

Extra Credit Problem, Engineering Statistics I - Spring 2024

***Note1** : When submitting your answer to question 2, please also submit the code you used.

***Note2** : If you solve a problem perfectly, you will receive up to 5 extra credits.

1. Suppose Y_1, Y_2, Y_3 are from $\text{Exp}(\beta)$. Let $X_1 = Y_{(1)}$, $X_2 = Y_{(2)} - Y_{(1)}$, $X_3 = Y_{(3)} - Y_{(2)}$. Where $Y_{(i)}$: i th order statistic.
 - (a) Find joint pdf of X_1, X_2, X_3
 - (b) Find marginal pdf of X_1, X_2, X_3
 - (c) Show that X_1, X_2, X_3 are independent
2. (**Use software such as Python, R, Matlab, etc ..**) Suppose X_1, \dots, X_n are from $N(0,1)$. The distribution of the maximum $X_{(n)}$ does not have a closed form pdf. However, we can consider two ways to approximate the pdf. Let $n = 30$.
 - (a) Approximate the distribution of $X_{(n)}$ by simulation, that is, repeat the data generation many times and draw a histogram of $X_{(n)}$.
 - (b) Write the pdf of $X_{(n)}$, as learned from the class. Draw the pdf by numerically evaluating the function over a grid. Compare this with the histogram in the above.