EE201 Circuit Theory
KAIST, Fall 2013

Dec. 20, 2013

학번(ID):________________ 이름(Name):________________

Final Exam (2.5 hours)   Assume all initial conditions are zero unless otherwise noted.

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1. [Phasor and Complex numbers] [**12pts**]

(a) Suppose $v(t) = \cos(10^3 t + 30^o)$. Let's represent this signal as $\mathbf{V} = 1\angle 0^o$. If $\mathbf{I} = 0.5\angle 45^o$, then what is $i(t)$? [2pts]

sol) $i(t) = 0.5\cos(10^3 t + 75^o)$

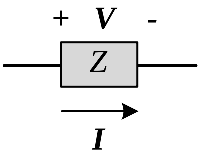
(b) Suppose $v(t) = \cos(10^3 t + 30^o)$. Let's represent this signal as $\mathbf{V} = 1 + j$. If $\mathbf{I} = 1 - j$, then what is $i(t)$? [2pts]

sol) $i(t) = \cos(10^3 t - 60^o)$

(c) Suppose $v(t) = \cos(10^3 t + 30^o)$ and $i(t) = \cos(2\times 10^3 t + 60^o)$. If $v(t)$ is represented as $\mathbf{V} = 1\angle 30^o$, then what is $\mathbf{I}$? [2pts]

sol) $\mathbf{I}$ cannot be represented as phasor, since $v(t)$ and $i(t)$ operating at different frequency.

(d) Suppose V=1+j and I=1–j. What is the average power consumption? [2pts]



sol)

$$\mathrm{V} = \sqrt{2}\angle 45°, \mathrm{I} = \sqrt{2}\angle -45°$$

$$\mathrm{P} = \frac{\sqrt{2}\sqrt{2}}{2}\cos\big(45° - (-45°)\big) = 1$$

(e) Suppose V and Z are complex numbers. To calculate the average power consumption in Z, what should be done? [2pts]

① $P_Z = \mathrm{Re}\left\{\frac{V\cdot V}{Z}\right\}$ ② $P_Z = \mathrm{Re}\left\{\frac{V\cdot V^*}{Z}\right\}$ ③ $P_Z = \mathrm{Re}\left\{\frac{V\cdot V^*}{Z^*}\right\}$ ④ $P_Z = \mathrm{Re}\{V\}\,\mathrm{Re}\left\{\frac{V^*}{Z^*}\right\}$

(f) Choose all that can be derived by analyzing power in complex numbers. (i.e. V=a+bj, I=c+dj) [2pts]

① **Average power** ② Instantaneous power ③ **Apparent power** ④ **Complex power**

#1	#2	#3	#4	#5	#6	#7	#8	Total
/12	/20	/10	/12	/13	/10	/13	/10	/100

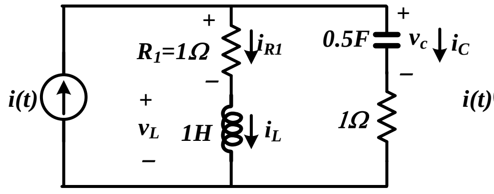
2. Consider the below RLC Circuit **[20pts]**

(a) Choose the circuits that have the same natural response. [3pts]

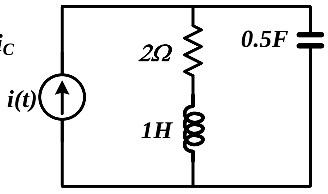
(1, 2, 4, 6, ,) have the same natural response.

(, , , , ,) have the same natural response.

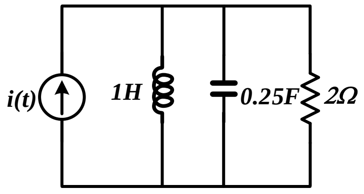
(, , , , ,) have the same natural response.



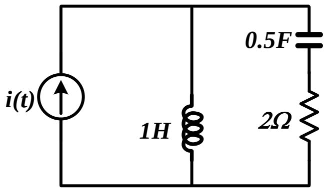
①



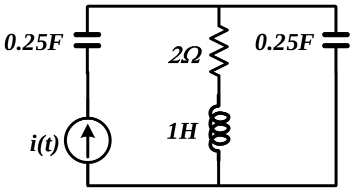
②



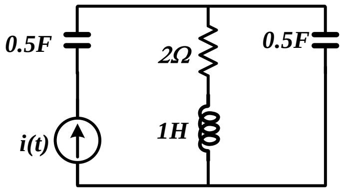
③



④



⑤



⑥

For (b) ~ (d), consider the circuit shown in ①, and suppose ***i(t) = 3u(-t) + 5u(t).***

(b) What kind of response does this circuit have? [2pts]

① Overdamped ② Underdamped ③ Critically-damped

sol)

$$0.5\frac{d^2v_c}{dt^2}+(1+1)\cdot 0.5\cdot\frac{dv_c}{dt}+v_c=0$$

$$s^2+2s+2s=0$$

(c) Please write down a general solution for $v_c(t)$. [2pts]

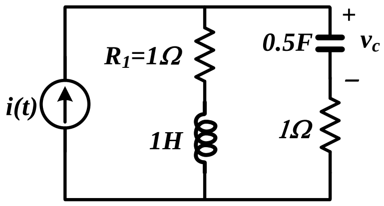
sol)

$$v_c(t)=(B_1\sin w_d t+B_2\sin w_d t)\cdot e^{-\alpha t}+C$$

(d) Find the initial values. Fill in the table below. [5pts]

$v_{R1}(0^+)$	$v_L(0^+)$	$v_C(0^+)$	$i_{R1}(0^+)$	$i_L(0^+)$
3V	2V	3V	3A	3A
$i_C(0^+)$	$\frac{dv_{R1}}{dt}(0^+)$	$\frac{dv_C}{dt}(0^+)$	$\frac{di_{R1}}{dt}(0^+)$	$\frac{di_L}{dt}(0^+)$
2A	2V/s	4V/s	2A/s	2A/s

(e) Find $v_c(t)$ for t>0 when $i(t) = 3u(-t) + 5u(t)$. [5pts]



sol)

$$w_d = 1, \quad \alpha = 1$$

$$v_c(t) = (B_1 \sin w_d t + B_2 \sin w_d t) \cdot e^{-\alpha t} + C$$

$$v_c(\infty) = 5V = C$$

$$v_c(0^+) = 3V = B_2 + C \rightarrow B_2 = -2$$

$$\frac{d}{dt} v_c(0^+) = B_1 - B_2 = 4 \rightarrow B_1 = 2$$

$$\therefore v_c(t) = (2\sin t - 2\cos t) \cdot e^{-t} + 5 \text{ [V]}$$

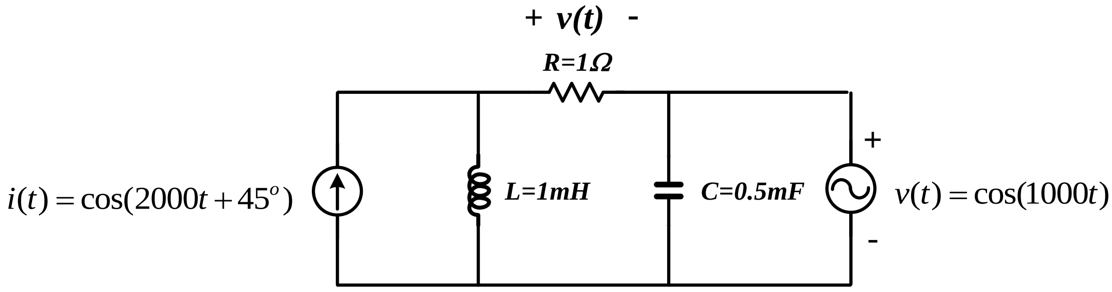
(f) If the circuit is overdamped or underdamped, change the value of C so that it is critically damped response. If the circuit is critically damped, then change the value of C so that it is overdamped. [3pts]

sol)

$$LC\frac{d^2 v_c}{dt^2} + (R+R) \cdot C \cdot \frac{dv_c}{dt} + v_c = 0$$

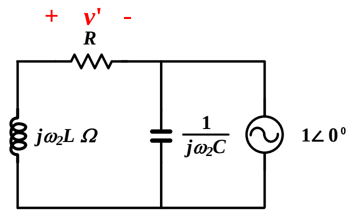
It is underdamped, and C = 1F makes it critically damped.

3. Suppose the below circuit is in the steady-state condition. [11pts]

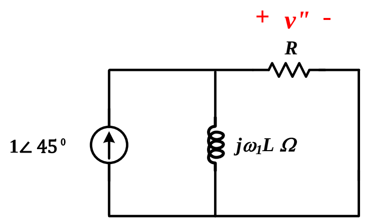


(a) Find the steady-state time-domain voltage across the resistor v(t). [6pts] (Use the function $tan^{-1}(\ )$ if necessary.)

sol)



$v' = -1\angle 0° \cdot \frac{R}{R + j\omega_2 L}$



$v'' = 1\angle 45° \cdot R \cdot \frac{j\omega_1 L}{R + j\omega_1 L}$

$$v' = -1\angle 0° \left[\frac{R}{R + j\omega_2 L}\right] = -1\angle 0° \left[\frac{1}{1+j}\right] = -1\angle 0° \left[\frac{1}{2}(1-\mathrm{j})\right] = -1\angle 0° \left[\frac{1}{\sqrt{2}}\angle \tan^{-1}(-1)\right] = -\frac{1}{\sqrt{2}}\angle -45°$$

$$= -\frac{1}{\sqrt{2}}\cos(1000t - 45°)\ (V)$$

$$v'' = 1\angle 45° \left[\frac{j\omega_1 RL}{R + j\omega_1 L}\right] = 1\angle 45° \left[\frac{2j}{1+2j}\right] = 1\angle 45° \left[\frac{2}{5}(2+j)\right]$$

$$= 1\angle 45° \left[\frac{2\sqrt{5}}{5}\angle \tan^{-1}\left(\frac{1}{2}\right)\right] = \frac{2\sqrt{5}}{5}\angle\left(45° + \tan^{-1}\left(\frac{1}{2}\right)\right) = \frac{2\sqrt{5}}{5}\angle(45° + \tan^{-1}(3))$$

$$= \frac{2\sqrt{5}}{5}\cos\left(2000t +\ 45° + \tan^{-1}\left(\frac{1}{2}\right)\right)(V) \quad \text{or} \quad \frac{2\sqrt{5}}{5}\cos(2000t + \tan^{-1}(3))\ (V)$$

by superposition

$$v(\mathrm{t}) = -\frac{1}{\sqrt{2}}\cos(1000\mathrm{t} - 45°) + \frac{2\sqrt{5}}{5}\cos\left(2000t +\ 45° + \tan^{-1}\left(\frac{1}{2}\right)\right)\ (V)$$

$$\text{or}\ \ v(\mathrm{t}) = -\frac{1}{\sqrt{2}}\cos(1000\mathrm{t} - 45°) + \frac{2\sqrt{5}}{5}\cos(2000t + \tan^{-1}(3))\ \ (V)$$

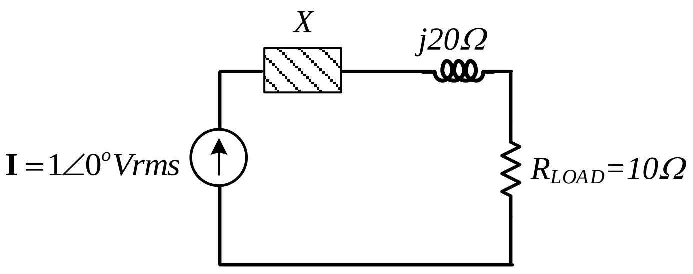
(b) What is the <u>average</u> power delivered to resistor R?

sol)

$$P = \frac{1}{2}\frac{v_{m1}^2 + v_{m2}^2}{R} = \frac{\frac{1}{2} + \frac{4}{5}}{2} = 0.65\ (W)$$

4. Consider the circuit below. [12pts]

(a) Determine the value of the device, ***X***, so that the power factor (PF) is 0.8. Give your answer in complex domain. [4pts]



sol)

$I = 1\angle 0°$

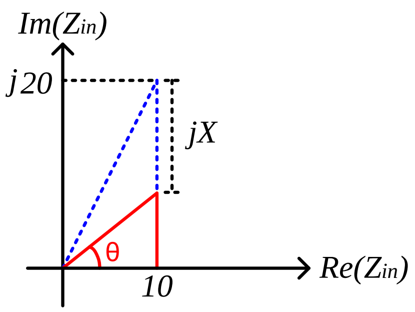
$Z_{in} = 10 + \mathrm{j}(20 + \mathrm{X})$

$$\mathrm{PF} = \frac{10}{\sqrt{10^2 + (20 - X)^2}} = 0.8$$

$\mathrm{X} = -12.5 \ \text{or} -27.5$

PF is 0.8 lagging

$\therefore \mathrm{X} = -\mathrm{j}12.5\ \Omega$



(b) If the AC source is at 100 rad/s and PF is 0.8 *lagging*, find the type and value of the device ***X***. [4pts]

sol)

$$\mathrm{X} = -\mathrm{j}12.5\ \Omega = \frac{1}{j\omega x}$$

$$\mathrm{x} = \frac{1}{1250} = 0.8\mathrm{m}$$

$\therefore$ Capacitor and 0.8mF

(c) By increasing the PF, power efficiency defined as P_{Load}/P_{SRC} is (increased, decreased, **stays the same**), where P_{load} is the average power consumption in the load resistor and P_{SRC} is the average power consumption in the source. [2pts] (1pt deducted if incorrect)

(d) How much money can you save by increasing the PF by 20% if the electric company charges you by the average power spent in your home? Give your answer in percent. [2pts]

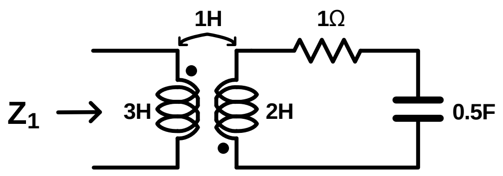
sol)

Although power factor is increased, the average power is the same.

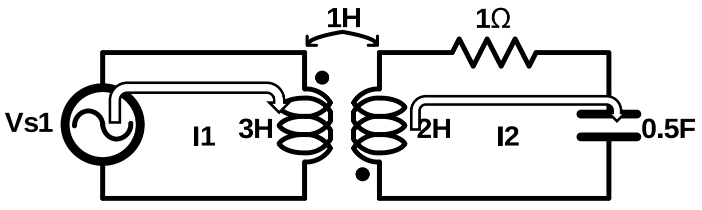
$\therefore$ 0%

5. Mutual inductance. [13pts]

(a) Find the impedance **Z**1. Assume ω = 1 rad/s. [3pts]



sol)



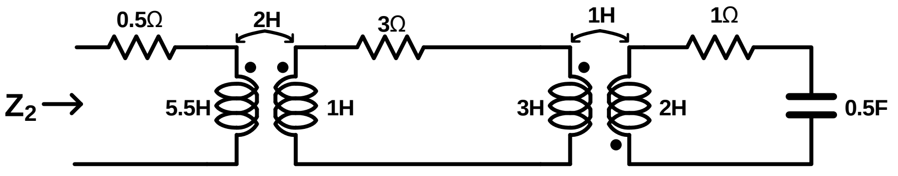
$V_{s1} = j3I_1 + jI_2$

$0 = jI_1 + (1 - j2 + j2)I_2 \Leftrightarrow 0 = jI_1 + I_2 \Leftrightarrow I_2 = -jI_1$

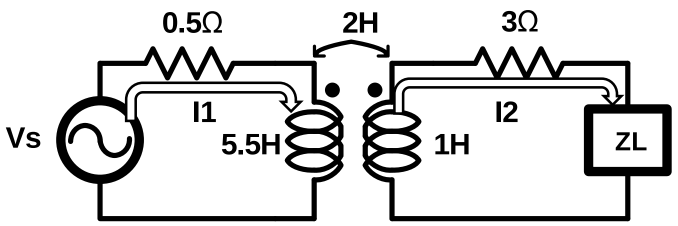
$\frac{V_{s1}}{I_1} = j3 + j \times (-j) = 1 + j3$

$\therefore \boldsymbol{Z_L = 1 + j3}$

(b) Find the impedance **Z**2. Assume ω = 1 rad/s. [3pts]



sol)



$V_s = (0.5 + j5.5)I_1 - j2I_2$

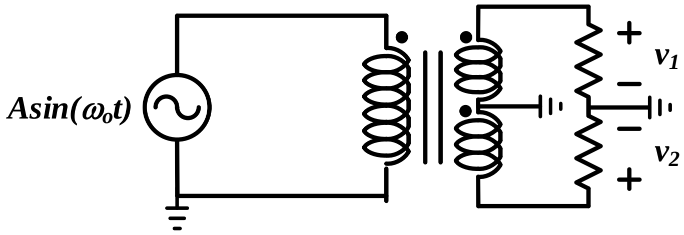
$0 = -j2I_1 + (Z_L + 3 + j)I_2 \Leftrightarrow 0 = -j2I_1 + (1 + j3 + 3 + j)I_2 \Leftrightarrow I_2 = \frac{j2}{4 + j4}I_1$

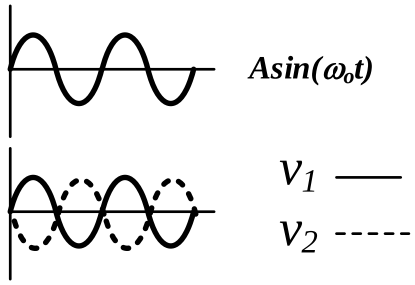
$\frac{V_s}{I_1} = (0.5 + j5.5) - j2 \times \left(\frac{j2}{4 + j4}\right) = (0.5 + j5.5) + \frac{4}{4 + j4} = 0.5 + j5.5 + \frac{1}{1 + j} = 0.5 + j5.5 + 0.5 - j0.5$

$= 1 + j5$

$\therefore \boldsymbol{Z_{in} = 1 + j5}$

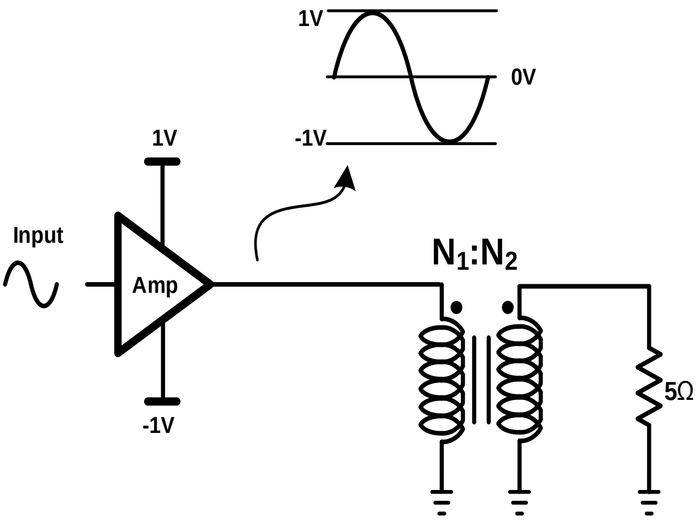
(c) Draw the waveform of v_1 and v_2. Absolute value of the amplitude is not important.[3pts]





sol) ***Any differential wave is correct***

(d) Suppose we have an amplifier shown below, whose output voltage is limited by the supply voltages of ±1V. Your job is to determine the ratio N_1:N_2 of the ideal transformer shown below, such that the average power delivered to the load is 10W for a sinusoidal output. [4pts]



sol)

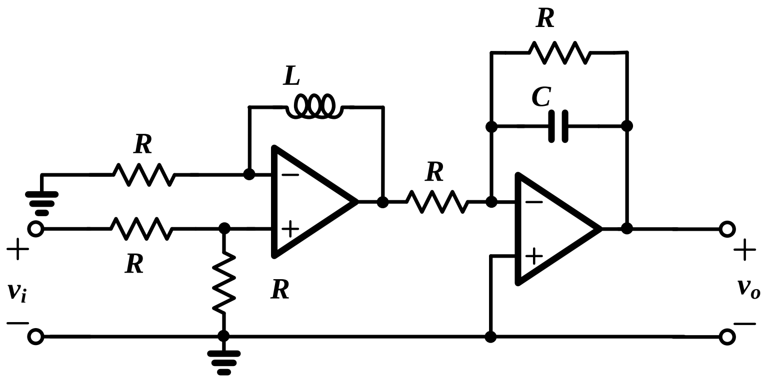
$$Power = 10W = \frac{1}{2}\times\frac{V_{peak}^2}{R} = \frac{1}{2}\times\frac{V_{peak}^2}{5} \Leftrightarrow V_{peak} = \sqrt{10\times 10} = 10\text{V}$$

$$\frac{\text{V}_{\text{peak}}}{V_{amplifier}} = \frac{N_2}{N_1} = \frac{10V}{1V} = 10$$

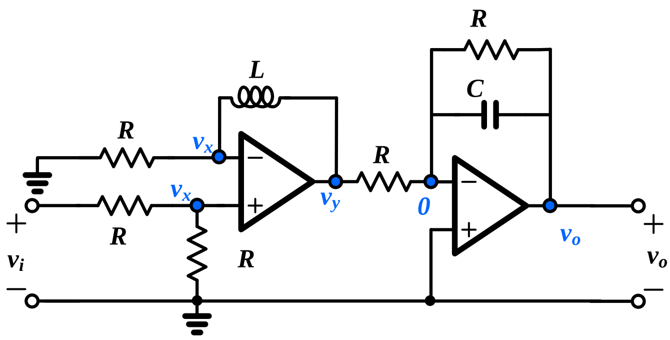
$$\textbf{ANS)}\boldsymbol{N_1 : N_2 = 1 : 10}$$

6. Analyze the below circuit in s-domain. [10pts]

(a) What is the transfer function from v_i to v_o? Express your answer in s-domain. [5pts]



Sol)



$$v_x = \frac{R}{2R} v_i \to \frac{v_x}{v_i} = \frac{1}{2} \quad \cdots \text{①}$$

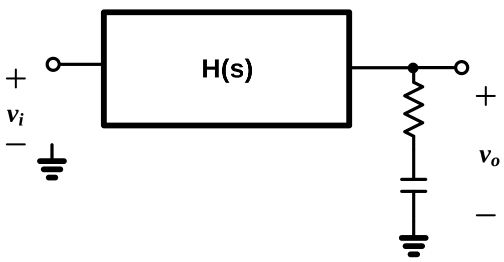
$$\frac{0 - v_x}{R} = \frac{v_x - v_y}{Ls} \to (\mathrm{R} + \mathrm{Ls}) \cdot v_x = R \cdot v_y \to \frac{v_y}{v_x} = \frac{R + Ls}{R} \quad \cdots \text{②}$$

$$\frac{v_y - 0}{R} = \frac{0 - v_o}{R \left\| \frac{1}{Cs} \right.} \to \frac{v_y}{R} = -\frac{RCs + 1}{R} v_o \to \frac{v_o}{v_y} = -\frac{1}{RCs + 1} \quad \cdots \text{③}$$

$$H(s) = \frac{v_o}{v_i} = \frac{v_x}{v_i} \cdot \frac{v_y}{v_x} \cdot \frac{v_o}{v_y} = \frac{1}{2} \cdot \frac{R + Ls}{R} \cdot -\frac{1}{RCs + 1} = -\frac{R + Ls}{2R(RCs + 1)} = -\frac{1}{2} \cdot \frac{L\left(s + \frac{R}{L}\right)}{R^2 C\left(s + \frac{1}{RC}\right)}$$

$$\therefore H(s) = -\frac{1}{2} \cdot \frac{L\left(s + \frac{R}{L}\right)}{R^2 C\left(s + \frac{1}{RC}\right)}$$

(b) Suppose there is a load at the output as shown below. As frequency is increased, how different will the gain be different from the original circuit. [2pts] (1pt deducted if incorrect)

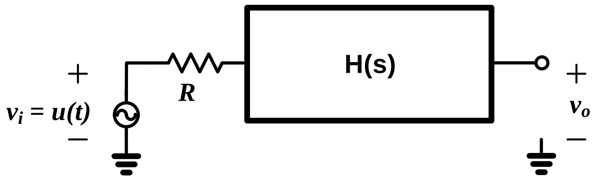


① Gain will be smaller.

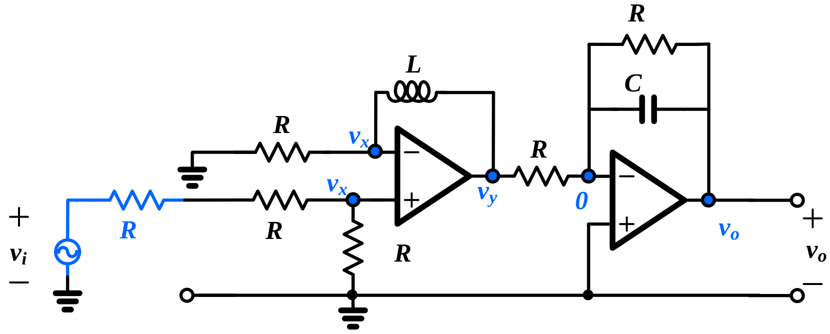
② Gain will be larger.

③ **Gain will stay the same.**

(c) What is the output voltage when the below voltage source is added. Express your answer in s-domain. [3pts]



Sol)



$$v_x = \frac{R}{3R} v_i \rightarrow \frac{v_x}{v_i} = \frac{1}{3} \quad \cdots \text{①}$$

$$\frac{0 - v_x}{R} = \frac{v_x - v_y}{Ls} \rightarrow (\mathrm{R} + \mathrm{Ls}) \cdot v_x = R \cdot v_y \rightarrow \frac{v_y}{v_x} = \frac{R + Ls}{R} \quad \cdots \text{②}$$

$$\frac{v_y - 0}{R} = \frac{0 - v_o}{R \| \frac{1}{Cs}} \rightarrow \frac{v_y}{R} = -\frac{RCs + 1}{R} v_o \rightarrow \frac{v_o}{v_y} = -\frac{1}{RCs + 1} \quad \cdots \text{③}$$

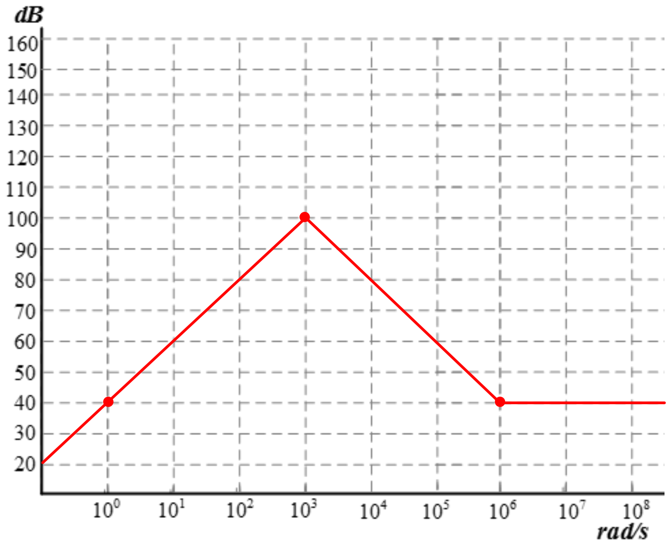
$$\frac{v_o}{v_i} = \frac{v_x}{v_i} \cdot \frac{v_y}{v_x} \cdot \frac{v_o}{v_y} = \frac{1}{3} \cdot \frac{R + Ls}{R} \cdot -\frac{1}{RCs + 1} = -\frac{R + Ls}{3R(RCs + 1)} = -\frac{1}{3} \cdot \frac{L\left(s + \frac{R}{L}\right)}{R^2 C\left(s + \frac{1}{RC}\right)} = H(s)$$

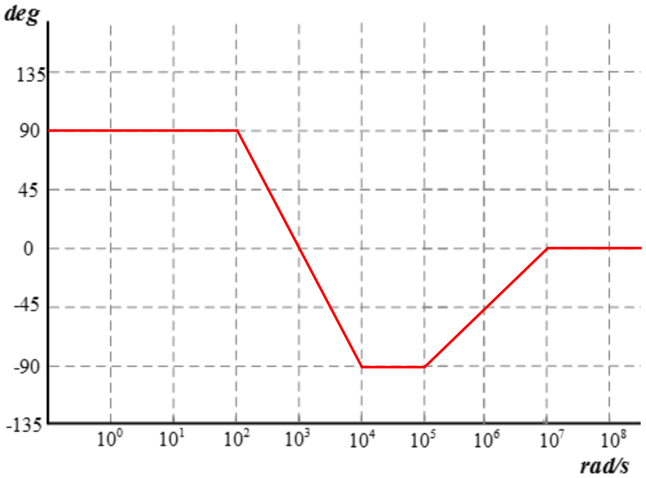
$$v_i(t) = u(t) \rightarrow v_i(s) = \frac{1}{s},$$

$$thus, v_o(s) = H(s) \times v_i(s) = -\frac{1}{3} \cdot \frac{1}{s} \cdot \frac{L\left(s + \frac{R}{L}\right)}{R^2 C\left(s + \frac{1}{RC}\right)}$$

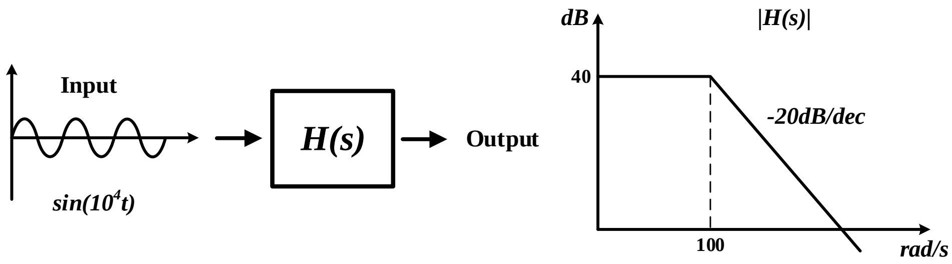
7. Frequency response [**13pts**]

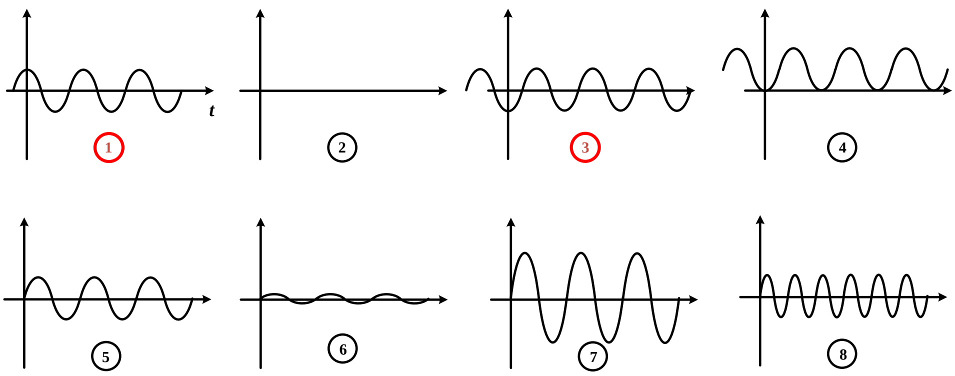
(a) Draw the asymptotic Bode plots of magnitude and phase of $H(s) = \left[100s(1+\frac{s}{10^6})\right] \Big/ \left[(1+\frac{s}{10^3})^2\right]$ [6pts]





(b) Suppose H(s) represents a circuit composed of linear elements and has the following magnitude plot. If an input of $\sin(10^4 t)$ is applied to H(s), choose which of the below responses can be achieved. Choose all that applies. [2pts]





(c) In problem (b), what is the pole frequency? What is the actual magnitude of |H(s)| at this frequency? [2pts]

Sol)

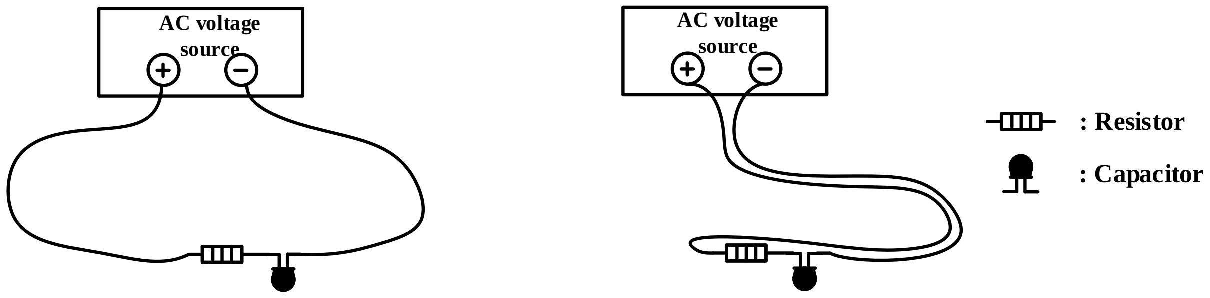
Pole frequency = 100 rad/s, magnitude of |H(s)| = 37dB

(d) Explain why H(s) =N(s)/D(s) does not blow up at the pole frequency, since by definition, pole results in D(s) =0. [3pts]

Sol)

In practical circuit, pole is a real value. However, when we draw Bode plot, s is imaginary (ie. S=jω). Hence the denominator of H(s) cannot be zero.

8. Consider the below experiment where AC voltage source of $\sin(\omega_0 t)$ is applied. Assume that both circuits have identical input source, resistor, capacitor, and wire length. [**10pts**]



(a) Explain the difference between the two circuits from the circuit perspective. You will receive 0 point if you write something that is incorrect. You will receive 1 point if you leave the answer sheet blank. [5pts]

Sol)

Magnetic field is formed when there is current. In the left figure, there is lots of spacing between the lines and thus, magnetic field formed in one wire is not affected by the other wire. However, in the right figure the lines are close and since the direction of currents are opposite, the magnetic field outside the wires is cancelled. Therefore, there is more inductance in the left figure.

(b) In the above circuit, can you think about a case when KCL does not hold? Please explain. You will receive 0 point if you write something that is incorrect. You will receive 1 point if you leave the answer sheet blank. [5pts]

Sol)

KCL does not hold if the frequency is high enough such that the wavelength of the signal is comparable to the length of the wire. (or due to finite speed of electrons, current at different points along the wire will be different. This effect will be pronounced as the length of the wire is increased.)