EE20011 2025 Fall
Homework - Problem Set #3
Due: October 15th, 11:59 pm
Please scan your homework and upload it on the KLMS website.

Problem 1 Effective mass, Hole, Band gap

(a) Explain the effective mass of electrons on top of valance band in a crystal.

- The parameter $m*$ is called the *effective mass*, taking into account the particle mass and also taking into account the effect of the internal forces.
- In a periodic crystal, the electron's dispersion near an extremum is well-approximated by a parabola. The (scalar) effective mass is defined from the E(k) curvature: $m^* = \frac{\hbar^2}{\frac{d^2E}{dk^2}}$
- At the valence-band maximum (VBM) the curvature is negative ($m^* = \frac{\hbar^2}{\frac{d^2E}{dk^2}} = -\frac{\hbar^2}{2C_2} < 0$), so an electron there has a negative effective mass. That is why describing transport with electrons at the VBM is inconvenient.

(b) Relate the effective mass of **(a)** to that of holes.

- We instead describe the absence of an electron (a hole) near the VBM. Its charge is +q, and its dispersion is again parabolic near the band edge.
- Hence the hole effective mass is the positive quantity: $m_p^* \equiv \frac{\hbar^2}{\left|\frac{d^2E}{dk^2}\right|} = |m_n^*|$
- Holes then accelerate in the direction of the electric field and can be treated as normal, positive-mass, positive-charge carriers.

(c) What are the differences of metal, insulator, semiconductor in terms of conduction band, band gap, and valance band?

- Metal: at least one band is partially filled (or bands overlap). States exist at the Fermi level E_F even at T=0 K → high conductivity.
- Insulator: VB full, CB empty, and a large band gap (few eV). Thermal excitation does not create many carriers → very low conductivity.
- Semiconductor: VB full and CB empty at T=0 K, but moderate bandgap (order of 1 eV). At finite T, thermally excited electrons (in CB) and holes (in VB) appear and conduct.

Problem 2 (Energy) Density of states

- Use the parabolic dispersion $E = \frac{\hbar^2 k^2}{2m}$ and include the usual factor of 2 for spin.
- **Any unsupported or skipped steps in your solution will result in a deduction.**

(a) Derive energy density of states of a particle in 3-, 2-, 1-dimensions.

- 3D: $g_{3D}(E) = \frac{1}{2\pi^2}\left(\frac{2m}{\hbar^2}\right)^{\frac{3}{2}}\sqrt{E}\,, E \geq 0$
- 2D: $g_{2D}(E) = \frac{m}{\pi\hbar^2}\,, E \geq 0$
- 1D: $g_{1D}(E) = \frac{1}{\pi\hbar}\sqrt{\frac{2m}{E}}\,, E > 0$ (In accordance with the lecture notes, $\frac{4}{\hbar\pi}\sqrt{\frac{m}{2E}}$ will also be considered correct.)

(b) Derive average energy of electrons at T = 0 with E_f.

- $N = \int_0^{E_F} g_{3D}(E)dE,\ \ U = \int_0^{E_F} E g_{3D}(E)dE \ \Rightarrow \langle E\rangle = \frac{U}{N} = \frac{3}{5}E_F$

Problem 3 Effective density of states

- Assumptions: parabolic bands near the edges, non-degenerate (Boltzmann) statistics, thermal equilibrium.

(a) Explain the effective density of state in a conduction band.

- The constants N_C and N_V are the effective density of states for the conduction and valence bands and utilize compact expression for carrier density.
- N_C (and N_V) summarize how many band-edge states are thermally available within ~kT of the edge.
- Near the conduction-band edge E_C with electron effective mass m_n^* and parabolic dispersion, the 3D DOS per energy is $g_C(E) = \frac{(2m_n^*)^{3/2}}{2\pi^2\hbar^3}\sqrt{E - E_C}\ (E \geq E_C)$
- In the non-degenerate limit, the Fermi–Dirac distribution reduces to the Boltzmann factor: $f(E) \approx \exp(-(E - E_F)/kT)$
- Integrate to get the electron density: $n = \int_{E_C}^{\infty} g_C(E) f(E) dE = \exp(-(E_C - E_F)kT)\int_0^{\infty}\frac{(2m_n^*)^{\frac{3}{2}}}{2\pi^2\hbar^3}\sqrt{E - E_C}\exp(-(E - E_C)/kT)d(E - E_C)$
- Let $N_C \equiv 2\left(\frac{2\pi m_n^* kT}{h^2}\right)^{\frac{3}{2}}$, then $n = N_C\exp(-(E_C - E_F)/kT)$

(b) Explain the physical reason why product of electron and hole concentrations of n-type Si semiconductor (for example, $N_d= 10^{16}$ cm^{-3}) is same as that of intrinsic semiconductor.

- $np = [N_c \exp(-(E_C - E_F)/kT)][N_V \exp(-(E_F - E_V)/kT)] = N_C N_V \exp(-(E_C - E_V)/kT) = N_C N_V \exp(-E_g/kT) = n_i^2$
- From the equation above, the Fermi level E_F cancels. Therefore, as long as the assumptions hold (non-degenerate, equilibrium), the product np is a material- and temperature-only constant, equal to the intrinsic carrier density squared n_i^2.
- In thermal equilibrium, the thermal generation rate is fixed by the band gap and temperature, while the recombination rate scales with np. Detailed balance (G_{th}=R) therefore forces np to a unique value at each T; that value is n_i^2. Doping moves E_F and changes n and p in opposite directions, but their product stays fixed.

(c) Explain the physical reason why the hole concentration of the n-type Si is much less than that of intrinsic Si Semiconductor.

- With donors $N_D \gg n_i$, $n \approx N_D$ from the mass-action law, $p = \frac{n_i^2}{n} \approx \frac{n_i^2}{N_D} \ll n_i$.
- Physically, moving E_F upward toward E_C suppresses the probability of empty states at the valence-band edge, so holes become very scarce.

Problem 4

(a) A semiconductor is doped with an impurity concentration N such that N >> n_i and all the impurities are ionized. Also, n = N and p = n_i^2/N. Is the impurity a donor or an acceptor? (2point)

Answer : The impurity is a donor

(b) The electron concentration in a piece of Si maintained at 300 K under equilibrium condition, is 10^5 cm^{-3}. What is the hole concentration? (2point)

Answer :

$$p_i = \frac{n_i^2}{n} = \frac{(1.5*10^{10})^2}{10^5} = 2.25*10^{15}(cm^{-3})$$

(c) For a silicon sample maintained at T = 300 K, the Fermi level is located 0.259 eV below the intrinsic Fermi level. What are the hole and electron concentrations? (3point)

Answer :

$$n = n_i e^{\frac{E_F - E_i}{kT}} = 1.5*10^{10} e^{\frac{-0.259}{0.0259}} = 1.5*10^{10} e^{-10} = 6.81*10^5 (cm^{-3})$$

$$p = \frac{n_i^2}{n} = \frac{(1.5*10^{10})^2}{6.81*10^5} = 3.3 * 10^{14}(cm^{-3})$$

(d) In a non-degenerate germanium sample maintained under equilibrium conditions near room temperature, it is known that $n_i = 10^{13} cm^{-3}$, $n = 2p$, and $N_A = 0$. Determine n and N_D. (3point)

Answer :

$$np = n_i^2 = 2p^2 = (10^{13})^2 \rightarrow p = 7.07 * 10^{12}(cm^{-3})$$

$$n = 2p = 1.414 * 10^{13}(cm^{-3})$$

$$\text{Due to charge neutrality,}\ \ n + N_a = p_0 + N_D$$

$$n + N_a = p + N_D \rightarrow N_D = n - p = p = 7.07 * 10^{12}(cm^{-3})$$

Problem 5. Intrinsic semiconductor

The electron concentration in silicon at T = 300 K is $n_0 = 2\times10^5$ cm^{-3}.

(a) Determine the position of the Fermi level with respect to the valence band energy level. (4point)

Answer :

Solution 1)

$$E_F - E_i = kT * \ln\left(\frac{n_0}{n_i}\right) = 0.0259 * \ln\left(\frac{2 * 10^5}{1.5 * 10^{10}}\right) = -0.29[eV]$$

$$E_i \approx E_v + \frac{E_g}{2} = E_v + 0.56[eV]$$

$$E_F - E_v = (E_F - E_i) + (E_i - E_v) \approx -0.29 + 0.56 = 0.27[eV]$$

Solution 2)

$$p_0 = N_V \exp\left(-\frac{E_F - E_V}{kT}\right) \rightarrow \frac{n_i^2}{n_0} = 1.04 * 10^{19} * \exp\left(-\frac{E_F - E_V}{0.0259}\right)$$

$$\rightarrow E_F - E_V = 0.236[eV]$$

(b) Determine p_0. (3point)

Answer :

$$p_0 = \frac{n_i^2}{n_0} = \frac{(1.5*10^{10})^2}{2*10^5} = 1.125 * 10^{15}(cm^{-3})$$

(c) Is this n- or p-type material? (3point)

Answer : this is a p-type material.

Problem 6. Extrinsic Semiconductor

Silicon at T =300 K is doped with boron atoms such that the concentration of holes is $p_0 = 5\times10^{15}\ cm^{-3}$.

(a) Find $E_F - E_v$. (2point)

Answer :

Solution 1)

$$E_i - E_F = kT * \ln\left(\frac{p_0}{n_i}\right) = 0.0259 * \ln\left(\frac{(5 * 10^{15})}{1.5 * 10^{10}}\right) = 0.329[eV]$$

$$E_F - E_v = (E_F - E_i) + (E_i - E_v) \approx -0.329 + 0.56 = 0.231[eV]$$

Solution 2)

$$E_F - E_V = -kT * \ln\left(\frac{p_0}{N_V}\right) = 0.198[eV]$$

(b) Determine $E_c - E_F$. (2point)

Answer :

Solution 1)

$$E_C - E_F = (E_C - E_i) + (E_i - E_F) \approx 0.56 + 0.329 = 0.889[eV]$$

Solution 2)

$$E_C - E_F = -kT * \ln\left(\frac{n_0}{N_C}\right) = 0.857[eV]$$

(c) Determine n_0. (2point)

Answer :

$$n_0 = \frac{n_i^2}{p_0} = \frac{(1.5*10^{10})^2}{5*10^{15}} = 4.5 * 10^4 (cm^{-3})$$

(d) Which carrier is the majority carrier? (2point)

Answer : Holes are the majority carrier.

(e) Determine $E_{Fi} - E_F$. (2point)

Answer :

$$E_i - E_F = kT * \ln\left(\frac{p_0}{n_i}\right) = 0.0259 * \ln\left(\frac{(5*10^{15})}{1.5*10^{10}}\right) = 0.329[eV]$$

Problem 7. Charge neutrality

Assume that silicon, germanium, and gallium arsenide each have dopant concentrations of $N_d = 1\times10^{13}\ cm^{-3}$ and $N_a = 2.5\times10^{13}\ cm^{-3}$ at T = 300 K. For each of the three materials:

(*a*) Is this material n type or p type?

(*b*) Calculate n_0 and p_0.

(a) Since $N_a > N_d$, all material is p type

(b) $n_0p_0=n_i^2$ (constant for given semiconductor material at a given T)

$$n_0 = \frac{(N_d - N_a)}{2} + \sqrt{\left(\frac{N_d - N_a}{2}\right)^2 + n_i^2} \quad , \quad p_0 = \frac{(N_a - N_d)}{2} + \sqrt{\left(\frac{N_a - N_d}{2}\right)^2 + n_i^2}$$

- Silicon : $n_i = 1.5\times10^{10}$ [cm^{-3}] → $n_0 = 1.5\times10^{7}$ [cm^{-3}], $p_0 = 1.5\times10^{13}$ [cm^{-3}]
- Germanium : $n_i = 2.5\times10^{13}$ [cm^{-3}] → $n_0 = 1.86\times10^{13}$ [cm^{-3}], $p_0 = 3.36\times10^{13}$ [cm^{-3}]
- Gallium arsenide : $n_i = 2.0\times10^{6}$ [cm^{-3}] → $n_0 = 2.7\times10^{-1}$ [cm^{-3}], $p_0 = 1.5\times10^{13}$ [cm^{-3}]

Problem 8. Charge neutrality

A particular semiconductor material is doped at $N_d = 2\times10^{14}$ cm^{-3} and $N_a = 1.2\times10^{14}$ cm^{-3}. The thermal equilibrium electron concentration is found to be $n_0 = 1.1\times10^{14}$ cm^{-3}. Assuming complete ionization, determine the intrinsic carrier concentration and the thermal equilibrium hole concentration.

- At complete ionization, $n_0 + N_A = p_0 + N_D$.
  $p_0 = n_0 + (N_A - N_D) = 3.0\times10^{13}$ [cm^{-3}]
- $n_i^2 = n_0 p_0 = 3.3\times10^{27}$ [cm^{-3}]
  $n_i = 5.7\times10^{13}$ [cm^{-3}]

Problem 9. Position of Fermi level

For a particular semiconductor, E_g = 1.50 eV, $m_p^* = 10\, m_n^*$, T = 300 K, and $n_i = 1\times10^5$ cm^{-3}.

(a) Determine the position of the intrinsic Fermi energy level with respect to the center of the bandgap.

(b) Impurity atoms are added so that the Fermi energy level is 0.45 eV below the center of the bandgap.

(*i*) Are acceptor or donor atoms added?

(*ii*) What is the concentration of impurity atoms added?

(a)

- $E_{Fi} = E_{midgap} + \frac{3}{4} kT \ln\left(\frac{m_p^*}{m_n^*}\right)$ → $E_{Fi} - E_{midgap} = \frac{3}{4} kT \ln\left(\frac{m_p^*}{m_n^*}\right)$
- $E_{Fi} - E_{mid} = \frac{3}{4} \times (0.0259) \times \ln(10) = 0.0447$ eV
- Thus, $E_{Fi} \approx$ 0.045 eV above the center of the bandgap

(b)

(*i*) Impurity atoms added 0.45eV below the center of the bandgap. So this impurity atoms is p-type, <u>accepter added</u>.

(*ii*)

- $E_{Fi} - E_F = 0.045 + 0.45 = 0.495$ eV
- $E_{Fi} - E_F = kT \ln\left(\frac{p_0}{n_i}\right) \rightarrow p_0 = n_i \ln\left(\frac{E_{Fi}-E_F}{kT}\right) = (1\times10^5)\times \ln\left(\frac{0.495}{0.0259}\right) \approx 2.0\times10^{13}$ [cm^{-3}]
- $n_i^2 = n_0\, p_0 \rightarrow n_0 = 5.0\times10^{-4}$ [cm^{-3}]
- $N_A - N_D = p_0 - n_0 \approx$ <u>2.0×10^{13} [cm^{-3}]</u> (cf. $N_A - N_D \approx p_0$)