

# 14. Wave Optics

## I. MALUS' LAW

### THEORY

Natural light (sunlight, fluorescent lamp) is usually unpolarized. If natural light is incident on the linear polarizer as in *Fig. 1*, the only component parallel to the transmission axis of the linear polarizer will be transmitted. *Fig. 1* shows that the unpolarized light is firstly incident on the linear polarizer, the transmitted light is secondly incident on the other polarizer (analyzer), and finally the intensity of the transmitted light is analyzed by the detector.

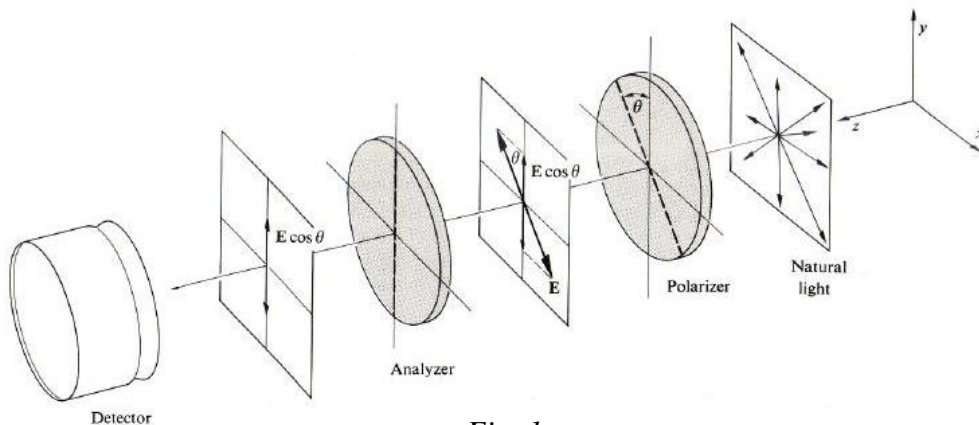
The only component parallel to the transmission axis of the analyzer will be allowed to be transmitted. If the angle between the transmission axis of the polarizer and analyzer is  $\theta$  and the electric field transmitted from the linear polarizer is  $E_1$ , the electric field of the transmitted light will be  $E_1 \cos \theta$ .

Now, we rotate the transmission axis of the analyzer and measure the irradiance (average energy per unit area per unit time) of the transmitted light from the analyzer. Since the irradiance is proportional to the square of the amplitude of the electric field, the irradiance  $I(0)$  of the transmitted light from the polarizer is proportional to  $E_1^2$  and irradiance  $I(\theta)$  leaving the analyzer is proportional to  $E_1^2 \cos^2 \theta$ . This can be simply expressed as *Eq. (1)* and called as **Malus' law**.

$$I(\theta) = I(0) \cos^2 \theta. \quad (1)$$

### METHODS

1. Observe the variation of the light intensity while rotating the transmission axis.
2. Write down the angle at which the light disappears.



*Fig. 1.*

## II. SINGLE SLIT DIFFRACTION

### THEORY

When the plane wave of light is incident on the narrow slit of width  $a$ , the diffraction pattern is shown in opaque screen  $B$ . We simply calculate the position at  $B$  where the first dark fringe appears. First, we divide the slit into two zones of equal width  $a/2$ , as shown in *Fig. 2*. We extend to  $P_1$  a light ray  $r_1$  from the top point of the top zone and a light ray  $r_2$  from the top point of the bottom zone. A central axis is drawn from the center of the slit to screen  $C$ , and  $P_1$  is located at an angle  $\theta$  to that axis.

The all points within the slit are in phase. When the light waves of  $r_1$  and  $r_2$  reach point  $P_1$ , then they are out of phase by  $\lambda/2$  (remember that  $P_1$  is the first position of the dark fringe at  $B$ ) because the light wave  $r_2$  travels a longer path than  $r_1$ . If we assume  $D \gg a$ , then we can approximate that two rays are parallel and the path difference between the two rays will be  $(a/2)\sin\theta$ .

We can repeat this analysis for any other pair of rays in the two zones. Each such pair of rays has the same path difference  $\lambda/2$ . Therefore, the position of the first dark fringe is simply described as shown below:

$$\begin{aligned}(a/2)\sin\theta &= \lambda/2, \\ a\sin\theta &= \lambda \quad (\text{first minimum}).\end{aligned}\tag{2}$$

We can find the second dark fringe above and below the central axis as we found the first dark fringes, except that we now divide the slit into four zones of equal width  $a/4$ , as shown in *Fig. 3*. We then extend rays  $r_1$ ,  $r_2$ ,  $r_3$ , and  $r_4$  from the top points of the zones to point  $P_2$ , the location of the second dark fringe above the central axis. The path differences between  $r_1$  and  $r_2$  and between  $r_3$  and  $r_4$  are each equal to  $\lambda/2$ . For  $D \gg a$ , we can approximate these four rays as being parallel and the path difference will be  $a/4\sin\theta$ . The position of the second dark fringe is described as shown below:

$$\begin{aligned}a/4\sin\theta &= \lambda/2, \\ a\sin\theta &= 2\lambda \quad (\text{second minimum}).\end{aligned}\tag{3}$$

We can continue this analysis for the third, fourth, or any number of dark fringes. The general equation can be expressed as shown below:

$$a\sin\theta = m\lambda,\tag{4}$$

for  $m = 1, 2, 3, \dots$  (minima), and the intensity is given by

$$I = I_m \left( \frac{\sin \alpha}{\alpha} \right)^2, \quad (5)$$

where  $\alpha = (\pi a / \lambda) \sin \theta$ , and  $I_m$  = the intensity of the center bright fringe. Eq. (5)

shows that the ratio  $a / \lambda$  determines the shape of the diffraction.

## METHODS

1. Cut a slit 1 m from the screen.
2. Make the laser light illuminate into the slit and observe the diffraction which appears on the screen.
3. Write down the shape and the brightness of the diffraction while varying the slit width.

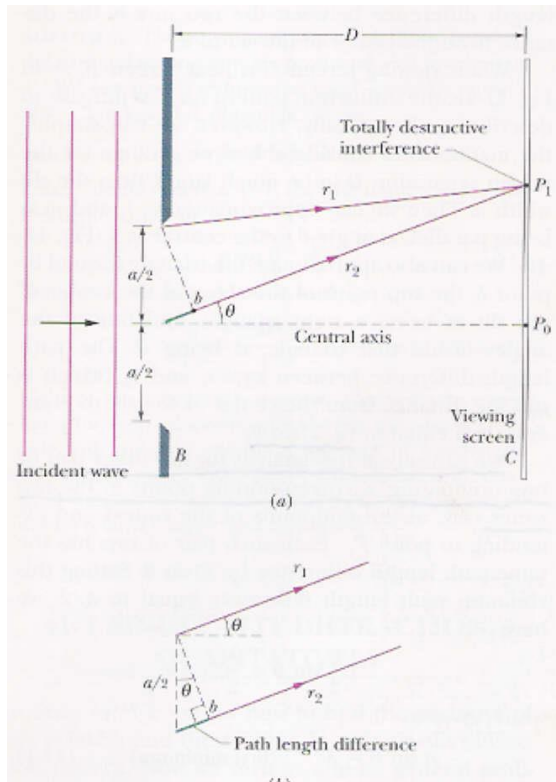


Fig. 2.

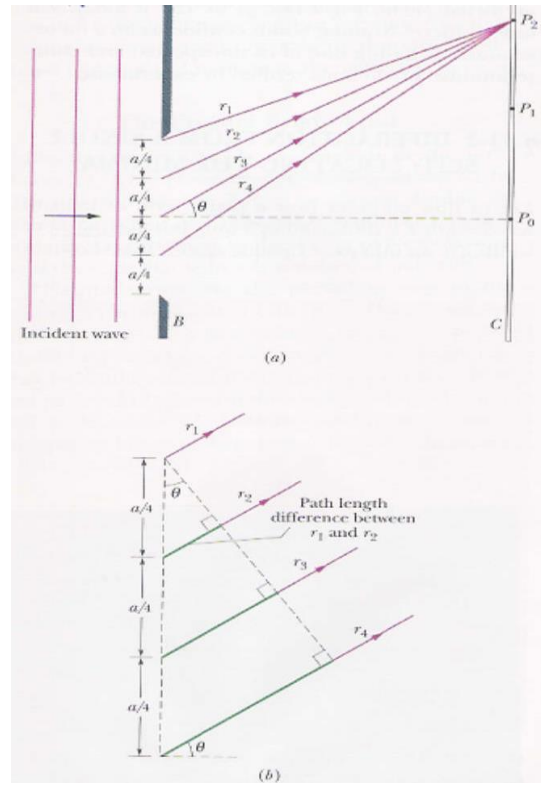


Fig. 3.

### III. CIRCULAR APERTURE DIFFRACTION

#### THEORY

Here, we consider diffraction by a circular aperture of diameter  $d$ , such as a circular converging lens. *Fig. 4* shows the image of a distant point source of light formed on a photographic film placed in the focal plane of a converging lens. This image is not a point, but a circular disk surrounded by several progressively faint secondary rings. As shown in *Eq. (5)*, the ratio  $d/\lambda$  determines the scale of the diffraction pattern.

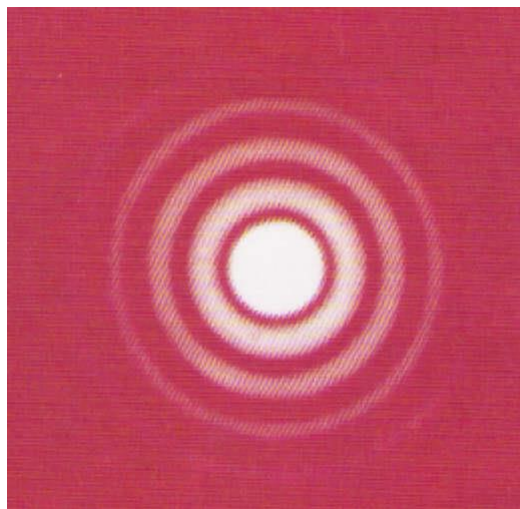
The first minimum for the diffraction pattern of a circular aperture of diameter  $d$  is given by

$$\sin \theta = 1.22\lambda / d. \quad (6)$$

The main difference from *Eq. (2)* is the factor 1.22, which exists because of the circular shape of the aperture.

#### METHODS

1. Cut a slit 1 m from the screen.
2. Make the laser light be incident on the slit and observe the diffraction pattern shown on the screen.
3. Observe the change of the diffraction pattern while varying the shape of the slit.
4. Observe the change of the diffraction pattern while varying the size of the aperture.



*Fig. 4.*

## IV. YOUNG'S DOUBLE SLIT

### THEORY

In Young's double slit experiment, they assumed that slits were narrow compared to the wave length of the light illuminating them; that is  $a \ll \lambda$  ( $a$  = width of the slit). For such narrow slits, the interference of light from the two slits produces bright fringes with approximately the same intensity.

In practice with visible light, however, the condition  $a \ll \lambda$  is often not met. For relatively wide slits, the interference of light from two slits produces bright fringes that do not all have the same intensity. In fact, their intensity is modified by the diffraction of light through the slit. As an example, *Fig. 5(a)* shows the double slit fringe pattern that would occur if the slits were infinitely narrow; all the bright interference fringes have the same intensity. *Fig. 5(b)* shows the diffraction pattern of an actual slit: the broad central maximum and one weaker secondary maximum. *Fig. 5(c)* shows the resulting interference pattern of the two slits. The pattern is found by using the diffraction curve of *Fig. 5(b)* as an envelope on the intensity curve in *Fig. 5(a)*. The position of the fringe is not modified; only the intensity is affected.

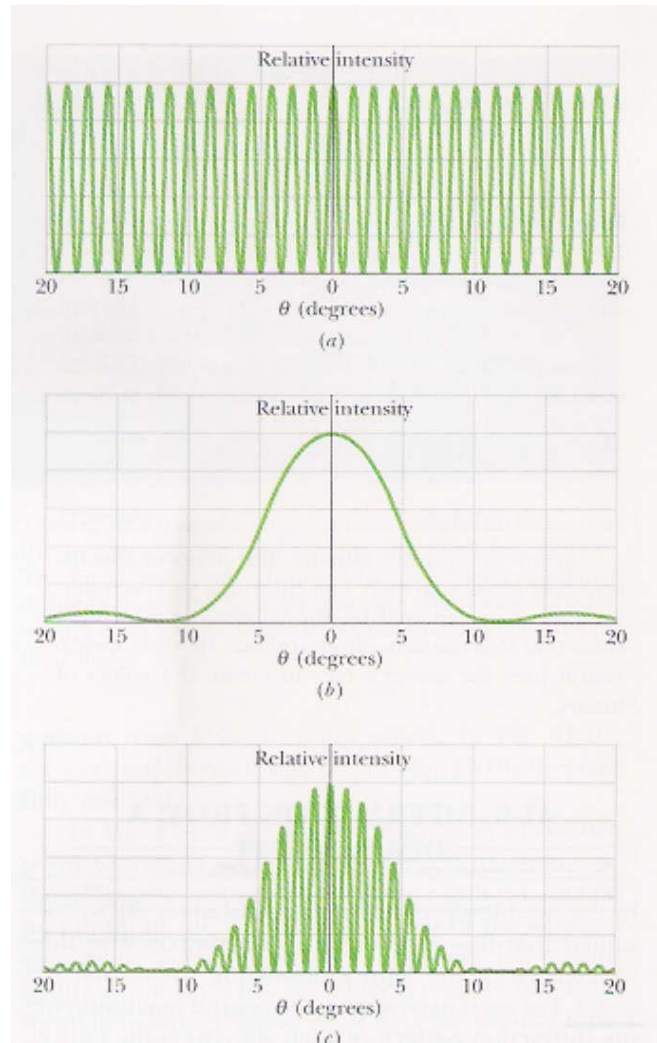
With diffraction effects are taken into account, the intensity of a double slit interference pattern is given by

$$I = I_m (\cos^2 \beta) \left( \frac{\sin \alpha}{\alpha} \right)^2, \quad (7)$$

where  $\beta = (\pi d / \lambda) \sin \theta$ , and  $\alpha = (\pi a / \lambda) \sin \theta$ . Here,  $d$  is the distance between the centers of the slits and  $a$  is the slit width. The factor  $\cos^2 \beta$  is entered due to the interference effect of the two slits and the factor  $(\sin \alpha) / \alpha$  is entered due to the diffraction effect of the slit.

### METHODS

1. Make the laser light illuminate the center positions of the two slits and observe the diffraction pattern on the screen.
2. Observe the change of the diffraction pattern while varying the width of the slit.
3. Observe the change of the diffraction pattern while varying the distance between the two slits.



*Fig. 5.*

## V. INTERFERENCE AND DIFFRACTION BY MULTIPLE SLITS

### THEORY

A logical extension of a double slit interference experiment is to increase the number of slits from two to a larger number  $N$ . An arrangement like of the one in *Fig. 6*, usually involving more slits is called a diffraction grating.

A sharply defined bright fringe will occur when  $d \sin \theta$ , which is the path difference between rays from adjacent slits in *Fig. 7*, is equal to an integer of wave length.

$$d \sin \theta = m\lambda, \quad (8)$$

for  $m = 0, 1, 2, \dots$  (maxima).

## METHODS

1. Observe the diffraction pattern shown on the screen.
2. Observe the change of the diffraction pattern while varying the number of slits.

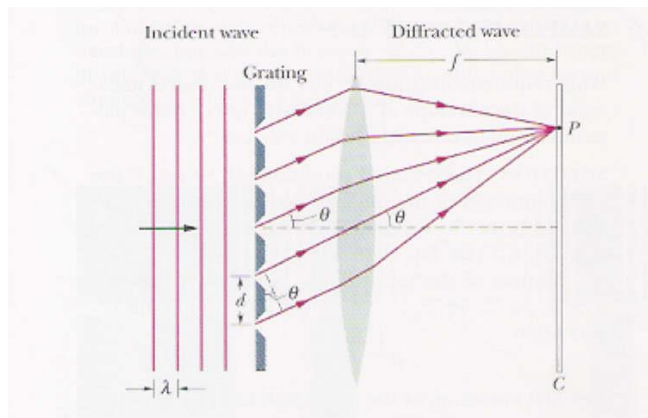


Fig. 6.

## V. NEWTON'S RING

### THEORY

If the two pieces of glass are forced together as a point, as might be done by pressing on them with a sharp pencil, a series of concentric, nearly circular, fringes is formed around that point. Known as Newton's ring, this pattern is more precisely examined with the arrangement of Fig. 7. The amount of uniformity in the concentric circular pattern is a measure of the degree of perfection in the shape lens. With  $R$  as the radius of curvature of the convex lens, the relation between the distance  $x$  and the film thickness  $d$  is given by

$$x^2 = R^2 - (R - d)^2 = 2Rd - d^2. \quad (9)$$

Since  $R \gg d$ , this becomes  $x^2 = 2Rd$ . If the reflected light  $E_1$  and  $E_2$  have a path difference of  $(m + 1/2)\lambda$ , the bright fringe occurs. (Notice the  $180^\circ$  relative phase shift between internally and externally reflected light.)

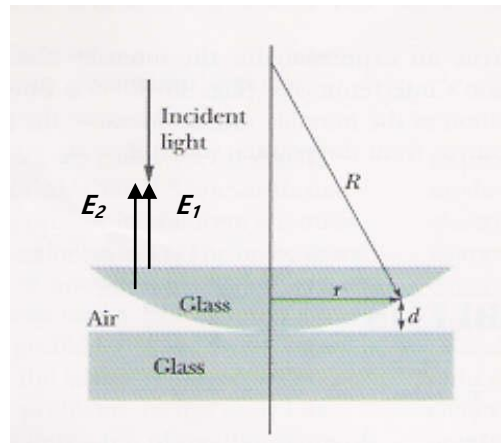
$$\text{Path difference of } m^{\text{th}} \text{ bright fringe} = 2d = (m + 1/2)\lambda. \quad (10)$$

Combining the last two expressions, the radius of the bright fringe ring is given by

$$X_m = [(m + \frac{1}{2})\lambda R]^{1/2}. \quad (11)$$

## METHODS

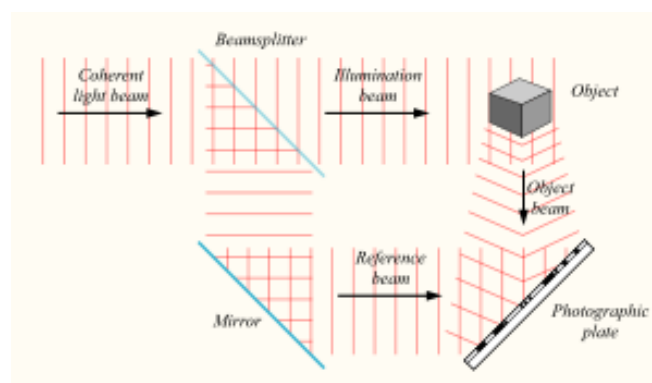
1. Observe the interference pattern.
2. Write down the radius of the first and second bright rings and compare it with the theoretical values.



*Fig. 7.*

## VII. HOLOGRAPHY

The procedure to make the hologram is shown in *Fig. 9*. The object light is incident on the object and scattered into the film which is very sensitive to light. A second light, known as the reference beam, also illuminates the film. Therefore, the interference between the two lights occurs in the film, and the interference pattern is developed on the film. After this, if the same light from the reference beam, called a reconstruction beam, illuminates the film, the light is diffracted by the grating formed in the film. The diffraction pattern depends on the interference pattern developed on the film. The effect of the diffraction is to reconstruct the image of the object. An observer looking into the other side of the film can see the image of the object.



*Fig. 8.*