2020 Fall General Physics 2 Homework #10

1. (5pts) When two events occur at the location A and B. The time interval between two locations is proper time interval. (O/X) **Answer: X Proper time is the time as measured by a clock with observe two events at the same location.**

2. (5pts) If two observers are in relative motion. One observer finds two events to be simultaneous, the other observer will not. (O/X) **Answer: O**

3. (5pts) A light source S moves toward the right direction. Which points the Doppler shift occur? **Answer: (e) In contrast with classical mechanics, Doppler shift occurs in the case that the source moves transverse to the detector.**

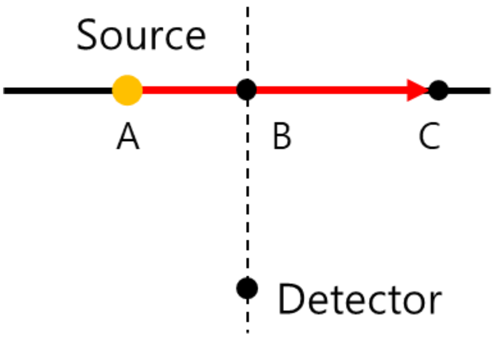
(a) A

(b) B

(c) C

(d) A,C

(e) All



4. (5pts) There are people A and B which have same age. B boards a spaceship and travels to the other star. B arrives at the stare, B returns to Earth immediately. Then, which one is older? **Answer: (a) By time dilation, time flows slowly in a moving frame.**

(a) A

(b) B

(c) same

5. (15pts) A spaceship moves away from Earth with speed v and fires a shuttle craft in the forward direction at a speed v relative to the spaceship. The pilot of the shuttle craft launches a probe in the forward direction at speed v relative to the shuttle craft. What is the speed of the probe relative to the Earth?

The speed of shuttle craft relative to the Earth is v′.

$$\mathrm{v} = \frac{\mathrm{v}' - \mathrm{v}}{1 - \frac{\mathrm{v}'\mathrm{v}}{\mathrm{c}^2}}$$ (+4)

Then, $\mathrm{v}' = \frac{2\mathrm{v}}{1 + \frac{\mathrm{v}^2}{c^2}}$ (+4)

Similarly, speed of the probe relative to the Earth is v″.

$$\mathrm{v} = \frac{\mathrm{v}'' - \mathrm{v}'}{1 - \frac{\mathrm{v}''\mathrm{v}'}{\mathrm{c}^2}}$$ (+4)

$$\mathrm{v}'' = \frac{\mathrm{v}' + \mathrm{v}}{1 + \frac{\mathrm{v}'\mathrm{v}}{\mathrm{c}^2}} = \frac{v^2 + 3c^2}{c^2 + 3v^2} v$$ (+3)

6. (15pts) When an object's speed increases by 10 %, its momentum increases by a factor of 10. What was its original speed?

The original speed is v and original momentum is $\gamma \mathrm{mv} = \frac{\mathrm{mv}}{\sqrt{1-\frac{\mathrm{v}^2}{\mathrm{c}^2}}}$.

If speed increases by 10 %, then the speed is $\frac{11}{10}v$.

Then, the momentum becomes $\gamma' \mathrm{m} * \frac{11}{10}v = \frac{11mv}{10\sqrt{1-\frac{121v^2}{100c^2}}}$ (+5)

And it is same with $\frac{10\mathrm{mv}}{\sqrt{1-\frac{\mathrm{v}^2}{\mathrm{c}^2}}}$.

$\frac{11mv}{10\sqrt{1-\frac{121v^2}{100c^2}}} = \frac{10\mathrm{mv}}{\sqrt{1-\frac{v^2}{c^2}}}$ (+5)

$$11\sqrt{1-\frac{v^2}{c^2}} = 100\sqrt{1-\frac{121v^2}{100c^2}}$$

$$121\left(1-\frac{v^2}{c^2}\right) = 10000\left(1-\frac{121v^2}{100c^2}\right)$$

v=0.91c (+5)

7. (15pts) The only invariant time in relativity is proper time which means that a length of time at a rest frame of an object. Show that infinitesimal proper time ($d\tau$) is proportional to dS ($dS^2 = c^2dt^2 - dx^2 - dy^2 - dz^2$). Calculate ratio between infinitesimal proper time and infinitesimal time in any inertial frame (ct, x, y, z), and explain the meaning of the ratio.

At rest frame $(dx_r = dy_r = dz_r = 0)$

$$c^2 d\tau^2 - dx_r^2 - dy_r^2 - dz_r^2 = c^2 d\tau^2$$

$$c^2 d\tau^2 = c^2 dt^2 - dx^2 - dy^2 - dz^2 = dS^2$$

(for any inertial frame)

$d\tau \propto dS$

4pts

$$c^2\left(\frac{d\tau}{dt}\right)^2 = c^2 - \left(\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 + \left(\frac{dz}{dt}\right)^2\right)$$

$$= c^2 - v^2 \qquad \beta = v/c$$

$$\frac{d\tau}{dt} = \sqrt{1-\beta^2} = \frac{1}{\gamma}$$

4pts

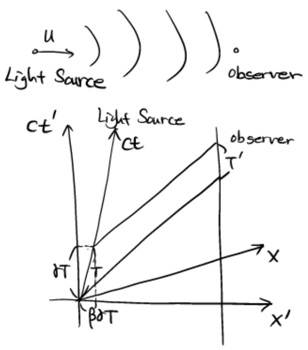
$\frac{dt}{d\tau} = \gamma \Rightarrow$ Time Dilation 2pts

8. (15pts) A source emitting light with frequency f moves toward you at speed u. By using the Lorentz transformation and drawing proper spacetime diagram, show that you measure a Doppler-shifted frequency given by,

$$f' = f\sqrt{\frac{c+u}{c-u}}$$

Use the binomial approximation to show that this result can be written in the form,

$$f' = \frac{f}{1-u/c}, (u \ll c)$$



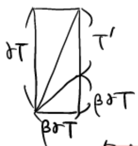


Diagram 5pts

$T = \frac{1}{f}$

$\beta = \frac{u}{c}$

$$\begin{pmatrix} c\Delta t \\ \Delta x \end{pmatrix} = \begin{pmatrix} \gamma & \beta\gamma \\ \beta\gamma & \gamma \end{pmatrix} \begin{pmatrix} T \\ 0 \end{pmatrix} = \begin{pmatrix} \gamma T \\ \beta\gamma T \end{pmatrix}$$

Lorentz Transformation 5pts

$T' = \gamma T - \beta\gamma T$

$\frac{1}{f'} = \gamma(1-\beta)\frac{1}{f}$

Calculation 3pts

$$f' = f\frac{1}{1-\beta}\sqrt{1-\beta^2} = f\sqrt{\frac{1+\beta}{1-\beta}}$$

for u<<c $\quad (1+\beta)^{1/2} \simeq (1+\frac{1}{2}\beta)$

$(1-\beta)^{-1/2} \simeq (1+\frac{1}{2}\beta)$

$$f' = f(1+\beta)^{1/2}(1-\beta)^{-1/2} \simeq f(1-\beta)^{-1} = \frac{f}{1-\beta}$$

Calculation 2pts

[Advanced Problem]

(20pts) Consider a line of positive charge with line charge density λ as measured in a frame S at rest with respect to the charges.

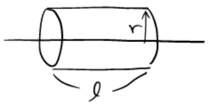
(a) (2pts) Show that the electric field a distance r from this charged line has magnitude $E = \lambda/2\pi\varepsilon_0 r$, and that there is no magnetic field (no relativity needed here).

(b) (5pts) Now consider the situation in a frame S′ moving at speed v parallel to the line of charge. Show that the charge density measured in S′ is given by $\lambda' = \gamma\lambda$, where $\gamma = 1/\sqrt{1-v^2/c^2}$.

(c) (4pts) Use the result of (b) to find the electric field in S′. Since the charge is moving with respect to S′, there is a current in S′. Also, find an expression for this current and the magnetic field it produces.

(d) (9pts) Determine the values of the quantities $\vec{E}\cdot\vec{B}$ and $E^2 - c^2B^2$ in both reference frames, and show that these quantities are invariant. Explain physical meaning of this invariance.

(a)



$\oint \vec{E}\cdot d\vec{A} = \frac{Q}{\varepsilon_0}$ 1pt

$|\vec{E}|(2\pi r l) = \frac{\lambda l}{\varepsilon_0}$

$|\vec{E}| = \frac{\lambda}{2\pi\varepsilon_0 r}$ 1pt

(b) $\lambda' = \frac{Q'}{l'} = \gamma\frac{Q}{l} = \gamma\lambda$ 1pt

$Q' = Q$ 2pts
Charge conservation

$l' = \frac{l}{\gamma}$ 2pts
Length Contraction

(c) $I = \frac{dQ'}{dt'} = \frac{dQ'}{dx'}\frac{dx'}{dt'} = \lambda' v$ 3pts

$\oint \vec{B}\cdot d\vec{l} = \mu_0 I$ $\quad |\vec{B}|(2\pi r) = \gamma\mu_0\lambda v$

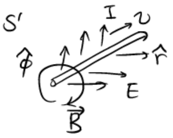
$|\vec{B}| = \frac{\gamma\mu_0\lambda v}{2\pi r}$ 1pt

(d) S

$\vec{E} = \frac{\lambda}{2\pi\varepsilon_0 r}\hat{r}$

$\vec{B} = 0$ 3pts

S′



$\vec{E}' = \frac{\gamma\lambda}{2\pi\varepsilon_0 r}\hat{r}$

$\vec{B}' = \frac{\gamma\mu_0\lambda v}{2\pi r}\hat{\phi}$ 3pts

$\vec{E}\cdot\vec{B} = \vec{E}'\cdot\vec{B}' = 0$ 1pt

$$|\vec{E}|^2 - c^2|\vec{B}|^2 = |\vec{E}'|^2 - c^2|\vec{B}'|^2$$

$$= \frac{\lambda^2}{4\pi^2\varepsilon_0^2 r^2} \qquad = \frac{\lambda^2}{4\pi^2 r^2}\left(\frac{\gamma^2}{\varepsilon_0^2} - c^2\mu_0^2\gamma^2 v^2\right)$$

$$= \frac{\lambda^2}{4\pi^2 r^2}\left(\frac{\gamma^2}{\varepsilon_0^2} - \frac{\gamma^2\beta^2}{\varepsilon_0^2}\right) \quad \beta = \frac{v}{c}$$

$$= \frac{\lambda^2}{4\pi^2\varepsilon_0^2 r^2}$$

2pts