

MAS 250 – MIDTERM

October 18, 2022

Time allowed: 165 minutes

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Honor Code: I will be academically honest in all of my academic work and will not tolerate academic dishonesty of others.

Instructions: Show all your work on your solutions. You will NOT receive credit if you do not justify your answers.

Good luck!

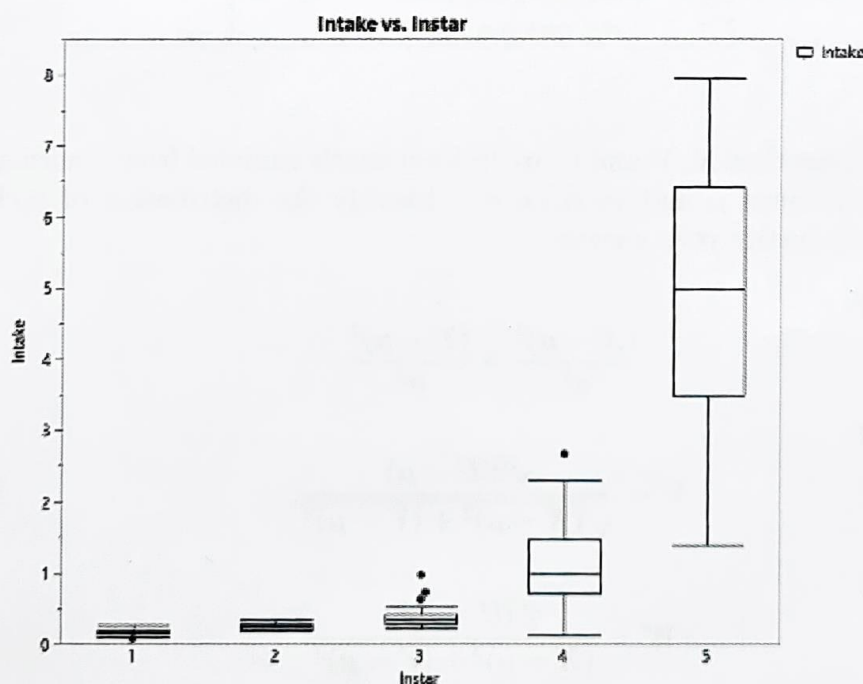
1. (12 points) According to data from the National Institute of Mental Health, the 12-month prevalence of bipolar disorder (otherwise known as manic-depression) among U.S. adults is about 3%. The 12-month prevalence means the proportion of people who experience bipolar symptoms in a 12-month period. A researcher is studying the prevalence of bipolar disorder in city A. The researcher took a random sample of 30 adults and checked whether they have bipolar disorder or not. The researcher found that 4 people experienced the bipolar symptoms in a 12-month period (sample proportion = $4/30=0.13$), which is a higher proportion than 3%.
 - (a) (4 points) What are the population and the sample in this problem?
 - (b) (4 points) What are the parameter and the statistic?
 - (c) (4 points) What is the variable of interest? Is it categorical or quantitative?

2. (13 points) Suppose there are two boxes. Box 1 contains 20 articles, of which 5 are defective, and Box 2 contains 30 articles, of which 6 are defective. One article is randomly selected from each box, and the selections from the two boxes are independent. Let X denote the total number of defective articles obtained.
- (a) (5 points) Determine the probability distribution of X .
 - (b) (4 points) Calculate $E(X)$.
 - (c) (4 points) Calculate $V(X)$.
3. (7 points) Two independent assembly lines I and II have the same rate of defectives in their production of voltage regulators. Five regulators are sampled from each line and tested. Among the total of ten tested regulators, four are defective. Find the probability that exactly two of the defective regulators came from line I.
4. (9 points) Resistors used in the construction of an aircraft guidance system have life lengths whose density is given as (with measurements in thousands of hours):

$$f(x) = \frac{2xe^{-x^2/10}}{10}, \quad x > 0.$$

- (a) (4 points) Find the probability that the life length of a randomly selected resistor of this type exceeds 5000 hours.
- (b) (5 points) If three resistors of this type are operating independently, find the probability that exactly one of the three will burn out prior to 5000 hours of use.

5. (16 points) Suppose that the moment generating function of the amount of time an adult in Korea gets sleep on any night is given as $\phi(t) = e^{7t+2t^2}$, $-\infty < t < \infty$.
- (3 points) What is the distribution of the amount of time an adult in Korea gets sleep on any night?
 - (3 points) What is the probability that a randomly selected adult in Korea gets more than 10 hours of sleep tonight?
 - (3 points) If only 1.5% of the adults sleep less than a specified hour w , what is the value of w ?
 - (3 points) Suppose 9 adults are selected at random. What is the probability that the average sleep of the 9 adults is more than 6 hours?
 - (4 points) If you survey 100 groups of 9 adults, what is the probability of more than ten groups having the mean sleep less than 6 hours? Answer it using the normal approximation and the continuity correction.
6. (5 points) The data are a subset of measurements taken from one type of caterpillar by a team of researchers interested in studying the biological growth of the species. There are 267 observations in the data set, which you may assume are a representative sample of this type of caterpillar. The variables are **Intake**, Food intake of the animal (in grams/day), and **Instar**, Stage of caterpillar's life (1, 2, 3, 4, 5). Compare the distribution shapes, centers, variations, and outliers from the boxplot below.



7. (9 points) Suppose that customers arrive at a checkout counter according to Poisson process with a rate of two per minute.

- (a) (4 points) What are the mean and variance of the waiting times between successive customer arrivals?
- (b) (5 points) If a clerk takes three minutes to serve the first customer arriving at the counter, what is the probability that at least one more customer will be waiting when the service to the first customer is completed? (Hint: What is the distribution of the number of customers arriving within **three** minutes?)

8. (18 points) The joint density function of X and Y is given by

$$f(x, y) = \begin{cases} k(1 - y), & 0 < x \leq y < 1, \\ 0, & \text{elsewhere.} \end{cases}$$

- (a) (3 points) Determine k .
- (b) (4 points) Derive the marginal density of X and Y .
- (c) (4 points) Find $P(X \leq 1/3 | Y = 1/2)$.
- (d) (4 points) Calculate $Cov(X, Y)$.
- (e) (3 points) Are X and Y independent? Why or why not?

9. (11 points) Suppose that X , Y and U are independently sampled from a normal distribution with mean μ and variance σ^2 . Identify the distribution of each question. Clearly justify your answer.

- (a) (3 points)

$$\frac{(X - \mu)^2}{\sigma^2} + \frac{(Y - \mu)^2}{\sigma^2}.$$

- (b) (4 points)

$$V = \frac{\sqrt{2}(U - \mu)}{\sqrt{(X - \mu)^2 + (Y - \mu)^2}}.$$

- (c) (4 points)

$$W = \frac{2(U - \mu)^2}{(X - \mu)^2 + (Y - \mu)^2}.$$

Discrete random variables

Distribution	Parameters	Probability function	$E(Y)$	$Var(Y)$	$m(t)$
Bernoulli	p	$p^y(1-p)^{1-y}$ $y = 0, 1, 0 < p < 1$	p	$p(1-p)$	$pe^t + 1 - p$ $-\infty < t < \infty$
Binomial	n, p	$\binom{n}{y} p^y (1-p)^{n-y}$ $y = 0, 1, \dots, n$	np	$np(1-p)$	$(pe^t + 1 - p)^n$ $-\infty < t < \infty$
Hyper-geometric	N, r, n	$\binom{r}{y} \binom{N-r}{n-y} / \binom{N}{n}$	$\frac{nr}{N}$	$\frac{nr}{N} \frac{N-r}{N} \frac{N-n}{N-1}$	
Poisson	λ	$\frac{e^{-\lambda} \lambda^y}{y!}$ $\lambda > 0, y = 0, 1, \dots$	λ	λ	$e^{\lambda(e^t-1)}$ $-\infty < t < \infty$

Continuous random variables

Distribution	Parameter	Density function	$E(Y)$	$Var(Y)$	$m(t)$
$U[\alpha, \beta]$	α, β	$\frac{1}{\beta-\alpha}$ $\alpha \leq y \leq \beta$	$\frac{\alpha+\beta}{2}$	$\frac{(\beta-\alpha)^2}{12}$	$\frac{e^{t\beta} - e^{t\alpha}}{t(\beta-\alpha)}$ $t \neq 0$
Exponential	λ	$\lambda e^{-\lambda y}$ $y \geq 0, \lambda > 0$	$1/\lambda$	$1/\lambda^2$	$\frac{\lambda}{\lambda-t}$ $t < \lambda$
Gamma	α, λ	$\frac{\lambda^\alpha y^{\alpha-1} e^{-\lambda y}}{\Gamma(\alpha)}$ $y \geq 0, \alpha, \lambda > 0$	$\frac{\alpha}{\lambda}$	$\frac{\alpha}{\lambda^2}$	$\left(\frac{\lambda}{\lambda-t}\right)^\alpha$ $t < \lambda$
Normal	μ, σ^2	$\frac{1}{\sigma\sqrt{2\pi}} e^{-(y-\mu)^2/2\sigma^2}$ $-\infty < y < \infty$ $-\infty < \mu < \infty, \sigma > 0$	μ	σ^2	$e^{\mu t + \sigma^2 t^2/2}$ $-\infty < t < \infty$

