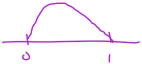
Beta Probability Distribution

· $\alpha=1,\ \beta=1$ · $f(y)=1$ ⇒ Uniform

· $\alpha>1,\ \beta>1$ e.g $f(y)=\cdot y^2(1-y)^3$

$\frac{\Gamma(3)\Gamma(4)}{\Gamma(7)}=\frac{2!3!}{6!}$



· $\alpha<1,\ \beta<1$.

$Y \sim Beta(\alpha, \beta)$: beta r.v.

$$f(y) = \frac{y^{\alpha-1}(1-y)^{\beta-1}}{B(\alpha,\beta)}, \quad \boxed{0 \le y \le 1}$$

for $\alpha > 0, \beta > 0$, and where the beta function is

$$B(\alpha,\beta) = \int_0^1 y^{\alpha-1}(1-y)^{\beta-1}dy = \frac{\Gamma(\alpha)\Gamma(\beta)}{\Gamma(\alpha+\beta)}. \Rightarrow \int_0^1 f(y)\,dy = 1$$

$$\mu = E(Y) = \frac{\alpha}{\alpha+\beta} \quad \text{and} \quad \sigma^2 = V(Y) = \frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}. \rightarrow \text{Check!!}$$

$\boxed{E(Y^k(1-Y)^m) = \ldots\ldots}$

$$\int_0^1 x \cdot \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} x^{\alpha-1}(1-x)^{\beta-1}\,dx$$

$$= \overbrace{\int \frac{\Gamma(\alpha+\beta+1)}{\Gamma(\alpha+1)\Gamma(\beta)} x^{(\alpha+1)-1}(1-x)^{\beta-1}\,dx}^{1} \times \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha+\beta+1)} \cdot \frac{\Gamma(\alpha+1)}{\Gamma(\alpha)} = \frac{\alpha}{\alpha+\beta}$$

$$\int_0^1 x^{\frac{1}{2}}(1-x)^2\,dx = \frac{\Gamma(\frac{3}{2})\Gamma(3)}{\Gamma(\frac{9}{2})} = \frac{\frac{1}{2}\cdot\cancel{\Gamma(\frac{1}{2})}\cdot 2!}{\frac{7}{2}\cdot\frac{5}{2}\cdot\frac{3}{2}\cdot\frac{1}{2}\cdot\cancel{\Gamma(\frac{1}{2})}}$$

$=$

- If $Y \sim Beta(\alpha, \beta)$

  then, $1-Y \sim Beta(\beta, \alpha)$

- Relation to Gamma ☆

$$\boxed{\begin{cases} Y_1 \sim Gamma(\alpha_1, \beta) \\ Y_2 \sim \quad '' \quad (\alpha_2, \beta) \end{cases} \text{ indep} \qquad X = \frac{Y_1}{Y_1+Y_2} \sim B(\alpha_1, \alpha_2)}$$