

## GEL4200: Communications numériques 2011 Examen final

*Mercredi le 27 avril 2011; Durée: 13h30 à 15h20  
Documentation fournie; une calculatrice permise*

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### Problème 1 (25 points sur 100)

Suppose we have a second order PLL whose loop filter has frequency response :

$$F(\omega) = \frac{1}{j\omega + 1}$$

and unitary loop gain (that is,  $K_0=1$ ).

- A. (10 points) Give the phase estimate  $\hat{\theta}(t)$  when there is a phase jump of height .1 radians at  $t=0$ .
- B. (5 points) What is the asymptotic error?
- C. (10 points) What is the maximal excursion of the phase estimate as a percentage of the phase jump of .1 radians?

$g(t)$	$G(j\omega)$
$\frac{1}{\omega_0} u(t) [1 - e^{-\omega_0 t}]$	$\frac{1}{j\omega} \frac{1}{j\omega + \omega_0}$
$\frac{1}{\omega_0} u(t) \left[ t - \frac{1 - e^{-\omega_0 t}}{\omega_0} \right]$	$\frac{1}{(j\omega)^2} \frac{1}{j\omega + \omega_0}$
$1 - \frac{e^{-\zeta\omega_n t}}{\sqrt{1-\zeta^2}} \sin\left(\omega_n t \sqrt{1-\zeta^2} + \cos^{-1} \zeta\right)$	$\frac{1}{j\omega} \frac{\omega_n^2}{(j\omega)^2 + j\omega 2\zeta\omega_n + \omega_n^2}$

**Problème 2 (30 points sur 100)**

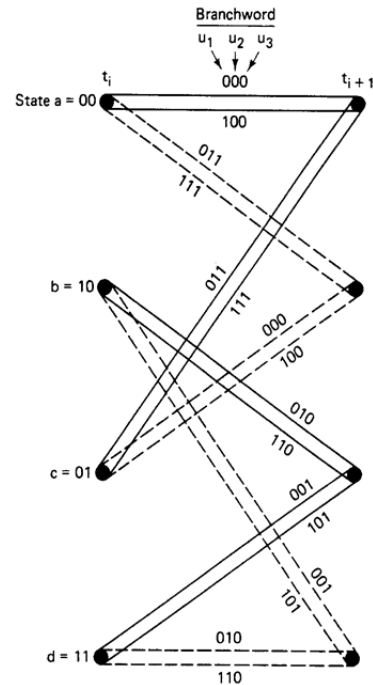
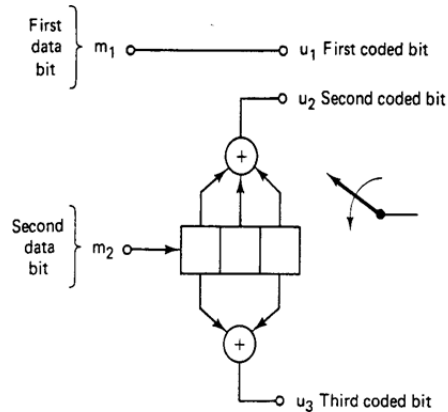
Consider the following parity control matrix for a block code:

$$H^T = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$$

- A. (10 points) What is the shift register implementation of the encoder?
- A. (10 points) What is the minimal distance?
- B. (10 points) Give the syndrome table.

### Problème 3 (20 points sur 100)

## TCM encoder



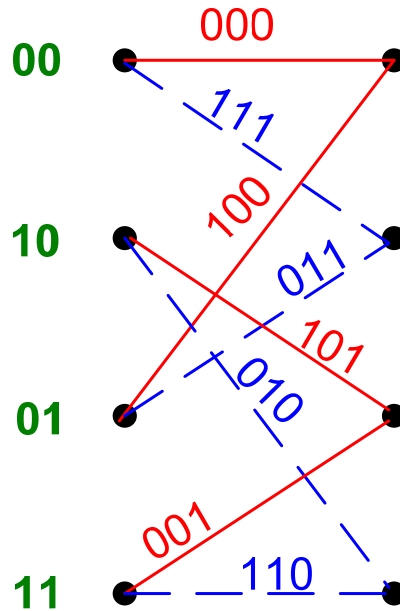
Consider the use of 8 QAM with either of the two following constellation mappings between the logical symbols and the I/Q coordinates. Which of the mappings is most appropriate for TCM and why?

Option A

Option B

**Problème 4 (25 points sur 100)**

Consider the following encoding trellis for a convolutional code.



- A. (5 points) What is the code rate? What is the constraint length?
- B. (20 points) Find the free distance (or minimal distance)  $d_f$  of the convolutional code using hard decisions. How many paths are there at the minimal distance?

Please use the trellis decoding sheets that are provided and place them in your exam booklet.

