3. Conservation of Momentum and Impulse

INTRODUCTION

Elastic and inelastic collisions are performed with two dynamics carts of different masses. Magnetic bumpers are used in the elastic collision and Velcro® bumpers are used in the completely inelastic collision. In both cases, momentum is conserved.

Cart velocities are recorded using two Rotary Motion Sensors connected to the carts by string wrapped around pulleys. This measurement method adds very little friction to the experiment and, since the velocities are continuously monitored, any deceleration due to friction can be measured. The total kinetic energy before and after the collision is also studied.

What happens at the moment of collision while momentum is conserved? A cart with a spring bumper runs down a track and collides with the endstop. The cart experiences a variable force during the time of the collision, causing it to change its velocity. In this experiment, the relationship between momentum, force, and impulse will be explored for the spring bumper, a clay bumper, and a magnetic bumper.

To determine the change in momentum (impulse), the speeds before and after the collision are measured using the Smart Cart position sensor. The force during the collision is measured using the Smart Cart force sensor. To confirm the impulse, the force versus time is plotted and the impulse is determined by finding the area under the curve.

I. CONSERVATION OF MOMENTUM

THEORY

The momentum of a cart depends on its mass and velocity.

$$\text{Momentum} = \overrightarrow{p} = m\overrightarrow{v}. \tag{1}$$

The direction of the momentum is the same as the direction of the velocity. During a collision, the total momentum of the system of both carts is conserved because the net force on the two-cart system is zero. This means that the total momentum just before the collision is equal to the total momentum just after the collision. If the momentum of one cart decreases, the momentum of the other cart increases by the same amount. This is true regardless of the type of collision, and even in cases where kinetic energy is not conserved. The law of conservation of momentum is stated as

$$\overrightarrow{p}_{Total\ Before\ Collision} = \overrightarrow{p}_{Total\ After\ Collision}. \tag{2}$$

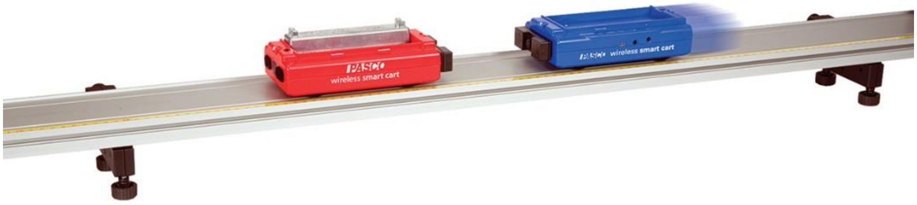
The kinetic energy of a cart also depends on its mass and speed but kinetic energy is a scalar.

$$KE = \frac{1}{2}mv^2 \tag{3}$$

The total kinetic energy of the system of two carts is found by adding the kinetic energies of the individual carts.

SETUP

1. Install the magnetic bumpers on the carts.
2. Level the track using the leveling screws on the track feet. When you place a cart at rest on the track, give it a little push in each direction. It should not accelerate in either direction.
3. Use the balance to find the mass of each cart.



4. Create a graph of *velocity* vs. *time*, putting both the Red Cart Velocity and the Blue Cart Velocity on the same vertical axis.
5. Check the signs of the velocities. The goal is to have the velocities of both carts be positive to the right.
   a. With the red cart to the left of the blue cart, face both carts with their magnetic bumpers to the right. Start recording and push both carts to the right. Both velocities should be positive.
   b. This establishes the coordinate system to have positive x to the right for both carts.

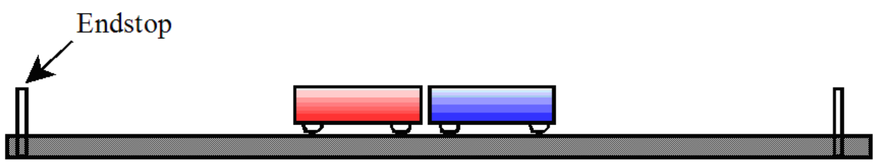
EQUIPMENT

1. Smart Cart - Red
2. Smart Cart Blue
3. 250 g Mass Bars (Set of 2)
4. 1.2 m Dynamics Track
5. Track End Stops (Set of 2)
6. Track Feet (Set of 2)

PROCEDURE

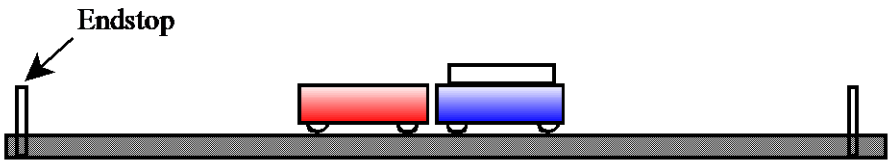
I. Explosions

A. Equal Mass Carts



1. Depress the plunger on one cart to position #2. Does it matter which cart has its plunger depressed as long as it is in contact with the other cart? Place the two carts in contact with other in the center of the track.
2. Start recording and tap the trigger release to launch the carts. Hitting the trigger with a mass bar works well.
3. Stop recording before either cart reaches the end of the track.
4. On the velocity vs. time graph, use the Multi-Coordinates Tool to find the velocity of the red and blue carts just after the explosion.

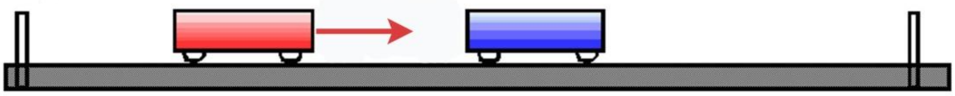
B. Unequal Mass Carts



Use the balance to find the mass of two mass bars, and then place them both in the blue cart. Repeat steps 1 through 4 of part A.

II. Completely Inelastic Collisions (Velcro® Bumpers for Inelastic Collisions)

A. Equal Mass Carts



1. Place the red and blue carts at rest on the track as shown above, with the Velcro® bumpers facing each other.

2. Because we are reversing the red cart, to keep the same frame of reference, open the Data Summary and click on the properties button next to the Red Smart Cart Position Sensor and select Change Sign.

3. Start recording and give the red cart a push toward the blue cart. Stop recording before either cart reaches the end of the track.

4. On the *velocity* vs. *time* graph, find the velocity of the red cart just before and just after the collision. It may be helpful to expand the graph, to see just that area you are interested in.

5. The initial velocity of the blue cart is zero and its final velocity is the same as the red cart because they stick together.

B. Unequal Mass Carts

Place the two mass bars in the blue cart and repeat the procedure from Part A.

III. Elastic Collisions (Magnetic Bumpers for Elastic Collisions)

A. Equal Mass Carts

1. Place the red and blue carts at rest on the track as shown above, with the magnetic bumpers facing each other. Now the red cart is in the original positive direction and the blue cart has been reversed so open the Data Summary and click on the properties button next to the Red Smart Cart Position Sensor and deselect Change Sign and open the Blue Smart Cart Position Sensor properties and select Change Sign.
2. Place the red and blue carts at rest on the track, with the magnetic bumpers facing each other.
3. Start recording and give the red cart a push toward the blue cart.
4. Stop recording before either cart reaches the end of the track.
5. On the *velocity* vs. *time* graph, find the velocity of the red cart just before and just after the collision. It may be helpful to expand the graph, to see just that area you are interested in.
6. The initial velocity of the blue cart is zero. Find the final velocity blue cart.

B. Unequal Mass Carts

1. Place the two mass bars in the blue cart.
2. Repeat the procedure from Part A.

ANALYSIS

1. Calculate the initial and the final momentum for each cart for each of the collisions.
2. Calculate the percent difference between the total initial momentum and the total final momentum for each collision.

$$\%difference = \frac{p_{before} - p_{after}}{p_{before}} \times 100\%$$

3. Calculate the initial and the final kinetic energy for each cart for each of the collisions.
4. Calculate the percent of the total kinetic energy lost for each collision.

IV. Total Momentum and Total Energy

1. Create these calculations in PASCO Capstone:

   $p_{total} = m_1\upsilon_1 + m_2\upsilon_2$

   $KE_{total} = KE_1 + KE_2$

   $KE_1 = \frac{1}{2} m_1 \upsilon_1^{\ 2}$

   $KE_2 = \frac{1}{2} m_2 \upsilon_2^{\ 2}$

   υ_1 = [Red Velocity, Ch P1(m/s)]

   υ_2 = [Blue Velocity, Ch P2(m/s)]

   m_1 = mass of Red cart

   m_2 = mass of Blue cart

2. Graph p_{total} vs. $time$ and add a second plot area for KE_{total} vs. $time$.

3. Examine the graphs to see what happens before, during, and after the collisions. Look at each type of collision and record your observations. You will have to change the masses in the calculations when you look at the unequal mass collisions.

CONCLUSION

In general, what did you learn about conservation of momentum and kinetic energy in different types of collisions?

1. Was momentum conserved for all types of collisions?
2. Was total velocity conserved for all types collisions?
3. Was energy conserved for all types of collisions? Where did the extra kinetic energy come from in the explosions? What happens to the initial kinetic energy that is lost in a collision?

II. IMPULSE

THEORY

According to Newton's Second Law,

$$\vec{F} = \frac{d\vec{p}}{dt} \tag{1}$$

where F is the force on an object, p is the momentum of the object, and t is time. Rearranging and solving for the impulse (Δp) gives

$$\Delta\vec{p} = \vec{p}_f - \vec{p}_i = \int \vec{F}dt \tag{2}$$

where the momentum is $\vec{p} = m\vec{v}$.

$$\Delta\vec{p} = m\vec{v}_f - m\vec{v}_i \tag{3}$$

and

$$\int \vec{F}dt = Area. \tag{4}$$

Area is the area under the F versus t curve.

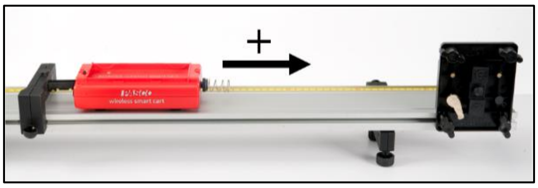


Figure 1: Definition of the Positive Direction

Note on Positive Direction:
We define the positive direction for the velocity to be away from the Smart Cart Force Sensor. See Figure 1. The force on the cart is negative during the collision.

EQUIPMENT

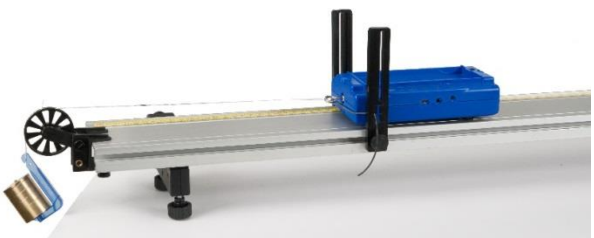
1. Smart Cart - Red 2. Smart Cart Blue
2. Force Sensor Bracket
3. 250 g Mass Bars (Set of 2)

4. Dynamics Track
5. Dynamics Track Endstops (Set of 2)
6. Dynamics Track Feet (Set of 2)

SETUP

1. Put the force sensor bracket on the end of the track. See Figure 2.
2. Use adjustable feet on both ends to level the track. Put the cart on the track and give it a little push in one direction to see if it coasts to a stop or accelerates and then push it in the direction to see if the cart coasts to a stop equally in both directions.
3. Turn on the Smart Cart and connect via Bluetooth in the Hardware Setup.
4. Put the weak spring bumper on the end of the Smart Cart force sensor.
5. Put an endstop approximately in the middle of the track. This will be the starting point of the cart for every run.

6. In PASCO Capstone, create a graph Force vs. time and add a plot area and put Velocity on the vertical axis. Set the Smart Cart Force Sensor sample rate to 500 Hz and the Smart Cart Position Sensor to 40 Hz.
7. Force Calibration (Optional)
   a. Open the Calibration Wizard at left and choose to calibrate one of the Smart Cart force sensors.
   b. Clamp the pulley on the end of the track. Put the Smart Cart on the dynamics track. Put the rubber bumper in front of the cart so it won't move. Connect a string to the hook and hold the Smart Cart in place while you hang 275 g of mass over the pulley. Adjust the pulley so the string is level with the track.



c. Choose a one-point calibration (offset) because we have already zeroed the force sensor.

d. The 275 g mass weighs 2.70 N. In the Calibration Wizard, enter 2.70 N and then click on "Set Current Value to Standard Value". Then click "Finish".

PROCEDURE

1. Measure the mass of the cart with the bumper installed.
2. Select the Force Sensor in the sample rate selector and press the Zero button beside the sample rate while nothing is touching the force sensor.
3. Open the Recording Conditions on the Sampling Control Bar and set the start condition to Measurement Based on Position is above 0.01 meters. Set the stop condition to Time-based for 1.5 seconds.

4. Compress the cart's plunger to its first position. Place and hold the cart plunger against the endstop.
5. Begin data recording, then hit the plunger release on the top of the cart with a mass bar. Remember, recording will start after the cart has moved 1 cm and will automatically stop after 1.5 seconds.

ANALYSIS

1. Using the Multi-Coordinate Tool, find the initial and final velocities for each trial and record them in Table I. In Capstone, create User Data Sets for all the columns except the last one. The %Diff is a calculation made in the Capstone calculator:

   %Diff = ([Δp]-[Impulse])/(0.5*([Δp]+[Impulse]))*100

Table I: Results from Graph Analysis

	Set	Set	Set	Set	Set	Set
	Bumper Type	Initial Velocity (m/s)	Final Velocity (m/s)	Impulse (Ns)	Δp (kg-m/s)	% Diff (%)
1	Spring					
2	Clay					
3	Magnet					

2. On the *Force* vs. *Time* graph, find the area under the curve to determine the impulse from the moment just before the collision to the moment just after the

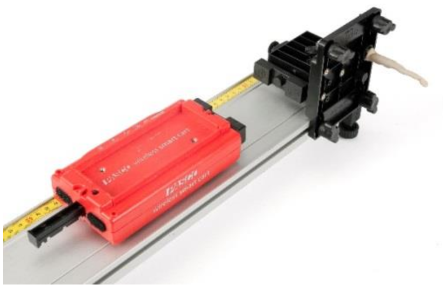
collision for each trial. You should make a selection to only include the area of the collision. You may have to increase the precision of the Area by right-clicking on the annotation and choosing the Tool Properties and changing the Numerical Format. Record each value in Table I.

3. Calculate the change in momentum Δp using the velocities for each trial and record the resulting values in Table I. Show a sample calculation.

CLAY AND MAGNETIC BUMPERS

1. Replace the weak spring bumper with the clay bumper. Shape the clay into a long protrusion so the clay will take a long time to collapse during the collision. Repeat the procedure and the analysis.

2. Replace the clay bumper with the magnetic bumper. Put another magnetic bumper on the force bracket at the end of the track. Repeat the procedure and the analysis.



QUESTIONS

1. How do the measured values for impulse compare to the calculated values for change in momentum?
2. What are some factors that may have caused error in your measured values, and how could these have been avoided?

3. How is the momentum of the cart before and after the collision related? Was energy lost in the collision? If so, where did the energy go? Kinetic Energy is $KE = \frac{1}{2} m\upsilon^2$. Calculate the kinetic energy before and after the collision.

4. How do the three different *Force* vs. *Time* graphs compare?
   a. Which had the highest maximum force?
   b. Which had the longest time of impact?
   c. Which had the greatest impulse?
   d. Which had the greatest change in momentum?
   e. Explain why the shapes of the curves are different.

5. How does the impulse from the clay compare to the impulse from the magnets? Why?
   a. Compare the initial velocities of the cart for the clay and magnetic bumpers.
   b. Compare the final velocities of the cart for clay and magnetic bumpers.

CONCLUSION

Summarize the differences and similarities of elastic, inelastic, and completely inelastic collisions. Include the numerical results for the impulse for each of the bumpers and the percent differences.

How do the measured values for impulse compare to the calculated values for change in momentum? What are some factors that may have caused error in your measured values, and how could these have been avoided?