Student ID Number: , Name:

EE210: Probability and Introductory Random Process - Midterm Exam.

Please provide detailed procedure to get your final answer.

Oct. 20, 2015

1. (15 points) In an experiment, A and B are mutually exclusive events with probabilities $\mathrm{P}[A \cup B] = 5/8$ and $\mathrm{P}[A] = 3/8$.

(a) (5 points) Find $\mathrm{P}[B]$, $\mathrm{P}[A \cap B^C]$, and $\mathrm{P}[A \cup B^C]$

(b) (10 points) Are A and B independent?

2. (15 points) An academic department offers 8 lower level courses: $\{L_1, L_2, \ldots, L_8\}$ and 10 higher level courses: $\{H_1, H_2, \ldots, H_{10}\}$. A valid curriculum consists of 4 lower level courses, and 3 higher level courses.

(a) (5 points) How many different curricula are possible?

(b) (10 points) Suppose that $\{H_1, \ldots, H_5\}$ have L_1 as a prerequisite, and $\{H_6, \ldots, H_{10}\}$ have L_2 and L_3 as a prerequisites, i.e., any curricula which involve, say, one of $\{H_1, \ldots, H_5\}$ must also include L_1. How many different curricula are there?

3. (20 points) If an aircraft is present in a certain area, a radar detects it and generates and alarm signal with probability 0.99. If an aircraft is not present, the radar generates a (false) alarm, with probability 0.10. We assume that an aircraft is present with probability 0.05.

(a) (10 points) Draw a tree diagram of this radar system with appropriate labels at the branches and ending points.

(b) (10 points) What is the probability of the presentation of an aircraft when we already know the alarm is generated?

4. (10 points) A binary message is transmitted as a signal s, which is either -1 or $+1$. The communication channel corrupts the transmission with additive Gaussian noise with mean $\mu = 0$ and variance $\sigma^2 = 1$. The receiver concludes that the signal -1 (or $+1$) was transmitted if the value received is < 0 (or ≥ 0, respectively). What is the probability of error?

MORE PROBLEMS ON THE OTHER SIDE !

5. (20 points) The random variable X and Y have the following joint PMF. Let

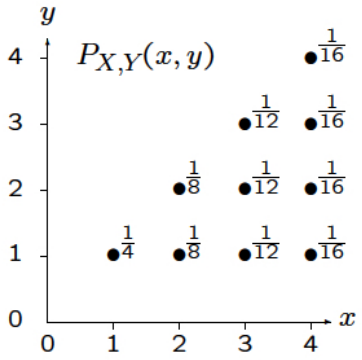

Figure 1: Joint PMF of the random variable X and Y

(a) (5 points) Find the expected values, E[X] and E[Y].

(b) (5 points) Find the variances, Var[X] and Var[Y].

(c) (5 points) Find the covariance, Cov[X, Y].

(d) (5 points) Find the correlation coefficient, $\rho_{X,Y}$.

6. (20 points) Let us consider random variables X and Y with the following joint PDF.

$$f_{X,Y}(x,y) = \begin{cases} ce^{-x}e^{-y} & 0 \le y \le x < \infty \\ 0 & \text{otherwise} \end{cases}$$

(a) (5 points) Find the constant c for $f_{X,Y}(x,y)$ to be a valid joint PDF.

(b) (5 points) Find the marginal PDFs, $f_X(x)$ and $f_Y(y)$.

(c) (5 points) Find $P[X + Y \le 1]$.

(d) (5 points) Find the correlation coefficient, $\rho_{X,Y}$, of X and Y.

THE END OF PROBLEMS! THANKS FOR YOUR HARD WORK!