MAS 250 Homework Assignment 4

Due: October 14 (Friday) 1 pm

Instruction: Turn in homework as a single pdf file.
Note: The solution set will be uploaded on KLMS by the evening of October 14.

1. If $X \sim B(n_1, p)$, $Y \sim B(n_2, p)$, and X and Y are independent each other, what is the distribution of $X + Y$? Answer the question using the moment generating function of the Binomial distribution.

2. (a) The number of people entering the intensive care unit at a hospital on any single day possesses a Poisson distribution with a mean equal to five persons per day. Let X be the number of people entering the intensive care unit on a particular day. Given that $X > 1$, what is the probability of $X > 2$?

   (b) If events are occurring in time according to a Poisson distribution with mean λt, then the interarrival times between events have an exponential distributions with mean $1/\lambda$. If calls come into a police emergency center at the rate of 10 per hour, what is the probability that more than 15 minutes will elapse between the next two calls?

3. According to Zimmels, the sizes of particles used in sedimentation experiments often have a uniform distribution. In sedimentation involving mixtures of particles of various sizes, the larger particles hinder the movements of the smaller ones. Thus, it is important to study both the mean and the variance of particle sizes. Suppose that spherical particles have diameters that are uniformly distributed between .01 and .05 centimeters. Find the mean and variance of the volumes of these particles.

4. Scores on a certain nationwide college entrance examination follow a normal distribution with a mean of 500 and a standard deviation of 100.

   (a) If a school only admits students who score over 680, what proportion of the student pool would be eligible for admission?

   (b) What limit would you set that makes 50% of the students eligible?

   (c) What should be the limit if only the top 15% are to be eligible?

5. The daily amount of money spent on maintenance and repairs by a company is observed, over a long period of time, to be normal distributed with mean \$200 and standard deviation \$100. Assume that each day spending is independent and \$210 is budgeted for each business day (from Monday to Friday).

   (a) What is the probability that the actual cost on a randomly selected day will exceed the budgeted amount?

   (b) Find the probability that the mean daily cost (of five business days) of a randomly selected week exceeds the budgeted daily amount.

(c) Assume that there are four weeks in October. What is the probability that the mean daily cost exceeds the budgeted daily amount for at least one week?

(d) Assume that this budget plan will be sustained next year (52 weeks). What is the probability that the mean daily cost will exceed the budgeted daily amount in more than 25 weeks during the next year?

6. Suppose that (X_1, X_2, X_3, X_4) is a random sample from a normal distribution with mean μ and variance σ^2.

   (a) Show that
   $$U = \frac{(X_1 - X_2)^2}{2\sigma^2}$$
   follows a χ^2 distribution with 1 df.

   (b) Show that
   $$V = \frac{(X_3 - X_4)}{\sqrt{(X_1 - X_2)^2}}$$
   follows a t distribution with 1 df.

   (c) Show that
   $$W = \frac{(X_3 - X_4)^2}{(X_1 - X_2)^2}$$
   follows a F distribution with (1,1) df.

7. From the exercise problems in Chapter 5:

   12, 18, 26, 41

8. From the exercise problems in Chapter 6:

   15

9. (Suggested in Chapter 5: no submission)

   5, 6, 9, 15, 18, 20, 21, 22, 25, 28, 29, 35, 39, 40, 45, 47

10. (Suggested in Chapter 6: no submission)

    2, 6, 8, 10, 12, 16, 18, 20, 25, 29