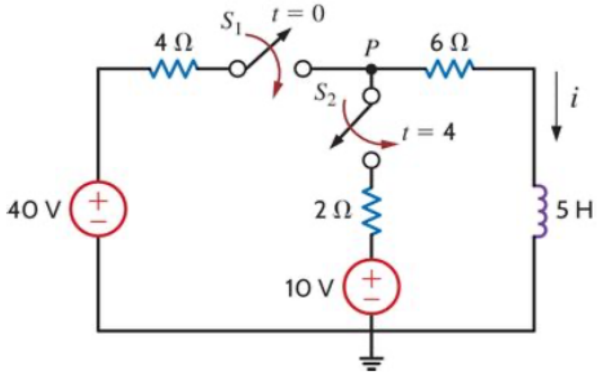
1. **(6 Points) At $t = 0$, switch 1 is closed as shown, and switch 2 is closed 4-second later. Find $i(t)$ for $t > 0$.**



For $t < 0$, switches S_1 and S_2 are open so that $i = 0$. Since the inductor current cannot change instantly,

$$i(0^-) = i(0) = i(0^+) = 0$$

For $0 \le t \le 4s$, S_1 is closed so that the 4-Ω and 6-Ω resistors arein series. Hence, assumingfor now that S_1 is closed forever,

$$i(\infty) = \frac{40}{4+6} = 4\ A, \qquad R_{Th} = 4 + 6 = 10\ \Omega$$

Thus, $$i(t) = i(\infty) + [i(0) - i(\infty)]e^{-\frac{t}{\tau}} = 4(1 - e^{-2t})\ A, \qquad 0 \le t \le 4s$$

i(∞) +1pt
τ +1pt
i(t) +1pt

For $t \ge 4s$, S_2 is closed; the 10-V voltage source is connected, andthe circuit changes. This sudden change does not affect the inductorcurrent because the current cannot change abruptly. Thus, the initialcurrent is,

$$i(4) = i(4^-) = 4(1 - e^{-8}) \approx 4\ A$$

To find $i(\infty)$, let v be the voltage at node P. Using KCL,

$$\frac{40-v}{4} + \frac{10-v}{2} = \frac{v}{6} \qquad => \qquad v = \frac{180}{11}\ V$$

$$i(\infty) = \frac{v}{6} = 2.727\ A$$

The Thevenin resistance at the inductor terminals is

$$R_{Th} = 4||2 + 6 = \frac{22}{3}\ \Omega$$

And $$\tau = \frac{L}{R_{Th}} = \frac{15}{22} s$$

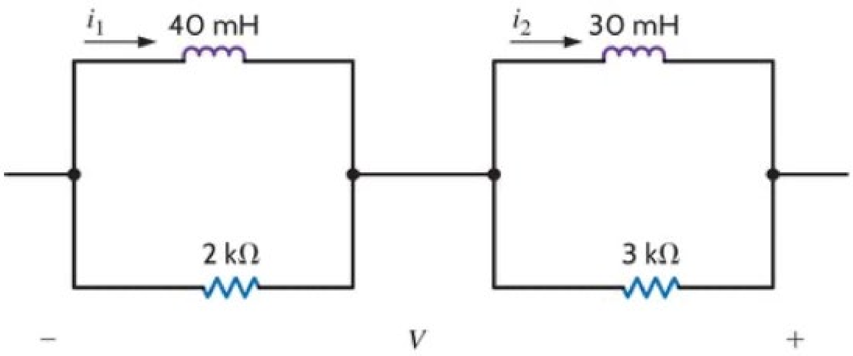
Hence,

$$i(t) = i(\infty) + [i(4) - i(\infty)]e^{-\frac{t-4}{\tau}}, \qquad t \ge 4$$

$$= 2.727 + 1.273\ e^{-1.467(t-4)}, \qquad t \ge 4$$

i(∞) +1pt
τ +1pt
i(t) +1pt

2. (6 Points) In the network shown below, initial values are $i_1(0)$ = 20mA and $i_2(0)$ = 15mA. Determine (a) $V(0)$, (b) $V(t$ = 15μs), and (c) the time t when $V(t)$ = 0.1$V(0)$.



By KVL,

$$\begin{aligned} 40m \cdot \frac{di_1}{dt} + 2k \cdot i_1 = 0 \\ 30m \cdot \frac{di_2}{dt} + 3k \cdot i_2 = 0 \end{aligned}\quad ,$$ **[+1 Points]**

and can be expressed

$$\begin{aligned} \frac{di_1}{dt} + 50k \cdot i_1 = 0 \\ \frac{di_2}{dt} + 100k \cdot i_2 = 0 \end{aligned}\quad .$$

Therefore,

$$\begin{aligned} i_1 = 20m \cdot \exp(-50k \cdot t) \\ i_2 = 15m \cdot \exp(-100k \cdot t) \end{aligned}$$

$$\therefore V(t) = 2k \cdot i_1 + 3k \cdot i_2 = 40 \cdot \exp(-50k \cdot t) + 45 \cdot \exp(-100k \cdot t)$$ **[+1 Points]**

(a) V(0) = 85V **[+1 Points]**

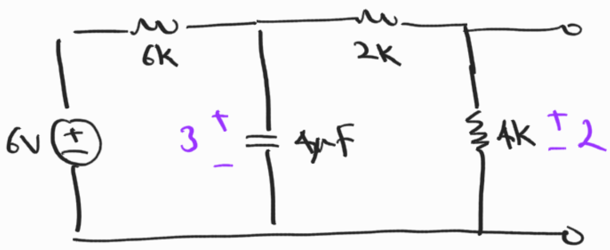
(b) $V(t$ = 15μs) = 28.94V **[+1 Points]**

(c) **[+2 Points]**

$$\begin{aligned} & s = \exp(-50k \cdot t) \\ & 40s + 45s^2 = 0.1 \times 85 \\ & s = 0.1772 = \exp(\text{-}50\text{k} \cdot t) \\ & \therefore t = 34.60 \mu s \end{aligned}$$

3.

i) $t=0$.

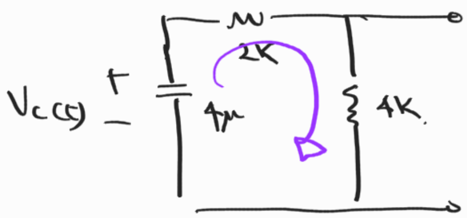


$$V_o(t)\big|_{t=0} = 6 \times \frac{4K}{6K+2K+4K}$$

$$= 2\ (V)$$

initial value of $V_o(t)$ ... 2 points

ii) $t>0$



$$\tau = RC$$

$$= 6k \cdot 4\mu$$

$$= 24\,ms.$$

$$V_c(t) = 3e^{-t/\tau}$$

$$V_o(t) = V_c(t) \times \frac{4K}{2K+4K}$$

$$= 2 \cdot e^{-t/\tau}$$

exponential decay & reason ... 2 points

right answer ... 2 points

4. **(6 Points) If $R = 20\ \Omega$, $L = 0.6$ H, what values of C will make a series RLC circuit : (a) overdamped, (b) critically damped, and (c) underdamped?**

$$L\frac{d^2 i}{dt^2} + R\frac{di}{dt} + \frac{1}{C}i = 0$$

$$\frac{d^2 i}{dt^2} + \frac{R}{L}\frac{di}{dt} + \frac{1}{LC}i = 0$$

+2 points

Here, $w_0 = \frac{1}{\sqrt{LC}}$, $2\xi w_0 = \frac{R}{L}, \xi = \frac{R\sqrt{LC}}{2L} = \frac{R}{2}\sqrt{\frac{C}{L}}$

Damping factor: $\frac{R}{2}\sqrt{\frac{C}{L}}$

Use equation +1 points

1) Damping factor > 1 (Overdamped), = 1 (Critically damped), < 1 (Underdamped)
2) Discriminant $(\left(\frac{R}{L}\right)^2 - 4\frac{1}{LC})$

   > 0 (Overdamped), = 0 (Critically damped), < 0 (Underdamped)

(a) Overdamped: $C > \frac{4L}{R^2} = 4 \times \frac{0.6}{400} = 6 \times 10^{-3}\ F, or\ C > 6mF$ +1 points

(b) Critically damped: $C = \frac{4L}{R^2} = 6mF$ +1 points

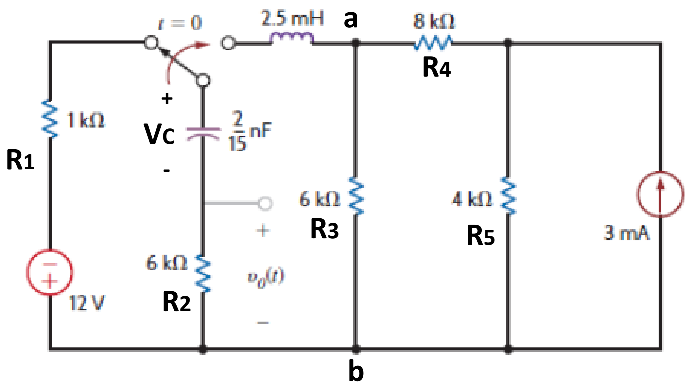
(c) Underdamped: $C < \frac{4L}{R^2} = 6mF$ +1 points

If condition of damping factor (Discriminant) is correct, but incorrect equation ➔ Total 3 points, (If condition is also incorrect ➔ 2points)

Any calculation error in (a), (b), (c): -1 points

No unit (or wrong unit): -0.5 points

5. **(6 points) Find V₀(t) for t>0 in the circuit and plot the response including the time interval just prior to moving the switch**

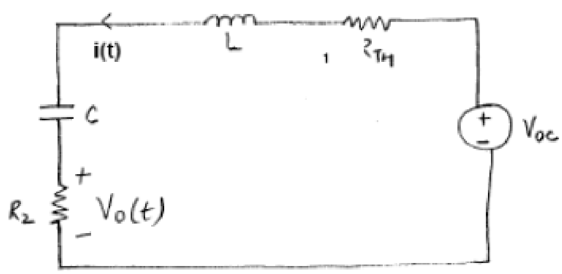


i) $t = 0^-$:

$V_C(0^-) = -12\,[V]$, $V_O(0^-) = 0\,[V]$, $I_L(0^-) = 0\,[A]$ ⋯ 0.5pts

$V_{ab,opencircuit} = \left(\frac{R_3}{R_3+R_4+R_5}\right) \times 12[V] = 4[V]$ ⋯ 0.5pts

ii) $t > 0$:



$R_{TH} = (R_4 + R_5)||R_3 = 4k\Omega$

$$(R_2 + R_{TH}) \cdot i(t) + L\frac{di(t)}{dt} + \frac{1}{C}\int i(t)dt = V_{OC}$$

$$\frac{d^2i(t)}{dt} + \frac{R_2+R_{TH}}{L}\frac{di(t)}{dt} + \frac{i(t)}{LC} = 0$$

⋯ 0.5pts

$$\lambda = \frac{-4\times10^6 \pm \sqrt{(4\times10^6)^2 - 4\cdot(1)\cdot(3\times10^{12})}}{2} = -2\times10^6 \pm 1\times10^6$$

∴ The given situation is an overdamped case :

$i(t) = K_1 e^{\lambda_1 t} + K_2 e^{\lambda_2 t}$

➔ $i(t) = K_1 e^{-1\times10^6 t} + K_2 e^{-3\times10^6 t}$ ⋯ 0.5pts

➔ $\frac{di(t)}{dt} = -1\times10^6 \times K_1 e^{-1\times10^6 t} + -3\times10^6 \times K_2 e^{-3\times10^6 t}$

utizling KVL, we obtain

$$V_{OC} = (R_{TH} + R_2)i(t) + L\frac{di(t)}{dt} + V_C(t)$$

$$\frac{di(t)}{dt} = \frac{V_{OC}}{L} - \frac{R_{TH}+R_2}{L}i(t) - \frac{1}{L}V_C(t) = \frac{4}{2.5m} - \frac{(4k+6k)}{2.5m}(0) - \frac{1}{2.5m}(-12)$$

$$\frac{di(t)}{dt}\Big|_{t=0} = 6400 = -1\times10^6K_1 - 3\times10^6K_2$$ ⋯ 0.5pts

$$i(0) = i_L(0^+) = i_L(0^-) = 0\ [A]$$

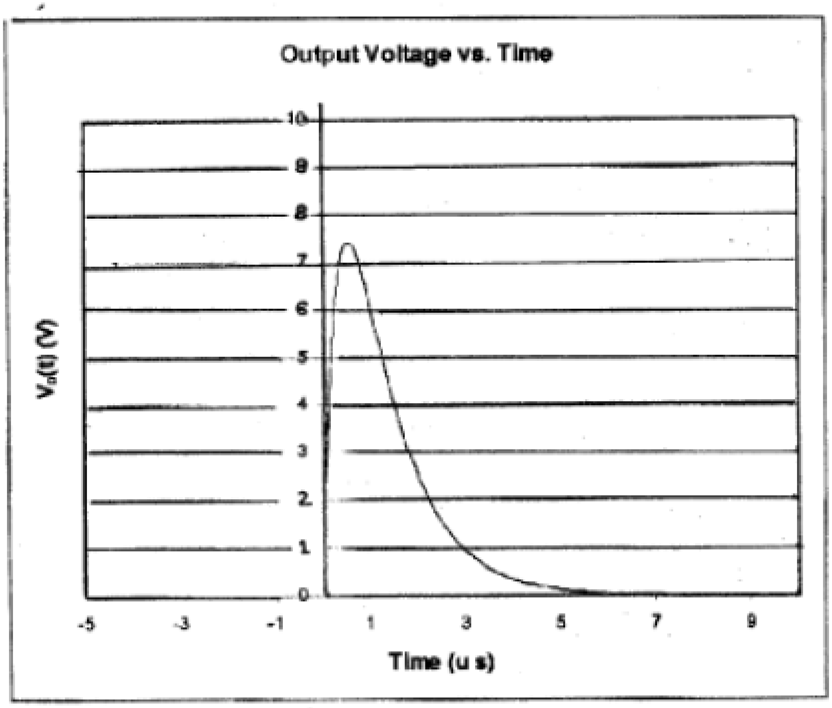
$$K_1 + K_2 = 0$$ ⋯ 0.5pts

$$K_1 = 0.0032\,,\ \ K_2 = -0.0032$$ ⋯ 1pts

$$i(t) = 3.2e^{-1\times10^6t} - 3.2e^{-3\times10^6t}\ [mA],\ \ t > 0$$

$$v_O(t) = i(t)R_2 = 19.2e^{-1\times10^6t} - 19.2e^{-3\times10^6t}\ [V]\,,\ \ t > 0$$ ⋯ 0.5pts

$$= 0[V]\qquad ,\ \ t < 0$$ ⋯ 0.5pts



⋯ 1pts