1. Work function is the minimum energy needed to remove an electron from a solid to the vacuum. (O / X) [5 pts]

2. The energy needed to remove an electron from the ground state ($n = 1$) to the free electron in hydrogen is 13.6 eV. (O / X) [5 pts]

3. Consider a matter wave. Is kinetic energy of a free particle linearly proportional to its wave vector ($k = 2\pi/\lambda$)? (O / X) [5 pts]

4. Uncertainty principle relates uncertainties in position and momentum. Can it also be interpreted uncertainties between energy and frequency? (O / X) [5 pts]

$$\Delta E\ \Delta\omega \geq \hbar$$

5. In all the direction of the universe, the radiation has been detected from the space that is characteristic of an ideal radiator at $T = 2.7\ \mathrm{K}$. For this temperature, what is the wavelength at the peak of Plank distribution?

By Wien's law, [15 pts, no partial point]

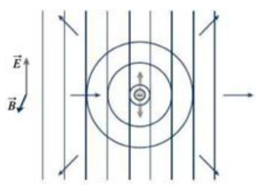
$$\lambda_m = \frac{2.90 \times 10^{-3}\ \mathrm{m \cdot K}}{T} = \frac{2.90 \times 10^{-3}\ \mathrm{m \cdot K}}{2.7\ \mathrm{K}} = 1.1\ \mathrm{mm}.$$

6. In a photoelectric effect experiment, we illuminate the metal film with certain wavelength. It is required to set $1.25\ \mathrm{V}$ reversely to reduce the current to zero. Obtain the maximum speed of the emitted photoelectrons without external voltage. ($m_e = 9.11 \times 10^{-31}\ \mathrm{kg}$, $e = 1.60 \times 10^{-19}\ C$)

Maximum kinetic energy $K_{\max} = eV_{\mathrm{rev}} = (1.60 \times 10^{-19}\ \mathrm{C}) \times (1.25\ \mathrm{V}) = 2.00 \times 10^{-19}\ \mathrm{J}$ [7 pts]

$$v_{\max} = \sqrt{\frac{2K_{\max}}{m}} = \sqrt{\frac{2 \times (2.00 \times 10^{-19}\ \mathrm{J})}{9.11 \times 10^{-31}\ \mathrm{kg}}} = 6.63 \times 10^{5}\ \mathrm{m/s} \quad [8\ \mathrm{pts}]$$

7. Under an oscillating electric field along x-direction with angular frequency ω_0 in electromagnetic radiation, an electron oscillates by $\mathrm{d} = \mathrm{d}_0 e^{i\omega_0 t}$. The oscillating electron generates an oscillating dipole with $\vec{p} = -e\vec{d} = -\hat{\imath}ed$. ($\hat{\imath}$ is unit vector along x)



(a) Using the equation (20.6b) in your main textbook, calculate an electric field at $x = x'$ (on-axis) generating by the oscillating dipole and find a frequency of this field. It is the classical prediction of Compton experiment. [5 pts]

The oscillating dipole is represented as

$$p = -ed = -ed_0 e^{i\omega_0 t}$$

The electric field at $x = x'$ generating by the dipole is $\vec{E} = \frac{2kp}{|x|^3}\hat{\imath}$, thus,

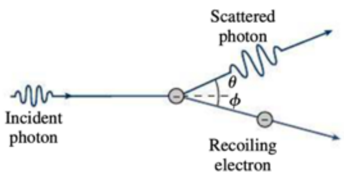
$$\vec{E} = \hat{\imath}\frac{2kp}{|x'|^3} = -\hat{\imath}\frac{2ked_0}{|x'|^3}e^{i\omega_0 t} \text{ [3 pts]}$$

So, the electric field generated by the oscillating dipole has frequency $f = \frac{\omega_0}{2\pi}$. [2 pts]

[Evaluation criteria]

There is no description of electric field, but if one state that the frequency of field after scattering is same as the one before, evaluate *2 points*.

However, the Compton experiment result did not agree with the classical prediction. To interpret this result, consider an *elastic collision* between a photon with initial wavelength λ_0 moving in the x-direction and a stationary electron, as depicted in the right figure.



(b) Using the relativistic energy-momentum relation in Chapter 33 and photon energy in the equation (34.6), write the energy and momentum conservation of this elastic collision. Since the photon is massless particle, the energy-momentum relation is written as $E = pc$. Using this relation, solve the equations of problem (c) to find the Compton shift (Equation 34.8). [8 pts]

The relativistic energy-momentum relation is

$$E = \sqrt{(pc)^2 + (mc^2)^2}$$

With the photon energy $E = hf$, the energy conservation is

$$E_{\text{before}} = E_{\gamma 0} + E_{e0} = hf_0 + m_e c^2$$

$$E_{\text{after}} = E_\gamma + E_e = hf + \sqrt{(p_e c)^2 + (m_e c^2)^2}$$

$$\rightarrow hf_0 + m_e c^2 = hf + \sqrt{(p_e c)^2 + (m_e c^2)^2} \;\cdots\; (1) \text{ [3 pts]}$$

The momentum conservation is written as

$$\vec{p}_{\text{before}} = \vec{p}_{\gamma 0}$$

$$\vec{p}_{\text{after}} = \vec{p}_\gamma + \vec{p}_e$$

$$\rightarrow \vec{p}_{\gamma 0} = \vec{p}_\gamma + \vec{p}_e \;\cdots\; (2) \text{ [2 pts]}$$

The equation (1) is rewritten as,

$$p_e^2 c^2 = (hf_0 + m_e c^2 - hf)^2 - m_e^2 c^4 \;\cdots\; (1')$$

Also, from the equation (2),

$$|\vec{p}_e|^2 = p_e^2 = (\vec{p}_\gamma - \vec{p}_{\gamma_0})\cdot(\vec{p}_\gamma - \vec{p}_{\gamma_0}) = p_\gamma^2 + p_{\gamma_0}^2 - 2p_\gamma p_{\gamma 0}\cos\theta \;\cdots\; (2')$$

Putting (2′) in the equation (1′),

$$p_e^2 c^2 = p_\gamma^2 c^2 + p_{\gamma_0}^2 c^2 - 2p_\gamma p_{\gamma 0} c^2 \cos\theta,$$

and with the relation $E_\gamma = hf = pc$,

$$p_e^2 c^2 = h^2 f^2 + h^2 f_0^2 - 2(hf)(hf_0)c^2 \cos\theta = (hf_0 + m_e c^2 - hf)^2 - m_e^2 c^4$$

$$2hf_0 m_e c^2 - 2hf m_e c^2 = 2h^2 f_0 f(1 - \cos\theta)$$

Dividing by $2hf_0 f m_e c$ in both sides of equation,

$$\frac{c}{f} - \frac{c}{f_0} = \frac{h}{m_e c}(1 - \cos\theta)$$

Since $c/f = \lambda$,

$$\Delta\lambda = \lambda - \lambda_0 = \frac{h}{m_e c}(1 - \cos\theta) \text{ [3 pts]}$$

This is called as Compton shift (Equation 34.8).

[Evaluation criteria]

The expression of relativistic energy for electron, $E_e = \gamma m_e c^2$, where γ is the Lorentz factor is also right answer.

Below is also possible answer of this problem:

EVALUATE The initial energy is the energy of the photon plus the rest energy of the electron:

$$E_i = hc/\lambda_0 + mc^2.$$

The final energy is the energy of the new photon plus the relativistic energy of the moving electron:

$$E_f = hc/\lambda + \gamma mc^2$$

Equating these two energies (by conservation of energy) gives us the first of the three desired equations:

$$\frac{hc}{\lambda_0} + mc^2 = \frac{hc}{\lambda} + \lambda mc^2.$$

The next two equations come from the initial momentum, $\vec{p}_i = \hat{i}\, h/\lambda_0$ and the components of final momentum $p_{fx} = \frac{h}{\lambda}\cos\theta + \gamma mu\cos\phi$ and $p_{fy} = 0 = \gamma mu\sin\phi - \frac{h}{\lambda}\sin\theta$. By conservation of momentum, we can equate the initial and final momentum in each direction, which leads to

$$p_{ix} = p_{fx} \quad \Rightarrow \quad \frac{h}{\lambda_0} = \frac{h}{\lambda}\cos\theta + \gamma mu\cos\phi$$

$$p_{iy} = p_{fy} = 0 \quad \Rightarrow \quad 0 = \frac{h}{\lambda}\sin\theta - \gamma mu\sin\phi$$

These are the second two of the desired relationships we were to derive. Solving these three equations for $\lambda_0 - \lambda$ directly is a lengthy algebraic process. An easier approach is to start with the momentum in vector form and use the law of cosines:

$$\vec{p}_\gamma = \vec{p}_{\gamma'} + \vec{p}_{e'} \rightarrow p_{e'}^2 = p_\gamma^2 + p_{\gamma'}^2 - 2p_\gamma p_{\gamma'}\cos\theta$$

We equate the two equations for $p_{e'}^2$ to obtain

$$\frac{\left(\frac{hc}{\lambda_0}-\frac{hc}{\lambda}+mc^2\right)^2-m^2c^4}{c^2}=\left(\frac{h}{\lambda_0}\right)^2+\left(\frac{h}{\lambda}\right)^2-2\frac{h}{\lambda_0}\frac{h}{\lambda}\cos\theta$$

$$\cancel{\left(\frac{hc}{\lambda_0}\right)^2}+\cancel{\left(\frac{hc}{\lambda}\right)^2}-\frac{2c^3hm}{\lambda}+\frac{2c^3hm}{\lambda_0}-\frac{2c^2h^2}{\lambda\lambda_0}=\cancel{\left(\frac{hc}{\lambda_0}\right)^2}+\cancel{\left(\frac{hc}{\lambda}\right)^2}-2\frac{hc}{\lambda_0}\frac{hc}{\lambda}\cos\theta$$

$$\frac{mc}{\lambda_0}-\frac{mc}{\lambda}=\frac{h}{\lambda\lambda_0}(1-\cos\theta)$$

$$\lambda-\lambda_0=\frac{h}{mc}(1-\cos\theta)$$

$$\vec{p}_\gamma=\vec{p}_{\gamma'}+\vec{p}_{e'}\rightarrow p_{e'}^2=p_\gamma^2+p_{\gamma'}^2-2p_\gamma p_{\gamma'}\cos\theta$$

$$p_{e'}^2=\left(\frac{h}{\lambda_0}\right)^2+\left(\frac{h}{\lambda}\right)^2-2\frac{h}{\lambda_0}\frac{h}{\lambda}\cos\theta$$

We now use conservation of energy in the form $\frac{hc}{\lambda_0}+mc^2=\frac{hc}{\lambda}+\sqrt{m^2c^4+p_{e'}^2c^2}$, and solve for $p_{e'}^2$ to obtain

$$p_{e'}^2=\frac{\left(\frac{hc}{\lambda_0}-\frac{hc}{\lambda}+mc^2\right)^2-m^2c^4}{c^2}$$

(c) Discuss the difference between classical prediction and the result of Compton experiment using the result of (a) and (b) in the view of energy conservation. [2 pts]

In problem (a), classical viewpoint, we can find that the scattered radiation has same frequency as the incident radiation. It means that the energy of radiation is conserved, thus, it is an elastic scattering process.

$$\vec{E}_{\text{incident}}=\hat{\imath}E_0e^{i\omega_0t}$$

$$\vec{E}_{\text{scattered}}=-\hat{\imath}\frac{2ked_0}{|x'|^3}e^{i\omega_0t}$$

However, in problem (b), the quantum mechanical viewpoint, the scattering process gives the Compton shift as

$$\Delta\lambda=\lambda-\lambda_0=\frac{h}{m_ec}(1-\cos\theta)$$

It means that the photon energy for scattered radiation is different from the one for incident radiation. It is an inelastic scattering, which does not satisfy the energy conservation.

[evaluation criteria]

There are no partial points. The main difference between classical prediction and Compton experiment result is which the photon energy (or frequency of radiation) is conserved or not. (elastic or inelastic scattering)

8.

(a) The Bohr model follows the three postulates below.

1. *An atomic system can only exist in a discrete set of stationary states*

2. *The radiation absorbed or emitted during a transition between two stationary states of energies E1 and E2 is characterized by a unique frequency amount of* $\frac{E_1-E_2}{h}$.

3. *The stationary states correspond to a set of allowed orbits in Rutherford atomic model.*

As you know, the Bohr model is not the exact model of atom. Among the three postulates, which one is the biggest problem do you think. *(Hint: One of the postulates have no logical reason and it just follows the past theory. That one is the answer I want you to choose)*

3 [3 pts]

(b) Let's assume that a hydrogen atom follows Bohr model. Transitions to a final state of the hydrogen atom characterized by some quantum number n. If $\Delta n = 1$, calculate the wavelengths of the biggest three energy radiation. Use the constants in the text book.

$$\frac{1}{\lambda} = \frac{ke^2}{2a_0hc}\left(\frac{1}{n^2} - \frac{1}{(n+1)^2}\right)$$

$$\frac{ke^2}{2a_0hc} = 1.09 \times 10^7\ m^{-1}$$

The biggest three energy occurs when n=1, 2, 3

$$\lambda = \frac{1}{(1.09 \times 10^7\ m^{-1}) \times \left(\frac{1}{n^2} - \frac{1}{(n+1)^2}\right)}$$

$$n = 1, \qquad \lambda = 1.22 \times 10^{-7}\ \mathrm{m}$$

$$n = 2, \qquad \lambda = 6.61 \times 10^{-7}\ \mathrm{m}$$

$$n = 3, \qquad \lambda = 1.89 \times 10^{-7}\ \mathrm{m}$$

[4 pts for each level]

9. Consider a heavy atom with an atomic number Z (number of protons), which has only single electron. Assume

(a) Write down quantization rule of angular momentum. [2 pts]

$$mvr = n\hbar$$

(b) What is the electric potential $V(r)$ due to nucleus? [3 pts]

$$V(r) = -\frac{kZe}{r} = -\frac{1}{4\pi\varepsilon_0}\frac{Ze}{r}$$

(c) Assume the circular orbit of an electron around the heavy nucleus. From the result (a) & (b), obtain the energy levels of a heavy atom. (Hint: Coulomb force acts as a centripetal force.) [10 pts]

$$\frac{1}{4\pi\varepsilon_0}\frac{Ze^2}{r^2} = \frac{mv^2}{r}$$

$$E = U + K = -\frac{1}{4\pi\varepsilon_0}\frac{Ze^2}{r} + \frac{mv^2}{2} = -\frac{1}{4\pi\varepsilon_0}\frac{Ze^2}{r} + \frac{1}{2}\frac{1}{4\pi\varepsilon_0}\frac{Ze^2}{r} = -\frac{1}{2}\frac{1}{4\pi\varepsilon_0}\frac{Ze^2}{r} = \frac{1}{2}U$$

From the force balance and angular momentum quantization one can get

$$\frac{1}{4\pi\varepsilon_0}\frac{Ze^2}{r^2} = \frac{mv^2}{r} = \frac{n^2\hbar^2}{mr^3},$$

$$\frac{1}{r} = \frac{Ze^2}{4\pi\varepsilon_0}\frac{m}{n^2\hbar^2}.$$

One can get the energy of a heavy atom.

$$E_n = -\frac{1}{2}\frac{1}{4\pi\varepsilon_0}\frac{Ze^2}{r} = -\frac{1}{2}\frac{1}{4\pi\varepsilon_0}\frac{e^2}{4\pi\varepsilon_0\hbar^2}\frac{Z^2e^2}{n^2} = -\frac{1}{8\pi\varepsilon_0}\frac{Z^2e^2}{a_0 n^2} = -\frac{Z^2 \times (13.6\ \mathrm{eV})}{n^2}$$

[Evaluation Criteria]

Force balance equation: 2 pts

Properly use momentum quantization relation: 3 pts

Right answer: 5 pts

(d) Suppose a He^+ atom is in its lowest energy state. How much energy is needed to remove electron from the atom? [5 pts]

$$-E_1 = \frac{2^2 \times (13.6\ \mathrm{eV})}{1^2} = 54.4\ \mathrm{eV}$$