EE201 Circuit Theory (Spring 2016)

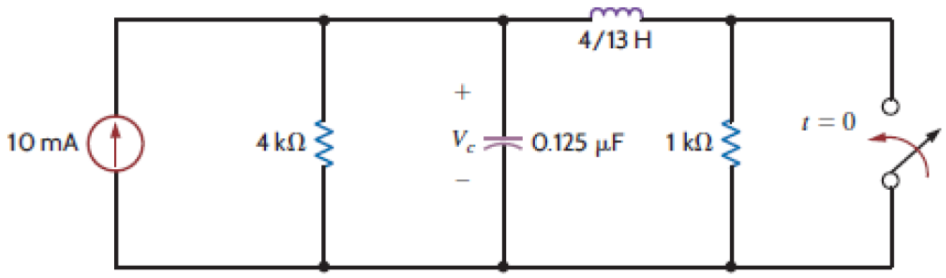
Final Exam.

(Total: 210 Points / 8 Problems)

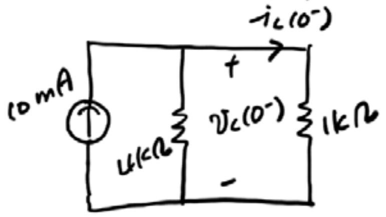
Student ID Number:

Name:

1. Find $V_c(t)$ for $t > 0$ in the circuit shown below. (20 points)



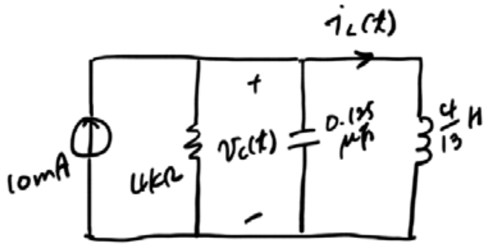
At $t = 0^-$, by replacing the capacitor and inductor by open circuit and short circuit respectively, the network can be drawn as follows.



$v_c(0^-) = 10mA \times (4k\Omega \parallel 1k\Omega) = 8V.$

$i_L(0^-) = 10mA \times \dfrac{4k}{4k+1k} = 8mA.$

For $t > 0$, the network is configured as shown below.



We can write the following differential equation:

$$\frac{d^2 v_c(t)}{dt^2} + \frac{1}{RC}\frac{d v_c(t)}{dt} + \frac{1}{LC} v(t) = 0.$$

Putting in the component values, $\dfrac{d^2 v_c(t)}{dt^2} + 2000\dfrac{d v_c(t)}{dt} + 26 \times 10^6 = 0.$

Solving the characteristic equation, $s^2 + 2000s + 26 \times 10^6 = 0$ yields

$s_{1,2} = -1000 \pm \sqrt{1000^2 - 26 \times 10^6} = -1000 \pm j5000.$

Therefore, $v_c(t) = K_1 e^{-1000t} \cos 5000t + K_2 e^{-1000t} \sin 5000t + K_3.$

By replacing the capacitor and inductor with the open circuit and short circuit respectively, we can obtain $v_C(\infty) = 0$.

Thus, $v_C(\infty) = K_3 = 0\ V$.

$v_C(0^+) = K_1 = v_C(0^-) = 8\ V$.

$$\frac{dv_C(t)}{dt} = -1000K_1 e^{-1000t}\cos 5000t - 5000K_1 e^{-1000t}\sin 5000t$$
$$-1000K_2 e^{-1000t}\sin 5000t + 5000K_2 e^{-1000t}\cos 5000t$$

Therefore, $\left.\frac{dv_C(t)}{dt}\right|_{t=0^+} = -1000K_1 + 5000K_2 = -8000 + 5000K_2$. ①

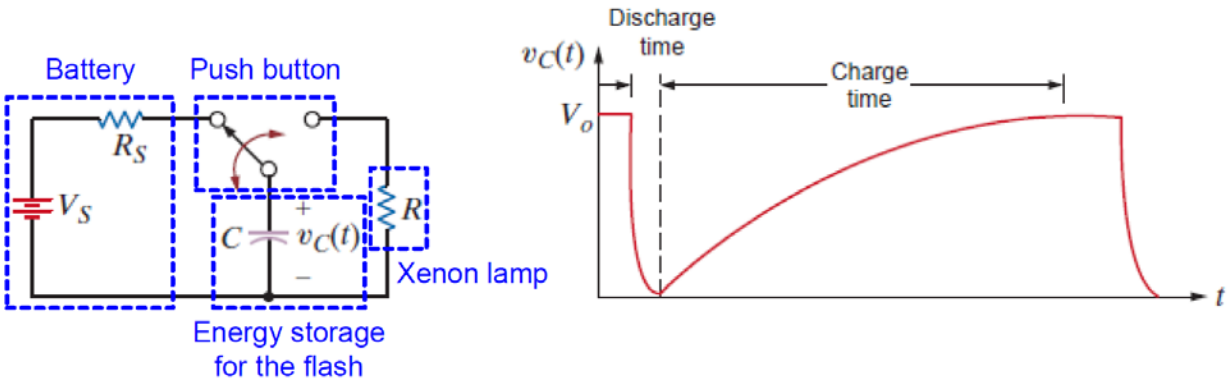
From KCL, we can write $10m = \frac{v_C(0^+)}{4k} + C\left.\frac{dv_C(t)}{dt}\right|_{t=0^+} + i_L(0^+)$.

$$\therefore \left.\frac{dv_C(t)}{dt}\right|_{t=0^+} = \frac{1}{0.125\mu} \times \left(10m - \frac{8}{4k} - 8m\right) = 0 \quad ②$$

From ① and ②, $K_2 = \frac{8}{5} = 1.6$.

As a result, $v_C(t) = 8e^{-1000t}\cos 5000t + 1.6e^{-1000t}\sin 5000t\ V$.

2. Consider the flash circuit in a camera which can be modeled using the circuit shown below. The operation of the flash circuit involves depressing the push button on the camera that triggers both the shutter and the flash and then waiting a few seconds before repeating the process to take the next picture. The voltage source V_S and resistor R_S model the battery that powers the camera and flash. The capacitor C models the energy storage, the switch models the push button, and finally the resistor R models the xenon flash lamp. Thus, if the capacitor is charged, when the button is pushed, the capacitor voltage drops and energy is released through the xenon lamp, producing the flash.



The xenon flash has the following specifications:

$50\text{ V} \le$ voltage required for successful flash $\le 70\text{ V}$

Equivalent resistance, $R = 80\ \Omega$

A time constant of 1 ms is required during flash time. In addition, the resistor R_S must dissipate no more than 100-mW peak power.

(a) Given that $V_S = 60$ V, determine values for C and R_S. (10 points)

During flash time, the discharging time constant is given by

$\tau_{discharge} = RC = 80 \times C = 1\text{ ms}$. $\therefore C = 12.5\,\mu F$.

At the beginning of the charge time, the capacitor voltage is zero and the power dissipated by R_S is at its maximum value.

Therefore, $\frac{V_S^2}{R_S} = \frac{(60)^2}{R_S} \le 0.1$, resulting in $R_S \ge 36k\Omega$.

(R_S can be any value equal to or larger than $36k\Omega$.
But in this solution, we will solve the rest of the problem assuming $R_S = 36k\Omega$.)

(b) Find the recharge time (= the time required for the capacitor voltage to reach 50 V after discharging to 0 V), the xenon lamp's voltage, current, power, and total energy dissipated during the flash. (20 points)

Let us define $t=0$ as the moment at which the switch moves from the lamp(=R) back to Rs.

At $t=0$, $v_c(0)=0$, and at $t=\infty$, $v_c(\infty)=V_s=60V$.

The charging time constant is $\tau_{charge} = R_S C = 36k \times 12.5\mu = 0.45\,s$

Therefore, we can write $v_c(t) = 60 - 60e^{-t/0.45}$.

We can obtain the recharge time, $t_{recharge}$, by solving

$50 = 60 - 60e^{-t_{recharge}/0.45}$, which leads to $t_{recharge} = 806\,ms$.

Now let us define $t=0$ as the moment when the flash is triggered.

Here we assume $v_c(0) = 50V$.

($v_c(0)$ can be any value between 50V and 60V.)

$v_c(t) = 50e^{-t/\tau_{discharge}} = 50e^{-t/1m}\,V$, which is the voltage applied across the lamp.

Hence, the lamp current is $\frac{v_c(t)}{R} = 1.6e^{-1000t}\,mA$.

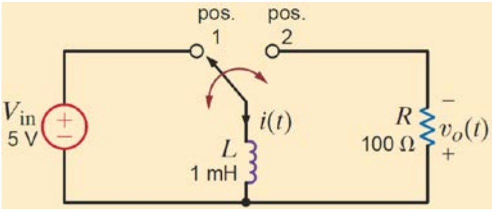
The power dissipated is calculated by

$\frac{1}{80}(50e^{-1000t})^2 = 31.25e^{-2000t}\,W$.

The total energy dissipated by the lamp during the flash is given by

$\int_0^\infty 31.25e^{-2000t}dt = -\frac{31.25}{2000}e^{-2000t}\Big|_0^\infty = 15.625\,mJ$.

3. As an application of the inductor's energy storage capability, let us consider the high-voltage pulse generator circuit shown below. This circuit is capable of producing high-voltage pulses from a small dc voltage. Let us design the circuit so that it can produce an output voltage peak of 500 V every 2 ms, that is 500 times per second.



(a) When $0 < t \le T_1$, the switch is in position 1. Find the inductor current $i(t)$ during this time period as well as the peak inductor current. (10 points)

When the switch is in position 1,

$$i(t) = \frac{1}{L}\int_0^t V_{in}\,dt = \frac{1}{L} V_{in} t = \frac{1}{1m} \times 5t = 5000t \text{ A} \quad (0 < t \le T_1)$$

The peak current is given by

$$i_{peak} = i(T_1) = 5000\,T_1 \text{ A}$$

(b) When $t > T_1$, the switch is in position 2. Find the output voltage $v_o(t)$ for $t > T_1$. Determine T_1 so that the output voltage peak becomes 500 V as targeted. (15 points)

For $t > T_1$, the inductor current flows into the resistor, producing the voltage $v_o(t) = i(t)R$.

We know that $v_o(t)$ is of the form

$v_o(t) = K e^{-(t-T_1)/\tau}$ where $\tau = L/R = 1m/100 = 10\mu s$.

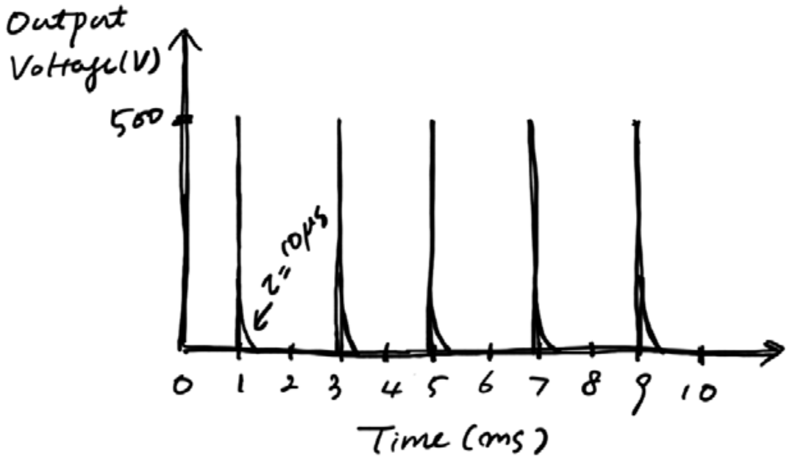
Since $v_o(T_1) = I_{peak} \times R = 5000\,T_1 \times 100 = 500000\,T_1$,

$$v_o(t) = 500000\,T_1\, e^{-(t-T_1)/10\mu} \text{ V}$$

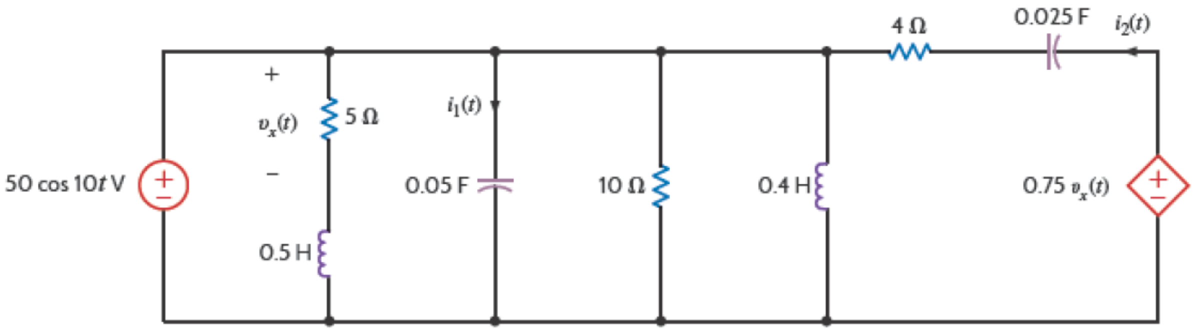
To obtain the output voltage peak of 500V, $v_o(T_1) = 500000\,T_1 = 500$.

Therefore $T_1 = 1ms$, and $v_o(t) = 500\, e^{-(t-1m)/10\mu}$ V

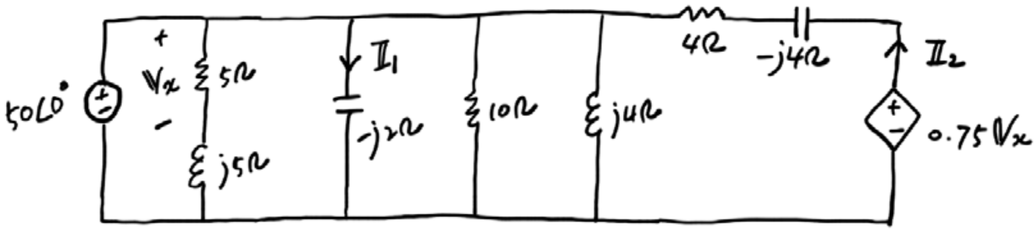
(c) If the switch moves back to the position 1 when $t = 2$ ms and repeats the same process, plot $v_o(t)$ for $0 < t < 10$ ms. (5 points)



4. Calculate $i_1(t)$ and $i_2(t)$ in the circuit shown below. (20 points)



At $\omega = 10$ rad/s, the impedance of 0.5-H inductor is $Z_{0.5H} = j\omega L = j(10\times0.5) = j5\Omega$.
Similarly, $Z_{0.4H} = j(10\times0.4) = j4\Omega$, $Z_{0.05F} = 1/j\omega C = -j/(10\times0.05) = -j2\Omega$,
$Z_{0.025F} = -j/(10\times0.025) = -j4\Omega$.
Therefore, the network can be redrawn as shown below.



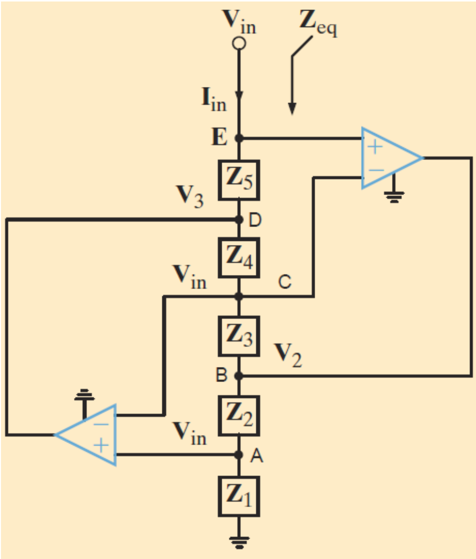
$I_1 = \dfrac{50\angle0^\circ}{-j2} = 25\angle90^\circ$ A, resulting in $i_1(t) = 25\cos(10t + 90^\circ)$ A.

$V_x = 50\angle0^\circ \times \left(\dfrac{5}{5+j5}\right) = 35.36\angle-45^\circ$ V.

$I_2 = \dfrac{0.75V_x - 50\angle0^\circ}{4-j4} = \dfrac{0.75\times35.36\angle-45^\circ - 50\angle0^\circ}{4-j4} = 6.44\angle-104^\circ$ A.

$\therefore i_2(t) = 6.44\cos(10t - 104^\circ)$ A.

5. The circuit shown below is called a general impedance converter. Derive an expression for the impedance $\mathbf{Z}_{eq}$ in terms of $\mathbf{Z}_1$, $\mathbf{Z}_2$, $\mathbf{Z}_3$, $\mathbf{Z}_4$ and $\mathbf{Z}_5$, and then using resistors of equal value and a 1-μF capacitor, create a 1-H equivalent inductance. (30 points)



Assuming ideal op-amps, the voltage at nodes A and C are V_{in} as shown in the figure.

By applying KCL at node A, we can write

$$\frac{V_2 - V_{in}}{Z_2} = \frac{V_{in}}{Z_1}$$, which can be rewritten as $V_2 = V_{in}\left(1 + \frac{Z_2}{Z_1}\right)$. —①

At node C, we can write

$$\frac{V_3 - V_{in}}{Z_4} = \frac{V_{in} - V_2}{Z_3}.$$

Solving this equation for V_3 yields

$$V_3 = V_{in}\left(1 + \frac{Z_4}{Z_3}\right) - V_2\left(\frac{Z_4}{Z_3}\right). \quad \text{—②}$$

Substituting ① into ② yields $V_3 = V_{in}\left(1 - \frac{Z_2 Z_4}{Z_1 Z_3}\right)$. —③

At node E, we can write

$$I_{in} = \frac{V_{in} - V_3}{Z_5}.$$

By using ③, $I_{in} = V_{in}\left(\frac{Z_2 Z_4}{Z_1 Z_3 Z_5}\right)$.

Therefore,

$$Z_{eq} = \frac{V_{in}}{I_{in}} = \frac{Z_1 Z_3 Z_5}{Z_2 Z_4}.$$

If $Z_1 = Z_3 = Z_5 = Z_2$ (or Z_4) $= R$ and Z_4 (or Z_2) $= 1/j\omega C$, the Z_{eq} becomes

$$Z_{eq} = j\omega C R^2 = j\omega L_{eq}.$$

Since $C = 1\mu F$ and $L_{eq} = 1H$, $R = \sqrt{L_{eq}/C} = \sqrt{1/10^{-6}} = 1k\Omega$.

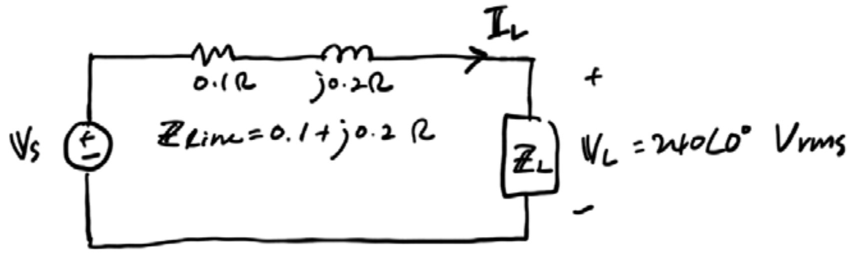
6. A transmission line with impedance $0.1 + j0.2\ \Omega$ is used to deliver power to a load. The load is capacitve and the load voltage is $240\angle 0°$ V at 60 Hz. If the load requires 15 kW and the real power loss in the line is 660 W, determine the input voltage to the line. (20 points)

This problem has an error: the load voltage should be given as $240\angle 0°$ Vrms, not $240\angle 0°$ V.

With the load voltage of $240\angle 0°$ V, the power factor becomes greater than 1, which is not possible.

Therefore, the full score of 20 points is given to all the students for this problem.

If the load voltage is given as $\mathbb{V}_L = 240\angle 0°$ Vrms correctly, then the solution is as follows.



$P_L = 15$kW

$P_{line} = 660$W

Since $P_{line} = I_L^2 R_{line}$, $I_L = \sqrt{P_{line}/R_{line}} = \sqrt{660/0.1} = 81.24$ Arms.

From $P_L = V_L I_L \times pf$, $pf = P_L / V_L I_L = 15k/(240 \times 81.24) = 0.769$.

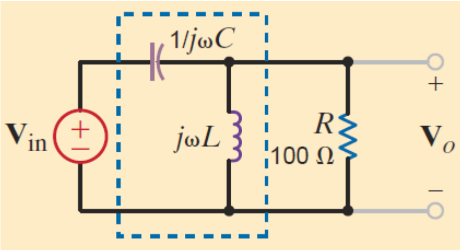
Since the load is capacitive, $pf = 0.769$ leading and the power factor angle is calculated as $\theta = \cos^{-1}(0.769) = 39.74°$.

$\therefore\ \mathbb{I}_L = 81.24\angle 39.74°$ Arms.

Therefore, $\mathbb{V}_S = \mathbb{V}_L + \mathbb{I}_L(0.1 + j0.2) = 240\angle 0° + 81.24\angle 39.74° \times (0.1 + j0.2)$

$= 236.52\angle 4.29°$ Vrms.

7. From earlier studies, we have come to expect gain from "active" devices. However, an experienced engineer has suggested that we could achieve some gain from the proper configuration of "passive" elements, and has proposed the circuit shown below. Derive the voltage gain expression of this circuit and determine values of *L* and *C* for a voltage gain of 10 at 1 kHz if the load is 100 Ω. (30 points)



The voltage gain of the network can be written as

$$\left|\frac{V_o}{V_{in}}\right| = \left|\frac{(R \| j\omega L)}{1/j\omega C + (R \| j\omega L)}\right| = \left|\frac{j\omega RL/(R+j\omega L)}{1/j\omega C + j\omega RL/(R+j\omega L)}\right| = \left|\frac{j\omega RL}{R/j\omega C + L/C + j\omega RL}\right|$$

$$= \left|\frac{j\omega L}{j\left(\omega L - \frac{1}{\omega C}\right) + L/RC}\right|. \quad -①$$

At resonance, $\omega L = 1/\omega C$ or $\omega^2 = 1/LC$.

At this resonant frequency, $\left|\frac{V_o}{V_{in}}\right| = |j\omega RC| = \omega RC$.

Therefore, to obtain the gain of 10 at 1kHz, $2\pi \times 1k \times 100 \times C = 10$.

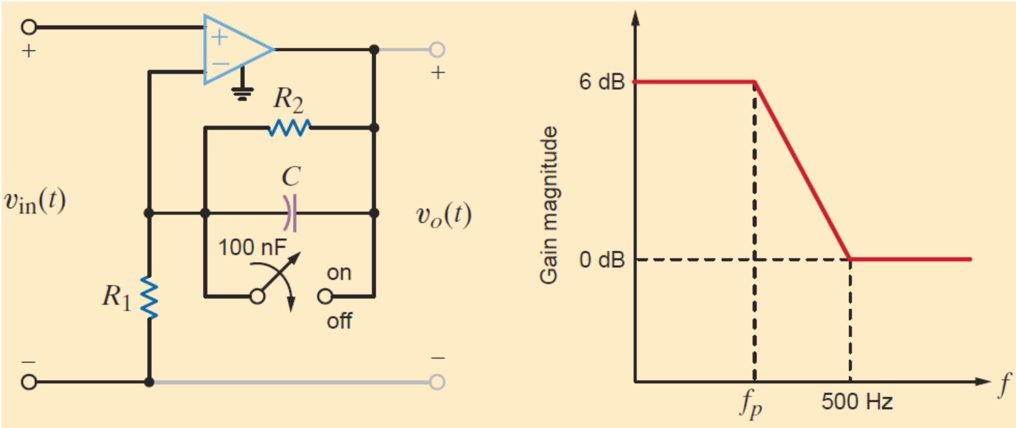
$\therefore C = 15.92\mu F$.

Since the resonance frequency should be 1kHz, $(2\pi \times 1k)^2 = 1/(L \times 15.92\mu)$.

$\therefore L = 1.59\,mH$.

(As long as the values of L and C are properly chosen so that the voltage gain of 10 is obtained at 1kHz by using ①, even if 1kHz is not designed to be the resonant frequency, the solution is still regarded to be correct.)

8. The circuit shown below is a “bass boost” amplifier that amplifies only low-frequency audio signals, as illustrated by the Bode plot.



(a) Derive the transfer function $\mathbf{V}_o/\mathbf{V}_{in}$ when the switch is open. (15 points)

When the switch is open, the circuit is in the noninverting op-amp configuration.

Therefore, $$\frac{V_o}{V_{in}} = 1 + \frac{R_2 \| 1/j\omega C}{R_1} = 1 + \frac{R_2/(1+j\omega R_2 C)}{R_1} = \frac{R_1 + R_2 + j\omega R_1 R_2 C}{R_1(1+j\omega R_2 C)}$$
$$= \left(\frac{R_1+R_2}{R_1}\right)\left[\frac{1+j\omega (R_1 \| R_2) C}{1+j\omega R_2 C}\right].$$

(b) From this transfer function and the given Bode plot, select appropriate values for R_1 and R_2. What is the resulting value of f_p in the Bode plot? (15 points)

Since the gain at dc ($\omega = 0$) should be 6 dB or a factor of 2,

$$\left.\frac{V_o}{V_{in}}\right|_{\omega=0} = \frac{R_1+R_2}{R_1} = 2,$$ resulting in $R_2 = R_1$.

From the Bode plot, the zero frequency is 500 Hz, and hence

$(R_1 \| R_2)C = \frac{R_1}{2} \times 100n = 1/(2\pi \times 500)$. $\therefore R_1 = R_2 = 6.366\ k\Omega$.

Thus, the pole frequency can be obtained as follows:

$$f_p = \frac{1}{2\pi R_2 C} = \frac{1}{2\pi \times 6.366k \times 100n} = 250\ Hz.$$