HW#7 Solution

E8.4 The frequency response for the system

$$G(s) = \frac{K}{(s+a)(s^2+5s+6.25)}$$

is shown in Figure E8.4. Estimate K and a by examining the frequency response curves.

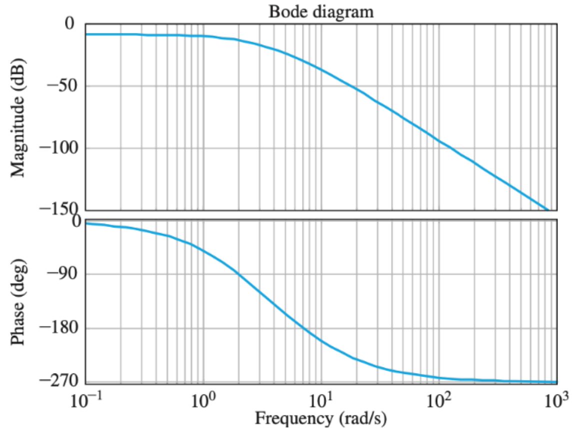


FIGURE E8.4
Bode plot.

(Ans)

The transfer function is

$$G(s) = \frac{K}{(s+a)(s^2+5s+6.25)} = \frac{K}{(s+a)(s+2.5)^2} \ .$$

Phase and Magnitude relationship from the transfer function is as follows:

$$\phi(w) = -\tan^{-1}\frac{\omega}{a} - 2\tan^{-1}\frac{\omega}{2.5}$$

$$20\log|G(j\omega)| = 20\log K - 20\log|j\omega + a| - 2\times 20\log|j\omega + 2.5| \ .$$

[Estimate] (*No exact number is required for the calculation below. And the method or value may vary depending on the measured value.)

Note that $\phi = -90^\circ$ at $\omega = 2\mathrm{rad/s}$. Substitute $\omega = 2\mathrm{rad/s}$ in Phase equation, $\phi(\omega)$, and solve it for a.

$$a = 8.88 \ .$$

Note that $20\log|G(j\omega)| = -50\text{dB}$ at $\omega = 20\text{rad/s}$. Substitute $\omega = 20\text{rad/s}$ in Magnitude relationship, $20\log|G(j\omega)|$, and solve it for K.

$$K = 28.0701 \ .$$

E8.9 The Bode plot of a system is shown in Figure E8.9. Estimate the transfer function $G(s)$.

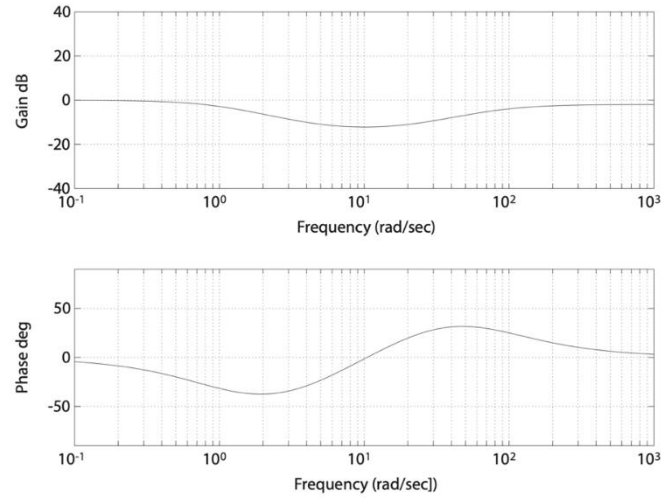


FIGURE E8.9

(Ans)

We can expect that there are two zeros and two poles. Therefore, the Transfer function can be represented as follow.

$$G(s) = K\frac{(s/z_1 + 1)(s/z_2 + 1)}{(s/p_1 + 1)(s/p_2 + 1)} .$$

Phase and magnitude equation is as follows:

$$\phi(w) = \tan^{-1}\frac{\omega}{z_1} + \tan^{-1}\frac{\omega}{z_2} - \tan^{-1}\frac{\omega}{p_1} - \tan^{-1}\frac{\omega}{p_1}$$

$$20\log|G(j\omega)| \\ = 20\log K + \sum_{i=1,2} 20\log|j\omega + z_1| - \sum_{i=1,2} 20\log|j\omega + p_i| .$$

Due to the initial value of $20\log|G(0)| = 0,\ K = 1$.

Consider that slope of $20\log|G(j\omega)|$ is declined at zeros and inclined at poles. Therefore, when $z_1 < z_2$ and $p_1 < p_2$, values should have the following order of magnitude.

$$p_1 < z_1 < z_2 < p_2 .$$

[Estimate] (*No exact number is required for the calculation below. And the method or value may vary depending on the measured value.)

The Bode plot of Figure E8.9 is for below equation.

$$G(s) = \frac{(s/5+1)(s/20+1)}{(s+1)(s/80+1)}\ .$$

E8.11 Consider the feedback control system in Figure E8.11. Sketch the Bode plot of $G_C(s)G(s)$ and determine the crossover frequency, that is, the frequency when $20\log_{10}|G_C(jw)G(jw)| = 0\text{dB}$.

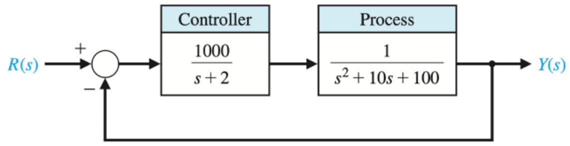


FIGURE E8.11 Unity feedback system.

(Ans)

The frequency response of the open loop system from the given control system is as follows.

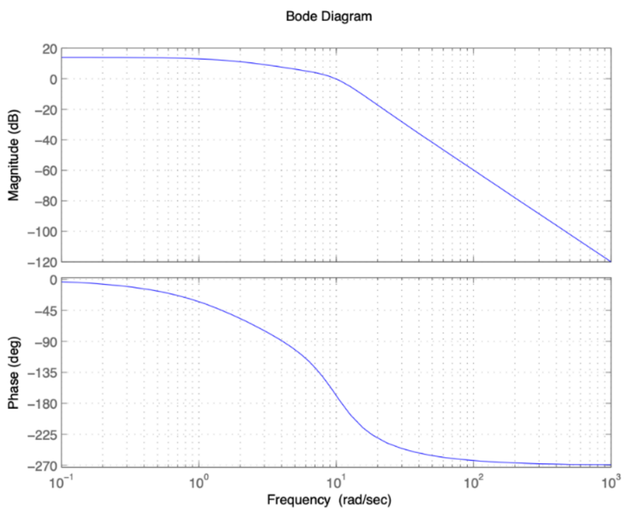
$$G_C(s)G(s) = \left(\frac{1000}{s+2}\right)\left(\frac{1}{s^2+10s+100}\right) = \frac{1000}{(s+2)(s^2+10s+100)} \; .$$

The magnitude relationship of the system is as follow.

$$20\log_{10}|G_c(j\omega)G(j\omega)|$$
$$= 20\log_{10}1000 - 20\log_{10}|j\omega+2| - 20\log_{10}|(j\omega)^2 + 10(j\omega) + 100| \; .$$

The frequency when $20\log_{10}|G_C(jw)G(jw)| = 0$ is $\omega = 9.9$ rad/s, which is the crossover frequency.

The Bode plot of this system is shown in the figure below.



E8.15 Consider the single-input, single-output system described by

$$\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}u(t)$$
$$y(t) = \mathbf{C}\mathbf{x}(t)$$

where

$$\mathbf{A} = \begin{bmatrix} 0 & 1 \\ -6-K & -1 \end{bmatrix}, \mathbf{B} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \mathbf{C} = [5 \quad 3].$$

Compute the bandwidth of the system for $K = 1, 2,$ and 10. As K increases, does the bandwidth increase or decrease?

(Ans)

The closed-loop transfer function is

$$T(s) = \frac{3s+5}{s^2+s+K+6} \ .$$

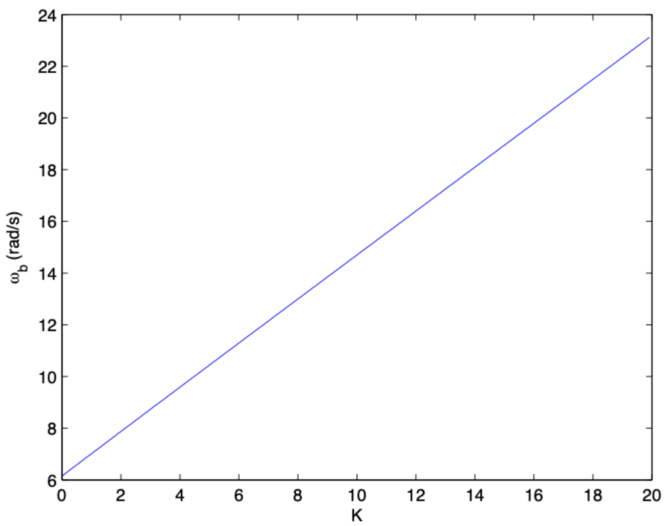
The bandwidth as a function of K is shown in the figure below. The bandwidth as a function of K is:

(a) $K = 1$ and $\omega_b = 7.0$ rad/sec.

(b) $K = 2$ and $\omega_b = 7.9$ rad/sec.

(c) $K = 10$ and $\omega_b = 14.7$ rad/sec.

The bandwidth increases as K increases.



P8.8 A feedback control system is shown in Figure P8.8. The specification for the closed-loop system requires that the percent overshoot to a step input be $P.O. \leq 10\%$. (a) Determine the corresponding specification $M_{p\omega}$ in the frequency domain for the closed-loop transfer function. (b) Determine the resonant frequency ω_r. (c) Determine the bandwidth of the closed-loop system ω_n.

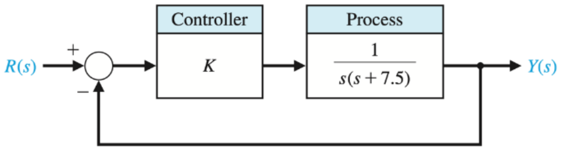


FIGURE P8.8 Second-order unity feedback system.

(Ans)

The transfer function is

$$T(s) = \frac{K}{s^2 + 7.5s + K} .$$

(a) When $P.O. = 10\%$, we determine that $\zeta = 0.5911$ by solving

$$10 = 100e^{-\pi\zeta/\sqrt{1-\zeta^2}} .$$

So, $2\zeta\omega_n = 7.5$ implies that $\omega_n = 6.343$, hence $K = \omega_n^2 = 40.24$. Also,

$$M_{p\omega} = \left(2\zeta\sqrt{1-\zeta^2}\right)^{-1} = 1.048 .$$

(b) For second-order systems we have

$$\omega_r = \omega_n\sqrt{1-2\zeta^2} = 3.481$$

when $\zeta = 0.5911$ and $\omega_n = 6.343$.

(c) The linear approximation $\omega_B/\omega_n = -1.19\zeta + 1.85$ is accurate for $0.3 \leq \zeta \leq 0.8$. So,

$$\omega_B \approx (-1.19\zeta + 1.85)\omega_n = 7.272 \text{ rad/s} .$$