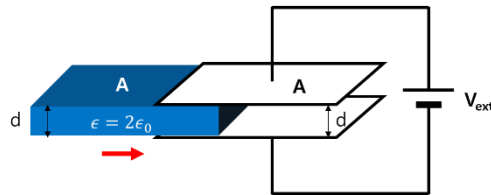


There is a parallel capacitor, with size **A** and distance **d** in the air (ϵ_0). Solid dielectric slab is inserted between the plates of a charged capacitor at each case. The slab thickness is same of the plate spacing, and its area is the same as the plates. Solve the following problems (1~2) about each parallel capacitor cases.

1. The parallel capacitor is initially connected with a fixed voltage source V_{ext} . the dielectric constant of the slab is $\epsilon = 2\epsilon_0$. In this case, the amount of energy stored in the capacitor increases 2 times larger than the case without the slab. (O / X) [5 pts]



Answer: O, no partial point

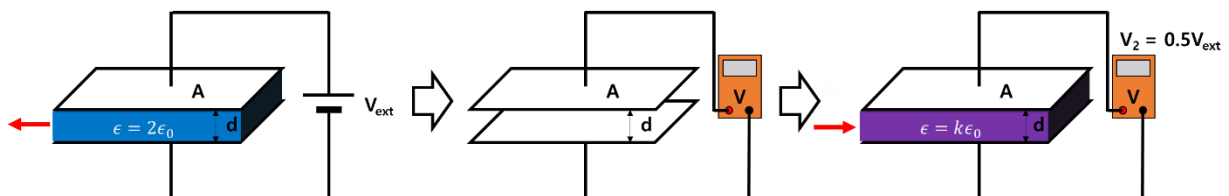
Solution :

Let C_0 is the capacitance without slab.

$$U_{\text{air}} = \frac{1}{2} CV^2 = \frac{1}{2} \left(\epsilon_0 \frac{A}{d} \right) V_{\text{ext}}^2$$

$$U_{\text{slab}} = \frac{1}{2} CV^2 = \frac{1}{2} \left(2\epsilon_0 \frac{A}{d} \right) V_{\text{ext}}^2 = 2 \cdot \frac{1}{2} C_0 V^2 = 2U_{\text{air}}$$

2. Now, we pull out the dielectric slab from Problem 1. And then, we remove the fixed voltage source and connect voltage meter instead to measure electric potential at the capacitor. (Assume that there is no current flow through voltage meter) We insert another dielectric slab with $\epsilon = k\epsilon_0$. If the final measured voltage is $0.5V_{\text{ext}}$, when a dielectric slab is inserted, what can we say about k ? [5 pts]



- ① $k < 0$ ② $0 \leq k < 1$ ③ $1 \leq k < 2$ ④ $2 \leq k < 4$ ⑤ $4 \leq k$

Answer: ④, no partial point

Solution :

- (1) *slab out* : $2C_0 \rightarrow C_0$; $V \rightarrow V_{\text{ext}}$; $\therefore Q \rightarrow \frac{1}{2}Q$
 (2) *voltage source remove* $\rightarrow Q$ would be conserved
 (3) *new slab in* : $C_0 \rightarrow kC_0$; $V_{\text{ext}} \rightarrow \frac{1}{2}V_{\text{ext}}$; $\frac{1}{2}Q \rightarrow \frac{1}{2}Q$ (conserved); $\therefore C_0 V_{\text{ext}}$
 $= kC_0 \cdot \frac{1}{2} V_{\text{ext}} \rightarrow k = 2$

For the AC voltage used by each country to supply electric power from powerhouse to people's house, the standard is different as some country uses 110V and others 220V or 230V. Solve the following problems (3~4).

3. There is no difference between these standards in terms of power loss during supplying the same amount of power. (O / X) [5 pts]

Answer: X, no partial point

Solution :

As $P = IV$ and power loss in a electrical transmission line increases as the square of the current I , there is difference in power loss between the standards supplying the same amount of power.

4. If we use four types of transmission lines of same material with cross-section A and length l as below, which type of line will have the lowest power loss during the transmission if we supply the same current? [5 pts]

- ① $A = 1 \text{ m}^2, l = 4 \text{ m}$ ② $A = 0.5\text{m}^2, l = 2 \text{ m}$ ③ $A = 1 \text{ m}^2, l = 3 \text{ m}$
④ $A = 1 \text{ m}^2, l = 5 \text{ m}$

Answer: ③, no partial point

Solution :

Resistance is linearly proportional to length and inverse linearly proportional to cross-sectional area A .

Thus, for the smallest l/A , the power loss is the smallest.

A capacitor consists of a conducting sphere of radius a surrounded by a concentric conducting shell of radius b . The inner sphere is charged with $+Q$ and outer shell is charged with $-Q$.

5. Show that its capacitance is $C = ab/k(b - a)$ (k is Coulomb constant). [15 pts]

6. Calculate stored potential energy of a capacitor. [15 pts]

Solution :

5. Place charge $+Q$ on the inner sphere, and $-Q$ on the outer one. With gauss law, the electric field between sphere and shell is written as

$$\frac{Q}{\epsilon_0} = \int \vec{E} \cdot \vec{a} = 4\pi r^2 \dots [4 \text{ pts}]$$

$$\vec{E} = \frac{Q}{4\pi\epsilon_0 r^2} \hat{r} = k \frac{Q}{r^2} \hat{r} \dots [4 \text{ pts}]$$

Then the potential difference between sphere and shell is given by

$$V = - \int_b^a \frac{kQ}{r^2} dr = kQ \left(\frac{1}{a} - \frac{1}{b} \right) = \frac{kQ(b - a)}{ab} \dots [4 \text{ pts}]$$

$$\therefore C = \frac{Q}{V} = \frac{ab}{k(b - a)} \dots [3 \text{ pts}]$$

6. The potential energy stored in a capacitor is given by

$$U = \frac{1}{2} CV^2 = \frac{Q^2}{2C} \dots [10 \text{ pts}]$$

$$= \frac{kQ^2(b - a)}{2ab} \dots [5 \text{ pts}]$$

7. A potential difference of 3 μV is set up across 15.00 cm length of wire that has a radius 2.00 mm. How much charge drifts through a cross section in 5.00 ms? The resistivity of the wire material is $1.6 \times 10^{-8} \Omega \cdot \text{m}$. Also, $\pi = 3$ [15 pts]

$$R = \frac{\rho L}{A} = \frac{(1.6 \times 10^{-8})(0.15)}{\pi(2.0 \times 10^{-3})^2} \Omega$$

With potential difference 3 μV , the current flowing through the wire is

$$I = \frac{V}{R} = \frac{3.00 \times 10^{-6}}{2 \times 10^{-4}} = 1.5 \times 10^{-2} \text{ A}$$

$$\Delta Q = i\Delta t = 1.5 \times 10^{-2} \times (5.00 \times 10^{-3}) = 7.5 \times 10^{-5} \text{ C}$$

$R = \frac{\rho L}{A}$, $I = \frac{V}{R}$, $\Delta Q = i\Delta t$ 5 pts for each equations, -2 pts for each time you make a mistake in calculation.

8. The current density in a particle beam with circular cross section of radius a points along the beam axis with a magnitude that decreases linearly from J_0 at the center ($r = 0$) to 1/4 of that value at the edge ($r = a$). Find an expression for the total current in the beam. [15 pts]

$$J = J_0 - \frac{3J_0}{4} \frac{r}{a}$$

Integrate over circular rings of radius r , each of which has area $dA = 2\pi r dr$

$$dI = J dA$$

$$I = \int_0^a J(r) dA = \int_0^a J_0 \left(1 - \frac{3r}{4a}\right) 2\pi r dr = 2\pi J_0 \int_0^a \left(1 - \frac{3r}{4a}\right) r dr = \frac{1}{2} J_0 \pi a^2$$

$$J = J_0 - \frac{3J_0}{4} \frac{r}{a} \text{ equation 5 pts}$$

$$dI = J dA \quad (I = \int_0^a J(r) dA) \text{ equation 5 pts}$$

$$dA = 2\pi r dr \text{ 5 pts}$$

calculation mistake -2 pts

9. Consider an ohmic object, of conductivity σ , bounded by the two concentric spherical surfaces of radius a and $2a$. Electric potential on each surface is kept constant so that the current flows through the object. How much power is needed to make this happen? [20pts]

Let V be the potential difference between the two surfaces and assume the potential on the object is inversely proportional to the distance from the center (actually this is true on the basis of some maths)

9 번 풀이

$$V(r) \equiv \frac{A}{r} \quad (\text{on the object})$$

$$\rightarrow \vec{E} = -\vec{\nabla} V = \frac{A}{r^2} \hat{r} \dots \dots \dots (1)$$

* Or if you don't get used to dealing with gradient, think in another way. (Refer to Ch.22.3 *Calculating field from potential.*)

Note that potential varies only radially on the object. The equipotentials are spherical surfaces, which implies electric field is in radial direction :

$$\vec{E} = f(r) \hat{r}$$

Now we can determine $f(r)$ by considering potential difference of the two points which is radially separated by dr , $\vec{r} + dr \hat{r}$ and \vec{r}

$$\begin{aligned} dV &= -\vec{E} \cdot d\vec{r} \hat{r} \\ &= -f(r) \hat{r} \cdot d\vec{r} \hat{r} \\ &= -f(r) dr \end{aligned}$$

$$\rightarrow f(r) = -\frac{dV}{dr} = \frac{A}{r^2}$$

$$\vec{E}(\vec{r}) = \frac{A}{r^2} \hat{r} \dots \dots \dots (1) , \text{ again}$$

For any closed surface S which encloses $r=a$ and lies between $r=a$ and $r=2a$,

$$I = \oint_S \vec{J} \cdot d\vec{A} = \oint_S \frac{\sigma A}{r^2} \hat{r} \cdot d\vec{A} = 4\pi\sigma A \dots \dots \dots (2)$$

On the other hand

$$V = \frac{A}{a} - \frac{A}{2a} = \frac{A}{2a} \dots \dots \dots (3)$$

$$\therefore P = IV = 4\pi\sigma A \cdot V = 4\pi\sigma(2aV) \cdot V = \mathbf{8\pi\sigma aV^2}$$

9 번 채 점 기 준

감점요인	배점(pts)
σ, a, V 외에 자신이 도입한 양으로 답을 표기한다	-1
유도과정이나 논리는 타당한데 답이 틀렸다.	-1
유도과정에 헛점이 있다:	
· 식(1)처럼 퍼텐셜로부터 전기장 알아내는 역할을 하는 과정이 없다.	-3
· 식(2)처럼 전류와 전기장을 연결하는 과정이 없다.	-5
· 식(3)처럼 문제에서 준 전위차를 자신이 도입한 퍼텐셜과 연결하는 과정이 없다.	-3
<p>답은 맞는데 유도과정의 핵심 아이디어가 잘못됐다.</p> <p>대표적인 오류: 단순히 축전기 전기용량 계산하듯이 전하량 $Q, -Q$ 를 도입해서 푼다. 이렇게 되면 전류를 가우스 법칙으로 구하게 돼버린다. 즉,</p> $I = \oint_S \vec{J} \cdot d\vec{A} = \oint_S \sigma \vec{E} \cdot d\vec{A} = \frac{\sigma Q}{\epsilon_0}$ <p>express Q in terms of V and a as follows</p> $V = \frac{kQ}{a} - \frac{kQ}{2a} = \frac{kQ}{2a}$ $\therefore P = IV = \dots = 8\pi\sigma a V^2$ <p>** 하지만 차원이 드러나도록 일부러 비례상수를 kQ 로 놓은 거면 무방하다, 전하량이 실재한다고 놓고 풀어버렸는지 확인할 것.</p>	-10