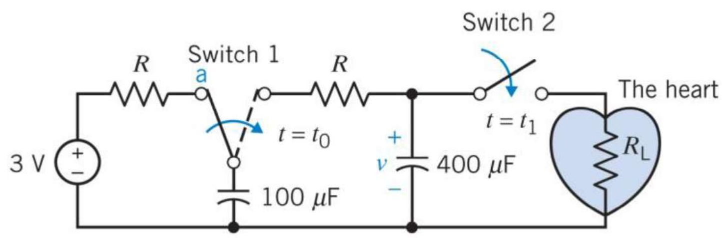
1. **(30 points) Cardiac Pacemakers:** Cardiac pacemakers are used by people to maintain regular heart rhythm when they have a damaged heart. The circuit of a pacemaker can be represented as shown below. The resistance of the wires, R, can be neglected because R is extremely small. The heart's load resistance R_L is 1 kΩ. The first switch is activated at $t = t_0$, and the second switch is activated at $t_1 = t_0 + 10$ ms. Note that the first switch returns to its position *a* when the second switch is activated, and the second switch returns to its open position when the first switch is activated. This cycle is repeated every second. Find $v(t)$ for $t_0 \le t \le t_0 + 1$ s.



Solution)

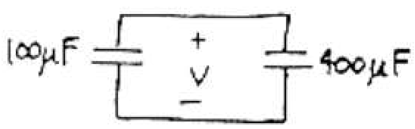
At $t = t_o^-$:

Assume that the circuit has reached steady state so that the voltage across the 100uF capacitor is 3V. The charge stored by the capacitor is

$$q(t_o^-) = (100 \times 10^{-6}\mathrm{F}) \times (3\mathrm{V}) = 300 \times 10^{-6}\ \mathrm{C} \quad (or\ 300\mathrm{uC}) \qquad \text{4pts (a)}$$

$t_o < t < t_o + 10\mathrm{ms}$:

With R is negligibly small, the circuit reaches steady state almost immediately(i.e. at $t = t_o^+$). The voltage across the parallel capacitors is determined by considering charge conservation :



$$q(t_o^+) = (100\mathrm{uF}) \times v(t_o^+) + (400\mathrm{uF}) \times v(t_o^+) = q(t_o^-) \qquad \text{10pts (b)}$$

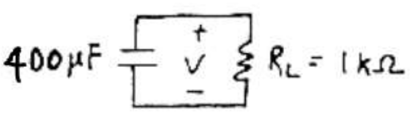
$$v(t_o^+) = \frac{q(t_o^+)}{100 \times 10^{-6}\mathrm{F} + 400 \times 10^{-6}\mathrm{F}}$$

$$= \frac{q(t_o^-)}{500 \times 10^{-6}\mathrm{F}} = \frac{300 \times 10^{-6}\mathrm{C}}{500 \times 10^{-6}\mathrm{F}} = 0.6\ \mathrm{V} \qquad \text{3pts (c)}$$

$t_o + 10\mathrm{ms} < t < t_o + 1\mathrm{s}$:

$$i(t) = \frac{v(t)}{R_L} = -C\frac{dv(t)}{dt} \qquad \text{7pts (d)}$$

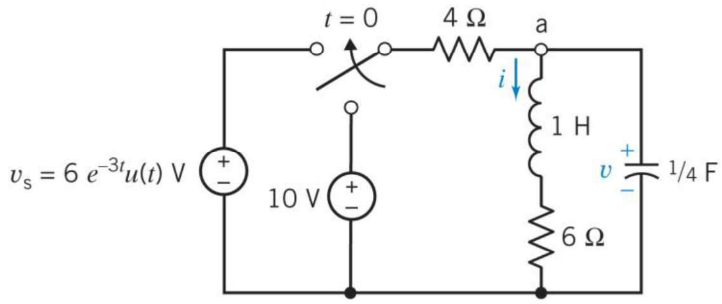
$$v(t) = v(t_o^+)\ e^{-(t - t_0 - 0.01)/RC}$$



$$= 0.6\ e^{-(t - t_0 - 0.01)/(1\mathrm{k}\Omega \times 400\mathrm{uF})}$$

$$\therefore v(t) = 0.6\ e^{-2.5(t - t_0 - 0.01)}\ \mathrm{V} \qquad \text{6pts (e)}$$

2. **(40 points) Complete Response of a Second-Order Circuit:** Find the complete response $v(t)$ including both the natural and forced responses for $t > 0$ for the circuit given below. Assume the circuit is at steady state at $t = 0^-$.



At $\mathrm{t} = 0^-$, $v(0^-) = 6\,V, i(0^-) = 1\,A$

After the switch is thrown,

KVL for the right-hand mesh: $-v + \frac{di}{dt} + 6i = 0$ …… *(i)*

KCL at node a: $\frac{v - v_s}{4} + i + \frac{1}{4}\frac{di}{dt} = 0$ …… *(ii)*

By *(i)*, *(ii)*, $\left(\frac{di}{dt} + 6i\right) - v = 0$ and $i + \left(\frac{v}{4} + \frac{1}{4}\frac{di}{dt}\right) = \frac{v_s}{4}$.

By using $s = d/dt$, $s^2 = d^2/dt^2$, and $1/s = \int dt$,

$(s + 6)i - v = 0,$ …… *(iii)*

$i + \frac{1}{4}(s + 1)v = \frac{v_s}{4}.$ …… *(iv)*

(a)3pts

By *(iii)*, *(iv)*,

$((\mathrm{s} + 6)(\mathrm{s} + 1) + 4)v = (s + 6)v_s$ or $(s^2 + 7\mathrm{s} + 10)v = (s + 6)v_s$.

Hence, the second-order differential equation is $\frac{d^2v}{dt^2} + 7\frac{dv}{dt} + 10v = \frac{dv_s}{dt} + 6v_s$. …… *(v)*

(b)5pts

The characteristic equation is $s^2 + 7\mathrm{s} + 10 = 0$ and the roots are $s_1 = -2$ and $s_2 = -5$.

The natural response is $v_n = A_1\mathrm{e}^{-2t} + A_2\mathrm{e}^{-5t}$. **(c)5pts**

The forced response is assumed to be of the form $v_f = B\mathrm{e}^{-3t}$. **(d)5pts**

By substituting v_f into *(v)*,

$9B\mathrm{e}^{-3t} - 21B\mathrm{e}^{-3t} + 10B\mathrm{e}^{-3t} = -18\mathrm{e}^{-3t} + 36\mathrm{e}^{-3t}$.

Therefore, $B = -9$ and $v_f = -9\mathrm{e}^{-3t}$. **(*e*)5pts**

The complete response is then $v = v_n + v_f = A_1 \mathrm{e}^{-2t} + A_2 \mathrm{e}^{-5t} - 9\mathrm{e}^{-3t}$. …… *(vi)* **(f)5pts**

Because $v(0^-) = 6\,V$, $A_1 + A_2 = 15$.

Because $i(0^-) = 1\,A$ and from *(iv)* $\frac{dv}{dt} = -4i - v + v_s$.

At $t = 0$, $\frac{dv(0)}{dt} = -4i(0) - v(0) + v_s(0) = -4 - 6 + 6 = -4$. …… *(vii)*

From *(vi)*, $\frac{dv}{dt} = -2A_1 \mathrm{e}^{-2t} - 5A_2 \mathrm{e}^{-5t} + 27\mathrm{e}^{-3t}$. …… *(viii)*

By *(vii)*, *(viii)*, $\frac{dv(0)}{dt} = -2A_1 - 5A_2 + 27 = -4$ so $2A_1 + 5A_2 = 31$.

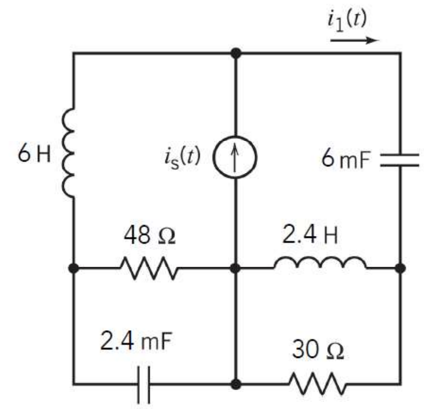
(g)5pts

Since $A_1 = \frac{44}{3}$ and $A_2 = \frac{1}{3}$, **(h)5pts**

$v = \frac{44}{3}\mathrm{e}^{-2t} + \frac{1}{3}\mathrm{e}^{-5t} - 9\mathrm{e}^{-3t}$. **(i)2pts**

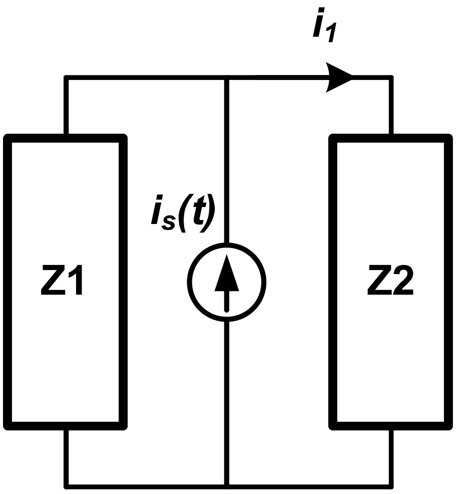
Prob 3.

(30 points) Sinusoidal Steady-State Analysis: The input to the circuit shown below is the current source current $i_s(t) = 30\cos(12t + 30°)$ mA. The output is the current $i_1(t)$. Determine the steady-state response $i_1(t)$.



(1) $\omega = 12$ [+5]

(2) Change circuit as follows, Total [+15]



Where Z1 is

$$j72 + \left(48 \parallel \frac{1}{j(12) \times 2.4 \times 10^{-3}}\right)$$

$$= 16.49 + j49.21 \quad [+7.5]$$

and Z2 is

$$\frac{1}{j(12) \times 6 \times 10^{-3}} + (30 \parallel j(12) \times 2.4)$$

$$= 14.38 + j1.1 \quad [+7.5]$$

(3) Calculate i_1 Total [+10]

$$\overline{i_1} = \overline{i_s} \times \frac{Z1}{Z1 + Z2} = 30\angle 30 \times \frac{16.49 + j(49.21)}{30.87 + j(50.31)} = 19.29 + 17.99j = 26.38\angle 43\ mA \quad [+7]$$

$$\therefore i_1(t) = 26.38\cos(12t + 43^o)\ mA \quad [+3]$$

Wrong answer: **[-3]**

Any calculation error: **[-1]**

No unit (wrong unit): **[-1]**

* If calculation error in 'step 2', and calculate $i_1(t)$ correctly based on step 2

  ➔ 3 points will be deducted in step 3

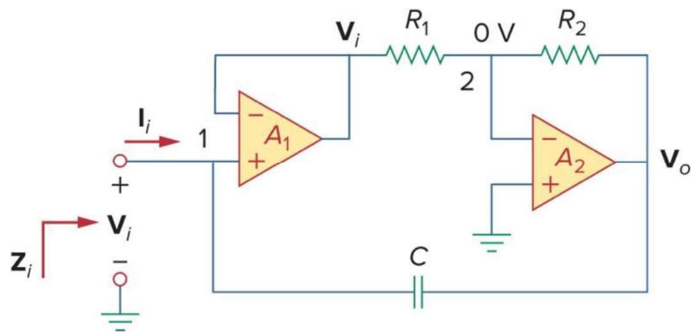
Using different method (Loop, KVL, KCL …)

  ➔ If there exist proper simultaneous equation: **[+5]**

  ➔ Correct answer will get full credit

4. **(40 points) Op Amps in AC Circuits:** Answer the following questions.

   (a) **(20 points) Capacitance Multiplier:** The op amp circuit shown below is known as a capacitance multiplier. Find $\mathbf{Z}_i = \mathbf{V}_i/\mathbf{I}_i$ and explain why it is called a capacitance multiplier.



At node 1, **(+5 Points)**

$$\mathrm{I_i} = \frac{V_i - V_o}{1/j\omega C} = j\omega C(V_i - V_o)$$

At node 2, **(+5 Points)**

$$\frac{V_i - 0}{R_1} = \frac{0 - V_o}{R_2} \Rightarrow V_o = -\frac{R_2}{R_1}V_i$$

By substituting,

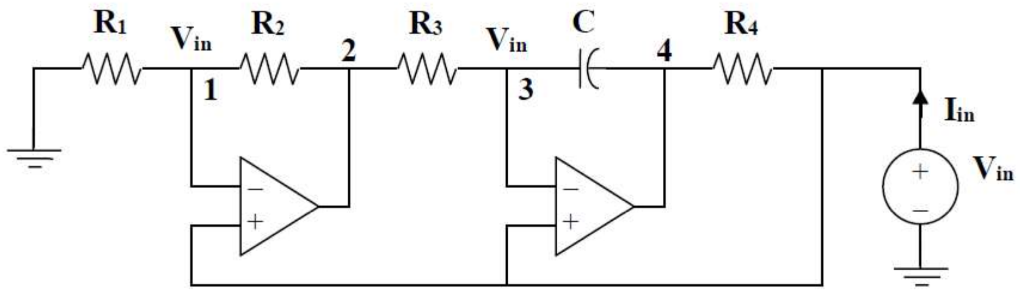
$$\mathrm{I_i} = j\omega C\left(1 + \frac{R_2}{R_1}\right)V_i$$

$$\therefore \frac{I_i}{V_i} = j\omega\left(1 + \frac{R_2}{R_1}\right)C$$

Finally, the input impedance is **(+ 5 Points each)**

$$\mathbf{Z_i} = \frac{\mathbf{V_i}}{\mathbf{I_i}} = \frac{\mathbf{1}}{\boldsymbol{j\omega C_{eq}}}, \boldsymbol{where}\ \boldsymbol{C_{eq}} = \left(\mathbf{1} + \frac{\boldsymbol{R_2}}{\boldsymbol{R_1}}\right)\boldsymbol{C}$$

(b) (20 points) Inductance Simulator: The op amp circuit shown below is known as an inductance simulator. Find $\mathbf{Z}_{in}$ = $\mathbf{V}_{in}/\mathbf{I}_{in}$ and explain why it is called an inductance simulator.



At node 1, **(+5 Points)**

$$\frac{0-\mathrm{V_{in}}}{\mathrm{R_1}}=\frac{V_{in}-V_2}{R_2}$$

At node 3, **(+5 Points)**

$$\frac{\mathrm{V_2}-\mathrm{V_{in}}}{\mathrm{R_3}}=\frac{V_{in}-V_4}{1/j\omega C}$$

By substituting,

$$-\mathrm{V_{in}}+\mathrm{V_4}=\frac{-\mathrm{R_2}}{\mathrm{j\omega}CR_3R_1}V_{in}$$

Thus, **(+5 Points each)**

$$\mathrm{I_{in}}=\frac{\mathrm{V_{in}}-\mathrm{V_4}}{\mathrm{R_4}}=\frac{R_2}{j\omega C\ \ R_3R_1R_4}V_{in}$$

$$\mathrm{Z_{in}}=\frac{V_{in}}{I_{in}}=\frac{j\omega C\ \ R_1R_3R_4}{R_2}=j\omega L_{eq},\ \text{where}\ \ \mathrm{L_{eq}}=\frac{R_1R_3R_4C}{R_2}$$

5. **(30 points) Parallel Loads**: A customer's plant has two parallel loads connected to the power utility's distribution lines. The first load consists of 50 kW of heating and is purely resistive. The second load is a set of motors that operate at 0.86 lagging power factor. The motors' load is 100 kVA. Power is supplied to the plant at 10 kV_{rms}. Determine the total current flowing from the utility's transmission lines into the plant and the plant's overall power factor.

First, consider the heating load. Because the load is purely resistive,

$$S_1 = 50 \text{ kW}$$

Next, consider the motors. The power factor is lagging, so $\theta_2 > 0^\circ$:

$$\boldsymbol{\theta_2 = \cos^{-1}(pf_2) = \cos^{-1}(0.86) = 30.7^\circ}$$ **+ 6 pts**

The complex power absorbed by the motors is

$$\mathbf{S_2 = |S_2| \angle\theta_2 = 100\angle 30.7 \text{ kVA}}$$ **+ 6 pts**

The average power and reactive power absorbed by the motors is obtained by converting the complex power to rectangular form:

$$\mathbf{S_2 = |S_2| \cos \theta_2 + j|S_2|\sin \theta_2 = 100 \cos 30.7^\circ + j100 \sin 30.7^\circ = 86 + j51 \text{ kVA}}$$

Therefore,

$$\mathbf{P_2 = 86 \text{ kW and } Q_2 = 51 \text{ kVAR}}$$

The total complex power S delivered to thet otal load is the sum of the complex power delivered to each load:

$$\mathbf{S = S_1 + S_2 = 50 + (86 + j51) = 136 + j51 \text{ kVA}}$$ **+ 6 pts**

The average power and reactive power of the customer's load is

$$\mathbf{P = 136 \text{ kW and } Q = 51 \text{ kVAR}}$$

To calculate the power factor of the customer's load, first convert S to polar form:

$$\mathbf{S = 145.2 \angle 20.6 \text{ kVA}}$$

Then,

$$\mathbf{pf = \cos 20.6 = 0.94^\circ \text{ lagging}}$$ **+ 6 pts**

The total current flowing from the utility's lines into the plant can be calculated from the apparent power absorbed by the customer's load and the voltage across the terminals of the customer's load. Recall that

$$\mathbf{|S| = V_m I_m / 2 = V_{rms} I_{rms}}$$

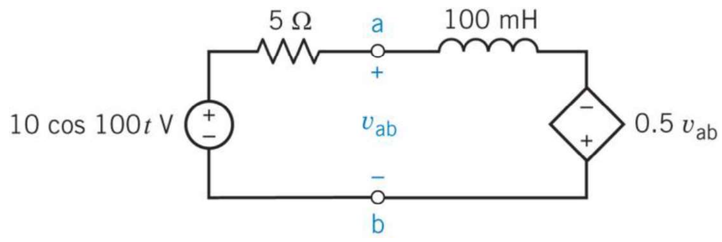
Solving for the current gives

$$\mathbf{I_{rms} = |S| / V_{rms} = 145{,}200 / 10^4 = 14.52 \text{ A rms}}$$ **+ 6 pts**

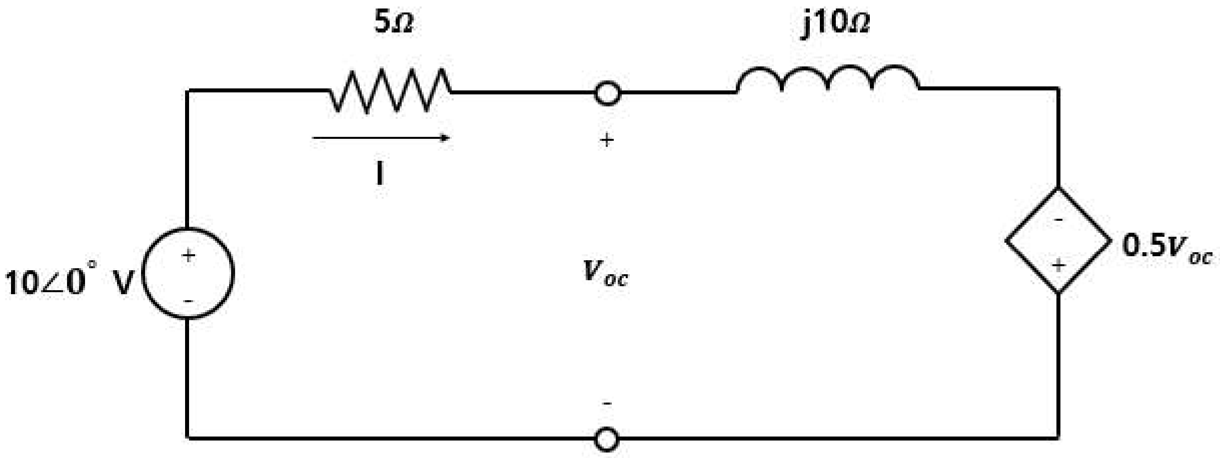
-2 pts for each calculation error

It's okay to answer with currents

6. **(30 points) Maximum Power Transfer:** Answer the following questions.



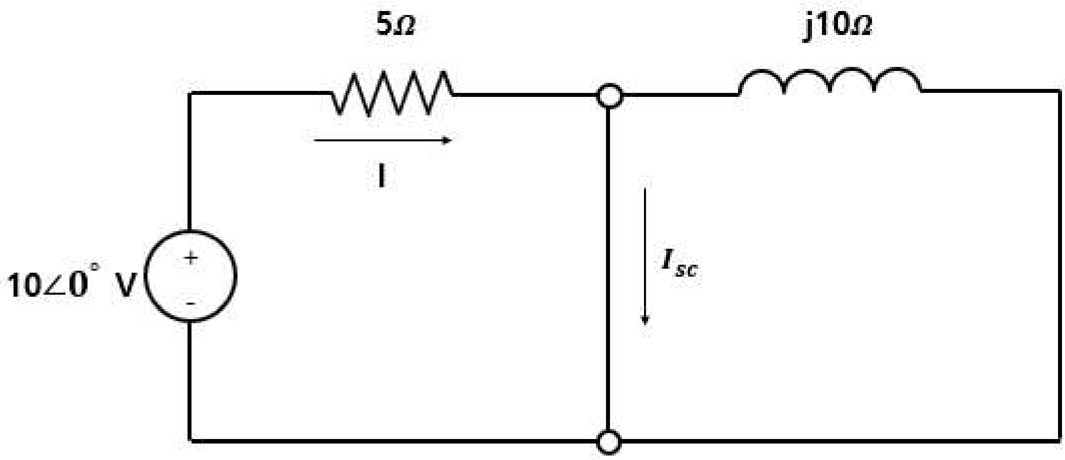
(a) **(15 points)** Determine the load impedance $\mathbf{Z}_{ab}$ that will absorb maximum power if it is connected to terminals a – b of the circuit shown above.



$$-10 + 5\mathrm{I} + j10\mathrm{I} - 0.5V_{oc} = 0 (+3\text{pts})$$

$$\mathrm{I} = \frac{10 - V_{oc}}{5} (+2\text{pts})$$

$$\Rightarrow V_{oc} = 8\angle 36.9^{\circ} = 6.4 + j4.8\ (\mathrm{V}) (+2\text{pts})$$



$$I_{sc} = \frac{10\angle 0^{\circ}}{5} = 2\angle 0^{\circ}\ (A)\color{red}{(+3\text{pts})}$$

$$\text{Thevenin impedance}, Z_t = \frac{V_{oc}}{I_{sc}} = 3.2 + j2.4(\Omega)\color{red}{(+2\text{pts})}$$

$$\text{Maximum power transfer requires } Z_{th} = Z_t^{*} = 3.2 - j2.4(\Omega)\color{red}{(+3\text{pts})}$$

(b) (10 points) Determine the maximum power absorbed by this load.

$$\text{P} = \frac{1}{2}|I_m|^2 Re\{Z_{th}\}\color{red}{(+5pts)} = \frac{1}{2}\frac{64}{\{(3.2 + j2.4) + (3.2 - j2.4)\}^2}3.2 = 2.5W\ \color{red}{(+5pts)}$$

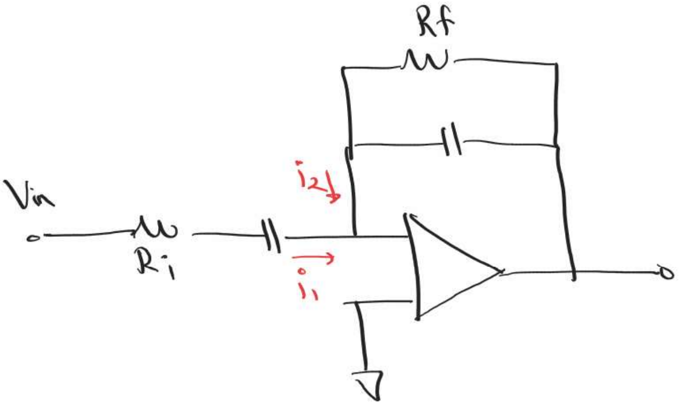
(c) (5 points) Determine a model of this load and indicate the element values.

$$Z_L\ can\ be\ implemented\ as\ the\ series\ combination\ of\ a\ resistor\ and\ a\ capacitor$$

$$with\ R = 3.2\Omega\ \color{red}{(+2pts)}\ and\ C = \frac{1}{100 \times 2.4} = 4.17mF\ \color{red}{(+3pts)}$$

7.

(a)



$$0 = \underbrace{\boxed{V_{in}\frac{1}{R_i+\frac{1}{sC_i}}}}_{} + \boxed{V_{out}\frac{1}{R_f \parallel \frac{1}{sC_f}}}$$

$0 = i_1 + i_2$

... KVL or KCL equation 5 points

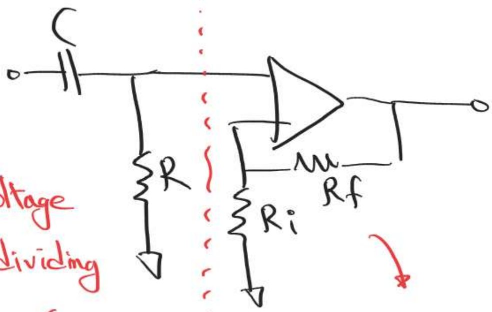
$$\Rightarrow \frac{V_{out}}{V_{in}} = -\frac{R_f \parallel \frac{1}{sC_f}}{R_i+\frac{1}{sC_i}}$$

$$= -\frac{sR_fC_i}{(1+sR_iC_i)(1+sR_fC_f)}$$ ... 10 points

pole : 2 zero : 1

$\Rightarrow$ Bandpass filter ... 5 points

(b)



voltage dividing

non-inverting amplifier.

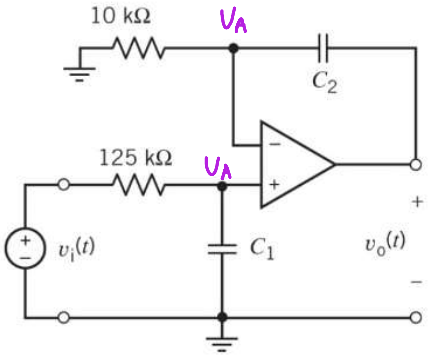
$$\boxed{\frac{R}{1/sC+R}} \times \boxed{\left(1+\frac{R_f}{R_i}\right)}$$ ... KVL or KCL equation 5 points

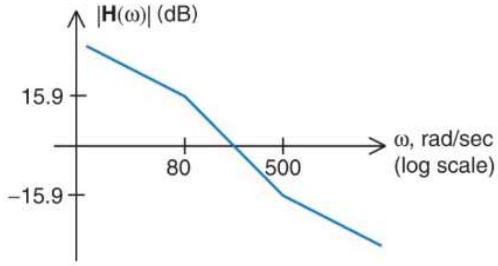
$$\Rightarrow \left(1+\frac{R_f}{R_i}\right)\frac{sRC}{1+sRC}$$ ... 10 points

$\therefore$ pole $= 1/RC$, zero $= 0$.

$\Rightarrow$ High-pass filter. ... 5 points

8. **(40 points) Bode Plot of a Circuit:** Consider the circuit shown below. The input to the circuit is the source voltage $v_i(t)$. The output is the node voltage at the output terminal of the op amp $v_o(t)$. The network function that represents this circuit is $\mathbf{H}(\omega) = \mathbf{V}_o/\mathbf{V}_i$ and the corresponding magnitude Bode plot is also shown below. Determine the values of the capacitances C_1 and C_2.





[Transfer function] 16

$$V_A = V_i \frac{\frac{1}{j\omega C_1}}{125k + \frac{1}{j\omega C_1}} = \frac{1}{j\omega(125k \cdot C_1) + 1} \qquad 8$$

$$V_o = V_A \cdot \frac{10k + \frac{1}{j\omega C_2}}{10k} = \frac{j\omega(10k \cdot C_2) + 1}{j\omega(10k\ C_2)} \qquad 8$$

$$\frac{V_o}{V_i}(\omega) = \frac{1 + j\omega(10k \cdot C_2)}{j\omega(10k \cdot C_2)\left[1 + j\omega(125k \cdot C_1)\right]}$$

[Pole & zeros] 16

pole : $0,\ \frac{1}{125k \cdot C_1}$ (rad/s) 8

Zero : $\frac{1}{10k \cdot C_2}$ (rad/s) 8

* Just write $H(s) = \frac{A(1 + s \cdot 500)}{s(1 + s \cdot 80)}$ with all wrong answers +8

[C_1 & C_2] 8

$$\frac{1}{125k \cdot C_1} = 80 \Rightarrow C_1 = 100\,nF \qquad 4$$

3 1

$$\frac{1}{10k \cdot C_2} = 500 \Rightarrow C_2 = 200\,nF \qquad 4$$

3 1