1. Design a circuit which will add a 4-bit 2's complement number to a 5-bit 2's complement number. Use full adders. Include a circuit to detect an overflow. [15 pts]

① 4-bit $A3A2A1A0$ → 5-bit $A4A3A2A1A0$ → 5-bit $A3A3A2A1A0$ [5 pts]

4-bit and 5-bit binary number can be represented as 4b'$A3A2A1A0$ and 5b'$B4B3B2B1B0$ respectively.

In 2's complement system,
4b'$A3A2A1A0$ can be converted to a 5-bit binary number: 5b'$A3A3A2A1A0$, which is called sign extension.

The sign extension can be proved as below:

i) if A3=0;

4b'$A3A2A1A0$ = 4b'0$A2A1A0$ = $A2 \cdot 2$^2 + $A1 \cdot 2$^1 + $A0 \cdot 2$^0
5b'$A3A3A2A1A0$ = 5b'00$A2A1A0$ = $A2 \cdot 2$^2 + $A1 \cdot 2$^1 + $A0 \cdot 2$^0

ii) if A3=1;

4b'$A3A2A1A0$ = 4b'1$A2A1A0$ = -2^3 + ($A2 \cdot 2$^2+ $A1 \cdot 2$^1 + $A0 \cdot 2$^0)
5b'$A3A3A2A1A0$ =5b'11 $A2A1A0$ = -2^4 + (2^3+ $A2 \cdot 2$^2+ $A1 \cdot 2$^1 + $A0 \cdot 2$^0)
= -2^3 + ($A2 \cdot 2$^2+ $A1 \cdot 2$^1 + $A0 \cdot 2$^0)

∴ 4b'$A3A2A1A0$ = 5b'$A3A3A2A1A0$

Therefore
4b'$A3A2A1A0$ + 5b'$B4B3B2B1B0$ = 5b'$A3A3A2A1A0$+5b'$B4B3B2B1B0$

(왜 A4 → A3이 되는지 안쓰면 2점 감점)

② Circuit diagram with 5 full adders [3 pts]

③ Correct circuit detecting overflow [7 pts]
(logic gate로 표현 안하고 Boolean expression만 했을 시 4점 감점)
(NAND, NOR 이외의 GATE 사용 시 2점 감점)

2. Minimum SOP is an SOP with minimum number of product terms and minimum number of literals. Minimum POS is a POS with minimum number of sum terms and minimum number of literals. Minimum POS of f is obtained by complementing minimum SOP of f'. Explain why. [10 pts]

i) complement of POS form is SOP form and their number of literals are the same

- By De Morgan's law, the number of literals is not changed. So (POS)' →SOP that the number of literals is the same

ii) the complement of minimum POS is minimum SOP form

- if (minimum POS)' is not minimum SOP, there would be another minimum SOP form that is minimized from (minimum POS)'. Then (minimum SOP)' would be the POS form. But the number of literals of this POS form would be reduced rather than original POS form. So, this is the contradiction

3. We try to prove the associative law of Boolean OR, i.e. $x + (y + z) = (x + y) + z$. Form $[x + (y + z)][(x + y) + z]$ and show that it is equal to $x + (y + z)$ in one way and $(x + y) + z$ in the other. You can use other properties of Boolean algebra except for associative laws. [15 pts]

[x+(y+z)][(x+y)+z] = x(x+y)+xz+(y+z)(x+y)+(y+z)z

=x+xy+xz+yx+y+zx+zy+yz+z

=x(1+y+z)+(y+z)x+y+zy+yz+z

=x+(y+z)x+(y+z)y+(y+z)z

=(1+y+z)x+(y+z)(y+z)

=x+(y+z) ... (a)

Also,

[x+(y+z)][(x+y)+z] = x(x+y)+xz+(y+z)(x+y)+(y+z)z

=x+xy+xz+yx+y+zx+zy+yz+z

=(x+y)x+(x+y)y+(x+y)z+z(x+y+1)

=(x+y)(x+y)+(x+y)z+z

=(x+y)+(x+y+1)z

=(x+y)+z ... (b)

Thus, x + (y + z) = [x+(y+z)][(x+y)+z] = (x + y) + z

4. The following prime implicant chart is for a four-variable function $f(A, B, C, D)$.
   a. List the maxterms of f. [5 pts]
   b. List the don't-cares of f, if any. [5 pts]

	2	3	7	9	11	13
–0–1		×		×	×	
–01–	×	×			×	
--11		×	×		×	
1--1				×	×	×

4-a

K-map

AB/CD	00	01	11	10
00	0	x	1	1
01	0	0	1	0
11	0	1	x	0
10	0	1	1	x

Correct answer: Pi M(0,4,5,6,8,12,14)

4-b

d(1, 10, 15)

5. Consider $F(A, B, C, D) = \Sigma m(0, 2, 5, 6, 7, 8, 9, 12, 13, 15)$. Derive two minimum POS forms. Identify hazards in each form. [10 pts]

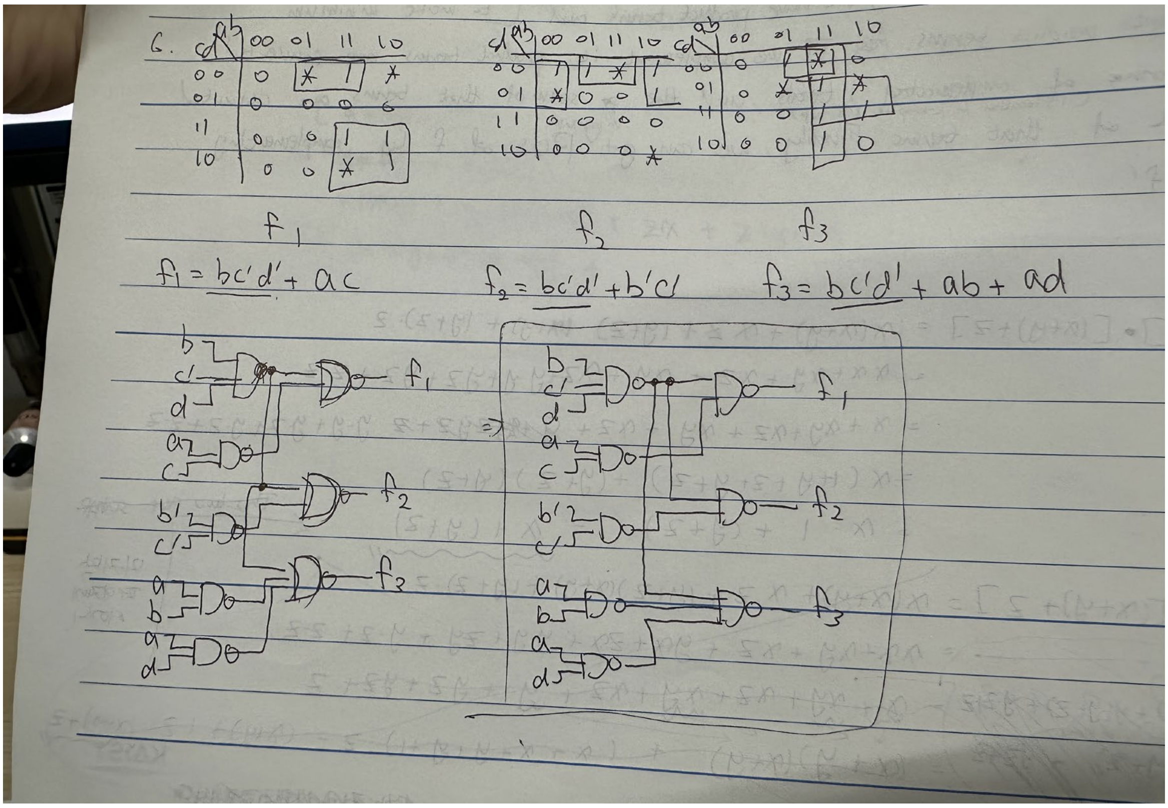
Form 1: $F = (A + B' + C + D)(A + B + D')(A' + C' + D)(A' + B + C')$

Form 2: $F = (A + B' + C + D)(A + B + D')(A' + C' + D)(B + C' + D')$

Static 0-hazard in form 1: 0011 → 1011

Static 0-hazard in form 2: 1011 → 1010

6. Find a minimum two-level, multiple-output NAND-NAND gate circuit to realize three Boolean functions: $f_1(a, b, c, d) = \Sigma m(10, 11, 12, 15) + \Sigma d(4, 8, 14)$, $f_2(a, b, c, d) = \Sigma m(0, 4, 8, 9) + \Sigma d(1, 10, 12)$, and $f_3(a, b, c, d) = \Sigma m(4, 11, 13, 14, 15) + \Sigma d(5, 9, 12)$. [10 pts]



7. Consider a full subtracter, which performs $x_i - y_i$ with b_i as input borrow; d_i is a difference and b_{i+1} is output borrow.
    a. Derive Boolean expressions for d_i and b_{i+1}. [10 pts]
    b. Implement a full subtracter using two 2-input exclusive OR gates, one inverter, and three 2-input NOR gates. [10 pts]

7-a.

INPUT			OUTPUT	
A	B	Bin	D	Bout
0	0	0	0	0
0	0	1	1	1
0	1	0	1	1
0	1	1	0	1
1	0	0	1	0
1	0	1	0	0
1	1	0	0	0
1	1	1	1	1

From above table we can draw the K-Map as shown for "difference" and "borrow".

A \ B Bin	00	01	11	10
0	0	1	0	1
1	1	0	1	0

D=A'B'Bin + AB'Bin' + A'BBin' + ABBin

A \ B Bin	00	01	11	10
0	0	1	1	1
1	0	0	1	0

Bout=A'Bin + A'B + BBin

Correct answer:

di= xi'yi'bi+ xiyi'bi'+ xi'yibi'+ xiyibi

bi+1= xi'bi+ xi'yi+ yibi

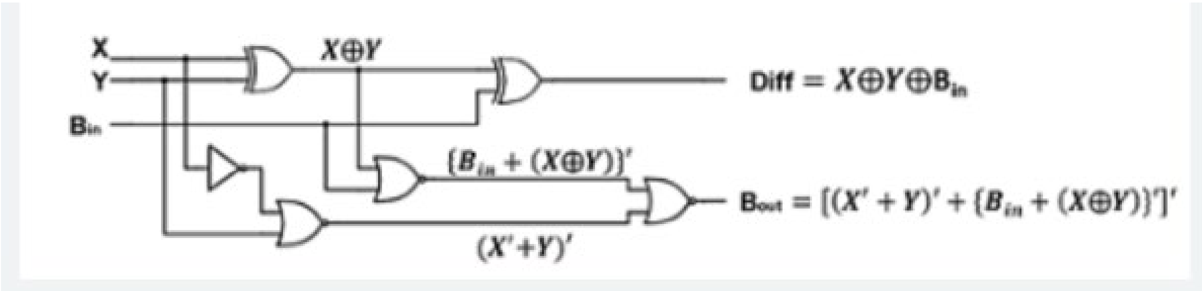
7-b.

Diff의 Logic table로부터 Diff = X XOR Y XOR B_{in}임을 알 수 있다.

B_{out}의 경우, output이 1이 되는 경우를 product of sum으로 표현하고, 논리식을 정리하면

$$
\begin{aligned}
\mathrm{B}_{out} &= (X' + Y + B_{in})(X' + Y + B'_{in})(X' + Y' + B_{in})(X + Y + B_{in}) \\
&= (X' + Y + B_{in})(X' + Y + B'_{in})\{B_{in} + (X' + Y')(X + Y)\} \\
&= (X' + Y)\{B_{in} + (X \oplus Y)\} \\
&= [(X' + Y)' + \{B_{in} + (X \oplus Y)\}']' \\
&= (X' \text{ NOR } Y) \text{ NOR } (B_{in} \text{ NOR } (X \oplus Y))
\end{aligned}
$$

따라서 최종 답은 아래와 같다.



8. Consider the expression: *abd'f' + b'cegh' + abd'f + acd'e + b'ce.*
   a. Algebraically simplify the expression to SOP of two product terms. [5 pts]
   b. Convert the SOP from (a) to (minimum) POS form. [5 pts]

8-a.

abd'(f+f')+b'ce(gh'+1)+acd'e=abd'+b'ce+acd'e(b+b')
=abd'(1+ce)+b'ce(ad'+1)=abd'+b'ce

8-b.

As we know from the factoring theorem,

$$AB + A'C = (A + C)(A' + B)$$

Applying this in 6

$$\begin{aligned} b(ad') + b'(ce) \\ &= (b + ce)(b' + ad') \end{aligned}$$

From distributive law

$$A + BC = (A + B)(A + C)$$

applying this in 8

$$(b + ce)(b' + ad') = [(b + c)(b + e)][(b' + a)(b' + d')]$$

So final expression in product of sum form

$$= \boxed{(b + c)(b + e)(b' + a)(b' + d')}$$