MAS 250 Homework Assignment 3

Due: October 4 (Tuesday) 1 pm

Instruction: Turn in homework as a **single pdf file**.

1. Suppose that two defective refrigerators have been included in a shipment of four refrigerators. The buyer begins to test the four refrigerators one at a time. Define the random variable X as the number of tests until the buyer locates both of the defective refrigerators.

   (a) Find the probability distribution of X.

   (b) Find $E[(X - \frac{10}{3})^2]$.

2. Let X have the density function given by

$$f(x) = \begin{cases} 0.2, & -1 < x \leq 0, \\ 0.2 + cx, & 0 < x \leq 1 \\ 0, & \text{elsewhere} \end{cases}$$

   (a) Find c.

   (b) Find the cumulative distribution fucntion $F(x)$.

   (c) Find $F(-1)$, $F(0)$, and $F(1)$.

   (d) Find $P(0 \leq X \leq 0.5)$.

   (e) Find $P(X > 0.5 | X > 0.1)$.

3. Let X and Y denote the proportions of time (out of one workday) during which employees I and II, respectively, perform their assigned tasks. The joint relative frequency behavior of X and Y is modeled by the density function

$$f(x, y) = \begin{cases} x + y, & 0 \leq x \leq 1, \ 0 \leq y \leq 1 \\ 0, & \text{elsewhere,} \end{cases}$$

   Find

   (a) $P(X < 1/2, Y > 1/4)$.

   (b) $P(X + Y \leq 1)$.

   (c) Find the marginal density functions for X and Y.

   (d) Find $P(X \geq 1/2 | Y \geq 1/2)$.

   (e) If employee II spends exactly 50% of the day working on assigned duties, find the probability that employee I spends more than 75% of the day working on similar duties.

(f) Are X and Y independent?

4. The joint density function of X and Y is given by

$$f(x,y) = \begin{cases} 3x, & 0 \le y \le x \le 1, \\ 0, & \text{elsewhere.} \end{cases}$$

   (a) Derive the marginal density of X and Y

   (b) Are X and Y independent?

   (c) Derive the conditional density of X given $Y = y$.

   (d) Find $P(X \le 3/4 | Y = 1/2)$.

   (e) Calculate $Cov(X, Y)$.

5. An engineer wishes to estimate the mean yield of a chemical process based on the yield measurements X_1, X_2, X_3 from three runs of an experiment ($E(X_i) = \mu$ and $V(X_i) = \sigma^2$ for $i = 1, 2, 3$). Consider the following two estimators of the mean yield μ:

$$\begin{aligned} T_1 &= \frac{X_1 + X_2 + X_3}{3} \quad \text{sample mean} \\ T_2 &= \frac{X_1 + 2X_2 + 2X_3}{5} \quad \text{weighted average} \end{aligned}$$

   (a) Compare $E(T_1)$ and $E(T_2)$.

   (b) Assume that X_i's are independent. Compare $V(T_1)$ and $V(T_2)$. Which one is smaller?

6. From the exercise problems in Chapter 4:

   9, 10, 11, 45, 52

7. (Suggested: no submission)

   4, 6, 13, 19, 23, 26, 29, 33, 34, 35, 38, 42, 43, 44, 49, 50, 51, 52, 54