(b) Find marginal pdf of X_1, X_2, X_3

a) 에서 $f_{X_1,X_2,X_3}(x_1, x_2, x_3) = \frac{6}{\beta^3} e^{-\frac{3x_1+2x_2+x_3}{\beta}}$ 을 구하였다.

이를 이용해, marginal pdf 를 구하면, 다음과 같다.

$$f_{X_1}(x_1) = \int_0^\infty \int_0^\infty \frac{6}{\beta^3} \exp\left(-\frac{3x_1+2x_2+x_3}{\beta}\right) dx_3 dx_2$$

$$= \int_0^\infty \frac{6}{\beta^3} e^{-\frac{3x_1+2x_2}{\beta}} \int_0^\infty e^{-\frac{x_3}{\beta}} dx_3 \, dx_2 = \frac{6}{\beta^2} e^{-\frac{3x_1}{\beta}} \int_0^\infty e^{-\frac{2x_2}{\beta}} dx_2 = \frac{3}{\beta} e^{-\frac{3x_1}{\beta}}$$

$$f_{X_2}(x_2) = \int_0^\infty \int_0^\infty \frac{6}{\beta^3} \exp\left(-\frac{3x_1+2x_2+x_3}{\beta}\right) dx_3 dx_1 = \int_0^\infty \frac{6}{\beta^2} e^{-\frac{3x_1+2x_2}{\beta}} dx_1 = \frac{2}{\beta} e^{-\frac{2x_2}{\beta}}$$

$$f_{X_3}(x_3) = \int_0^\infty \int_0^\infty \frac{6}{\beta^3} \exp\left(-\frac{3x_1+2x_2+x_3}{\beta}\right) dx_2 dx_1 = \int_0^\infty \frac{3}{\beta^2} e^{-\frac{3x_1+x_3}{\beta}} dx_1 = \frac{1}{\beta} e^{-\frac{x_3}{\beta}}$$

따라서,

$$f_{X_1}(x_1) = \frac{3}{\beta} e^{-\frac{3x_1}{\beta}}, \quad f_{X_2}(x_2) = \frac{2}{\beta} e^{-\frac{2x_2}{\beta}}, \quad f_{X_3}(x_3) = \frac{1}{\beta} e^{-\frac{x_3}{\beta}}$$