EE201 Circuit Theory (Fall 2018)
Mid-Term Exam.

(Total: 190 Points / 8 Problems)

Student ID Number:

Name:

Prob. 1	Prob. 2	Prob. 3	Prob. 4	Prob. 5	Prob. 6	Prob. 7	Prob. 8	Total
/10	/20	/20	/50	/20	/20	/35	/15	/185

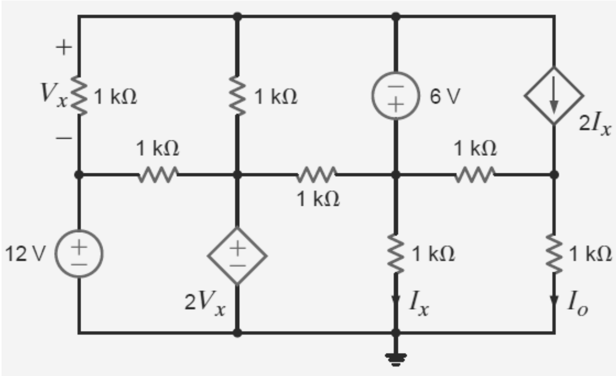
1. **(10 points)** The charge entering the positive terminal of an element is given by the expression $q(t) = -12e^{-2t}$ mC. The power delivered to the element is $p(t) = 2.4e^{-3t}$ W. Compute the current in the element, the voltage across the element, and the energy delivered to the element in the time interval $0 < t < 100$ ms.

$$i(t) = \frac{dq(t)}{dt} = \frac{d(-12e^{-2t})}{dt} = (-2) \times (-12e^{-2t}) = 24e^{-2t}\ \mathrm{mA}$$

$$v(t) = \frac{p(t)}{i(t)} = \frac{(2.4e^{-3t}\ \mathrm{W})}{(24e^{-2t}\ \mathrm{mA})} = 100e^{-t}\ \mathrm{V}$$

$$w(100\ \mathrm{ms}) = \int_0^{100\ \mathrm{ms}} p(t)dt = \int_0^{100\ \mathrm{ms}} 2.4e^{-3t}dt = -\frac{2.4}{3}e^{-3t}\Big|_0^{100\ \mathrm{ms}} = -\frac{2.4}{3}\left[e^{-300\times 10^{-3}} - e^{0}\right] = 207\ \mathrm{mJ}$$

2. (20 points) Find I_o in the network shown below using nodal analysis.



Since the network has six nodes, five linear independent equations are needed to determine the unknown node voltages.

The equation for the supernode are

$$V_1 - V_4 = -6$$

$$\frac{V_1 - V_2}{1\text{k}} + \frac{V_1 - V_3}{1\text{k}} + 2I_x + \frac{V_4 - V_3}{1\text{k}} + \frac{V_4}{1\text{k}} + \frac{V_4 - V_5}{1\text{k}} = 0$$

The three remaining equations are

$$V_2 = 12$$

$$V_3 = 2V_x$$

$$\frac{V_5 - V_4}{1\text{k}} + \frac{V_5}{1\text{k}} = 2I_x$$

The equations for the control parameters are

$$V_x = V_1 - 12$$

$$I_x = \frac{V_4}{1\text{k}}$$

Combining these equations yields the following set of equations:

$$-2V_1 + 5V_4 - V_5 = -36$$

$$V_1 - V_4 = -6$$

$$-3V_4 + 2V_5 = 0$$

Solving these equations yields

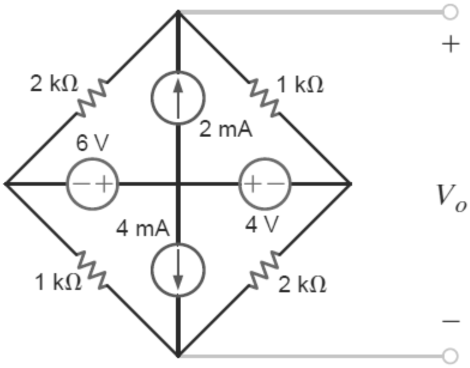
$$V_1 = -38\text{ V}$$

$$V_4 = -32\text{ V}$$

$$V_5 = -48\text{ V}$$

Then, since $V_3 = 2V_x$, $V_3 = -100$ V. I_o is -48 mA.

3. (20 points) Find V_o in the network shown below using loop analysis.



Let us assign the loops as follows.

Loop 1 (I_1): 2 kΩ – 2 mA – 6 V

Loop 2 (I_2): 1 kΩ – 6 V – 4 mA

Loop 3 (I_3): 2 kΩ – 1 kΩ – 4 V – 6 V

Loop 4 (I_4): 1 kΩ – 6 V – 4 V – 2 kΩ

For two current sources, we can write

$$I_1 = -2 \text{ mA}$$
$$I_2 = 4 \text{ mA}$$

Applying KVL for loop 2 yields

$$2\text{k}\,(I_1 + I_3) + 1\text{k}\,I_3 - 4 + 6 = 0$$

Writing a KVL equation for loop 4 yields

$$1\text{k}\,(I_2 + I_4) - 6 + 4 + 2\text{k}\,I_4 = 0$$

We can rewrite these KVL equations as follows.

$$2\text{k}\,I_1 + 3\text{k}\,I_3 = -2$$
$$1\text{k}\,I_2 + 3\text{k}\,I_4 = 2$$

Substituting the values of I_1 and I_2 into these equations yields

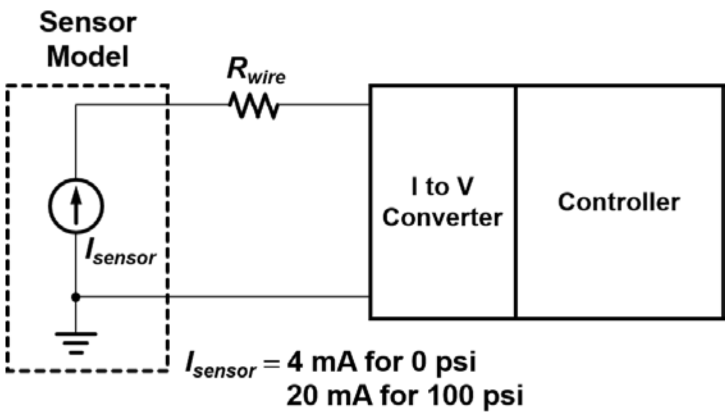
$$I_3 = \frac{2}{3} \text{ mA}$$
$$I_4 = -\frac{2}{3} \text{ mA}$$

Therefore,

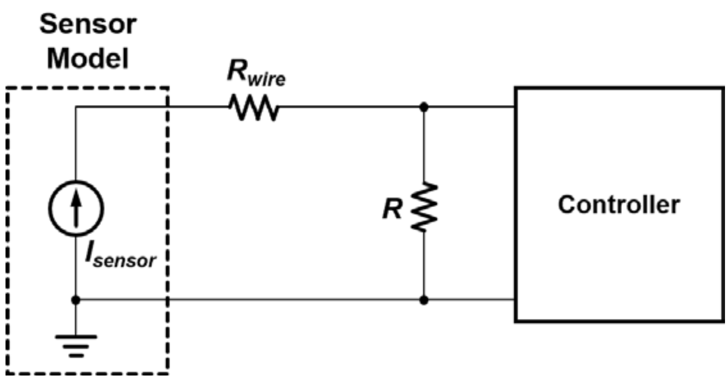
$$V_o = 1\text{k}\,I_3 + 2\text{k}\,I_4 = \frac{2}{3} - \frac{4}{3} = -\frac{2}{3} \text{ V}$$

4. **(50 points)** In factory instrumentation, process parameters such as pressure and flow rate are measured, converted to electrical signals, and sent some distance to an electronic controller. The controller then decides what actions should be taken. One of the main concerns in these systems is the physical distance between the sensor and the controller. An industry standard format for encoding the measurement value is called the 4–20 mA standard, where the parameter range is linearly distributed from 4 to 20 mA. For example, a 100 psi pressure sensor would output 4 mA if the pressure were 0 psi, 20 mA at 100 psi, and 12 mA at 50 psi. But most instrumentation is based on voltages between 0 and 5 V, not on current.

Therefore, let us design a current-to-voltage converter that will output 5 V when the current signal is 20 mA. The circuit shown below is a model of the given situation. The wiring from the sensor unit to the controller has some resistance, R_{wire}, which increases by 1 Ω with 1-m increase of the distance.



(a) **(5 points)** As shown below, for the current-to-voltage converter, we use an extremely simple circuit – a resistor. What should be the value of R? (It is assumed that the controller does not load the remaining portion of the circuit.)



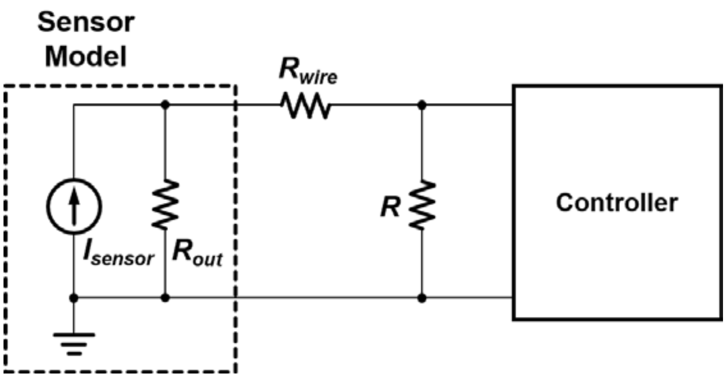
Since the current-to-voltage converter should output 5V for $I_{sensor} = 20mA$,

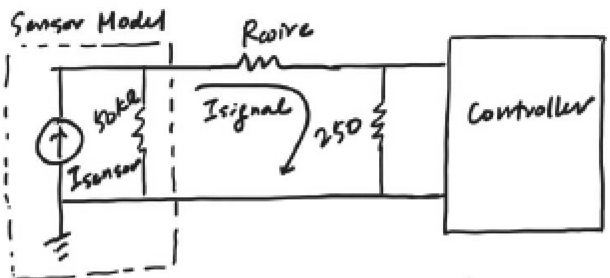
$R = 5V / 20mA = \underline{250\Omega}$.

(b) (5 points) For the configuration shown in (a), considering that the sensor output information is contained in the current value, will R_{wire} affect the accuracy at the controller? If yes, how much error (in percentage) will be introduced by R_{wire} when the distance between the sensor and controller is 50 m?

No. Since the data are contained in the current value, R_{wire} does not affect the accuracy at the controller as long as the sensor acts as an ideal current source as modeled in Fig. 3(b).
If the sensor output were a voltage proportional to pressure, the voltage drop in the line would cause measurement error even if the sensor output were an ideal voltage source.

(c) (15 points) Introducing a more accurate model for the sensor, the circuit can be redrawn as shown below. When R_{out} is 50 kΩ, if we want to keep the error at the controller within 1%, how far can we place the sensor from the controller? Assume the same value as in (a) for R.





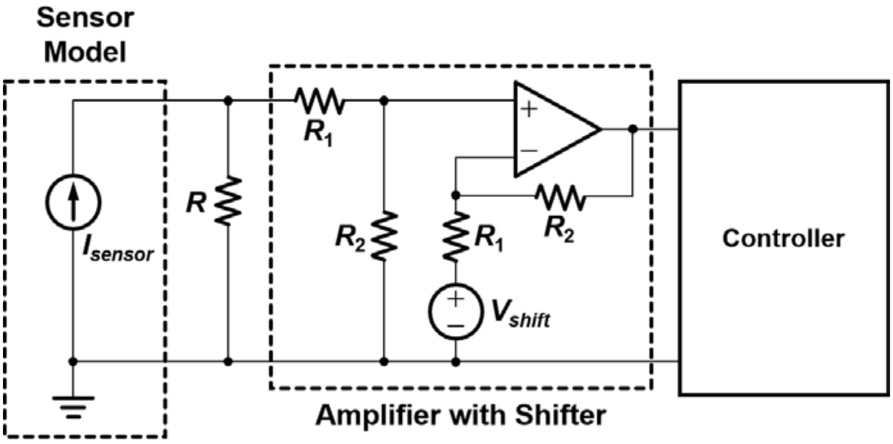
$$I_{signal} = I_{sensor} \times \frac{50k}{50k + (R_{wire} + 250)}$$

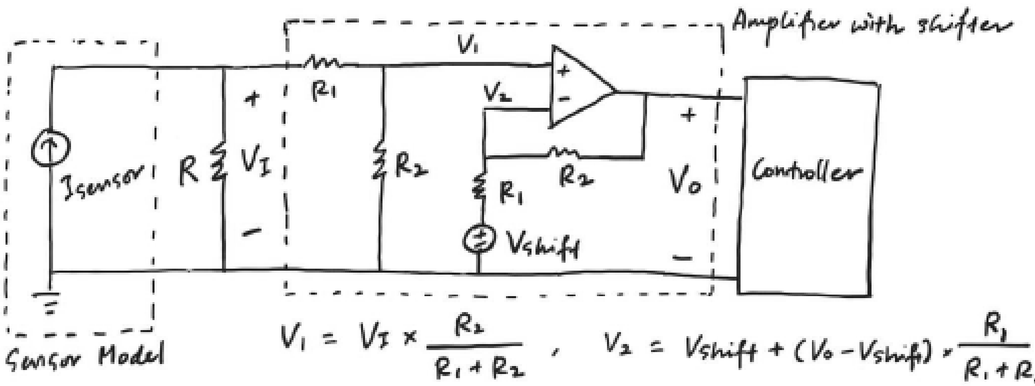
$$\therefore \frac{I_{sensor} - I_{signal}}{I_{sensor}} = \frac{R_{wire} + 250}{50k + (R_{wire} + 250)} < 0.01$$

$$0.99\, R_{wire} < 500 + 2.5 - 250 = 252.5$$

$$\therefore R_{wire} < 255.05\,\Omega \rightarrow \text{distance} \leq \sim 255\text{m}$$

(d) (25 points) In (a), the value of R was chosen to convert a current in the 4- to 20-mA range to a voltage such that a 20-mA input produced a 5-V output. In this case, the minimum current (4 mA) produces a resistor voltage of 1 V. Unfortunately, many control system operate on a 0- to 5-V range rather than a 1- to 5-V range. Let us design a new converter that will output 0 V at 4 mA and 5 V at 20 mA, using the configuration shown below. Assuming the same value as in (a) for R and ignoring any effect from R_{out} and R_{wire}, determine the values for R_1, R_2, and V_{shift}.





$V_1 = V_I \times \dfrac{R_2}{R_1 + R_2}$, $\quad V_2 = V_{shift} + (V_o - V_{shift}) \times \dfrac{R_1}{R_1 + R_2}$

Since $V_1 = V_2$, $\quad V_I \times \dfrac{R_2}{R_1 + R_2} = V_{shift} + (V_o - V_{shift}) \times \dfrac{R_1}{R_1 + R_2}$.

Solving for V_o, $\quad V_o = (V_I - V_{shift}) \times \dfrac{R_2}{R_1}$.

For $V_I = 1$, $\quad V_o = (1 - V_{shift}) \times \dfrac{R_2}{R_1} = 0$. — ①

For $V_I = 5$, $\quad V_o = (5 - V_{shift}) \times \dfrac{R_2}{R_1} = 5$. — ②

From ① and ②, $\underline{V_{shift} = 1}$ and $\dfrac{R_2}{R_1} = \dfrac{5}{4} = 1.25$

Since we don't want the converter resistor, R, to affect the amplifier, the vast majority of the 4-20mA current should flow entirely through R and not through the amplifier resistors, R_1 and R_2.

Therefore, R_1 and $R_2 \gg R$.

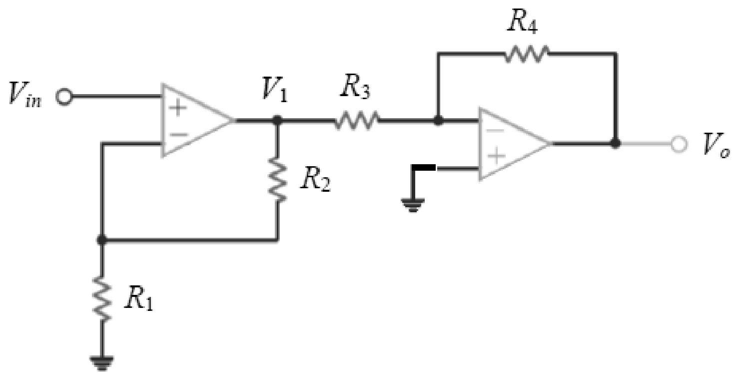
Considering all the necessary conditions to satisfy, we can determine the values of R_1 and R_2 as follow.

$\underline{R_1 = 100\,k\Omega}$, $\underline{R_2 = 125\,k\Omega}$.

5. **(20 points)** Design a two-stage op-amp network that has a gain of –50,000 while drawing no current into its input terminal. Use no resistors smaller than 1 kΩ.

Since no current should be drawn into its input terminal, we can employ a non-inverting amplifier configuration as the first stage of the two-stage circuit.

Then, an inverting amplifier can follow as the second stage to provide the negative voltage gain.



The voltage gain of this circuit is given by

$$\frac{V_o}{V_{in}} = -\left(1 + \frac{R_2}{R_1}\right)\frac{R_4}{R_3}$$

If we assign a gain of 500 to the first stage and the remaining gain of -100 to the second stage,

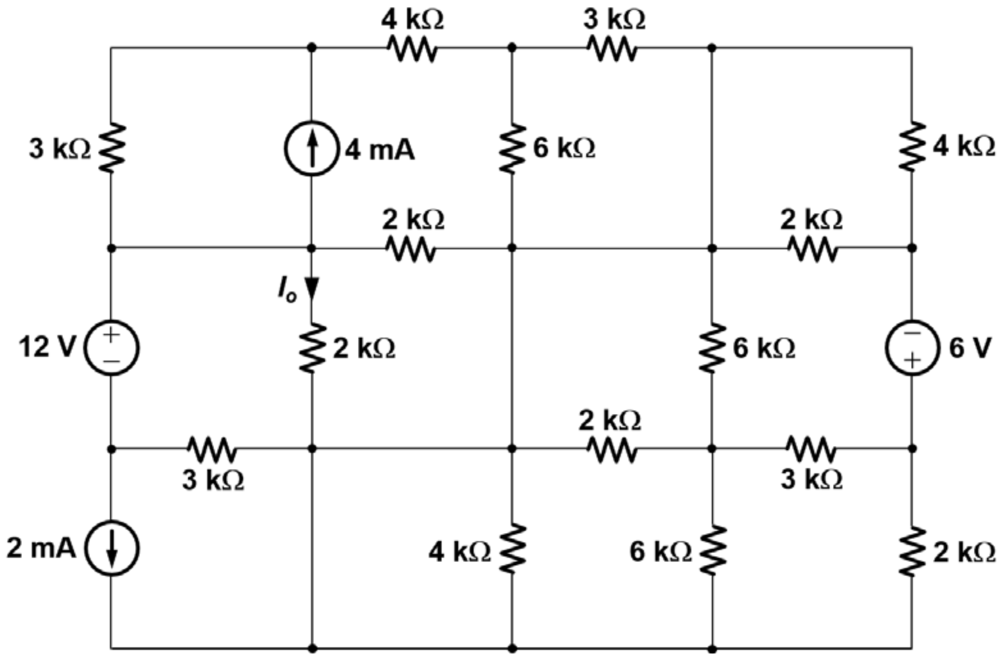
$$\frac{R_2}{R_1} = 499$$

and

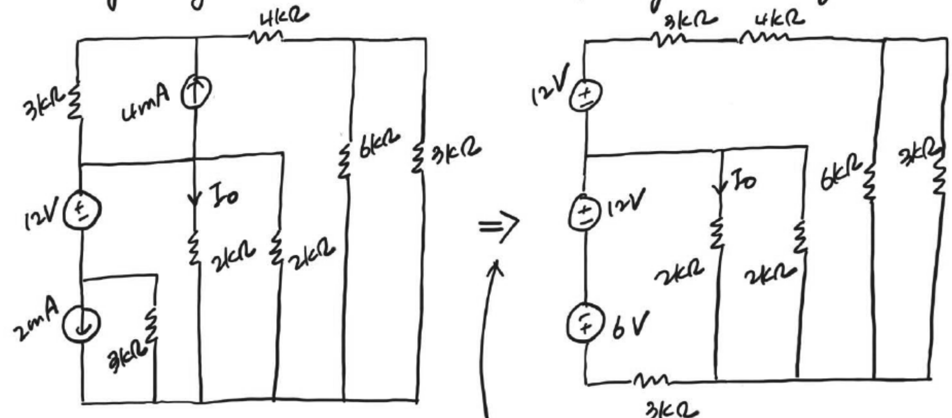
$$\frac{R_4}{R_3} = 100$$

If we choose $R_1 = R_3 = 2$ kΩ, $R_2 = 998$ kΩ and $R_4 = 200$ kΩ.

6. (20 points) Use source transformation to find I_o in the network shown below.

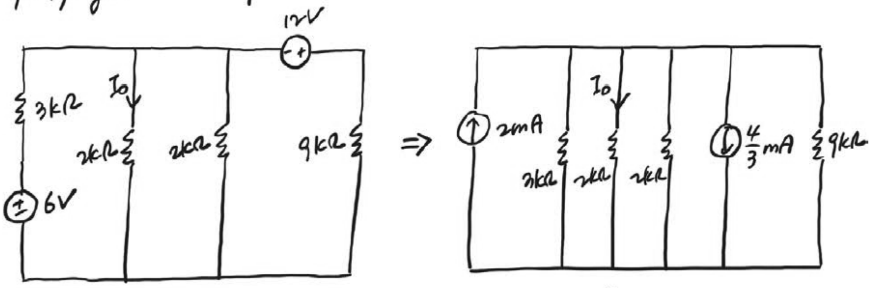


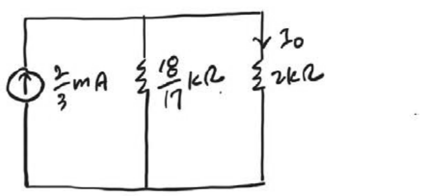
Redrawing the given circuit results in the following circuit diagram.



Source transformation

Simplifying the circuit further,





$$\therefore I_o = \frac{2}{3}m \times \frac{\frac{18}{17}k}{2k + \frac{18}{17}k}$$

$$= \frac{2}{3}m \times \frac{9}{26} = \frac{3}{13}m = \underline{0.23\text{ mA}}$$

7. **(35 points)** A network of pre-charged capacitors is given as shown below.

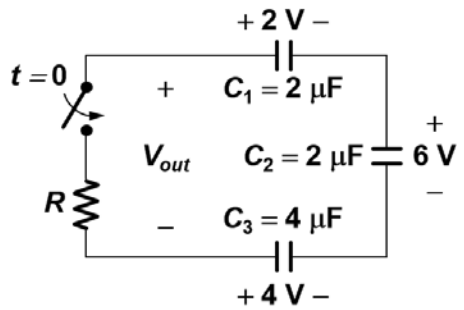


Fig. 5

(a) **(10 points)** Before $t = 0$, what is the value of V_{out}? How much energy is initially stored in the network?

$$V_{out}(0-) = V_{c1}(0-) + V_{c2}(0-) + V_{c3}(0-) = 2+6-4 = 4V$$

Initially stored energy:

$$E(0-) = \frac{1}{2}C_1 V_{c1}(0-)^2 + \frac{1}{2}C_2 V_{c2}(0-)^2 + \frac{1}{2}C_3 V_{c3}(0-)^2$$

$$= \frac{1}{2}\times 2\mu \times 2^2 + \frac{1}{2}\times 2\mu \times 6^2 + \frac{1}{2}\times 4\mu \times (-4)^2 = 4\mu + 36\mu + 32\mu = 72\mu J$$

(b) **(15 points)** At $t = 0$, the switch is closed and the network starts to discharge through R. After long enough time after the switch is closed, what value does V_{out} reach? At this steady state, find the voltage across each capacitor.

At $t=\infty$, no current flows in this network.

$$\rightarrow V_{out}(\infty) = V_{c1}(\infty) + V_{c2}(\infty) + V_{c3}(\infty) = 0 \quad \text{—①}$$

Since all the capacitors are connected in series, their amount of charge change should be the same as Δq.

$$① \rightarrow \left(2+\frac{\Delta q}{2\mu}\right) + \left(6+\frac{\Delta q}{2\mu}\right) + \left(-4+\frac{\Delta q}{4\mu}\right) = 0$$

$$\rightarrow \left(\frac{1}{2\mu}+\frac{1}{2\mu}+\frac{1}{4\mu}\right)\Delta q = -4 \rightarrow \Delta q = -\frac{16}{5}\mu C$$

$$\therefore V_{c1}(\infty) = 2 - \frac{16}{5}\mu \times \frac{1}{2\mu} = 0.4V$$

$$V_{c2}(\infty) = 6 - \frac{16}{5}\mu \times \frac{1}{2\mu} = 4.4V$$

$$V_{c3}(\infty) = -4 - \frac{16}{5}\mu \times \frac{1}{4\mu} = -4.8V$$

(c) (10 points) At $t = \infty$, how much energy is left in the network? How much energy has been dissipated by discharging through R?

Total energy left in the network:

$$E(\infty) = \frac{1}{2}C_1 V_{C1}(\infty)^2 + \frac{1}{2}C_2 V_{C2}(\infty)^2 + \frac{1}{2}C_3 V_{C3}(\infty)^2$$

$$= \frac{1}{2} \times 2\mu \times (0.4)^2 + \frac{1}{2} \times 2\mu \times (4.4)^2 + \frac{1}{2} \times 4\mu \times (-4.8)^2$$

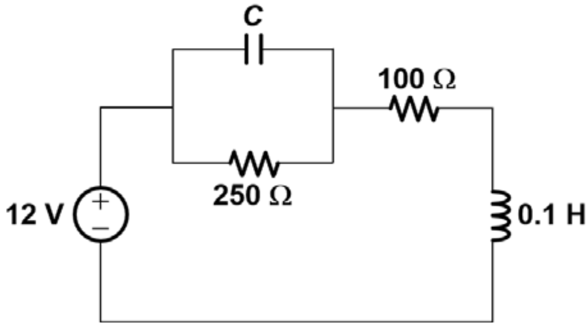
$$= 0.16\mu + 19.36\mu + 46.08\mu$$

$$= 65.6\ \mu J$$

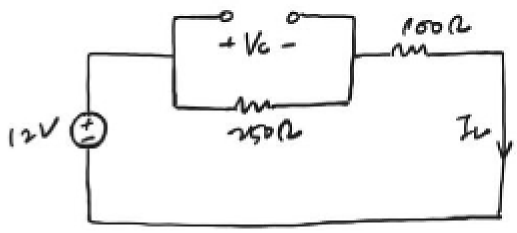
Energy dissipated by discharging through R:

$$E(0^-) - E(\infty) = 72\mu - 65.6\mu = 6.4\mu J$$

8. (15 points) In the circuit shown below, find the value of *C* if the energy stored in the capacitor equals the energy stored in the inductor.



In steady state, the capacitor can be regarded as an open circuit and the inductor as a short circuit.



$$V_c = 12 \times \frac{250}{250+100} = 8.57V$$

$$I_L = 12/(250+100) = 34.29mA$$

Since the energy stored in the capacitor equals the energy stored in the inductor,

$$\frac{1}{2}C \times (8.57)^2 = \frac{1}{2} \times 0.1 \times (34.29m)^2$$

$$\therefore C = \underline{1.6\mu F}$$