^2}{\varepsilon_0}\int_a^b \frac{1}{r}\,dr$$

$$\frac{W_e}{h} = \frac{1}{4\pi}\frac{\rho_L^2}{\varepsilon_0}\left(\frac{1}{4}\right) + \frac{1}{8\pi}\frac{\rho_L^2}{\varepsilon_0}\ln(b/a)$$

$$\boxed{\frac{W_e}{h} = \frac{1}{8\pi}\frac{\rho_L^2}{\varepsilon_0}\left(\frac{1}{2} + \ln(b/a)\right)}$$ 5 pts

<u>Question 3</u> :

a) $NI = \mathcal{R}\varphi$

$\mathcal{R} = \dfrac{x}{\mu_0 A} \Rightarrow \varphi = \dfrac{\mu_0 N I A}{x}$

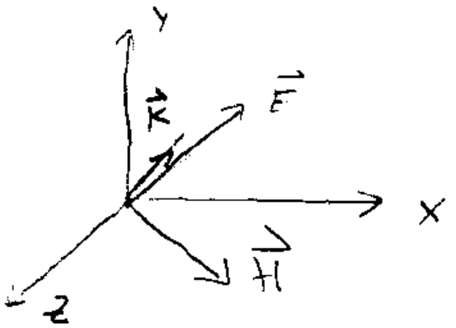
b) $L = N\dfrac{d\varphi}{dI} = \dfrac{\mu_0 N^2 A}{x}$

Question 2

$$\vec{E} = (\hat{e}_x + \hat{e}_y)/\sqrt{2}$$
$$\vec{H} = (\hat{e}_x - \hat{e}_y)/\sqrt{2}$$

a) $\vec{E} \times \vec{H} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 1/\sqrt{2} & -1/\sqrt{2} & 0 \end{vmatrix} \Rightarrow -\hat{z}$ donc direction $-\hat{z}$

b)



c) polarisation linéaire

d) ici on doit vérifier

$$\frac{\sigma}{\omega\varepsilon} = \frac{2}{2\pi \times 100 \times 10^{6} \times 2{,}5 \times 8{,}85 \times 10^{-12}} = 144$$

donc bon conducteur

e) $$\alpha = \beta = \sqrt{\frac{\omega\mu\sigma}{2}} = \sqrt{\pi \times 100 \times 10^{6} \times 4\pi \times 10^{-7} \times 2} = 28$$

$\alpha = 28 \ m^{-1}$

$\beta = 28$ rad/m

$$\eta = \sqrt{\frac{\omega\mu}{\sigma}}\, e^{j\pi/4} = 19{,}9\, e^{j\pi/4} \ \Omega$$

f) $v_p = \frac{c\sqrt{2}}{\sqrt{2,5}} \frac{1}{(1,44)^{1/2}} = 2,2\times10^{7}\ m/s = \sqrt{\frac{400\pi\times10^{6}}{4\pi\times10^{-7}\times 2}}$

g) on sait que à $z=0$

$$P_o = \frac{|E_o|^2}{2|\eta|}\cos(\tau)$$

$$|E_o| = \left|\frac{2|\eta| P_o}{\cos(\pi/4)}\right|^{1/2}$$

$$E_o = \left|\frac{2\times 19,9 \times 1\times10^{-3}}{\cos(45)}\right|^{1/2} = 0,24\ V/m$$

h) $$\vec{E} = E_o\frac{(\hat{\imath}_x+\hat{\imath}_y)}{\sqrt{2}}\, e^{\alpha z}\,(\cos(\omega t+\beta z))$$

$$\vec{H} = \frac{E_o}{\eta}\frac{(\hat{\imath}_x-\hat{\imath}_y)}{\sqrt{2}}\, e^{\alpha z}\cos(\omega t+\beta z)$$

avec le valeurs trouvées

$$\vec{E} = \underbrace{\frac{0,24}{\sqrt{2}}}_{\to 0,17}(\hat{\imath}_x+\hat{\imath}_y)\, e^{28z}\cos(\underbrace{200\pi\times10^{6}}_{6,28\times10^{8}}\, t+28z)$$

$$\vec{H} = \underbrace{\frac{0,24}{19\sqrt{2}}}_{8,5\times10^{-3}}(\hat{\imath}_x-\hat{\imath}_y)\, e^{28z}\cos(200\pi\times10^{6}\, t+28z-\pi/4)$$

i) $P(z) = 0,01\,P_o \Rightarrow \frac{P(z)}{P(0)} = e^{-2\alpha z}$ $\quad z = \frac{\ln(0,01)}{-2\alpha} = 8,1\ cm$

Question 4.

a) Dans les parties sans diélectrique on a

$$\vec{E}_1 = -\frac{\rho_s}{2\varepsilon_0}\hat{\imath}_x \quad \text{pour la plaque supérieure}$$

et $$\vec{E}_2 = -\frac{\rho_s}{2\varepsilon_0}\hat{\imath}_x \quad \text{pour la plaque inférieure}$$

donc $$\vec{E}_{tot} = -\frac{\rho_s}{\varepsilon_0}\hat{\imath}_x \quad \text{pour} \quad 0<x<d/3$$
$$\frac{2d}{3}<x<d$$

$$\vec{E}_{tot} = \frac{-\rho_s}{5\varepsilon_0}\hat{\imath}_x \quad \text{pour} \quad \frac{d}{3}<x<\frac{2d}{3}$$

b) $$V(x=3d) - V(x=0) = -\int_0^d \vec{E}\cdot d\vec{l}$$

$$= \frac{\rho_s}{\varepsilon_0}\frac{d}{3} + \frac{\rho_s}{5\varepsilon_0}\frac{d}{3} + \frac{\rho_s}{\varepsilon_0}\frac{d}{3}$$

$$\boxed{V(x=3d) = \frac{11\rho_s d}{15\varepsilon_0}}$$

c) $$C = \frac{Q}{\Delta V} = \frac{15\rho_s A}{11\rho_s d}\varepsilon_0 = \frac{15A\varepsilon_0}{11d}$$

d) pour une capacité plan ce serait

$$C = \frac{A\varepsilon_0}{d} \quad \text{donc } C_{\text{diélectrique}} > C_{\text{sans}}$$