# MAS 250 – FINAL

December 13, 2022 Time allowed: 165 minutes

**Name:** Phachara Jumpa

**Honor Code:** I will be academically honest in all of my academic work and will not tolerate academic dishonesty of others.

**Pledge**: *I have neither given nor received any unauthorized aid during this exam.*

**Instructions:** Show all your work on your solutions. You will NOT receive credit if you do not justify your answers.

**Good luck!**

t-values:

- $t_{7,0.005} = 3.4995$, $t_{7,0.01} = 2.9980$
- $t_{8,0.005} = 3.3554$, $t_{8,0.01} = 2.8965$
- $t_{14,0.025} = 2.1448$, $t_{14,0.05} = 1.7613$
- $t_{15,0.025} = 2.1315$, $t_{15,0.05} = 1.7531$
- $t_{24,0.025} = 2.0639$, $t_{24,0.05} = 1.7109$
- $t_{25,0.025} = 2.0596$, $t_{25,0.05} = 1.7081$

$\chi^2$-values:

- $\chi^2_{1,0.05} = 3.8415$, $\chi^2_{1,0.025} = 5.0239$
- $\chi^2_{2,0.05} = 5.9915$, $\chi^2_{2,0.025} = 7.3778$

1. (11 points) A plant physiologist conducted an experiment to determine whether mechanical stress would affect the growth of soybean plants. Young plants were randomly assigned to one of two treatment groups: control or stress. Each group had 13 plants. After the treatment, the researcher waited 16 days and measured how much the plants had grown. **Does mechanical stress affect plant growth?** We will answer this question in steps using the data (summarized in the table below). You may assume the data are approximately bell-shaped and equal population standard deviations.

| | Control | Stress |
|---|---|---|
| $n$ | 13 | 13 |
| sample mean | 30.59 cm | 27.78 cm |
| sample standard deviation | 2.13 | 1.73 |

   (a) (3 points) What are $\mu_1$ and $\mu_2$ in the context of this problem? Assume the control group is group 1, and the stress group is group 2. Also, state the null and alternative hypotheses for the problem.

   (b) (5 points) Test the hypotheses the null and alternative hypotheses in (a) at significance level $\alpha = 0.05$. Make sure to state your conclusion and interpretation in the context of the problem. That is, make sure the conclusion and interpretation address the original question (in bold font at the beginning of the problem).

   (c) (3 points) Without creating a 95% two-sided confidence interval for $\mu_1 - \mu_2$, will the interval include 0? How do you know?

2. (12 points) In a recent national election, a newspaper conducted an exit poll of voters and asked which candidate they voted for. Out of a random sample of $n = 1000$ voters, 547 voted for the Republican candidate and 453 voted for the Democratic candidate. Let $p$ denote the population proportion of voters who voted for the Republican candidate. **Based on the exit poll, should the newspaper report that the Republican candidate won?**

   (a) (3 points) State the null and alternative hypotheses and calculate the corresponding test statistic.

   (b) (3 points) Calculate the $p$-value for the test statistic. What do you conclude at significance level $\alpha = 0.01$? Make sure your conclusion and interpretation addresses the original question (in bold font).

   (c) (3 points) Create a 95% two-sided confidence interval for $p$ and interpret the interval.

   (d) (3 points) How large a sample should be chosen to estimate the proportion $p$ with a length of 95% two-sided confidence interval no more than 0.05? Assume that you do not have previous information about $p$.

3. (8 points) Let $X_1, \cdots, X_{36}$ be a random sample from a normal distribution having known ~~variance~~ $\sigma = 3$. Consider two simple hypotheses:
standard deviation

$$H_0 : \mu = 2 \quad H_1 : \mu = 0.$$

(a) (4 points)When $\alpha = .05$, what is the power of the test?

(b) (4 points) To achieve at least 95% of power at $\mu = 0$, how large does a random sample need to be? Use $\sigma = 3$ and $\alpha = .05$.

4. (8 points) Does drinking coffee affect blood flow, particularly during exercise? Doctors studying healthy subjects measured myocardial blood flow (MBF) during bicycle exercise before and after giving the subjects a dose of caffeine that was equivalent to drinking two cups of coffee. The following table shows the MBF levels before and after the subjects took a tablet containing 200 mg of caffeine.

| Subject | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Before | 6.37 | 5.69 | 5.58 | 5.27 | 5.11 | 4.89 | 4.70 | 3.53 |
| After | 4.52 | 5.44 | 4.70 | 3.81 | 4.06 | 3.22 | 2.96 | 3.20 |
| W (Before-After) | 1.85 | 0.25 | 0.88 | 1.46 | 1.05 | 1.67 | 1.74 | 0.33 |

(a) (4 points) Do the data support the difference of the means between before and after taking caffeine? Interpret the result in the context at $\alpha = 0.01$. Use $\bar{W} = 1.15$ and $S_W = 0.63$.

(b) (4 points) Construct a 99% two-sided confidence interval for the difference between two population means. What is your interpretation about the interval?

5. (11 points) For women who are pregnant with twins, complete bed rests in late pregnancy is commonly prescribed. To test the value of this practice, 212 women with twin pregnancies were randomly allocated to a bed-rest group or a control group. The accompanying table shows the incidence of preterm delivery (less than 37 weeks of gestation). Let $p_1$ be the proportion of preterm deliveries for the bed rest group and $p_2$ be the proportion of preterm deliveries for the control group.

| | Bed Rest | Controls |
|---|---|---|
| # of preterm deliveries | 32 | 20 |
| # of regular deliveries | 73 | 87 |

(a) (4 points) Do the data suggest that there is a difference between the bed rest and control groups? Test the two sample proportion $Z$-test at $\alpha = 0.05$. (Keep four decimal places in calculating for accuracy.)

(b) (4 points) Are the resting and premature delivery associated? Answer the question using the $\chi^2$-test at $\alpha = 0.05$. (Keep four decimal places in calculating for accuracy.) You can use the following:

$52 \times 105/212 = 25.7547$, $52 \times 107/212 = 26.2453$

$160 \times 105/212 = 79.2453$, $160 \times 107/212 = 80.7547$

(c) (3 points) What is the relationship between (a) and (b)?

6. (28 points) How well does the number of beers a student drinks predict his or her blood alcohol content? Sixteen student volunteers at Ohio State University drank a randomly assigned number of 12-ounce cans of beer. Thirty minutes later, a police officer measured their blood alcohol content (BAC).

| Beers | 5 | 2 | 9 | 8 | 3 | 7 | 3 | 5 |
|---|---|---|---|---|---|---|---|---|
| BAC | 0.10 | 0.03 | 0.19 | 0.12 | 0.04 | 0.095 | 0.07 | 0.06 |
| Beers | 3 | 5 | 4 | 6 | 5 | 7 | 1 | 4 |
| BAC | 0.02 | 0.05 | 0.07 | 0.10 | 0.085 | 0.09 | 0.01 | 0.05 |

$$n = 16 \quad \sum x = 77 \quad \sum y = 1.15$$
$$\sum x^2 = 443 \quad \sum y^2 = 0.11115 \quad \sum xy = 6.5$$
$$S_{xx} = 72.4375 \quad S_{xY} = 0.9656 \quad S_{YY} = 0.0285$$

(a) (3 points) Write the statistical model for a simple linear regression (before starting analysis) and describe the assumptions for the error term.

(b) (4 points) Calculate the sample correlation coefficient and interpret the result. Conduct a test for the population correlation coefficient $\rho$, i.e. $H_0 : \rho = 0$ and $H_1 : \rho \neq 0$ at $\alpha = 0.05$.

(c) (3 points) Find the least squares line.

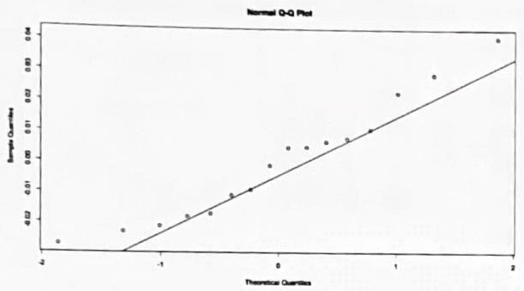
(d) (3 points) Estimate the standard deviation of the error term, $\hat{\sigma}$.

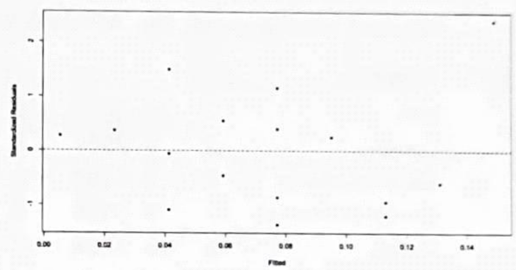
(e) (5 points) Is there sufficient evidence that BAC is linearly related to the number of beers? Answer this question by **testing the slope** at two sides using $\alpha = .05$. Specify the null and alternative hypotheses, calculate $t$ test statistic, and draw your conclusion. What is the relationship with the test in (b)?

(f) (3 points) Calculate the coefficient of determination, and interpret the number. Relate the result with the sample correlation coefficient in (b).

(g) (3 points) Predict BAC for 5.5 beers and find a 95% ~~confidence~~ prediction interval for the new $Y$.

(h) (4 points) Interpret the two plots and check the assumptions of the regression model.





7. (14 points) Iron-deficiency anemia is the most common form of malnutrition in developing countries, affecting about 50% of children and women and 25% of men. Iron pots for cooking foods had traditionally been used in many of these countries, but they have been largely replaced by aluminum pots, which are cheaper and lighter. One study designed to investigate this issue compared the iron content of some Ethiopian foods cooked in aluminum, clay, and iron pots. The iron in the food is measured in milligrams of iron per 100 grams of cooked food. The following table summarizes some numerical measures.

| Group | Sample Size | Sample Mean | Sample standard deviation |
|---|---|---|---|
| Aluminum | 12 | 1.8733 | 0.5223 |
| Clay | 12 | 2.0367 | 0.6013 |
| Iron | 12 | 3.7133 | 0.8843 |

The overall sample mean $x_{..} = 2.5411$.

(a) (3 points) Write an appropriate statistical model. Specify factor and the response variable.

(b) (5 points) Complete the following ANOVA table. Specify the null hypothesis of the test and conduct an analysis of variance at $\alpha = 0.01$.

| Source | df | Sum of Squares | Mean of Squares | $F$ | $P$-value |
|---|---|---|---|---|---|
| Between | | | | | 0.000 |
| Within | | | | | |
| Total | | 40.4740 | | | |

(Hint: Calculate SSw first, then SSb can be obtained easily.)

(c) (3 points) During the experiment the researchers actually cooked three different types of food (meat, legumes, and vegetables). Therefore, four samples of each food were cooked in each type of pot. Although the effect of different types of food was not the main interest of the study, the researchers wanted to measure both effects (caused by different types of pots and food) and their interaction effect on the iron content. In this case, write an appropriate statistical model.

(d) (3 points) Specify the null hypotheses of the three tests below and interpret the test results using the ANOVA table at $\alpha = 0.05$.

| Source | DF | SS | MS | F | P |
|---|---|---|---|---|---|
| pot | 2 | 24.8940 | 12.4470 | 92.26 | 0.000 |
| food | 2 | 9.2969 | 4.6484 | 34.46 | 0.000 |
| Interaction | 4 | 2.6404 | 0.6601 | 4.89 | 0.004 |
| Error(Within) | 27 | 3.6425 | 0.1349 | | |
| Total | 35 | 40.4738 | | | |

8. (8 points) Let $X_1, \ldots, X_n$ denote independent and identically distributed random variables from the following distribution with parameters $\theta$. Assume that $m$ is known. Then, if $\theta > 0$ and $m > 0$,

$$f(x|\theta) = \frac{mx^{m-1}}{\theta} e^{-x^m/\theta}, \quad x \geq 0.$$

(a) (4 points) Find the MLE for $\theta$.

(b) (4 points) Using the fact that $Y = \sum_{i=1}^{n} X_i^m \sim Gamma(n, \theta)$, find the mean squared error (MSE) of the MLE in (a).

$\alpha = n, \ \lambda = \frac{1}{\theta}$

Discrete random variables

| Distribution | Parameters | Probability function | $E(Y)$ | $Var(Y)$ | $m(t)$ |
|---|---|---|---|---|---|
| Bernoulli | $p$ | $p^y(1-p)^{1-y}$<br>$y = 0, 1,\ 0 < p < 1$ | $p$ | $p(1-p)$ | $pe^t + 1 - p$<br>$-\infty < t < \infty$ |
| Binomial | $n, p$ | $\binom{n}{y} p^y(1-p)^{n-y}$<br>$y = 0, 1, \ldots, n$ | $np$ | $np(1-p)$ | $(pe^t + 1 - p)^n$<br>$-\infty < t < \infty$ |
| Hyper-geometric | $N, r, n$ | $\binom{r}{y}\binom{N-r}{n-y}/\binom{N}{n}$ | $\frac{nr}{N}$ | $\frac{nr}{N}\frac{N-r}{N}\frac{N-n}{N-1}$ | |
| Poisson | $\lambda$ | $\frac{e^{-\lambda}\lambda^y}{y!}$<br>$\lambda > 0,\ y = 0, 1, \ldots$ | $\lambda$ | $\lambda$ | $e^{\lambda(e^t-1)}$<br>$-\infty < t < \infty$ |

Continuous random variables

| Distribution | Parameter | Density function | $E(Y)$ | $Var(Y)$ | $m(t)$ |
|---|---|---|---|---|---|
| $U[\alpha, \beta]$ | $\alpha, \beta$ | $\frac{1}{\beta-\alpha}$<br>$\alpha \le y \le \beta$ | $\frac{\alpha+\beta}{2}$ | $\frac{(\beta-\alpha)^2}{12}$ | $\frac{e^{t\beta}-e^{t\alpha}}{t(\beta-\alpha)}$<br>$t \neq 0$ |
| Exponential | $\lambda$ | $\lambda e^{-\lambda y}$<br>$y \ge 0, \lambda > 0$ | $1/\lambda$ | $1/\lambda^2$ | $\frac{\lambda}{\lambda-t}$<br>$t < \lambda$ |
| Gamma | $\alpha, \lambda$ | $\frac{\lambda^\alpha y^{\alpha-1}e^{-\lambda y}}{\Gamma(\alpha)}$<br>$y \ge 0, \alpha, \lambda > 0$ | $\frac{\alpha}{\lambda}$ | $\frac{\alpha}{\lambda^2}$ | $\left(\frac{\lambda}{\lambda-t}\right)^\alpha$<br>$t < \lambda$ |
| Normal | $\mu, \sigma^2$ | $\frac{1}{\sigma\sqrt{2\pi}}e^{-(y-\mu)^2/2\sigma^2}$<br>$-\infty < y < \infty$<br>$-\infty < \mu < \infty, \sigma > 0$ | $\mu$ | $\sigma^2$ | $e^{\mu t+\sigma^2t^2/2}$<br>$-\infty < t < \infty$ |

**STANDARD NORMAL DISTRIBUTION: Table Values Represent AREA to the LEFT of the Z score.**

| Z | .00 | .01 | .02 | .03 | .04 | .05 | .06 | .07 | .08 | .09 |
|---|---|---|---|---|---|---|---|---|---|---|
| 0.0 | .50000 | .50399 | .50798 | .51197 | .51595 | .51994 | .52392 | .52790 | .53188 | .53586 |
| 0.1 | .53983 | .54380 | .54776 | .55172 | .55567 | .55962 | .56356 | .56749 | .57142 | .57535 |
| 0.2 | .57926 | .58317 | .58706 | .59095 | .59483 | .59871 | .60257 | .60642 | .61026 | .61409 |
| 0.3 | .61791 | .62172 | .62552 | .62930 | .63307 | .63683 | .64058 | .64431 | .64803 | .65173 |
| 0.4 | .65542 | .65910 | .66276 | .66640 | .67003 | .67364 | .67724 | .68082 | .68439 | .68793 |
| 0.5 | .69146 | .69497 | .69847 | .70194 | .70540 | .70884 | .71226 | .71566 | .71904 | .72240 |
| 0.6 | .72575 | .72907 | .73237 | .73565 | .73891 | .74215 | .74537 | .74857 | .75175 | .75490 |
| 0.7 | .75804 | .76115 | .76424 | .76730 | .77035 | .77337 | .77637 | .77935 | .78230 | .78524 |
| 0.8 | .78814 | .79103 | .79389 | .79673 | .79955 | .80234 | .80511 | .80785 | .81057 | .81327 |
| 0.9 | .81594 | .81859 | .82121 | .82381 | .82639 | .82894 | .83147 | .83398 | .83646 | .83891 |
| 1.0 | .84134 | .84375 | .84614 | .84849 | .85083 | .85314 | .85543 | .85769 | .85993 | .86214 |
| 1.1 | .86433 | .86650 | .86864 | .87076 | .87286 | .87493 | .87698 | .87900 | .88100 | .88298 |
| 1.2 | .88493 | .88686 | .88877 | .89065 | .89251 | .89435 | .89617 | .89796 | .89973 | .90147 |
| 1.3 | .90320 | .90490 | .90658 | .90824 | .90988 | .91149 | .91309 | .91466 | .91621 | .91774 |
| 1.4 | .91924 | .92073 | .92220 | .92364 | .92507 | .92647 | .92785 | .92922 | .93056 | .93189 |
| 1.5 | .93319 | .93448 | .93574 | .93699 | .93822 | .93943 | .94062 | .94179 | .94295 | .94408 |
| 1.6 | .94520 | .94630 | .94738 | .94845 | .94950 | .95053 | .95154 | .95254 | .95352 | .95449 |
| 1.7 | .95543 | .95637 | .95728 | .95818 | .95907 | .95994 | .96080 | .96164 | .96246 | .96327 |
| 1.8 | .96407 | .96485 | .96562 | .96638 | .96712 | .96784 | .96856 | .96926 | .96995 | .97062 |
| 1.9 | .97128 | .97193 | .97257 | .97320 | .97381 | .97441 | .97500 | .97558 | .97615 | .97670 |
| 2.0 | .97725 | .97778 | .97831 | .97882 | .97932 | .97982 | .98030 | .98077 | .98124 | .98169 |
| 2.1 | .98214 | .98257 | .98300 | .98341 | .98382 | .98422 | .98461 | .98500 | .98537 | .98574 |
| 2.2 | .98610 | .98645 | .98679 | .98713 | .98745 | .98778 | .98809 | .98840 | .98870 | .98899 |
| 2.3 | .98928 | .98956 | .98983 | .99010 | .99036 | .99061 | .99086 | .99111 | .99134 | .99158 |
| 2.4 | .99180 | .99202 | .99224 | .99245 | .99266 | .99286 | .99305 | .99324 | .99343 | .99361 |
| 2.5 | .99379 | .99396 | .99413 | .99430 | .99446 | .99461 | .99477 | .99492 | .99506 | .99520 |
| 2.6 | .99534 | .99547 | .99560 | .99573 | .99585 | .99598 | .99609 | .99621 | .99632 | .99643 |
| 2.7 | .99653 | .99664 | .99674 | .99683 | .99693 | .99702 | .99711 | .99720 | .99728 | .99736 |
| 2.8 | .99744 | .99752 | .99760 | .99767 | .99774 | .99781 | .99788 | .99795 | .99801 | .99807 |
| 2.9 | .99813 | .99819 | .99825 | .99831 | .99836 | .99841 | .99846 | .99851 | .99856 | .99861 |
| 3.0 | .99865 | .99869 | .99874 | .99878 | .99882 | .99886 | .99889 | .99893 | .99896 | .99900 |
| 3.1 | .99903 | .99906 | .99910 | .99913 | .99916 | .99918 | .99921 | .99924 | .99926 | .99929 |
| 3.2 | .99931 | .99934 | .99936 | .99938 | .99940 | .99942 | .99944 | .99946 | .99948 | .99950 |
| 3.3 | .99952 | .99953 | .99955 | .99957 | .99958 | .99960 | .99961 | .99962 | .99964 | .99965 |
| 3.4 | .99966 | .99968 | .99969 | .99970 | .99971 | .99972 | .99973 | .99974 | .99975 | .99976 |
| 3.5 | .99977 | .99978 | .99978 | .99979 | .99980 | .99981 | .99981 | .99982 | .99983 | .99983 |
| 3.6 | .99984 | .99985 | .99985 | .99986 | .99986 | .99987 | .99987 | .99988 | .99988 | .99989 |
| 3.7 | .99989 | .99990 | .99990 | .99990 | .99991 | .99991 | .99992 | .99992 | .99992 | .99992 |
| 3.8 | .99993 | .99993 | .99993 | .99994 | .99994 | .99994 | .99994 | .99995 | .99995 | .99995 |
| 3.9 | .99995 | .99995 | .99996 | .99996 | .99996 | .99996 | .99996 | .99996 | .99997 | .99997 |