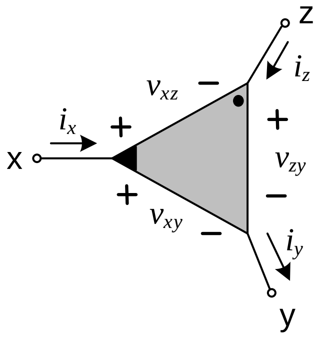
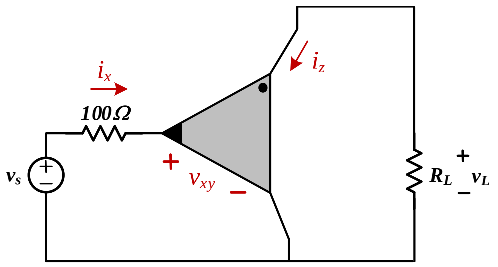
Mid-Term Exam Solution

1. **[Basic Circuit]** Consider the following 3-terminal device which has the following characteristics. [8pts]



$$i_x = \frac{v_{xy}}{100}$$
$$i_z = 100 i_x$$
$$i_y = i_x + i_z$$

(a) Find v_L/v_s of the below circuit. [4pts]



(sol)

$$v_{xy} = v_s - 100 \cdot i_x = v_s - v_{xy} \quad \rightarrow \quad v_{xy} = \frac{v_s}{2}$$

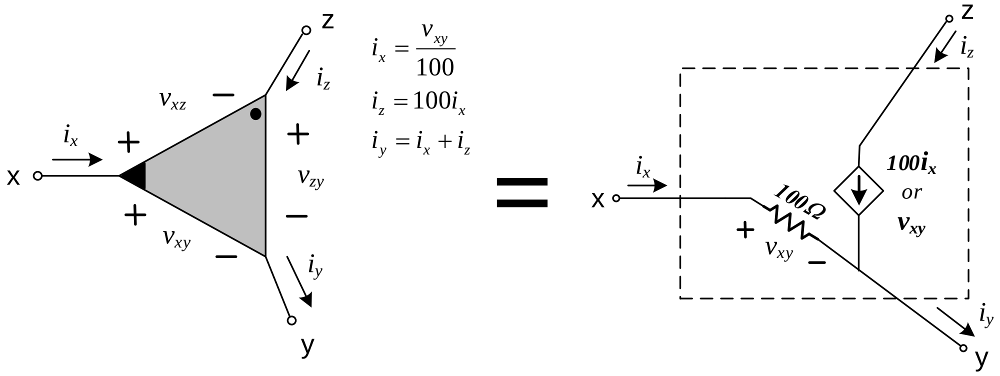
$$i_z = 100 \cdot i_x = v_{xy} = \frac{v_s}{2}$$

$$v_L = (-i_z) \cdot R_L = \left(-\frac{v_s}{2}\right) \cdot R_L$$

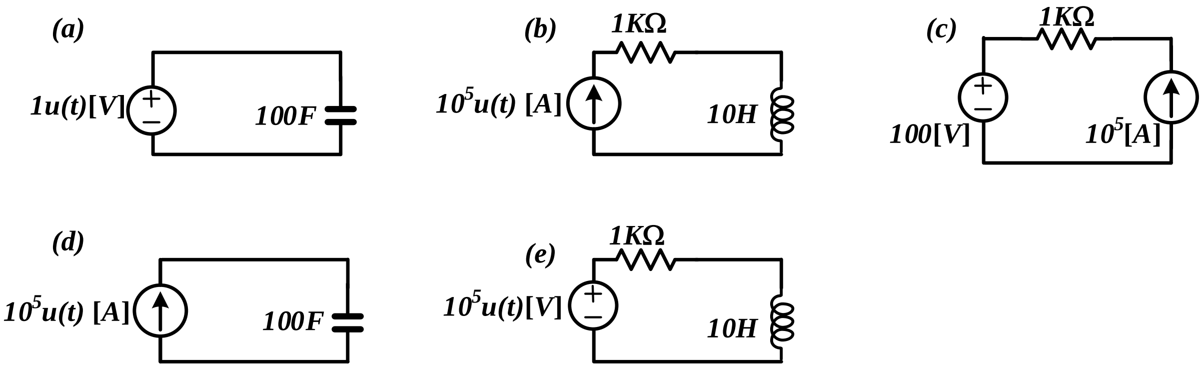
$$\therefore \frac{v_L}{v_s} = -\frac{R_L}{2}$$

(b) Draw an equivalent circuit model of the above device using one resistor and one dependent source. Specify the value of resistor and dependent source. [4pts]

(sol)



2. Consider the circuits shown below. Which circuits are physically possible without damaging the wire? Assume that wire can handle current up to 10^6A. [5pts]



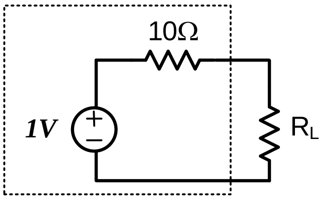
ANS: ___C_____,_____D_____,___E_____,__________,__________

A: Infinite current at t=0+;

B: Infinite voltage at t=0+;

3. Consider the circuit shown below. [12pts]

(a) Suppose the supply and the resistor in the dotted box represents a battery. What is the maximum power that can be drawn from this battery and what is the value of R_L for this condition? [2pts]



It is about the maximum power delivery question, not a maximum power efficiency.

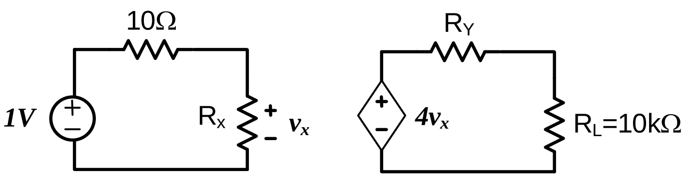
Maximum power delivery: $R_S=R_L$ ➔ $R_L=10\Omega$, $P_{max\_drawn}=1/40W$

When the R_L is 0Ω, the power consumption of the battery is maximized. HOWEVER, the power is not to be DRAWN from the battery. The power is consumed by battery itself.

\+ There are some vague terms, there can be a confusion.

If you see that the source is only a battery without R_S (10Ω), maximum power drawn happens at the $R_L=0\Omega$. So, we concede that the $R_L=0\Omega$, and it can be another correct answer. ($R_L=0\Omega$, $P_{max}=1/10W$)

(b) Find values of R_X and R_Y between 0.1Ω ~ 1MΩ, so that power is delivered to the load is maximum. [4pts]

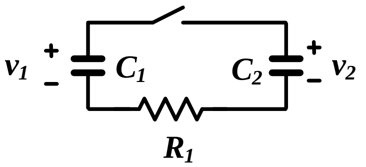


For the maximum power delivery R_Y should be 0.1Ω.

To maximize dependent source at the second stage, v_x should be maximized.

Therefore, the first stage should have Rx as large as possible. Rx=1MΩ.

For (c) and (d), consider the circuit shown below. The switch turns on at t=0. Assume that that C_1 and C_2 have initial voltages of V_i and 0, respectively.



$$v_1(0) = V_i$$
$$v_2(0) = 0$$

(c) What should be the value of C_2 and R_1 such that maximum energy is delivered to C_2 at steady-state? Express your answer using C_1 and/or V_i. [3pts]

Initial charge: $Q_{init} = V_i C_1$

Steady-stage charge = initial charge = $Q_{init} = Q_{SS} = V_f (C_1 + C_2)$

Steady-stage Energy at C_2 =

$$E_{ss2} = \frac{1}{2} C_2 V_f^{\ 2} = \frac{1}{2} C_2 \cdot \left(\frac{C_1}{C_1 + C_2} V_i \right)^2$$

$$\frac{dE_{ss2}}{dC_2} = \frac{C_1^{\ 2} V_i^2}{2} \cdot \frac{d \dfrac{C_2}{(C_1+C_2)^2}}{dC_2} = \frac{C_1^{\ 2} V_i^2}{2} \cdot \frac{(C_1+C_2)(C_1+C_2-2C_1)}{(C_1+C_2)^4}$$

$$\frac{dE_{ss2}}{dC_2} = 0$$ At $C_1 = C_2$

Steady-stage energy will be maximized when the C_1 is equal to C_2.

Energy delivery is independent to the R_1.

(d) Suppose $C_2 = 0.5C_1$ and the switch has a resistance of R_S when it is turned on. What is the energy lost through R_1 and R_S as the circuit reaches steady-state? Express your answer using C_1 and/or V_i. [3pts]

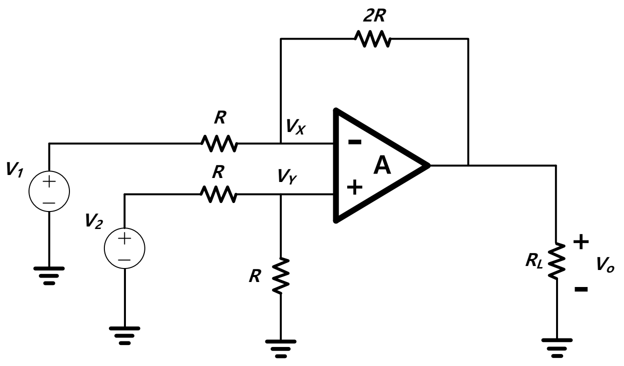
$$E_{init} = \frac{1}{2} C_1 V_i^2$$

$$E_{ss} = \frac{1}{2}(C_1 + C_2) V_f^{\ 2} = \frac{1.5}{2} C_1 V_f^{\ 2} = \frac{1.5}{2} C_1 \left(\frac{1}{1.5} V_i \right)^2 = \frac{1}{3} C_1 \cdot V_i^2$$

$$E_{lost} = E_{ss} - E_{init} = \frac{1}{6} C_1 \cdot V_i^2$$

It does not need to specify the lost energy at each resistor.

For (a)~(c), consider the circuit shown below. [19pts]



(a) Assume that the opamp is ideal. (i.e. A=∞). Express v_O in terms of v_1 and v_2 . [4pts]

sol)

$$V_o = -\frac{2R}{R}V_X + \left(1+\frac{2R}{R}\right)(V_Y/2)$$
$$V_o = -2\,V_X + \frac{3}{2}V_Y$$

(b) Suppose A < ∞. How does the gain $|v_O/v_1|$ change compared to your answer in (a)? [2pts]

(i) Increases (ii) Decreases (iii) Stays the same

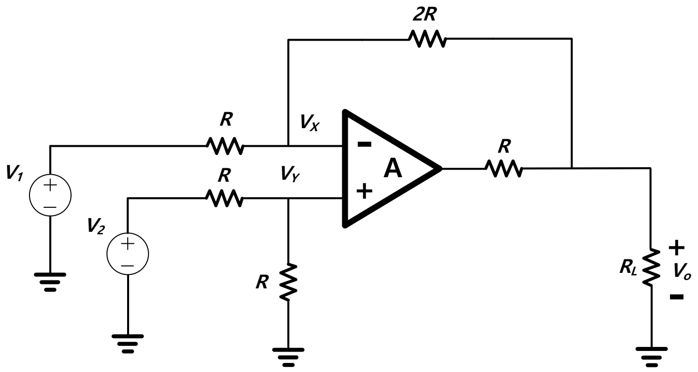
(c) Suppose A < ∞. What is the relation between V_X and V_Y? [2pts]

(i) $V_X = V_Y$ (ii) $V_X \neq V_Y$

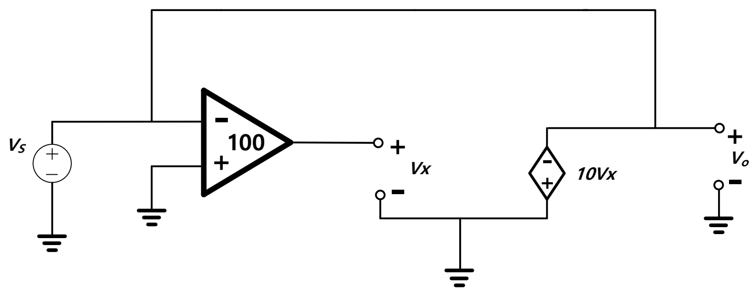
(d) Suppose A = ∞ in the below circuit. How does the gain $|v_O/v_X|$ change compared to your answer in (a) and what is the relation between V_X and V_Y ? [4pts]

- Gain $|v_O/v_X|$: (i) Increases (ii) Decreases (iii) Stays the same

- Relation between V_X and V_Y : (i) $V_X = V_Y$ (ii) $V_X \neq V_Y$



(e) Find the gain, $|v_O/v_S|$ of the below circuit. [3pts]



sol)

The problem has a contradiction.

(f) In this course, we learned a feedback circuit using an opamp that provides a voltage gain of 2. Why don't people just design an open-loop amplifier that has a gain of 2 without using feedback? Please fill in the below. [4pts]

In a feedback circuit, feedback factor(β) is ____________than 1 and thus, it can be made ____________________. The open-loop amplifier in the feedback circuit should have __________________________.

First blank: (i) smaller (ii) larger

Second blank: (i) very accurately (ii) with low power consumption (iii) with low-cost

Third blank: (i) large gain and high power consumption.

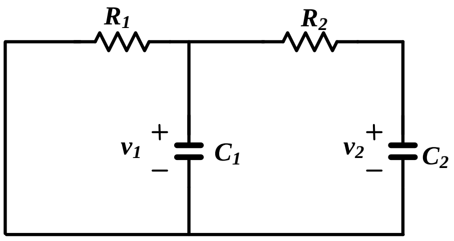
(ii) large gain and low power consumption.

(iii) small gain and low-power consumption

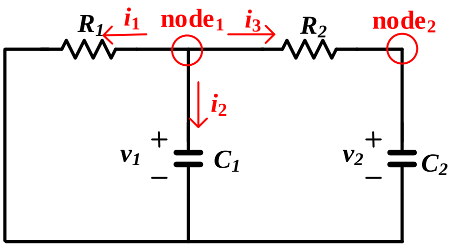
(iv) small and accurate gain

(v) large gain and can be inaccurate.

4. Consider the circuit shown below [8pts]



(a) Assume $R_1=R_2=1\Omega$ and $C_1=C_2=1F$. Write down one differential equation for v_1 that describes this circuit. (Do NOT include v_2 in your equation.) [4pts]



Sol 1)

[KCL 1]

$$i_1(t)+i_2(t)+i_3(t)=0$$

$$(i_1(t)=v_1(t),\, i_2(t)=\dot{v}_1(t),\ \text{and}\ i_3(t)=v_1(t)-v_2(t))$$

$$v_1(t)+\dot{v}_1(t)+v_1(t)-v_2(t)=0$$

$$v_2(t)=2v_1(t)+\dot{v}_1(t)$$

[KCL 2]

$$(v_2(t)-v_1(t))+\dot{v}_2(t)=0$$

$$(2v_1(t)+\dot{v}_1(t))-v_1(t)+(2\dot{v}_1(t)+\ddot{v}_1(t))=0$$

$$\ddot{v}_1(t)+3\dot{v}_1(t)+v_1(t)=0$$

Sol 2)

[KCL]

$$i_1(t)+i_2(t)+i_3(t)=0$$

$$(i_1(t)=v_1(t),\, i_2(t)=\dot{v}_1(t),\ \text{and}\ i_3(t)=v_1(t)-v_2(t)=\dot{v}_2(t))$$

$$v_1(t)+\dot{v}_1(t)+\dot{v}_2(t)=0 \qquad \ldots(1)$$

$$v_1(t)+\dot{v}_1(t)+v_1(t)-v_2(t)=0 \qquad \ldots(2)$$

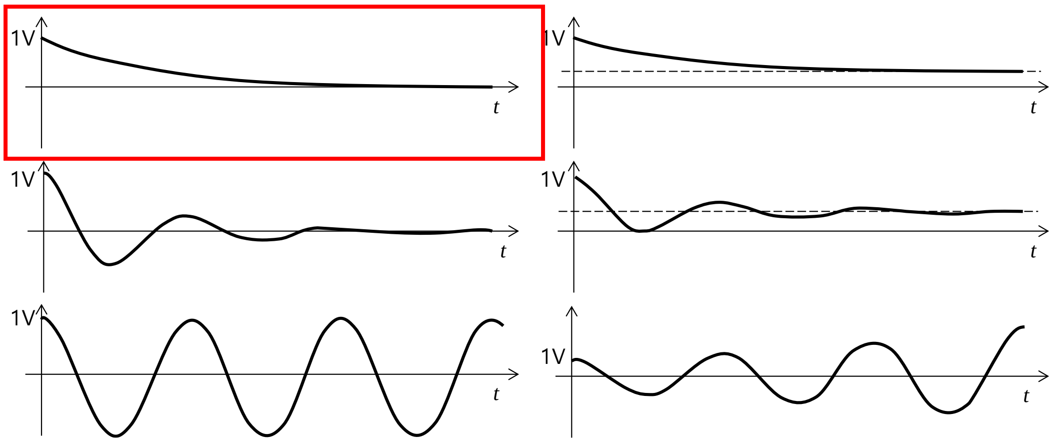
By taking derivative to the equation (2),

$$2\dot{v}_1(t)+\ddot{v}_1(t)-\dot{v}_2(t)=0 \qquad \ldots(2')$$

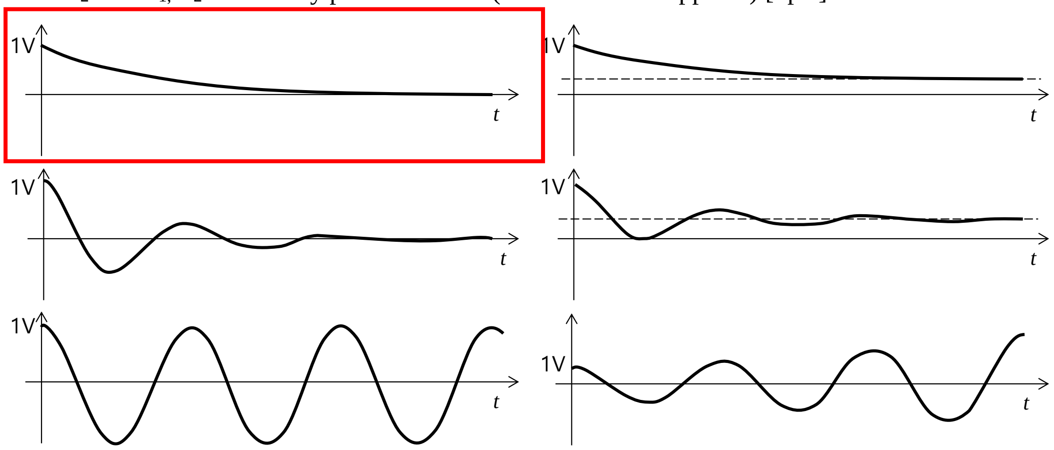
By adding (1) and (2'),

$$\ddot{v}_1(t)+3\dot{v}_1(t)+v_1(t)=0$$

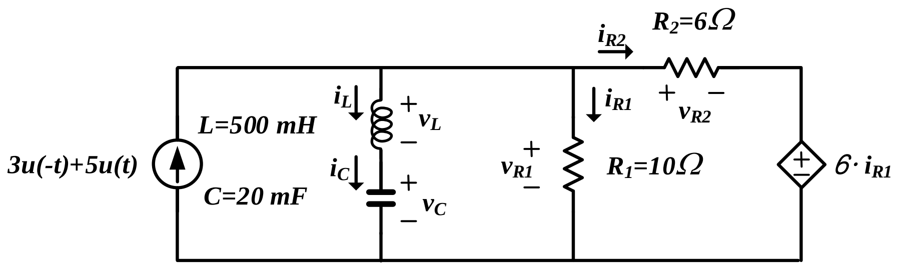
(b) Suppose there is initial voltage of 1V on C_1 and 0V on C_2. Which of the below can be a solution to this circuit?[2pts]



(c) Suppose there is initial voltage of 1V on C_1 and 0V on C_2. Which of the below can be a solution to this circuit if R_1, R_2 and C_1, C_2 can be any positive value. (Choose all that applies.) [2pts]



5. **Consider the circuit shown below. [16pts]**



(a) Fill in the table below. [8pts]

$v_L(0^-)$	$i_L(0^-)$	$v_{R1}(0^-)$	$v_C(0^-)$
0	**0**	**18**	**18**
$i_{R1}(0^-)$	$i_C(0^+)$	$i_{R1}(0^+)$	$i_{R2}(0^+)$
1.8	**0**	**3**	**2**
$v_C(0^+)$	$v_{R1}(0^+)$	$v_{R2}(0^+)$	$v_L(0^+)$
18	**30**	**12**	**12**
$\frac{dv_C}{dt}(0^+)$	$\frac{di_L}{dt}(0^+)$	$\frac{di_{R1}}{dt}(0^+)$	$\frac{di_{R2}}{dt}(0^+)$
0	**24**	**-14.4**	**-9.6**

(b) Find $i_L(t)$ for t>0. [6pts]

$$R_{th} = 6$$

$$\alpha = \frac{R_{th}}{2L} = 6\,, \omega_0 = \frac{1}{\sqrt{LC}} = 10$$

$$\omega_d = \sqrt{\omega_0^2 - \alpha^2} = 8$$

$$i_L(t) = e^{-\alpha t}(B_1 cos\omega_d t + B_2 sin\omega_d t) + I_F$$

$$i_L(\infty) = 0 = I_F$$

$$i_L(0^+) = e^{-\alpha 0}(B_1 cos\omega_d 0^+ + B_2 sin\omega_d 0^+) + I = 0 = B_1$$

$$B_1 = 0$$

$$v_L(0^+) = R_{th} \times (5-3) = L\frac{di_L}{dt}\bigg|_{t=0^+} = L\omega_d B_2 cos\omega_d t|_{t=0^+} = L\omega_d B_2$$

$$B_2 = \frac{R_{th} \times (5-3)}{L\omega_d} = 3$$

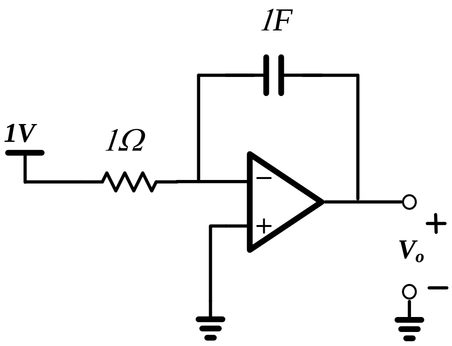
$$i_L(t) = e^{-6t}(3sin8t)$$

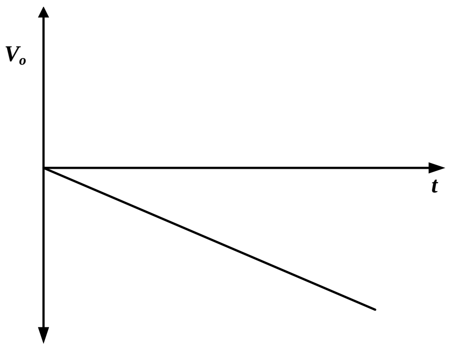
(c) What kind of response does this circuit have? [2pts]

(i) Overdamped (ii) **<u>Underdamped</u>** (iii) Critically-damped

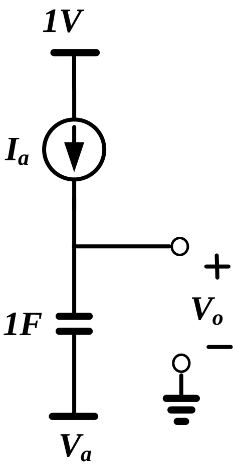
7. Assume that the opamp is ideal and initial charge of the capacitor is 0. [15pts]

(a) Sketch $V_o(t)$ for t>0. [4pts]





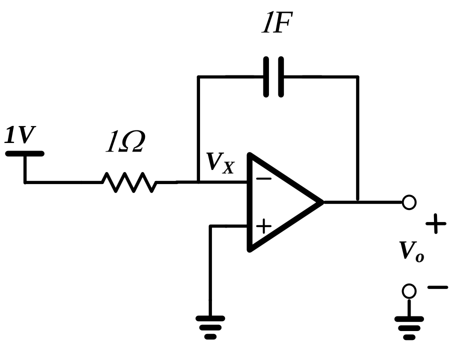
(b) Find the values of Va and Ia such that the above two circuits are identical. [4pts]



Sol) V_a= 0V, I = -1 A

(c) Suppose that the opamp has finite gain of ***G***. Derive an expression of V_O (t) as a function of ***G***. [7pts]

Sol)



$$G(0 - V_X) = V_O \quad \cdots ①$$

$$\frac{1 - V_X}{1} = C \cdot \frac{d(V_X - V_O)}{dt} \quad \cdots ②$$

$$1 + \frac{1}{G}V_O = -(\frac{1}{G} + 1)\frac{dV_O}{dt}$$

$$\left(\frac{1}{G} + 1\right)\frac{dV_O}{dt} + \frac{1}{G}V_O + 1 = 0$$

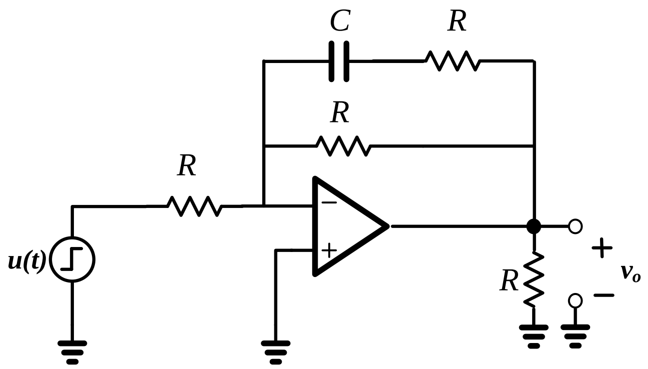
$$\therefore V_O = G(e^{-\frac{t}{(G+1)}} - 1)$$

8. Assume that the opamp is ideal and initial charge of the capacitor is 0 [10pts] (Hint: Use 7(b))

A. What is the time-constant of the below circuit? [2pts]

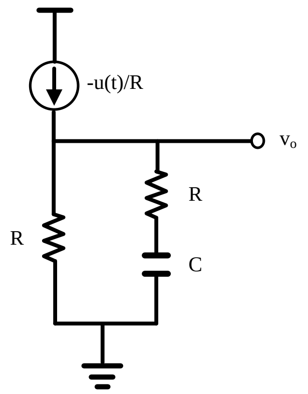
B. Find the values of $v_o(0)$ and $v_o(\infty)$ [4pts]

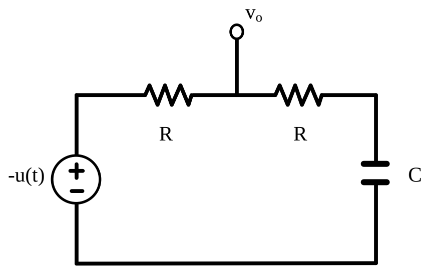
C. Find an expression for $v_o(t)$. Assume that initial charge on the capacitor is 0. [4pts]



Sol)

(a)





Time constant = C x (R+R) = 2RC

(b)

vo(0-) = 0V , vo(0+) = -1/2V

vo(∞) = -1V

(c)

Let's denote the voltage of node between C capacitor and R resistor as vx.

KCL:

$$-\frac{u(t)}{R} + C\frac{d}{dt}(-v_x) + \frac{-v_o}{R} = 0 \quad - ①$$

$$C\frac{dv_x}{dt} + \frac{v_x - v_o}{R} = 0 \quad - ②$$

Substitute ② into ① then you get the following eq.

$$-\frac{u(t)}{R}+\frac{v_x - v_o}{R}+\frac{-v_o}{R} = 0$$

$$\Rightarrow \ \mathrm{v_x} = 2v_o + u(t)$$

Substitute this eq. into ②

$$C\frac{d}{dt}(2v_o + v_{in}) + \frac{v_o + v_{in}}{R} = 0$$

$$\Rightarrow \ \frac{d}{dt}v_o + \frac{v_o}{2RC} = -\frac{v_{in}}{2RC}$$

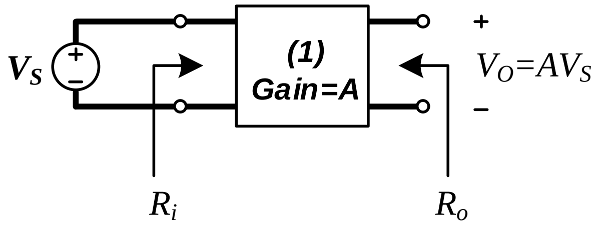
The general eq. for output voltage is

$$\mathrm{v_o} = A\, e^{-\frac{t}{2RC}} + \mathrm{B}$$

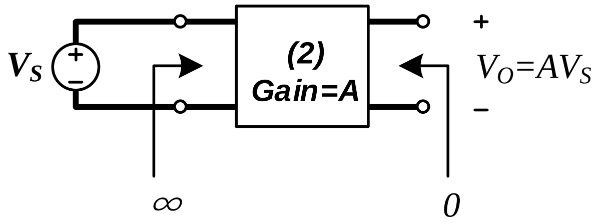
From the answers to (b), we can decide the values of A and B.

$$\mathrm{v_o} = \left(\frac{1}{2}\, e^{-\frac{t}{2RC}} - 1\right)u(t)$$

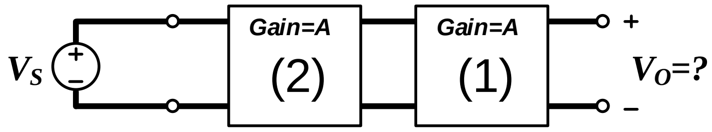
9. Consider the below system which has a voltage gain of A and some input and output resistances. [7pts]



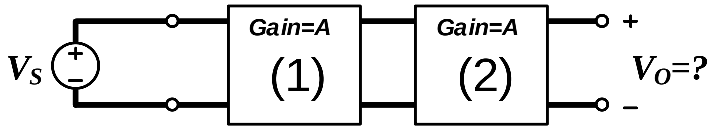
$$Cascaded\ Gain = A\cdot\left(\frac{R_i}{R_i+R_o}\right)\cdot A$$



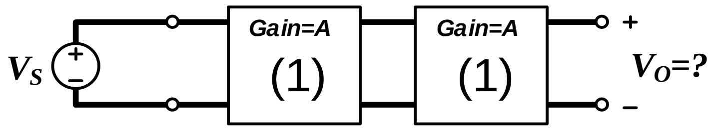
Find the gain of the below cascaded networks. Express your answer using A, R_i and R_O.



$$V_o = A^2\left(\frac{R_i}{R_i+0}\right)V_i = A^2V_i$$



$$V_o = A^2\left(\frac{\infty}{\infty+R_o}\right)V_i = A^2V_i$$



$$V_o = A^2\left(\frac{R_i}{R_i+R_o}\right)V_i$$

학번(ID):__________________ 이름(Name):____________________

#1	#2	#3	#4	#5	#6	#7	#8	#8	Total
/8	/5	/12	/19	/8	/16	/15	/10	/7	/100