HW#4 Solution

E5.6 Consider the block diagram shown in Figure E5.6 [16]. (a) Calculate the steady-state error for a ramp input. (b) Select a value of K that will result in zero percent overshoot to a step input. Provide rapid response.

Plot the poles and zeros of this system and discuss the dominance of the complex poles. What overshoot for a step input do you expect?

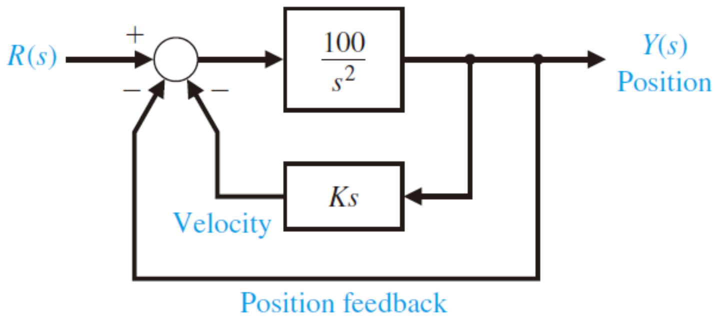


FIGURE E5.6 Block diagram with position and velocity feedback.

(Ans)

(a) The feedback paths, Ks and 1 are in parallel with each other.

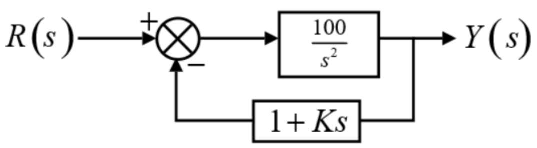
Thus, the equivalent feedback gain is

$$H(s) = 1 + Ks.$$

The forward gain is

$$G(s) = \frac{100}{s^2}.$$

The reduced lock diagram is as follows:



Recall that the transfer funcion is the ratio of the output to the input, and the expression for the transfer funtion for the closed loop negative feedback system is

$$T(s) = \frac{Y(s)}{R(s)} = \frac{G(s)}{1 + G(s)H(s)} = \frac{100}{s^2 + 100Ks + 100}.$$

The input is a ramp. Thus,

$$r(t) = At$$

and

$$R(s) = \frac{A}{s^2}.$$

The steady-state error is computed as follows:

$$e_{ss} = \lim_{s \to 0} s[R(s) - Y(s)] = \lim_{s \to 0} s[1 - T(s)]\frac{A}{s^2}$$

$$= \lim_{s \to 0}\left[1 - \frac{\frac{100}{s^2}}{1 + \frac{100}{s^2}(1 + Ks)}\right]\frac{A}{s} = KA.$$

(b) Recall the transfer function $T(s)$ with the standard second order transfer funtion:

$$T(s) = \frac{{\omega_n}^2}{s^2 + 2\zeta\omega_n s + {\omega_n}^2}.$$

Thus,

$${\omega_n}^2 = 100,\ \ \omega_n = 10\, rad/s$$

$$2\zeta\omega_n = 100K$$

$$\zeta = \frac{100K}{2\omega_n} = 5K.$$

The critically damped response has zero overshoot. Therefore, the system is critically dampled. That is, $\zeta = 1$.

Thus,

$$K = 0.2.$$

The closed-loop system has no zeros and poles are at

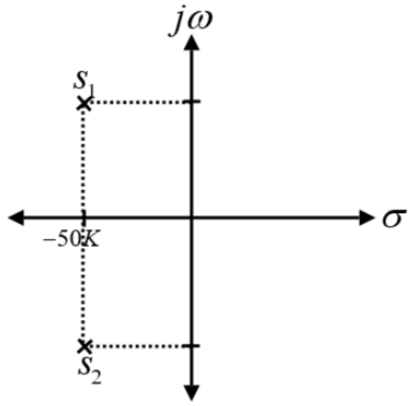
$$s_{1,2} = -50K \pm 10\sqrt{25K^2 - 1}\,.$$

The percent overshoot to a step input is

$$P.O. = 100e^{\frac{-5\pi K}{\sqrt{1-25K^2}}} \quad \text{for} \quad 0 < K < 0.2$$

and $P.O. = 0$ for $K \geq 0.2$.

Plot the poles on the complex plane.



E5.10 A second-order control system has the closed-loop transfer function $T(s) = Y(s)/R(s)$. The system specifications for a step input follow:

1. Percent overshoot $P.O. \leq 5\%$.
2. Settling time $T_s < 4$s.
3. Peak time $T_p < 1$s.

Show the desired region for the poles of $T(s)$ in order to achieve the desired response. Use a 2% settling criterion to determine settling time.

(Ans)

The Transfer function of a second order system is

$$\frac{Y(s)}{R(s)} = \frac{{\omega_n}^2}{s^2 + 2\zeta\omega_n s + {\omega_n}^2}.$$

Write the formula for percent overshoot. Percent overshoot should be less than or equal to 5.

$$P.O. = 100e^{-\frac{\pi\zeta}{\sqrt{1-\zeta^2}}} \leq 5$$

Then it implies

$$\zeta \geq 0.69.$$

Write the formula for settling time with 2% criterion. Settling time should be less than 4.

$$T_s = \frac{4}{\zeta\omega_n} < 4$$

Then it implies

$$\omega_n\zeta > 1.$$

Write the formula for peak time. Peak time should be less than 1.

$$T_p = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}} < 1$$

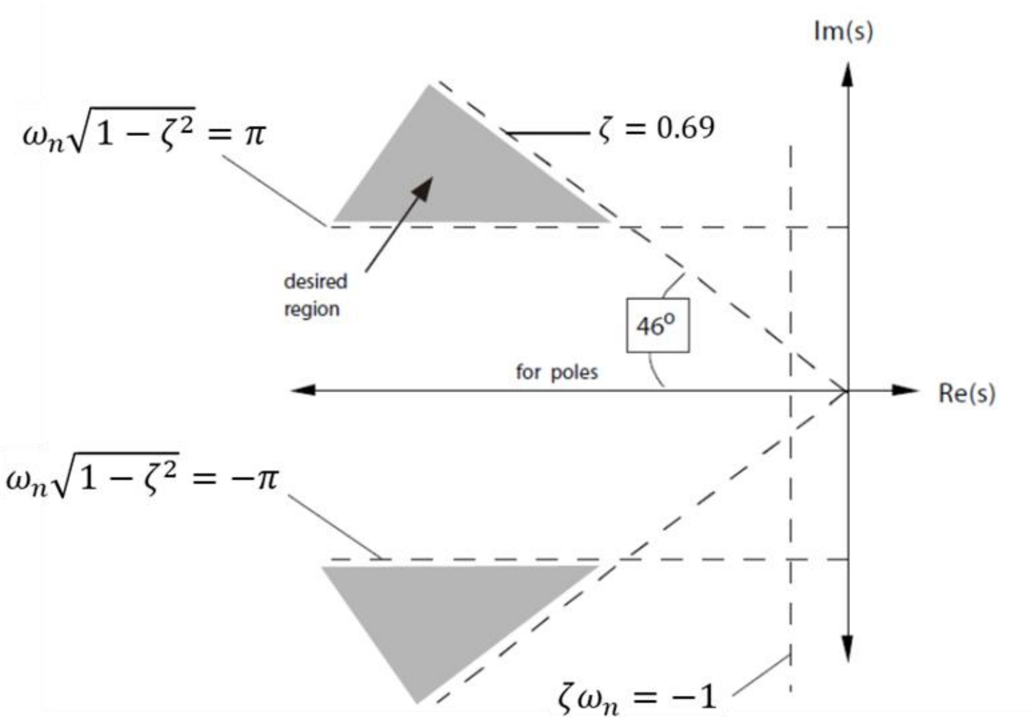
Then it implies

$$\omega_n\sqrt{1-\zeta^2} > \pi.$$

Note that two poles of a second-order system are

$$s = -\omega_n \zeta \pm j\omega_n\sqrt{1-\zeta^2}.$$

The constraints imposed on ζ and ω_n by the performance specifications define the permissible area for the poles of T_s, as shown in the figure below.



E5.12 The Ferris wheel is often featured at state fairs and carnivals. George Ferris was born in Galesburg, Illinois, in 1859; he later moved to Nevada and then graduated from Rensselaer Polytechnic Institute in 1881. By 1891, Ferris had considerable experience with iron, steel, and bridge construction. He conceived and constructed his famous wheel for the 1893 Columbian Exposition in Chicago [8]. Consider the requirement that the steady-state speed must be controlled to within 5% of the desired speed for the Ferris wheel speed control system shown in Figure E5.12.

(a) Determine the required gain K to achieve the steady-state requirement.

(b) For the gain of part (a), determine and plot the tracking error for a unit step disturbance. Does the speed change more than 5%? (Set $R(s) = 0$ and recall that the tracking error $E(s) = R(s) - T(s)$.)

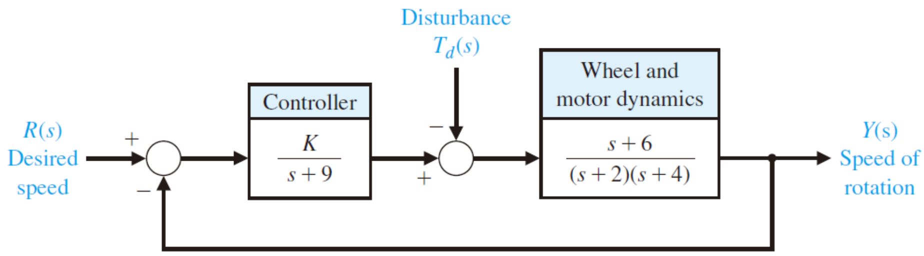


FIGURE E5.12 Speed control of a Ferris wheel.

(Ans)

(a) The tracking error is givien by

$$E(s) = \frac{R(s)}{1 + G_c G(s)} = \frac{(s+9)(s+2)(s+4)}{(s+9)(s+2)(s+4) + K(s+6)} R(s).$$

The steady-state tracking error (with $R(s) = 1/s$) is

$$\lim_{s \to 0} sE(s) = \frac{72}{72 + 6K}.$$

We require $e_{ss} < 0.05$, so solving for K yields $K > 228$.

(b) Find the transfer function for the disturbance input $T_d(s)$ by putting $K = 288$ and by assuming $R(s) = 0$.

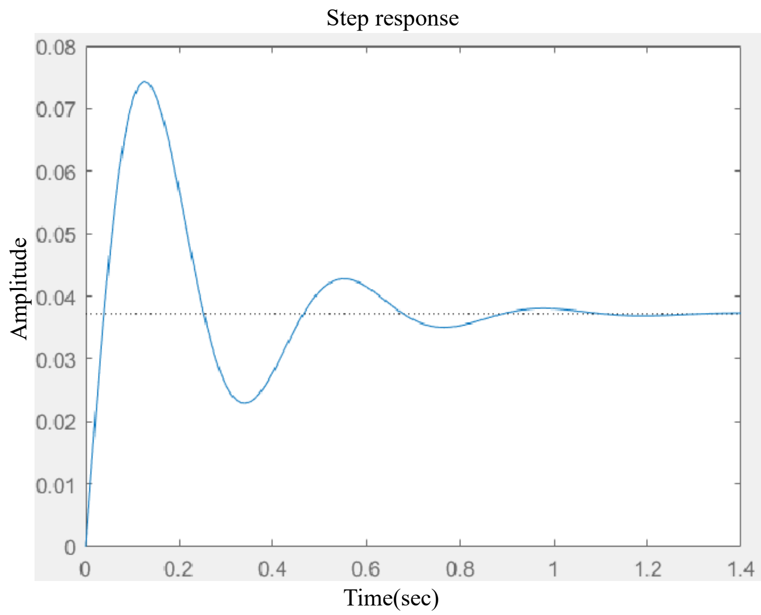
$$Y(s) = \frac{-\dfrac{s+6}{(s+2)(s+4)}}{1 + \dfrac{(s+6)}{(s+2)(s+4)} \cdot \dfrac{K}{(s+9)}} T_d(s)$$

$$= \frac{-(s+9)(s+6)}{s^3 + 15s^2 + 290s + 1440} T_d(s)$$

For tracking error for the disturbance input $T_d(s)$ is,

$$E(s) = R(s) - Y(s) = \frac{(s+9)(s+6)}{s^3 + 15s^2 + 290s + 1440} \cdot \frac{1}{s}.$$

Consider the plot for the tracking error as shown in figure below.



From the plot of $E(s)$, we can check that $e_{ss} < 0.05$ which means that speed change is less than 5%.

E5.20 Consider the closed-loop system in Figure E5.20, where

$$L(s) = \frac{(s+2)}{(s^2+5s)}K_a.$$

(a) Determine the closed-loop transfer function $T(s) = Y(s)/R(s)$.
(b) Determine the steady-state error of the closed-loop system response to a unit ramp input.
(c) Select a value for K_a so that the steady-state error of the system response to a unit step input is zero.

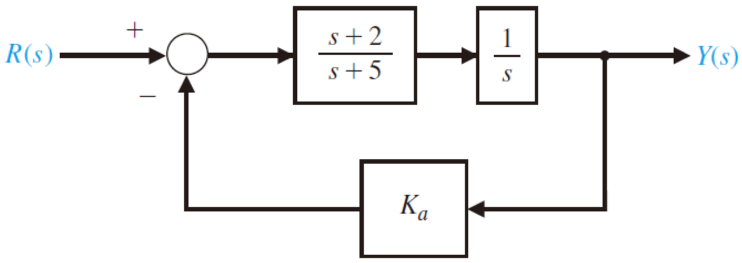


FIGURE E5.20 Non-unity closed-loop feedback control system with parameter K_a.

(Ans)

(a) The closed-loop transfer function is

$$T(s) = \frac{s+2}{s^2+(5+K_a)s+2K_a}.$$

(b) Define the steady-state error to be $E(s) = R(s) - Y(s)$. Then,

$$E(s) = \frac{s^2+(4+K_a)s+2(K_a-1)}{s^2+(5+K_a)s+2K_a} R(s).$$

From the final value theorem, it follows that for a ramp input

$$e_{ss} = \lim_{s\to 0} sE(s) = \lim_{s\to 0} s\frac{s^2+(4+K_a)s+2(K_a-1)}{s^2+(5+K_a)s+2K_a}\cdot\frac{1}{s^2}.$$

If $K_a = 1$, then $e_{ss} = 2.5$; otherwise, if $K_a \neq 1$, then $e_{ss} \to \infty$.

(c) With $R(s) = 1/s$, it follows that

$$e_{ss} = \lim_{s\to 0} sE(s) = \lim_{s\to 0} s\frac{s^2+(4+K_a)s+2(K_a-1)}{s^2+(5+K_a)s+2K_a}\cdot\frac{1}{s} = \frac{K_a-1}{K_a}.$$

If $K_a = 1$, then $e_{ss} = 0$.

P5.1 An important problem for television systems is the jumping or wobbling of the picture due to the movement of the camera. This effect occurs when the camera is mounted in a moving truck or airplane. The Dynalens system has been designed to reduce the effect of rapid scanning motion; see Figure P5.1. A maximum scanning motion of 25°/s is expected. Let $K_g = K_t = 1$ and assume that τ_g is negligible. (a) Determine the error of the system $E(s)$. (b) Determine the necessary loop gain $K_aK_mK_t$ when a 1°/s steady-state error is allowable. (c) The motor time constant is $\tau_m = 0.40$ s. Determine the necessary loop gain so that the settling time (to within 2% of the final value of v_b) is $T_s \le 0.03$ s.

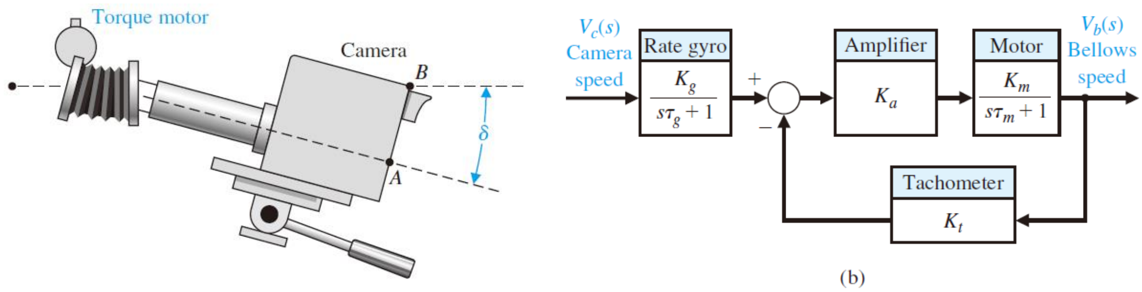


FIGURE P5.1 Camera wobble control.

(Ans)

(a) As $K_g = 1$, τ_g is neglible, $\frac{K_g}{s\tau_g+1} \approx 1$. The system error is

$$E(s) = V_c(s) - V_b(s) = \frac{1}{1 + \frac{K_aK_m}{s\tau_m + 1}} V_c(s)$$

where

$$V_c(s) = \frac{25°/sec}{s}.$$

So,

$$\lim_{t\to 0} e(t) = \lim_{s\to 0} sE(s) = \frac{25}{1 + K_aK_m}.$$

(b) If we desire $e_{ss} \leq 1°/sec$, then

$$\frac{25°/s}{1 + K_a K_m} \leq 1°/sec$$

and solving for $K_a K_m$ yields

$$K_a K_m \geq 24.$$

(c) The closed-loop transfer function is

$$T(s) = \frac{V_b(s)}{V_c(s)} = \frac{K_a K_m}{s\tau_m + 1 + K_a K_m}\ .$$

To obtain settling time, set $v_c(t) = A \quad (t \geq 0)$. Then, the step response of the system is

$$V_b(s) = T(s) \cdot V_c(s) = \frac{K_a K_m}{s\tau_m + 1 + K_a K_m} \cdot \frac{A}{s}$$

$$= \frac{AK_a K_m}{\tau_m} \cdot \left(\frac{1}{s} \cdot \frac{1}{s + \frac{1 + K_a K_m}{\tau_m}} \right)$$

$$= \frac{AK_a K_m}{1 + K_a K_m} \cdot \left(\frac{1}{s} - \frac{1}{s + \frac{1 + K_a K_m}{\tau_m}} \right)$$

Therefore,

$$v_b(t) = \frac{AK_a K_m}{1 + K_a K_m} \left(1 - e^{\frac{-(K_a K_m + 1)}{\tau_m} t} \right), \quad (t \geq 0)$$

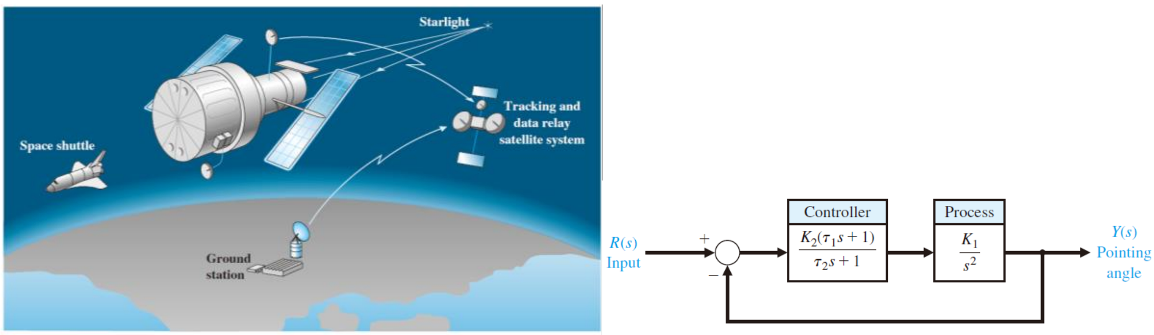
So, at settling time, we have

$$1 - e^{\frac{-(K_a K_m + 1)}{\tau_m} t} \geq 0.98$$

where $\tau_m = 4$. Setting $t = 0.03$ and solving for $K_a K_m$ yields

$$K_a K_m \geq 52$$

P5.5 A space telescope is to be launched to carry out astronomical experiments [8]. The pointing control system is desired to achieve 0.01 minute of arc and track solar objects with apparent motion up to 0.21 arc minute per second. The system is illustrated in Figure P5.5(a). The control system is shown in Figure P5.5(b). Assume that $\tau_1 = 1$ s and $\tau_2 = 0$. (a) Determine the gain $K = K_1K_2$ required so that the response to a unit step command is as rapid as reasonable with a percent overshoot of $P.O. \leq 5\%$. (b) Determine the steady-state error of the system for a step and a ramp input.



(a) (b)

(Ans)

(a) The closed-loop transfer function is

$$T(s) = \frac{K_1K_2(s+1)}{s^2 + K_1K_2 s + K_1K_2}.$$

A percent overshoot less than 5% implies $\zeta \geq 0.69$. So, choose $\zeta = 0.69$. Then set $2\zeta\omega_n = K_1K_2$ and $\omega_n^2 = K_1K_2$.

Then

$$2(0.69)\omega_n = \omega_n^2,$$

and solving for ω_n yields

$$\omega_n = 1.38.$$

Therefore $K_1K_2 = \omega_n^2 = 1.9$. When $K_1K_2 \geq 1.9$ it follows that $\zeta \geq 0.69$.

(b) Write the transfer function for K_1K_2 equal to 1.9

$$T(s) = \frac{Y(s)}{R(s)} = \frac{1.9(s+1)}{s^2 + 1.9s + 1.9}.$$

Write the formula for steady state error and apply $E(s)$ for a unit step input.

$$\begin{aligned} e_{ss} &= \lim_{s\to 0} sE(s) \\ &= \lim_{s\to 0} sR(s)\left(1 - \frac{Y(s)}{R(s)}\right) \\ &= \lim_{s\to 0} s\frac{1}{s}\left(1 - \frac{1.9(s+1)}{s^2 + 1.9s + 1.9}\right) \\ &= \lim_{s\to 0} \frac{s^2}{s^2 + 1.9s + 1.9} \quad = 0 \end{aligned}$$

Thus, the steady state error to unit step input is 0.

Write the formula for steady state error and apply $E(s)$ for unit ramp input.

$$\begin{aligned} e_{ss} &= \lim_{s\to 0} sE(s) \\ &= \lim_{s\to 0} sR(s)\left(1 - \frac{Y(s)}{R(s)}\right) \\ &= \lim_{s\to 0} s\frac{1}{s^2}\left(1 - \frac{1.9(s+1)}{s^2 + 1.9s + 1.9}\right) \\ &= \lim_{s\to 0} \frac{1}{s}\left(1 - \frac{1.9(s+1)}{s^2 + 1.9s + 1.9}\right) \\ &= \lim_{s\to 0} \frac{s}{s^2 + 1.9s + 1.9} \quad = 0 \end{aligned}$$

Thus, the steady state error to unit ramp input is 0.

P5.14 For the original system of Problem P5.13, we want to find the lower-order model when the poles of the second-order model are specified as −1 and −2 and the model has one unspecified zero. Show that this low-order model is

$$G_L(s) = \frac{0.986s + 2}{s^2 + 3s + 2} = \frac{0.986(s + 2.028)}{(s + 1)(s + 2)}.$$

(Ans)

Consider

$$L(s) = \frac{2(c_1 s + 1)}{(s+1)(s+2)}.$$

After cancellation of like factors, we compute $H(s)/L(s)$,

$$\frac{H(s)}{L(s)} = \frac{s^3 + 7s^2 + 24s + 24}{(s+3)(s+4)2(c_1 s + 1)}.$$

Therefore,

$$M(s) = s^3 + 7s^2 + 24s + 24,$$

$$\Delta(s) = 2[c_1 s^3 + (7c_1 + 1)s^2 + (12c_1 + 7)s + 12].$$

Then, following the procedure outlined in Section 5.10, we have

$$M^0(0) = 24, \quad M^1(0) = 24, \quad M^2(0) = 14, \quad M^3(0) = 6, \text{and}$$

$$\Delta^0(0) = 24, \quad \Delta^1(12c_1 + 7)2, \quad \Delta^2(0) = 2\big(2(7c_1 + 1)\big), \quad \Delta^3(0) = 12c_1.$$

For $q = 1$:

$$M_2 = 240, \text{and}$$

$$\Delta_2 = 4[144c_1^2 + 25].$$

The, equating Δ_2 and M_2, we find c_1,

$$c_1 = 0.493$$

So,

$$L(s) = \frac{2(0.493s + 1)}{(s+1)(s+2)} = \frac{0.986s + 2}{s^2 + 3s + 2} = \frac{0.986(s + 2.028)}{(s+1)(s+2)}.$$