

## Extra Credit Problem, Engineering Statistics I - Spring 2024

**\*Note1** : When submitting your answer to question 2, please also submit the code you used.

**\*Note2** : If you solve a problem perfectly, you will receive up to 5 extra credits.

1. Suppose  $Y_1, Y_2, Y_3$  are from  $\text{Exp}(\beta)$ . Let  $X_1 = Y_{(1)}$ ,  $X_2 = Y_{(2)} - Y_{(1)}$ ,  $X_3 = Y_{(3)} - Y_{(2)}$ .  
Where  $Y_{(i)}$  :  $i$  th order statistic.

(a) Find joint pdf of  $X_1, X_2, X_3$

(b) Find marginal pdf of  $X_1, X_2, X_3$

(c) Show that  $X_1, X_2, X_3$  are independent

2. (Use software such as Python, R, Matlab, etc ..) Suppose  $X_1, \dots, X_n$  are from  $N(0,1)$ . The distribution of the maximum  $X_{(n)}$  does not have a closed form pdf. However, we can consider two ways to approximate the pdf. Let  $n = 30$ .

(a) Approximate the distribution of  $X_{(n)}$  by simulation, that is, repeat the data generation many times and draw a histogram of  $X_{(n)}$ .

(b) Write the pdf of  $X_{(n)}$ , as learned from the class. Draw the pdf by numerically evaluating the function over a grid. Compare this with the histogram in the above.