2020 Fall General Physics 2 Homework #06

Question 1. [5 points]

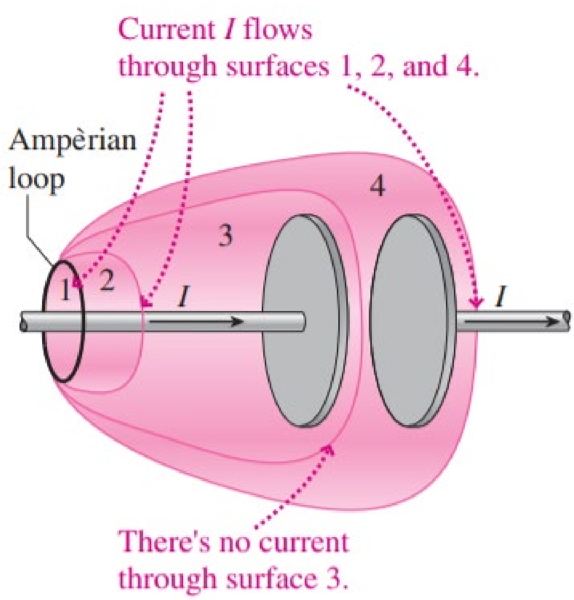
Propagation direction of electromagnetic wave, electric field and magnetic field are orthogonal each other. [O / X]
(Optional: Can you prove it generally?)

Possible Solution for Question 1.

O (+5pt)

Question 2. [5 points]

Considering each surface 1, 2, 3 and 4 in the figure* below, the sum of (1) the current I flowing in the wire, penetrating the surface and (2) the displacement current on the surface, are the same each other. [O / X]



* Wolfson, R. (2012). Essential University Physics.

Possible Solution for Question 2

The answer : O

From the Ampère's law with Maxwell's modification, the sum of the current and the displacement current is the same for all surfaces since the loop to which each surface is attached are the same. For surfaces 1, 2 and 4, the contribution mainly comes from the current flowing in the wire, whereas for surface 3, the displacement current makes the whole contribution.

Question 3. [5 points]

We can completely block unpolarized light using two polarizers. [O / X]

Possible Solution for Question 3.

Answer: (O) Setting two polarizers orthogonally.

Question 4. [5 points]

Let's compare the speed of light in several mediums. The relative permittivity of air, teflon, and quartz are 1, 2.8, and 3.8. The relative permeability of air, teflon and quartz are almost 1. Then, where is the light slower in?

a. Air

b. Teflon

c. Quartz

d. The speed of light is same in these three mediums

Question 5. [15 points]

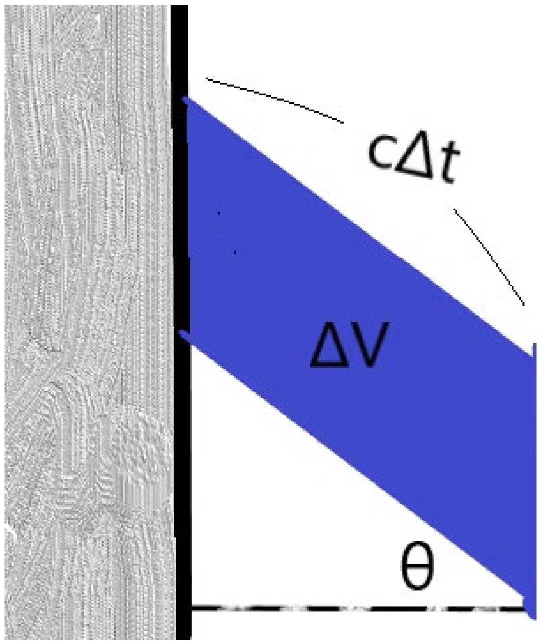
For given electric field and magnetic field, calculate Poynting vector $\vec{S}$.

$\vec{E} = E_0(cos(kz - \omega t)\hat{x} + sin(kz - \omega t)\hat{y})$, $\vec{B} = (\vec{k} \times \vec{E})/\omega$ where $\vec{k} = k\hat{z}$, $\omega = ck$

Density of field momentum is given by $\vec{S}/c^2$ where c is the speed of light.

Assuming that there is no reflection from a plane, calculate radiation pressure on the plane. The normal vector of the plane is $(1/\sqrt{3})(1,1,1)$.

(Hint: See figure below! θ is the angle between Poynting vector and plane normal vector. Light sweeps over the volume ΔV within Δt.)



Solution)

$$\vec{S} = \frac{\vec{E}\times\vec{B}}{\mu_0} = \frac{\vec{E}\times(\vec{k}\times\vec{E})}{\mu_0\omega} = \frac{\vec{k}(\vec{E}\cdot\vec{E}) - \vec{E}(\vec{k}\cdot\vec{E})}{\mu_0\omega} = \frac{1}{c\mu_0}E_0^2\hat{z}$$

Force per area A is

$$\frac{\vec{F}}{A} = \frac{\Delta\vec{p}}{\Delta t}\frac{1}{A} = \frac{\vec{\rho}}{A}\frac{\Delta V}{\Delta t}$$

where $\vec{\rho}$ is the field momentum density.

$$\Delta V = Ac\Delta t\cos\theta$$

$$\vec{F}/A = \vec{\rho}c\ \cos\theta$$

where θ is the angle between Poynting vector and plane normal vector.

Radiation pressure is the normal component of force per area.

$$P_{rad} = \frac{\vec{F}\cdot\hat{n}}{A} = \vec{\rho}\cdot\hat{n}c\cos\theta = \frac{\vec{S}\cdot\hat{n}}{c}\cos\theta = \frac{E_0^2\hat{z}\cdot\hat{n}}{c^2\mu_0}\cos\theta = \frac{E_0^2}{c^2\mu_0}(\cos\theta)^2$$

$$= \frac{E_0^2}{3c^2\mu_0} = \frac{\epsilon_0 E_0^2}{3}$$

Question 6. [15 points]

When we talk about *average* intensity, what does it precisely mean? Let us be more specific on a possible definition of a time average for a time-dependent quantity $f(t)$, given as the following:

$$\langle f \rangle \equiv \langle f(t) \rangle \equiv \lim_{T\to\infty} \frac{1}{T} \int_0^T f(t')dt'$$

To evaluate the (time) average intensity, one may need to get the average electric field $\bar{E}$ given as:

$$\bar{E} \equiv \sqrt{\left\langle \left|\vec{E}\right|^2 \right\rangle}$$

which is nothing but a root of a mean of a square (i.e. RMS) of a electric field's magnitude. Using given formulas, let us evaluate the average electric field for the following electric field vector:

$$\vec{E} \equiv \vec{E}(x,t) = E_p \cos(kx - \omega t)\,\hat{k}$$

where E_p is the peak amplitude, k is the wave number, $\hat{k}$ is the unit vector along propagation direction of the field and ω is the angular frequency of the electric field.

(a) Evaluate $\left|\vec{E}\right|^2$ as a function of x and t, for the given electric field vector $\vec{E}$. [5 points]

(b) Evaluate $\left\langle \left|\vec{E}\right|^2 \right\rangle$ by using the definition of time average given above. [5 points]

(c) Finally, evaluate $\bar{E}$, the average electric field's magnitude. Check whether $\bar{E}$ has any dependence on x. [5 points]

Possible Solution for Question 6

(a) [5 points]

$$\left|\vec{E}\right|^2 \equiv \left|\vec{E}(x,t)\right|^2 = \left|E_p\right|^2 \cos^2(kx - \omega t)$$

(b) [5 points]

$$\begin{aligned}
\left\langle \left|\vec{E}\right|^2 \right\rangle &= \lim_{T\to\infty} \frac{1}{T} \int_0^T \left|\vec{E}(x,t)\right|^2 dt' \\
&= \lim_{T\to\infty} \frac{1}{T} \int_0^T \left|E_p\right|^2 \cos^2(kx - \omega t)\, dt' \\
&= \lim_{T\to\infty} \frac{1}{T} \left|E_p\right|^2 \left[\frac{1}{2}T + \frac{1}{2\omega}\{\sin(kx - \omega T) - \sin(kx)\}\right] \\
&= \frac{1}{2}\left|E_p\right|^2
\end{aligned}$$

(c) [5 points]

$$\bar{E} \equiv \sqrt{\left\langle \left|\vec{E}\right|^2 \right\rangle} = \frac{E_p}{\sqrt{2}}$$

It has no dependence on x.

Question 7. [15 points]

Agent Jiheon get lost in a space while secret mission. He found the spaceship was 50 m away, but he only has an hour's worth of oxygen. He has a laser gun, which produces and average power of 480 kW. Will he be able to get to the spaceship before he dies? If he cannot, how much time will take Jiheon reach to the spaceship? Explain your answer.

Let's assume that total mass of agent Jiheon and space suit is 200 kg, and we can neglect initial relative velocity between Jiheon and the spaceship. $\mathrm{c} = 3.0 \times 10^8\ m/s.$

Possible Solution for Question 7.

$$\mathrm{F} = \frac{\mathrm{P}}{\mathrm{c}} \Rightarrow \mathrm{x} = \frac{at^2}{2} = \frac{Ft^2}{2m} = \frac{Pt^2}{2mc} = \frac{(4.8 \times 10^5\ W/m^2) \cdot (3.6 \times 10^3\ s)^2}{2 \cdot (200\ kg) \cdot (3.0 \times 10^8 m/s)} = 51.84\ \mathrm{m}$$

So, he can survive.

Question 8. [15 points]

Imagine the two concentric metal spherical shells. The inner shell with the radius "a" has the uniform charge Q(t) and the outer shell with the radius "b" has the uniform charge -Q(t). The material with conductivity "σ" is filled between two spherical shells. The current flows from the inner shell to the outer shell.

(a) Find the magnitude of current density between two shells in terms of σ, Q(t), and r. "r" is the distance from the center and a<r<b. (Hint : Ohm's law) (+5)

(b) Find the current between two shells in terms of σ and Q(t). Then, construct the differential equation of Q(t). (caution : when you construct the differential equation, you have to choose the correct sign.) (+3)

(c) Find the displacement current between two shells in terms of σ and Q(t). You will have to use the result of (b) to solve this problem. (+2)

(d) Find the magnetic field between two shells. (+5)

Possible Solution for Question 8

(a) $J = \sigma E = \sigma \mathrm{k} \frac{\mathrm{Q}}{r^2}$ (+5)

(b) $I = J \cdot 4\pi r^2 = 4\pi k \sigma Q$ (+2)

$\mathrm{I} = -\frac{\mathrm{dQ}}{\mathrm{dt}} = 4\pi k \sigma Q$ (+1)

(c) $\mathrm{I_{dis}} = \epsilon_0 \frac{d\Phi_E}{dt} = \epsilon_0 \frac{d}{dt}\left(\frac{Q}{\epsilon_0}\right) = \frac{\mathrm{dQ}}{\mathrm{dt}}$

Using the result of (b), $\mathrm{I_{dis}} = -\mathrm{I} = -4\pi k \sigma Q$ (+2)

(d) Using the Ampere's law with Maxwell's modification,

$\int \vec{B} \cdot \vec{dl} = \mu_0 (I + I_{dis}) = 0$

$\therefore \vec{B} = 0$ (+5)

Question 9. [20 points]

Through this chapter, we have learned about an electromagnetic field. Then how would a charged particle behave under the influence of the field? Although an exact calculation may be out of the scope for this course, one may consider a limit case where the speed of the charged particle is small enough than the speed of light. Let us consider a field given as the following:

$$\vec{E}(x,t) = E_p \sin(kx - \omega t)\,\hat{j}$$

$$\vec{B}(x,t) = B_p \sin(kx - \omega t)\,\hat{k}$$

We consider an electron with charge $-e$ in a free space with the field given above. Since we are dealing with the case where the speed of the particle is small enough, one may assume that the work done by the field on the particle is negligible so that the amplitude of the fields, namely E_p and B_p, are essentially constants of time.

(a) Evaluate each component of the net force $\vec{F}$ by the fields acting on the electron, i.e. evaluate F_x, F_y, F_z. Notice that $\vec{E}$ has y-component only and $\vec{B}$ has z-component only. Express your answer in terms of v_x, v_y, v_z etc. (Hint. Lorentz force) [4 points]

(b) Write Newton's equation of motion for each component of the particle velocity, i.e. write the equations of motion in the following form:

$$\frac{dv_x}{dt} = (\dots), \qquad \frac{dv_y}{dt} = (\dots), \qquad \frac{dv_z}{dt} = (\dots)$$

One may notice that the particle do not accelerate along the z-direction. [4 points]

(c) Using the relation $B_p = E_p/c$, express the equation of motion for v_x and v_y in terms of v_x/c and v_y/c. Express the equations of motion in the limit of $v_x \ll c$ and $v_y \ll c$, i.e. neglect all terms which vanish when v_x/c and v_y/c go to zero (but not exactly being zero). One may see only the equation for v_y remains. [4 points]

(d) Integrate dv_y/dt to get $v_y(t)$ with $v_y(0)$ given as an initial condition. [4 points]

(e) Evaluate the time-average of the particle's kinetic energy $\langle K \rangle$:

$$\langle K \rangle \equiv \left\langle \frac{1}{2} m v^2 \right\rangle = \left\langle \frac{1}{2} m v_y^2(t) \right\rangle$$

One may refer to Question #6 for the definition of time averaged value. What is the value of $v_y(0) = v_0$ that makes the average kinetic energy minimized? [4 points]

The obtained value is often called the ponderomotive energy which is an average energy a charged particle would have under the influence of a monochromatic (i.e. single frequency) electromagnetic field.

Possible Solution for Question 9

[For each sub-question, simple calculation mistake but with correct expression for e.g. Lorentz force, Newtonian equation of motion etc. results in {-1 point} instead of no point]

(a) [4 points]

$$F_x = -ev_y B_p \sin(kx - \omega t)$$
$$F_y = -e\left(E_p - v_x B_p\right) \sin(kx - \omega t)$$
$$F_z = 0$$

(b) [4 points]

$$\frac{dv_x}{dt} = -\frac{e}{m} E_p \frac{v_y}{c} \sin(kx - \omega t)$$
$$\frac{dv_y}{dt} = -\frac{e}{m} E_p \left(1 - \frac{v_x}{c}\right) \sin(kx - \omega t)$$
$$\frac{dv_z}{dt} = 0$$

(c) [4 points]

$$\frac{dv_x}{dt} \cong 0$$
$$\frac{dv_y}{dt} \cong -\frac{e}{m} E_p \sin(kx - \omega t)$$

(d) [4 points]

$$v_y(t) \cong -\frac{eE_p}{m\omega} \cos(kx - \omega t) + C$$

with

$$C \equiv C(x) \equiv v_y(0) + \frac{eE_p}{m\omega} \cos(kx)$$

(e) [4 points]

$$\begin{aligned}
\langle K \rangle &\equiv \left\langle \frac{1}{2} m v_y^2(t) \right\rangle \\
&\cong \frac{1}{2} m \left\langle \left(\frac{eE_p}{m\omega}\right)^2 \cos^2(kx - \omega t) - 2C \frac{eE_p}{m\omega} \cos(kx - \omega t) + C^2 \right\rangle \\
&= \frac{1}{2} m \left[\left(\frac{eE_p}{m\omega}\right)^2 \langle \cos^2(kx - \omega t) \rangle - 2C \frac{eE_p}{m\omega} \langle \cos(kx - \omega t) \rangle + \langle C^2 \rangle \right] \\
&= \frac{1}{2} m \left[\left(\frac{eE_p}{m\omega}\right)^2 \frac{1}{2} - 2C \frac{eE_p}{m\omega} 0 + C^2 \right] \\
&= \frac{\mathrm{e}^2 E_p^2}{4m\omega^2} + \frac{1}{2} m C^2 \\
&\geq \frac{\mathrm{e}^2 E_p^2}{4m\omega^2}
\end{aligned}$$

when the last inequality becomes equality when $C \equiv v_y(0) + \frac{eE_p}{m\omega} \cos(kx) = 0$, i.e.

$$v_y(0) = -\frac{eE_p}{m\omega} \cos(kx)$$