EE321 Final Exam

Write your names and student ids first!

1. (35) Consider a system has a noisy observation of x expressed as $y = x + n$ where $x \in \{-1, +1\}$, and n is a Gaussian random variable with zero mean and variance σ^2. The system makes its decision on x based on the observation y in such a way that $\hat{x} = +1$ when $y > k$, and $\hat{x} = -1$ when $y \leq k$.

(a) (5) Find the conditional probability density function $f_Y(y|x = +1)$ and $f_Y(y|x = -1)$.
(b) (10) Express the optimal threshold, k minimizing the decision error probability, $\Pr(\hat{x} \neq x)$ when $\Pr(x = +1) = \pi$.

Hint: the probability density function of Gaussian random variable is expressed as

$$f_N(n) = \frac{1}{\sqrt{2\sigma^2}} e^{-\frac{n^2}{2\sigma^2}}$$

In a BPSK system, two waveforms $s_1(t) = A\cos\omega_c t$ and $s_2(t) = -A\cos\omega_c t$ are used to represent user message bit $m = 1$ and 0, respectively. That is, $m = 1 \leftrightarrow s_1(t) = A\cos\omega_c t$ and $m = 0 \leftrightarrow s_2(t) = -A\cos\omega_c t$. Either $s_1(t)$ or $s_2(t)$ is transmitted over an additive-white-Gaussian-noise (AWGN) channel, and the received signal is coherently detected with a matched filter. The value of A is 20mV, and the bit rate is 1Mb/s. Assume that the noise power spectral density $N_0/2 = 10^{-11}$ W/Hz.

(c) (10) When $\Pr(m = 0) = \Pr(m = 1)$, find the optimal decision threshold k and find the symbol-error-rate, $\Pr(\hat{m} \neq m)$.

Use the approximation of Q function: $Q(x) \approx \frac{1}{x\sqrt{2\pi}} e^{-\frac{x^2}{2}}$ for $x > 0$.

(d) (10) Repeat (c) with the probabilities of message bits: $\Pr(m = 0) = 0.018$ and $\Pr(m = 1) = 0.982$.

2. (20) Consider a quadrature phase shift keying (QPSK) system with

$$s(t) = \sqrt{\frac{E_s}{T_s}} c_1 \cos\omega_c t + \sqrt{\frac{E_s}{T_s}} c_2 \sin\omega_c t$$

where $c_1, c_2 \in \{-1, +1\}$. Then, we have the signal constellations where each signal point represents a sequence of two bits. For example, the signal point s_1 represent the binary bit sequence of 11. The signal $s(t)$ is transmitted over an additive-white Gaussian noise (AWGN) channel, and the signal after a matched filter output for is expressed as a vector, $\bar{y} = \left(c_1\sqrt{E_s/2}, c_2\sqrt{E_s/2}\right) + (n_1, n_2)$ where $n_1 \sim \mathcal{N}(0, N_0/2)$ and $n_1 \sim \mathcal{N}(0, N_0/2)$.

(a) (5) Find the symbol-error-rates (SERs) for the signal constellations Fig. 1 (a) and Fig. 1 (b), i.e., $P_s = \Pr(\hat{s} \neq s)$ where $\hat{s}$ is the estimate of the transmitted symbol s.

(b) (10) Find the bit-error-rates (BERs) for the for the signal constellations (a) and (b), i.e., $P_b = \Pr(\hat{b} \neq b) = \frac{1}{2}\sum_{i=1}^{2} \Pr(\hat{b}_i \neq b_i)$.

(c) (5) Which constellation has a better BER? Justify your answer.

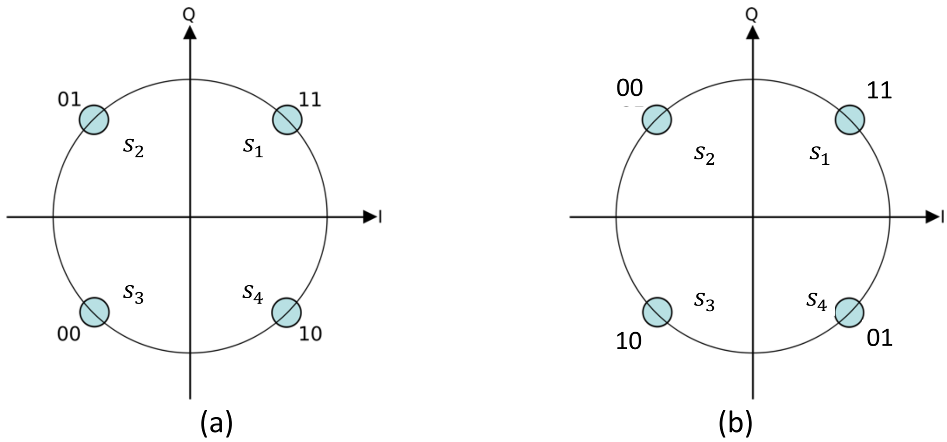

Fig. 1. Signal constellations for QPSK schemes

3. (20) Consider a communication system with an input data stream of data rate $R = 144\text{Mbits/s}$ transmits the data over an RF channel with M-ary quadrature-amplitude modulation (QAM) scheme. The RF channel has a bandwidth which allows 36M QAM symbols per second.
(a) (10) Find the minimum modulation order M with which the input data can be transmitted over the RF channel.
(b) (10) Suppose that the available E_b/N_0 is 20 (=13dB). Then, what is the symbol error rate?
Hint: Use the approximation of symbol error rate

$$P_E \approx 4\left(1 - \frac{1}{\sqrt{M}}\right) Q\left(\sqrt{\frac{2a^2}{N_0}}\right)$$

$$Q(x) \approx \frac{1}{x\sqrt{2\pi}} \cdot e^{-\frac{x^2}{2}} \text{ for } x > 0$$

$$a = \sqrt{\frac{3E_s}{2(M-1)}}$$

4. (10) Consider a source S with an alphabet $\mathcal{X} = \{x_1, x_2, x_3, x_4, x_5, x_6\}$ with probabilities $\Pr(S = x_i) = 0.30, 0.25, 0.20, 0.12, 0.08, 0.05$ for $i = 1,2,\ldots,6$, respectively.
(a) (5) Find the entropy of the source
(b) (5) Design a Huffman code and find the average code length.

5. (15) Consider a linear code $\mathcal{C} = \{\bar{c}_i | 1 \le i \le 2^k\}$ whose minimum distance is given by $d_{\min}$ and length is n.
(a) (5) Remember the minimum distance is defined as $d_{\min} = \min_{i \neq j} d_H(\bar{c}_i, \bar{c}_j)$ for all $1 \le i, j \le 2^k$. Then, prove that if the code is a linear code, $d_{\min} = \min_i w_H(\bar{c}_i)$ for all $1 \le i \le 2^k$.
(b) (5)Let us construct a code $\mathcal{C}' = \{\bar{c}_i' | 1 \le i \le 2^k\}$ of length $n+1$ in such a way that $\bar{c}_i' = (c_{i1}', c_{i2}', \ldots, c_{in}', c_{i(n+1)}')$ and $c_{ij}' = c_{ij}$ for $1 \le j \le n$ and $c_{i(n+1)}' = \sum_{j=1}^{n} c_{ij}$. Find the minimum distances of $\mathcal{C}'$ when $d_{\min}$ of $\mathcal{C}$ is odd and even each.
(c) (5) Suppose that the code $\mathcal{C}$ in (a) is a Hamming code. Note that the minimum distance of Hamming code is 3. Then, what are the error-detection and correcting capabilities of $\mathcal{C}'$?