EE20011 Introduction to Physical Electronics

Fall 2025

Final Exam

December 17, 2025

Wednesday 10:00 – 12:00 pm (A)

- **CLOSED BOOK**
- **Show all your work! Partial credit may be given.**
- **Use a calculator but no cell phones.**
- **You have 2 hours to complete this exam.**
- **There are total of 8 problems.**

Problem	Score Assigned	Score Earned
1 (T, F)	20	
2	10	
3	30	
4	20	
5	30	
6	20	
Total	130	

GOOD LUCK!

Problem 1. Conceptual True/False (20 points (2 points each))

Read following descriptions and answer with writing T or F depending on your decision. Correct *the inclined part* when your answer is F. When the answer is F, the correction is 1 pt.

		T/F	Corrections
1	The n_0p_0 product of a semiconductor is *increased* as the temperature is increased.	**T**	
2	The resistivity of a semiconductor in extrinsic region *decreases* as the temperature increases, which prevents the thermal runaway.	**F**	increases
3	The dominant scattering mechanism in a semiconductor at room temperature is *ionized impurity scattering* which makes the mobility to have $\sim T^{-3/2}$ dependency.	**F**	phonon scattering
4	Hole diffusion coefficient in a semiconductor *decreases* as the temperature increases.	**F**	increases
5	The Hall voltage becomes *the larger* when the mobility of the carriers in the measured semiconductor increases.	**T**	
6	Excess electrons are injected at a certain position in a n-type semiconductor which will be disappear at the position with the time constant of *recombination time*.	**F**	dielectric relaxation time
7	Excess holes are injected at a certain position in a n-type semiconductor which will be disappear at the position by the *ambipolar diffusion process* as well as *recombination process.*	**T**	
8	The junction capacitance of a reverse biased pn junction is *larger* than that of a forward biased one.	**F**	smaller
9	The diode voltage of a pn junction with a constant current bias is *reduced* with the increase of temperature.	**T**	
10	The reverse saturation current of a pn junction *is increased* as the doping densities are decreased.	**T**	

Problem 2. Non-uniformly doped semiconductor

In a GaAs in equilibrium, the donor impurity concentration varies as N_d (x)= N_{d0}exp (-x/L) for $0 \le x \le L$, where L = 0. 5 μm and $N_{d0} = 8 \times 10^{16}$ cm^{-3}. Assume μ_n = 5000 cm^2/V-s, e =$1.6x10^{-19}$ C and T = 300 K. [10 pts]

(a) Derive the expression for the electron diffusion current density versus distance over the given range of x. [5 pts]

(b) Determine the induced electric field that generates a drift current density that compensates the diffusion current density. [5 pts]

<Solution>

(a) Derive the expression for the electron diffusion current density versus distance over the given range of x. [5pts]

$J_n = J_{drf} + J_{dif} = 0$

$J_{dif} = eD_n\frac{dn}{dx} = eD_n\frac{dN(x)}{dx} = \frac{eD_n}{-L}N_{do}exp\left(-\frac{x}{L}\right)$ [1pts]

$D_n = \mu_n(kT/e) = (5000)(0.0259) = 129.5cm^2/s$ [derivation 1pt, answer 1pt]

Then,

$J_{dif} = \frac{(1.6x10^{-19})(129.5)(8x10^{16})}{-0.5x10^{-4}}exp(-x/L)\ A/cm^2$

$=-3.32x10^4\ exp(-x/L)\ A/cm^2$ [2pts]

(b) Determine the induced electric field that generates a drift current density that compensates the diffusion current density. [5pts]

$0 = J_{drf} + J_{dif}$

$Jdrf = e\mu_n nE$ [1pts]

$J_{drf} = (1.6x10^{-19})(5000)(8x10^{16})[exp(-x/L)]E = 64[exp(-x/L)]E$ [2pts]

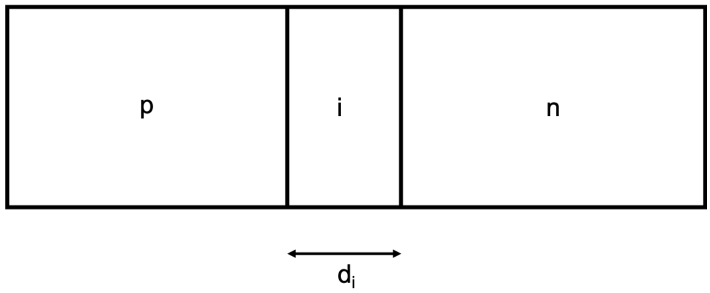
Therefore,

$64[exp(-x/L)]E = 3.32\ x10^4\ exp(-x/L)$

$E = 518.8\ V/cm$ [2pts]

Problem 3. The p-i-n Junction [30 pts]

Consider *p-i-n* type diode. *p* and *n* types semiconductors are uniformly doped at $N_a = 10^{16}$ cm^{-3} and $N_d = 10^{16}$ cm^{-3}, respectively. *i*-layer is intrinsic and *di* = 0.5 um. Assume that the Debye length is much larger than 0.5um in the i region.



a) Draw the charge density, electric field, electrostatic potential, and energy band diagram within the diode qualitatively (without any numbers). [10pts]

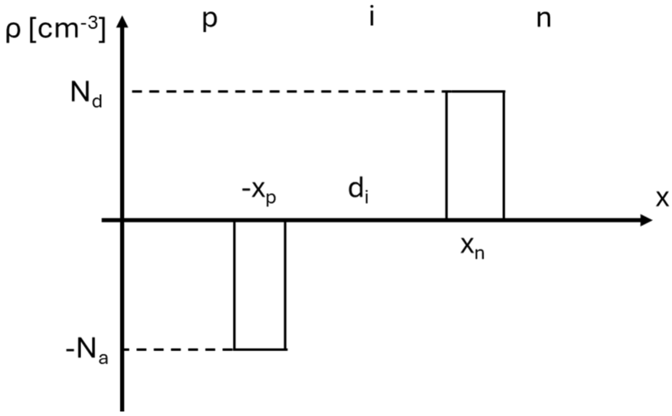
b) What is the maximum electric field? What is the built-in potential across the device? $V_T = kT/q = 26$mV, $n_i = 1.5 \times 10^{10}$ cm^{-3}. [10pts]

c) When d_i = zero, this diode becomes abrupt pn junction. Compare the properties of the p-i-n junction and pn junction qualitatively in terms of 1) junction capacitance and 2) reverse breakdown voltage. [10pts]

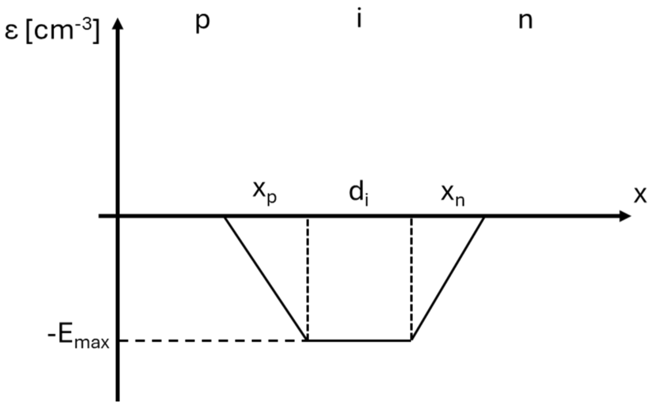
<Solution>

a) Draw the charge density, electric field, electrostatic potential, and energy band diagram within the diode. [10pts]

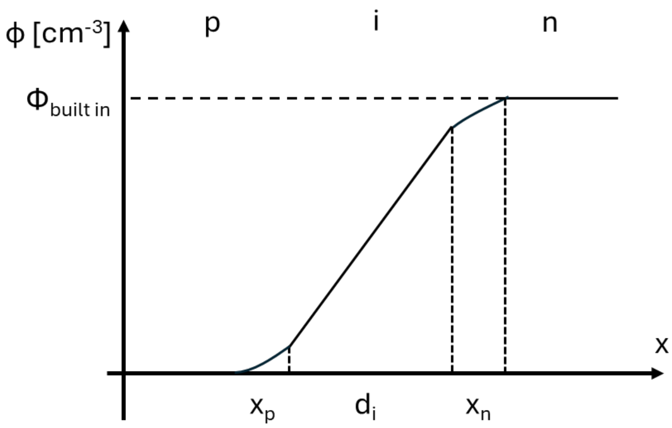
charge density [2pt]



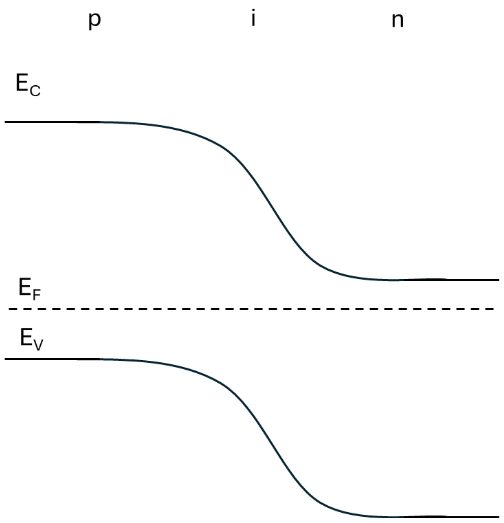
electric field [2pt]



electrostatic potential [2pt]



energy band diagram [2pt]



If they try to draw i region is strait line, and the space charge regions are convex up and down, provide [2pt].

b) What is the maximum electric field? What is the built-in potential across the device? [10 pts]

built-in potential [5pt]

$$V_{bi} = \frac{kT}{e} \ln \frac{N_a N_d}{n_i^2}$$

maximum electric field [5pt]

$$V_{bi} = \frac{e}{2\epsilon_s}\left(N_a x_p^2 + 2N_a d_i x_p + N_d x_n^2\right) = \frac{eN_a}{2\epsilon_s}\left((1 + \frac{N_a}{N_d})x_p^2 + 2d_i x_p\right)$$

$$x_p = \frac{-d_i + \sqrt{d_i^2 + \left(1 + \frac{N_a}{N_d}\right)\frac{2\epsilon_s V_{bi}}{eN_a}}}{1 + \frac{N_a}{N_d}}$$

$$E_{max} = \frac{eN_a x_p}{\epsilon_s}$$

c) Compare the properties of the p-i-n junction and pn junction qualitatively in terms of 1) junction capacitances and 2) reverse breakdown voltages [10 pt]

The p-i-n diode has the smaller junction capacitance [5 pts] and the larger breakdown voltage [5 pts].

Problem 4. The pn Junction [20pts]

Consider a silicon *p-n* diode doped uniformly. At zero bias, under equilibrium condition, 75 % of the total space charge region is in the n-region. The built-in potential is V_{bi} = 0.93 V. The intrinsic carrier density n_i of silicon is 1.5×10^{10} cm^{-3} at 300 K.

a) Determine the depletion width W and each space charge region (x_p, x_n). [10pts]

b) If -1 V of an external bias is applied, what percentage of depletion width (W) will be changed, compared to depletion width at zero bias? [10pts]

<Solution>

a) Determine the depletion width W and each region (x_p, x_n). [10pts]

At zero bias,

$\mathrm{x_n} = 0.75\, W$

$\mathrm{W} = \mathrm{x_n} + x_p$

$\mathrm{x_n} = 3x_p$

Under uniform doping condition,

$\mathrm{N_D} x_n = N_A x_p$ [2pts]

$\mathrm{N_A} = 3N_D$

$\mathrm{V_{bi}} = V_T \ln\left(\frac{N_A N_D}{n_i^2}\right) = 0.93\ \mathrm{V}$ [2pts]

; where $\mathrm{V_T} = \frac{k_B T}{e} = 0.0259\ \mathrm{V},\ \mathrm{n_{i,Si}} = 1.5 \times 10^{10}\ \mathrm{cm^{-3}}$

$\mathrm{N_D} = 5.43 \times 10^{17}\ \mathrm{cm^{-3}}$

$\mathrm{N_A} = 1.63 \times 10^{18}\ \mathrm{cm^{-3}}$

$\mathrm{x_n} = \left[\frac{2\varepsilon_{Si} V_{bi}}{e}\left(\frac{N_A}{N_D}\right)\left(\frac{1}{N_A + N_D}\right)\right]^{0.5} = 4.077 \times 10^{-6}\ \mathrm{cm} = 40.77\ \mathrm{nm}$ [2pts]

$$x_p = \left[\frac{2\varepsilon_{Si}V_{bi}}{e}\left(\frac{N_D}{N_A}\right)\left(\frac{1}{N_A+N_D}\right)\right]^{0.5} = 1.359 \times 10^{-6} \text{ cm} = 13.59 \text{ nm}$$ [2pts]

Total depletion width $W = x_n + x_p = 54.36$ nm [2pts]

NOTE: If the answer is incorrect, partial credit will be given for using the correct equations.

Calculation errors will be ignored.

b) If -1 V of an external bias is applied, what percentage of depletion width (*W*) will be changed, compared to depletion width at zero bias? [10pts]

At -1 V external bias,

$$W = \left[\frac{2\varepsilon_{Si}(V_{bi}-V_{bias})}{e}\left(\frac{1}{N_A}+\frac{1}{N_D}\right)\right]^{0.5}$$ [4pts]

$$W = \left[\frac{2 \times 11.7 \times (8.854 \times 10^{-14}) \times (0.93+1)}{1.6 \times 10^{-19}}\left(\frac{1}{5.43 \times 10^{17}} + \frac{1}{1.63 \times 10^{18}}\right)\right]^{0.5}$$

; where $\varepsilon_{Si} = 11.7 \times (8.854 \times 10^{-14})$

$V_{bias} = -1\ V$ [2pts]

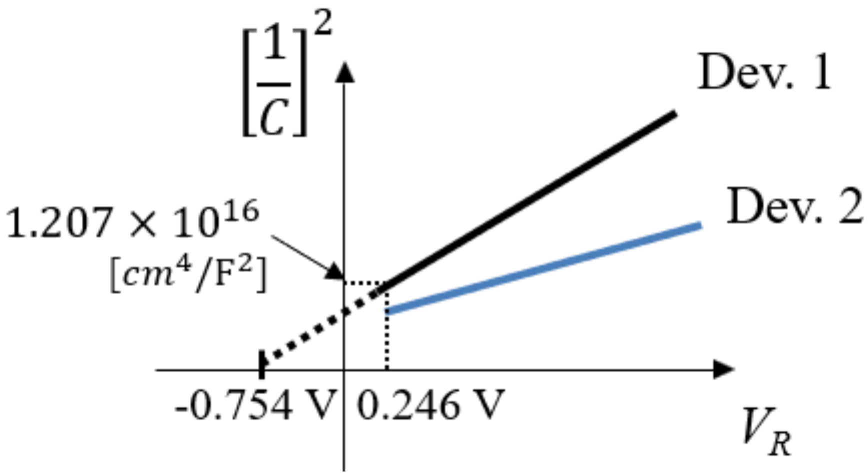
$W = 7.831 \times 10^{-6} \text{ cm} = 78.31 \text{ nm}$ [2pts]

$$\Delta W = \frac{78.31 - 54.36}{54.36} = 0.4406 = 44.06\%$$ [2pts]

NOTE: If the answer is incorrect, partial credit will be given for using the correct equations.

Calculation errors will be ignored.

Problem 5. C-V characteristics of p+n Junctions [30pts]



Consider two different devices having one-sided junction within silicon substrate. Two abrupt junctions have relatively high boron (p-type) concentration compared to that of phosphorous(n-type). Capacitance plots of the junctions are shown above. One can estimate the doping concentrations from the slopes and x-intercept points of the graphs.

Permittivity of free space value is $8.85 \times 10^{-14}\ F/cm$ and T=300 K. You may use the given parameters in the next page.

(a) Calculate the N_d value of Dev. 1. [5 pts]

(b) Calculate the N_a value of Dev. 1. [5 pts]

(c) Calculate the depletion width of Dev. 1. [5 pts]

(d) Between Dev. 1 and Dev. 2, which device has longer depletion width? And which one has higher doping concentration in n-type region? Explain as precisely as you can. [5pts]

(e) Which device will have higher breakdown voltage? Explain why. [5 pts]

(f) Now let's imagine a new device, Dev. 3. This device has GaAs substrate and the same doping concentration as Dev. 1. Draw the C-V plots of Dev. 3 and Dev 1 together, and denote the *x*-intercept and slope values. [5 pts]

<Solution>

(a) Calculate the Nd value of Dev. 1. (5 pts)

slope $= 1.207 \times 10^{16} = \frac{2}{e\varepsilon_s N_d}$ [2 pts]

$N_d = \frac{2}{(1.6\times10^{-19})(11.7)(8.85\times10^{-14})(1.207\times10^{16})} = 1 \times 10^{15} cm^{-3}$ [3 pts]

(b) Calculate the Na value of Dev. 1. (5 pts)

$V_{\rm bi} = \frac{kT}{q} \ln \frac{N_a N_d}{n_i^2}$ [2 pts]

$N_a = \frac{n_i^2}{N_d} exp\left[\left(\frac{q}{kT}\right) V_{bi}\right] = \frac{(1.5\times10^{-10})^2}{1\times10^{15}} exp\left[\frac{0.754}{0.0259}\right] = 9.89 \times 10^{17} cm^{-3}$ [3 pts]

(c) Calculate the depletion width of Dev. 1. (5 pts)

$\mathrm{W} \cong x_{\rm n} \cong \sqrt{\frac{2\varepsilon_s V_{bi}}{e} \frac{1}{N_d}} = \sqrt{\frac{2(11.7)(8.85\times10^{-14})(0.754)}{(1.6\times10^{-19})(1\times10^{15})}} = 0.988\mu\mathrm{m}$

[Derivation 1 pts, Answer 4 pts]

(d) Between Dev. 1 and Dev. 2, which device has longer depletion width? And which one has higher doping concentration in n-type region? Explain as precisely as you can.

(Hint: $y = \frac{x}{\ln x}$ is an increasing function at sufficiently large x) (5 pts)

Dev. 2 has higher N_d value because it exhibits lower slope (slope $= \frac{2}{e\varepsilon_s N_d}$).

Due to the relatively high Na concentration, depletion width can be approximated below.

$$\mathrm{W} \cong x_{\rm n} \cong \sqrt{\frac{2\varepsilon_s V_{bi}}{e} \frac{1}{N_d}} = \sqrt{\frac{2\varepsilon_s}{e} \frac{kT}{q} \frac{\ln\left(\frac{N_a N_d}{n_i^2}\right)}{N_d}} \quad \text{[2~3 pts]}$$

Since Dev. 1 has lower N_d value and $y = \frac{x}{\ln x}$ is a increasing function at large x, [0~1 pts]

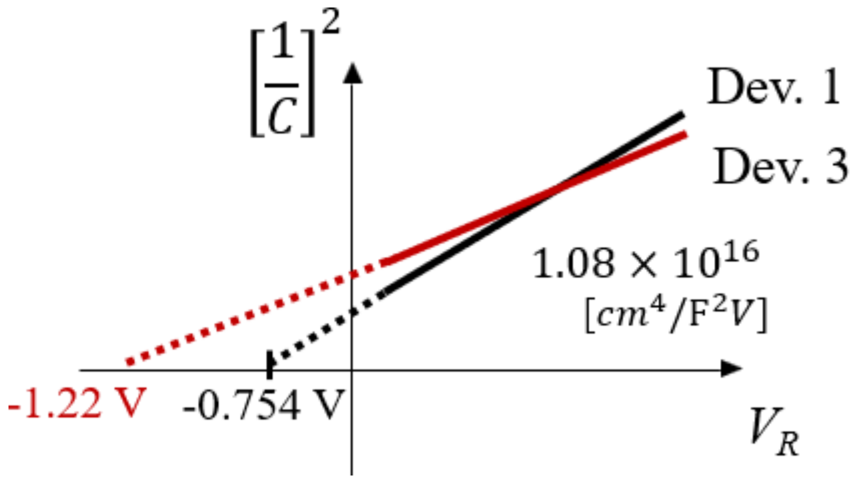
Dev 1. has longer depletion width. [3 pts]

(e) Which device will have higher breakdown voltage? Explain why. (5 pts)

Since breakdown voltage is high at low N_d concentration, Dev. 1 has higher breakdown voltage. [Explanation 2 pts, Answer 3 pts]

(f) Now let's imagine a new device, Dev. 3. This device has GaAs substrate and the same doping concentration as Dev. 1. Draw the C-V plots of Dev. 3 and Dev 1 together, and denote the x-intercept and slope values. (5 pts)

Due to the dielectric constant difference, GaAs shows smaller slope (slope = $\frac{2}{e\varepsilon_s N_d}$). And at the same doping level, V_{bi} of GaAs is larger due to the lower intrinsic carrier value (n_i).



[3 pts, 1 point deducted if axis labels are missing.]

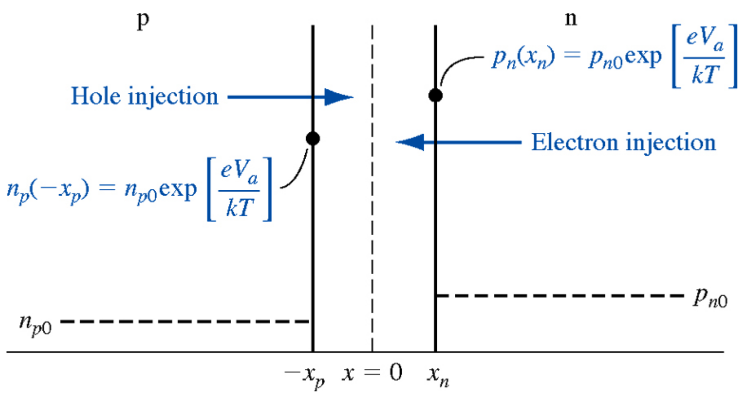
$$slope = \frac{2}{(1.6 \times 10^{-19})(13.1)(8.85 \times 10^{-14})(10^{15})} = 1.08 \times 10^{16} cm^4/F^2 \cdot V$$

$$V_{\mathrm{bi}} = \frac{kT}{q} \ln \frac{N_a N_d}{n_i^2} = (0.0259) \ln \frac{(9.89 \times 10^{17})(1 \times 10^{15})}{(1.8 \times 10^6)^2} = 1.22\ V$$

[2 pt for slope, 1 pt for V_{bi}]

Problem 6. I-V relationship of a pn junction diode [20pts]

By applying forward bias Va, the excess minority carriers are created in each space charge region edge of the pn junction as shown below figure.



a) According to the above figure, which side has the higher doping density? p region and n region? How do you decide it? [5 pts]

b) Find the minority diffusion currents as a function of x at both sides. [5 pts]

c) Find the total current equation. [5 pts]

d) Explain how the total current is calculated and assumptions. [5 pts]

<solutions>

a) p- type doping is the higher since the minority carrier density is the smaller compared to the other side.[5 pts]

b) [5 pts]

$$\Rightarrow J_p(x) = \frac{eD_p p_{n0}}{L_p}\left[\exp\left(\frac{eV_a}{kT}\right) - 1\right] e^{\frac{(x_n - x)}{L_p}} \quad (x \geq x_n)$$

$$\Rightarrow J_n(x) = \frac{eD_n n_{p0}}{L_n}\left[\exp\left(\frac{eV_a}{kT}\right) - 1\right] e^{\frac{(x_p + x)}{L_n}} \quad (x \leq -x_p)$$

c)[5pts]

$$J = J_p(x_n) + J_n(-x_p) = \frac{eD_p p_{n0}}{L_p}\left[\exp\left(\frac{eV_a}{kT}\right) - 1\right] + \frac{eD_n n_{p0}}{L_n}\left[\exp\left(\frac{eV_a}{kT}\right) - 1\right] = \left[\frac{eD_p p_{n0}}{L_p} + \frac{eD_n n_{p0}}{L_n}\right]\left[\exp\left(\frac{eV_a}{kT}\right) - 1\right]$$

d) [5 pts] assumptions

1. The individual electron and hole currents are continuous functions through the pn structure [1 pt]
2. The individual electron and hole currents are constant throughout the depletion region [1 pt]
3. Abrupt pn junction [1 pt]
4. Low level injection [1 pt]
5. Maxwell Boltzman distribution[1 pt] – Nondegenerated semiconductor

------Useful equations and physical constants--------------------

$$N_a x_p = N_d x_n$$

$$x_p = \left\{\frac{2\varepsilon_s(V_{bi} + V_R)}{e}\left[\frac{N_d}{N_a}\right]\left[\frac{1}{N_a + N_d}\right]\right\}^{1/2}$$

$$x_n = \left\{\frac{2\varepsilon_s(V_{bi} + V_R)}{e}\left[\frac{N_a}{N_d}\right]\left[\frac{1}{N_a + N_d}\right]\right\}^{1/2}$$

$$W = \left\{\frac{2\varepsilon_s(V_{bi} + V_R)}{e}\left[\frac{N_a + N_d}{N_a N_d}\right]\right\}^{1/2} \approx \left\{\frac{2\varepsilon_s(V_{bi} + V_R)}{eN_d}\right\}^{1/2}$$

$$E_{\max} = -\frac{eN_d x_n}{\varepsilon_s} = -\frac{eN_a x_p}{\varepsilon_s} = -\left\{\frac{2e(V_{bi} + V_R)}{\varepsilon_s}\left(\frac{N_a N_d}{N_a + N_d}\right)\right\}^{1/2} = -\frac{2(V_{bi} + V_R)}{W}$$

$$C' = \frac{dQ'}{dV_R} = \left\{\frac{e\varepsilon_s N_a N_d}{2(V_{bi} + V_R)(N_a + N_d)}\right\}^{1/2} \approx \left\{\frac{e\varepsilon_s N_d}{2(V_{bi} + V_R)}\right\}^{1/2}$$

$$V_{bi} = |\phi(x = x_n)| = \frac{e}{2\varepsilon_s}(N_d x_n^2 + N_a x_p^2)$$

$$V_o = \psi 1 - \psi 2 = V_T \ln(\frac{N_{D1}}{N_{D2}})$$

$$V_o = V_T \ln \frac{N_D N_A}{ni^2}$$

$$E = -\frac{V_T}{n(x)}\frac{dn(x)}{dx} \left(= \frac{V_T}{p(x)}\frac{dp(x)}{dx}\right)$$

Dielectric constant of Si : 11.7 ε_0
Dielectric constant of GaAs : 13.1 ε_0
$\varepsilon_0 = 8.85 \times 10^{-14}$ F/cm

$$\delta p_n(x) = p_n(x) - p_{n0} = Ae^{x/L_p} + Be^{-x/L_p} \quad (x \geq x_n)$$

$$\delta n_p(x) = n_p(x) - n_{p0} = Ce^{x/L_n} + De^{-x/L_n} \quad (x \leq -x_p)$$

$$\boxed{p_n(x_n) = p_{n0} \exp\left(\frac{eV_a}{kT}\right)} \qquad \boxed{p_n(x \to +\infty) = p_{n0}}$$

$$\boxed{n_p(-x_p) = n_{p0} \exp\left(\frac{eV_a}{kT}\right)} \qquad \boxed{n_p(x \to -\infty) = n_{p0}}$$

$$\boxed{J_p(x_n) = \frac{eD_p p_{n0}}{L_p}\left[\exp\left(\frac{eV_a}{kT}\right) - 1\right]}$$

$$\boxed{J_n(-x_p) = \frac{eD_n n_{p0}}{L_n}\left[\exp\left(\frac{eV_a}{kT}\right) - 1\right]}$$