학번:________________ 이름:_______________

Final Exam (2.5 hours)

For all multiple questions,

- ✓ Indicate your confidence level to the first decimal between 0 ~ 1 (0: wild guess ~ 1: very confident).
- ✓ -1 point if incorrect. Your score = point x confidence level
- ✓ e.g.) 2pts x 0.6 (confidence level) = 1.2pt e.g.) –1pt x 1 (confidence level) = –1 pt

1. Consider a device whose I-V characteristic is $V = K I^2$, where K is a constant. [4pts]

(a) Can you use superposition to analyze a circuit that has this device? [2pts]

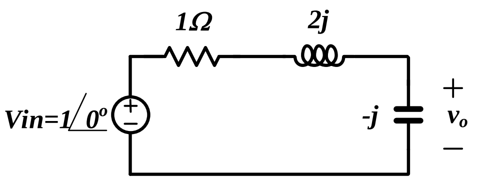
(Yes, **No**) Confidence level: ( )

(b) Can you use KCL or KVL to analyze this circuit? [2pts]

1) You can use KCL only. 2) You can use KVL only. **3) You can use both KCL and KVL.** 4) You cannot use KCL nor KVL.

Confidence level: ( )

2. Consider the circuit shown below where the input voltage (v_i) is sin(2t) which is represented in phasor as $1\angle 0^\circ$. [8pts]



(a) What is the output voltage, v_o? Express your answer in complex numbers. [2pts]

Sol) $1\angle 0^\circ \times \frac{-j}{1+2j-j} = -\frac{1}{2} - \frac{1}{2}j$

(b) Suppose the input voltage is changed to cos(t), what is the output voltage, v_o? Express your answer in complex numbers. [3pts]

Sol) 아래 세 경우 모두 정답 Answer is right in those 3 cases below,

Case1) $1\angle 0^\circ \times \frac{-2j}{1+j-2j} = 1 - 1j$

Case2) $1\angle 90^\circ \times \frac{-2j}{1+j-2j} = 1 + 1j$

Case3) **주파수가 바뀌었음으로 표현할 수 없음 (can't express it since w has changed)**

부분점수) 아래와 같이 inductor와 capacitor 값을 변경한 것이 보일 경우: +1.5점

$$\frac{-j(\text{capacitor})}{1+2j(\text{inductor})-j(\text{capacitor})} \rightarrow \frac{-2j(\text{capacitor})}{1+j(\text{inductor})-2j(\text{capacitor})}$$

(c) Suppose the input voltage is changed sin(2t) + cos(t). What is the output voltage, v_o? Assume that your answer to (a) and (b) are $a+bj$ and $c+dj$, respectively. Express your answer in time domain. Use a,b,c,d to express your answer. [3pts]

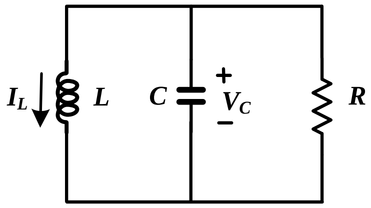
Sol) $v_{o,1} = \sqrt{a^2+b^2} \cdot \sin\left(2t + \tan^{-1}\frac{b}{a}\right)$, $v_{o,2} = \sqrt{c^2+d^2} \cdot \cos\left(t + \tan^{-1}\frac{d}{c}\right)$

$$\therefore v_o = \sqrt{a^2+b^2} \cdot \sin\left(2t + \tan^{-1}\frac{b}{a}\right) + \sqrt{c^2+d^2} \cdot \cos\left(t + \tan^{-1}\frac{d}{c}\right)$$

부분점수 1) 답에 $\sqrt{a^2+b^2}$ 또는 $\sqrt{c^2+d^2}$ 가 존재할 경우: +0.5점 (if those terms exists, +0.5 points)

부분점수 2) 답에 $\sin\left(2t + \tan^{-1}\frac{b}{a}\right)$ 또는 $\cos\left(t + \tan^{-1}\frac{d}{c}\right)$ 가 존재할 경우: +0.5점

3. Consider the below RLC circuit, where the initial conditions are $V_C(0) = 1V$, $I_L(0) = 0A$. [14pts]



(a) What kind of response does this circuit have when R=12.5Ω, L=4H and C = 0.01F? [2pts]

1) Overdamped 2) Underdamped 3) Critically damped Confidence level: ( )

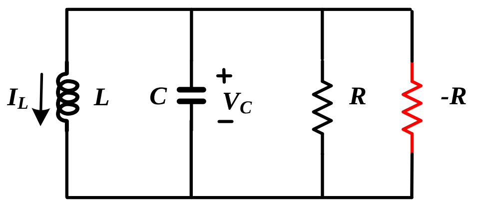
(b) What is the transient waveform of V_C for t>0? Please provide a mathematical expression. [3pts]

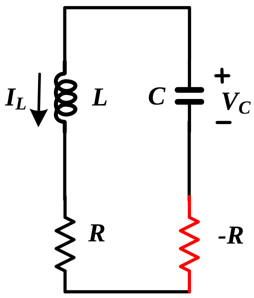
$$V_c(t) = e^{-4t}(\cos 3t - 4/3 sin3t)$$
$$V_c(t) = e^{-A_1 t}(B_1 \cos C_1 t + B_2 sinC_2 t)$$

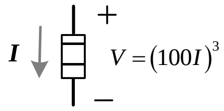
부분점수: A_1, B_1, B_2, C_1, C_2 틀릴 경우 각 경우 마다 -0.5 점

(c) Suppose it is possible to use negative resistance. By using a negative resistance, please modify the circuit so that it produces a sustained oscillation with finite amplitude. [3pts]

두 경우를 제외 하고 0 점





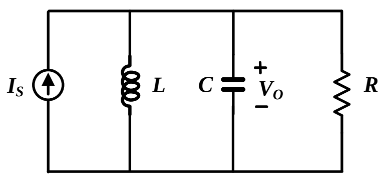
(d) Suppose the resistor is replaced by a device that has an I-V characteristic of $V = (100\, I)^3$. () What kind of response do you expect to see? [3pts]

1) Overdamped 2) Underdamped 3) Critically damped 4) First-order system 5) Underdamped or critically to overdamped

6) Overdamped to critically or underdamped. 7) None of the above. Confidence level: ( )

(e) Suppose a current source is added as shown below, where $I_S = I_0 \sin\left(\frac{t}{\sqrt{LC}}\right)$. [3pts]

What is the amplitude of the steady-state output voltage? (Definition of amplitude: $A_0 \sin(t)$ has amplitude of A_0.)



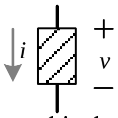
$$V_O = I_S \times R = I_0 R \sin\left(\frac{1}{\sqrt{LC}} \cdot t\right),\ at\ \omega = \sqrt{\frac{1}{LC}}$$

$$A_O = I_O \times R$$

$$V_O = I_S \times Z = I_S (j\omega L \,||\, \frac{1}{j\omega C} \,||\, R)$$

부분 점수: 1) Impedance Z 를 적어만 놓은 학생 0.5 점(+0.5 points for those who only wrote impedance Z right)

2) 공진 주파수에서 L-C 에 의한 Impedance 가 무한대임을 인지하였으나, V = I/R 로 정답을 작성한 학생 1 점 (recognized that LC impedance is infinite, but wrote the answer as V=I/R, +1 point)

4. Consider a two terminal device  whose I-V characteristic is $v(t) = K\dfrac{d^2 i(t)}{dt^2}$, where K is a constant. [17pts]

(a) What is the total phase shift due to this device for a sinusoid voltage or current? Express your answer using K. [2pts]

Sol) K≥0: 180° or -180° or π or –π & K < 0: 0

(b) Find a frequency domain representation (e.g. capacitor is 1/jωC) of this device for the case when input voltage is sin(10t). Express your answer using K. [2pts]

Sol) -100K : pt

(Partial score) 100K : 1pt -100 : 1pt 100 : 0pt

(c) Find the I-V characteristic of this device in s-domain (Laplace transform) when all initial conditions are zero. (e.g. express V(s) using I(s), s, and K) [2pts]

Sol) $\mathrm{V(s)} = Ks^2I(s) - sf(0^-) - f'(0^-) = Ks^2I(s)$ ∵ all initail conditions are zero

or $\mathrm{Z(s)} = \mathrm{V}(s)/\mathrm{I}(s) = Ks^2$

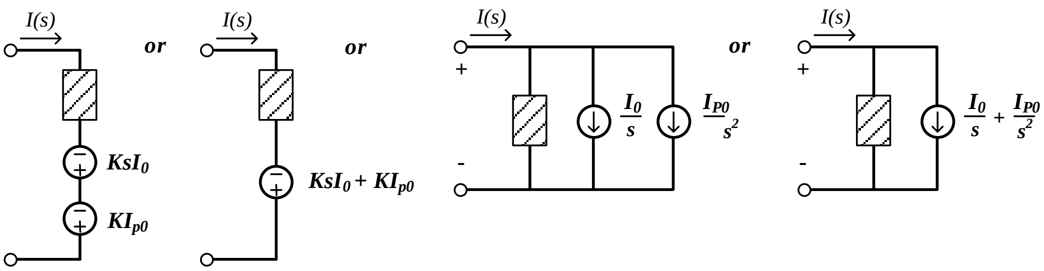
(d) Repeat (c) when initial conditions are not zero. Express your answer using K, I_{P0} = di/dt | $_{t=0}$ and $I_0 = i(0)$ [3pts]

Sol) $\mathrm{V(s)} = Ks^2I(s) - KsI_0 - KI_{P0}$

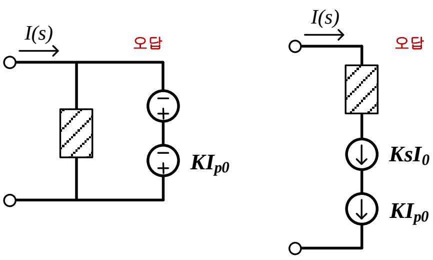
(Partial score) 부호 틀릴 시 개당 -1pt

1pt per a term : ex1) $\mathrm{V(s)} = Ks^2I(s) - KI_{P0}$: 2pt ex2) $\mathrm{V(s)} = Ks^2I(s) + KsI_0$: 1pt

(e) Draw the equivalent circuit model for (d). [3pts]

Sol) 

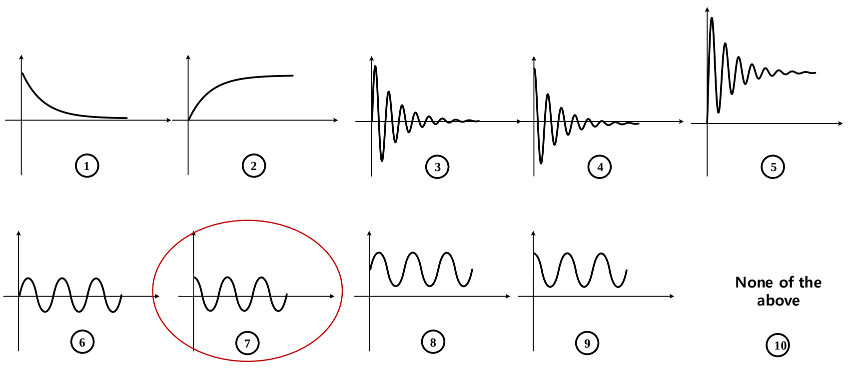
(Partial score) 부호 or voltage source 방향 틀릴 시 개당 -1pt



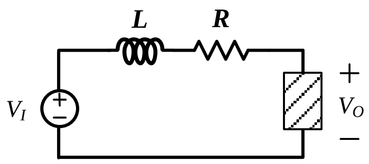
(f) What are possible waveforms of the below circuit in v_o? Choose all that applies. Assume zero initial condition. [3pts]

Sol) $V_O(s) = \frac{Ks}{Ks^2+R}$

$v(t) = \cos(\sqrt{\frac{R}{k}}t)$



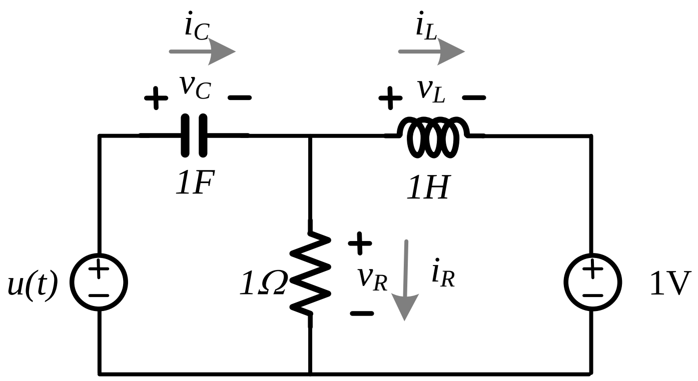
(g) Consider the circuit shown below. What kind of filtering function does the above circuit provide from V_I to V_O? [2pts]



1) low-pass 2) band-pass **3) high-pass** 4) band-stop

Confidence level ( )

5. Please fill in the below table. RLC circuits. Only the underlined cells will be graded. [13pts]



at $t=0^-$

i_R	1 [A]	v_R	1 [V]
i_C	0 [A]	v_C	-1 [V]
i_L	-1 [A]	v_L	0 [V]

at $t=0^+$

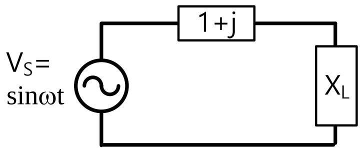
i_R	2 [A]	v_R	2 [V]
i_C	1 [A]	v_C	-1 [V]
i_L	-1 [A]	v_L	1 [V]

at $t=0^+$

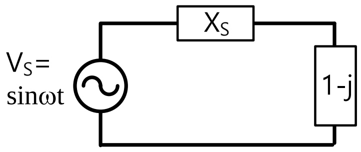
di_R/dt	-1 [A/s]	dv_R/dt	-1 [V/s]
di_C/dt	0 [A/s]	dv_C/dt	1 [V/s]
di_L/dt	1 [A/s]	dv_L/dt	-1 [V/s]

Ans) each cell is worth 1point (Correct: 1pt, Incorrect: 0pt). No deduction for missing units.

6. [AC power] Consider the circuits shown below. [10pts]



Problem (a)



Problem (b)

(a) In the left circuit (a), find X_L that will result in maximum power transfer to the load X_L. [2pts]

Ans) 1-j

(b) In the right circuit (b), find X_S that will result in maximum power transfer to the load 1-j. [2pts]

Ans) j

(c) What is the power factor (PF) for your answer to (a)? [2pts]

Ans) 1

(d) Suppose the input voltage Vs is changed to $\sin(\omega t + 90^{O})$ in Fig. (a). [2pts]

Does you answer in (a) still result in maximum power transfer? (Yes, No) Confidence level ( ) [1pts]

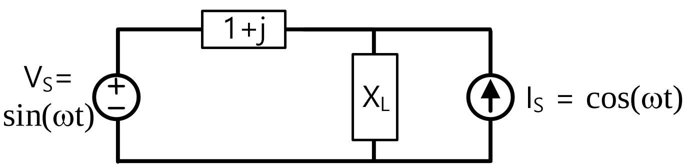
How about PF? (Increases, Decreases, stays the same) Confidence level ( ) [1pts]

(e) Suppose there is another current source Is = cos(ωt) as shown below [2pts]

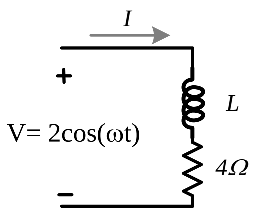
Does your answer in (a) still result in maximum power transfer for both V_S and I_S?

(Yes, No) Confidence level ( ) [1pts]

How about PF for V_S and I_S? (Increases, Decreases, stays the same) Confidence level ( ) [1pts]



7. Consider the circuit shown below. [11pts]



(a) What is the apparent power? Express your answer using ω and L. [2pts]

Ans) $\frac{1}{2}\frac{4}{\sqrt{w^2L^2+16}}$

+1pts if 1/2 is omitted

+0.5pts if only denominator is correct

(b) Suppose the power factor (PF) is 0.8. What is L when ω=3? [2pts]

Ans) 1H

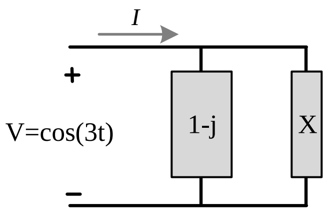
Correct even if there is no unit

(c) Consider the circuit shown below. What circuit element should X be and what is its value so that PF = 1. [5pts]

X is a (resistor, inductor, capacitor) (1pt) Confidence level ( )

Current through X represented in complex number is ___-0.5j____ (2pt)

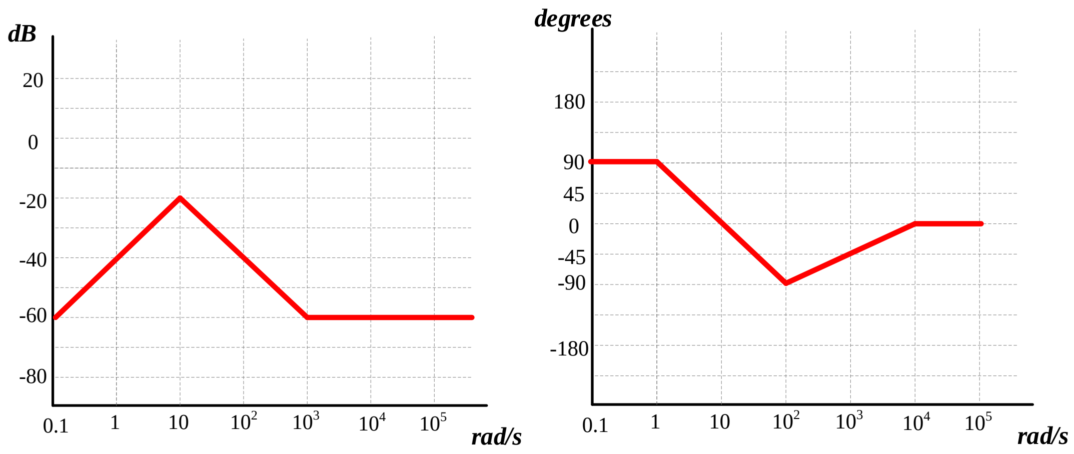
The value of X is __2/3__ Ω/H/F (2pt).



(d) In Korea, does your electric bill increase when you use devices of power factor less than 1 ? [2pts]

(Yes, No) Confidence level ( )

8. Draw the bode plot of the below transfer function. $H(s)=\dfrac{s\cdot(1+s/1000)}{100\cdot(1+s/10)^2}$ [8pts]



채점기준(dB)

Right answer: 4점

If suited in those cases +x points per each cases (Overall 3 points for all right cases)

0.1~10 range +20dB/dec: +0.5점

10~1000 range: −20dB/dec: +0.5점

1000~ to the end: 0dB/dec: +0.5점

At 0.1rad/s crosses −60dB point: +0.5점

At 10rad/s crosses −20dB point: +0.5점

At 1000rad/s crosses −60dB point: +0.5점

채점기준(dB)

Right answer: 4점

If suited in those cases +x points per each cases (Overall 3.5 points for all right cases)

0.1~1 range: +0°/dec: +0.5점

10~100 range: −90°/dec: +0.5점

100~10000 range: +45°/dec: +0.5점

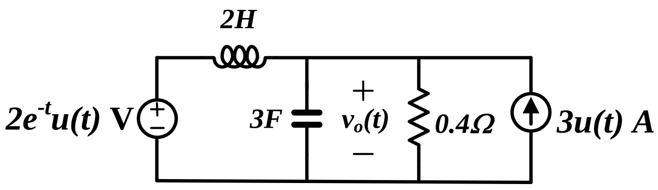
10000~ to the end: 0/dec: +0.5점

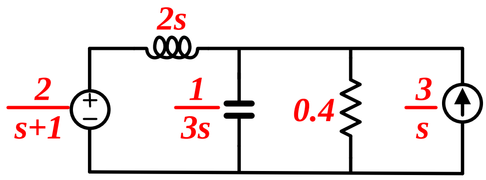
At 0.1rad/s crosses +90° point: +0.5점

At 100rad/s crosses −90° point: +0.5점

At the last frequency crosses +0° point: +0.5점

9. Consider the circuit shown below. [6pts]





(a) Find the values of the circuit shown aboe in s-domain by using Laplace transform. Assume all initial conditions are zero. [2pts]

Sol) **저항은 점수 없음. 나머지 0.5점씩 (No points for Resistance. Others are 0.5 points each)**

(b) Express V_O in s-domain. [2pts]

Sol) **(a)가 틀린 경우, (b) 무조건 틀림 (If (a) is wrong, (b) is also wrong)**

Case1)

$$\frac{\frac{2}{s+1}-v_O}{2s}+\frac{3}{s}=\frac{v_O}{1/3s}+\frac{v_O}{0.4}$$

$$\frac{2}{s+1}+6=(6s^2+5s+1)\cdot v_O \rightarrow v_O=\frac{6s+8}{s+1}\cdot\frac{1}{6s^2+5s+1}=\frac{6s+8}{s+1}\cdot\frac{1}{(2s+1)(3s+1)}$$

Case2)

$$v_O=\frac{2}{s+1}\cdot\frac{\frac{1}{3s}\parallel 0.4}{\left(2s+\frac{1}{3s}\parallel 0.4\right)}+\frac{3}{s}\cdot\left(2s\parallel\frac{1}{3s}\parallel 0.4\right)$$

$$v_O=\frac{2}{s+1}\cdot\frac{2}{12s^2+10s+2}+\frac{3}{s}\cdot\frac{4s}{12s^2+10s+2}=\left(\frac{4}{s+1}+12\right)\cdot\frac{1}{12s^2+10s+2}=\frac{6s+8}{s+1}\cdot\frac{1}{(2s+1)(3s+1)}$$

$$\therefore \boldsymbol{v_o}=\frac{\mathbf{6s+8}}{\mathbf{(s+1)(2s+1)(3s+1)}}=\frac{\mathbf{1}}{\mathbf{s+1}}-\frac{\mathbf{20}}{\mathbf{2s+1}}+\frac{\mathbf{27}}{\mathbf{3s+1}}=\frac{\mathbf{1}}{\mathbf{s+1}}-\frac{\mathbf{10}}{\mathbf{s+1/2}}+\frac{\mathbf{9}}{\mathbf{s+1/3}}$$

부분점수 1) 분모에 (2s+1) 또는 (s+1/2) 가 존재할 경우: +0.5점 (if those term exist in Denominator, +0.5 points)

부분점수 2) 분모에 (3s+1) 또는 (s+1/3) 가 존재할 경우: +0.5점

부분점수 3) 계산식 전개 과정 중 계산 틀릴 경우: 감점 없음. (Calculation mistake: no points deducted)

(c) Find the inverse Laplace transform of $\dfrac{2s^2+3s+2}{s^3+2s^2+2s}$ [2pts]

Sol)

$$\frac{2s^2+3s+2}{s(s^2+2s+2)}=\frac{A}{s}+\frac{B}{s^2+2s+2}=\frac{1}{s}+\frac{s+1}{s^2+2s+2}=\frac{1}{s}+\frac{s+1}{(s+1)^2+1}$$

$$\therefore \textbf{Laplace transform of } \frac{\mathbf{1}}{\boldsymbol{s}}+\frac{\boldsymbol{s}+\mathbf{1}}{(\boldsymbol{s}+\mathbf{1})^2+\mathbf{1}}=(\mathbf{1}+\boldsymbol{e}^{-\boldsymbol{t}}\mathbf{cos}\,\boldsymbol{t})\boldsymbol{u}(\boldsymbol{t})$$

부분점수 1) u(t) 없는 경우: -0.5점 (no u(t): -0.5 points)

부분점수 2) cos 을 sin 으로 적은 경우: -0.5점 (wrote sin in place of cos: -0.5 points)

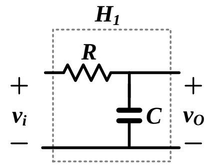
부분점수 3) cos 없는 경우: -1.0점 (didn't write cos term: -1point)

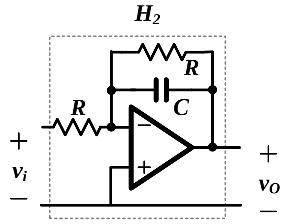
부분점수 4) a, b, c, d가 틀릴 경우 각 경우 마다: -0.5점 (-0.5 points for each a, b, c, d)

$$(\boldsymbol{a}+\boldsymbol{b}\boldsymbol{e}^{-\boldsymbol{c}\boldsymbol{t}}\,\mathbf{cos}\,\boldsymbol{d}\boldsymbol{t})\boldsymbol{u}(\boldsymbol{t})$$

부분점수 5) 그외의 경우: 점수 0점 (other cases: 0 point)

10. Consider the circuit shown below. [9pts]





(a) What is the transfer function $H_1 = V_O(s)/V_I(s)$? [2pts]

Sol)

$$H_1(s) = \frac{V_O(s)}{V_I(s)} = \frac{\frac{1}{sC}}{R + \frac{1}{sC}} = \frac{1}{1 + sCR}$$

채점기준

2점: $\frac{\frac{1}{sC}}{R + \frac{1}{sC}}$, $\frac{1}{1 + sCR}$, $\frac{\frac{1}{j\omega C}}{R + \frac{1}{j\omega C}}$, $\frac{1}{1 + j\omega CR}$ (전개 하지 않아도 됨, no need for expansion)

0.5점: $\frac{\frac{1}{\omega C}}{R + \frac{1}{\omega C}}$, $\frac{1}{1 + \omega CR}$ ($s = j\omega$가 아니라 $s = \omega$로 쓴 경우, wrote s as w instead of jw)

이외의 경우 모두 0점 (otherwise, 0 points)

(b) What is the transfer function $H_2 = V_O(s)/V_I(s)$? [2pts]

Sol)

$$-\frac{V_I}{R} - \frac{V_O}{R \,||\, {}^{1}\!/_{sC}} = 0$$

$$H_2(s) = \frac{V_O(s)}{V_O(s)} = -\frac{R \,||\, {}^{1}\!/_{sC}}{R} = -\frac{1}{R} \times \frac{1}{{}^{1}\!/_{R} + sC} = -\frac{1}{R} \times \frac{R}{1 + sCR} = -\frac{1}{1 + sCR}$$

채점기준

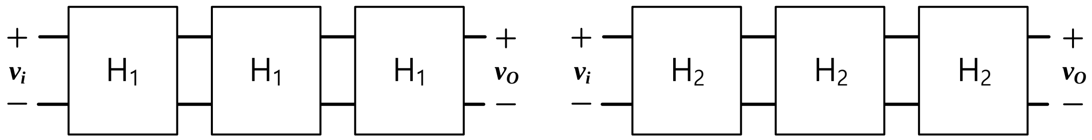
2점: $-\frac{R \,||\, {}^{1}\!/_{sC}}{R}$, $-\frac{1}{1 + sCR}$, $-\frac{R \,||\, {}^{1}\!/_{j\omega C}}{R}$, $-\frac{1}{1 + j\omega CR}$ (전개 하지 않아도 됨, no need for expansion)

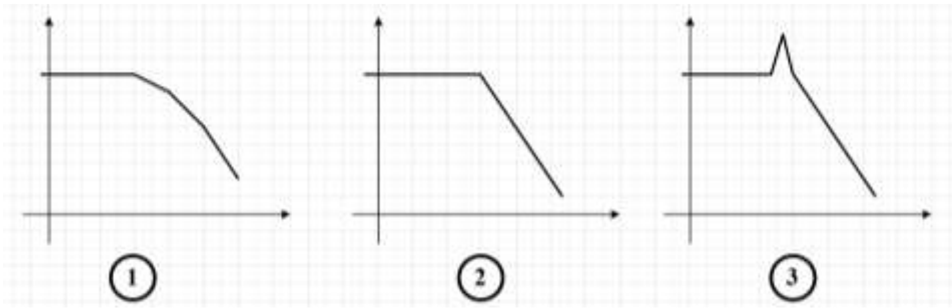
1.5점: 부호가 틀린 경우

0.5점: $-\frac{R \,||\, {}^{1}\!/_{\omega C}}{R}$, $-\frac{1}{1 + \omega CR}$ ($s = j\omega$가 아니라 $s = \omega$로 쓴 경우, wrote s as w instead of jw)

이외의 경우 모두 0점 (otherwise, 0 points)

(c) Suppose a sharp roll-off low-pass filter is desired. One student designed it using cascaded H_1 while another used cascaded H_2. Choose the correct magnitude Bode plots of cascaded H_1 and cascaded H_2. [3pts]

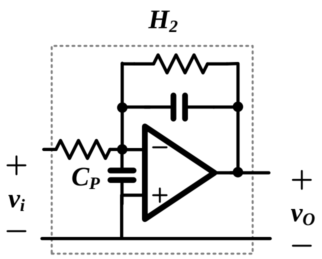




Cascaded H_1: ①, ②, ③ Confidence level ( ) [1.5pt]

Cascaded H_2: ①, ②, ③ Confidence level ( ) [1.5pt]

(d) Suppose the opamp has input capacitance as shown below. Compared to the transfer function in (b), What will happen to the magnitude of new H_2 as frequency is increased? [2pts]



|H2| (increases, decreases, stays the same) due to C_P. Confidence level ( )