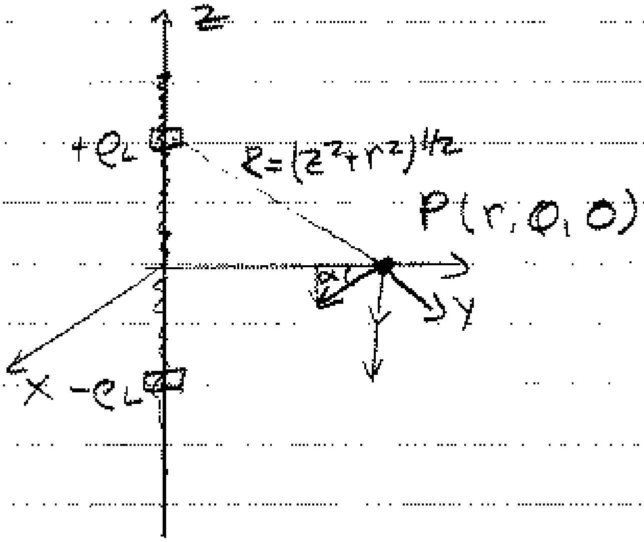
Question 1 :



en considérant deux éléments de charge situés symétriquement en $+z$ et $-z$, on constate que le champ résultant est orienté suivant $-\hat{a}_z$.

$$dE_z = 2\,\frac{\rho_L\,dz}{4\pi\varepsilon_0 R^2}\,\sin\alpha \quad \text{avec} \quad \sin\alpha = \frac{z}{R}$$

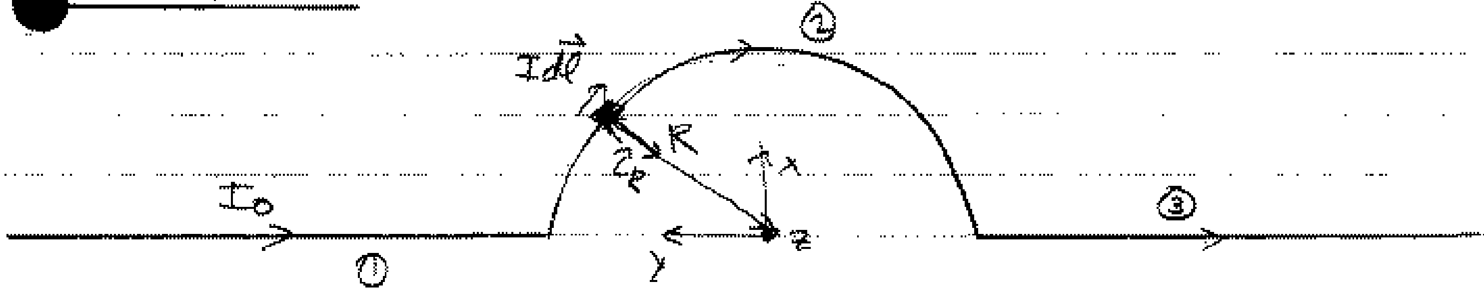
$$dE_z = 2\,\frac{\rho_L\, z\,dz}{4\pi\varepsilon_0 (z^2+r^2)^{3/2}}$$

$$E_z = \frac{2\rho_L}{4\pi\varepsilon_0}\int_0^\infty \frac{z\,dz}{(z^2+r^2)^{3/2}}$$

$$E_z = \frac{2\rho_L}{4\pi\varepsilon_0}\left.\frac{-1}{\sqrt{z^2+r^2}}\right|_0^\infty$$

$$E_z = \frac{\rho_L}{2\pi\varepsilon_0 r} \Rightarrow \boxed{\vec{E}(r) = \frac{\rho_L}{2\pi\varepsilon_0 r}(-\hat{a}_z)}$$

Question 2



$$\vec{dB} = \frac{\mu_0}{4\pi} \frac{I\vec{dl} \times \hat{r}_R}{R^2}$$

$$\vec{dB} = \frac{\mu_0}{4\pi} \frac{I R d\phi}{R^2} \underbrace{(-\hat{r}_\phi \times -\hat{r}_r)}_{-\hat{r}_z}$$

$$\vec{B} = \left[\frac{\mu_0 I}{4\pi R} \int_0^{-\pi} d\phi\right] -\hat{r}_z$$

$$\boxed{\vec{B} = \frac{\mu_0 I}{4R} \hat{r}_z}$$

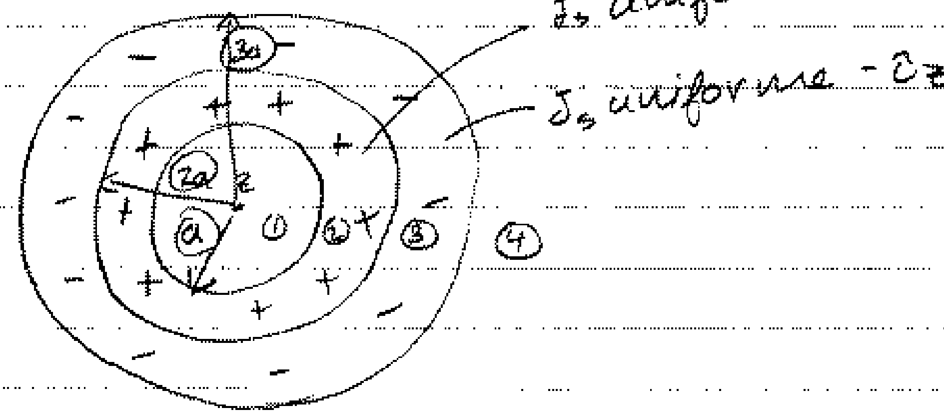
- Les segments de fils droits ne contribuent pas au champ.

- aussi anneau de courant sur son axe

$$\vec{B} = \frac{\mu_0 a^2 I}{2[z^2+a^2]^{3/2}} \hat{r}_z = \frac{\mu_0 I}{2a} \hat{r}_z$$

pour 1/2 anneau $\boxed{\vec{B} = \frac{\mu_0 I}{4a} \hat{r}_z}$

Question 3 :



Question → quel est le champ magnétique de $r=0$ à $r=\infty$.

On doit appliquer la loi d'Ampère dans chacune des régions de l'espace

$$\oint \vec{H} \cdot d\vec{l} = \int \vec{J} \cdot d\vec{s}$$

Étant donné la symétrie du problème on sait que

$$\vec{H} = H_\phi(r)\,\hat{a}_\phi \quad \text{et} \quad \oint \vec{H} \cdot d\vec{l} = H_\phi(r)\, 2\pi r$$

Il faut donc trouver $\int \vec{J} \cdot d\vec{s}$ pour chacune des régions

① $r < a$ $\quad \int \vec{J} \cdot d\vec{s} = 0$

② $2a < r < a$ $\quad \int \vec{J} \cdot d\vec{s} = \pi (r^2 - a^2) J_{so}$

③ $2a < r < 3a$ $\quad \int \vec{J} \cdot d\vec{s} = \pi ((2a)^2 - a^2) J_{so} - \pi (r^2 - (2a)^2) J_{so}$

$$= \pi J_{so} \left[4a^2 - a^2 - r^2 + 4a^2 \right]$$

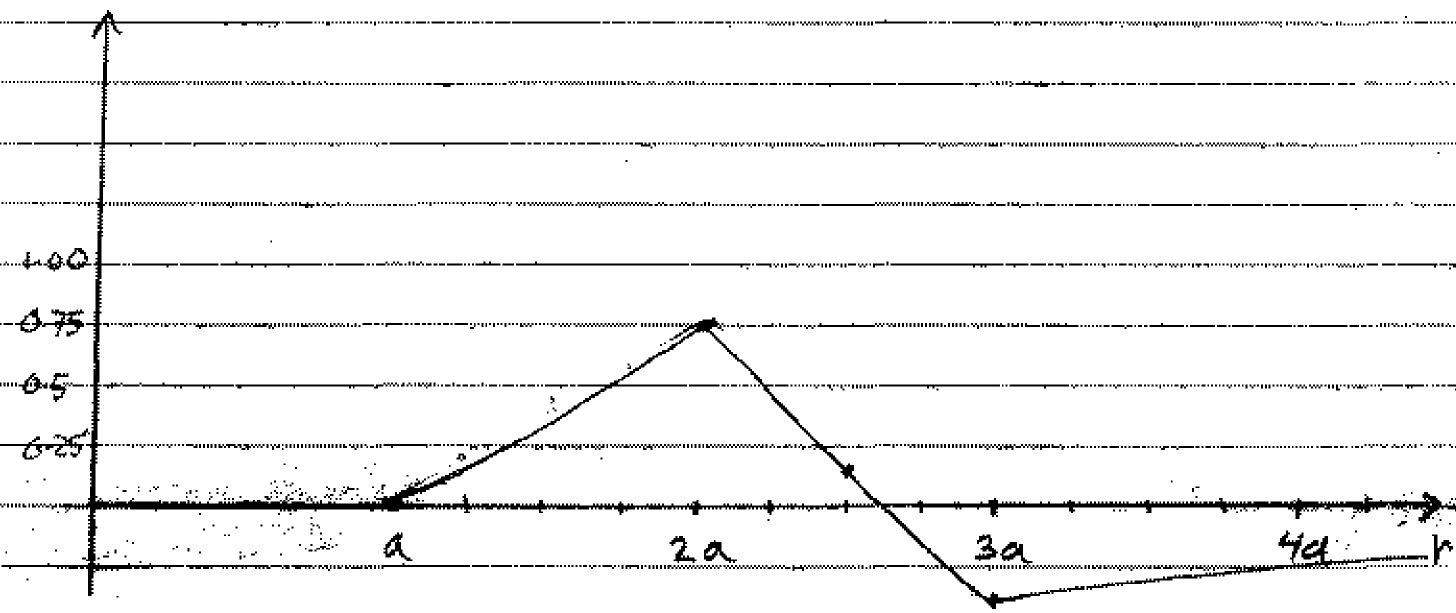
$$= \pi J_{so} \left[7a^2 - r^2 \right]$$

(4) $r > 3a$ $\quad \int \vec{J} \cdot d\vec{S} = \pi J_{s0}\left[7a^2 - 9a^2\right] = -2\pi a^2 J_{s0}$

Donc

$$H_\varphi(r)\,2\pi r = \begin{cases} 0 & r < a \\ \pi(r^2 - a^2)J_{s0} & a < r < 2a \\ \pi(7a^2 - r^2)J_{s0} & 2a < r < 3a \\ -2\pi a^2 J_{s0} & 3a < r \end{cases}$$

$$H_\varphi(r) = \begin{cases} 0 & r < a \\ \frac{1}{2}\left(r - \frac{a^2}{r}\right)J_{s0} & a < r < 2a \\ \frac{1}{2}\left(\frac{7a^2}{r} - r\right)J_{s0} & 2a < r < 3a \\ -\frac{a^2}{r} J_{s0} & r > 3a \end{cases}$$

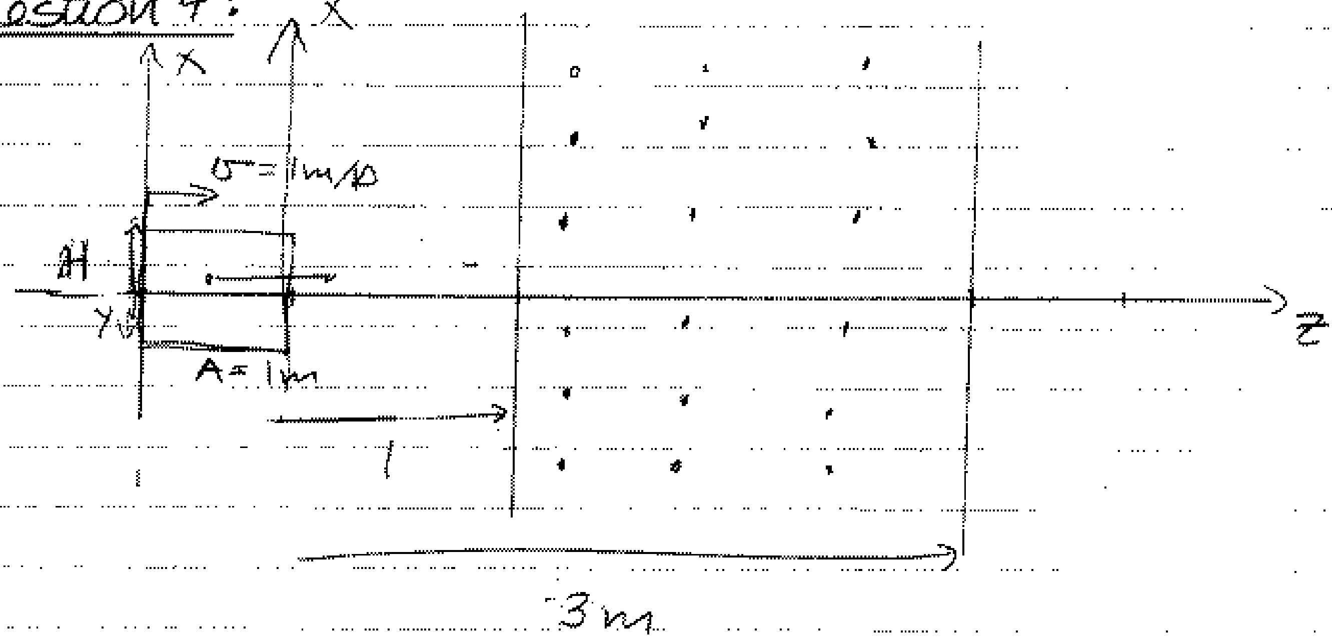


en quantités normalisées (pour aider au graphique)

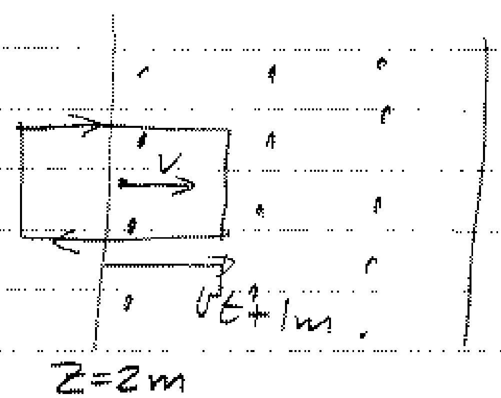
$$H_\phi(r) = \begin{cases} 0 \\ \frac{a}{2}\left((r/a) - \frac{1}{(r/a)}\right) J_{s0} & 1 < \frac{r}{a} < 2 \\ \frac{a}{2}\left(\frac{7}{(r/a)} - (r/a)\right) J_{s0} & 2 < \frac{r}{a} < 3 \\ -\frac{a}{(r/a)} J_{s0} & r > 3 \end{cases}$$

$$\frac{H_\phi(r)}{aJ_{s0}} = \begin{cases} 0 \\ \frac{1}{2}\left(r' - \frac{1}{r'}\right) & 1 < r' < 2 \\ \frac{1}{2}\left(\frac{7}{r'} - r'\right) & 2 < r' < 3 \\ -\frac{1}{r'} & r' > 3 \end{cases}$$

Question 4 :



à $t = \frac{1m}{1m/s} = 1s$ la boucle commence à traverser la région où il y a un champ magnétique.



Fém dans sens horaire pour s'opposer à la variation de flux.

$$fém = -\frac{d}{dt}\int \vec{B}.ds = -\frac{d}{dt} H \int_2^{vt} (B a_y).(-a_y dz)$$

$$fém = H\ B \frac{d}{dt}\left[vt + 1 - 2\right]$$

$\boxed{fém = 0}$ pour $0 < t < 1$

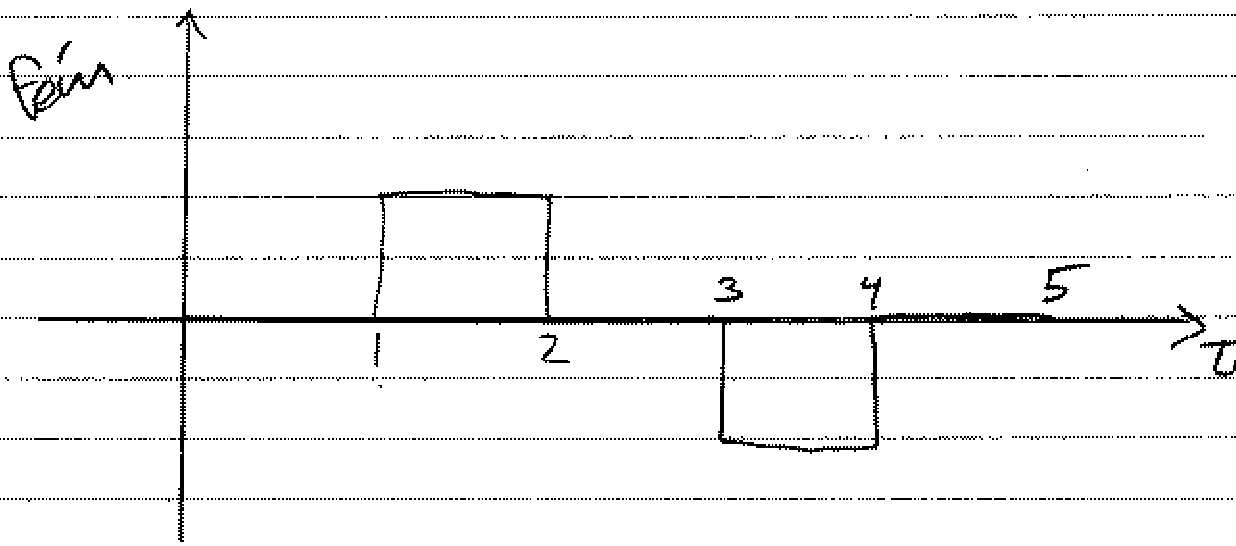
$\boxed{fém = HBv}$ pour $1 < t < 2$

$\boxed{fém = 0}$ pour $2 < t < 3$

et $f\acute{e}m = -HBv$ $\quad 3 < t < 4$

avec $v = 1$ m/s

$f\acute{e}m = 0$ $\quad t > 4$



Question boni :

$$E_1 = E_{01}(\cos\theta \, \hat{e}_r - \sin\theta \, \hat{e}_\theta)$$

$$E_2 = E_{02}\left[\left(1 + \frac{a^3}{3r^3}\right)\cos\theta \, \hat{e}_r - \left(1 - \frac{a^3}{2r^3}\right)\sin\theta \, \hat{e}_\theta\right]$$

rapport ε_1 et ε_2

composante tangentielle continue à $r = a$

$$-E_{01}\sin\theta \, \hat{e}_\theta = -E_{02}\left(1 - \frac{1}{2}\right)\sin\theta \, \hat{e}_\theta$$

$$E_{01} = \frac{1}{2}E_{02}$$

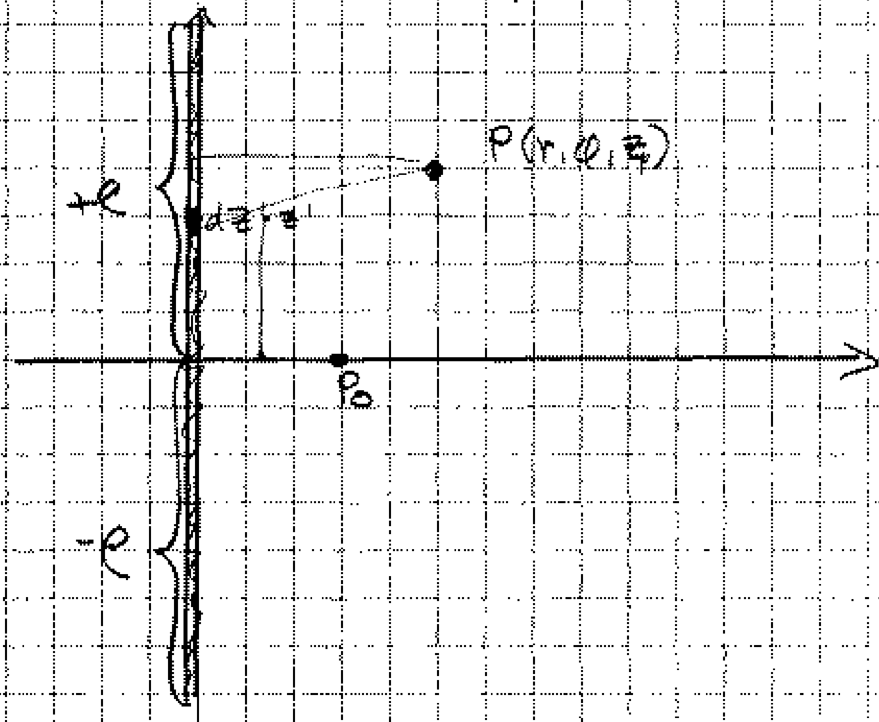
composante normale de D continue

$$\varepsilon_1 E_{01} \cancel{\cos\theta} \, \hat{e}_r = \varepsilon_0 E_{02} \frac{4}{3} \cancel{\cos\theta \, \hat{e}_r}$$

$$\varepsilon_1 E_{01} = \varepsilon_0 \, 2E_{01} \, \frac{4}{3}$$

$$\boxed{\varepsilon_1 = \frac{8}{3}\varepsilon_0}$$

En utilisant le potentiel:



à P_0 , la contribution au potentiel est

$$dV_{\pm} = \frac{\pm \rho_L \, dz}{4\pi\varepsilon_0 \sqrt{r_0^2 + z^2}}$$

à P , la contribution au potentiel est

$$dV_{\pm} = \frac{\pm \rho_L \, dz}{4\pi\varepsilon_0 \sqrt{r^2 + (z_p - z)^2}}$$

$$V = \frac{\rho_L}{4\pi\varepsilon_0} \int_0^{\infty} \left[\frac{1}{\sqrt{r^2 + (z_p - z)^2}} - \frac{1}{\sqrt{r_0^2 + z^2}} \right] dz$$

$$- \frac{\rho_L}{4\pi} \int_{-\infty}^{0} \left(\frac{1}{\sqrt{r^2 + (z_p - z)^2}} - \frac{1}{\sqrt{r_0^2 + z^2}} \right) dz$$

②

$$V = \frac{\rho_L}{4\pi\varepsilon_0}\left[\int_0^\infty \frac{dz}{\sqrt{r^2+(z_p-z)^2}} - \cancel{\int_0^\infty \frac{dz}{\sqrt{r_0^2+z^2}}} - \int_{-\infty}^0 \frac{dz}{\sqrt{r^2+(z_p-z)^2}} + \cancel{\int_{-\infty}^0 \frac{dz}{\sqrt{r_0^2+z^2}}}\right]$$

$$V = \frac{\rho_L}{4\pi\varepsilon_0}\left[\int_0^\infty \frac{dz}{\sqrt{r^2+(z_p-z)^2}} - \int_{-\infty}^0 \frac{dz}{\sqrt{r^2+(z_p-z)^2}}\right]$$

$$z' = z_p - z$$

$$z = 0 \Rightarrow z' = z_p$$

$$dz' = -dz$$

$$V = \frac{\rho_L}{4\pi\varepsilon_0}\left[\int_{z_p}^{-\infty} \frac{-dz'}{\sqrt{r^2+z'^2}} - \int_{\infty}^{z_p} \frac{-dz'}{\sqrt{r^2+z'^2}}\right]$$

$$V = \frac{\rho_L}{4\pi\varepsilon_0}\left[\int_{-\infty}^{z_p} \frac{dz'}{\sqrt{r^2+z'^2}} - \int_{z_p}^{\infty} \frac{dz'}{\sqrt{r^2+z'^2}}\right]$$

$$V = \frac{\rho_L}{4\pi\varepsilon_0}\left[\int_{-\infty}^{z_p} \frac{dz'}{\sqrt{r^2+z'^2}} - \int_{-z_p}^{-\infty} \frac{-dz''}{\sqrt{r^2+z''^2}}\right]$$

$$z'' = -z'$$

$$V = \frac{\rho_L}{4\pi\varepsilon_0}\left[\int_{-\infty}^{z_p} \frac{dz'}{\sqrt{r^2+z'^2}} - \int_{-\infty}^{-z_p} \frac{dz''}{\sqrt{r^2+z''^2}}\right]$$

$$V = \frac{\rho_L}{4\pi\varepsilon_0}\left[\int_{-z_p}^{z_p} \frac{dz'}{\sqrt{r^2+z'^2}}\right]$$

$$V = \frac{\rho_L}{2\pi\varepsilon_0}\left[\int_{0}^{z_p} \frac{dz'}{\sqrt{r^2+z'^2}}\right]$$

$$V = \frac{\rho_L}{2\pi\varepsilon_0}\left[\ln\left(\sqrt{r^2+z^2}+z\right)\right]\Big|_0^{z_p}$$

$$\boxed{V = \frac{\rho_L}{2\pi\varepsilon_0}\left[\ln\left(\sqrt{r^2+z^2}+z\right) - \ln(r)\right]}$$

$$\nabla\Phi = \frac{\partial\Phi}{\partial r}a_r + \frac{1}{r}\frac{\partial\Phi}{\partial\phi}a_\phi + \frac{\partial\Phi}{\partial z}a_z$$

$$\frac{\partial V}{\partial r} = \frac{\rho_L}{2\pi\varepsilon_0}\left[\frac{1\,(1/2)(r^2+z^2)^{-1/2}(2r)}{\sqrt{r^2+z^2}+z} - \frac{1}{r}\right]$$

$$\boxed{\frac{\partial V}{\partial r} = \frac{\rho_L}{2\pi\varepsilon_0}\left[\frac{r}{\left[\sqrt{r^2+z^2}+z\right](r^2+z^2)^{1/2}} - \frac{1}{r}\right]}$$

$$\frac{\partial V}{\partial z} = \frac{\rho_L}{2\pi\varepsilon_0}\left[\frac{1\left[(1/2)(2z)(r^2+z^2)^{-1/2}+1\right]}{\left(\sqrt{r^2+z^2}+z\right)}\right]$$

$$\boxed{\frac{\partial V}{\partial z} = \frac{\rho_L}{2\pi\varepsilon_0}\left[\frac{z}{\left(\sqrt{r^2+z^2}+z\right)(r^2+z^2)^{1/2}} + \frac{1}{\sqrt{r^2+z^2}+z}\right]}$$

$$\boxed{\vec{E} = -\frac{\rho_L}{2\pi\varepsilon_0}\left\{\left(\frac{r}{\left[\sqrt{r^2+z^2}+z\right](r^2+z^2)^{1/2}} - \frac{1}{r}\right)\hat{a}_r - \left(\frac{z}{\left(\sqrt{r^2+z^2}+z\right)(r^2+z^2)^{1/2}} + \frac{1}{\sqrt{r^2+z^2}+z}\right)\hat{a}_z\right\}}$$

@ $z = 0$

$$\vec{E} = -\frac{\rho_L}{2\pi\varepsilon_0}\left\{\left(\frac{1}{r} - \frac{1}{r}\right)\hat{a}_r + \left(0 + \frac{1}{r}\right)\hat{a}_z\right\}$$

$$\boxed{\vec{E} = \frac{-\rho_L}{2\pi\varepsilon_0 r}\,\hat{a}_z}$$