Université Laval
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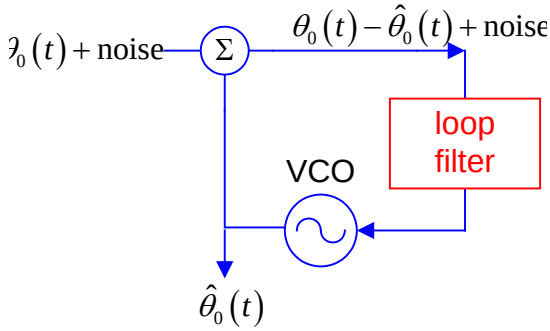
Département de génie électrique
et de génie informatique

GEL10280: Communications numériques
2003 Final Exam

Wednesday 30 April 2003; Time: 10h30 to 12h20
One page of notes allowed; one calculator allowed

Problem 1 (25 points out of 100)



A diagram of a linearized PLL is given here. The gain of the VCO is *K*. Suppose the loop filter is a low pass filter with the following frequency response

$$F(\omega)=\frac{\omega_0}{\omega_0+j\omega}$$

Find the response $\hat{\Theta}_0(\omega)$ and the asymptotic error for inputs:

1. step function, $\Theta_0(j\omega)=1/j\omega$ and also find $\hat{\theta}_0(t)$
2. a ramp, $\Theta_0(j\omega)=1/(j\omega)^2$.

f(*t*)	*F*(*j*ω)
$u(t)=\begin{cases}1 & t>0\\ 0 & t<0\end{cases}$	$\frac{1}{j\omega}$
$\frac{1}{\omega_0}u(t)\left[1-e^{-\omega_0 t}\right]$	$\frac{1}{j\omega}\frac{1}{j\omega+\omega_0}$
$\frac{1}{\omega_0}u(t)\left[t-\frac{1-e^{-\omega_0 t}}{\omega_0}\right]$	$\frac{1}{(j\omega)^2}\frac{1}{j\omega+\omega_0}$
$1-\frac{e^{-\varsigma\omega_n t}}{\sqrt{1-\varsigma^2}}\sin\left(\omega_n t\sqrt{1-\varsigma^2}+\cos^{-1}\varsigma\right)$	$\frac{1}{j\omega}\frac{\omega_n^2}{(j\omega)^2+j\omega 2\varsigma\omega_n+\omega_n^2}$

Problem 2 (10 points out of 100)

Consider the improvement in message error probability for a BPSK system with coherent detection with and without error correction. We use a linear block code with rate (24,12) that can correct two errors.

A. (10 points) For $\frac{E_b}{N_0} = 10\,\text{dB}$, is there an improvement in the message error probability? If yes, how much? If no, why not?

You may use the approximation

$$Q(x) \approx \frac{1}{x\sqrt{2\pi}} e^{-x^2/2}$$

Problem 3 (15 points out of 100)

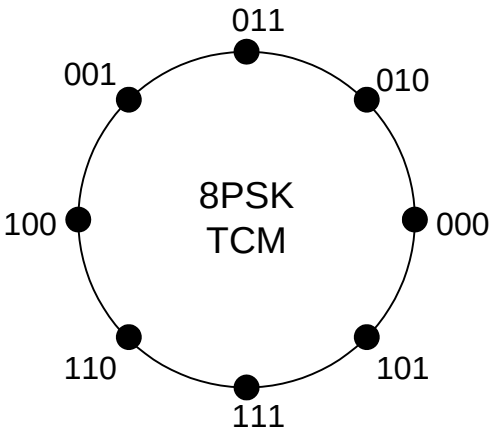
A. (5 points) Give three ways to generate a phase reference for a PLL.

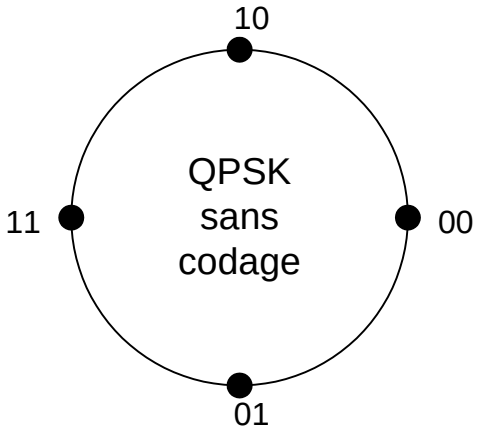
B. (5 points) What calculations are used in the Viterbi algorithm?

C. (5 points) How is trellis coded modulation (TCM) able to correct error while remaining bandwidth efficient?

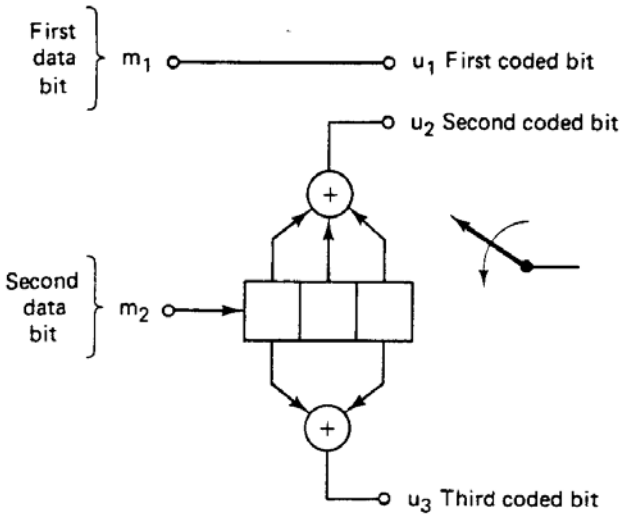
Problem 4 (25 points out of 100)

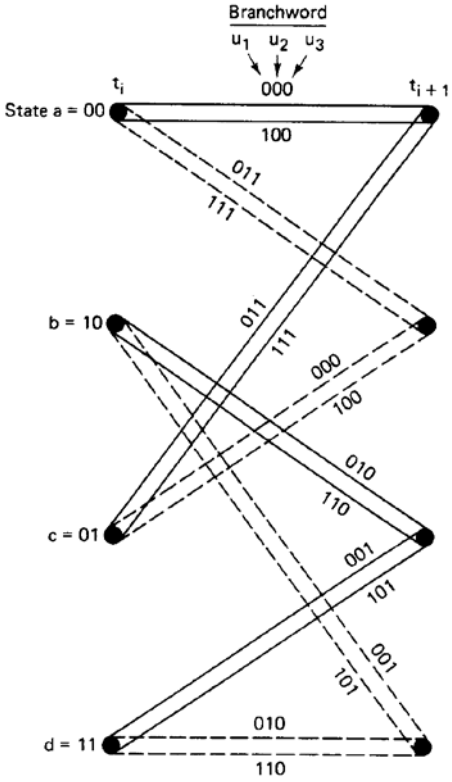
In class we found the asymptotic coding gain for 8PAM TCM vs. 4PAM without coding. Find the asymptotic coding gain for 8PSK TCM vs. QPSK without coding. Here is the code word mapping for the associated 8PSK symbols.





This is the convolutional code and the encoder trellis. Don't forget to use Euclidean distance in your calculations.





Problem 5 (25 points out of 100)

Here is the parity matrix for a block code (7,4) :

$$P = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

A. (5 points) Complete the standard array in the attached sheet.

B. (5 points) Give the parity check matrix.

C. (5 points) Complete the syndrome table in the attached sheet.

D. (10 points) What is the output (message bits) from the decoder when the received signal is

1 1 0 1 1 1 0

Nom : _____ Matricule :_____

error	Syndrome
0000001	
0000010	
0000100	
0001000	
0010000	
0100000	
1000000	