EE202A Signals and Systems, Fall 2023
December 11, 2023
Time Limit: 09:00 - 11:45 Lecturer: Jung-Woo Choi

Score Table (for teacher use only)

Question:	1	2	3	4	5	Total
Points:	10	20	20	15	35	100
Score:						

This is a CLOSED-BOOK exam.
Please provide ALL DERIVATIONS and EXPLANATIONS with your answers.
Any communication with others during the exam will be regarded as a cheating case.
This exam contains 2 pages (including this cover page) and 5 questions.
[ST] indicates "Student-made Questions" (Questions for Questions)

1. (10 points) Discrete Fourier transform

   (a) (2 points) Given that $x[n]$ has Fourier transform $X(e^{j\omega})$, express the Fourier transform of the following signal in terms of $X(e^{j\omega})$.

   $$x_2[n] = x[1-n] + x[-1-n]$$

   (b) (8 points) Use the properties of Fourier transform to derive A of the following sum

   $$A = \sum_{n=0}^{\infty} n \left(\frac{1}{2}\right)^n$$

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2. (20 points) Consider a causal LTI system described by the following LCCDE.

   $$y[n] + y[n-1] + \frac{1}{4}y[n-2] = x[n-1] - \frac{1}{2}x[n-2]$$

   (a) (5 points) Find the frequency response $H(e^{j\omega})$ of this system.

   (b) (10 points) Find the impulse response of this system. In particular, derive the response in the form of $h[n] = B \cdot nC^n u[n]$ and find the constants B and C.

   (c) (5 points) Determine whether the LTI inverse of this system is causal or not.

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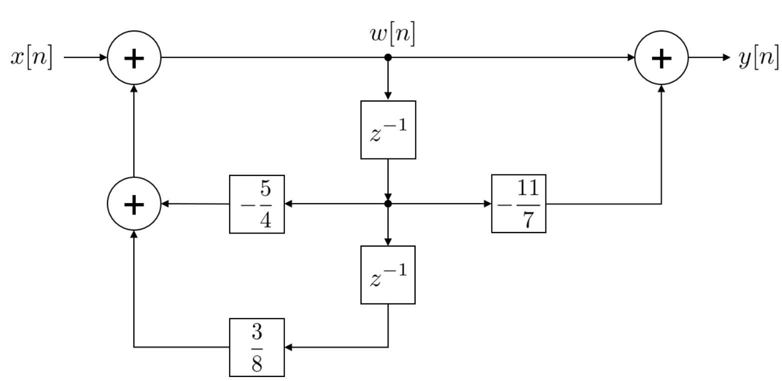
3. (20 points) [ST]



The input $x[n]$ and output $y[n]$ of a causal LTI system are related through the block diagram representation as shown in the figure.

(a) (5 points) Determine the difference equation relating $y[n]$ and $x[n]$.

(b) (5 points) Determine the system function $H(z)$ for this causal LTI system.

(c) (5 points) Determine the poles and ROC of $H(z)$

(d) (5 points) Determine the BIBO stability of this system.

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4. (15 points) [ST] Consider an LTI system with its transfer function $H(s)$.
The output of the system that corresponds to the input $x(t) = e^{-t}u(t)$ is $y(t) = K(e^{-3t}u(t) + e^{t}u(-t))$, where K is a constant value.

(a) (5 points) Derive the Laplace transform $Y(s)$ of $y(t)$ and region of convergence (ROC).

(b) (5 points) Determine the impulse response $h(t)$ of the system. Assume that the ROC of the system is identical to the ROC of $Y(s)$.

(c) (5 points) The output of the system that corresponds to the input $x(t) = 1$ is $y(t) = \frac{8}{3}$. Determine the value of K.

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5. (35 points) [2019 Final Problem] Consider an LTI system with the condition of initial rest. The system satisfies the following differential equation.

$$\frac{d^2y(t)}{dt^2} + 2\frac{dy(t)}{dt} + 2y(t) = 2\frac{d^2x(t)}{dt^2}$$

(a) (5 points) Determine the frequency response $H(j\omega) = Y(j\omega)/X(j\omega)$.

(b) (5 points) Determine whether this system is (a) underdamped (b) critically-damped (c) overdamped.

(c) (10 points) Draw the bode plot of the frequency response $H(j\omega)$ derived in the problem (a). Show both magnitude and phase responses.
(Specify (1) the break frequency, (2) slopes of the magnitude (in dB/decade) curve, (3) magnitude (in dB) at the pass-band, and (4) phases at $\omega = 0,\ \infty$, break frequency.)

(d) (10 points) A signal $x(t) = \cos(0.01t)\cos(\sqrt{2}t)$ is fed into the system. Estimate the delay of its envelope $\cos(0.01t)$ at the output $y(t)$ of this system. (Hint: if necessary, you can use $\frac{d\tan^{-1} g(x)}{dx} = \frac{1}{1+g(x)^2} \cdot \frac{dg(x)}{dx}$)

(e) (5 points) In problem (d), what is the Nyquist rate to sample the output $y(t)$ without aliasing?

[End of Problem]