Midterm Exam Fall 2021

1. (15) Find the Fourier transform of $z(t) = Ae^{-|t|}\cos(\omega_c t) = x(t)\cos(\omega_c t)$
   a. (5) Find the Fourier transform of $x(t) = Ae^{-|t|}$
   b. (5) Find the band width of the signal $x(t)$ where the bandwidth $B > 0$ is defined as
   $$X(B) = X(-B) = \frac{1}{\sqrt{2}}X(0)$$
   c. (5) Find the Fourier transform of $z(t)$

2. (20) Transmit power of the double side band (DSB) modulations

The DSB with carrier signal is expressed as

$$x_c(t) = A_c[1 + am_n(t)]\cos\omega_c t$$

where the message signal, $m(t) = \cos(2\pi t/3) - 2\sin(\pi t)$, is depicted in FIGURE 2.1.

   a. (5) Express the scaled version of message signal $m_n(t)$ in terms of $m(t)$.
   b. (5) What is the message signal power?
   c. (10) Find the modulation index a which results in the modulation efficiency, $E_{ff} = 50\%$.

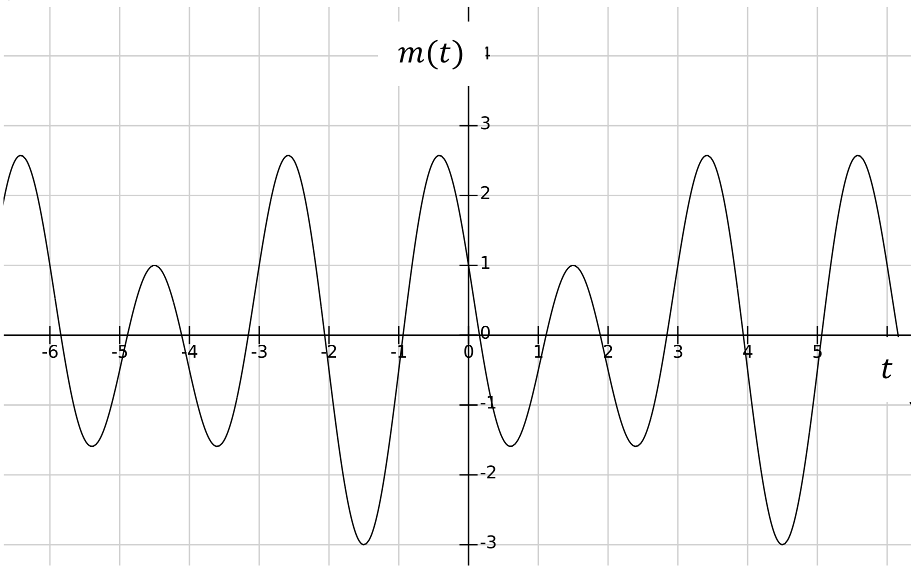


FIGURE 2.1

3. (20) A carrier is angle-modulated by the sum of two sinusoids
   $$x_c(t) = A\cos(\omega_c t + \beta_1\sin(\omega_1 t) + \beta_2\sin(\omega_2 t)).$$
   a. (15) Find the spectrum of $x_c(t)$. That is, express the modulated signal $x_c(t)$ in terms of the modified Bessel function, $J_n(\beta)$.
   b. (5) β_1 and β_2 are so small that $J_n(\beta_1) = J_n(\beta_2) = 0$ for $|n| > 2$. When $f_c = 100\text{Hz}$, $f_1 = 3\text{Hz}$, and $f_2 = 5\text{Hz}$, find the power at frequency 107Hz.

4. (10) Consider a random process $X(t)$ given by
$$X(t) = A\cos(\omega t + \theta)$$

where ω and θ are constants and A is a random variable. Is this stochastic process wide-sense stationary? Justify your answer.

5. (10) Let $Y(t) = X(t - d)$, where d is a constant delay and $X(t)$ is Wide-Sense Stationary. Express the followings in terms of $R_X(\tau)$ and $S_X(f)$:
   a. (5) $R_Y(\tau)$, and $S_{YX}(f)$
   b. (5) $R_{XY}(\tau)$, and $S_Y(f)$

6. (25) Consider an analog baseband communication system with additive white noise having power spectral density $N_0/2$. The signal $x(t)$ is transmitted over a distorting channel having the frequency response

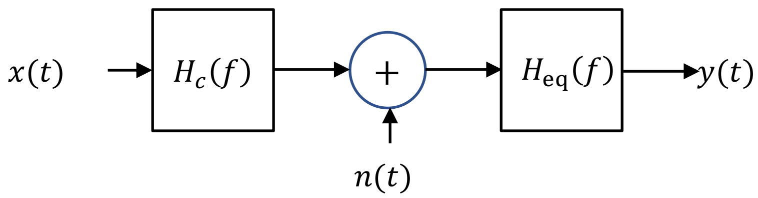
$$H_c(f) = \frac{1}{1 + j\frac{f}{B}}.$$

The additive white noise is added to the output of the distorting channel. Then, the distortion is equalized by a receiver filter having the frequency response

$$H_{\mathrm{eq}}(f) = \begin{cases} \sqrt{\frac{3}{8}} \cdot \frac{1}{H_c(f)}, 0 \le |f| \le B \\ 0, \text{otherwise} \end{cases}.$$

Finally, the system has $y(t)$ as its output.

a. (10) Express the output SNR when the transmitted signal $x(t)$ has its power spectral density $S_X(f) = S_x$ for $|f| \le B$ and 0, otherwise.



b. (10) Express the output SNR when the equalizer is a simple low-pass filter, namely $H_{\mathrm{eq}}(f) = 1/\sqrt{2}$ for $|f| \le B$ and 0, otherwise.

c. (5) Find the SNR gain due to the equalizer, namely
Gain = SNR (with the equalizer in a)/SNR (without the equalizer in b)

Hint: $\int 1/(1 + x^2)dx = \arctan(x) + C$