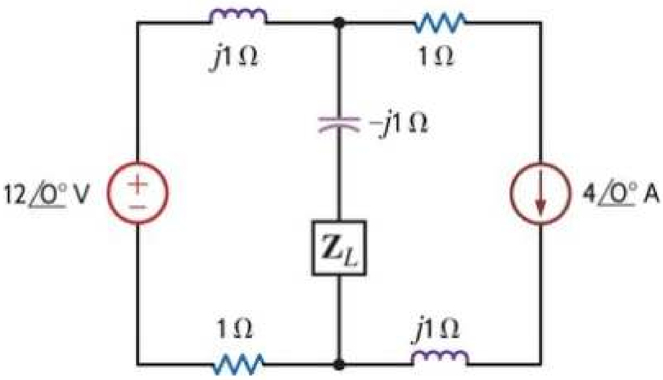
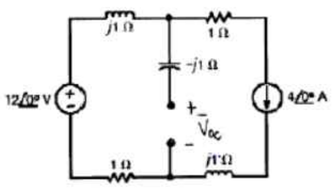
1. (6 Points) In the network shown below, find $\mathbf{Z}_L$ for maximum average power transfer and the maximum average power transferred.



Solution)



$$\bar{V}_{oc} + (1+j1)(4\angle 0^\circ) = 12\angle 0^\circ$$

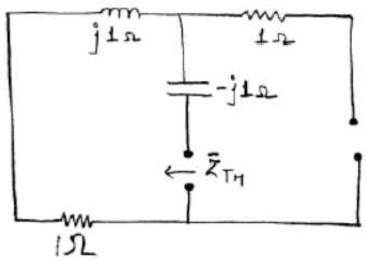
$$\bar{V}_{oc} = 8.94 \angle -26.57^\circ \text{ V}$$

조건 1: 올바른 ZL 값을 구할 것(최대 +3 점)

조건 1-1: 올바른 ZTH 를 구하지 못했다면 ZL 값에 상관없이 +0 점

조건 1-2: ZL 값은 틀렸으나 ZTH 를 올바르게 구했다면 +1 점

조건 1-3: ZTH 를 올바르게 구하고 ZL 을 올바르게 구했다면 +3 점



조건 2: 올바른 Pmax 값을 구할 것(최대 +3 점)

조건 2-1: 올바른 Voc 나 I 를 구하지 못했다면 Pmax 값에 상관없이 +0 점

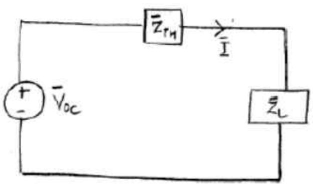
조건 2-2: Pmax 값은 틀렸으나 Voc 나 I 를 올바르게 구했다면 +1 점

조건 2-3: Pmax 를 올바르게 구하고 Voc 나 I 를 올바르게 구했다면 +3 점

$$\bar{Z}_{TH} = 1 + j1 - j1$$

$$\bar{Z}_{TH} = 1\,\Omega$$

$$\bar{Z}_L = \bar{Z}_{TH}^* = 1\,\Omega \text{ for maximum power transfer.}$$



참고: Voc = 8-4j 와 동일

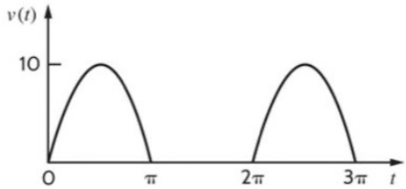
$$\bar{I} = \frac{8.94 \angle -26.57^\circ}{1+1}$$

$$\bar{I} = 4.47 \angle -26.57^\circ \text{ A}$$

$$P_{max} = \frac{I^2 R_L}{2} = \frac{(4.47)^2(1)}{2}$$

$$P_{max} = 10 \text{ W}$$

2. (6 Points) The waveform shown below is a half-wave rectified sine wave. Find the rms voltage value and the amount of average power dissipated in a 10-Ω resistor.



Period of the voltage waveform : $T = 2\pi$

$$v(t) = \begin{cases} 10\sin t & 0 \le t < \pi \\ 0 & \pi \le t < 2\pi \end{cases} \quad (+1)$$

$$V_{rms}^2 = \frac{1}{T}\int_0^T v^2(t)\,dt = \frac{1}{2\pi}\left[\int_0^{\pi}(10\sin t)^2 dt + \int_{\pi}^{2\pi} 0^2 dt\right]$$

$$= 25 \quad (+2)$$

$$\Rightarrow V_{rms} = 5V \quad (+1)$$

$$(\text{average power}) = \frac{V_{rms}^2}{R} = \frac{25}{10} = 2.5\,W \quad (+2)$$

3. (6 Points) A capacitive impedance $\mathbf{Z}_C = -j120\ \Omega$ is in parallel with a load impedance $\mathbf{Z}_{load}$. The parallel combination is supplied by the source $\mathbf{V}_S = 400\angle 0^\circ\ V_{rms}$ that generates a complex power of $1.6 + j0.5$ kVA. Find (a) the complex power delivered to $\mathbf{Z}_{load}$, (b) pf of $\mathbf{Z}_{load}$, and (c) pf of the source.

a. Current from the source:

$$I_S^* = \frac{1600 + j500}{400} = 4 + j1.25 \rightarrow I_S = 4 - j1.25$$ **+0.5**

Current of capacitive impedance node:

$$I_C = \frac{400}{-j120} = j\frac{10}{3}\ Arms$$ **+0.5**

Current of load impedance node:

$$I_L = I_S - I_C = (4 - j1.25) - j\frac{10}{3} = 4 - j\frac{55}{12}\ Arms$$ **+0.5**

The complex power delivered to ***Z**load* :

$$400 \times \left(4 + j\frac{55}{12}\right) = 1600 + j\frac{5500}{3} = 1600 + j1833.33\ \text{VA}$$ **+0.5**

b. Power factor of the load:

$$PF_L = \cos(\tan^{-1}(\frac{1833.33}{1600})) = 0.6575 lag$$ **+2**

c. Power factor of the source:

$$S_S = 1600 + j500$$

$$\cos(\tan^{-1}(\frac{500}{1600})) = 0.9645 lag$$

or

$$S_S = 1600 + j500 = 1676.3\angle 17.354$$

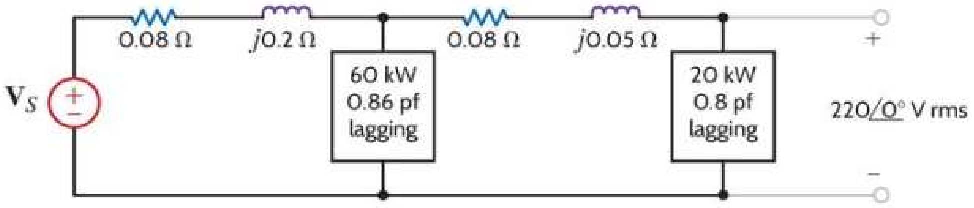
$$PF_S = \cos(17.354) = 0.9545 lag$$ **+2**

For b, c if only θ is calculated: 1 point

Any calculation error: -0.5 point

Wrong unit (incorrect unit): -0.5 point

4. (6 Points) Given the network shown below, compute (a) the input source voltage $\mathbf{V}_S$ and (b) input power factor. Find the input source voltage at $t = 0$.



Solutions)

Load 2 : $|I_2| = \dfrac{P_{L_2}}{|V_L|(pf_2)} = \dfrac{20\times10^3}{220(0.8)} = 114$ A rms

$\theta_{I_2} = \theta_{V_L} - \cos^{-1}(pf_2) = -36.9°$

$I_2 = 114 \angle -36.9°$ A rms

Load 1 : $|I_1| = \dfrac{P_{L_1}}{|V_1|(pf_1)}$

$V_1 = (0.08 + j0.05) I_2 + V_2$

$= 230.72 \angle -0.23°$ Vrms

$\therefore |I_1| = 302.4$ Arms $\quad \theta_{I_2} = \theta_{V_1} - \cos^{-1}(pf_1)$

$= \angle -30.9°$

$I_1 = 302.4 \angle -30.9$ A rms

$I_s = I_1 + I_2 = 415.45 \angle -32.54°$

$V_s = (0.08 + j0.2) I_s + V_1$

$= 307.8 \angle 9.59°$ Vrms

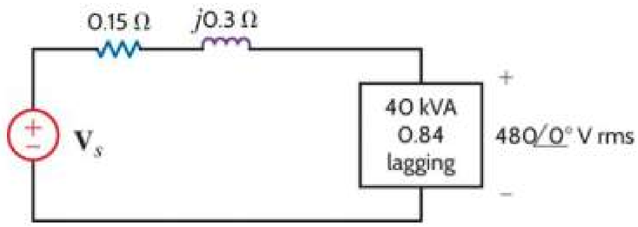
(a) $V_s(0) \simeq 303$ Vrms

$pf_s = \cos(\theta_{V_s} - \theta_{I_s})$

$= 0.74$

(b) $\boxed{pf_s = 0.74 \text{ lagging}}$

5. (6 points) For the network shown below, determine the value of capacitance that must be connected in parallel with the load so that the power factor of the combined load and capacitor is unity. Calculate the complex power supplied by the source after the power factor has been corrected to unity. The frequency f = 60 Hz.



$S_{old} = 40\angle \cos^{-1}(0.84) = 40\angle 32.86° \ kVA = 33.6 + j21.7 \ kVA$

$S_{new} = 33.6\angle 0° = 33.6 + j0 \ kVA$ (a)0.5pts

$S_{cap} = S_{new} - S_{old} = 33.6 - (33.6 + j21.7) = -j21.7 \ kVA$ (b)0.5pts

$$Q_{cap} = \frac{V_{out}^2}{X_c} \rightarrow X_c = \frac{V_{out}^2}{Q_{cap}} = \frac{480^2}{21.7 \cdot 10^3} = 10.62 \ ohm$$ (c)0.5pts

$$X_c = \frac{1}{\omega C} \rightarrow \boldsymbol{C = \frac{1}{\omega X_c} = \frac{1}{2\pi \cdot 60 \cdot 10.62} = 249.8uF}$$ (d)1.5pts

$$|\bar{I}| = \frac{S_{new}}{V_{rms}} = \frac{33.6 \ kVA}{480 \ V_{rms}} = 70 \ A_{rms}$$ (e)1pts

$S_{0.15+j0.3} = \bar{I}^2 \cdot (0.15 + j0.3) = 70^2 \cdot (0.15 + j0.3) = 735 + j1470 \ VA$ (f)0.5pts

$\boldsymbol{S_{source}} = S_{0.15+j0.3} + S_{new} = 735 + j1470 + 33.6k = \boldsymbol{34.335 + j1.47 \ kVA}$ (g)1.5pts

$= \boldsymbol{34.37\angle 2.45° \ kVA}$

In the partial score (d),(g)

-0.5pts No unit