

## GEL4200: Communications numériques 2017 Examen final

Wednesday 3 May 2017; Time: 13h30 to 15h20  
Documentation provided; calculator permitted

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### Problem 1 (25 points over 100)

Consider the following parity check matrix (control matrix).

$$H = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 1 & 0 & 1 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 & 0 & 1 & 0 & 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 1 & 0 & 1 & 0 & 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & 1 & 0 & 1 & 0 & 1 & 1 & 1 & 1 \end{bmatrix}$$

A. (5 points) Is this code systematic? What is the rate of the code?

Here is the parity matrix of another block code:

$$P = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

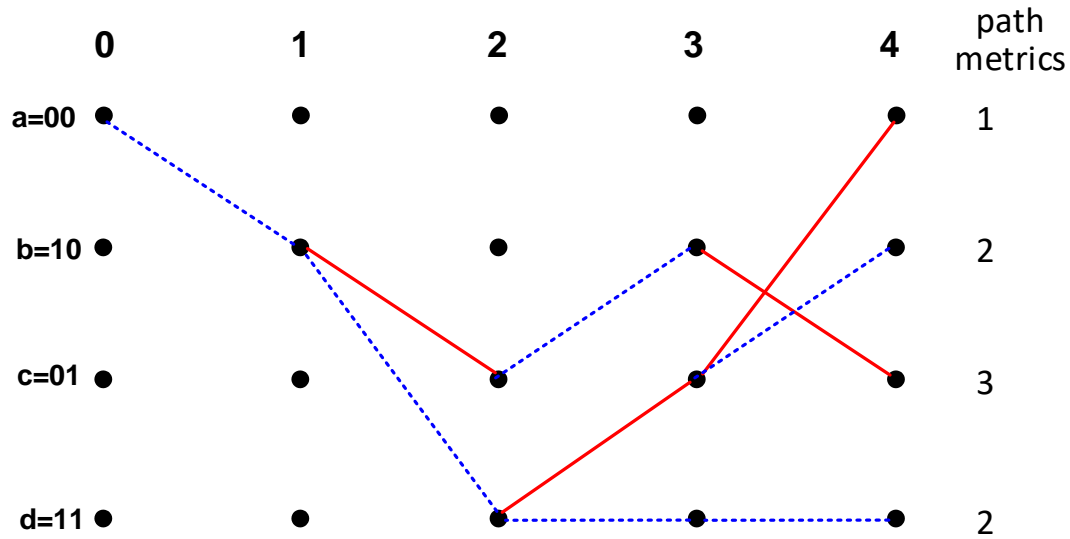
- B. (5 points) Give the generating matrix of a systematic version of the code.
- C. (5 points) How many bit errors can be corrected?
- D. (5 points) Give the syndrome table.
- E. (5 points) What is the output (the 2 message bits) of the decoder for received block  $[1 \ 1 \ 1 \ 1 \ 0]$  ?

**Problem 2 (26 points over 100)**

- A. (15 points) Consider symbol-by-symbol detection vs sequence detection. Give an example of an equalizing technique for each type of detection. Which detection/equalizer is more effective? Under what circumstances? At what cost?
- B. (11 points) DMT was first applied to telephone line modems. OFDM is frequently used in wireless communications, and DMT is almost unheard of in this application. Why was DMT a good choice for telephone line modems, rather than OFDM? Why is DMT inappropriate for wireless channels? Justify your response.

**Problem 3 (26 points over 100)**

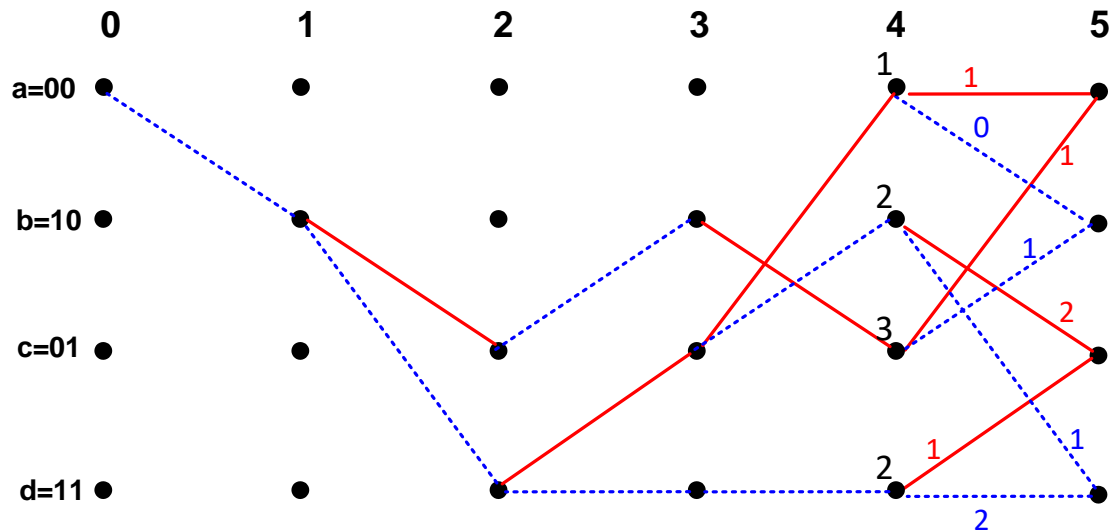
For the Viterbi Algorithm trellis below, the surviving paths up to time 4 are indicated (solid lines for logical zero, dotted lines for logical one). The path metric (using the Hamming distance) of each surviving path is indicated.



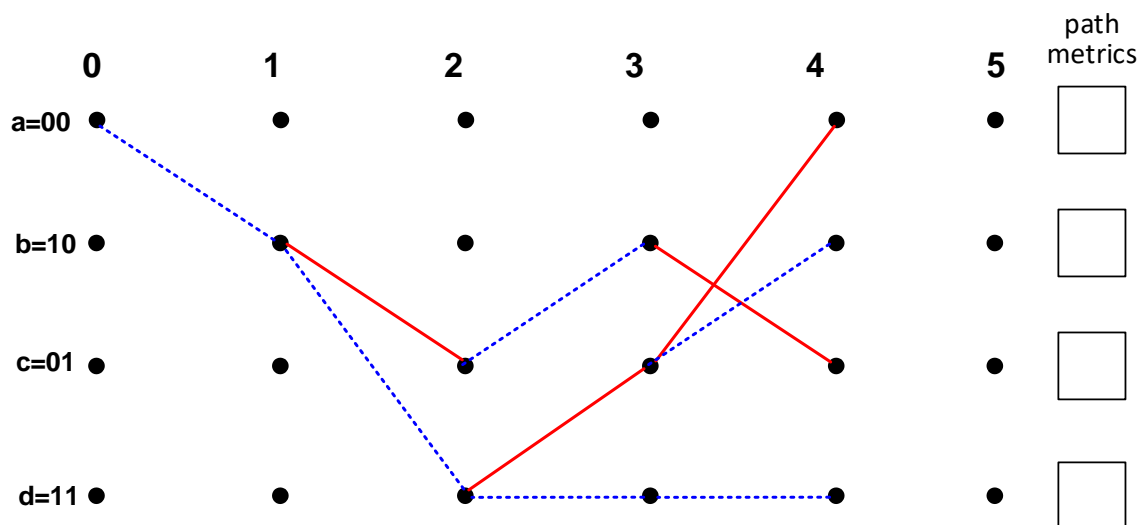
- A. (10 points) What is the most likely bit sequence transmitted? How many error(s), if any, occurred in the most likely bit sequence? Were the error(s) corrected?

**Problem 3 (continued)**

We continue the trellis for the next time interval. The branch metrics (using the Hamming distance) are indicated next to each possible transition in the trellis for this time interval.



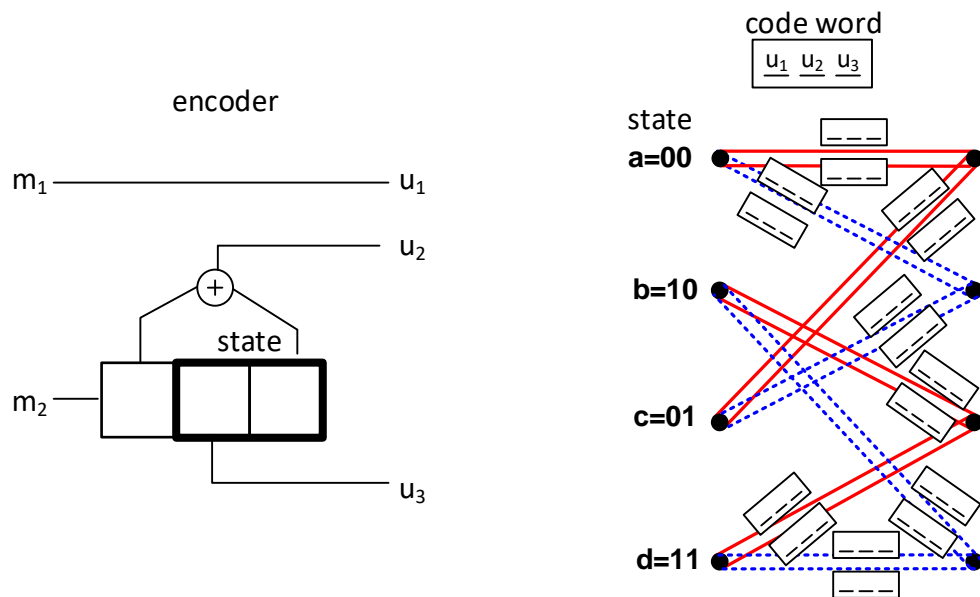
B. (16 points) Complete the following diagram for the surviving paths at time 5 and the path metric (using the Hamming distance) for each surviving path.



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**Problem 4 (23 points over 100)**

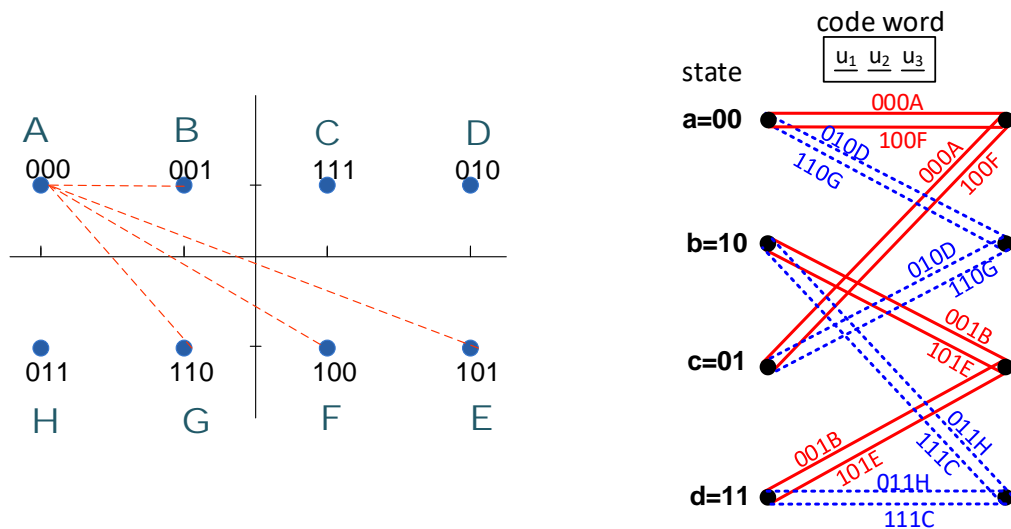
- A. (8 points) Complete the table for the code words that go in the trellis diagram for the following TCM (trellis coded modulation) encoder.



	upper code word ( $m_1 = 0$ )	upper code word ( $m_1 = 1$ )
Transition $a \rightarrow a$		
Transition $a \rightarrow b$		
Transition $b \rightarrow c$		
Transition $b \rightarrow d$		
Transition $c \rightarrow a$		
Transition $c \rightarrow b$		
Transition $d \rightarrow c$		
Transition $d \rightarrow d$		

### Problem 4 (continued)

Consider this TCM system using coded 8QAM to replace uncoded QPSK



To help you with your calculations, you can use the following information

Distances 8QAM

$$\text{dist}^2(\text{AB}) = \text{dist}^2(\text{AH}) = 2$$

$$\text{dist}^2(\text{AE}) = 6.32$$

$$\text{dist}^2(\text{AF}) = 4.47$$

$$\text{dist}^2(\text{AG}) = 2.83$$

$$E_s = 3$$

Distances QPSK

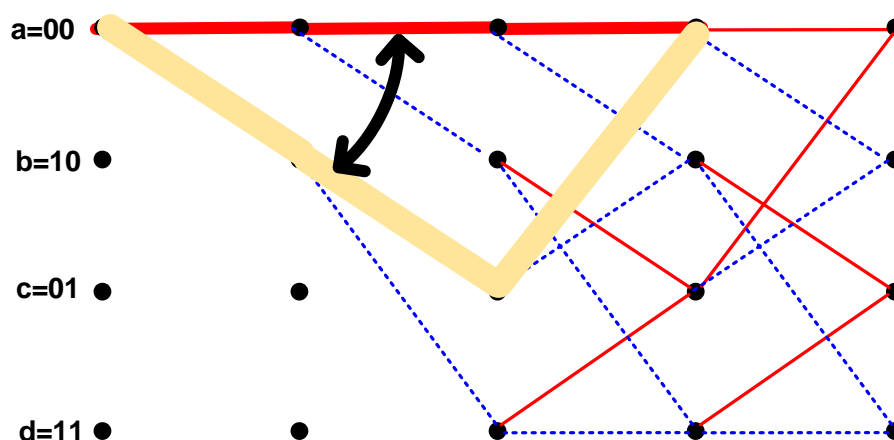
$$\text{dist}^2(\text{BC}) = 2$$

$$\text{dist}^2(\text{BF}) = 2.83$$

$$d_{\min} = \sqrt{2}$$

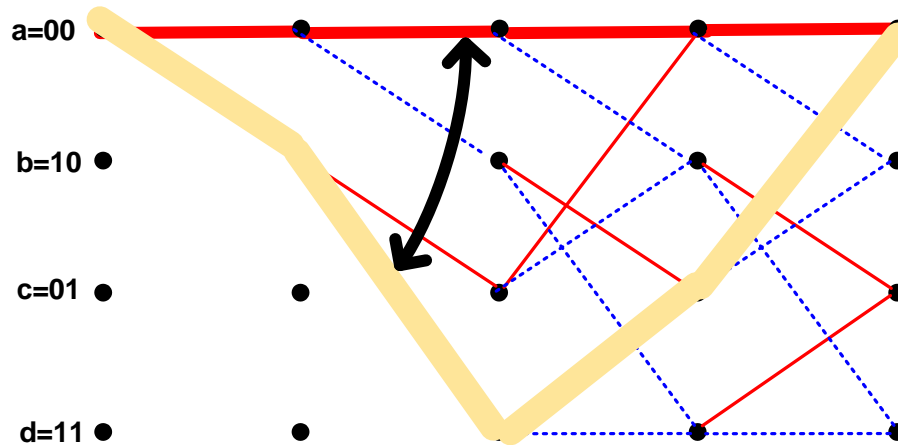
$$\tilde{E}_s = 2$$

- B. (15 points) Find the minimum Euclidean distance between the path of all state "a" transitions and the highlighted path



**Problem 4 (continued)**

- C. (5 points) Find the minimum Euclidean distance between the path of all state "a" transitions and the highlighted path.



- D. (5 points) Find the gain of the TCM 8QAM system vis-à-vis the uncoded QPSK system using

$$G \text{ (dB)} = 10 \log_{10} \left( \frac{d_f^2 / E_s}{d_{\min}^2 / \tilde{E}_s} \right)$$