Midterm Exam Fall 2023

There are seven problems

You have to justify your answers. Otherwise, you will not have partial credits.

Do not forget your names first!

1. (10) Consider the signal

$$x(t) = A\mathrm{sinc}(2Wt)\cos(\pi Wt).$$

What is the energy of the signal?

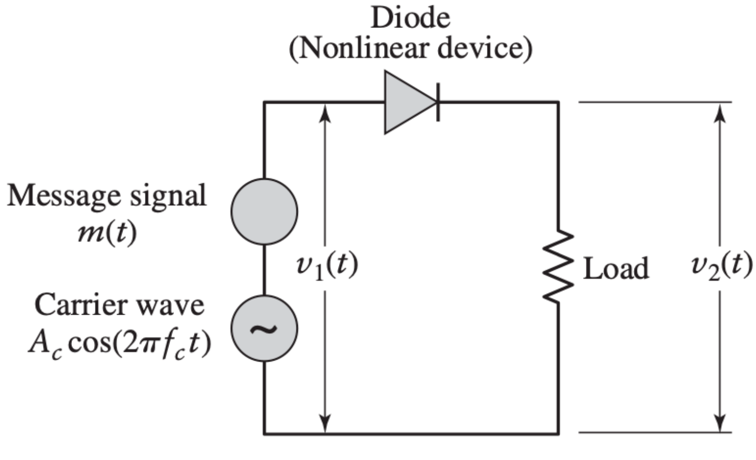
2. (20) For $g(t) = \exp(-\pi t^2)$, find its Fourier transform $G(f)$.

3. (20) The diode in the following circuit is a non-linear device whose output is expressed as

$$v_2(t) = a_1 v_1(t) + a_2 v_1^2(t)$$

where

$$v_1(t) = A_c\cos(2\pi f_c t) + m(t).$$



a) (10) The signal $v_2(t)$ is given to a bandpass filter whose center frequency is f_c and band width is $2W$ where W is the signal bandwidth of $m(f)$. The output from the bandpass filter is denoted by $\tilde{v}_2(t)$. Then, determine the spectral content of the bandpass filter output, i.e., $\tilde{V}_2(f)$ in terms of $M(f)$ where $\tilde{V}_2(f) = \mathcal{F}[\tilde{v}_2(t)]$ and $M(f) = \mathcal{F}[m(t)]$. It is assumed that the signal bandwidth of $v(f)$, W, is much smaller than f_c, i.e., $W \ll f_c$.

b) (5) Determine the bandpass filter output in the time domain signal, i.e., $\tilde{v}_2(t)$.

c) (5) If the message signal is normalized, find the modulation efficiency when the message energy is normalized to unity, i.e., $\langle m^2(t) \rangle = 1$.

4. (10) The maximum value of frequency deviation f_d is fixed at 75KHz for commercial FM broadcasting by radio. We assume that the bandwidth of modulating signal is $W = 15$KHz, which typically the maximum audio frequency of interest in FM transmission.

a) (2) Find the deviation ratio, D when the magnitude of the modulation signal is normalized to 1, i.e., $\max|m(t)| = 1$.

b) (3) Using the Carlson rule, find the transmission bandwidth of the FM signal

c) (5) For the modulating signal, $m(t) = \cos(2\pi W t)$ with $W = 15$KHz, find the transmission bandwidth of the FM signal which is defined as the minimum required bandwidth containing 95% of the total power

Table 4.1 Values of Selected Bessel Functions

n	$\beta = 0.05$	$\beta = 0.1$	$\beta = 0.2$	$\beta = 0.3$	$\beta = 0.5$	$\beta = 0.7$	$\beta = 1.0$	$\beta = 2.0$	$\beta = 3.0$	$\beta = 5.0$	$\beta = 7.0$	$\beta = 8.0$	$\beta = 10.0$
0	0.999	0.998	0.990	0.978	0.938	0.881	0.765	0.224	−0.260	−0.178	0.300	0.172	−0.246
1	0.025	0.050	0.100	0.148	0.242	0.329	0.440	0.577	0.339	−0.328	−0.005	0.235	0.043
2		0.001	0.005	0.011	0.031	0.059	0.115	0.353	0.486	0.047	−0.301	−0.113	0.255
3				0.001	0.003	0.007	0.020	0.129	0.309	0.365	−0.168	−0.291	0.058
4						0.001	0.002	0.034	0.132	0.391	0.158	−0.105	−0.220
5								0.007	0.043	0.261	0.348	0.186	−0.234
6								0.001	0.011	0.131	0.339	0.338	−0.014
7									0.003	0.053	0.234	0.321	0.217
8										0.018	0.128	0.223	0.318
9										0.006	0.059	0.126	0.292
10										0.001	0.024	0.061	0.207
11											0.008	0.026	0.123
12											0.003	0.010	0.063
13											0.001	0.003	0.029
14												0.001	0.012
15													0.005
16													0.002
17													0.001

5. (10) What is the Nyquist rate of the signal $x(t) = \mathrm{sinc}(200t) + \mathrm{sinc}^2(200t)$?

6. (10) A discrete-time random process Y_n for $n \in \mathbb{Z}$ is defined by

$$Y_n = \alpha_0 Z_n + \alpha_1 Z_{n-1}$$

where $\{Z_n\}$ is a wide-sense stationary random process with autocorrelation function $R_Z(n) = \sigma^2 \delta(n)$ where $\delta(n)$ is the Kronecker delta function.

a) (5) What is the autocorrelation function $R_Y(n, m) = E[Y_n Y_m]$?

b) (5) Is the process $\{Y_n\}$ wide-sense stationary?

7. (20) Let $X(t)$ be a random process defined by

$$X(t) = A\cos(2\pi f t)$$

where A is uniformly distributed between 0 and 1, and f is constant.

a) (10) Determine the autocorrelation of X.

b) (5) Is $X(t)$ wide-sense stationary?

c) (5) Consider a revised random process

$$Y(t) = A\cos(2\pi f t + \theta)$$

where θ is a random variable following a uniform distribution $\mathcal{U}[-\pi, \pi)$. Is the revised random process $Y(t)$ wide-sense stationary? Justify your answer.