EE201 Circuit Theory, (Fall 2023)

Midterm Exam.

Solution

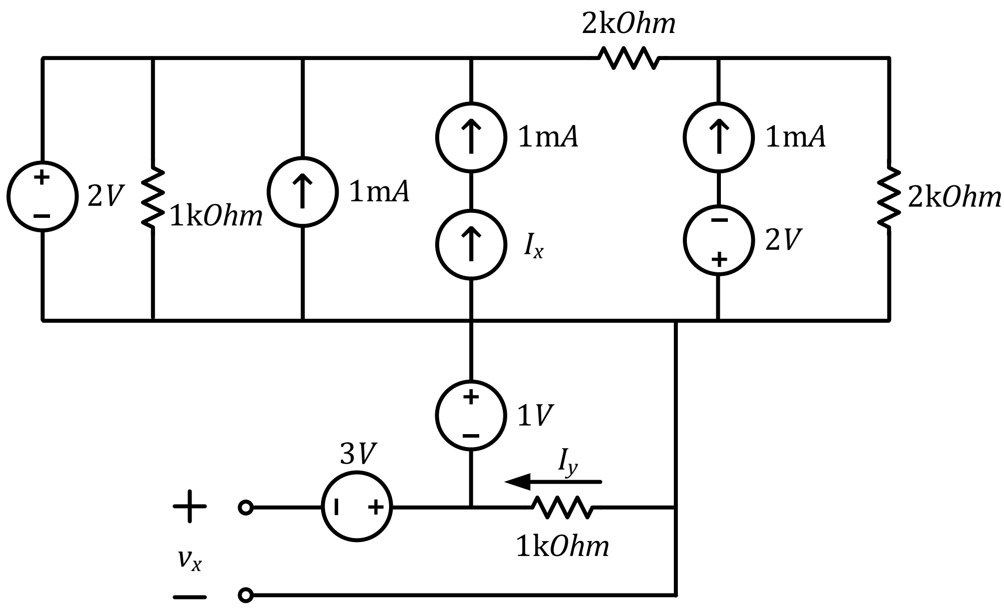
(Total: 100 Points / 8 Problems)

Student ID Number:

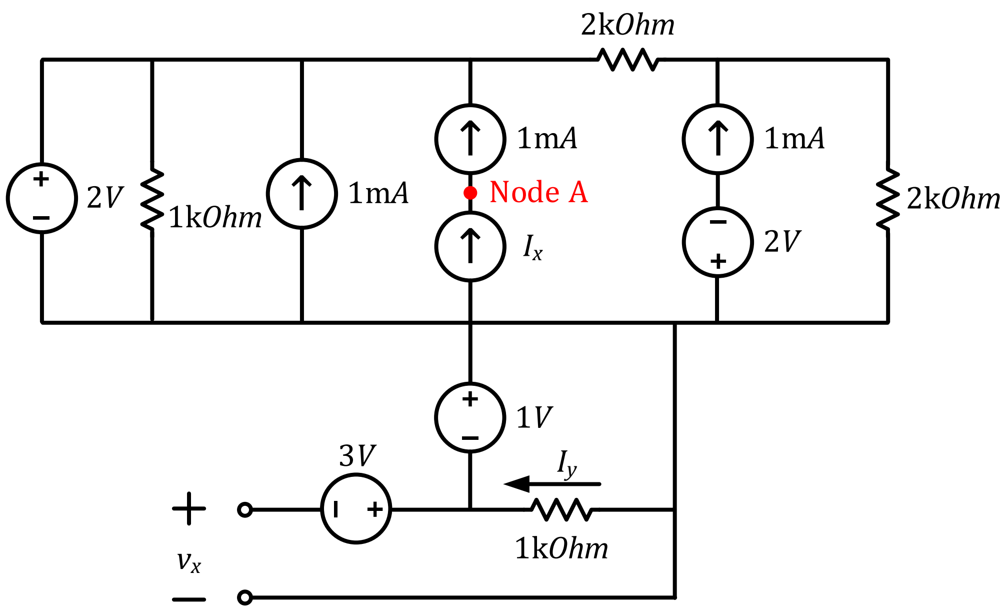
Name:

Prob. 1	Prob. 2	Prob. 3	Prob. 4	Prob. 5	Prob. 6	Prob. 7	Prob. 8	Total
/15	/16	/14	/10	/12	/10	/8	/15	/100

1. (15 points) Answer the following questions.



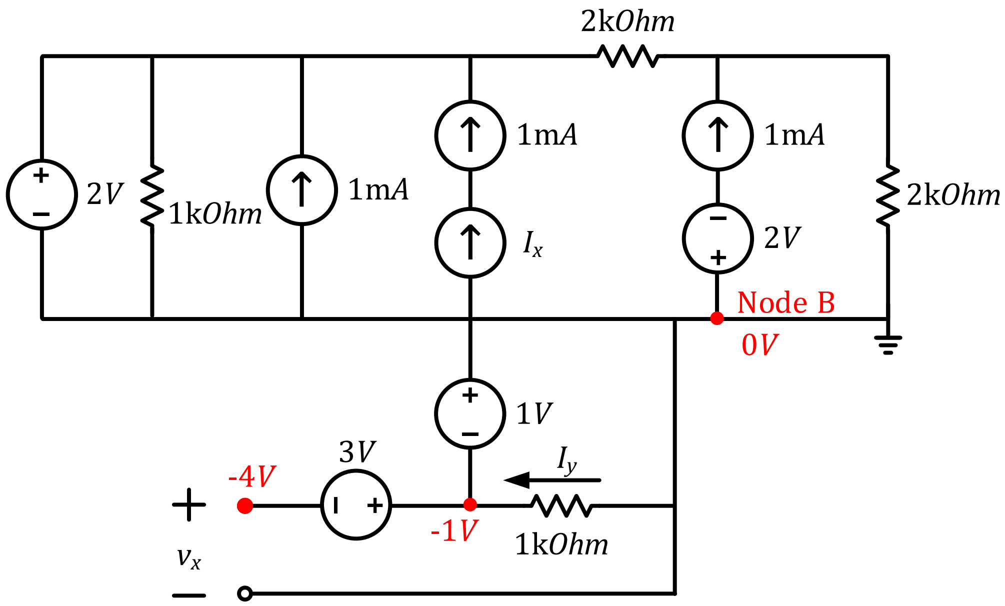
(a) (3 points) Find I_x of the network.



By KCL at node A, I_x should be 1 mA.

A. 1 mA (부분 점수 없음)

(b) (3 points) Find v_x of the network.

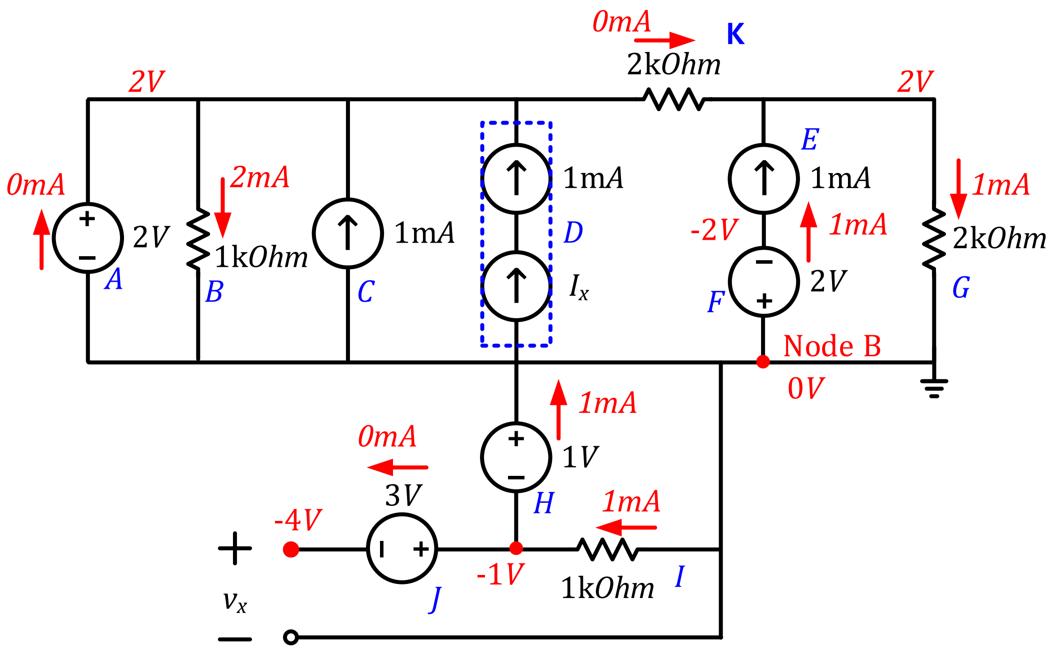


Let assume the node B as ground (0 V). then, the marked nodes' potentials are determined as above. Therefore, v_x= -4 [V] (부분 점수 없음)

(c) (3 points) Find I_y of the network.

As shown in problem (b), voltage drop across 1 kOhm is 1 [V]. by Ohm's law, $I_y = \frac{0-(-1)}{1\ kOhm} = 1\ [mA]$ (부분 점수 없음)

(d) (6 points) Calculate absorbing or supplying powers of the components in the network and verify the Tellegen's theorem.



By KCL, we can determine the voltage and currents in the circuit as above.
Determine all the absorbing, supplying power.

$$P_A = 0mA \times 2V = 0, P_B = 2mA \times 2V = 4mW, P_C = -1mA \times 2V = -2mW$$
$$P_D = -1mA \times 2V = -2mW, P_E = -1mA \times 4V = -4mW, P_F = 1mA \times 2V = 2mW$$
$$P_G = 1mA \times 2V = 2mW, P_H = -1mA \times 1V = -1mW$$
$$P_I = 1mA \times 1V = 1mW, P_J = 0mA \times 3V = 0$$
$$\sum P = 0 + 4mW + (-2mW) + (-2mW) + (-4mW) + 2mW + 2mW + (-1mW) + 1mW = 0$$

(부분 점수 있음)

A, B, C, D, E, F, G, H, I, J, K 11개 중 3개 이하 0점, 4~5개 1점, 6~7개 2점, 8~10개 3점, 11개 다 맞았을 시 4점.

11개를 다 맞추고 power의 계산까지 완벽히 했을 시 6점.

11개의 값을 다 맞게 구했지만 power를 계산하다가 문제가 하나씩 발견될 때마다 감점 1점이고 최하점은 4점

D의 경우를 보면 current source 사이의 전압은 정확히 구할 수 없기 때문에 적절한 임의의 값으로 놓고 D를 두 부분으로 나누어 계산한 것도 하나의 방법이므로 정답으로 인정.

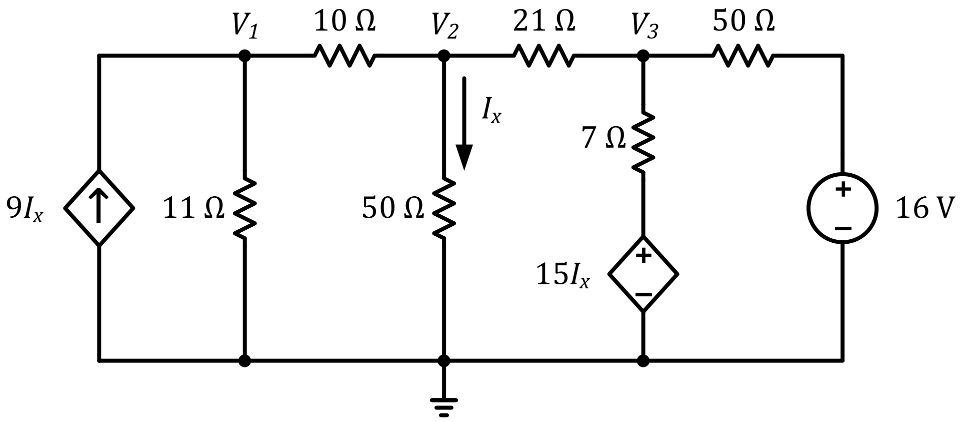
Current + Voltage Source = 7mW
Resistance = 7mW

Absorbing : 4(B) + 2(F) + 2(G) + 1(I) = 9mW
Supplying : 2(C) + 2(D) + 4(E) + 1(H) = 9mW

2. (16 points) Use **the nodal analysis** to answer the following questions.

(Solving without nodal analysis (e.g. the loop analysis) gets -0.5 point for each prob.)

(a) (5 points) Find V_1, V_2, and V_3 in the circuit shown below.



By Ohm's law, $I_x = \frac{V_2}{50}$ $(1\ points)$

By KCL,

$$9I_x = \frac{V_1}{11} + \frac{V_1 - V_2}{10}$$

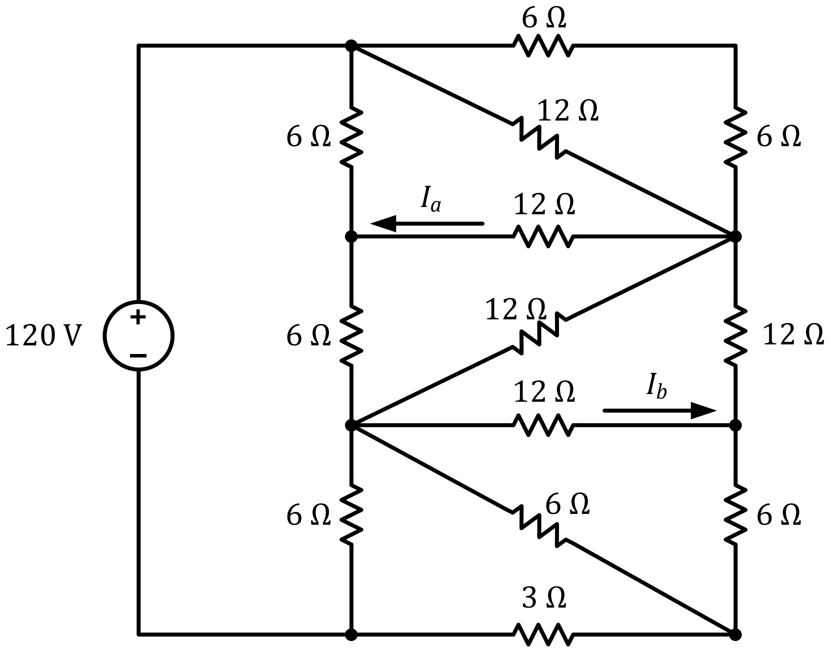
$$0 = \frac{V_2 - V_1}{10} + \frac{V_2}{50} + \frac{V_2 - V_3}{21}$$

$$0 = \frac{V_3 - V_2}{21} + \frac{V_3 - 15I_x}{7} + \frac{V_3 - 16}{50} \qquad (3\ points)$$

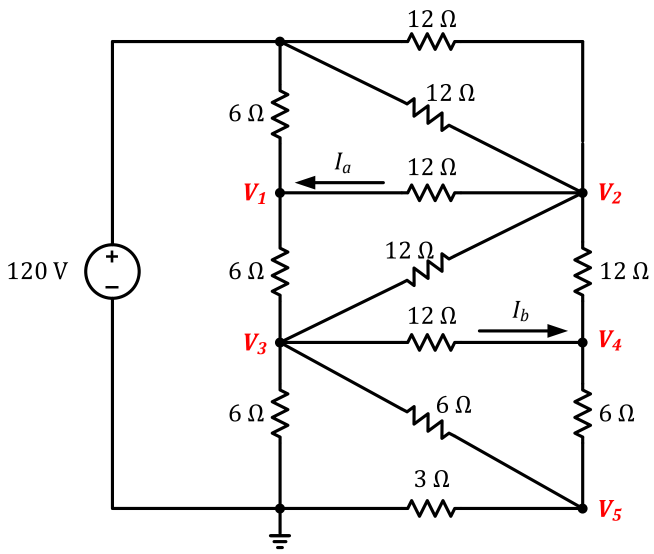
From all equations, $V_1 = 220\text{ V}, V_2 = 150\text{ V}, V_3 = 66\text{ V}$ $(1\ points)$

$\therefore \boldsymbol{V_1 = 220\text{ V}, V_2 = 150\text{ V}, V_3 = 66\text{ V}}$

(b) (5 points) Find I_a and I_b in the circuit shown below.



We assigned the node voltage and reference node as indicated below.



By KCL,

$$0 = \frac{V_1 - 120}{6} + \frac{V_1 - V_2}{12} + \frac{V_1 - V_3}{6}$$

$$0 = \frac{V_2 - 120}{12} + \frac{V_2 - 120}{12} + \frac{V_2 - V_1}{12} + \frac{V_2 - V_3}{12} + \frac{V_2 - V_4}{12}$$

$$0 = \frac{V_3}{6} + \frac{V_3 - V_1}{6} + \frac{V_3 - V_2}{12} + \frac{V_3 - V_4}{12} + \frac{V_3 - V_5}{6}$$

$0 = \frac{V_4 - V_2}{12} + \frac{V_4 - V_3}{12} + \frac{V_4 - V_5}{6}$

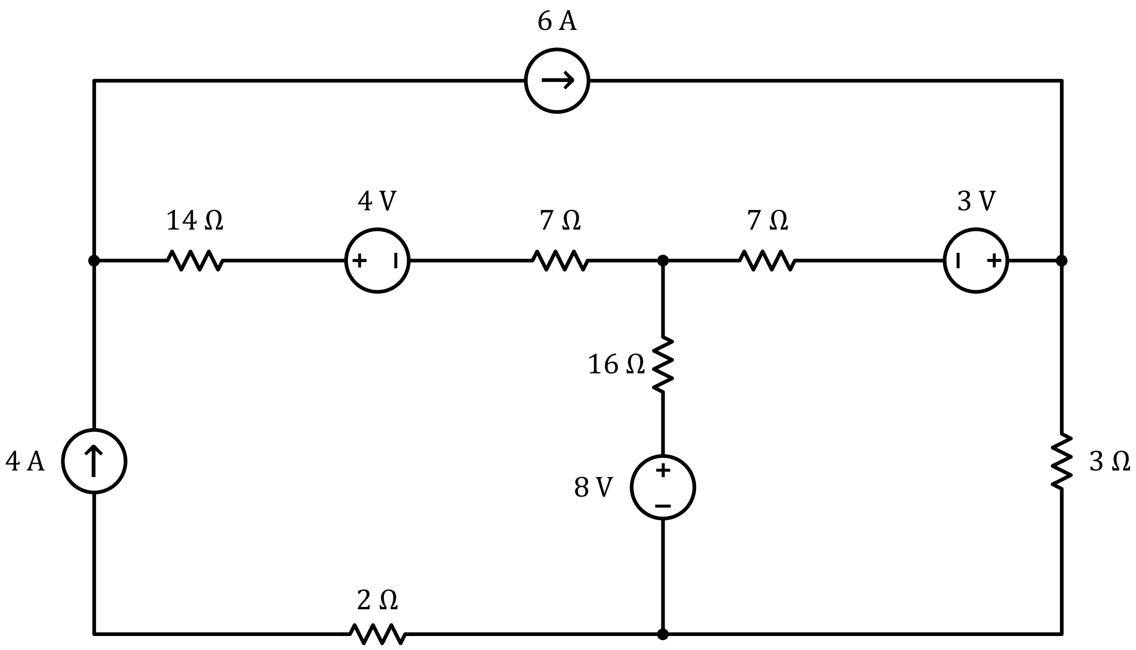
$0 = \frac{V_5}{3} + \frac{V_5 - V_3}{6} + \frac{V_5 - V_4}{6}$ $(3\ points)$

From all equations, $V_1 = 80\text{V}, V_2 = 80\text{V}, V_3 = 40\text{V}, V_4 = 40\text{V}, V_5 = 20\text{V}$ $(1\ points)$

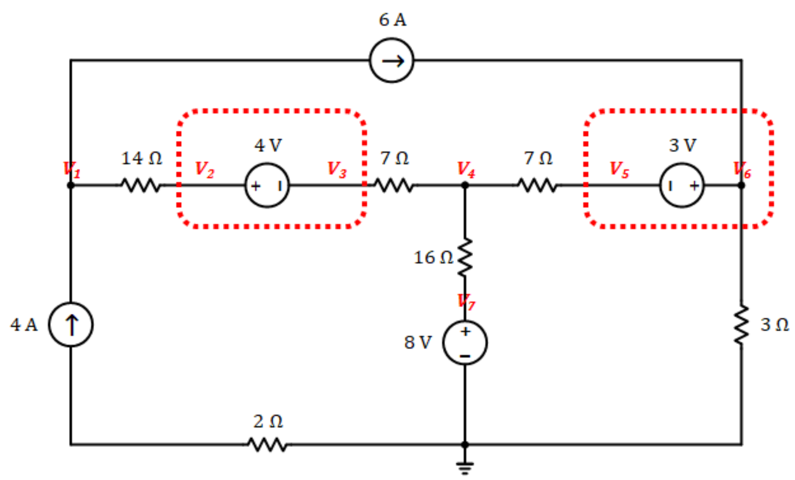
Thus, i_a and i_b are 0 A. ($\because \frac{V_2 - V_1}{12} = 0\ \text{A}, \frac{V_3 - V_4}{12} = 0\ \text{A}$) $(1\ points)$

$\therefore \boldsymbol{I_\mathbf{a} = 0\ \mathbf{A}, I_\mathbf{b} = 0\ \mathbf{A}}$

(c) (6 points) Calculate the power supplied by the 8 V source.



We assigned the node voltage and reference node as indicated below.



By KVL,

$V_2 - V_3 = 4$

$V_6 - V_5 = 3$

$V_7 = 8$ $\qquad\qquad (2\ points)$

By KCL,

$4-6=\frac{V_1-V_2}{14}$

$0=\frac{V_2-V_1}{14}+\frac{V_3-V_4}{7}$

$0=\frac{V_4-V_3}{7}+\frac{V_4-V_5}{7}+\frac{V_4-V_7}{16}$

$6=\frac{V_5-V_4}{7}+\frac{V_6}{3}$ $\quad (2\ points)$

From all equations, $V_1=-38\text{ V}, V_2=-10\text{ V}, V_3=-14\text{ V}, V_4=0\text{ V}, V_5=10.5\text{ V}, V_6=13.5\text{ V}, V_7=8\text{ V}$ $\quad (1\ points)$

Thus, the current flowing from the 8V source is 500 mA. ($\because \frac{V_7-V_4}{16}=\frac{8-0}{16}=0.5\text{ A}$)

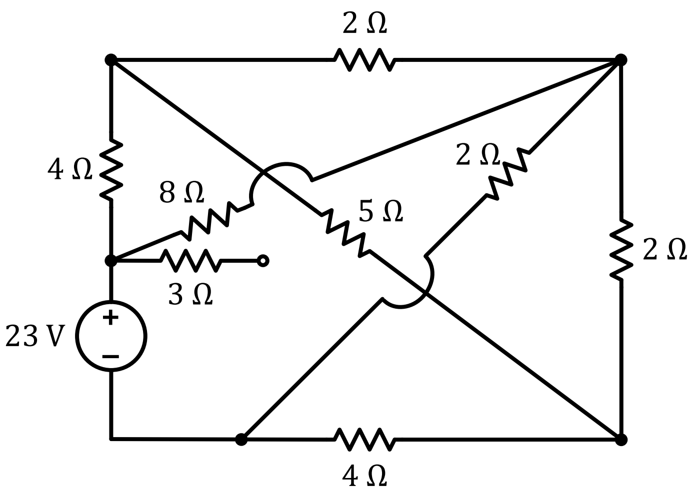
As a result, the power supplied by the 8V source is $8\text{ V}\cdot 500\text{ mA}=4\text{W}$ $\quad (1\ points)$

$\therefore$ **4W**

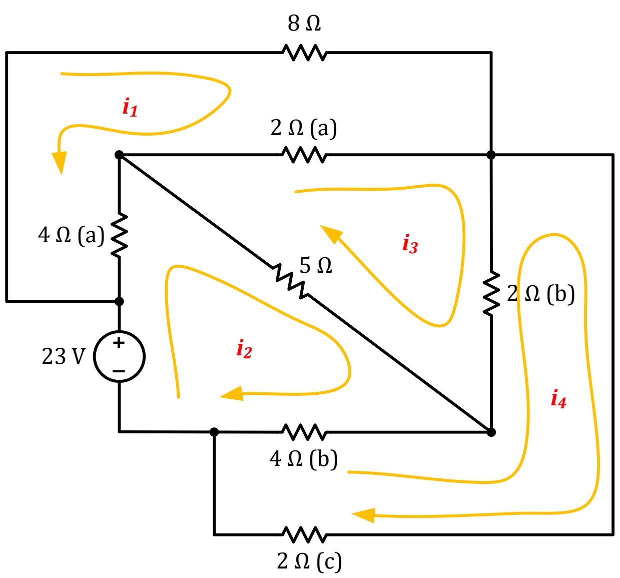
3. (14 points) Use **the loop analysis** to answer the following questions.

(Solving without loop analysis (e.g. the nodal analysis) gets -0.5 point for each prob.)

(a) **(6 points)** Calculate the power supplied by the 23 V source and verify that it exactly equals the total power absorbed in the circuit network.



We assigned the mesh current as indicated below.



By KVL,

$$8i_1 + 2 \cdot (i_1 - i_3) + 4 \cdot (i_1 - i_2) = 0$$

$-23+4\cdot(i_2-i_1)+5\cdot(i_2-i_3)+4\cdot(i_2-i_4)=0$

$2\cdot(i_3-i_1)+2\cdot(i_3-i_4)+5\cdot(i_3-i_2)=0$

$2i_4+4\cdot(i_4-i_2)+2\cdot(i_4-i_3)=0$

From all equations, $i_1=2\ \mathrm{A}, i_2=5\ \mathrm{A}, i_3=4\ \mathrm{A}, i_4=3.5\ \mathrm{A}$ (+2 points) (a)

The power absorbed by resistor:

$P_{2\Omega(\mathrm{a})}=2\cdot(i_1-i_3)^2=8\ \mathrm{W}$

$P_{2\Omega(\mathrm{b})}=2\cdot(i_3-i_4)^2=0.5\ \mathrm{W}$

$P_{2\Omega(\mathrm{c})}=2\cdot{i_4}^2=24.5\ \mathrm{W}$

$P_{4\Omega(\mathrm{a})}=4\cdot(i_1-i_2)^2=36\ \mathrm{W}$

$P_{4\Omega(\mathrm{b})}=4\cdot(i_4-i_2)^2=9\ \mathrm{W}$

$P_{5\Omega}=5\cdot(i_2-i_3)^2=5\ \mathrm{W}$

$P_{8\Omega}=8\cdot{i_1}^2=32\ \mathrm{W}$

$P_{3\Omega}=0\ \mathrm{W}$ ($\because$ one node of the 3Ω resistor is open.)

Thus, $\sum P_{absorbed}=8+0.5+24.5+36+9+5+32+0=115\ \mathrm{W}$ (+2 points) (b)

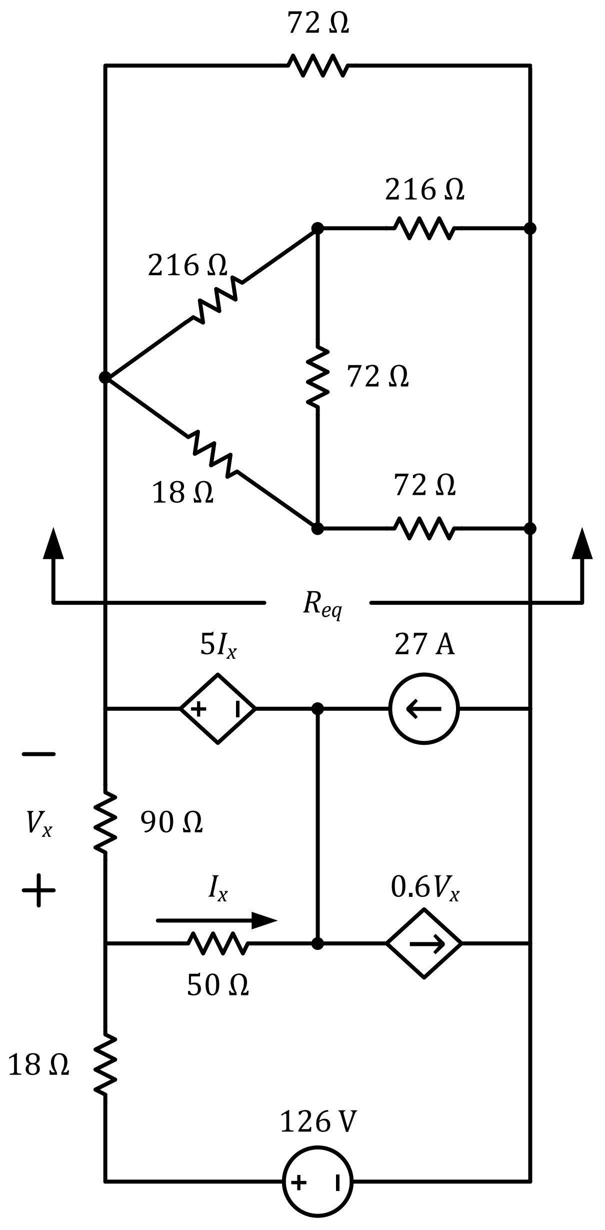
The power supplied by 23 V source: $P_{23\ \mathrm{V}}=23\cdot i_2=115\ \mathrm{W}$ (+2 points) (c)

$\therefore\ \boldsymbol{\sum P_{absorbed}=\sum P_{supplied}=115\ \mathrm{W}}$

단위 틀리면 -1 점

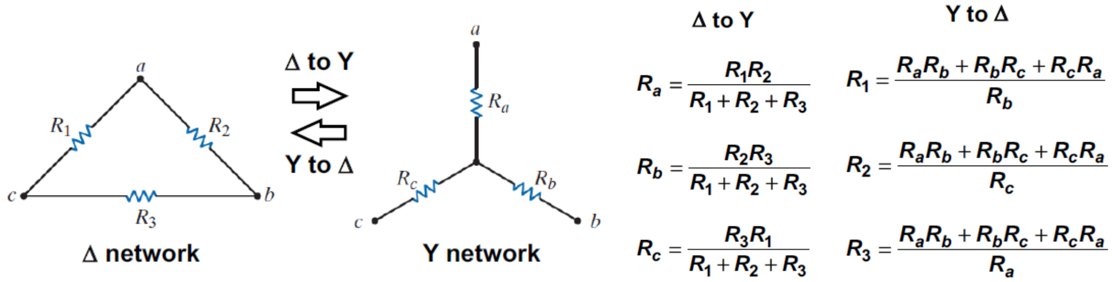
(b) **(8 points)** Find I_x, V_x and R_{eq} in the circuit shown below.

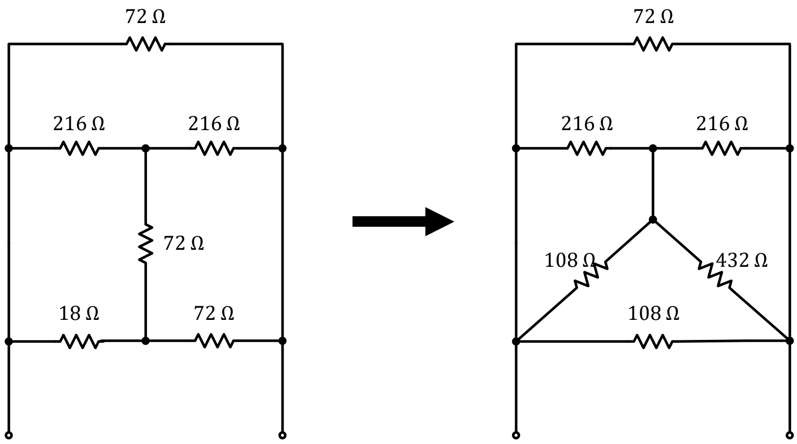
(Find R_{eq} +4 points, find I_x, V_x +2 points each))



1. Find R_{eq}

By Y-Δ transformation,

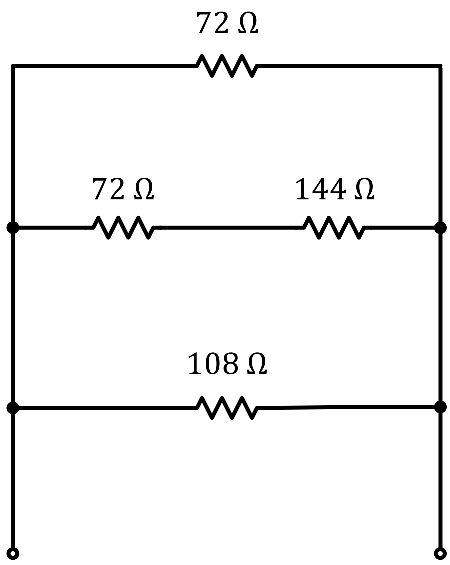




$R_1 = \frac{72\cdot72+72\cdot18+18\cdot72}{72} = 108\,\Omega$, $R_2 = \frac{72\cdot72+72\cdot18+18\cdot72}{18} = 432\,\Omega$, $R_3 = \frac{72\cdot72+72\cdot18+18\cdot72}{72} = 108\,\Omega$

(+2 points) (a)

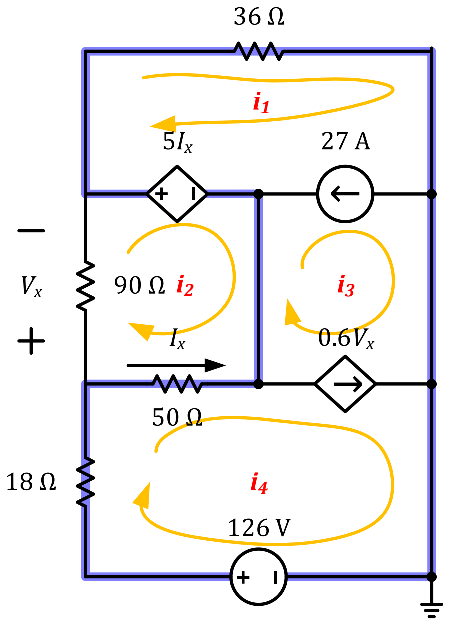
By combining parallel resistors,



Thus, $R_{eq} = 72\,\Omega \parallel 216\,\Omega \parallel 108\,\Omega = 36\,\Omega$ (+2 points) (b)

2. Find I_x and V_x

We assigned the mesh current as indicated below.



By Ohm's law, $V_x = 90i_2$

By KCL,

$i_1 - i_3 = 27$

$i_3 - i_4 = -0.6V_x$

$I_x = i_4 - i_2$

By KVL,

$90i_2 + 5I_x + 50 \cdot (i_2 - i_4) = 0$

$-126 + 18i_4 + 50 \cdot (i_4 - i_2) - 5I_x + 36 \cdot i_1 = 0$

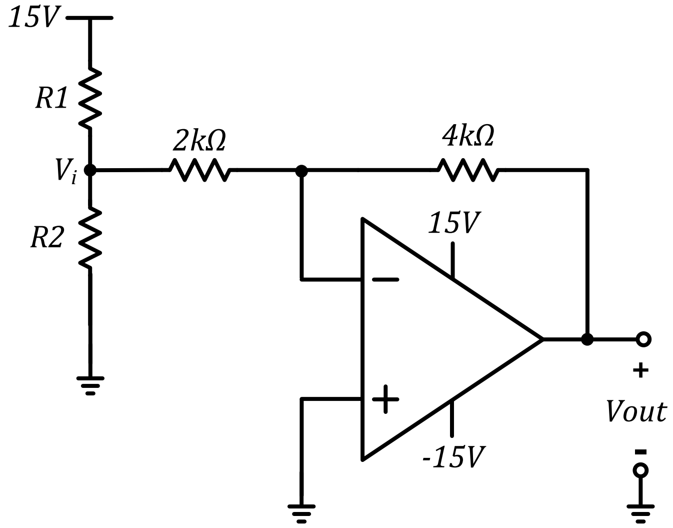
From all equations, $i_1 = 1.5\ \mathrm{A},\ i_2 = 0.5\ \mathrm{A}, i_3 = -25.5\ \mathrm{A}, i_4 = 1.5\ \mathrm{A}$

Thus, $I_x = i_4 - i_2 = 1\ \mathrm{A},$ (+2 points) (c)

$V_x = 45\ \mathrm{V}$ (+2 points) (d)

$\therefore\ \boldsymbol{I_x = 1\ \mathrm{A},\ V_x = 45\ \mathrm{V},\ R_{eq} = 36\ \Omega}$

4. (10 points) Consider the following circuit. To find v_{out}, let's consider the op-amp as ideal.

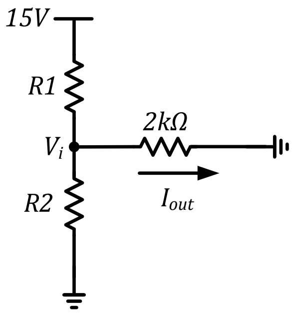


(a) (5 points) Find v_{out} of the circuit, (R_1=40Ω, R_2=10Ω)

$$V_i = \frac{R1}{R1+R2} \times 15V = 3V \;\; (+2.5 \text{ points})$$

$$V_{out} = -\frac{4k\Omega}{2k\Omega} \times 3V = -6V \;\; (+2.5 \text{ points})$$

(b) (5 points) Find v_{out} of the circuit, (R_1=8kΩ, R_2=2kΩ)

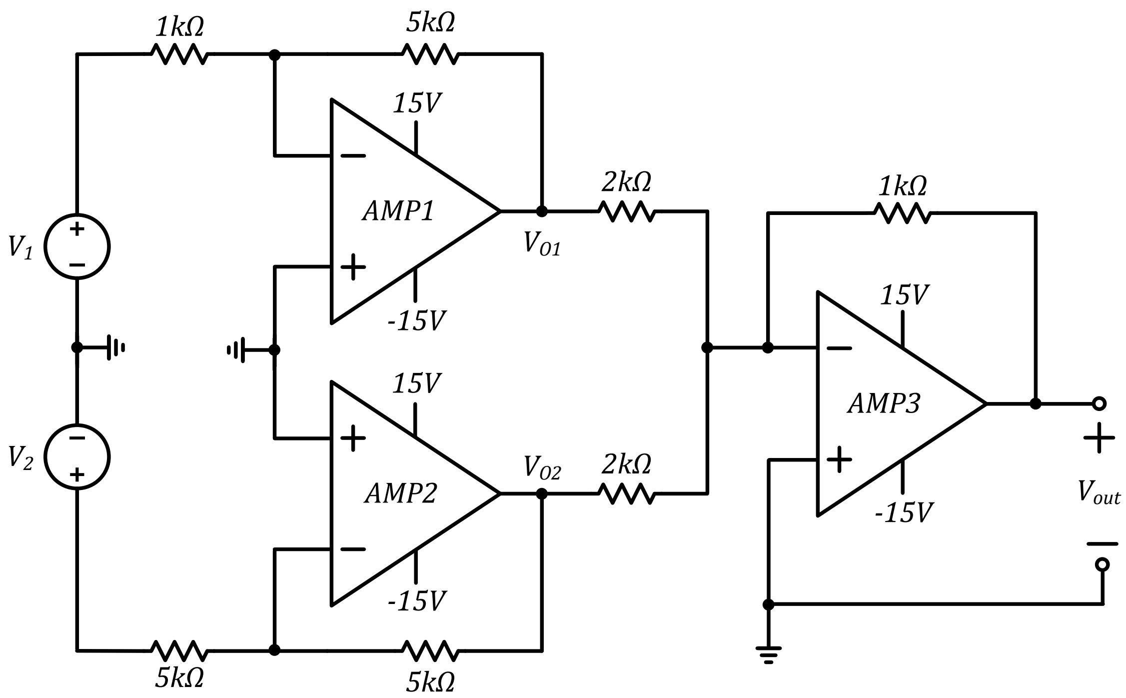


(figure for solution)

$$V_i = \left(\frac{(R2||2k\Omega)}{(R2||2k\Omega)+R1}\right) \times 15V = 1.67V$$ → V_i, I_{OUT} is not same with prob.(a) (+2.5 points)

$$V_{out} = -\frac{4k\Omega}{2k\Omega} \times 1.67V = -3.34V \;\; (+2.5 \text{ points})$$

5. (12 points) Consider the following circuit. Let's first consider the op-amp as ideal.



(a) (4 points) Find v_{out} of the circuit, (V_1=1V, V_2=3V)

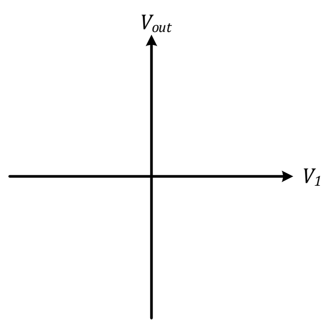
$$V_{O1} = -\frac{5k}{1k} \times 1V = -5V \;\; (+1 \text{ points})$$

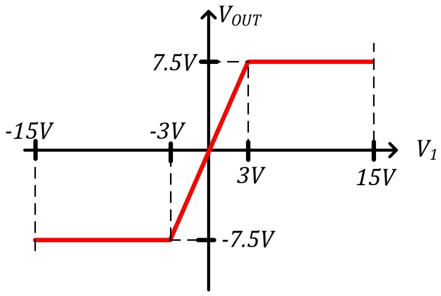
$$V_{O2} = -\frac{5k}{5k} \times 3V = -3V \;\; (+1 \text{ points})$$

$$\frac{VO1}{2k} + \frac{VO2}{2k} = -\frac{VOUT}{1k} \rightarrow V_{OUT} = 4V \;\; (+2 \text{ points})$$

(b) (8 points) Draw the V_{out} - V_1 curve in range of -15V< V_1 <15V. ~~What is the maximum and minimum V_1 value for this circuit?~~

(The output range of all Op-Amp is ±15V and V_2=0V.)





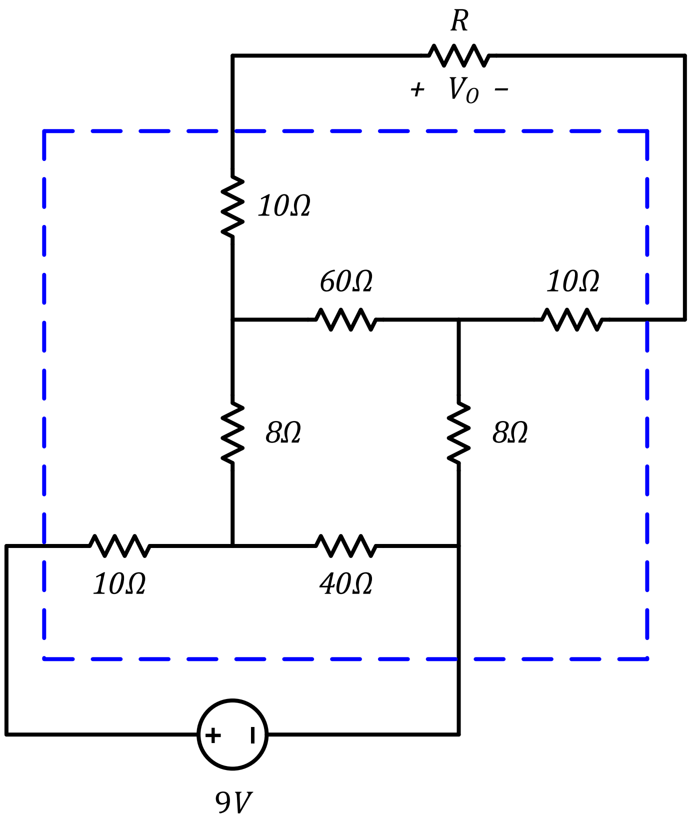
-3V<V_1<3V → $V_{OUT} = \frac{5}{2} \times V1$ (+3 points)

-3V>V_1→ $V_{OUT} = -7.5V$ (+2.5 points)

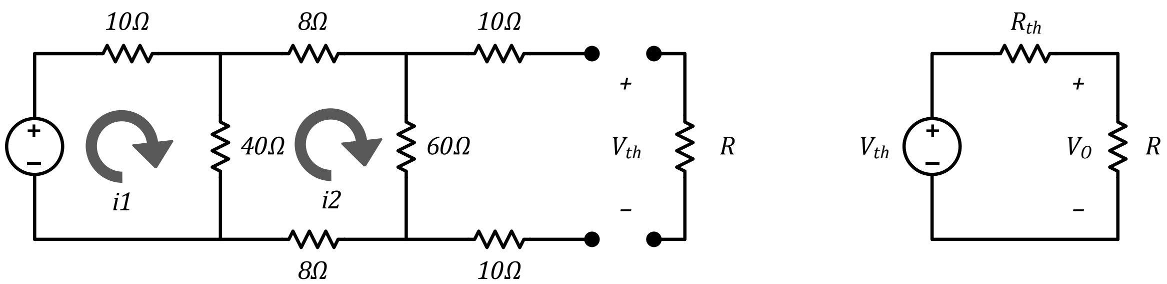
3V<V_1→ $V_{OUT} = 7.5V$ (+2.5 points)

(AMP1 is saturated at V_1=±3V)

6. (10 points) A resistance array is connected to a load resistor ***R*** and a ***9-V*** battery as shown in the figure below.



(a) (6 points) Find the value of ***R*** such that **V_O = 1.8V**



$$R_{th} = 10 + 10 + 60||(8 + 8 + 40||10) = 20 + 60||24 = 37.14\,\Omega \qquad (2\,points)$$

Using mesh analysis for figure on the left:

$$-9 + 50i1 - 40i2 = 0$$

$$116i2 - 40i1 = 0 \; or \; i1 = 2.9i2$$

$$\rightarrow i2 = 9/105$$

$$V_{th} = 60 * i2 = 5.143\,V \qquad (2\,points)$$

From figure on the right:

$$V_O = V_{th} * \frac{R}{R + R_{th}} = 1.8\,V$$

$$\rightarrow \frac{R}{R + 37.14} = \frac{1.8}{5.143}$$

$$\rightarrow R = 20\,\Omega \qquad (2\,points)$$

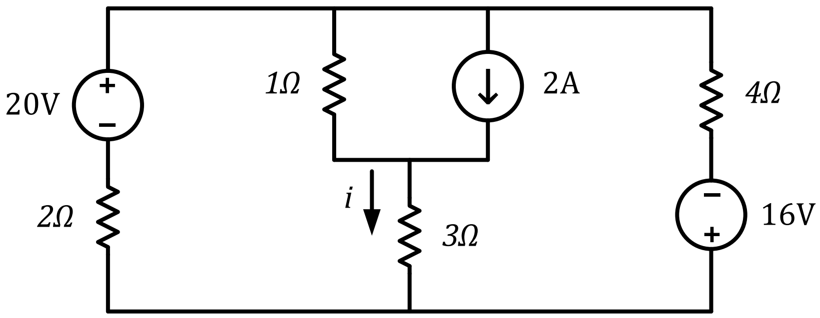
(b) (4 points) Calculate the value of ***R*** that will draw the maximum current. What is the ***maximum current***?

$$R = 0\Omega \qquad (2\,points)$$

$$(Note, R = R_{th}\ is\ a\ condition\ for\ the\ maximum\ power\ transfer\ to\ load\ R\ )$$

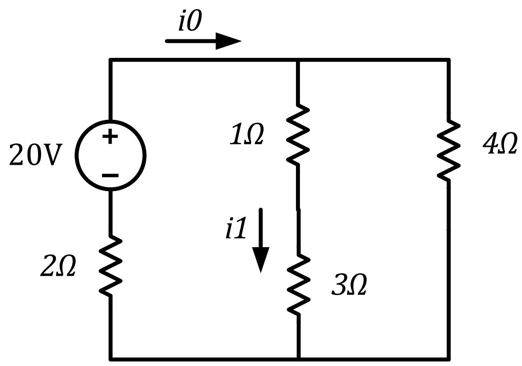
$$I\_\max = \frac{V_{th}}{R_{th}} = \frac{5.143}{37.14} = 138.48\,mA \qquad (2\,points)$$

7. (8 points) For the circuit below, use superposition to find the value of ***i***. Calculate the power delivered to the **3Ω** resistor.



Let $i = i1 + i2 + i3$, where $i1, i2$, and $i3$ are due to the $20V, 2A$, and 16V sources, respectively.

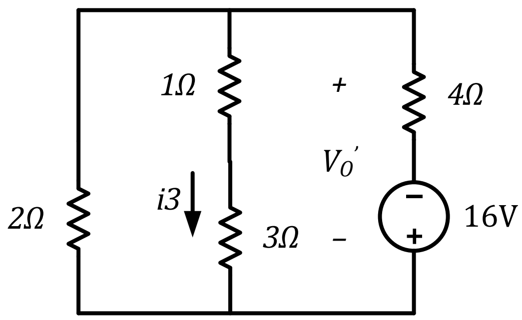
For $i1$, considering:



$$4||(3+1) = 2\ \Omega \ \rightarrow i0 = 20/(2+2) = 5\ A$$

$i1 = i0/2 = 2.5\ A$ $(2\ points)$ (a)

For $i3$, considering:

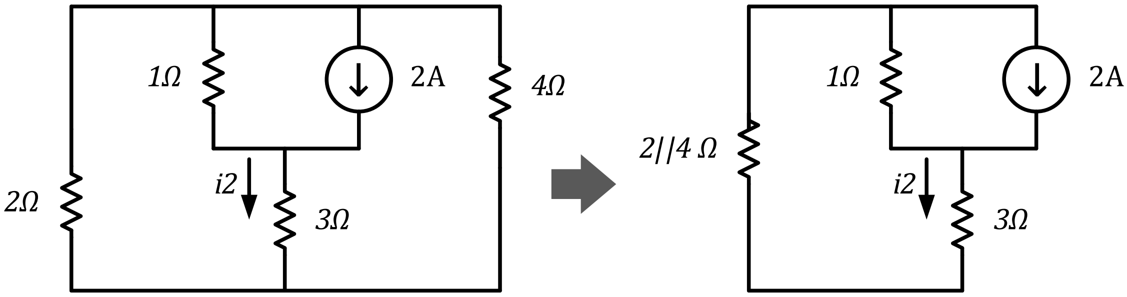


$$2||(1+3) = 4/3\ \Omega$$

$$V_0' = (-16)\frac{\left(\frac{4}{3}\right)}{\frac{4}{3}+4} = -4\ V$$

$i3 = \frac{V_0'}{4} = -1\ A$ (2 points) (c)

For $i2$, considering:



$$2||4 = 4/3\,, 3 + 4/3 = 13/3$$

$i2 = (2)\frac{1}{1+13/3} = 3/8 = 0.375\ A$ (2 points) (b)

Combining the results:

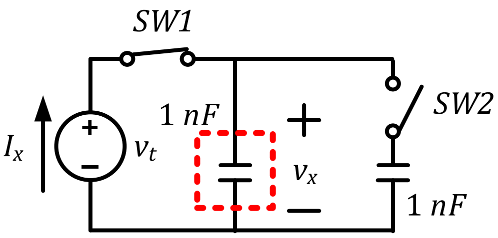
$i = i1 + i2 + i3 = 2.5 + 0.375 - 1 = 1.875\ A$ (1 points) (d)

$p = i^2R = (1.875)^2(3) = 10.55\ W$ (1 points) (e)

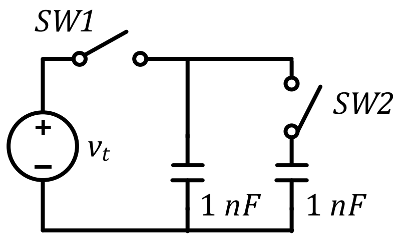
단위 틀리면 -1 점

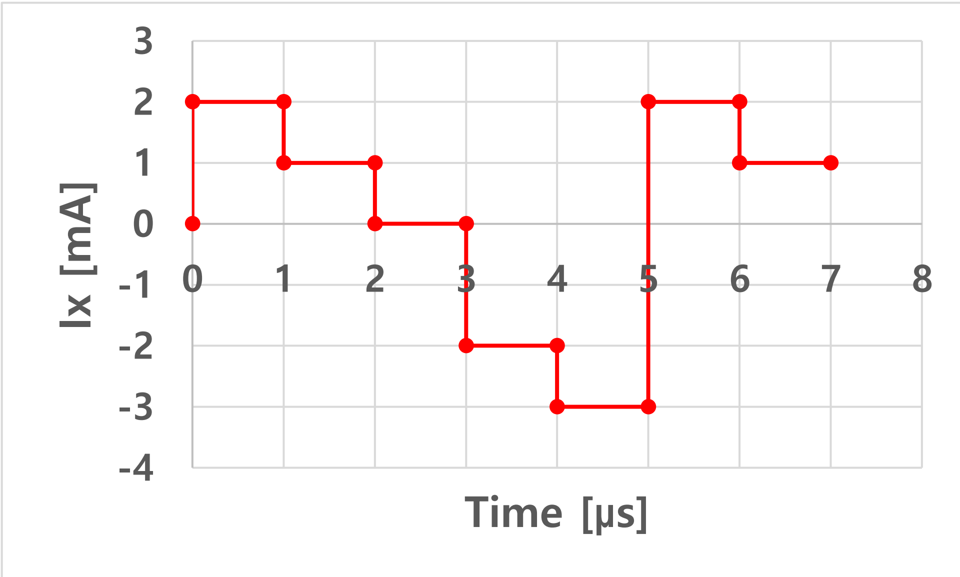
8. (15 points) Consider the following circuit. From 0 μs to 7 μs, switch 1 (SW1) is closed and switch 2 (SW2) is opened. From 7 μs to 8 μs, switch 1 (SW1) is opened and switch 2 (SW2) is opened. Answer the following questions.

$0\ \mu s \le t \le 7\ \mu s$

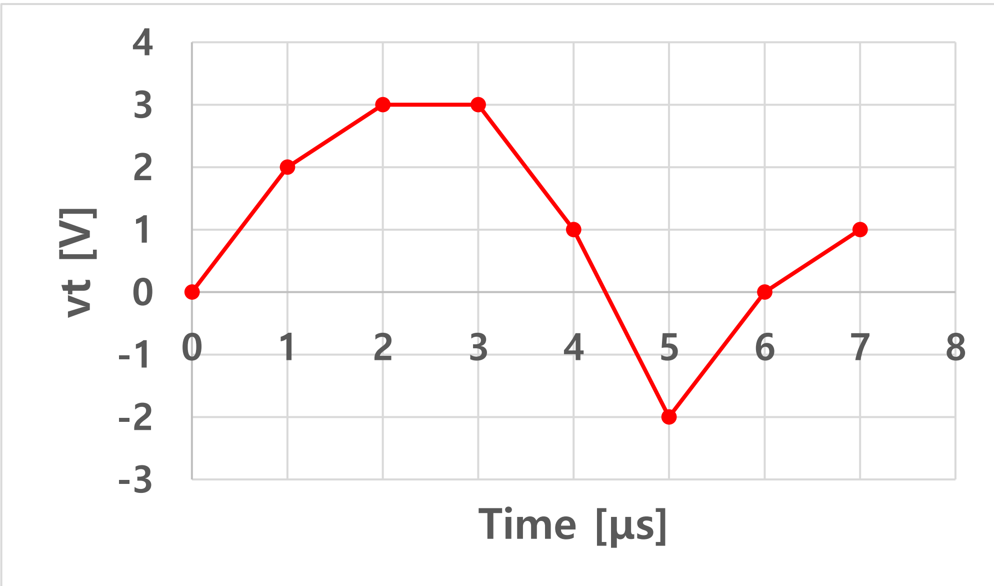


$7\ \mu s \le t \le 8\ \mu s$





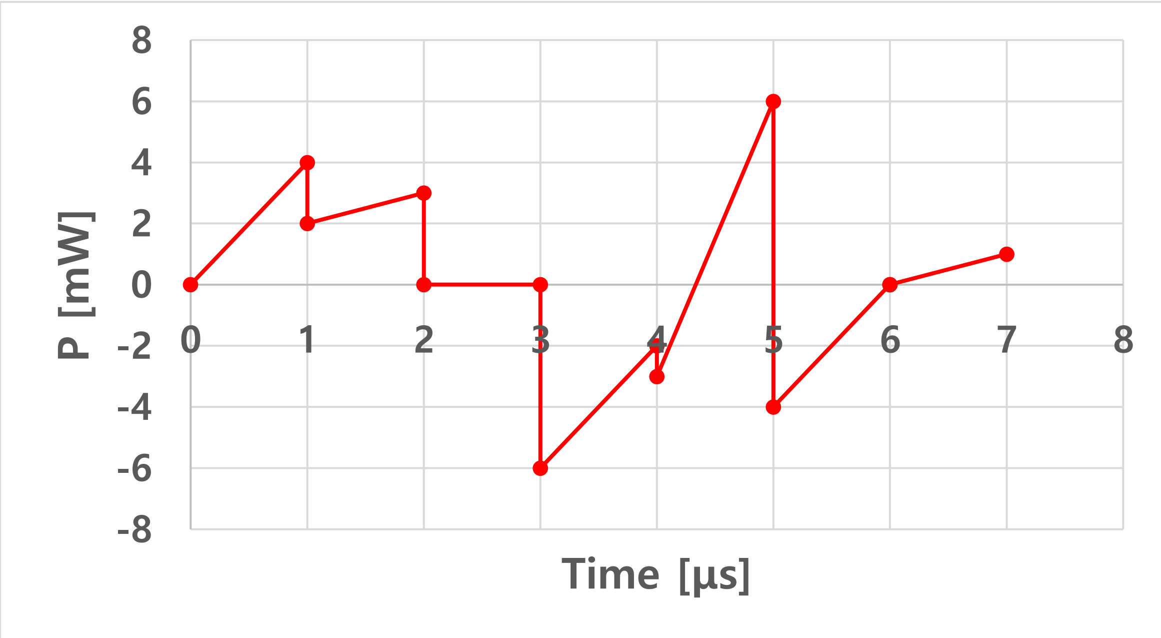
(a) (3 points) The waveform for the current $\mathrm{I_x}$ is shown in above graph. Determine the waveform for the voltage v_t. ($v_x = 1\ \mathrm{V}$ at t 7 μs)



(부분 점수 있음)

7 개의 구간 중 1 개의 구간 틀릴 때 마다 -1 점

(b) (3 points) Determine the supplying power (from 0 μs to 7 μs) of the voltage source v_t by time.



(부분 점수 있음)

7 개의 구간 중 1 개의 구간 틀릴 때 마다 -1 점

(c) (3 points) Determine the stored energy of the 1 nF capacitor (highlighted) at 7 μs, and explain Tellegen's theorem in the circuit. **(부분 점수 있음)**

At 7μs, voltage drop across capacitor 1 nF is 1 V.

$$\therefore \mathrm{E}_{\mathrm{C,7us}} = \frac{1}{2}CV^2 = \frac{1}{2} \times 1\ [nF] \times 1\ [V]^2 = 0.5\ [nJ]$$

————————(+1 점)

Integrate supplying power in (b),

$$E = 2\ [nJ]\ (0 \sim 1\mu s)$$

$$E = 2.5\ [nJ]\ (1 \sim 2\mu s)$$

$$E = 0\ [nJ]\ (2 \sim 3\mu s)$$

$$E = -4\ [nJ]\ (3 \sim 4\mu s)$$

$$E = -0.5\ [nJ]\ (4 \sim 4.333\mu s)$$

$$E = 2\ [nJ]\ (4.333 \sim 5\mu s)$$

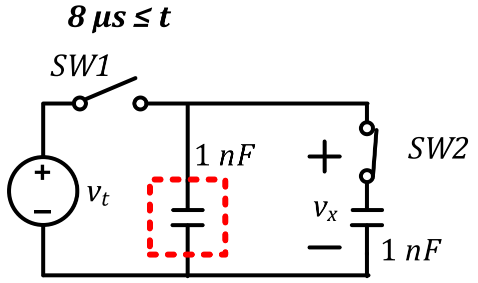
$$E = -2\ [nJ]\ (5 \sim 6\mu s) E = 0.5\ [nJ]\ (6 \sim 7\mu s)$$

$$\therefore \sum E = \sum E_C = 0.5\ [nJ] = \mathrm{E}_{\mathrm{C,7us}}$$

————————(+2 점)

회로에서 $0 \sim 7\mu s$ 시간 동안의 과정을 계산하지 않고 Tellegen's theorem 에 관해서만 언급한 경우 점수 부여 없음.

(d) (6 points) At 8 μs, switch 2 (SW2) is closed. Determine voltage across capacitor (v_x) and stored charge, energy in 1 nF capacitor (highlighted) after 8 μs.



By conservative law of charge, total charge quantity should be same.

Before closing SW2, Q = 1nC,

After closing SW2, $Q_T = Q_1 + Q_2$

$$Q_1 = 1\ [\mathrm{nF}] \times v_x, Q_2 = 1\ [nF] \times v_x, Q_T = 1[nC] = Q_1 + Q_2$$

$$\therefore 2v_x = 1, \therefore v_x = 0.5\ [V]$$

$$\therefore E_1 = \frac{1}{2}CV^2 = \frac{1}{2} \times 1\ [nF] \times 0.5^2 = 0.125\ [nJ]$$

Q1 = 0.5 nC, (+2 점)

vx = 0.5 V, (+2 점)

E1 = 0.125 nJ(+2 점)