Formal Languages and Automata (CS322)

Mid-term exam (2nd November 2024)

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[20 points] Q1. Consider the following languages A_1 and A_2 over $\Sigma = \{\mathsf{a}, \mathsf{b}\}$:

$$A_1 = \{w \mid w \text{ has even number of } \mathsf{a}\text{'s}\}$$

and

$$A_2 = \{w \mid \text{ in } w, \text{ each } \mathsf{a} \text{ is immediately followed by at least one } \mathsf{b}\}.$$

Note that ϵ is included in both A_1 and A_2. To present your DFA for (a)-(b), present the 5-tuple of DFA, the transition table and the transition diagram.

(a) (5 points) Construct a **deterministic** finite automaton M_1 with two states which recognizes A_1.

(b) (5 points) Construct a **deterministic** finite automaton M_2 with three states which recognizes A_2.

(c) (5 points) Present a transition diagram of a **deterministic** finite automaton M which recognizes the following language over $\Sigma = \{\mathsf{a}, \mathsf{b}\}$:

$$A = \{w \mid w \text{ has even number of } \mathsf{a}\text{'s and each } \mathsf{a} \text{ is immediately followed by at least one } \mathsf{b}\}.$$

(d) (5 points) Present a regular expression for A along with a sequence of transition diagrams of generalized NFAs, starting from your DFA for (c) ending in a two-states GNFA.

[28 points] Q2. Prove or disprove: if the language is regular, present a transition diagram of an NFA recognizing the language. If not, prove that the language is not regular using pumping lemma. All languages are over $\{0, 1\}$.

(a) (7 points) $A = \{w \mid w \text{ can be written as } 1^k \cdot y \text{ for some } k \geq 1 \text{ and } y \in \{0,1\}^* \text{ s.t. } y \text{ contains at least } k \text{ 1's}\}$

(b) (7 points) $B = \{w \mid w \text{ can be written as } 1^k \cdot y \text{ for some } k \geq 1 \text{ and } y \in \{0,1\}^* \text{ s.t. } y \text{ contains at most } k \text{ 1's}\}$

(c) (7 points) $C = \{w \mid w \neq w^R\}$

(d) (7 points) $D = \{w \mid$ the number of 1's in the even-numbered positions is even and the number of 1's in the odd-numbered positions is odd $\}$

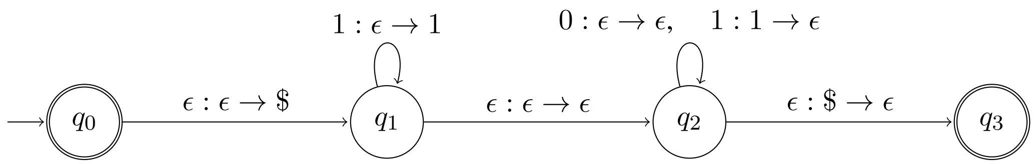
[16 points] Q3. Write a context-free grammar for each of the following languages.

(a) (8 points) The set of strings over $\{\mathsf{a}, \mathsf{b}\}$ of even length where the number of occurrences of b in even positions is the same as the number of occurrences of b in odd positions.

(b) (8 points) The set of strings of the form $a^i b^j c^p d^q$ with $i + j = p + q$.

[14 points] Q4. Consider the following pushdown automaton $P = (Q, \{0, 1\}, \{0, 1, \$\}, \delta, q_0, \{q_0, q_3\})$ where the transition function is given as the transition diagram below. Let L be the set of strings recognized by P.

δ:



(a) (4 points) For each of the following string, write if P accepts it or not:

(i) 01101, (ii) 1110010001, (iii) 110011000011, (iv) 1111010010011.

(b) (10 points) Present the transition diagram of a pushdown automaton P' which recognizes the language

$$\mathsf{Prefix}(L) = \{w \mid \text{ there exists a string } x \text{ such that } wx \in L\}$$

and explain why your pushdown automaton recognizes $\mathsf{Prefix}(L)$.

[8 points] Q5. A grammar $G = (V, \Sigma, S, R)$ is said to be *right-linear* (*left-linear*, respectively) if every rule is of the form $X \to aY$ ($X \to Ya$ respectively), where $a \in \Sigma \cup \{\epsilon\}$ and $Y \in V \cup \{\epsilon\}$. Consider the following right-linear grammar G; V_0 is the start variable.

- $V_0 \to 0V_0 \mid 1V_1$
- $V_1 \to 0V_0 \mid 1V_2 \mid \epsilon$
- $V_2 \to 1V_1 \mid 0V_2$

Write a deterministic finite automaton with three states which recognizes the language $L(G)$ of the grammar G. (Present it as 5-tuple and describe the transition function as a transition diagram.)

[14 points] Q6. Consider a pushdown automaton $P = (Q, \Sigma, \Gamma, \delta, q_0, F)$ which

- empties the stack when it accepts a string,
- has a single accept state, i.e. $|F| = 1$, and
- does not push a stack symbol of Γ onto the stack and pops a symbol of Γ off the stack simultaneously.

Justify the following statement by constructing an NFA, regular expression, or a suitable grammar for the language of stack contents:

> the set of strings over Γ that are the possible contents, read from top to bottom, of the stack in accepting computations of P is regular.

You can use the fact that a language generated by a left-linear grammar (respectively, by a right-linear grammar) is regular.