1 Prove that $x - y = (x^* + y)^*$, where superscript * denotes two's-complement. Prove also for one's-complement. [15 pts]

2 Prove (or disprove) the following identities for Boolean algebra, without using truth tables. For those equalities that are not identities, give a counterexample.

(a) $x + y' + x'yz' = x + y' + z'$ [5 pts]

(b) $abc'd + acd = abd + acd$ [5 pts]

(c) $(a + b')(b + c')(c + a') = (a' + b)(b' + c)(c' + a)$ [5 pts]

3 Is an "irredundant and prime cover" (i.e. a cover, where all cubes are prime and irredundant) a "minimum cover"? Prove or show a counterexample. [15 pts]

4 Simplify 3-outputs circuit (f_1 = xz', f_2 = xz' + x'z + x'y'z', f_3 = x'yz + y'z') using Quine-McClusky Method. [15 pts]

5 For a 1-variable Boolean function f, prove the following using the expansion theorem: [15 pts]

$$f(x+y) + f(xy) = f(x) + f(y)$$

6 Answer the followings:

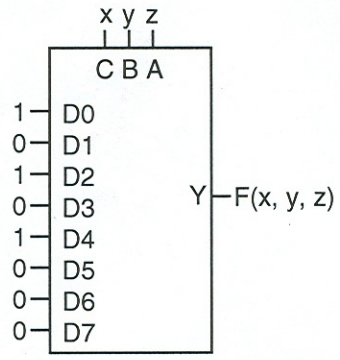
(a) Find a minimum two-level, multiple-output OR-AND circuit to realize $f_1 = b'd + a'b' + c'd$ and $f_2 = a'd' + bc' + bd'$ [10 pts]

(b) Realize the same functions with a minimum two-level NAND-NAND circuit. [10 pts]

7 Two expressions $(x + y)(x' + z)$ and $xz + x'y$ are equivalent in terms of logic. However, they are not equivalent in terms of hazard. Explain why. [15 pts]

8 Answer the followings:

(a) For the following 8-to-1 MUX circuit (x, y, and z are control input, where z is least significant), derive the simplified expression for the output F. [5 pts]



(b) For a 16K ROM with 8-bit word, how many bits do we need for an address? [5 pts]