Student ID Number: ________________, Name: ________________

EE210: Probability and Introductory Random Process Midterm Exam

Please provide detailed procedure to get your final answer.

Oct. 25, 2016

1. (15 points) Answer the following questions on probabilistic models.

(a) (5 points) Out of the students in a class, 60% are geniuses, 70% love chocolate, and 40% fall into both categories. Determine the probability that a randomly selected student is neither a genius nor a chocolate lover.

(b) (5 points) A six-sided die is loaded in a way that each even face is twice as likely as each odd face. All even faces are equally likely, as are all odd faces. Construct a probabilistic model for a single roll of this die and find the probability that the outcome is less than 4.

(c) (5 points) A four-sided die is rolled repeatedly, until the first time (if ever) that an even number is obtained. What is the sample space for this experiment?

2. (10 points) An urn contains 3 red balls and 2 blue balls. A ball is drawn. If the ball is red, it is kept out of the urn and a second ball is drawn from the urn. If the ball is blue, then it is put back in the urn and a red ball is added to the urn. Then a second ball is drawn from the urn.

(a) (4 points) What is the probability that both balls drawn are red?

(b) (6 points) If the second drawn ball is red, what is the probability that the first drawn ball was blue?

3. (20 points) The experiment consists of throwing a fair die once. The sample space for the experiment is $\Omega = \{1, 2, 3, 4, 5, 6\}$. We define two RVs as follows:

$$X(\zeta) \triangleq \begin{cases} 1+\zeta & \text{for outcomes } \zeta = 1 \text{ or } 3 \\ 0 & \text{for all other values of } \zeta. \end{cases}$$

$$Y(\zeta) \triangleq \begin{cases} 1-\zeta & \text{for outcomes } \zeta = 1, 2, \text{ or } 3 \\ 0 & \text{for all other values of } \zeta. \end{cases}$$

(a) (8 points) Compute the relevant single and joint PMFs, i.e. $P_X(x)$, $P_Y(y)$, and $P_{XY}(x, y)$.

(b) (6 points) Conpute the joint CDFs $F_{XY}(1, 1)$, $F_{XY}(3, -0.5)$, $F_{XY}(5, -1.5)$.

(c) (6 points) Are the RVs X and Y are independent?

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4. (25 points) Consider the simple but nonfactorable joint PDF

$$f_{XY}(x,y) = \begin{cases} A(x+y) & 0 < x \le 1, 0 < y \le 1 \\ 0 & \text{otherwise,} \end{cases}$$

and answer the following questions.

(a) (4 points) What is A?

(b) (6 points) What are the marginal PDFs, i.e. $f_X(x)$ and $f_Y(y)$?

(c) (9 points) What is $F_{XY}(x,y)$?

(d) (6 points) Compute $\mathrm{P}[X+Y \le 1]$.

5. (20 points) Let X be a uniform $(-\pi, \pi)$ random variable. Find the PDF of $Y = \sin X$

6. (10 points) The temperature T, in Celsius, on a January day at the peak of Mt. Baekdu is a Gaussian random variable with a variance of 64. With probability 0.5, the temperature T exceeds -24 degrees.

(a) (5 points) What is $\mathrm{P}[X > -16]$

(b) (5 points) What is $\mathrm{P}[-30 < X \le -20]$

The end of problems. Thanks for your hard work :)