1

(a) Let μ be the parameter indicating the population mean weight of adult salmon when the new method is used. Then, the null and alternative hypotheses are

$$H_0 : \mu = 7.6$$
$$H_1 : \mu > 7.6$$

(b) Since

$$\frac{\sqrt{n}(\overline{X} - \mu_0)}{S} = \frac{\sqrt{40}(8.1 - 7.6)}{1.5} = 2.1082 > 1.6849 = t_{0.05,39},$$

we reject the null hypothesis. Also, the p-value is

$$P(T_{39} \geq 2.1082) = 0.0207.$$

(c) The type II error is the probability of not rejecting H_0 when it is false. Since $t_{0.05,39} = 1.6849$,

$$t = 7.6 + 1.6849 \times \frac{1.5}{\sqrt{40}} = 7.9996$$

and the type II error β is

$$\beta = P\left(T_{39} < \frac{\sqrt{40}(7.9996 - 8.2)}{1.5}\right) = P(T_{39} < -0.8450) = 0.2016.$$

(d) In this problem, assume that we know the population standard deviation $\sigma = 1.5$. Since $z_{0.05} = 1.645$,

$$t = 7.6 + 1.645 \times \frac{1.5}{\sqrt{n}}$$

and the power will be

$$P\left(Z \geq \frac{\sqrt{n}(t - 8.2)}{1.5}\right) = P\left(Z \geq 1.645 + \frac{\sqrt{n}(7.6 - 8.2)}{1.5}\right).$$

To achieve at least 90% of power, we need

$$1.645 + \frac{\sqrt{n}(7.6 - 8.2)}{1.5} \leq -1.2815$$

since $P(Z \geq -1.2815) = 0.9$. Therefore, we need $n \geq 53.5275$, i.e., at least 54 samples.

2 Assume that the populations of two types of automobiles have the common variance.

(a) Then, the pooled estimate of the common variance will be

$$s_p^2 = \frac{(14-1)\times 102 + (12-1)\times 87}{14+12-2} = 95.125.$$

Since $t_{0.025,24} = 2.064$, the 95% confidence interval for $\mu_1 - \mu_2$ is

$$\begin{aligned}&(118-109) \pm 2.064 \times \sqrt{95.125}\sqrt{\frac{1}{14}+\frac{1}{12}}\\ &= (1.0807, 16.9193)\end{aligned}$$

(b) Since

$$|t| = \frac{|118-109|}{\sqrt{95.125}\sqrt{\frac{1}{14}+\frac{1}{12}}} = 2.3457 > 2.064,$$

we reject H_0 at $\alpha = 0.05$. That is, there is statistical evidence that the two population means are different.

(c) Checking if 0 is included in the confidence interval $(1.0807, 16.9193)$ in (a) and comparing $|t|$ with $t_{0.025,24}$ in (b) are the same method to perform the hypothesis test. Since 0 is not included in the CI in (a), we get the same result with (b); we reject H_0.

(d) Let X_i and Y_i's be the random samples of recorded measurement for each type of automobiles. Then we need the following assumptions.

$$\begin{aligned}X_1, \cdots, X_{n_1} &\overset{\text{i.i.d.}}{\sim} \mathcal{N}(\mu_1, \sigma^2),\\ Y_1, \cdots, Y_{n_2} &\overset{\text{i.i.d.}}{\sim} \mathcal{N}(\mu_2, \sigma^2),\end{aligned}$$

and X_i's and Y_i's are also independent. Note that we assumed the common variance σ^2.

3 Let w_i be the difference of weight gains of two diets. Then,

$$\{w_1, \cdots, w_9\} = \{98, -38, 56, -4, 8, 70, -50, -34, 100\}.$$

Also, the sample mean is $\overline{w} = 22.89$ and the sample standard deviation is $S_w = 59.2715$.

(a) Since $t_{0.05,8} = 1.86$, the 90% confidence interval is

$$22.89 \pm 1.86 \times \frac{59.2715}{\sqrt{9}} = (-13.86, 59.64).$$

That is, with 90% certainty, $\mu_1 - \mu_2$ lies in (-13.86, 59.64).

(b) Let μ_i be the population mean weight gains of the animals over a 140-day test period on diet i, $i = 1, 2$. Let $\mu_w = \mu_1 - \mu_2$. Then, $H_0 : \mu_w = 0$ and $H_1 : \mu_w \neq 0$. Since 0 is in the CI we found in (a), there is no evidence of $\mu_1 \neq \mu_2$ with 90% certainty.

(c) Since

$$|t| = \frac{22.89}{59.2715/\sqrt{9}} = 1.1586 < 1.86,$$

we fail to reject H_0 at $\alpha = 0.1$. The data do not support the difference of two means.

4 (a) The hypotheses are $H_0 : \mu = 5$ vs $H_1 : \mu > 5$. The sample mean and standard deviation are $\overline{x} = 5.65, s = .8106$, respectively. We need to assume that data population follows normal distribution and is independent with each other. The test statistic is

$$t = \frac{\overline{x} - \mu}{s/\sqrt{n}} = \frac{5.65 - 5}{.8106/\sqrt{8}} = 2.268 > t_{\alpha,n-1} = t_{.05,7} = 1.895$$

Thus, we reject H_0 at $\alpha = .05$. That is, the data contradict the assertion.

(b) The 95% confidence interval is

$$\begin{aligned}&\left(\overline{x} - t_{\alpha/2,n-1} \cdot s/\sqrt{n}, \overline{x} + t_{\alpha/2,n-1} \cdot s/\sqrt{n}\right)\\ =&\left(5.65 - 2.365 \times \frac{.8106}{\sqrt{8}}, 5.65 + 2.365 \times \frac{.8106}{\sqrt{8}}\right)\\ =&(4.9722, 6.3278)\end{aligned}$$

where $t_{\alpha/2,n-1} = t_{.025,7} = 2.365$.

5

(a) The hypotheses are $H_0 : p = .8$ vs $H_1 : p \neq .8$. The proportion of the dwarf variety is $\hat{p} = (200 - 64)/200 = 136/200 = .68$. Thus, the test statistic is

$$|z| = \left| \frac{.68 - .8}{\sqrt{.8 \times (1 - .8)/200}} \right| = 4.24 > 1.96$$

Therefore, we reject H_0 at $\alpha = .05$. That is, this observation strongly contradict the genetic model.

(b) The 95% confidence interval is

$$\hat{p} \pm z_{\alpha/2} \sqrt{\frac{\hat{p}(1-\hat{p})}{200}} = .68 \pm 1.96 \sqrt{\frac{.68 \times .32}{200}} = (.6153, .7447)$$

6

(a) Let p_1 be the proportions of pets having lymphoma from herbicide-used home and p_2 be the proportions of pets having lymphoma from no herbicide-used home. Our interest is to estimate $p_1 - p_2$. An estimate of $p_1 - p_2$ can be $473/827 - 19/130$.

(b) $H_0 : p_1 - p_2 = 0$ vs $H_1 : p_1 - p_2 > 0$.

(c) $n_1 = 827(> 25)$, $x_1 = 473(> 5)$, and $n_1 - x_1 = 354(> 5)$.
$n_2 = 130(> 25)$, $x_2 = 19(> 5)$, and $n_2 - x_2 = 111(> 5)$.

(d) The pooled estimate under the null hypothesis is

$$\hat{p} = \frac{473 + 19}{827 + 130} = 0.5141,$$

and so the test statistic is

$$z = \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\hat{p}(1-\hat{p}) \left(\frac{1}{n_1} + \frac{1}{n_2} \right)}} = 9.0296$$

where $\hat{p}_1 = \frac{473}{827}$ and $\hat{p}_2 = \frac{19}{130}$. Therefore, p-value is very close to 0.

(e) Since the p-value is less than 0.01, we reject the null hypothesis. Thus, we conclude that there is a strong evidence that pets are more likely to be diagnosed with lymphoma as a result of regular use of herbicides.

(f) The 99% confidence interval is

$$\hat{p}_1 - \hat{p}_2 \pm z_{0.005} \sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \frac{\hat{p}_2(1-\hat{p}_2)}{n_2}} = (0.3345, 0.5171)$$

where $z_{0.005} = 2.575$. The CI would cover $p_1 - p_2$ with 99% certainty in terms of relative frequency. Note that it does not include zero.

7 (a) The 95% confidence interval is $(9.7345, 9.8542)$. Also, using the one-sided test, the p-value is 0.03426 and since it is smaller than 0.05, we reject the null hypothesis.

```
> t.test(gallup$lsalary, mu = 9.85, alternative = 'two.sided',
+        conf.level = 0.95)

        One Sample t-test

data:  gallup$lsalary
t = -1.8238, df = 893, p-value = 0.06851
alternative hypothesis: true mean is not equal to 9.85
95 percent confidence interval:
 9.734527 9.854233
sample estimates:
mean of x
  9.79438
> t.test(gallup$lsalary, mu=9.85, alternative = "less",
+        conf.level = 0.95)

        One Sample t-test

data:  gallup$lsalary
t = -1.8238, df = 893, p-value = 0.03426
alternative hypothesis: true mean is less than 9.85
95 percent confidence interval:
     -Inf 9.844594
sample estimates:
mean of x
  9.79438
```

(b) The 95% confidence interval is $(0.7802, 1.1319)$. Also, since the p-value 0.511 is greater than 0.05, we fail to reject the null hypothesis. That is, we have not enough evidence to say that two variances are not equal.

```
> var.test(lsalary~gend, data = gallup, alternative = 'two.sided',
+          conf.level = 0.95)

        F test to compare two variances

data:  lsalary by gend
F = 0.93942, num df = 427, denom df = 465, p-value = 0.511
alternative hypothesis: true ratio of variances is not equal to 1
95 percent confidence interval:
 0.7802492 1.1319029
sample estimates:
ratio of variances
         0.9394184
```

7 (c) The 95% confidence interval is $(0.3765, 0.6074)$. Since the p-value is very close to 0, we can say that there is a difference in mean between male and female.

```
> t.test(lsalary~gend , data = gallup , var.equal = T, conf.level = 0.95)

        Two Sample t-test

data:  lsalary by gend
t = -8.3636, df = 892, p-value = 2.325e-16
alternative hypothesis: true difference in means between group 0 and group 1
 is not equal to 0
95 percent confidence interval:
 -0.6073840 -0.3765029
sample estimates:
mean in group 0 mean in group 1
       9.537953       10.029896
```

(d) In both exact and approximated test, the p-value is close to 0. Therefore, we reject the null hypothesis.

```
> binom.test(length(gallup$train[gallup$train == 1]),
+            length(gallup$train), p=0.5, alternative='two.sided')

        Exact binomial test

data:  length(gallup$train[gallup$train == 1]) and length(gallup$train)
number of successes = 374, number of trials = 894, p-value =
1.175e-06
alternative hypothesis: true probability of success is not equal to 0.5
95 percent confidence interval:
 0.3857646 0.4514643
sample estimates:
probability of success
             0.4183445

> prop.test(length(gallup$train[gallup$train == 1]),
+           length(gallup$train), p=0.5, alternative='two.sided',
+           correct=TRUE)

        1-sample proportions test with continuity correction

data:  length(gallup$train[gallup$train == 1]) out of length(gallup$train),
 null probability 0.5
X-squared = 23.518, df = 1, p-value = 1.238e-06
alternative hypothesis: true p is not equal to 0.5
95 percent confidence interval:
 0.3858749 0.4515249
sample estimates:
        p
0.4183445
```

8(8.46) Let $H_0 : \sigma \geq 0.1$ vs $H_1 : \sigma < 0.1$. Then the value of the test statistic is $\frac{(n-1)S^2}{0.01} = \frac{49\times 0.0064}{0.01} = 31.36$ and so p-value is

$$P(\chi^2_{49} < 31.36) = 0.023.$$

Since $0.023 < 0.1$, the null hypothesis that $\sigma \geq 0.1$ is rejected and the apparatus can be utilized. Note that the apparatus can be utilized even if we use a significance level as 0.05.