HW#1 Solution

E2.1 A unity, negative feedback system has a nonlinear function $y = f(e) = e^2$, as shown in Figure E2.1. For an input r in the range of 0 to 4, calculate and plot the open-loop and closed-loop output versus input and show that the feedback system results in a more linear relationship.

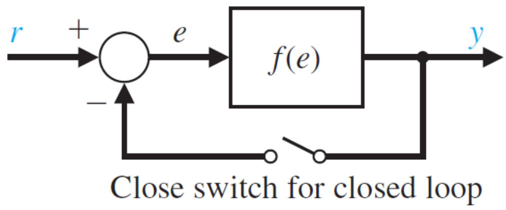


FIGURE E2.1 Open and closed loop.

(Ans)

When the system has a nonlinear function $y = e^2$, we have for the open-loop

$$e = r$$

and for the closed-loop

$$e = r - y.$$

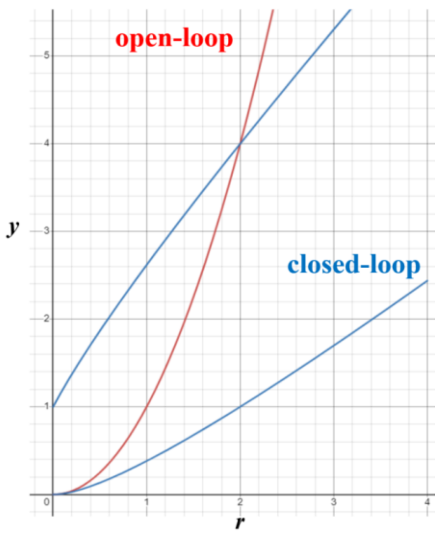
Thus, the system has

$$y = r^2 \text{ for the open-loop}$$

and

$$\begin{cases} y = e^2 \\ e^2 + e - r = 0 \end{cases} \text{ for the closed-loop}$$

for an input r in the range of 0 to 4.



The derivative of y with respect to r gives for the open-loop

$$\frac{dy}{dr} = 2r$$

and for the closed-loop

$$\frac{dy}{dr} = \frac{dy}{de}\frac{de}{dr} = \frac{2e}{2e+1}.$$

Or

$$y = e^2 = \frac{1}{4}\left(2 + 4r \pm 2\sqrt{1+4r}\right) \rightarrow \frac{dy}{dr} = 1 \pm \frac{1}{\sqrt{1+4r}}.$$

Thus, the system with a closed-loop is more linear than the system with an open-loop.

E2.10 One of the beneficial applications of an automotive control system is the active control of the suspension system. One feedback control system uses a shock absorber consisting of a cylinder filled with a compressible fluid that provides both spring and damping forces [17]. The cylinder has a plunger activated by a gear motor, a displacement-measuring sensor, and a piston. Spring force is generated by piston displacement, which compresses the fluid. During piston displacement, the pressure imbalance across the piston is used to control damping. The plunger varies the internal volume of the cylinder. This system is shown in Figure E2.10. Develop a block diagram model.

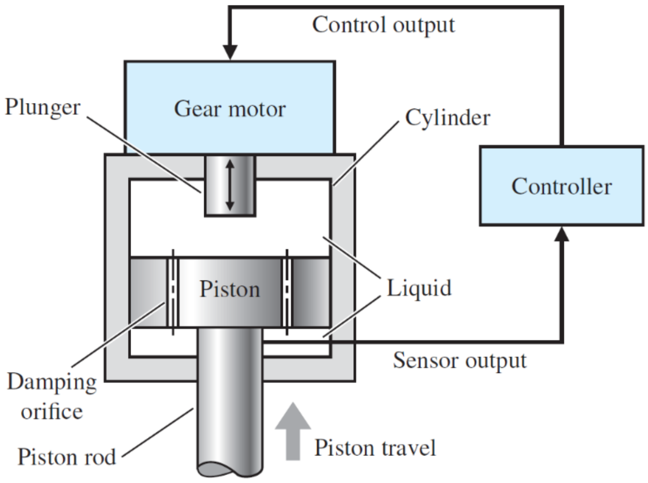


FIGURE E2.10 Shock absorber.

(Ans)

The block diagram of the shock absorber is shown in Figure E2.10.

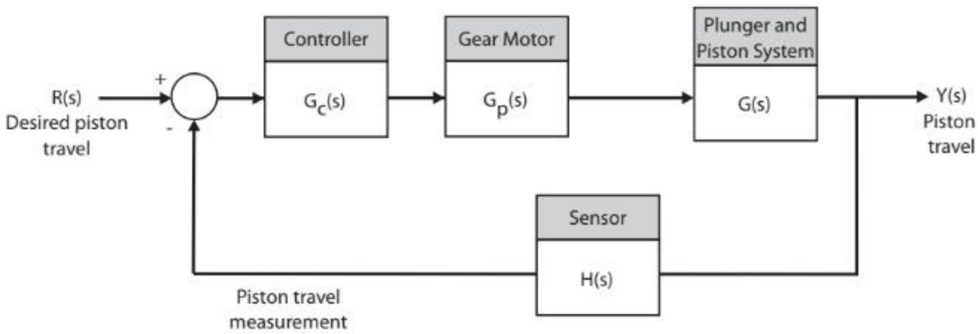


FIGURE E2.10
Shock absorber block diagram.

The closed-loop transfer function model is as follows:

$$Ans: T(s) = \frac{G_c(s)G_p(s)G(s)}{1 + H(s)G_c(s)G_p(s)G(s)}$$

E2.12 Off-road vehicles experience many disturbance inputs as they traverse over rough roads. An active suspension system can be controlled by a sensor that looks "ahead" at the road conditions. An example of a simple suspension system that can accommodate the bumps is shown in Figure E2.12. Find the appropriate gain K_1 so that the vehicle does not bounce when the desired deflection is $R(s) = 0$ and the disturbance is $T_d(s)$.

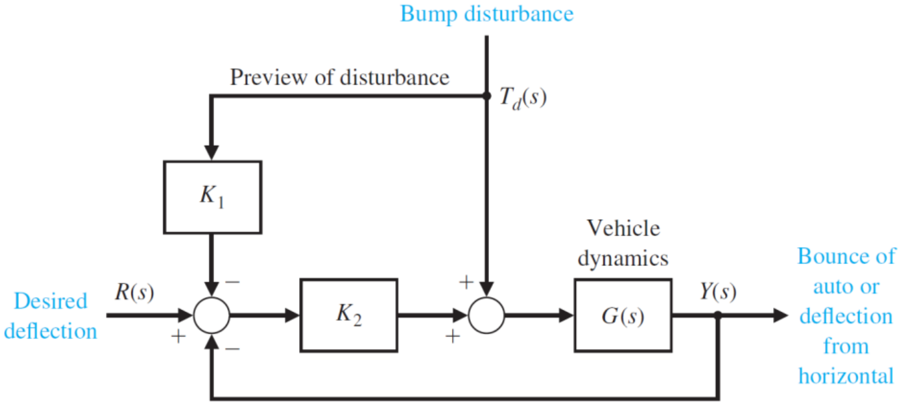


FIGURE E2.12 Active suspension system.

(Ans)

Find $Y(s)$ when $R(s) = 0$.

Then,

$$Y(s) = G(s) \cdot \left[T_d(s) + K_2 \cdot \left(-K_1 T_d - Y(s)\right)\right]$$
$$[1 + K_2 G(s)] \cdot Y(s) = G(s) \cdot [T_d(s) - K_1 K_2 T_d(s)].$$

By solving the above equation, we can obtain the transfer function as

$$Y(s) = \frac{(1 - K_1 K_2) G(s)}{1 + K_2 G(s)} T_d(s).$$

If we set $K_1 K_2 = 1$, then $Y(s) = 0$ for any T_d, which means that the vehicle does not bounce with any disturbance.

$$Ans: K_1 = \frac{1}{K_2}$$

E2.21 A high-precision positioning slide is shown in Figure E2.21. Determine the transfer function $X_p(s)/X_{in}(s)$ when the drive shaft friction is $b_d = 0.7$, the drive shaft spring constant is $k_d = 2$, $m_c = 1$, and the sliding friction is $b_s = 0.8$.

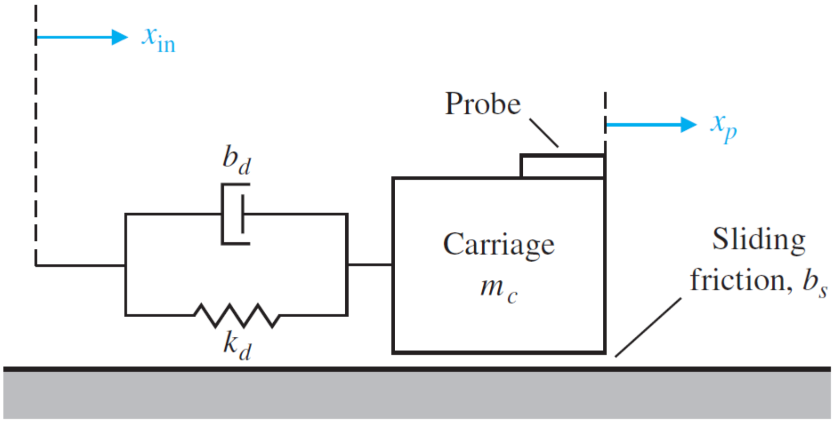


FIGURE E2.21 Precision slide.

(Ans)

The equation of system is

$$m_c\ddot{x}_p + \left(b_s\dot{x}_p + b_d\dot{x}_p + k_d x_p\right) = b_d\dot{x}_{in} + k_d x_{in}.$$

To determine the transfer function $\frac{X_p(s)}{X_{in}(s)}$, take the Laplace transform with zero initial conditions

$$[m_c s^2 + (b_d + b_s)s + k_d]X_p(s) = [b_d s + k_d]X_{in}(s).$$

Therefore, the transfer function is

$$Ans: \frac{X_p(s)}{X_{in}(s)} = \frac{b_d s + k_d}{m_c s^2 + (b_d + b_s)s + k_d} = \frac{0.7s + 2}{s^2 + 1.5s + 2}$$

E2.25 The block diagram of a system is shown in Figure E2.25. Determine the transfer function $T(s) = Y(s)/R(s)$.

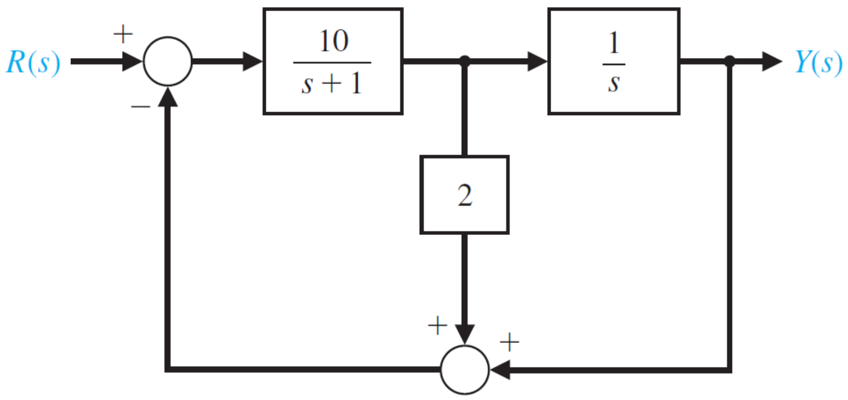
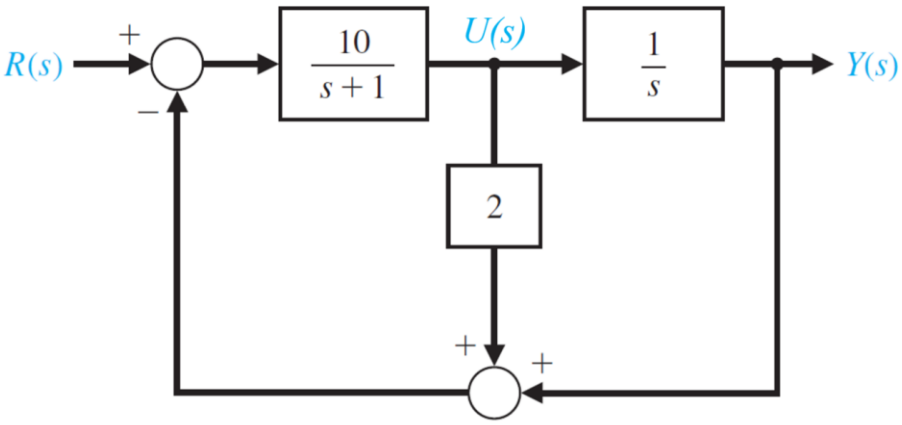


FIGURE E2.25 Multiloop feedback system.

(Ans)

Let's define $U(s)$ as shown above.



Then,

$$Y(s) = \frac{1}{s}U(s)$$
$$U(s) = sY(s). \quad (5.1)$$

Also, considering the block diagram in the figure, we can say

$$U(s) = \frac{10}{s+1}[R(s) - \{Y(s) + 2U(s)\}]$$
$$U(s) = \frac{10}{s+21}[R(s) - Y(s)]. \quad (5.2)$$

Putting (5.1) into (5.2):

$$sY(s) = \frac{10}{s+21}[R(s) - Y(s)].$$

By solving the above equation, we can get

$$Y(s) = \frac{10}{s^2+21s+10}R(s).$$

$$Ans: T(s) = \frac{Y(s)}{R(s)} = \frac{10}{s^2+21s+10}$$

E2.28 Determine the transfer function $V_0(s)/V(s)$ for the op-amp circuit shown in Figure E2.28 [1]. Let $R_1 = 167\ \text{k}\Omega$, $R_2 = 240\ \text{k}\Omega$, $R_3 = 1\ \text{k}\Omega$, $R_4 = 100\ \text{k}\Omega$, and $C = 1\ \mu\text{F}$. Assume an ideal op-amp.

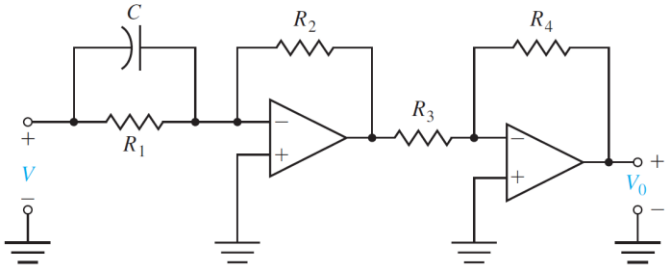
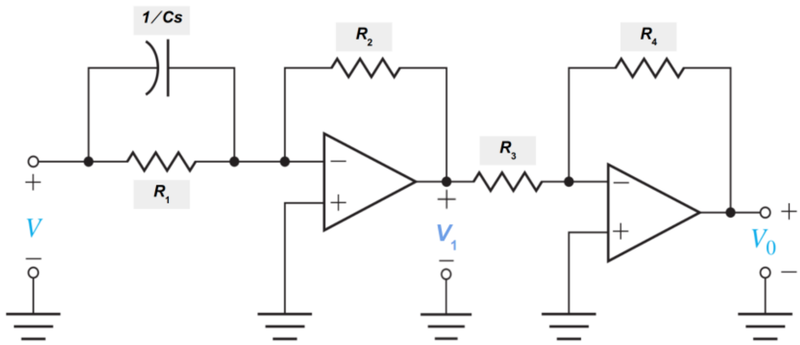


FIGURE E2.28
Op-amp circuit.

(Ans)

Using the Laplace transform, Figure 2.28 is



With an ideal op-amp and defined $V_1(s)$, we have the two relationship equations

$$\frac{1+R_1Cs}{R_1}V(s) = \frac{1}{R_2}V_1(s) \quad (6.1)$$

$$\frac{1}{R_3}V_1(s) = \frac{1}{R_4}V_0(s). \quad (6.2)$$

Putting (6.2) into (6.1), we get

$$\frac{1+R_1Cs}{R_1}V(s) = \frac{R_3}{R_2R_4}V_0(s).$$

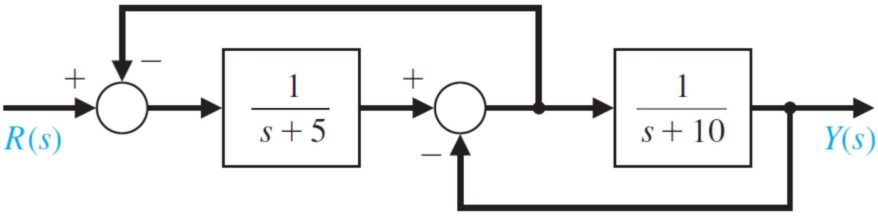
A transfer function $V_0(s)\,/\,V(s)$ is

$$\frac{V_0(s)}{V(s)} = \frac{R_2R_4(1+R_1Cs)}{R_1R_3} = \frac{R_2R_4}{R_1R_3} + \frac{R_2R_4C}{R_3}s$$

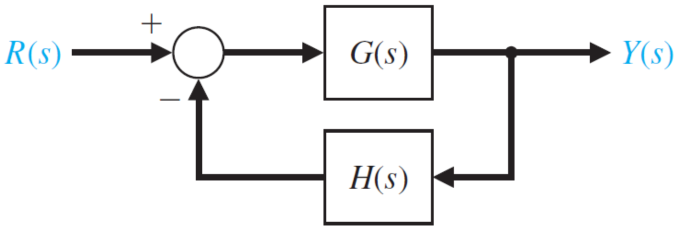
where $R_1 = 167k\Omega, R_2 = 240k\Omega, R_3 = 1k\Omega, R_4 = 100k\Omega, and\ C = 1\mu F$.

$$Ans: \frac{V_0(s)}{V(s)} = \frac{R_2R_4}{R_1R_3} + \frac{R_2R_4C}{R_3}s = 24s + 144$$

E2.29 A system is shown in Fig. E2.29(a).
(a) Determine $G(s)$ and $H(s)$ of the block diagram shown in Figure E2.29(b) that are equivalent to those of the block diagram of Figure E2.29(a).
(b) Determine $Y(s)/R(s)$ for Figure E2.29(b).



(a)



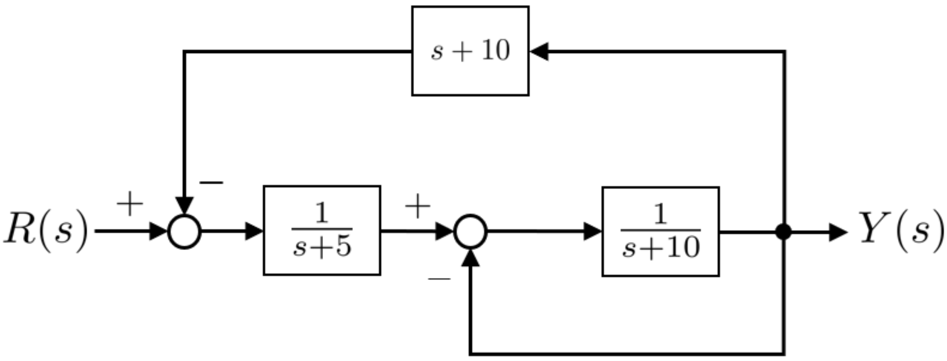
(b)

FIGURE E2.29 Block diagram equivalence.

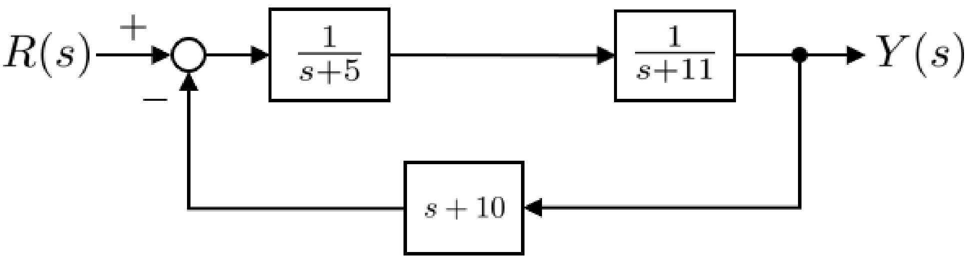
(Ans)

(a)

By moving the pick off point behind the $\frac{1}{s+10}$ block, we get:



And it is equivalent to



Then,

$$Ans: G(s) = \frac{1}{(s+5)(s+11)}$$

and

$$H(s) = s + 10 \ .$$

(b)

$$Ans: \frac{Y(s)}{R(s)} = \frac{G(s)}{1 + G(s)H(s)} = \frac{1}{(s+10) + (s+5)(s+11)}$$
$$= \frac{1}{s^2 + 17s + 65} \ .$$

P2.12 For the open-loop control system described by the block diagram shown in Figure P2.12, determine the value of K such that $y(t) \to 1$ as $t \to \infty$ when $r(t)$ is a unit step input. Assume zero initial conditions.

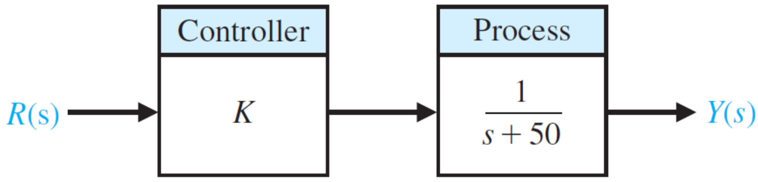


FIGURE P2.12 Open-loop control system.

(Ans)

The transfer function of the open-loop system is

$$\frac{Y(s)}{R(s)} = \frac{K}{s+50} \ .$$

Because the input is a unit step, $R(s) = 1/s$ and we have

$$Y(s) = \frac{K}{s(s+50)}$$
$$= \frac{K}{50}\left(\frac{1}{s} - \frac{1}{s+50}\right),$$

and the inverse Laplace transform is

$$y(t) = \frac{K}{50}(1 - e^{-50t})\,.$$

Therefore, as $t \to \infty$, it follows that $y(t) \to K/50$.

So, we choose $K = 50$ so that $y(t)$ approaches to 1.

P2.15 Consider the spring-mass system depicted in Figure P2.15. Determine a differential equation to describe the motion of the mass m. Obtain the system response $x(t)$ with the initial conditions $x(0) = x_0$ and $\dot{x}(0) = 0$.

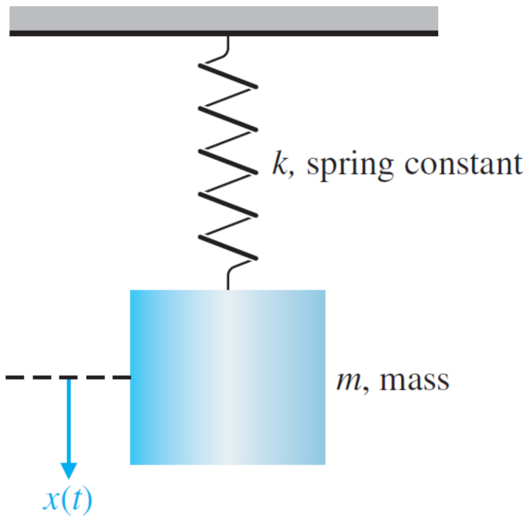


FIGURE P2.15 Suspended spring–mass system.

(Ans)

By summing the vertical direction forces and using Newton's Second law, we can obtain the below equation

$$\ddot{x} + \frac{k}{m}x = 0.$$

Because the system has no damping and external inputs, we can get the initial conditions as below.

$$x(0) = x_0, \dot{x}(0) = 0.$$

Taking the Laplace transform yields

$$X(s) = \frac{x_0 s}{s^2 + k/m}.$$

Then taking the inverse Laplace transform, we can get the answer

$$Ans: x(t) = x_0 \cos\sqrt{\frac{k}{m}}t.$$

P2.32 A system consists of two electric motors that are coupled by a continuous flexible belt. The belt also passes over a swinging arm that is instrumented to allow measurement of the belt speed and tension. The basic control problem is to regulate the belt speed and tension by varying the motor torques.

An example of a practical system similar to that shown occurs in textile fiber manufacturing processes when yarn is wound from one spool to another at high speed. Between the two spools, the yarn is processed in a way that may require the yarn speed and tension to be controlled within defined limits. A model of the system is shown in Figure P2.32. Find $Y_2(s)/R_1(s)$. Determine a relationship for the system that will make Y_2 independent of R_1.

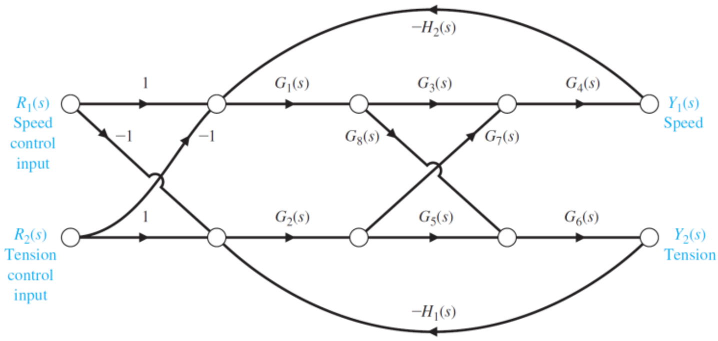


FIGURE P2.32 A model of the coupled motor drives.

(Ans)

With the observation, we can get the three loops:

$$L_1 = -G_1G_3G_4H_2$$
$$L_2 = -G_2G_5G_6H_1$$
$$L_3 = G_1G_8G_6H_1G_2G_7G_4H_2.$$

And we can also find two paths starting from $R_1(s)$ to $Y_2(s)$.

$$P_1 = G_1G_8G_6$$
$$P_2 = -G_2G_5G_6$$
$$P_3 = G_2G_7G_4H_2G_1G_8G_6$$

Using Masons' rule, transfer function is as below.

$$\frac{Y_2(s)}{R_1(s)} = \frac{G_1G_8G_6\Delta_1 - G_2G_5G_6\Delta_2 + G_2G_7G_4H_2G_1G_8G_6\Delta_3}{1-(L_1+L_2+L_3)+L_1L_2}$$

For the path 1,

$$\Delta_1 = 1$$

And for path 2,

$$\Delta_2 = 1 - L_1$$

Lastly for path 3,

$$\Delta_3 = 1.$$

Since we want independent relationship between R_1 and Y_2, transfer function should be zero. Therefore, we require

$$G_1 G_8 G_6 \Delta_1 - G_2 G_5 G_6 \Delta_2 + G_2 G_7 G_4 H_2 G_1 G_8 G_6 \Delta_3 = 0$$

$$Ans: G_1 G_8 G_6 - G_2 G_5 G_6 (1 + G_1 G_3 G_4 H_2) + G_2 G_7 G_4 H_2 G_1 G_8 G_6 = 0.$$