2.

(a) A & B are mutually exclusive $\Rightarrow$ $P(A \cap B) = 0$.

Since $P(A), P(B) > 0$, $P(A \cap B) \neq P(A)P(B) > 0$ $\therefore$ not independent.

(b) $A \perp B \Leftrightarrow P(A)P(B) = P(A \cap B)$ $\iff 1 - P(A) - P(B) + P(A)P(B) = 1 - P(A) - P(B) + P(A \cap B)$

$\iff (1 - P(A))(1 - P(B)) = 1 - P(A \cup B)$ $\iff P(A^c)P(B^c) = P((A \cup B)^c) = P(A^c \cap B^c)$

$\iff A^c \perp B^c$

(c) $\iff 1 \geq P(A) + P(B) - P(A \cap B) = P(A \cup B)$. $\Big\}$ axiom.

(d) $\iff P(A) + P(B) - P(A \cap B) \leq P(A) + P(B) \iff 0 \leq P(A \cap B)$

4.

F: two fails from five tests.

$$P(III \mid F) = \frac{P(F \mid III) \cdot P(III)}{P(F \mid III)P(III) + P(F \mid II)P(II) + P(F \mid I)P(I)}$$

$\underbrace{5C_2 \cdot \left(\frac{3}{10}\right)^2 \left(\frac{7}{10}\right)^3}$ $\underbrace{5C_2 \cdot \left(\frac{2}{10}\right)^2 \left(\frac{8}{10}\right)^3}$ $\underbrace{5C_2 \left(\frac{1}{10}\right)^2 \left(\frac{9}{10}\right)^3}$

$$\approx 0.5264$$

6.

P(defective) = 0.1 , P(Inspection | defective) = 0.6, P(Inspection | defective^c) = 0.2

P(defective | Inspection)

$$= \frac{P(Inspection \mid defective) \, P(defective)}{P(Inspection \mid defective) \, P(defective) + P(Inspection \mid defective^c) \, P(defective^c)}$$

$$= \frac{0.6 \cdot 0.1}{0.6 \cdot 0.1 + 0.2 \cdot 0.9} = \frac{1}{4}$$

8.

A B C D
       ‿ ‿
     defective

(a)

Y	0	1	2	3	4
P(Y)	0	0	4/24	8/24	12/24

(b) $E[Y] = \dfrac{8+24+48}{24} = \dfrac{10}{3}$, $E[(Y-\frac{10}{3})^2] = V[Y]$

$E[Y^2] = \dfrac{16+72+192}{24} = \dfrac{35}{3}$, $V[Y] = \dfrac{105-100}{9} = \underline{\dfrac{5}{9}}$ //

10.

(A) $P(Y=y) = \left(\dfrac{5}{6}\right)^{y-1} \cdot \dfrac{1}{6}$, $y = 1,2,3,\cdots$

$\llcorner_{\#}$ # of trials.

(b) $\displaystyle\sum_{y=4}^{\infty} P(Y=y) = \dfrac{1}{6}\sum_{y=4}^{\infty}\left(\dfrac{5}{6}\right)^{y-1} = \dfrac{1}{6}\cdot\dfrac{\left(\frac{5}{6}\right)^3}{1-\frac{5}{6}} = \left(\dfrac{5}{6}\right)^3$

Probability of rolling the dice at least 4 times

(c) $\dfrac{P(Y>6 \cap Y>3)}{P(Y>3)} = \dfrac{P(Y>6)}{P(Y>3)} = \dfrac{\left(\frac{5}{6}\right)^6}{\left(\frac{5}{6}\right)^3} = \left(\dfrac{5}{6}\right)^3$

Probability of rolling the dice at least n times   when we know
the dice is rolled at least 4 times.

12.

(a) $_5C_1 \cdot \left(\frac{3}{4}\right)^4 \left(\frac{1}{4}\right)^1 = \frac{405}{1024}$

(b) $\dfrac{\binom{5}{1}\binom{15}{4}}{\binom{20}{5}} = \dfrac{\frac{5 \times 15 \times 14 \times 13 \times 12}{4 \times 3 \times 2 \times 1}}{\frac{20 \times 19 \times 18 \times 17 \times 16}{5 \times 4 \times 3 \times 2 \times 1}}$

$$= \frac{5 \times 15 \times 14 \times 13 \times 12 \times \cancel{5}}{19 \times 18 \times 17 \times 16 \times \cancel{4}} = \frac{6825}{15504}$$

14.

(a) $P(Y = y) = \frac{1}{N}$, $\quad y = 1, \cdots, N$

(b) $E(Y) = \sum_1^N y \cdot \frac{1}{N} = \frac{N+1}{2}$, $\quad E(Y^2) = \sum_1^N y^2 \cdot \frac{1}{N} = \frac{(N+1)(2N+1)}{6}$

$$V(Y) = E(Y^2) - (E(Y))^2 = \frac{N+1}{2}\left(\frac{2N+1}{3} - \frac{N+1}{2}\right) = \frac{(N+1)(N-1)}{12}$$

16.

$f(y) = \dfrac{\lambda^y e^{-\lambda}}{y!}$, $\quad y = 0, 1, 2, \cdots$

$E[e^Y] = \displaystyle\sum_{y=0}^{\infty} e^y \frac{\lambda^y e^{-\lambda}}{y!} = \sum_{y=0}^{\infty} \frac{(e\lambda)^y \cdot e^{-\lambda}}{y!}$

$$= \sum_{y=0}^{\infty} \frac{(e\lambda)^y \, e^{-e\lambda} \cdot e^{(e-1)\lambda}}{y!} = e^{(e-1)\lambda} \underbrace{\sum_{y=0}^{\infty} \frac{(e\lambda)^y \, e^{-e\lambda}}{y!}}_{= 1, \text{ since } Y \sim \text{Poisson} \, (e\lambda)}$$

$$= e^{(e-1)\lambda}$$

18.

(a)
$$F'(y) = f(y) = \begin{cases} 0 & y \leq 0 \\ \frac{1}{12} & 0 < y < 3 \\ \frac{y}{18} & 3 \leq y < 6 \\ 0 & y \geq 6 \end{cases}$$

(b)
$$P(Y \geq 1 \mid Y \leq 3) = \frac{P(1 \leq Y \leq 3)}{P(Y \leq 3)}$$

$$= \frac{\frac{3}{12} - \frac{1}{12}}{\frac{3}{12}} = \frac{2}{3}$$

(c)
$$E(Y) = \int_0^3 \frac{1}{12} y \, dy + \int_3^6 \frac{y}{18} \cdot y \, dy = \left[\frac{y^2}{24}\right]_0^3 + \left[\frac{y^3}{54}\right]_3^6 = \frac{3}{8} + \frac{7}{2} = \frac{31}{8}$$

20.

$$P(Y=y) = \frac{5^y e^{-5}}{y!} \quad (y=0,1,2,\cdots) \quad \cdots \circledast$$

(a) $$P(Y>5 \mid Y>2) = \frac{P(Y>5)}{P(Y>2)} = \frac{1-P(Y=0,1,2,3,4,5)}{1-P(Y=0,1,2)} \longrightarrow \text{by } \circledast$$

(b) $$P(Y>1) = 1-P(Y=0)-P(Y=1) = 1-e^{-5}-5e^{-5} = 1-6e^{-5}$$

(c) $$E[C] = E[3^{Y+1}] = 3E[3^Y]$$

$$\sum_{y=0}^{\infty} 3^y \cdot \frac{5^y e^{-5}}{y!} = e^{10} \sum_{y=0}^{\infty} \frac{15^y e^{-15}}{y!} = e^{10}$$

(d) $$\sum_{y=0}^{\infty} e^{(2y+1)t} \cdot P(Y=y) = e^t \sum_{y=0}^{\infty} \frac{(5 \cdot e^{2t})^y \, e^{-5}}{y!} = e^{5e^{2t}+t-5} \underbrace{\sum_{y=0}^{\infty} \frac{(5e^{2t})^y \cdot e^{-5e^{2t}}}{y!}}_{=1}$$

$$\therefore e^{5e^{2t}+t-5}$$

(e) $$M_W'(0) = (10 e^{2t}+1) e^{5e^{2t}+t-5} \Big|_{t=0} = 11$$

22.

(a) $$\int_0^x f(y)dy = \tfrac{1}{2}x^2 \text{ when } 0\le x \le 1, \quad \int_0^x f(y)dy = \int_0^1 y\,dy + \int_1^x 1\,dy$$

$$F(x) \qquad\qquad\qquad F(x) \qquad = \tfrac{1}{2} + (x-1) \text{ when } 1 \le x \le 1.5$$

$$\therefore \; F(y) = \begin{cases} 0 & y<0 \\ \tfrac{1}{2}y^2 & 0\le y \le 1 \\ y-\tfrac{1}{2} & 1<y\le 1.5 \\ 1 & 1.5<y \end{cases}$$

(b) $$P(Y\le 1.2)-P(Y\le 0.5) = 0.7 - 0.125 = 0.575$$

(c) $$E[Y] = \int_0^1 y^2\,dy + \int_1^{1.5} y\,dy = \tfrac{1}{3} + \tfrac{5}{8} = \tfrac{23}{24}$$

24.

Let $T \sim \exp(\lambda)$, $\quad f(t) = \begin{cases} \frac{1}{\lambda} e^{-\frac{t}{\lambda}} & t \geq 0 \\ 0 & t < 0 \end{cases}$ ← parameter of distribution

$F(t) = \int_0^t \frac{1}{\lambda} e^{-\frac{1}{\lambda} x} \, dx = \left[-e^{-\frac{1}{\lambda} x} \right]_0^t = 1 - e^{-\frac{1}{\lambda} t}$

$r(t) = \frac{f(t)}{1 - F(t)} = \frac{\frac{1}{\lambda} e^{-\frac{1}{\lambda} t}}{e^{-\frac{1}{\lambda} t}} = \frac{1}{\lambda} \quad \text{constant}$

26.

Inter arrival time $T \sim \exp\left(\frac{1}{10}\right)$, $\quad f(t) = \begin{cases} 10 \, e^{-10t} & t \geq 0 \\ 0 & t < 0 \end{cases}$

$P\left(T > \frac{1}{4}\right) = \int_{\frac{1}{4}}^{\infty} 10 \, e^{-10t} \, dt = \left[-e^{-10t} \right]_{\frac{1}{4}}^{\infty} = \boxed{e^{-\frac{5}{2}}}$

↓
minute ⇒ hour

28.

$f(y) = \frac{1}{\sqrt{2\pi}\, b} e^{-\frac{(y-M)^2}{2b^2}}$, $\quad E(|Y - M|) = \int_{-\infty}^{\infty} |Y - M| f(y) \, dy = 2 \int_{M}^{\infty} (y - M) f(y) \, dy$

$= 2 \int_{M}^{\infty} (y - M) f(y) \, dy = 2 \int_{M}^{\infty} \frac{y - M}{\sqrt{2\pi}\, b} \cdot e^{-\frac{(y-M)^2}{2b^2}} \, dy = 2 \int_{0}^{\infty} \frac{t}{\sqrt{2\pi}\, b} e^{-\frac{t^2}{2b^2}} \, dt$

$= \sqrt{\frac{2}{\pi}} \cdot \frac{1}{b} \int_{0}^{\infty} t \, e^{-\frac{t^2}{2b^2}} \, dt$, $\quad$ Let $t^2 = k \rightarrow 2t\,dt = dk$, $t\,dt = \frac{dk}{2}$

$= \frac{1}{\sqrt{2\pi}} \cdot \frac{1}{b} \int_{0}^{\infty} e^{-\frac{k^2}{2b^2}} \, dk = \frac{1}{\sqrt{2\pi}\, b} \left[-2b^2 \cdot e^{-\frac{k}{2b^2}} \right]_{0}^{\infty} = \boxed{\sqrt{\frac{2}{\pi}} \cdot b}$

30.

(a) $P(Y=y) = q^{y-1} p$, $y = 1, 2, \cdots$

$$M_Y(t) = \sum_{y=1}^{\infty} e^{ty} q^{y-1} \cdot p = \frac{P}{q} \sum_{y=1}^{\infty} (e^t q)^y \quad \rightarrow \text{converses when}$$

$$\underbrace{-1 < e^t q < 1}_{\text{trivial}}$$

$$= \frac{P}{q} \cdot \frac{e^t q}{1 - e^t q} = \frac{Pe^t}{1 - qe^t} \qquad \underbrace{}_{t < -\ln q}$$

(b)

$$M_W(t) = e^{-t} M_Y(t) = P/(1 - qe^t)$$

$$\underbrace{}_{> 0}, \quad t < -\ln q$$

(c) $M_W'(0) = \left. \dfrac{P q e^t}{(1 - qe^t)^2} \right|_{t=0} = \dfrac{Pq}{P^2} = \dfrac{q}{P}$

(d) $M_W''(0) = \left. \dfrac{P q e^t (1 - qe^t)^2 + P q e^t \cdot 2(1 - qe^t) qe^t}{(1 - qe^t)^4} \right|_{t=0} = \dfrac{P^3 q + 2P^2 q^2}{P^4} = \dfrac{Pq + 2q^2}{P^2}$

$$\dfrac{Pq + 2q^2}{P^2} - \dfrac{q^2}{P^2} = \dfrac{Pq + q^2}{P^2} = \dfrac{q(P+q)}{P^2} = \dfrac{q}{P^2}$$

32.

(a) $\displaystyle \int_0^{\infty} cy e^{-y^2} dy = 1$, $\displaystyle \int_0^{\infty} y e^{-y^2} dy = \frac{1}{2} \int_0^{\infty} e^{-t} dt = \frac{1}{2} \left[-e^{-t} \right]_0^{\infty} = \frac{1}{2}$

$\therefore C = 2$

(b) $\displaystyle \int_0^y f(x)\, dx = 2 \int_0^y y e^{-y^2} dy = \left[-e^{-t} \right]_0^y = 1 - e^{-y}$

$\therefore F(y) = \begin{cases} 0 & y < 0 \\ 1 - e^{-y} & 0 \leq y \leq 1 \\ 1 & 1 \leq y \end{cases}$

(c) $F(y) = 0.5$ when $1 - e^{-y} = 0.5$, $y = -\ln 0.5 = \boxed{\ln 2}$

34.

Same as 28.

36.

(a) X : # of policies.

$E[X] = 9, \ V[X] = 9, \ 6[X] = 3$

$\Rightarrow P(3 \leq x \leq 15) = P(|X-9| < 2 \cdot 3)$

$= P(|X - E[X]| < 2 \cdot 6[X])$

$\geq 1 - \dfrac{1}{4} = \dfrac{3}{4}$.

$\therefore P(3 \leq x \leq 15) \geq \dfrac{3}{4}$

(b) $P(3 \leq x \leq 15) = P(X=3) + \cdots + P(X=15)$ where

$P(X=x) = \dfrac{9^x e^{-9}}{x!}$. It would be larger than (a) since (a) is lower bound.

38.

$P(T > t) \Longleftrightarrow P(N(t) = 0 \ \text{or} \ N(t)=1) = e^{-\lambda t} + \lambda t \, e^{-\lambda t}$

Since $P(N(t) = k) = \dfrac{(\lambda t)^k e^{-\lambda t}}{k!}$

$F(t) = P(T \leq t) = 1 - P(T > t) = 1 - e^{-\lambda t} - \lambda t \, e^{-\lambda t}$

$\Rightarrow f(t) = \dfrac{d}{dt} F(t) = \lambda e^{-\lambda t} - (\lambda e^{-\lambda t} - \lambda^2 t \, e^{-\lambda t}) = \lambda^2 t \, e^{-\lambda t} \quad (t \geq 0)$

40

(a) $f(y) = 1$, $0 \leq y \leq 1$

$$\int_0^1 e^{ty} \cdot 1 \, dy = \int_0^1 e^{-2t \ln y} \, dy = \int_0^1 e^{\ln(y)^{-2t}} \, dy = \int_0^1 y^{-2t} \, dy$$

$$= \frac{1}{1-2t} \left[y^{1-2t} \right]_0^1 = \frac{1}{1-2t} \quad \text{when} \quad 1-2t \neq 0, \ t \neq \frac{1}{2}$$

(b) <u>mgf of Gamma</u>: $(1-\beta t)^{-\alpha}$ when $t < \frac{1}{\beta}$

$$\Rightarrow \alpha = 1, \ \beta = 2$$

by uniqueness of mgf.

$$f(y) = \frac{y^{\alpha-1} e^{-y/\beta}}{\beta^\alpha \Gamma(\alpha)}, \qquad \int_0^\infty e^{ty} f(y) \, dy = \int \frac{y^{\alpha-1} e^{\left(\frac{t\beta-1}{\beta}\right) y}}{\beta^\alpha \Gamma(\alpha)} \, dy$$

$$= \frac{\left(\frac{\beta}{1-t\beta}\right)^\alpha}{\beta^\alpha} \int \underbrace{\frac{y^{\alpha-1} e^{\frac{t\beta-1}{\beta} y}}{\left(\frac{\beta}{1-t\beta}\right)^\alpha \Gamma(\alpha)} \, dy}_{\text{when } t\beta-1 < 0} = (1-\beta t)^{-\alpha}$$

$$\hookrightarrow \text{Gamma}\left(\alpha, \frac{\beta}{1-t\beta}\right)$$

42.

(a)

$$f_{Y_1}(y_1) = \int_{-y_1}^{y_1} f(y_1,y_2)\,dy_2 = \frac{1}{8}e^{-y_1}\int_{-y_1}^{y_1}(y_1^2-y_2^2)\,dy_2 = \frac{1}{8}e^{-y_1}\left[y_1^2 y_2 - \frac{1}{3}y_2^3\right]_{-y_1}^{y_1}$$

$$= \frac{1}{8}e^{-y_1}\left(2y_1^3 - \frac{2}{3}y_1^3\right) = \frac{1}{6}e^{-y_1}\cdot y_1^3 , \qquad 0 \le y_1 < \infty$$

$$f_{Y_2}(y_2) = \int_{-y_2}^{\infty} f(y_1,y_2)\,dy_1 = \underbrace{\int_{y_2}^{\infty}\frac{1}{8}y_1^2 e^{-y_1}\,dy_1}_{\frac{1}{8}e^{y_2}y_2^2 + \frac{1}{4}(e^{y_2}-e^{y_2}y_2)} - y_2^2\underbrace{\int_{-y_2}^{\infty}\frac{1}{8}e^{-y_1}\,dy_1}_{\frac{1}{8}e^{y_2}} = \frac{1}{4}e^{y_2}(1-y_2)$$

$$\text{when } y_2 < 0$$

(b)

$$\int_{y_2}^{\infty} f(y_1,y_2)\,dy_1 = \frac{1}{8}e^{-y_2}y_2^2 + \frac{1}{4}(e^{-y_2}+e^{-y_2}y_2) - \frac{1}{8}e^{-y_2}y_2^2 = \frac{1}{4}e^{-y_2}(1+y_2)$$

$$\text{when } y_2 > 0$$

$$\therefore f_{Y_2}(y_2) = \begin{cases} \frac{1}{4}e^{y_2}(1-y_2) , & y_2 < 0 \\[2mm] \frac{1}{4}e^{-y_2}(1+y_2) , & y_2 \ge 0 \end{cases}$$

(b)

no, since $f(y_1,y_2) \ne f(y_1,y_2)$.

(c)

$$f(y_2|y_1) = \frac{f(y_1,y_2)}{f(y_1)} = \frac{3(y_1^2-y_2^2)}{4y_1^3} , \qquad -y_1 < y_2 < y_1$$

(d)

$$P(Y_2 \le 1 \mid Y_1 = 2) = \int_{-2}^{1} \frac{3(4-y_2^2)}{32}\,dy_2 = \frac{3}{32}\left[4y_2 - \frac{1}{3}y_2^3\right]_{-2}^{1}$$

$$= \frac{3}{32}\left[12 - \frac{1}{3}\cdot 9\right] = \frac{27}{32}$$

44.

$$f(y_1) = \int_0^{y_1} \frac{1}{4} e^{-y_1/2} \, dy_2 = \frac{1}{4} y_1 e^{-y_1/2}, \qquad 0 \le y_1 < \infty$$

$$f(y_2) = \int_{y_2}^{\infty} \frac{1}{4} e^{-y_1/2} \, dy_1 = \left[-\frac{1}{2} e^{-y_1/2} \right]_{y_2}^{\infty} = \frac{1}{2} e^{-y_2/2}, \qquad 0 \le y_2 < \infty$$

$$E[Y_1] = \int_0^{\infty} \frac{1}{4} y_1^2 e^{-y_1/2} \, dy_1 = 4, \qquad E[Y_2] = \int_0^{\infty} \frac{1}{2} y_2 e^{-y_2/2} = 2$$

$$E[Y_1 Y_2] = \frac{1}{4} \int_0^{\infty} \int_{y_2}^{\infty} y_1 y_2 e^{-y_1/2} \, dy_1 dy_2 = 12$$

$$\llcorner \!\!\!\rightarrow = y_2 \int_{y_2}^{\infty} y_1 e^{-y_1/2} \, dy_1 = \left[-2e^{-\frac{y_1}{2}} y_1 - 4e^{-\frac{y_1}{2}} \right]_{y_2}^{\infty} \cdot y_2$$

$$= \left(2e^{-\frac{y_2}{2}} y_2 + 4e^{-\frac{y_2}{2}} \right) \cdot y_2$$

$$Cov(Y_1, Y_2) = E[Y_1 Y_2] - E[Y_1]E[Y_2] = 4$$

46.

GPA: $X \sim N(2.4, 0.8^2)$, Let $Z = \frac{X - 2.4}{0.8}$, then $Z \sim (0, 1^2)$

$$P(Z < 1.645) = 0.95 \quad \langle \Rightarrow \rangle \quad P\left(\frac{X-2.4}{0.8} < 1.645 \right) = 0.95$$

∴ 95 percentile : $2.4 + 0.8 \cdot 1.645 = 3.716$

48.

(a)

when $0 \le y_1 \le 1$, $y_1 - 1 \le y_2 \le 1 - y_1$, $f(y_1) = \int_{y_1-1}^{1-y_1} \frac{1}{2} dy_2 = \left[\frac{1}{2}y_2\right]_{y_1-1}^{1-y_1} = 1 - y_1$

$\Rightarrow f(y_1) = 1 - y_1$ when $0 \le y_1 \le 1$

when $-1 \le y_1 < 0$, $-1-y_1 \le y_2 \le 1+y_1$, $f(y_1) = \int_{-1-y_1}^{1+y_1} \frac{1}{2} dy_2 = 1 + y_1$

$\therefore f(y_1) = \begin{cases} 1+y_1, & -1 \le y_1 < 0 \\ 1-y_1, & 0 \le y_1 \le 1 \end{cases}$

by symmetry, $f(y_1) = f(y_2)$.

(b)

$f(y_1, y_2) \ne f(y_1) f(y_2) \Rightarrow$ not Independent.

(c)

$E[Y_1 Y_2] = \underbrace{\int_{-1}^{1} \int_{|y_2|-1}^{1-|y_2|} \frac{1}{2} y_1 y_2 \, dy_1 \, dy_2 = 0}_{=0 \text{ Since } y_1 \to \text{odd function}, \ 1-|y_2| + |y_2|-1 = 0}$

$E[Y_1] = \int_{-1}^{0} (1+y_1) y_1 \, dy_1 + \int_{0}^{1} (1-y_1) y_1 \, dy_1 = 0 = E[Y_2]$

$\therefore \text{Cov}(Y_1, Y_2) = E[Y_1 Y_2] - E[Y_1] E[Y_2] = 0.$

(d)

$E[Y_1^2 + 4Y_1 Y_2 + 4Y_2^2] - E[Y_1 + 2Y_2]^2 = E[Y_1^2] + 4E[Y_2^2] = 5E[Y_1^2]$

Since $E[Y_1 Y_2] = E[Y_1] = E[Y_2] = 0$ & $E[Y_1^2] = E[Y_2^2]$

$E[Y_1^2] = \int_{-1}^{0} y_1^2 (1+y_1) + \int_{0}^{1} y_1^2 (1-y_1) = \frac{1}{6}$

(e)

$$f(Y_2 \mid Y_1 = y_1) = \begin{cases} \dfrac{1}{2(1+y_1)}, & -1-y_1 \le y_2 \le 1+y_1 & \text{if } y_1 < 0 \\[3mm] \dfrac{1}{2(1-y_1)}, & -1+y_1 \le y_2 \le 1-y_1 & \text{if } y_1 \ge 0 \end{cases}$$

$$E[Y_2 \mid Y_1 = y_1] = \begin{cases} \displaystyle\int_{-1-y_1}^{1+y_1} \dfrac{y_2}{2+2y_1} dy_2 = 0 \\[6mm] \qquad\qquad\qquad \text{since } y_2 \text{ i odd function} \\ \qquad\qquad\qquad\quad \& \text{ symmetric Interval} \\[4mm] \displaystyle\int_{-1+y_1}^{1-y_1} \dfrac{y_2}{2-2y_1} dy_2 = 0 \end{cases}$$

$$= 0 \qquad \underbrace{\forall\, y_1}_{\text{Possible}}$$

50.

$P(X=2, Y=3) = 0$, $P(X=2) \ne 0$, $P(Y=3) \ne 0$

$\Rightarrow P(X=2, X=3) \ne P(X=2) P(X=3)$ $\Rightarrow$ not Independent   (a)

(b)

V	1	2	3	
2	$\frac{1}{12}$	$\frac{1}{6}$	$\frac{1}{12}$	$\frac{1}{3}$
V 3	$\frac{1}{12}$	$\frac{1}{6}$	$\frac{1}{12}$	$\frac{1}{3}$
4	$\frac{1}{12}$	$\frac{1}{6}$	$\frac{1}{12}$	$\frac{1}{3}$
	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{4}$	