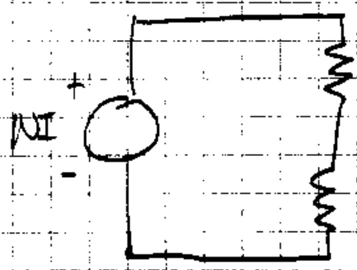
Examen final 2001

Question 1 (25)

a) Le circuit magnétique équivalent est



$$\mathcal{R}_{air} = \frac{2x}{\mu_0 A}$$

(5)

$$\mathcal{R}_{mag} = \frac{4l}{\mu_r \mu_0 A}$$

$$\mathcal{R}_{tot} = \frac{2}{\mu_0 A}\left(x + \frac{2l}{\mu_r}\right)$$

$$\phi = \frac{NI}{\mathcal{R}_{tot}}$$

$$\phi = \frac{NI\mu_0 A}{2(x + 2l/\mu_r)} \quad (5)$$

$$\boxed{L = \frac{N\phi}{I} = \frac{N^2 \mu_0 A}{2x\left(1 + \frac{2l}{x\mu_r}\right)}} \quad (5)$$

b) sans considérer l'armature

$$L_s = \frac{N^2 \mu_0 A}{2x}$$

$$\text{erreur} = \frac{L_s - L_a}{L_a} = \frac{1 - \frac{1}{(1+2l/x\mu_r)}}{\frac{1}{(1+2l/x\mu_r)}} \quad (5)$$

$$\text{erreur} = 1 + \frac{2l}{x\mu_r} - 1 = \frac{2l}{x\mu_r}$$

$$\text{erreur} = \frac{20 \times 10^{-2}}{2 \times 10^{-3} \times 450} = \frac{0,2}{0,002 \times 450} = 0,22$$

$$\boxed{\text{erreur} = 22\%} \quad (5)$$

Question 2 (20)

(2.5 par section)

a) selon $+\hat{a}_z$

b) $v_p = \frac{\omega}{\beta} = \frac{6{,}28\times10^6}{0{,}0272} = 2{,}31\times10^8$

c) $n = \frac{c}{v_p} = \frac{3}{2.31} = 1{,}3 = \sqrt{\varepsilon_r} \Rightarrow \varepsilon_r = 1{,}69$

$\varepsilon = 1{,}69\,\varepsilon_0 = 1{,}5\times10^{-11}$

d) $\eta = \eta_0 \sqrt{\frac{\mu_r}{\varepsilon_r}} = \frac{\eta_0}{1{,}3} = \frac{377}{1.3} = 290\ \Omega$

e) $f = \frac{6{,}28\times10^6}{2\pi} = 1\times10^6$ Hz

f) $\lambda = \frac{2\pi}{\beta} = \frac{2\pi}{0{,}0272} = 231$ m

g) $\vec{H} = \frac{1\times10^{-3}}{290}\cos(6{,}28\times10^6 t - 0{,}0272 z)\,\hat{a}_y$

$\vec{H} = 3{,}45\times10^{-6}\cos(6.28\times10^6 t - 0.0272 z)\,\hat{a}_y$

h) $\langle\vec{P}\rangle = \frac{1}{2\eta}|E|^2$

$\langle\vec{P}\rangle = \frac{1}{2\times290}|1\times10^{-3}|^2 = 1{,}72\times10^{-9}\ \frac{W}{m^2}$

Question 3

a) $\eta_1 = \eta_0 \quad \eta_2 = \eta_0/\sqrt{2.5}$

$r = \frac{1/\sqrt{2.5} - 1}{1/\sqrt{2.5} + 1} = -0{,}225$ 5

$t = \frac{2/\sqrt{2.5}}{1/\sqrt{2.5} + 1} = 0{,}775$

$$R = r^2 = 0{,}0507$$

$$I_r = I_0 R = 0{,}0507 \ mW/cm^2$$ (5)

$$I_t = 1 - I_r = 0{,}949 \ mW/cm^2$$

$$I_t = \frac{1}{1/\eta_{2.5}} t^2 = 0{,}949 \ mW/cm^2$$

b) Dans ce cas

$$\frac{\sigma}{\omega\varepsilon} = \frac{4 \times 10^7}{200 \times 10^{12} \times 8{,}85 \times 10^{-12}} = 2{,}26 \times 10^4$$

$$\frac{\sigma}{\omega\varepsilon} \gg 1$$

$$\eta = \sqrt{\frac{\omega\mu}{\sigma}} \ e^{j\pi/4}$$

$$\eta = \sqrt{\frac{400\pi \times 10^{12} \times 4\pi \times 10^{-7}}{4 \times 10^7}} \ e^{j\pi/4}$$

2.5 $$\eta = 2\pi \ e^{j\pi/4}$$

2.5 $$r = \frac{2\pi e^{j\pi/4} - 377}{2\pi e^{j\pi/4} + 377} = \frac{j4{,}44 - 372{,}56}{j4{,}44 + 381{,}44}$$

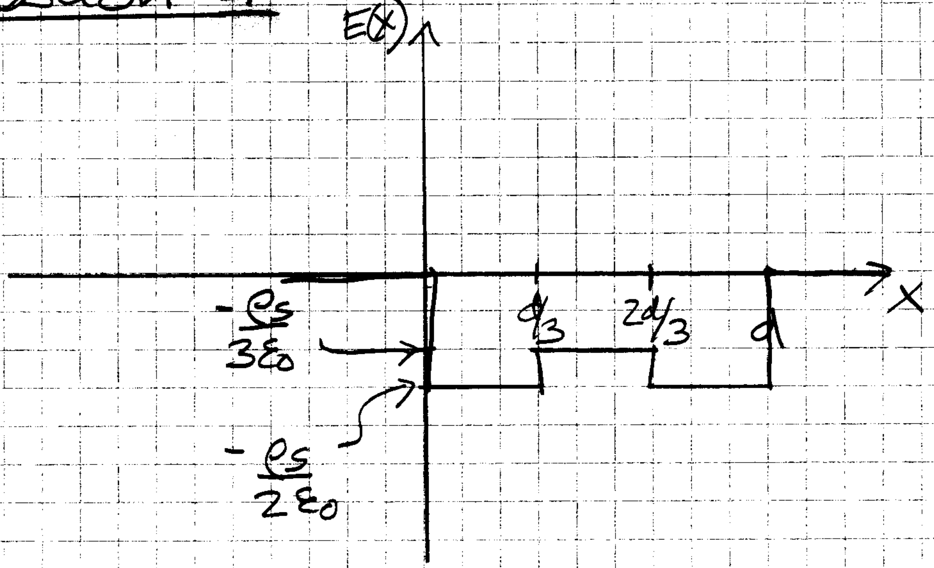
5.0 $$R = r r^* = \frac{(4{,}44)^2 + (372{,}56)^2}{(4{,}44)^2 + (381{,}44)^2} = 0{,}95$$

$$\boxed{R = 95\%}$$

$$P_r = 0{,}95 \ mW/cm^2$$

Question 4

a)

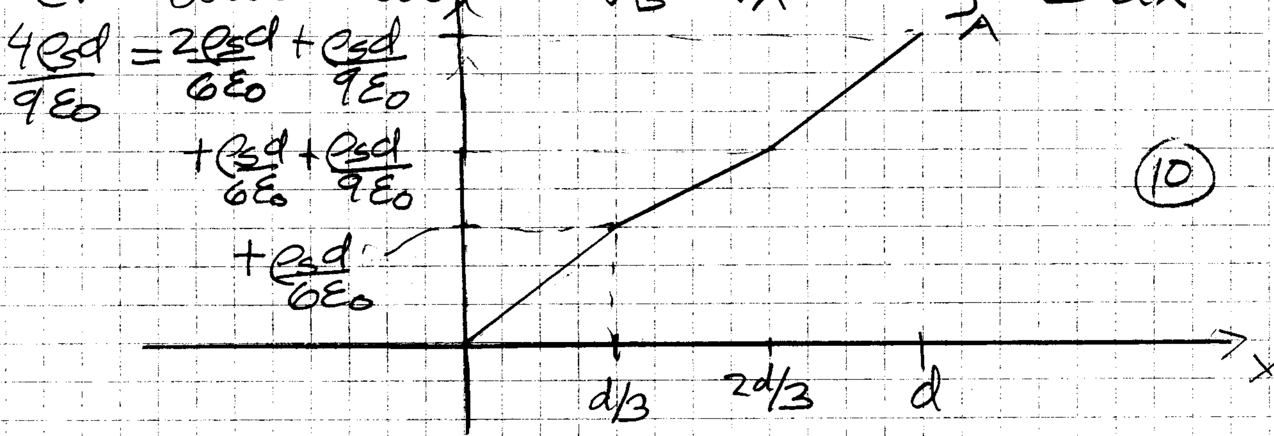


région ① $-\frac{\rho_s}{2\varepsilon_0}$ (10)

région ② $-\frac{\rho_s}{3\varepsilon_0}$

région ③ $-\frac{\rho_s}{2\varepsilon_0}$

b) en utilisant $V_B - V_A = -\int_A^B \vec{E}\cdot\vec{dx}$



(10)

c) Capacité $= \frac{Q}{\Delta V} = \frac{\rho_s A}{\frac{4\rho_s d}{9\varepsilon_0}} = \frac{9}{4}\frac{A\varepsilon_0}{d}$

ou avec $C_1 = \frac{6\varepsilon_0 A}{d}$, $C_2 = \frac{9\varepsilon_0 A}{d}$, $C_3 = \frac{6\varepsilon_0 A}{d}$

$$\frac{1}{C_{tot}} = \frac{2d}{6\varepsilon_0 A} + \frac{1}{9}\frac{d}{\varepsilon_0 A} = \frac{4}{9}\frac{d}{\varepsilon_0 A}$$

$$C_{tot} = \frac{9}{4}\frac{\varepsilon_0 A}{d}$$ (10)

d) $U_e = \frac{1}{2} C V^2$

$$U_e = \frac{1}{2}\frac{9}{4}\frac{\varepsilon_0 A}{d}\left(\frac{4\rho_s d}{9\varepsilon_0}\right)^2$$

$$U_e = \cancel{\frac{1}{2}}\,\cancel{\frac{9}{4}}\,\frac{\cancel{\varepsilon_0} A}{\cancel{d}}\,\frac{\cancel{16}^{\,4\,2}\rho_s^2 d^{\cancel{2}}}{9\,\cancel{81}\,\varepsilon_0^{\cancel{2}}}$$

$$\boxed{U_e = \frac{2\rho_s^2 A d}{9\varepsilon_0}}$$

(5)

ou $U_e = \frac{1}{2}\iiint \varepsilon |E|^2 dV$

$$U_e = \frac{1}{2}\frac{dA}{3}\left[\frac{2\cancel{\varepsilon_0}\rho_s^2}{2\,\cancel{4}\,\varepsilon_0^{\cancel{2}}} + \frac{3\cancel{\varepsilon_0}\rho_s^2}{3\,\cancel{9}\,\varepsilon_0^{\cancel{2}}} + \frac{2\cancel{\varepsilon_0}\rho_s^2}{2\,\cancel{4}\,\varepsilon_0^{\cancel{2}}}\right]$$

$$U_e = \frac{d}{6}A\frac{\rho_s^2}{\varepsilon_0}\left[1 + \frac{1}{3}\right]$$

$$\boxed{U_e = \frac{2}{9}\frac{dA\rho_s^2}{\varepsilon_0}}$$ $\frac{1}{3\cancel{6}} \times \frac{4}{3}^{2}$

boni elliptique (5)