# 10. Measuring the Earth's Magnetic Field

## OBJECTIVES

1. Determination of the components of the earth's magnetic field
2. Determination of the inclination angle of the earth's magnetic field

## PRINCIPLES

When circular induction coil with $N$ turns and an area of $A = \pi \cdot R^2$ rotates at a constant angular velocity $\omega$ in a homogenous magnetic field $B$ around its diameter $d$ as an axis, it is permeated by a magnetic flux

$$\Phi = \pi R^2 NB \cos(\omega t), \tag{1}$$

where $\omega$ is the angular velocity, $R$ is the radius of the induction coil, and $N$ is the turns of the induction coil. *Eq.* (1) assumes that the axis of rotation is perpendicular to the magnetic field $B$. The magnetic field $B$ can be determined from the amplitude of the induced voltage $U$ by

$$U = -\frac{d\Phi}{dt} = -\pi R^2 NB\omega \sin(\omega t). \tag{2}$$

Using the revolution time $T = 2\pi / \omega$, we find the following for the peak value of the induced AC voltage:

$$\widehat{U} = \frac{2\pi^2 NR^2}{T} B = aB, \tag{3}$$

$$a = \frac{2\pi^2 NR^2}{T}. \tag{4}$$

For a rotation of the coil around the z-direction of a Cartesian coordinate system (*Fig. 1*), the voltage amplitude

$$U_z = a\sqrt{B_x{}^2 + B_y{}^2} \tag{5}$$

is induced in the earth's magnetic field

$$\vec{B} = \begin{pmatrix} B_x \\ B_y \\ B_z \end{pmatrix}. \tag{6}$$

For reasons of symmetry, the following equations apply for orientations in the *x*- or *y*-direction :

$$U_x = a\sqrt{B_y{}^2 + B_z{}^2}, \tag{7}$$

$$U_y = a\sqrt{B_z{}^2 + B_x{}^2}. \tag{8}$$

The components of the earth's magnetic field can be calculated by resolving the system of *Eqs*. (5), (7), and (8):

$$B_x = \sqrt{\frac{-U_x{}^2 + U_y{}^2 + U_z{}^2}{2a^2}}, \tag{9}$$

$$B_y = \sqrt{\frac{U_x{}^2 - U_y{}^2 + U_z{}^2}{2a^2}}, \tag{10}$$

$$B_z = \sqrt{\frac{U_x{}^2 + U_y{}^2 - U_z{}^2}{2a^2}}. \tag{11}$$

For the total value of the earth's magnetic field, we obtain:

$$B = \sqrt{B_x{}^2 + B_y{}^2 + B_z{}^2} = \sqrt{\frac{U_x{}^2 + U_y{}^2 + U_z{}^2}{2a^2}}. \tag{12}$$

The angle of the dip $\theta$ of the earth's magnetic field can be obtained by the following relation:

$$\tan\theta = \frac{B_z}{\sqrt{B_x{}^2 + B_y{}^2}} = \sqrt{\frac{U_x{}^2 + U_y{}^2 - U_z{}^2}{2U_z{}^2}}. \tag{13}$$

## APPARATUS

| | |
|---|---|
| 1 Pair of Helmholtz coils | 555 604 |
| 1 Sensor CASSY | 524 010 |
| 1 µV-box | 524 040 |
| 1 CASSY Lab | 524 200 |
| 1 Connecting lead φ 2.5 mm$^2$, 200 cm, red | 501 35 |
| 1 Connecting lead φ 2.5 mm$^2$, 200 cm, blue | 501 36 |
| 1 Experiment motor | 347 35* |
| 1 Control unit for experiment motor | 347 36* |
| 1 Bench clamp with pin | 301 05* |
| *additionally recommended | |

In this experiment, the axis of rotation of the induction coil is successively aligned in the x-, y-, and z-direction of a rectangular coordinate system. The amplitude of the induced voltage is measured as a function of time with CASSY in each case. From the recorded signals, the amplitude and frequency is used to determine the strength and angle of the dip of the earth's magnetic field.

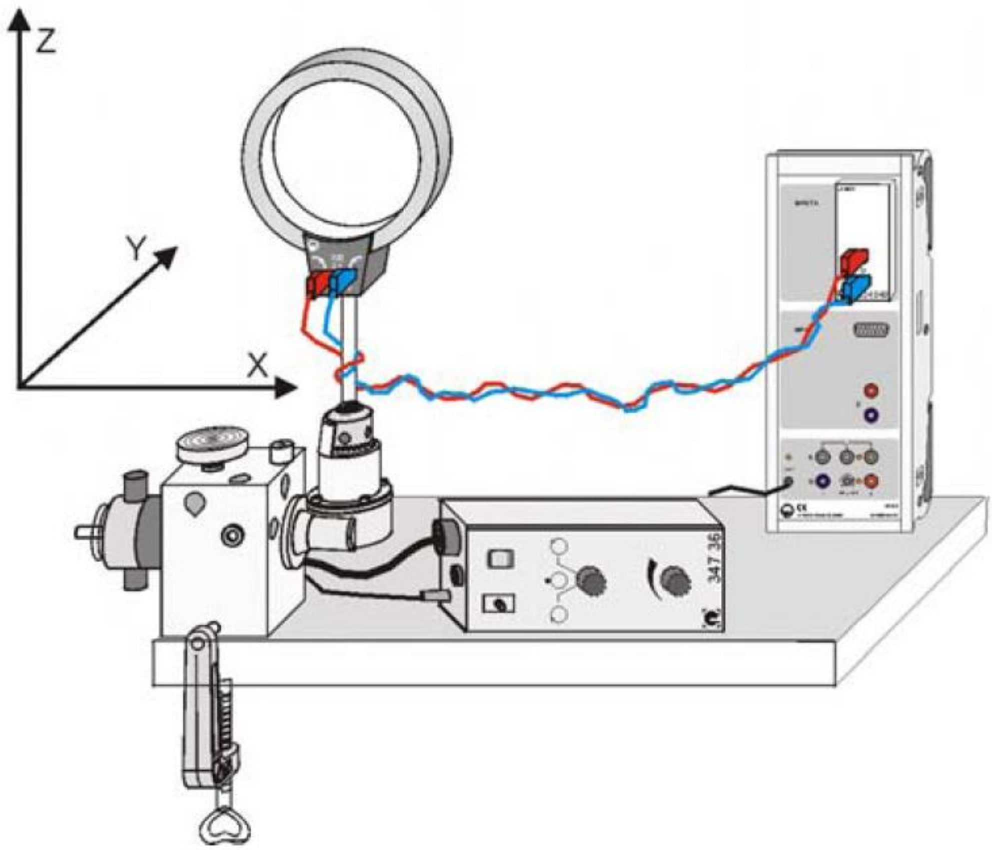


*Fig. 1.* Experimental setup with experiment motor (schematically).

## SETUP

1. Place the experiment motor on the corner of the bench, as shown in *Fig. 1*, so that an alignment in three directions (*x*, *y*, *z*) is possible.
2. Use two 2 m long twisted connecting leads for the connection between the μV-box and the induction loop.

## CARRYING OUT THE EXPERIMENT WITH THE MOTOR

1. Load the CASSY example file "earth magnetic field." Note: This file is not listed in the CASSY experiment example files. It has to be loaded from the hard disk by using the button or pressing the function key F3.
2. Clear the example data by using the button or pressing the function key F4.
3. Start the measurement of the induced voltage as function of time by pressing the function key F9 or using the button alternatively.
4. Rotate the induction coil manually around the *z*-direction.
5. **Note**: The measurement stops automatically after 20 s. For details of the measurement parameters press two times to see the settings in the menu "measuring parameter."
6. Repeat the measurement for the rotation about the *y*- and *x*-directions.

## MEASURING EXAMPLE

In *Fig. 2*, as an example, the voltage curve for the rotation around the x-axis is shown. For the result of the rotation around the *y*- and *z*-axes see *Fig. 5* and *Fig. 6*.

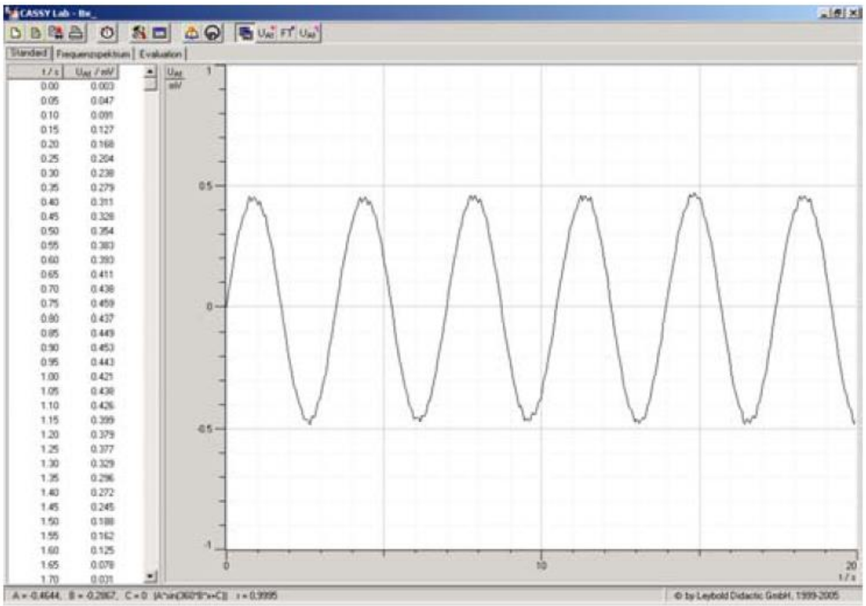


*Fig. 2.* Induced voltage $U_x$ as a function of time(*z*-direction = axis of rotation).

## EVALUATION AND RESULTS

To determine the components of the earth’s magnetic field, the frequency and the amplitude have to be predetermined.

### I. Determining the frequency

<METHOD #1>

1. Click the right mouse button in the display and choose “Set Marker/Measure Difference” (*Fig. 3*).
2. Click e.g. on zero voltage position and repeat this at the position after period $T$.
3. Alt T allows you to display the result of the status line in the display (*Fig. 3*).

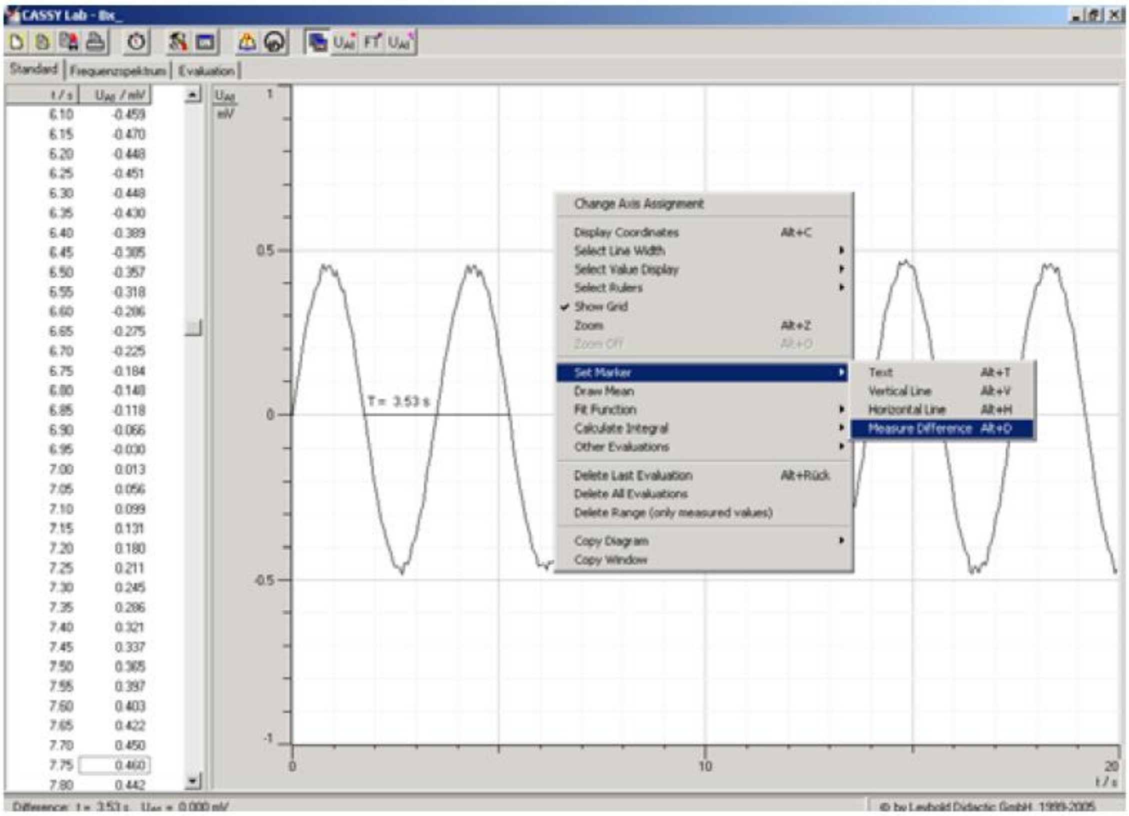


*Fig. 3.* Determination of the frequency (method #1). Compare *Fig. 4* and *Fig. 5*.

<METHOD #2>

1. Click on the tab “Frequency Spectrum”
2. Click the right mouse button in the display and choose “Other Evaluations” / “Calculate Peak Center” (*Fig. 4*).
3. Mark the peak by dragging the mouse over the experimental data.
4. Alt T allows you to display the result of the status line in the display (*Fig. 4*).

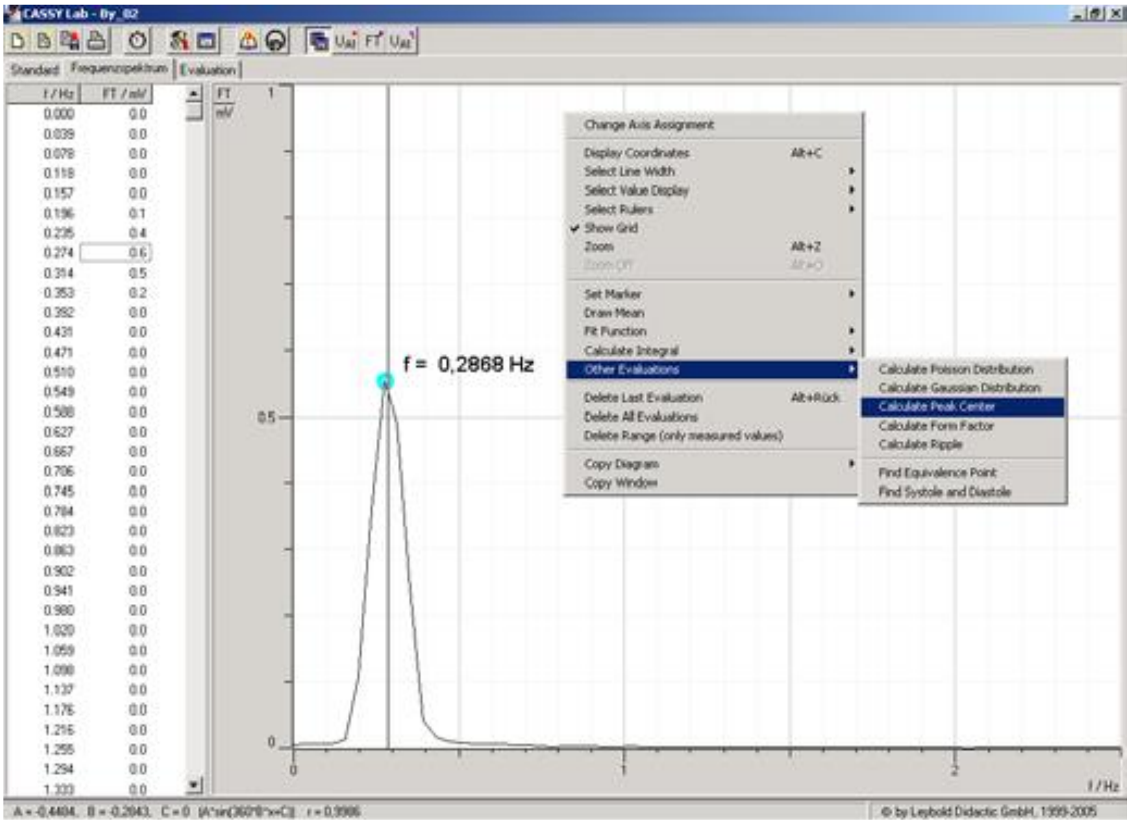


*Fig. 4.* Determination of the frequency (method #1). Compare *Fig. 3* and *Fig. 5*.

## II. Determining the amplitude of the induced voltage

<METHOD #1>

1. Click the right mouse button in the display and choose “Set Marker/Measure Difference.”
2. Click e.g. on a zero voltage position and repeat this at a position of maximum voltage.
3. Alt T allows you to display the result of the status line in the display.

<METHOD #2>

1. This method uses the fitting tool to determine the frequency and amplitude.
2. Invoke the fitting tool (*Fig. 5*) by clicking the right mouse button in the display and choosing “Fit Function” / “Free Fit” or use the speed up key, Alt F.
3. First select the appropriate fit function - here:

$$f(x, A, B, C, D) = A * \sin(360 * B * x + C) \tag{14}$$

4. Enter appropriate estimate values (starting values) for the fit parameter (*Fig. 6*).
   A = U0 ≈ 1 mV (amplitude − read from y-axis of the diagram)
   B ≈ 0.3 Hz (frequency − about 1 Hz)
   C = 0 (phase shift − zero due to measuring conditions)
   D = 0 (additional fit parameter − not used)

5. Select “Display result automatically as a new channel”
6. Continue with the “Continue with Range Marking” button.
7. Alt T allows you to display the result of the fit quickly in the display.

**Note:** The fit algorithm needs to have good estimate values to perform the fit successfully. For this fit model, i.e. *Eq.* (14), the initial value of the frequency has to be chosen close to the experimental value.

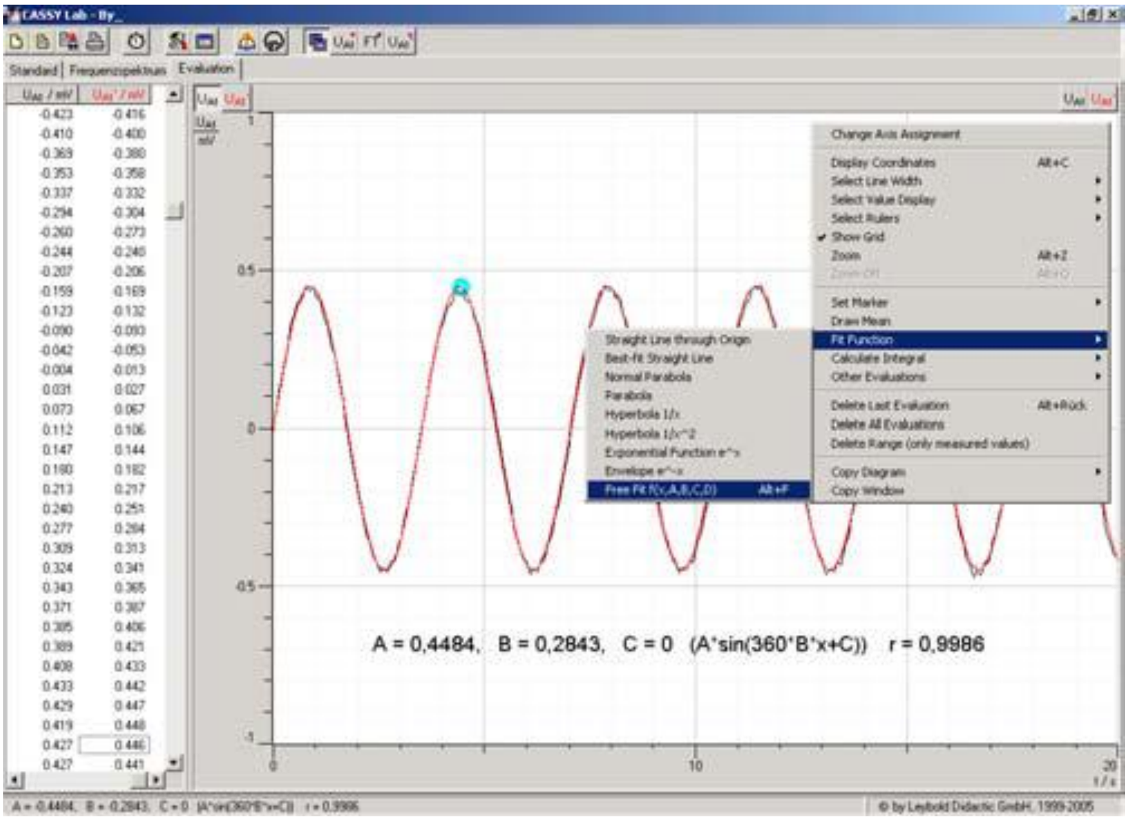


*Fig. 5.* Induced voltage $U_y$ (black line) as function of time (y-direction = axis of rotation). The red line corresponds to a fit according to the parameter listed below the voltage curve. Note: The fitting tool can be invoked by clicking the right mouse button in the display and choosing “Fit Function” / “Free Fit.”

*Fig. 5* and *Fig. 6* show a fit to the measured voltage values. Within the error limit, the sinusoidal form is verified nicely. For further data evaluation tools of CASSY Lab, refer to the supplementary information.

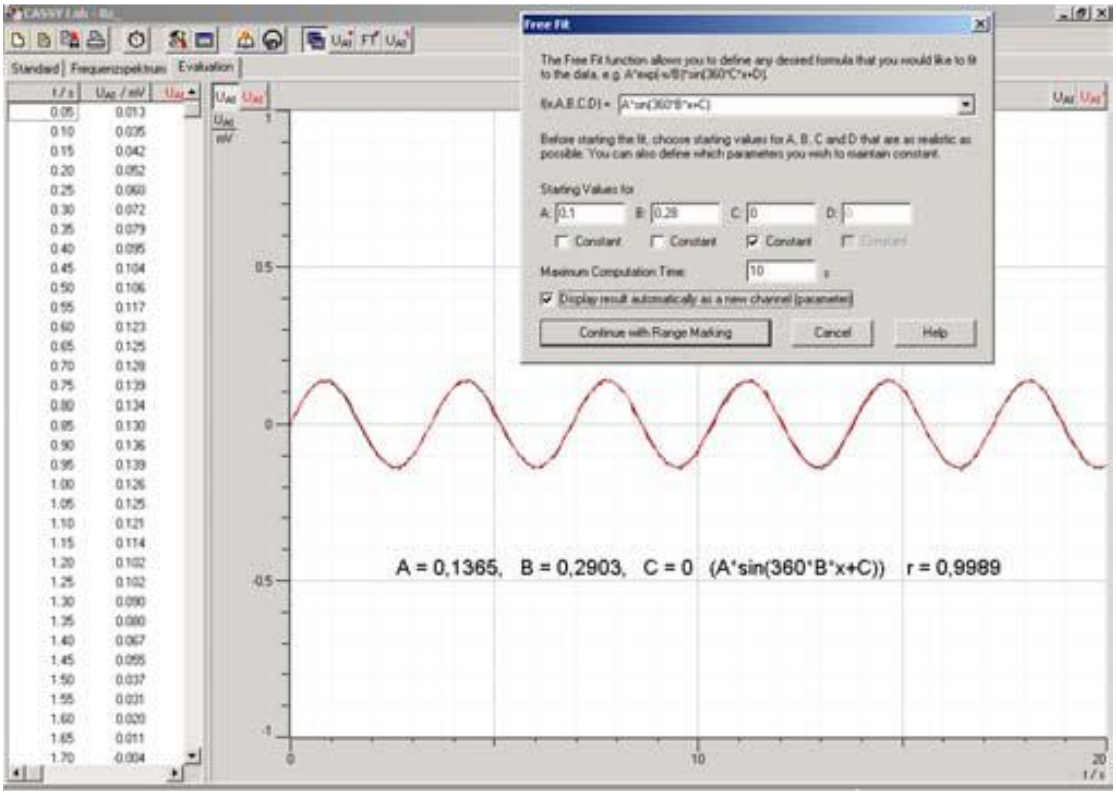


*Fig. 6.* Induced voltage $U_z$ (black line) as function of time (z-direction = axis of rotation). The red line corresponds to a fit according to the parameter listed below the voltage curve. Note: The fitting tool can be invoked by the speed up key, “Alt-F”.

The results of the fit for the three different axes of rotation are summarized in the following table:

| $\frac{U_x}{\mathrm{mV}}$ | $\frac{U_y}{\mathrm{mV}}$ | $\frac{U_z}{\mathrm{mV}}$ | $\frac{T}{\mathrm{s}}$ |
|---|---|---|---|
| 0.47 | 0.45 | 0.13 | 3.51 |

Table 1. Components of the induced voltage obtained by fitting *Eq.* (14) to the experimental data. The period corresponds to the mean average over the axes of rotation.

With the coil parameters $d$ = 13.5cm and $N$ = 320, the strength of the earth’s magnetic field can be determined using *Eqs.* (9) *and* (12):

$$B = \sqrt{\frac{0.47^2 + 0.45^2 + 0.13^2}{2 \cdot 8.2^2}} = 73.9\mu\mathrm{T} \qquad (15)$$

Using *Eq.* (13), the angle of the dip is calculated as follows:

$$\tan\theta = \sqrt{\frac{0.47^2 + 0.45^2 - 0.13^2}{2\cdot(0.13)^2}} = 3.48 \quad . \tag{16}$$

The obtained result can be checked easily by using the dip circle. The value obtained using the dip circle is $\theta$ = 70°.

## SUPPLEMENTARY INFORMATION

The main source of error in the experiment is the distortion of the magnetic field due to magnetized steel parts near the conductor loop. To achieve the maximum measuring accuracy, the coil parameters should be as large as possible.

The magnitude of the magnetic field on earth where no inclination occurs (magnetic equator) is about 31.2 $\mu$T and about a factor of 2 larger at the magnetic poles.