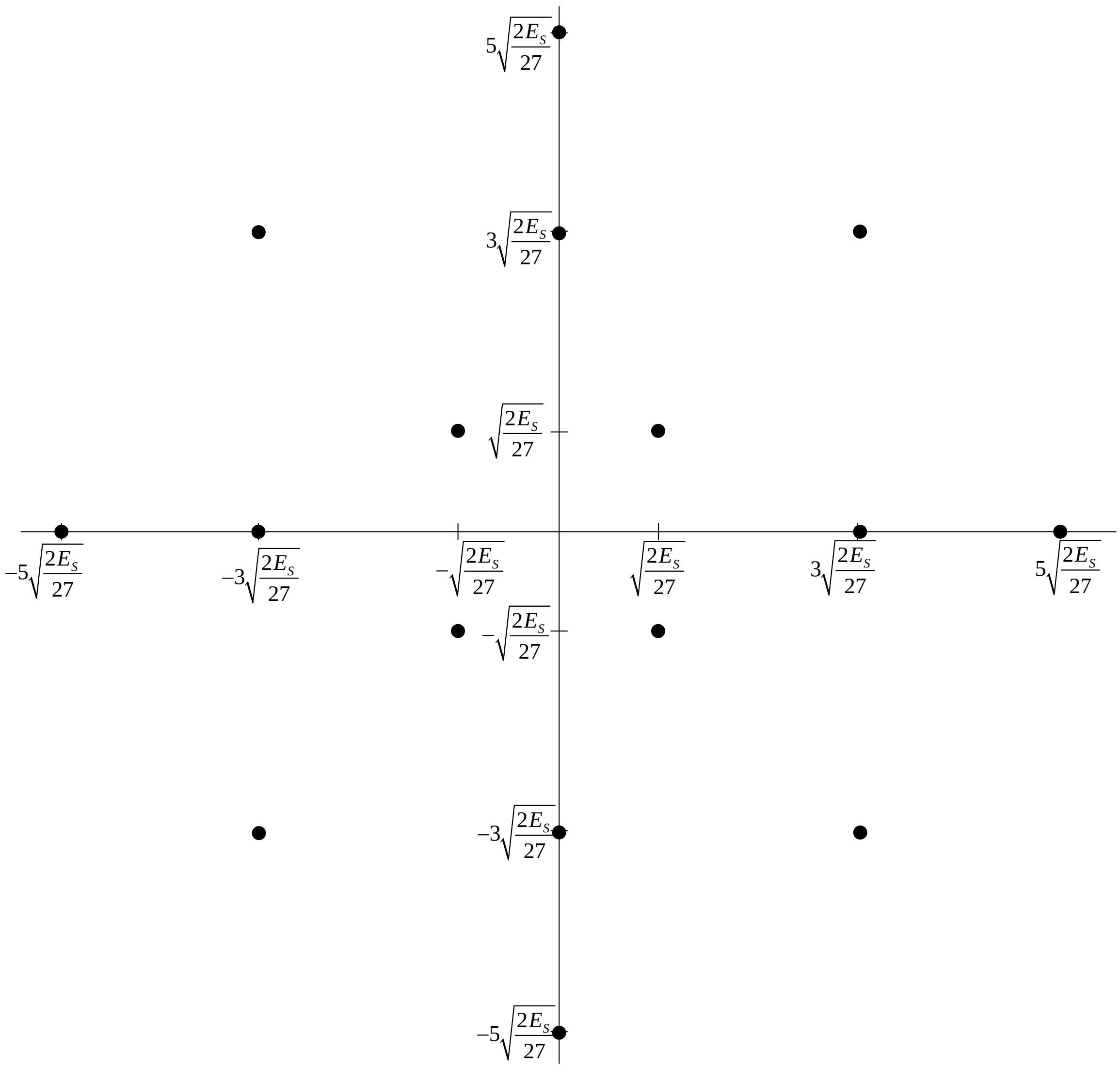
Wednesday 1 March 2017; Duration: 13:30 to 15:20
Two pages of documentation provided; a calculator is permitted.

Problem 1 (15 points out of 100)

Consider the following (non-square) 16QAM constellation shown in signal space.



A. (5 points) Give the minimal distance.

B. (8 points) Give the probability of error using the approximation from the union bound.

Problem 2 (10 points out of 100)

Consider the "Bandwidth Efficiency Plane". Find the coordinates for 16QAM (square) et 8FSK (coherent) for a probability of error of 10^{-6}.

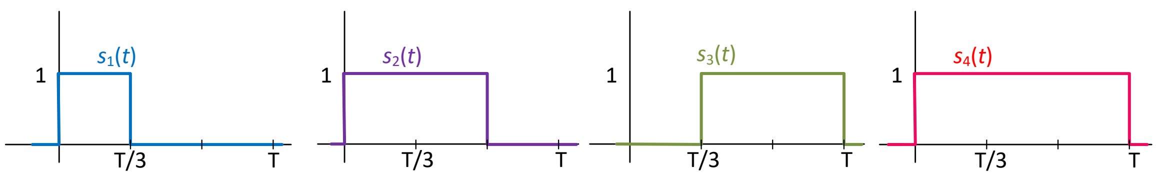
Problem 3 (20 points out of 100)

A. (10 points) Describe the phenomenon known as ISI, intersymbol interference. Describe circumstances where ISI might be present. How does a Nyquist pulse combat ISI?

B. (10 points) What is *a priori* probability in a communications system? How can knowledge of the *a priori* probability be used to improve system performance?

Problem 4 (30 points out of 100)

For the following set of four signals



A. (20 points) Find a set of orthonormal basis vectors.

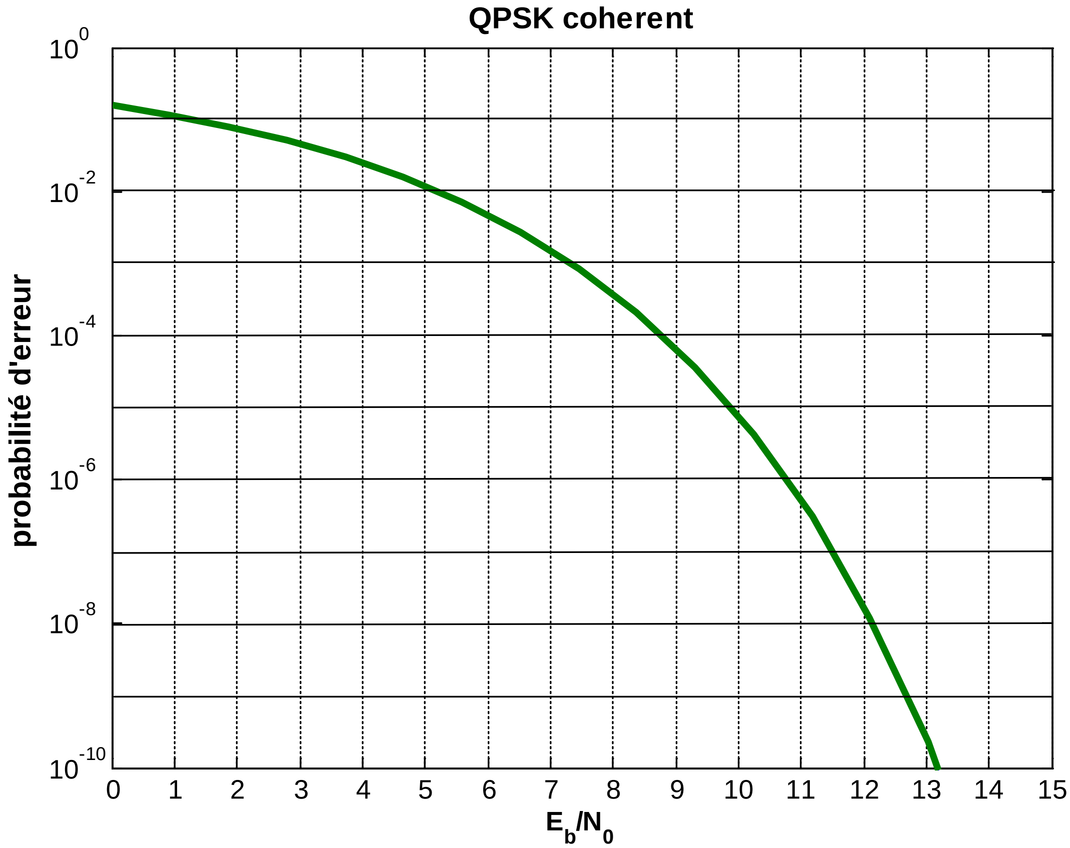
B. (10 points) Give the signal constellation coordinates in this basis for the signal space. The coordinates should be in terms of the average energy per bit, E_b.

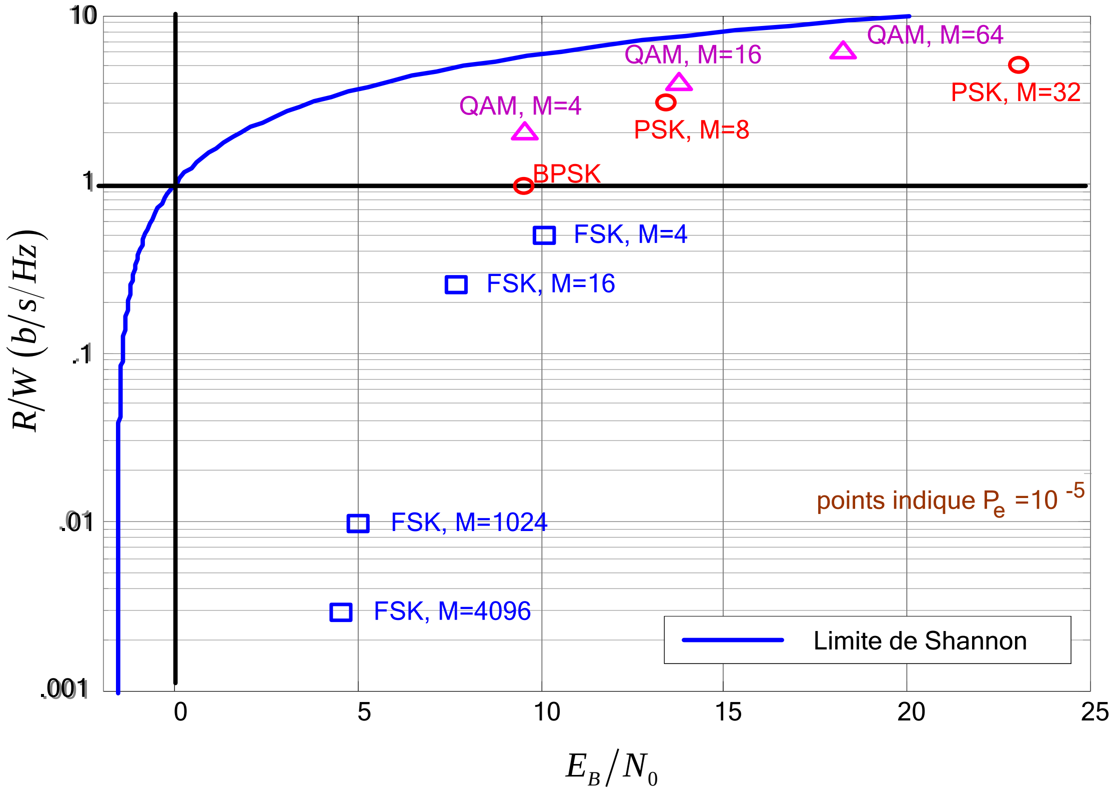
Problem 5 (25 points out of 100)

Identify a communications system from the table that is band limited. Justify your choice, that is, explain the qualities of a band limited system. Suggest an appropriate modulation format.

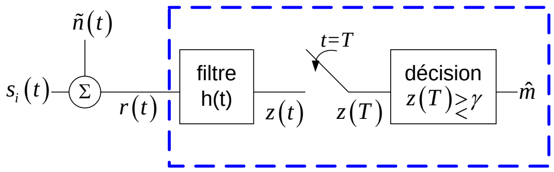
Identify a communications system from the table that is power limited. Justify your choice, that is, explain the qualities of a power limited system. Suggest an appropriate modulation format.

	Bandwidth	Bit rate	Power source	Signal strength
Twisted pair	3 kHz	10 kb/s	grid	~30 dB
Car door opener	20 MHz	10 b/s	battery	varies greatly with distance; requires line of sight
Microwave link	30 MHz	100 Mb/s	grid	good; distance chosen for good signal strength and line of sight
Mars lander	4 GHz	20 kb/s	solar	very weak, even with very high gain antennas
Cellular data	100 kHz	1 Mb/s	battery	varies greatly with distance
Cable internet	10 MHz	50 Mb/s	grid	~25 dB


QPSK coherent
probabilité d'erreur
10^{0}
10^{-2}
10^{-4}
10^{-6}
10^{-8}
10^{-10}
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15
E_b/N_0


Plan de l'efficacité spectrale (Bandwidth Efficiency Plane)
$R/W\ (b/s/Hz)$
10
1
.1
.01
.001
0 5 10 15 20 25
E_B/N_0
QAM, M=64
QAM, M=16
PSK, M=32
QAM, M=4
PSK, M=8
BPSK
FSK, M=4
FSK, M=16
points indique $P_e = 10^{-5}$
FSK, M=1024
FSK, M=4096
Limite de Shannon

Récepteur d'échantillonnage



MAP: i qui maximise $p(z|s_i)\,p(s_i)$

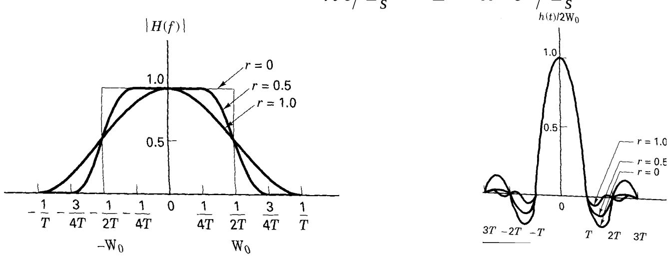
i qui minimise $\left\|\mathbf{r}-\mathbf{s}_i\right\|^2 - N_0 \ln P(\mathbf{s}_i)$

$P(\mathbf{s}_i) =$ probabilité a priori de symbole $\mathbf{s}_i$

ML: i qui maximise $p(z|s_i)$

i qui minimise $\left\|\mathbf{r}-\mathbf{s}_i\right\|^2$

Raised cosine $v(t) = \dfrac{\sin(\pi t/T_s)}{\pi t/T_s} \dfrac{\cos(r\pi t/T_s)}{1-4r^2t^2/T_s^2}$



Énergie moyenne

$$E_{moy} = \frac{1}{M}\sum_{i=1}^{M}\left\|\mathbf{s}_i\right\|^2 = \frac{1}{M}\sum_{i=1}^{M}\left[\text{énergie du signal } i\right]$$

Énergie par bit v. énergie par symbole

$$E_b \log_2 M = E_s$$

Conversion de l'espace I/Q vers espace du signal

$$\left(\tilde{a}_n^I, \tilde{a}_n^Q\right) = \sqrt{\frac{M \cdot E_s}{\sum_{i=1}^{M}\left[\left(a_n^I\right)^2 + \left(a_n^Q\right)^2\right]}}\left(a_n^I, a_n^Q\right)$$

coordonnées, espace du signal

coordonnées, espace I/Q

QAM cas rectangulaire (carrée) $M=L^2$

$$P_e = 2\left(1-\frac{1}{\sqrt{M}}\right)Q\left(\sqrt{\frac{3\log_2 M}{(M-1)}\frac{E_b}{N_0}}\right) \qquad d_{\min} = \sqrt{\frac{6\log_2 L}{L^2-1}}$$

Borne d'union

$$P_e \approx \frac{2K}{M}Q\left(\frac{D_{\min}}{\sqrt{2N_0}}\right) = \frac{2K}{M}Q\left(d_{\min}\sqrt{\frac{E_b}{N_0}}\right)$$

K est le nombre des paires des signaux séparés par la distance minimale D_{min}

Distance minimale dans l'espace du signal

$$D_{\min} = \min_{i\neq k}\left\|\mathbf{s}_i - \mathbf{s}_k\right\| \quad \text{et} \quad d_{\min} = \frac{D_{\min}}{\sqrt{2E_b}}$$

$$P_e(BPSK) = Q\left(\sqrt{\frac{2E_b}{N_0}}\right)$$

$$P_e(OOK) = Q\left(\sqrt{\frac{E_b}{N_0}}\right)$$

$$P_e(QPSK) \approx 2Q\left(\sqrt{\frac{2E_b}{N_0}}\right)$$

Perte par rapport à QPSK

$$d_{\min} = \sqrt{x}\sqrt{2} \qquad \text{perte} = -10\log_{10} x$$

Pour une modulation orthogonale

$$P_e(bit) = P_b = P_e(symbol)\frac{M/2}{M-1}$$

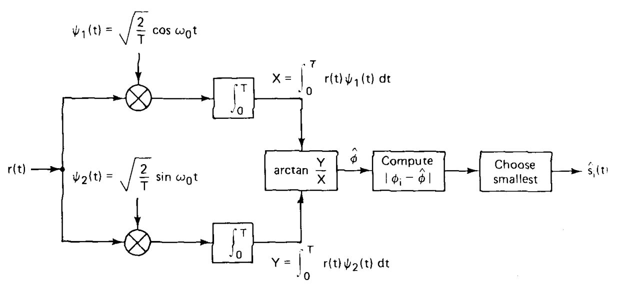
Pour une modulation non-orthogonale avec codage de gray

$$P_e(bit) = P_b = \frac{P_e(symbol)}{\log_2 M}$$

Efficacité spectrale

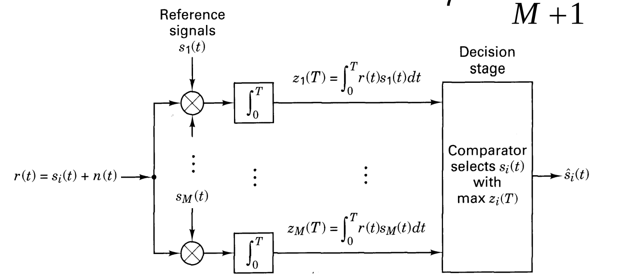
$$\eta = \frac{R_b}{W} = \frac{1}{T_b}\frac{1}{W} \text{bits/s}$$

MPSK cohérent $\eta = \log_2 M$ †



$$P_e(M) \approx 2Q\left(\sqrt{\frac{2E_s}{N_0}} \sin\frac{\pi}{M}\right) = 2Q\left(\sqrt{\frac{2E_b \log_2 M}{N_0}} \sin\frac{\pi}{M}\right)$$

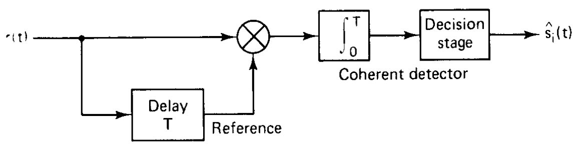
MFSK cohérent $\eta = \dfrac{2\log_2 M}{M+1}$ †



$$P_e = (M-1)Q\left(\sqrt{\frac{E_s}{N_0}}\right) = (M-1)Q\left(\sqrt{\frac{E_b \log_2 M}{N_0}}\right)$$

Séparation minimale $1/2T_s$

DPSK incohérent



~1 dB de perte entre DPSK et BPSK $\quad P_e = \dfrac{1}{2} e^{-E_b/N_0}$

Processus Gram Schmidt

$$\psi_1(t) = \frac{1}{\sqrt{E_1}} s_1(t) \text{ où } E_1 \triangleq \int_0^T s_1^2(t)\,dt$$

$$\theta_2(t) \triangleq s_2(t) - \langle s_2(t), \psi_1(t)\rangle \psi_1(t)$$

$$E_2 \triangleq \int_0^T \theta_2^2(t)\,dt \qquad \psi_2(t) = \frac{\theta_2(t)}{\sqrt{E_2}}$$

$$i. \qquad \theta_i(t) = s_i(t) - \sum_{k=1}^{i-1} \langle s_i(t), \psi_k(t)\rangle \psi_k(t)$$

$$E_i \triangleq \int_0^T \theta_i^2(t)\,dt \qquad \psi_i(t) = \frac{\theta_i(t)}{\sqrt{E_i}}$$

Relations trigonométriques

$$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$$

$$\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$$

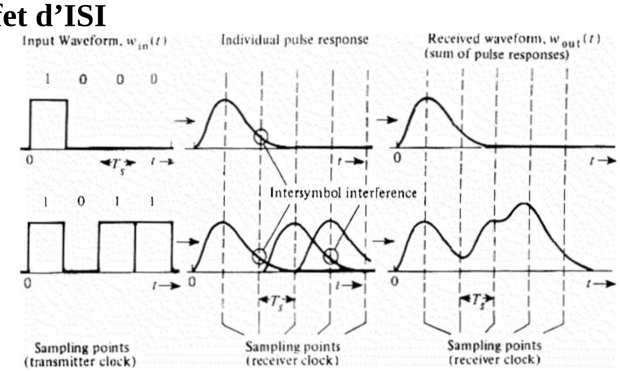
$$\cos 2\theta = 2\cos^2\theta - 1 = 1 - \sin^2\theta \qquad \tan\theta = \sin\theta/\cos\theta$$

Loi de Shannon

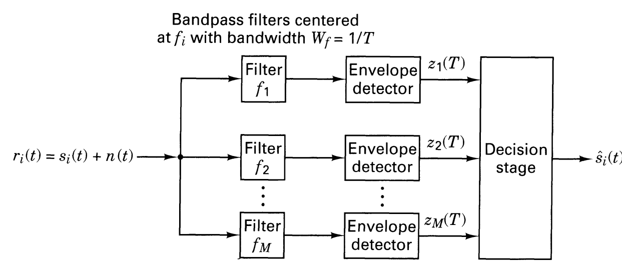
$$C = W\log_2(1+SNR) \qquad SNR = \frac{E_b}{N_0}\frac{R_b}{W}$$

$$\frac{E_b}{N_0} = \frac{W}{C}\left(2^{C/W} - 1\right) \qquad \frac{C}{W} \to 0 \;\Rightarrow\; \frac{E_b}{N_0} \to -1.6dB$$

L'effet d'ISI



MFSK incohérent $\eta = \dfrac{\log_2 M}{M}$ †



$$P_e(BFSK) = \frac{1}{2} e^{-E_b/2N_0}$$

~1 dB de perte entre BFSK cohérente et incohérente

Séparation minimale $1/T_s$

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† en supposant une impulsion Nyquist idéale