# 5. Moment of Inertia

## INTRODUCTION

With moment, a rigid body can circulate the axis of rotation where friction is low. The objective of this experiment is to learn how to measure rotation angle and angular velocity according to time, and to determine angular acceleration by a function of the moment.

## THEORY

Rotary motion is governed by potential energy, kinetic energy, and rotary energy. Rotary energy can change according to the mass, shape, and size. It is convenient to use the concept of the moment of inertia in an expression of rotary energy.

In rotary motion, the moment of inertia takes the roll of inertia, as in the mass of an object taking a roll of inertia in a straight line motion.

When a rigid body consisting of $n$ particles circulates the axis of rotation with the angular velocity of $\omega$, the total kinetic energy $K$ is

$$K = \frac{1}{2}\left(\sum m_i r_i^2\right)\omega^2 = \frac{1}{2} I\omega^2 \,, \tag{1}$$

where $I$ is known as the moment of inertia

$$I = \sum m_i r_i^{\,2} \; (r_i\text{: distance from the axis of rotation}). \tag{2}$$

For a continuous mass distribution, the moment of inertia is

$$I = \int r^2 dm \,. \tag{3}$$

In *Fig. 1*, if a weight with mass $M$ falls distance h and causes the rotation of a disk during time $t$, the principle of the conservation of energy is

$$Mgh = \frac{1}{2} M\upsilon^2 + \frac{1}{2} I\omega^2 \,, \tag{4}$$

where $\upsilon$ is the velocity when the weight falls a distance $h$, assuming that the weight moves with uniform acceleration as expressed by $\upsilon = at$, $h = \frac{1}{2}at^2$ ($a$ : acceleration) and so,

$$\upsilon = \frac{2h}{t}. \tag{5}$$

Here, $\omega$ is the acceleration of the axis of rotation, and

$$\omega = \frac{\upsilon}{R} \quad (R\text{ : radius of the axis of rotation}). \tag{6}$$

If *Eqs.* (5) and (6) are substituted with *Eq.* (4), the moment of inertia $I$ of a rotating body becomes

$$I = MR^2\left(\frac{gt^2}{2h} - 1\right). \tag{7}$$

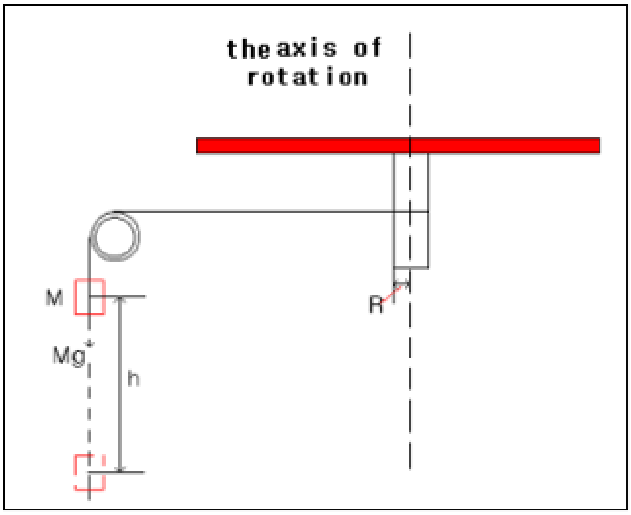


*Fig. 1*. The moment of inertia.

The following figures show various moments of inertia.

- The moment of inertia of a disk : $I = \frac{1}{2}ML^2$ .

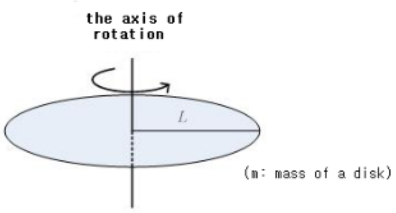


*Fig. 2*. Moment of inertia of a disk.

- The moment of inertia of a bar : $I = \frac{1}{12}ML^2$.

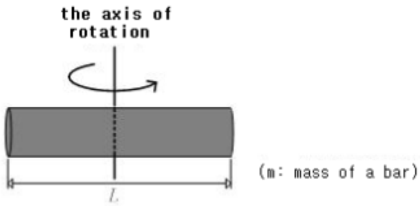


*Fig. 3*. Moment of inertia of a bar.

- The moment of inertia of a particle: $I = MR^2$.

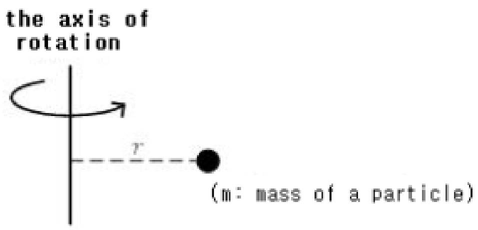


*Fig. 4*. Moment of inertia of a disk.

- The moment of inertia of a ring: $I = \frac{1}{2}M(R_1^{\ 2} + R_2^{\ 2})$.

($R_1$: inner radius, $R_2$ : outer radius )

## PROCEDURE

### I. The moment of inertia of an axis of rotation

*Fig. 5*. The moment of inertia of an axis of rotation.

(a) Create the set up shown in *Fig. 5* and make sure that it does not reach the pulley when the weight is placed at the end of its position.

(b) Connect webcam to the computer and run the video capture program. Drag the camera icon in the column on the right side to the center and determine if the apparatus shown in *Fig. 5* is displayed.

(c) Align the apparatus and the camera so that both the standard ruler of the standing post and the weight are in the same plane.

(d) Capture the motion for few seconds.

(e) Execute the [Coordinate set up] menu on the menu bar. Place the coordinate axis and origin. Place each ends of the ruler to the middle of each taping. Input the distance (50 cm).

(f) Rotate the disk and roll up the thread where the weight is hanging. If the thread is rolled up, the hanger of the weight moves upward. Ensure that it does not to reach the upper bound pulley. Do not add another weight to the hanger.

(g) Start the camera capture and drop the weight after the apparatus is stable.

(h) End the capture.

(i) Define the position of the weight in each frame by clicking, or by using auto-tracking menu (Tip: Include the black bowl when defining the object to trace down. It will lower the number of false tracking)

(j) Load graph by dragging and dropping the graph icon on the left icon bar. Set the horizontal axis as time and vertical axis as y position of the object.

(k) Fit the graph with the second order polynomial by using fit function menu. You can set appropriate ranges of data for better fit results.

(l) Capture the graph and export the data.

(m) Repeat steps (f) through (l) for three times and calculate the average acceleration.

(n) Measure the radius of the rotational axis with caliper and measure the mass of the weight hanger with a balance.

**II. The moment of inertia of a disk for horizontal rotation**

*Fig. 6*. The moment of inertia of a rotating disk: horizontal rotation.

(a) Create the set up shown in *Fig. 6* and make sure that it does not reach the pulley when the weight is placed at the end of its position.

(b) Connect webcam to the computer and run the video capture program. Drag the camera icon in the column on the right side to the center and determine if the apparatus shown in *Fig. 6* is displayed.

(c) Repeat (c) to (f) of experiment I with the apparatus shown in *Fig. 6*.

(d) Rotate the disk and roll up the thread where the weight is hanging. If the thread is rolled up, the hanger of the weight moves upward. Ensure that it does not reach the upper bound pulley. Hang three 100g weights onto the weight hanger.

(e) Repeat steps (g) through (n) of experiment I with the apparatus shown in *Fig. 6* and calculate the average acceleration.

(f) Calculate the moment of inertia with the acceleration. In the calculated moment of inertia, the moment of inertia of the axis of rotation is included. For this reason, it is necessary to subtract the moment of inertia of step (m) in experiment I to obtain the moment of inertia of the rotating disk.

(g) Measure the mass and radius of the disk.

**III. The moment of inertia of a disk for vertical rotation**

*Fig. 7*. The moment of inertia of a rotating disk: vertical rotation.

(a) Create the set up shown in *Fig. 7* and make sure that it does not reach the pulley when the weight is placed at the end of its position.

(b) Connect webcam to the computer and run the video capture program. Drag the camera icon in the column on the right side to the center and determine if the apparatus shown in *Fig. 7* is displayed..

(c) Repeat steps (c) through (f) of experiment I with the apparatus shown in *Fig. 7*.

(d) Rotate the disk and roll up the thread where the weight is hanging. If the thread is rolled up, the hanger of the weight moves upward. Ensure that it does not to reach the upper bound pulley. Hang three 100g weights onto the weight hanger.

(e) Repeat steps (g) through (n) of experiment I with the apparatus shown in *Fig. 7* and calculate the average acceleration.

(f) Calculate the moment of inertia with the acceleration. In the calculated moment of inertia, the moment of inertia of the axis of rotation is included. For this reason, it is necessary to subtract the moment of inertia in step (m) in experiment I to obtain the moment of inertia of the rotating disk.

(g) Measure the mass and radius of the disk.

## IV. The moment of inertia of a ring

*Fig. 8*. The moment of inertia of a ring.

(a) Create the set up shown in *Fig. 8* and make sure that it does not reach the pulley when the weight is placed at the end of its position.

(b) Connect webcam to the computer and run the video capture program. Drag the camera icon in the column on the right side to the center and determine if the apparatus shown in *Fig. 8* is displayed..

(c) Repeat steps (c) through (f) of experiment I with the apparatus shown in *Fig. 8*.

(d) Rotate the disk and roll up the thread where the weight is hanging. If the thread is rolled up, the hanger of the weight moves upward. Ensure that it does not reach the upper bound pulley. Hang four 100g weights onto the weight hanger.

(e) Repeat steps (g) through (n) of experiment I with the apparatus shown in *Fig. 8* and calculate the average acceleration.

(f) Calculate the moment of inertia with the acceleration. In the calculated moment of inertia, the moment of inertia of the axis of rotation is included. For this reason, it is necessary to subtract the moment of inertia of step (m) in experiment I and that of step (f) in experiment II to obtain the moment of inertia of the rotating disk.

(g) Measure the mass, the external diameter, and the internal diameter of the ring.

**V. The moment of inertia of a bar**

*Fig. 9*. The moment of inertia of a bar.

(a) Create the set up shown in *Fig. 9* and make sure that it does not reach the pulley when the weight is placed at the end of its position.

(b) Connect webcam to the computer and run the video capture program. Drag the camera icon in the column on the right side to the center and determine if the apparatus shown in *Fig. 9* is displayed.

(c) Repeat (c) to (f) of experiment I with the apparatus in *Fig. 9*.

(d) Rotate the disk and roll up the thread where the weight is hanging. If the thread is rolled up, the hanger of the weight moves upward. Ensure that it does not reach the upper bound pulley. Hang three 100g weights onto the weight hanger.

(e) Repeat (g) to (n) of experiment I with the apparatus shown in *Fig. 9* and calculate the average acceleration.

(f) Calculate the moment of inertia with the acceleration. In the calculated moment of inertia, the moment of inertia of the axis of rotation is included. For this reason, it is necessary to subtract the moment of inertia of step (m) in experiment I to obtain the moment of inertia of the rotating disk.

(g) Measure the length and mass of the bar.