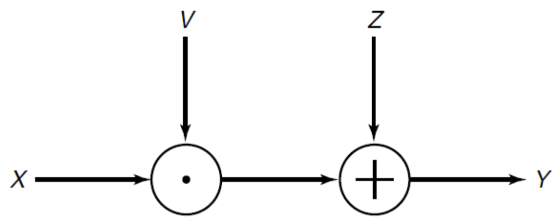
Exercise

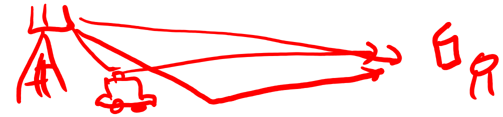
9.5 *Fading channel*. Consider an additive noise fading channel



$$Y = XV + Z,$$

where Z is additive noise, V is a random variable representing fading, and Z and V are independent of each other and of X. Argue that knowledge of the fading factor V improves capacity by showing that

$$I(X;Y|V) \geq I(X;Y).$$



$$I(X;Y,V)$$
$$= \cancel{I(X;V)}^{\,0} + I(X;Y|V)$$
$$= I(X;Y|V)$$
$$= H(X|V) - H(X|Y,V)$$
$$\geq H(X) - H(X|Y)$$
$$= I(X;Y)$$

Exercise

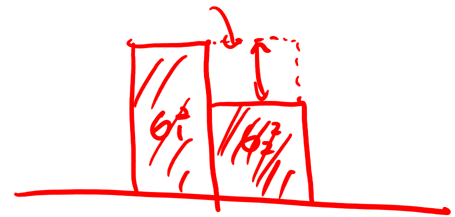
9.6 *Parallel channels and water-filling.* Consider a pair of parallel Gaussian channels:

$$\begin{pmatrix} Y_1 \\ Y_2 \end{pmatrix} = \begin{pmatrix} X_1 \\ X_2 \end{pmatrix} + \begin{pmatrix} Z_1 \\ Z_2 \end{pmatrix}, \tag{9.174}$$

where

$$\begin{pmatrix} Z_1 \\ Z_2 \end{pmatrix} \sim \mathcal{N}\left(0, \begin{bmatrix} \sigma_1^2 & 0 \\ 0 & \sigma_2^2 \end{bmatrix}\right), \tag{9.175}$$

and there is a power constraint $E(X_1^2 + X_2^2) \leq 2P$. Assume that $\sigma_1^2 > \sigma_2^2$. At what power does the channel stop behaving like a single channel with noise variance σ_2^2, and begin behaving like a pair of channels?



$$2P = \sigma_1^2 - \sigma_2^2$$

$$P = \frac{\sigma_1^2 - \sigma_2^2}{2}$$