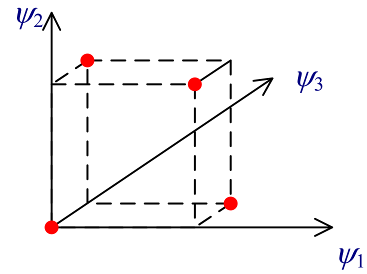
Wednesday 9 March 2016; Duration: 13:30 à 15:20
Two pages of documentation provided; a calculator is permitted.

Problem 1 (20 points out of 100)

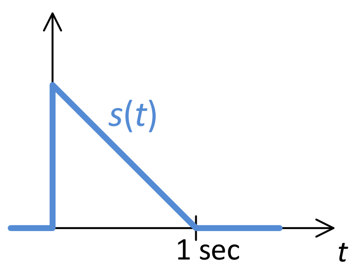
A. (5 points) Is the following modulation orthogonal? Justify your answer.

$$
\begin{aligned}
s_1 &= [0 \quad 0 \quad 0]\sqrt{E_b} \\
s_2 &= [4/3 \quad 4/3 \quad 0]\sqrt{E_b} \\
s_3 &= [0 \quad 4/3 \quad 2/3]\sqrt{E_b} \\
s_4 &= [4/3 \quad 0 \quad 2/3]\sqrt{E_b}
\end{aligned}
$$



B. (5 points) Under what circumstances are the ML (maximum likelihood) and MAP (maximum a posteriori) receivers equivalent?

C. (5 points) Give an equation and a sketch of the impulse response *h*(*t*) of the matched filter for



D. (5 points) Give a sketch of the correlator receiver that is equivalent to the matched filter receiver for the signal in problem 1C.

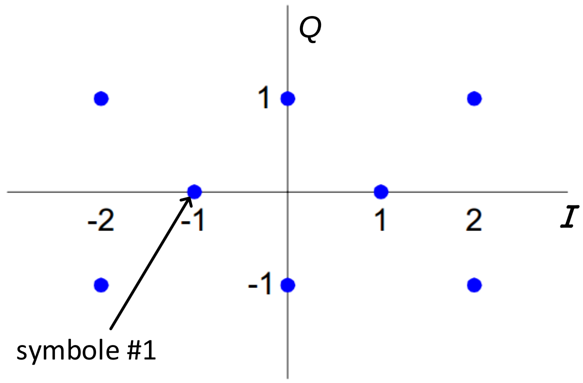
Problem 2 (15 points out of 100)

E. (15 points) Complete the following table in your blue exam book.

Modulation format	Dimension of the signal space	Equal energy symbols (yes/no)	Orthogonal modulation (yes/no)
OOK			
BPSK			
16QAM			
8FSK			
DBPSK			

Problem 3 (35 points out of 100)

Consider the following ***NON***-rectangular 8QAM modulation.



A. (15 points) Supposing all symbols have the same a priori probability, provide a sketch of the decision region in IQ space for the symbol #1? Indicate whether the following received signals in IQ space fall in the decision region for symbol 1.

IQ coordinates (r_1, r_2)	Choose symbol 1?
$(-1.5_1, 0)$	
$(-0.5, .75)$	
$(-1, -0.95)$	

B. (10 points) What are the signal space coordinates of the symbols?

C. (10 points) What is the symbol error probability using the approximation derived from the union bound?

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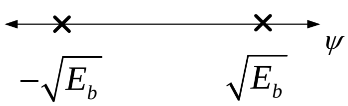
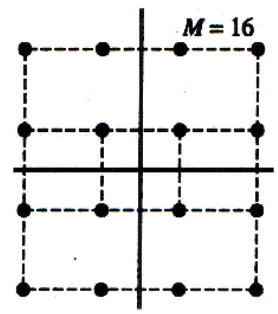
Problem 4 (30 points out of 100)

Assume a bit rate of 1024 b/s and ideal Nyquist pulses for all modulations, and assume the use of coherent detection for 16QAM and non-coherent detection for DBSPK and 8FSK.

	$s_1 = [1\ 0\ 0\ 0\ 0\ 0\ 0\ 0]\sqrt{E_s}$ $s_2 = [0\ 1\ 0\ 0\ 0\ 0\ 0\ 0]\sqrt{E_s}$ $s_3 = [0\ 0\ 1\ 0\ 0\ 0\ 0\ 0]\sqrt{E_s}$ $s_4 = [0\ 0\ 0\ 1\ 0\ 0\ 0\ 0]\sqrt{E_s}$ $s_5 = [0\ 0\ 0\ 0\ 1\ 0\ 0\ 0]\sqrt{E_s}$ $s_6 = [0\ 0\ 0\ 0\ 0\ 1\ 0\ 0]\sqrt{E_s}$ $s_7 = [0\ 0\ 0\ 0\ 0\ 0\ 1\ 0]\sqrt{E_s}$ $s_8 = [0\ 0\ 0\ 0\ 0\ 0\ 0\ 1]\sqrt{E_s}$	
DBPSK	8FSK	16QAM

A. (15 points) For each modulation give the following

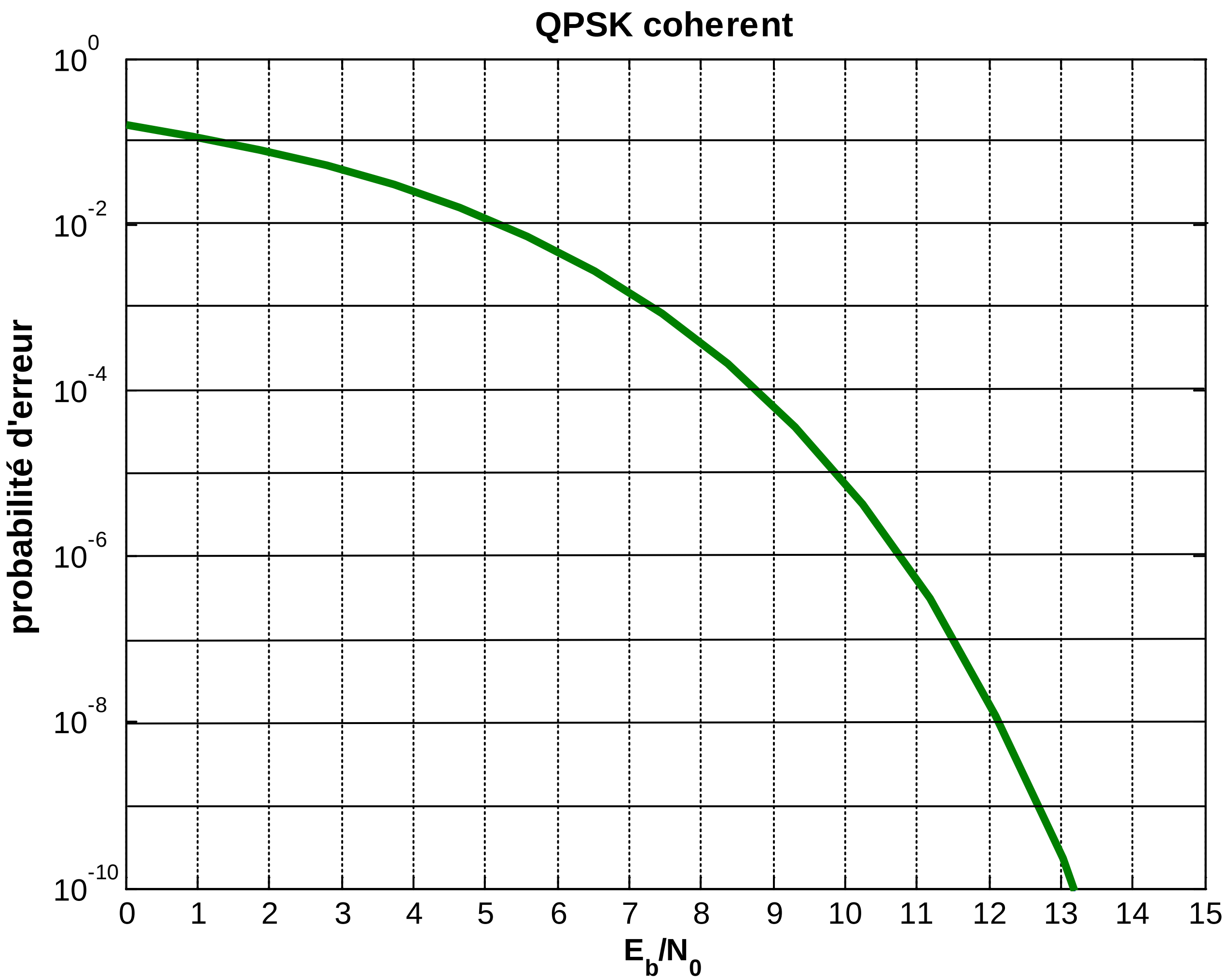
i. Total occupied bandwidth

ii. A sketch of the spectrum

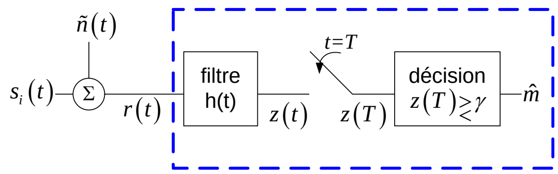
iii. Spectral efficiency in b/s/Hz

B. (15 points) Discuss trade-offs made for each modulation using the following matrix. For instance, you might indicate which modulation is best/worst for each criterion, or how they differ in a criterion.

DBPSK	8FSK	16QAM	
			BER vs. Eb/N0
			Spectral efficiency
			complexity


QPSK coherent
probabilité d'erreur
10^{0}
10^{-2}
10^{-4}
10^{-6}
10^{-8}
10^{-10}
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15
E_b/N_0

Récepteur d'échantillonnage



MAP: i qui maximise $p(z|s_i)\, p(s_i)$

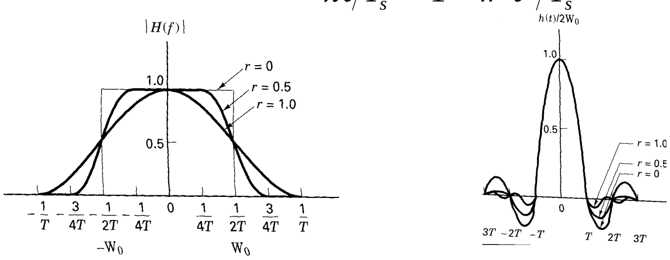
i qui minimise $\left\|\mathbf{r}-\mathbf{s}_i\right\|^2 - N_0 \ln P(\mathbf{s}_i)$

$P(\mathbf{s}_i) =$ probabilité a priori de symbole $\mathbf{s}_i$

ML: i qui maximise $p(z|s_i)$

i qui minimise $\left\|\mathbf{r}-\mathbf{s}_i\right\|^2$

Raised cosine $v(t) = \dfrac{\sin(\pi t/T_s)}{\pi t/T_s} \dfrac{\cos(r\pi t/T_s)}{1-4r^2t^2/T_s^2}$



Énergie moyenne

$$E_{moy} = \frac{1}{M}\sum_{i=1}^{M}\left\|\mathbf{s}_i\right\|^2$$

$$= \frac{1}{M}\sum_{i=1}^{M}\left[\text{énergie du signal } i\right]$$

Énergie par bit v. énergie par symbole

$$E_b \log_2 M = E_s$$

QAM

Conversion de l'espace I/Q vers espace du signal

$$\left(\tilde{a}_n^I, \tilde{a}_n^Q\right) = \sqrt{\frac{M \cdot E_s}{\sum_{i=1}^{M}\left[\left(a_n^I\right)^2 + \left(a_n^Q\right)^2\right]}}\left(a_n^I, a_n^Q\right)$$

coordonnées, espace du signal

coordonnées, espace I/Q

cas rectangulaire (carrée) $M=L^2$

$$P_e = 2\left(1-\frac{1}{\sqrt{M}}\right)Q\left(\sqrt{\frac{3\log_2 M}{(M-1)}\frac{E_b}{N_0}}\right) \qquad d_{\min} = \sqrt{\frac{6\log_2 L}{L^2-1}}$$

Borne d'union

$$P_e \approx \frac{2K}{M}Q\left(\frac{D_{\min}}{\sqrt{2N_0}}\right) = \frac{2K}{M}Q\left(d_{\min}\sqrt{\frac{E_b}{N_0}}\right)$$

K est le nombre des paires des signaux séparés par la distance minimale D_{min}

Distance minimale dans l'espace du signal

$$D_{\min} = \min_{i\neq k}\left\|\mathbf{s}_i - \mathbf{s}_k\right\| \text{ et } d_{\min} = \frac{D_{\min}}{\sqrt{2E_b}}$$

Pour une modulation orthogonale

$$P_e(bit) = P_b = P_e(symbol)\frac{M/2}{M-1}$$

$$P_e(BPSK) = Q\left(\sqrt{\frac{2E_b}{N_0}}\right)$$

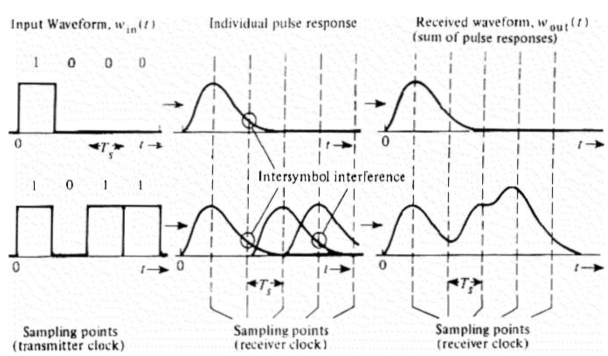
$$P_e(OOK) = Q\left(\sqrt{\frac{E_b}{N_0}}\right)$$

$$P_e(QPSK) \approx 2Q\left(\sqrt{\frac{2E_b}{N_0}}\right)$$

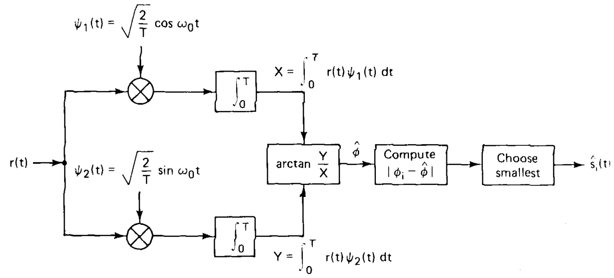
Perte par rapport à QPSK

$$d_{\min} = \sqrt{x}\sqrt{2} \qquad \text{perte} = -10\log_{10} x$$

L'effet d'ISI



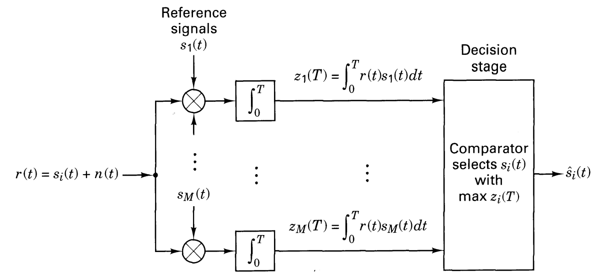
MPSK cohérent $\eta = \log_2 M$ †



$$P_e(M) \approx 2Q\left(\sqrt{\frac{2E_s}{N_0}} \sin\frac{\pi}{M}\right)$$

$$= 2Q\left(\sqrt{\frac{2E_b \log_2 M}{N_0}} \sin\frac{\pi}{M}\right)$$

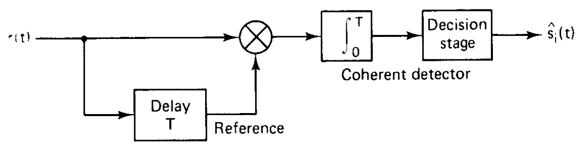
MFSK cohérent $\eta = \dfrac{2\log_2 M}{M+1}$ †



$$P_e = (M-1)Q\left(\sqrt{\frac{E_s}{N_0}}\right) = (M-1)Q\left(\sqrt{\frac{E_b \log_2 M}{N_0}}\right)$$

Séparation minimale $1/2T_s$

DPSK incohérent $P_e = \dfrac{1}{2} e^{-E_b/N_0}$



~1 dB de perte entre DPSK et BPSK

Processus Gram Schmidt

$$\psi_1(t) = \frac{1}{\sqrt{E_1}} s_1(t) \text{ où } E_1 \triangleq \int_0^T s_1^2(t)\,dt$$

$$\theta_2(t) \triangleq s_2(t) - \langle s_2(t), \psi_1(t)\rangle \psi_1(t)$$

$$E_2 \triangleq \int_0^T \theta_2^2(t)\,dt \qquad \psi_2(t) = \frac{\theta_2(t)}{\sqrt{E_2}}$$

$$i. \qquad \theta_i(t) = s_i(t) - \sum_{k=1}^{i-1} \langle s_i(t), \psi_k(t)\rangle \psi_k(t)$$

$$E_i \triangleq \int_0^T \theta_i^2(t)\,dt \qquad \psi_i(t) = \frac{\theta_i(t)}{\sqrt{E_i}}$$

Loi de Shannon

$$C = W \log_2(1 + SNR) \qquad SNR = \frac{E_b}{N_0}\frac{R_b}{W}$$

Relations trigonométriques

$$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$$

$$\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$$

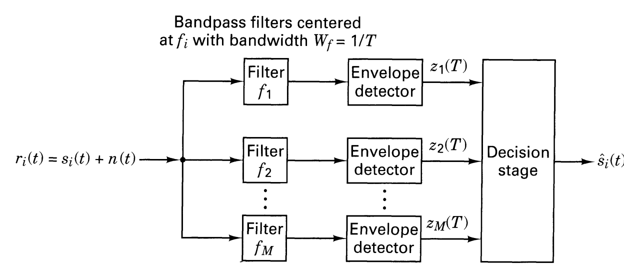
$$\cos 2\theta = 2\cos^2\theta - 1 = 1 - \sin^2\theta$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

Efficacité spectrale

$$\eta = \frac{R_b}{W} = \frac{1}{T_b}\frac{1}{W} \text{ bits/s}$$

MFSK incohérent $\eta = \dfrac{\log_2 M}{M}$ †



$$P_e(BFSK) = \frac{1}{2} e^{-E_b/2N_0}$$

~1 dB de perte entre BFSK cohérente et incohérente

† en supposant une impulsion Nyquist idéale

† en supposant une impulsion Nyquist idéale

	Séparation minimale $1/T_s$