# Formal Languages and Automata (CS322)

Final exam (18 December 2024)

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**[25 points]** Q1. Determine if the following statements hold. Answer in YES or NO.

(a) (5pt) Regular languages are closed under union, intersection, concatenation, complement and kleene star.

(b) (5pt) Context-free languages are closed under at least two of the following operations: union, intersection and complement.

(c) (5pt) Languages in P are closed under union, intersection, and complement.

(d) (5pt) Turing-recognizable languages are closed under union, intersection, and complement.

(e) (5pt) Decidable languages are closed under union, intersection, and complement.

**[30 points]** Q2. Show that the following languages are not context-free by using the pumping lemma.

(a) (15pt) $A = \{s\#t \mid s, t \in \{a, b\}^* \text{ and } s \text{ is a substring of } t\}$

(a) (15pt) $B = \{0^n\#0^{2n}\#0^{3n} \mid n \geq 0\}$

**[25 points]** Q3. Show that the following languages are undecidable *without using Rice's theorem*. You can use any undecidable language we learned in the class for the reduction.

(a) (10pt) $A = \{\langle M \rangle \mid M \text{ is a Turing machine and } L(M) \text{ is regular.}\}$.

(b) (15pt) $B = \{\langle M \rangle \mid M \text{ is a Turing machine that accepts } w^R \text{ whenever it accepts } w\}$.

**[20 points]** Q4. Consider the following scheduling problem: There are $n$ final exams $F_1, \ldots, F_n$ to be scheduled and there are $m$ students $s_1, \ldots, s_m$. Each of the students needs to take a specified subset of $n$ final exams. There are only $h$ slots for scheduling $n$ exams. The hard constraint is that no two exams can be allocated to the same slot if there is a student taking both exams. We want to decide if scheduling all the exams, i.e. assigning the exams to $h$ slots while satisfying the hard constraint, is possible or not.

(a) (10pt) Formulate the above problem as a decision problem. You can present your decision problem in the form of a language, or in the form with clearly indicated input and a question requiring YES or NO answer.

(b) (10pt) Show that your decision problem in (a) is NP-complete. You can use one of the NP-complete problems we learned in the class for the polynomial-time transformation.