EE303 Digital System Prof. Y. Shin

Fall 2018

Midterm Exam

October 16, 2018

Name _____

Student ID _____

Problem	Max Points	Your Points
1	10	
2	10	
3	15	
4	10	
5	10	
6	10	
7	10	
8	10	
9	15	
Total	100	

1. Left/right shifting is used to double/halve the magnitude of unsigned binary integers. For example, shifting 0011 (=3) to the left gives 0110 (=6), which is 3 doubled, and to the right gives 0001 (=1), which is 3 halved if fractional number (0.5) is ignored. Explain how shifting can be applied to 2's-complement numbers; your description (or algorithm) should be "complete" (i.e. cover all the cases).

2. Suppose that we want to add $+11$ in 6-bits (001011) and -5 in 4-bits (1011) of 2's complement. Before the addition, 1011 has to be converted to 6-bits format. This can simply be done by so-called "MSB extension", i.e. MSB of 1011 is extended to 2 more bits to yield 111011. Explain (or prove) why MSB extension works.

3. Uniqueness of the complement says "If x+y = 1 and x·y=0, then y = x' holds." Prove it. You can only use the following basic Boolean theorems (x+0=x, x·1=x, x+1=1, x·0=0, x+x'=1, x·x'=0, commutative, associative, and distributive laws). Specify which theorem you use at each step.

4. Using the "uniqueness of the complement" in Problem 3, prove De Morgan's Laws: $(x + y)' = x'y'$. Only the basic theorems from Problem 3 are allowed. Specify which theorem you use at each step.

5. (a) Derive a simplified Boolean expression for "difference" and "borrow" of a full subtracter (FS). (b) Compare FS and FA (full adder). How do you compare difference and sum, as well as borrow and carry?

6. Minimum SOP is an SOP with minimum number of product terms and minimum number of literals. Minimum POS is a POS with minimum number of sum terms and minimum number of literals. Minimum POS of f is obtained by complementing minimum SOP of f'. Explain why.

7. $f(a, b, c, d) = \sum m(1, 3, 5, 6, 8, 9, 12, 14, 15) + \sum d(4,10,13)$. Use the Quine-McCluskey method to find all minimum SOP expressions of f'.

8. (a) Find a minimum two-level, multiple-output NAND-NAND circuit to realize $f_1 = \sum m(3,6,7,11,13,14,15)$ and $f_2 = \sum m(3,4,6,11,12,13,14)$. (b) Repeat for a minimum two-level, NOR-NOR circuit.

9. Consider the following logic function.

$$F(A, B, C, D) = \Sigma m(0, 2, 5, 6, 7, 8, 9, 12, 13, 15)$$

   a. Find two different minimum AND-OR circuits which implement *F*. Identify two hazards in each circuit. Then find an AND-OR circuit for *F* that has no hazards.
   b. The minimum OR-AND circuit for *F* has one hazard. Identify it, and then find an OR-AND circuit for *F* that has no hazards.