1. Suppose that we want to add +11 in 6-bits (001011) and -5 in 4-bits (1011) of 2’s complement. Before the addition, 1011 has to be converted to 6-bits format. This can be simply done by so-called “MSB extension”, i.e. MSB of $\underline{1}011$ is extended to 2 more bits to yield $\underline{111}011$. Prove why MSB extension works. [10 pts]

   If we represent a negative number x in k-bits using 2’s complement, 2^k-x is used. Now, we want to represent x in k'-bits where k' is larger than k. Then x is represented by $2^{k'}$-x. Thus, If we want to change from k-bits to k'-bits, we must add $\underline{2^{k'}-2^k}$; it is equal to $2^{k'-1}+2^{k'-2}+\cdots+2^k$. This is equivalent to MSB extension.

   If you wrote -x =2^k-x, -1pts
   If you wrote x^* =2^k-x+1, -1pts

2. Answer the followings:
   a. Consider a K-map of 3-variables. What is the maximum number of essential primes you can discover? Show an example. [5 pts]

      4

1	
	1
1	
	1

or

	1
1	
	1
1	

      There are many other examples.

   b. Repeat (a) for 4-variable K-map. [10 pts]

      8

1		1	
	1		1
1		1	
	1		1

or

	1		1
1		1	
	1		1
1		1	

      There are many other examples.

   c. Now, what is the maximum number of essential primes a Boolean function f: $\{0, 1\}^n \rightarrow \{0, 1\}$ can have? Justify your answer. [10 pts]

      2^{n-1}

      Let’s assume that variables are represented by 1 and complements are 0, e.g. ab’c is 101. Then different essential primes must have more than 2 different numbers, e.g. 000 and 011 each are essential primes. Therefore the maximum number of essential primes can be obtained by computing $\sum_{i=0}^{\lfloor n/2 \rfloor} {}_nC_{2i}$ or $\sum_{i=0}^{\lfloor n/2 \rfloor} {}_nC_{2i+1}$. $\sum_{i=0}^{\lfloor n/2 \rfloor} {}_nC_{2i} + \sum_{i=0}^{\lfloor n/2 \rfloor} {}_nC_{2i+1} = \sum_{i=0}^{n} {}_nC_i = 2^n$ and $\sum_{i=0}^{\lfloor n/2 \rfloor} {}_nC_{2i} = \sum_{i=0}^{\lfloor n/2 \rfloor} {}_nC_{2i+1}$. The maximum number of essential primes is 2^{n-1}.

If you just made an answer without reasonable explanation, you got 3pts.
If you solved this problem using only answer from 2.a and 2.b, you got 4pts.
If you have some missing in your explanation, you got 7pts. (Explanation that EPI's are not adjacent each other earns 7pts.)
If you explained the reason why the example for 2.a and 2.b are general cases for maximum number of essential primes, you got 10pts. (You had to show why EPIs are not adjacent each other or show that although EPIs have common minterm, this problem can be also changed to find maximum number of minterms which are not adjacent each other.)

3. Find both the minterm expansion and maxterm expansion for the following functions, using *algebraic manipulations*:
   a. $f(A, B, C, D) = AB + A'CD$ [5pts]

      $AB(CD + CD' + C'D + C'D') + A'CD(B + B')$
      $= \sum m(3,7,12,13,14,15)$

      $(A + CD)(A' + B) = (A + C)(A + D)(A' + B) = (A + C + (B + D')(B' + D'))$
      $(A + D + (C + D')(C' + D'))(A + C + D + BB')$
      $(A' + B + (C + D)(C + D')(C' + D)(C' + D'))$
      $= \prod M(0,1,2,4,5,6,8,9,10,11)$

   b. $f(A, B, C, D) = (A + B + D')(A' + C)(C + D)$ [5pts]

      $(A + B + D' + CC')(A' + C + D' + BB')(A + C + D + BB')(A' + C + D + BB')$
      $= \prod M(0,1,3,4,8,9,12,13)$

      $(A + B + D')(C + A'D) = AC + BC + A'BD + CD' = ABCD' + A'BCD +$
      $ABC(D) + ACD'(B') + BCD'(A') + AC(B'D) + A'BD(C') + CD'(A'B')$
      $= \sum m(2,5,6,7,10,11,14,15)$

      If you correct either maxterm expansion or minterm expansion, you got 3pts.
      Although you wrote the correct answer, if you solved it using K-map, you got 2pts.

4. Given $F = AB'D' + A'B + A'C + CD$

   a. Use a K-map to find the maxterm expansion for F. Find also the minimum product of sums for F. [5 pts]

CD \ AB	00	01	11	10
00	0	1	0	1
01	0	1	0	0
11	1	1	1	1
10	1	1	0	1

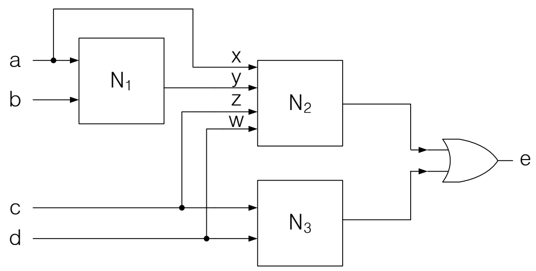
$\prod M(0,1,9,12,13,14)$
$= (A + B + C)(A' + C + D')(A' + B' + D)$

   b. Use a K-map to find the minimum SOP for F. [5 pts]

$AB'D' + A'B + A'C + CD$ or $AB'D' + A'B + B'C + CD$

If you correct either maxterm expansion or minmum POS, -2pts

5. In the following circuit, illustrated by a connection of blocks, we try to design block N_2.



   a. N_2 is given by $\Sigma m(1, 5, 7, 10, 13)$. Derive simplified POS. [5 pts]

zw \ xy	00	01	11	10
00	0	0	0	0
01	1	1	1	0
11	0	1	0	0
10	0	0	0	1

$N_2 = (z + w)(y' + w)(x + y + z')(x' + z' + w')(x' + y + z)$
or $(z + w)(y' + w)(x + y + z')(x' + z' + w')(x' + y + w')$
or $(z + w)(y' + w)(x + y + z')(x' + y' + z')(x' + y + w')$
or $(z + w)(x + w)(x' + y' + z')(y + z' + w')(x' + y + z)$
or $(z + w)(x + w)(x' + y' + z')(y + z' + w')(x' + y + w')$
or $(z + w)(x + w)(x' + y' + z')(x + y + z')(x' + y + w')$

If K-map is incorrect, -2pts
If simplification is incorrect, -1pts for each term (max -3pts)
If the expression is not represented by x, y, z, w, -1pts
If you have the wrong concept for POS, -2pts
If you found incorrect answer for a., -1pts in b. and -2pts in c. will be deducted respectively.

b. Assume that N_1 is an OR gate. Using this fact, realize N_2 using only 2-input NAND and INV. [5 pts]

Since $x = a$ and $y = a + b$, $x = 1$ and $y = 0$ never happen. Thus <u>xy' is don't care term</u>.

zw \ xy	00	01	11	10
00	0	0	0	X
01	1	1	1	X
11	0	1	0	X
10	0	0	0	X

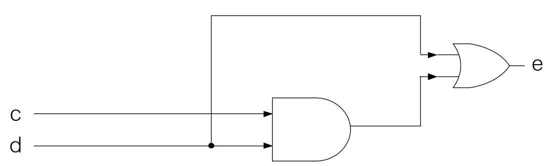
Let's assume that ⊙ denotes NAND operator.
$N_2 = z'w + x'yw = w(z' + x'y) = (w \odot (z \odot (x' \odot y)))' =$
or $(z' \odot w) \odot (x'y \odot w) = (z' \odot w) \odot ((x' \odot y)' \odot w)$

If you didn't mention about don't care, -2pts, but another method for using the fact that N_1 is an OR gate is written, only -1pts
If you have the wrong concept for don't care -1pts
If you made a mistake for minterm expansion, -2pts
If you have the wrong concept for NAND, -2pts

c. Assume that N_3 is an AND gate. Using the knowledge of N_1 from (b) and N_3, derive the simplified expression for N_2. Draw the connection of gates for whole circuit (minimize the number of gates). [10 pts]

Since the function of e is equal to $N_2 \cup N_3$, if N_3 is equal to 1, e is always 1 irrelevantly to N_2. Thus $N_3 = 1$ is don't care, i.e. <u>zw is don't care term</u>.

zw \ xy	00	01	11	10
00	0	0	0	X
01	1	1	1	X
11	X	X	X	X
10	0	0	0	X



If you didn't mention don't care, -5pts, but
1) if you find the fact that e = d and try applying the fact to N_2 simplification, -2pts
2) if you wrote the correct answer,-3pts.

6. Answer the followings:
   a. In complement number systems, a constant M is used to define a negative number, i.e. negative number of x is represented by $M - x$, which is non-negative. What is the value for M such that all the integers in the range $[-N, +P]$ are uniquely represented? [5 pts]

      After representing integers $[0, +P]$ using positive numbers $[0, +P]$, we want to represent negative integers $[-N, -1]$ using complement numbers.
      We have to represent negative integers using $M - x$, where x in the range $[1, +N]$, so $M - N$ is the least complement number of all complement numbers. After representing positive numbers, the least number of remained positive numbers is $P + 1$. Since we want to be uniquely represented, $M - N$ must be equal to $P + 1$. Thus, M is equal to $N + P + 1$.

      If you mentioned that M is equal to $N + P + 1$ due to integer range is $-N$ to $+P$, you got 2pts.

   b. Develop a multiplication method for biased representation, i.e. propose a method to obtain $xy + bias$ from $x + bias$ and $y + bias$. (a step in the method should be simple, e.g. $+bias$ is MSB flip.) [5 pts]

      $(x + bias) \times (y + bias) = xy + bias \times (x + y + bias)$
      $= xy + bias + bias \times ((x + bias) +(y + bias) - bias) - bias$

      1: Multiply two inputs
      --- $(x + bias) \times (y + bias)$
      2: Add two inputs, then flip the MSB
      --- $(x + bias) +(y + bias) - bias$
      3: Shift the result of 2 left ($\log_2 bias$) times
      --- $bias \times ((x + bias) +(y + bias) - bias)$
      4: Subtract the result of 3 from the result of 1, then flip the MSB
      --- $(x + bias) \times (y + bias) - bias \times ((x + bias) +(y + bias) - bias) + bias$

      If you didn't mention that $+bias$ means 2^{N-1} bit, -3pts.
      If you didn't start the above proof from $x + bias$ and $y + bias$, -2pts.
      If you had trivial mistakes, -1pts.

   c. When we derive minimum POS of f, we derive minimum SOP of f' and take its complement. Explain why this is true (i.e. Explain why complement of minimum SOP of f' is minimum POS of f). [5 pts]

Let's consider a Karnaugh map for *f'*. It can be obtained by exchanging 0 and 1 from the Karnaugh map for *f*.
If we find prime implicants for POS of *f*, those are also found for SOP of *f'*.
For example,

1	1	1	0
0	1	1	0
0	0	1	0
1	1	0	0

f

0	0	0	1
1	0	0	1
1	1	0	1
0	0	1	1

f'

Thus, complement for SOP of *f'* is equal to POS of *f*. And since SOP of *f'* and POS of *f* have the same prime implicants, if we find minimum SOP of *f'*, POS of *f* is also minimum.

If you mentioned about K-map or other similar concepts, you got 2pts.
If you mentioned that the reason why SOP of *f'* and POS of *f* are all minimum is they have the same prime implicants, you got 3pts.
c.f) Explanation without reason will earn 0 pts.

d. In Petrick's method, we derive a Boolean expression, in which each variable indicates whether a particular prime is picked or not. We then simply this expression to discover the minimum number of primes. Simplifying this expression is easier than simplifying a general Boolean expression because no variables are complemented. Explain why. [10 pts]

Because all prime implicants are essential.
(As long as a product term isn't contained in another term, e.g. *abc* is contained in *ab*, the product term is essential.)

It is very hard to minimize Boolean expression because it is difficult to find the minimum number primes which cover all remained minterms after finding essentials. Therefore, Boolean simplification is very easy in Petrick's method since all primes are essential.

If you wrote that all prime implicants are essential, you got 7pts.
If you wrote explanations supporting underlined sentence, you got rest of 3pts.
c.f) Even though the answer does not mention about the underlined sentence, if the answer somehow explains the reason well will earn 5 pts.
Students who simply describe the procedure of Petrick's method will get 0 pts.