EE201 Circuit Theory May 25, 2014

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Quiz #2 [Solution]

1. Phasor and Complex numbers [6pts]

(a) Suppose $v_1(t) = \cos(10^3t + 60^o)$. Let's represent this signal as $\mathbf{V_1} = 1 \angle 0^o$. If $\mathbf{V_2} = 0.5 \angle 10^o$, then what is $v_2(t)$?

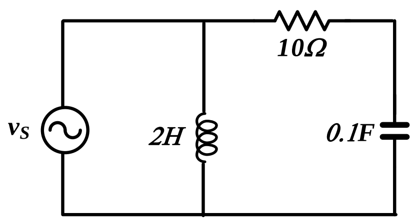
Sol) $v_1(t) = 0.5\cos(10^3t + 70^o)$

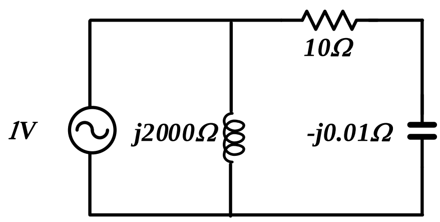
(b) Suppose $v_1(t) = \cos(10^3t + 60^o)$. Let's represent this signal as $\mathbf{V_1} = 1+j$. If $v_2(t) = \cos(10^3t + 105^o)$, then what is $\mathbf{V_2}$?

Sol) $v_1(t) = \cos(10^3t + 60^o) \rightarrow \mathbf{V_1} = 1+j = \sqrt{2} \angle 45^o$

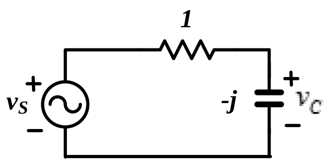
$\therefore$ $v_2(t) = \cos(10^3t + 105^o) \rightarrow \mathbf{V_1} = \sqrt{2} \angle 90^o = \sqrt{2}\, j$

2. Represent the impedance of each circuit component in complex numbers when $v_S = \cos(10^3t\ )$. [10pts]





3. Find the sinusoidal steady-state voltage, $v_C(t)$, of the below circuit, when $v_S = \cos(10^3t\ )$. **<u>Express your answer in time-domain.</u>** [10pts]



Sol)

$\mathbf{V_C} = \mathbf{V_S} * (-j)/(1-j)$

$= 1 \angle 0^o * 1/\sqrt{2} \angle -45^o$

$v_C = (1/\sqrt{2}) * \cos(10^3t - 45^o)$

4. Suppose the sinusoid voltage and current of a load is represented using the below vectors. Note that the magnitude of the vectors represents the **rms** values. [16pts]

(a) What are the apparent power, average power and power factor? [9pts]

Complex power: $\mathbf{S} = \mathbf{V}_{rms}\mathbf{I}^*_{rms}$

Apparent power: $|\mathbf{S}| = V_{rms}I_{rms}$

$$\mathbf{V}_{rms} = 2+2j, \ \rightarrow V_{rms} = \sqrt{4+4} = \sqrt{8}$$
$$\mathbf{I}_{rms} = 2+j, \ \rightarrow I_{rms} = \sqrt{4+1} = \sqrt{5}$$

Apparent power $= |\mathbf{S}| = |V_{rms}| \cdot |I_{rms}| = 2\sqrt{2} \cdot \sqrt{5} = 2\sqrt{10}$

Average power $= P = \overrightarrow{V_{rms}} \cdot \overrightarrow{I_{rms}} = (2,2)\cdot(2,1) = 6$

Power factor $= \ PF = \dfrac{P}{|\mathbf{S}|} = \dfrac{6}{2\sqrt{10}} = \dfrac{3}{\sqrt{10}} = 0.3\sqrt{10}$

(b) How will the above answer change if the vector represents the amplitude and not the rms value? [3pts]

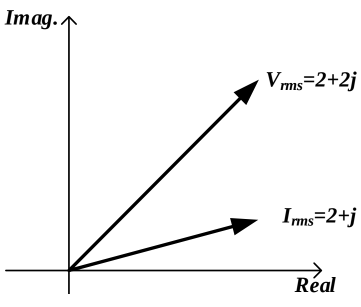
$\dfrac{V_m}{\sqrt{2}} = V_{rms}, \quad \dfrac{I_m}{\sqrt{2}} = I_{rms}$

Apparent power $= \ |\mathbf{S}| = |V_{rms}| \cdot |I_{rms}| = \sqrt{10}$

Average power $= P = 2+1 = 3$

Power factor $= \ PF = \dfrac{P}{|\mathbf{S}|} = \dfrac{3}{\sqrt{10}} = 0.3\sqrt{10}$

(c) What is the voltage vector that is needed to achieve a power factor of 1? [4pts]



Sol 1> $V_{rms} = k(2+j)$, k can be any number.

In the case with the same apparent power:

$$\sqrt{5k^2} = \sqrt{8} \rightarrow k = \sqrt{\frac{8}{5}}$$

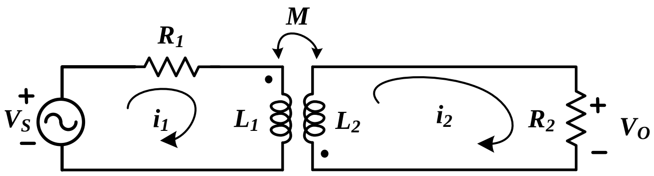
So $V_{rms} = \sqrt{\dfrac{8}{5}}(2+j) = \dfrac{4\sqrt{10}}{5} + \dfrac{2\sqrt{10}}{5}j$

<Sol 2>

Insert power factor correction vector

: V_{corr}= - j

5. Consider the circuit shown below. [8pts]



(a) Suppose $v_S(t) = \cos(10^3 t)$. Write down two equations that can be used to analyze the above circuit. You may write the equations in differential form.

Sol)

$$v_s = i_1 \cdot R_1 + L_1 \frac{di_1}{dt} + M \frac{di_2}{dt}$$

$$i_2 \cdot R_2 + L_2 \frac{di_2}{dt} + M \frac{di_1}{dt} = 0$$

(b) Suppose $V_S = 1$V. Express the steady-state voltage V_O in terms of R_1, R_2, L_1, L_2 and M.

Sol)

DC input $= \frac{di_1}{dt} = 0$

V_o=0 V