Solve following problems (1~4) with referring figure 1. There are two point charges. One is at point M with charge q, and another is at point N with charge rq. Let r is real number, q>0, $\overline{MA} = \overline{AB} = \overline{BC} = \overline{CN} = d$, $\overline{BX} = 2d$, and $\overline{MN} \perp \overline{BX}$.

1. Assume that the two point charges are fixed. When r = 4.9, direction of electric field at point A is right. (O / X) [5 pts]

Answer: O

2. Assume that the two point charges are not fixed and r = -1.8. Then, two point charges repel each other. (O / X) [5 pts]

Answer: X

3. Let r=-9. Where is the point that net electric field is 0? [5 pts]

① A ② B ③ C ④ No answer

Answer: ④

4. Let r=1. If we put charge Q<0 at the point X, what is direction of force that charge Q feels? [5 pts]

① +x direction ② +y direction ③ -x direction ④ -y direction

Answer: ④

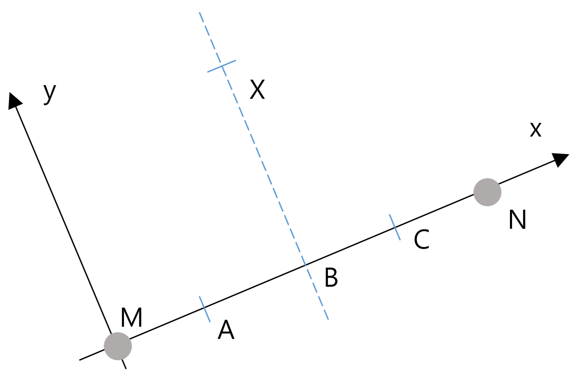


Figure 1

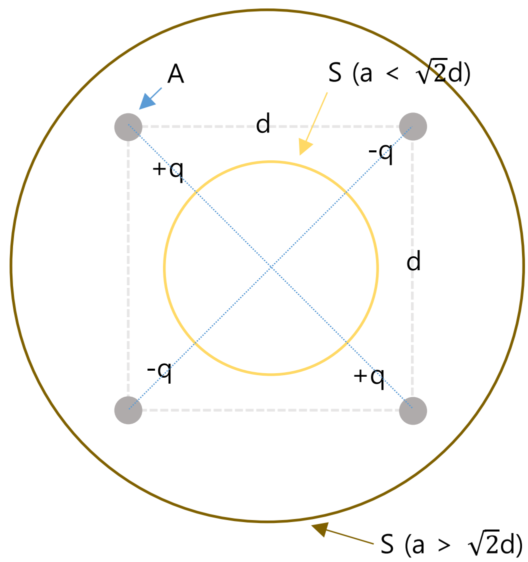


Figure 2

There is a quadrupole. Solve following problems. (5~7)

(Quadrupole: 4 point charges q, -q, -q, q are located on vertexes of square like figure 2)

5. Draw electric field lines. [15 pts]

-10 pts when direction is wrong.

6. Evaluate the force that A feels due to other charges. Note that **k** is Coulomb constant and define →, ↑ directions as x, y direction, respectively. [15 pts]

Force due to right (→) charge: $\vec{F}_1 = \frac{kq^2}{d^2}\hat{\imath} \ldots 4 \text{ pts}$

Force due to underneath (↓) charge: $\vec{F}_2 = -\frac{kq^2}{d^2}\hat{\jmath} \ldots 4 \text{ pts}$

Force due to ↘ charge: $\vec{F}_3 = \frac{kq^2}{(\sqrt{2}d)^2}\left(-\frac{1}{\sqrt{2}}\ \hat{\imath} + \frac{1}{\sqrt{2}}\hat{\jmath}\right) \ldots 4 \text{ pts}$

$$\therefore \vec{F} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 = \frac{kq^2}{d^2}\left\{\left(1 - \frac{1}{2\sqrt{2}}\right)\hat{\imath} - \left(1 - \frac{1}{2\sqrt{2}}\right)\hat{\jmath}\right\} \ldots 3 \text{ pts}$$

7. Evaluate total electric flux on the surface S. S is spherical shell (or surface of sphere) with diameter a, and the center of this circle is coincident with the center of the quadrupole. This diameter can be both bigger and smaller than $\sqrt{2}$d. [15 pts]

$$\Phi_E = \oint_{\text{enclosed}} \vec{E} \cdot d\vec{A} = \frac{q_{enc}}{\epsilon_0} \ldots 5 \text{ pts}$$

In this problem, $q_{enc} = 0$ … 5 pts

Therefore, electric flux is also 0 … 5 pts

8. There is charged sphere with radius R whose charge density is $\rho(\vec{r}) = \frac{Q}{\pi R^3}\frac{r}{R}$. Using Gauss' law, evaluate the electric field inside the sphere. (i.e. $\vec{E}(\vec{a})$ where a<R.) [15 pts]

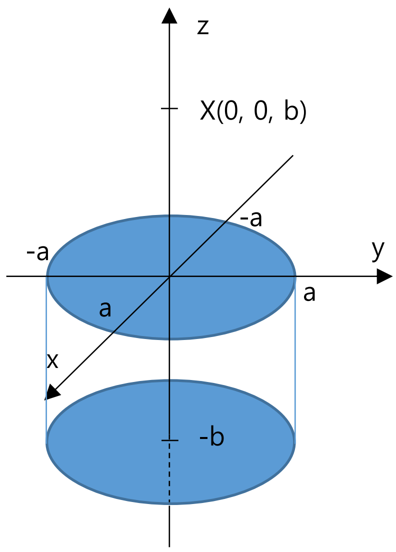
Due to spherical symmetry, the amplitude of electric field is constant on the same radius and its direction is always radial. Therefore,

$$\oint_{\text{enclosed}} \vec{E}\cdot d\vec{A} = \frac{q_{\text{enc}}}{\epsilon_0} \ldots 5 \text{ pts}$$

$$\Rightarrow E(a)\times 4\pi a^2 = \frac{1}{\epsilon_0}\int_{enclosed} \rho(\vec{r})dV = \frac{1}{\epsilon_0}\int_0^a \frac{Q}{\pi R^3}\frac{r}{R}\times 4\pi r^2 dr = \frac{Q}{\epsilon_0}\left(\frac{a}{R}\right)^4 \ldots 5 \text{ pts}$$

$$\therefore \vec{E}(\vec{a}) = \frac{Q}{4\pi a^2\epsilon_0}\left(\frac{a}{R}\right)^4 \vec{e}_r \ldots 5 \text{ pts}$$

(Advanced Problem!!) There is a uniformly charged cylinder: $\{(x,y,z)|x^2+y^2\leq a^2, -b\leq z\leq 0\}$. It is charged with charge Q. Evaluate the electric field at the point X(0, 0, b). [20 pts]



We can divide this cylinder into infinitely many congruence disks whose center is located (0, 0, -h) (0<h<b), and each disk is also divided into infinitely many rings with various radius: 0 to a.

Each ring whose center is located z=-h (0<h<b) with radius r (0<r<a) makes electric field (refer example 20.6 in textbook)

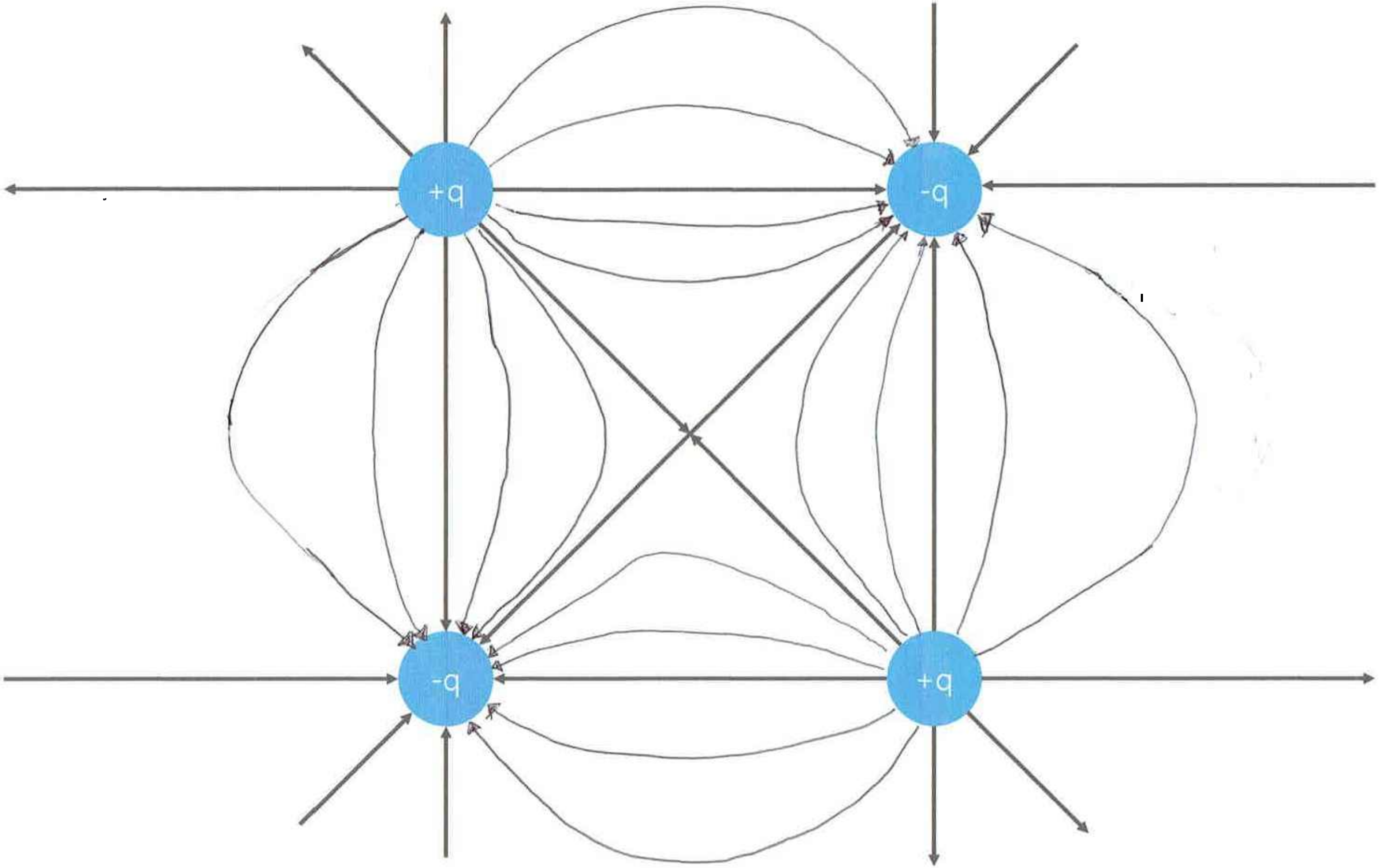
$$d\vec{E} = \frac{k(b+h)dQ}{\{(b+h)^2+r^2\}^{\frac{3}{2}}} = \frac{k(b+h)\frac{Q}{\pi a^2 b}\times 2\pi r dr dh}{\{(b+h)^2+r^2\}^{\frac{3}{2}}}\hat{k} \ldots 10 \text{ pts}$$

$$\therefore \vec{E} = \int d\vec{E}$$

$$= \int_0^b\int_0^a \frac{k(b+h)\frac{Q}{\pi a^2 b}\times 2\pi r dr dh}{\{(b+h)^2+r^2\}^{\frac{3}{2}}}\hat{k} = \int_0^b\int_0^{\tan^{-1}\frac{a}{b+h}} \frac{2kQ}{a^2 b}\sin\theta\, d\theta dh\hat{k} \qquad (r\equiv(b+h)\tan\theta)\ldots 4 \text{ pts}$$

$$= \int_0^b \frac{2kQ}{a^2 b}\left(1-\frac{b+h}{\sqrt{a^2+(b+h)^2}}\right)\hat{k}dh = \left(\frac{2kQ}{a^2} - \int_{\tan^{-1}\frac{b}{a}}^{\tan^{-1}\frac{2b}{a}} \frac{2kQ}{ab}\frac{\sin\phi}{\cos^2\phi}d\phi\right)\hat{k} \qquad (b+h\equiv a\tan\phi)\ldots 4 \text{ pts}$$

$$= \frac{2kQ}{a^2} - \frac{2kQ}{ab}\left\{\sqrt{1+\left(\frac{2b}{a}\right)^2} - \sqrt{1+\left(\frac{b}{a}\right)^2}\right\}\hat{k} = \frac{2kQ}{a^2}\left\{1+\sqrt{1+\left(\frac{a}{b}\right)^2} - \sqrt{4+\left(\frac{a}{b}\right)^2}\right\}\hat{k} \ldots 2 \text{ pts}$$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