EE201 Circuit Theory (Spring 2017)
Mid-Term Exam.

(Total: 310 Points / 10 Problems)

Student ID Number:

Name:

1. **(10 points)** Element 2 in Fig. 1 absorbed 32 W. Find the power absorbed or supplied by elements 1 and 3.

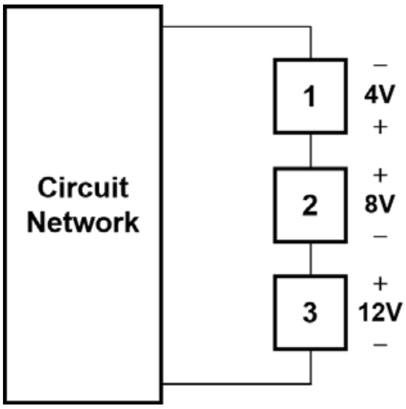
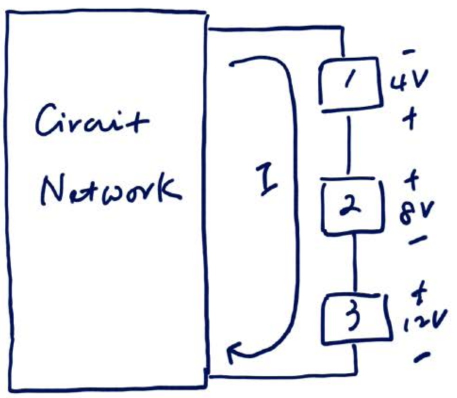


Fig. 1



The energy absorbed by the element 2

$P_2 = I \times 8V = 32W \quad \therefore I = 4A.$

Therefore

$P_1 = 4A \times (-4V) = -16W$

$P_3 = 4A \times 12V = 48W$

The power supplied by the element 1 is 16W.

The power absorbed by the element 3 is 48W.

2. (a) (10 points) Determine the value of the voltage v in Fig. 2(a).

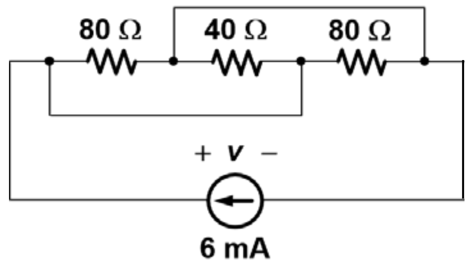
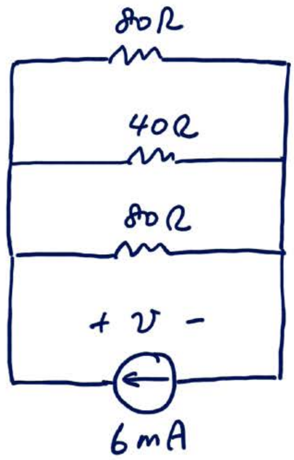


Fig. 2(a)

Redrawing the given circuit results in the following circuit diagram.



$\therefore R_{eq} = 80 \parallel 40 \parallel 80 = 20\Omega$

Therefore,

$v = 6mA \times 20\Omega = \underline{120 mV}$.

(b) (20 points) Find the equivalent resistance R_{eq} in the network shown in Fig. 2(b).

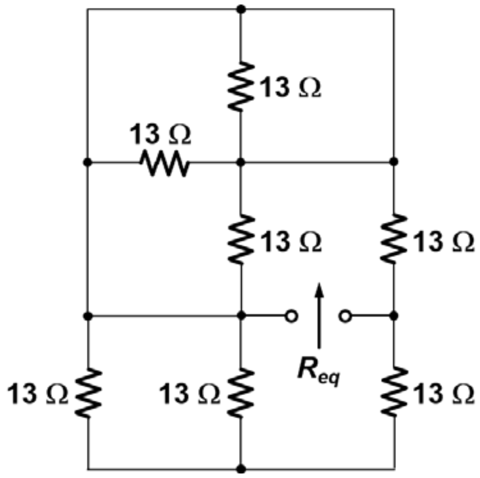
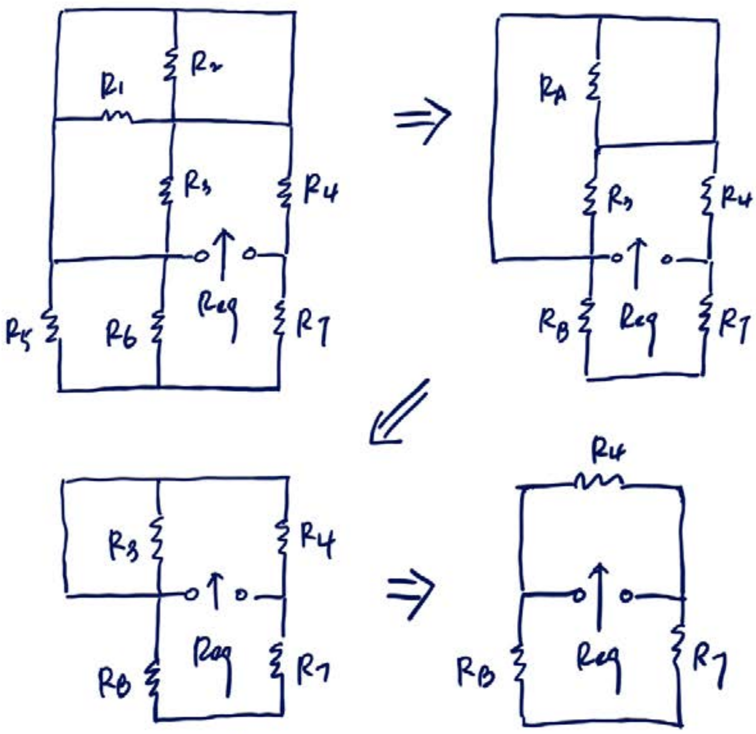


Fig. 2(b)



$R_A = R_1 \,//\, R_2 = 6.5\,\Omega$, but shorted.

$R_B = R_5 \,//\, R_6 = 6.5\,\Omega$

R_3 is shorted.

$$\therefore R_{eq} = R_4 \,//\, (R_B + R_7) = 13 \,//\, (6.5 + 13) = \underline{7.8\,\Omega}$$

3. In factory instrumentation, process parameters such as pressure and flow rate are measured, converted to electrical signals, and sent some distance to an electronic controller. The controller then decides what actions should be taken. One of the main concerns in these systems is the physical distance between the sensor and the controller. An industry standard format for encoding the measurement value is called the 4–20 mA standard, where the parameter range is linearly distributed from 4 to 20 mA. For example, a 100 psi pressure sensor would output 4 mA if the pressure were 0 psi, 20 mA at 100 psi, and 12 mA at 50 psi. But most instrumentation is based on voltages between 0 and 5 V, not on current.

Therefore, let us design a current-to-voltage converter that will output 5 V when the current signal is 20 mA. The circuit shown in Fig. 3(a) is a model of the given situation. The wiring from the sensor unit to the controller has some resistance, R_{wire}, which increases by 1 Ω with 1-m increase of the distance.

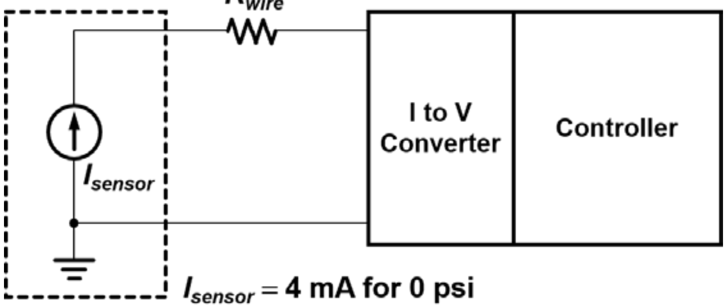


Fig. 3(a)

(a) **(10 points)** As shown in Fig. 3(b), for the current-to-voltage converter, we use an extremely simple circuit – a resistor. What should be the value of R? (It is assumed that the controller does not load the remaining portion of the circuit.)

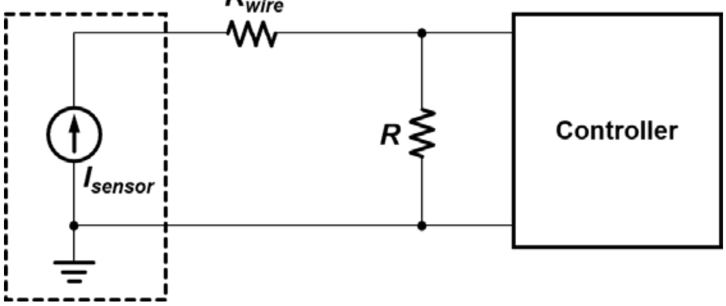


Fig. 3(b)

Since the current-to-voltage converter should output 5V for $I_{sensor} = 20mA$.

$R = 5V / 20mA = \underline{250\Omega}$

(b) (10 points) In Fig. 3(b), considering that the sensor output information is contained in the current value, will R_{wire} affect the accuracy at the controller? If yes, how much error (in percentage) will be introduced by R_{wire} when the distance between the sensor and controller is 50 m?

No. Since the data are contained in the current value, R_{wire} does not affect the accuracy at the controller as long as the sensor acts as an ideal current source as modeled in Fig. 3(b).
If the sensor output were a voltage proportional to pressure, the voltage drop in the line would cause measurement error even if the sensor output were an ideal voltage source.

(c) (20 points) Introducing a more accurate model for the sensor, the circuit can be redrawn as shown in Fig. 3(c). When R_{out} is 50 KΩ, if we want to keep the error at the controller within 1%, how far can we place the sensor from the controller?

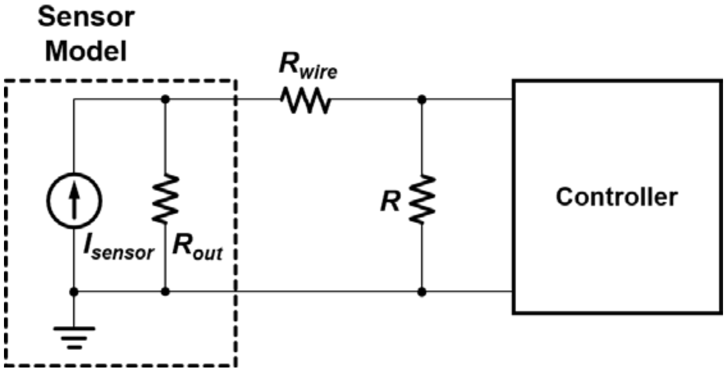
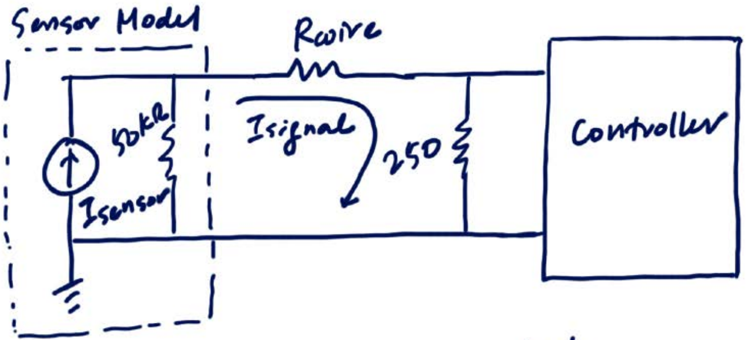


Fig. 3(c)



$$I_{signal} = I_{sensor} \times \frac{50k}{50k + (R_{wire} + 250)}$$

$$\therefore \frac{I_{sensor} - I_{signal}}{I_{sensor}} = \frac{R_{wire} + 250}{50k + (R_{wire} + 250)} < 0.01$$

$$0.99\, R_{wire} < 500 + 2.5 - 250 = 252.5$$

$$\therefore R_{wire} < 255.05\,\Omega \rightarrow \text{distance} \leq \sim 255m$$

(d) (30 points) In (a), the value of R was chosen to convert a current in the 4- to 20-mA range to a voltage such that a 20-mA input produced a 5-V output. In this case, the minimum current (4 mA) produces a resistor voltage of 1 V. Unfortunately, many control system operate on a 0- to 5-V range rather than a 1- to 5-V range. Let us design a new converter that will output 0 V at 4 mA and 5 V at 20 mA, using the configuration shown in Fig. 3(d). Assuming the same value as in (a) for R and ignoring any effect from R_{out} and R_{wire}, determine the values for R_1, R_2, and V_{shift}.

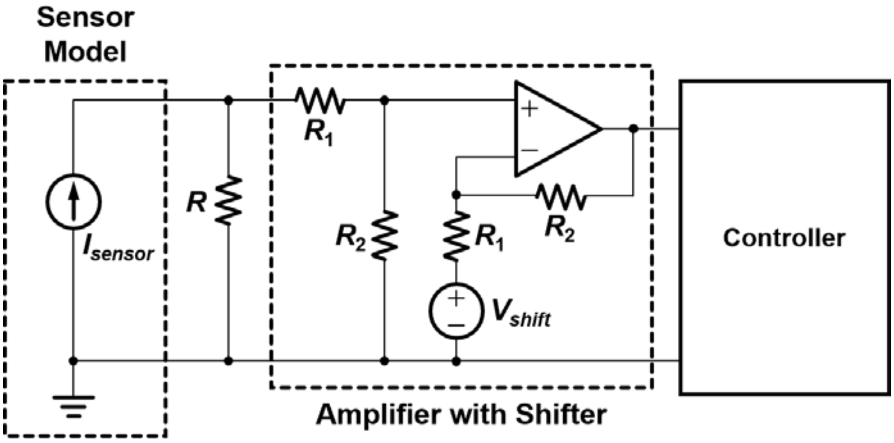
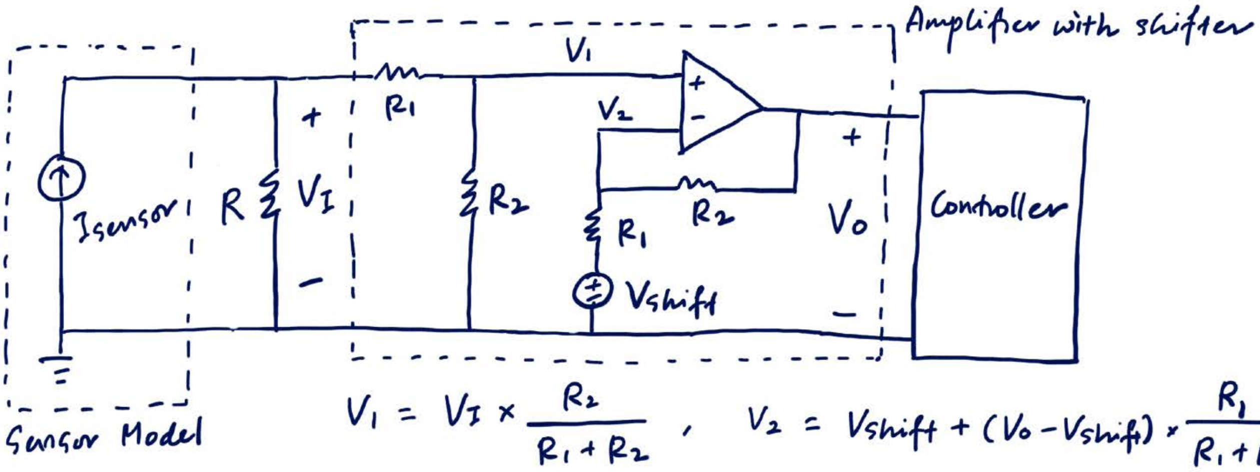


Fig. 3(d)



$V_1 = V_I \times \frac{R_2}{R_1 + R_2}$, $V_2 = V_{shift} + (V_o - V_{shift}) \times \frac{R_1}{R_1 + R_2}$

Since $V_1 = V_2$, $V_I \times \frac{R_2}{R_1+R_2} = V_{shift} + (V_o - V_{shift}) \times \frac{R_1}{R_1+R_2}$.

Solving for V_o, $V_o = (V_I - V_{shift}) \times \frac{R_2}{R_1}$.

For $V_I = 1$, $V_o = (1 - V_{shift}) \times \frac{R_2}{R_1} = 0$. — ①

For $V_I = 5$, $V_o = (5 - V_{shift}) \times \frac{R_2}{R_1} = 5$. — ②

From ① and ②, $\underline{V_{shift} = 1}$ and $\frac{R_2}{R_1} = \frac{5}{4} = 1.25$

Since we don't want the converter resistor, R, to affect the amplifier, the vast majority of the 4-20mA current should flow entirely through R and not through the amplifier resistors, R_1 and R_2.

Therefore, R_1 and $R_2 \gg R$.

Considering all the necessary conditions to satisfy, we can determine the values of R_1 and R_2 as follow.

$\underline{R_1 = 100\,k\Omega}$, $\underline{R_2 = 125\,k\Omega}$.

4. (20 points) Use nodal analysis to find V_1, V_2, V_3, and V_4 for the circuit shown in Fig. 4.

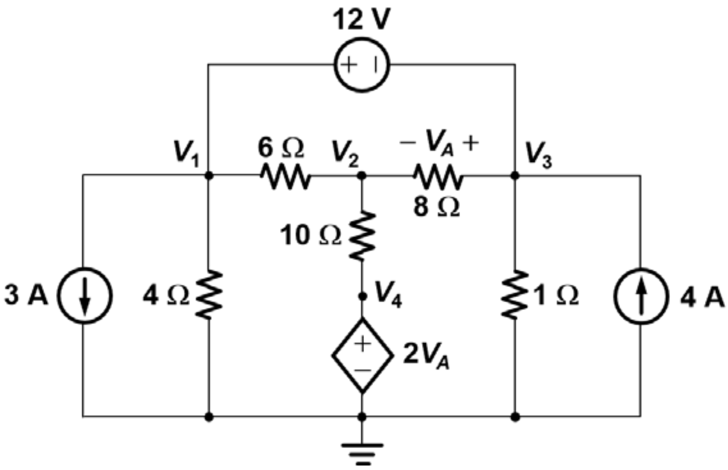
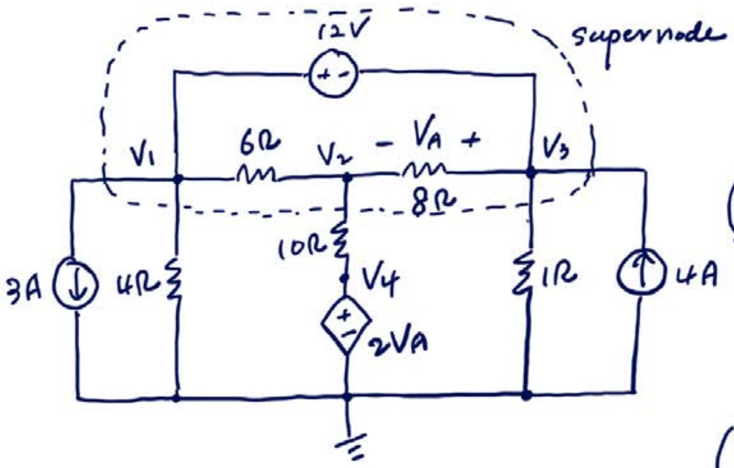


Fig. 4



Applying KCL at the node of V_2,

$$\frac{V_2-V_1}{6}+\frac{V_2-V_4}{10}+\frac{V_2-V_3}{8}=0$$

$$20V_1-47V_2+15V_3+12V_4=0 \quad -①$$

Applying KCL at the supernode,

$$3+\frac{V_1}{4}+\frac{V_2-V_4}{10}+\frac{V_3}{1}-4=0$$

$$5V_1+2V_2+20V_3-2V_4=20 \quad -②$$

Constraint equation:

$V_1-V_3=12. \quad -③$

controlling equation:

$V_4=2V_A=2(V_3-V_2) \quad -④$

Solving ①.②.③ and ④ simultaneously,

$\underline{V_1=9.68V}$

$\underline{V_2=1.45V}$

$\underline{V_3=-2.32V}$

$\underline{V_4=-7.54V}$

5. **(20 points)** Use loop analysis to find the power delivered by the current-controlled voltage source in the circuit shown in Fig. 5.

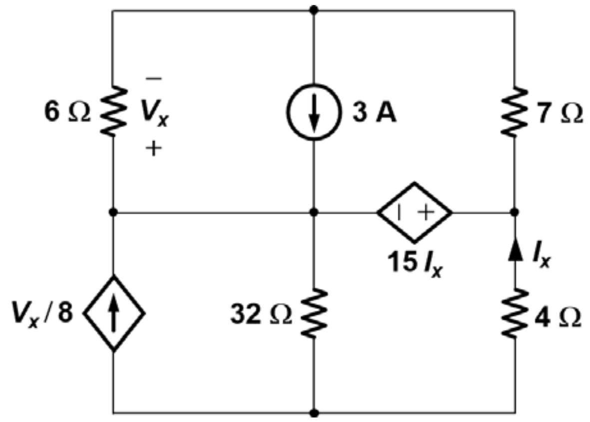
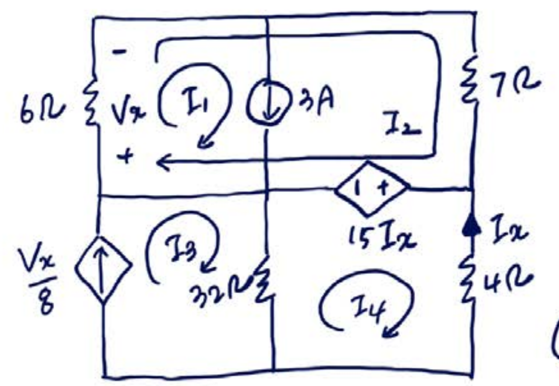


Fig. 5



Constraint equations:

$I_1 = 3$ —①

$I_3 = V_x/8 = 6(I_1 + I_2)/8 = \frac{3}{4}I_1 + \frac{3}{4}I_2$ —②

Applying KVL at the loop of I_2,

$6 \times (I_1 + I_2) + 7 \times I_2 + 15\, I_x = 0$

$6 I_1 + 13 I_2 + 15 I_x = 0$ —③

Applying KVL at the loop of I_4,

$-15 I_x + 4 I_4 + 32(I_4 - I_3) = 0$

$32 I_3 - 36 I_4 + 15 I_x = 0$ —④

$I_x = -I_4$ —⑤

Solving ①, ②, ③, ④ and ⑤ simultaneously,

$I_1 = 3\,A$, $I_2 = 0.53\,A$, $I_3 = 2.65\,A$,

$I_4 = 1.66\,A$.

∴ The power delivered by the current-controlled voltage source

$P = 15 I_x \times (I_4 - I_2) = 15 \times (-1.66) \times (1.66 - 0.53) = \underline{-28.14\,W}$.

6. If an 8-kΩ load is connected to the terminals of the linear circuit network in Fig. 6, $V_{AB} = 16$ V. If a 2-kΩ load is connected to the terminals, $V_{AB} = 8$ V.

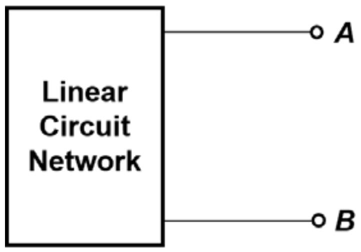
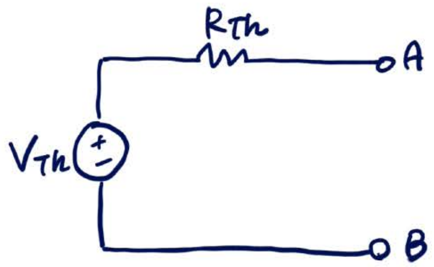


Fig. 6

(a) (20 points) Find V_{AB}, if a 20-kΩ load is connected to the terminals.

If we replace the linear circuit network with its Thevénin equivalent circuit, the circuit diagram can be redrawn as follows.



If an 8-kΩ load is connected,

$V_{AB} = V_{Th} \times \frac{8k}{R_{Th}+8k} = 16$ — ①

If a 2-kΩ load is connected,

$V_{AB} = V_{Th} \times \frac{2k}{R_{Th}+2k} = 8$ — ②

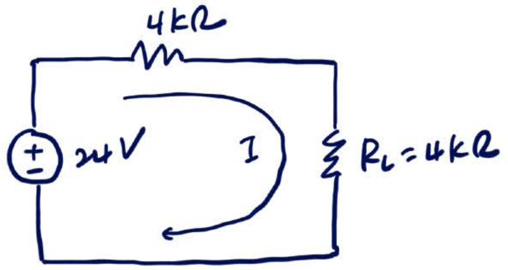
Solving ① and ② simultaneously,

$R_{Th} = 4k\Omega$ and $V_{Th} = 24V$

$\therefore$ If a 20-kΩ load is connected,

$V_{AB} = V_{Th} \times \frac{20k}{R_{Th}+20k} = 24 \times \frac{20k}{4k+20k} = \underline{20V}$

(b) (20 points) Find the value of the load resistor for maximum power transfer. How large power can be delivered to the load at that condition?



Since $R_{Th} = 4k\Omega$, the maximum power is transferred when the load resistor is $\underline{4k\Omega}$.

The maximum power delivered, $P_{max} = I^2 R_L = \left(\frac{24}{8k}\right)^2 \times 4k = \underline{36\,mW}$

7. **(30 points)** Use source transformation to find I_o in the network shown in Fig. 7.

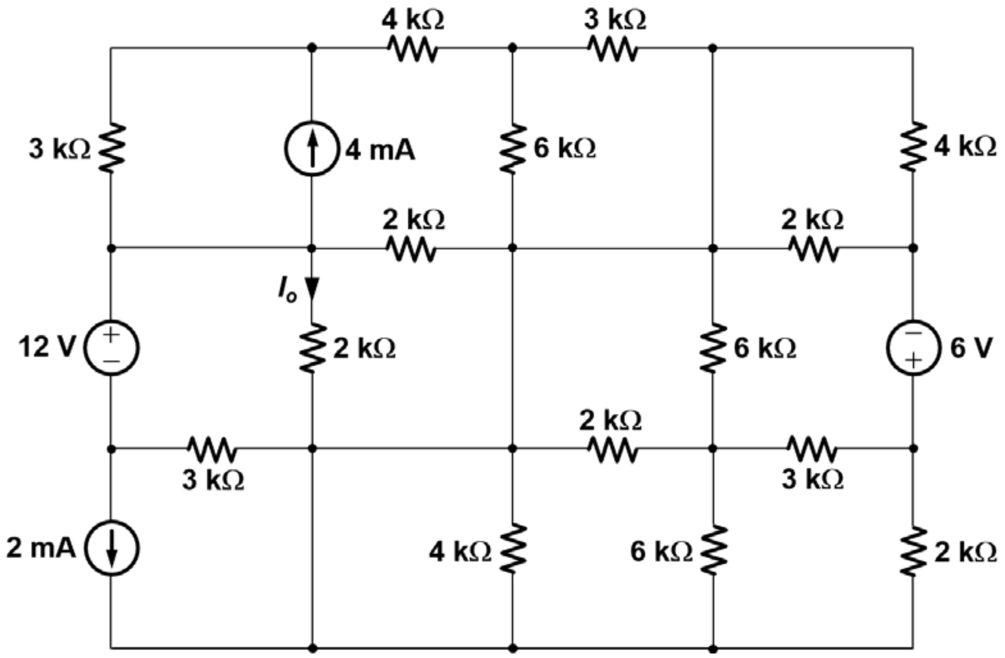
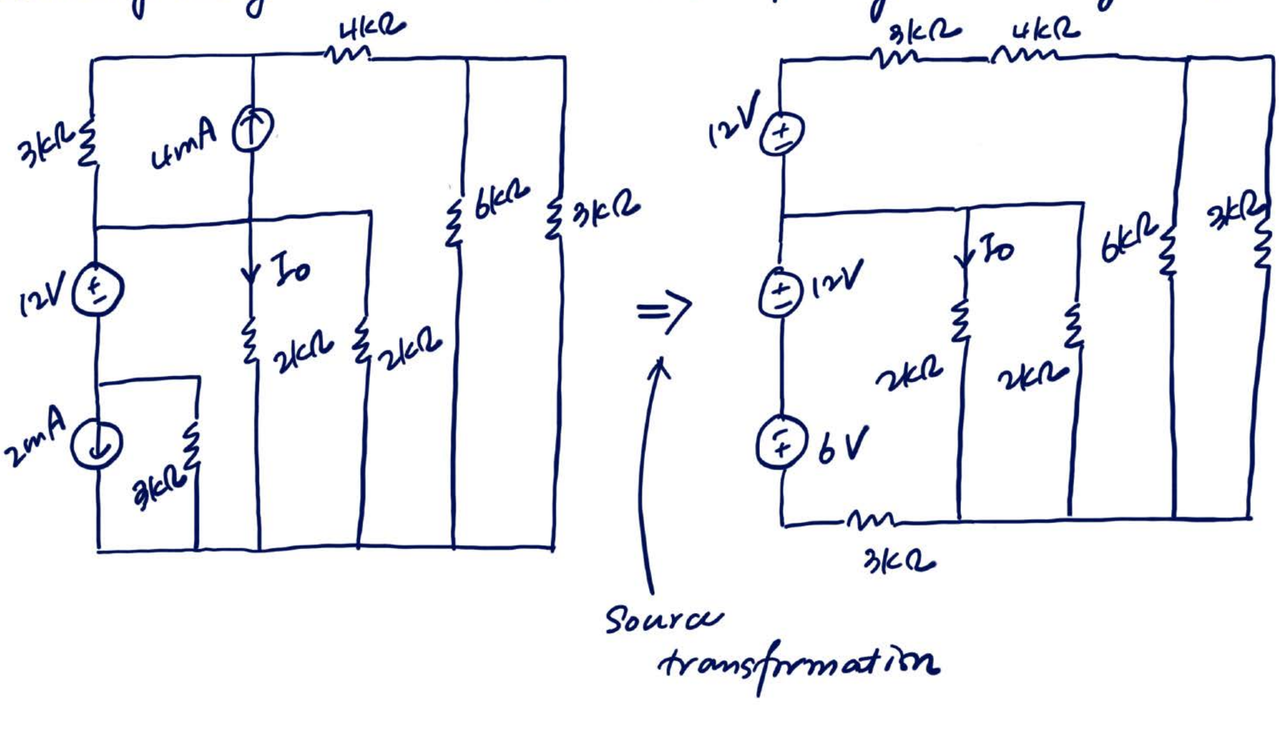
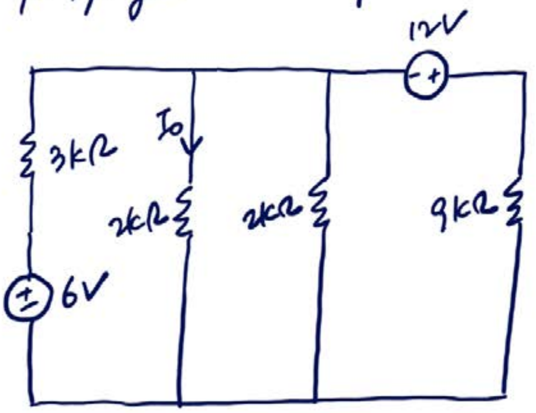


Fig. 7

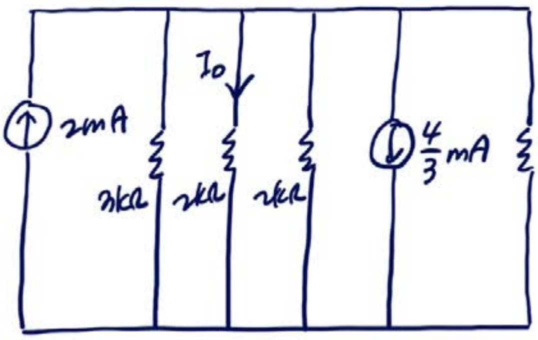
Redrawing the given circuit results in the following circuit diagram.



Simplifying the circuit further,



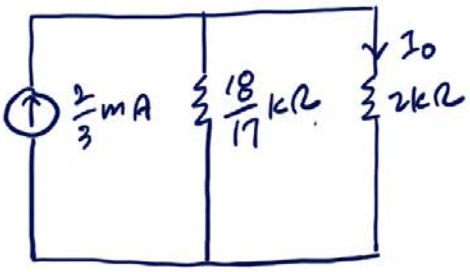
$\Rightarrow$



$\Downarrow$

$$\therefore I_o = \frac{2}{3}m \times \frac{\frac{18}{17}k}{2k + \frac{18}{17}k}$$

$$= \frac{2}{3}m \times \frac{9}{26} = \frac{3}{13}m = \underline{0.23\,mA}.$$



8. **(30 points)** We often find that in the use of electronic equipment, there is a need to adjust some quantity such as voltage, frequency, contrast, or the like. For very accurate adjustments, it is most convenient if coarse- and fine-tuning can be separately adjusted. Therefore, let us design a circuit in which two inputs (i.e., coarse and fine voltages) are combined to produce a new voltage of the form

$$V_{tune} = \frac{1}{2}V_{coarse} + \frac{1}{20}V_{fine},$$

employing the circuit configuration shown in Fig. 8. Determine the values for R, R_1, and R_2.

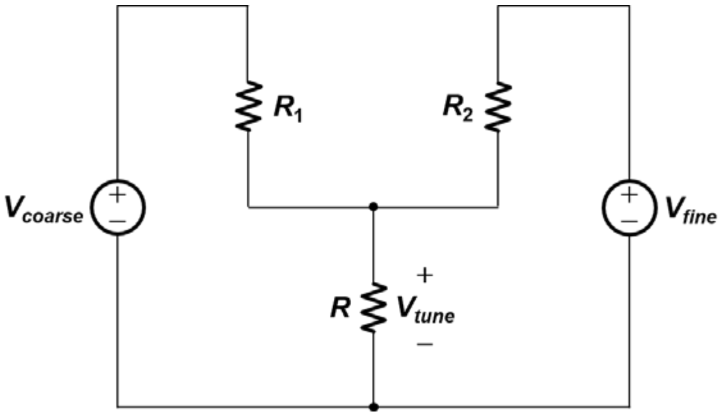
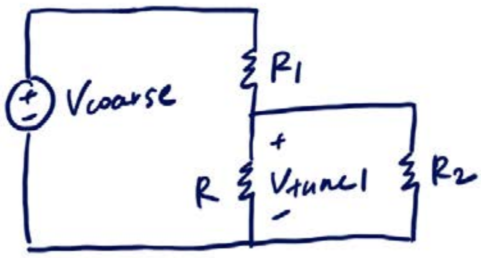


Fig. 8

The superposition technique can be applied.

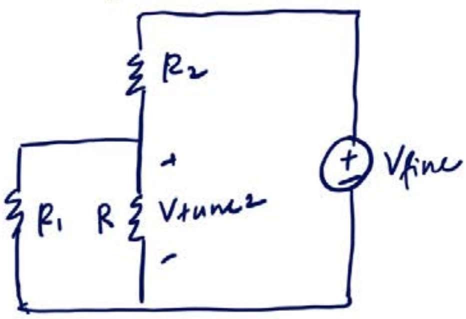
Deactivating V_{fine} leads to the following circuit.



$$V_{tune1} = V_{coarse} \times \frac{(R /\!/ R_2)}{R_1 + (R /\!/ R_2)} = \frac{1}{2} V_{coarse}$$

$$\therefore\ R /\!/ R_2 = R_1 \quad -①$$

Setting V_{coarse} as zero results in the circuit shown below.



$$V_{tune2} = V_{fine} \times \frac{(R /\!/ R_1)}{R_2 + (R /\!/ R_1)} = \frac{1}{20} V_{fine}$$

$$\therefore\ R_2 = 19 \times (R /\!/ R_1) \quad -②$$

If we choose $R = 1k\Omega$, from ① and ②, $\underline{R_1 = 900\Omega, \text{ and } R_2 = 9k\Omega}$.

9. Fans are frequently needed to keep electronic circuits cool. They vary in size, power requirement, input voltage, and air-flow rate. In a particular application, three fans are connected in parallel to a 24-V source as shown in Fig. 9(a).

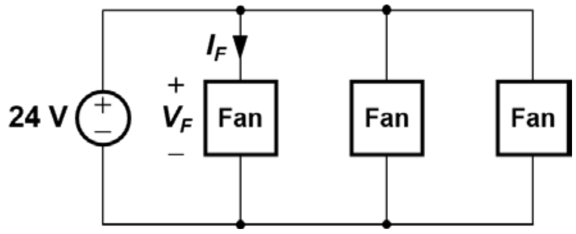


Fig. 9(a)

A number of tests were run on this configuration, and it was found that the air flow, fan current, and input voltage are related by the following equations:

$$F_{CFM} = 200 I_F, \quad V_F = 100 I_F$$

where F_{CFM} represents the air-flow rate in cubic feet per minute, V_F is the fan voltage in volts, and I_F is the fan current in amperes. Note that the fan current is related to fan speed, which in turn is related to the air flow. A popular and inexpensive method for monitoring currents in applications where high accuracy is not critical involves placing a low-value sense resistor in series with the fan to sense the current by measuring the sense-resistor's voltage.

We wish to design a circuit shown in Fig. 9(b) that will measure the air flow in this three-fan system.

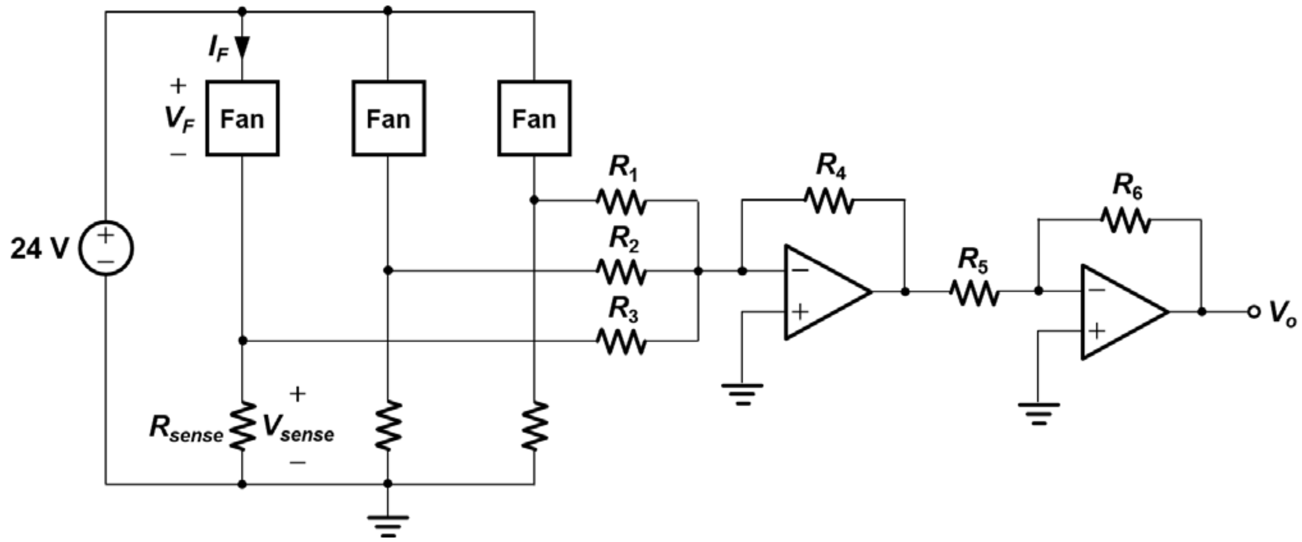


Fig. 9(b)

(a) (10 points) Determine the value of the sense resistor, placed in series with each fan, such that its voltage is 2% of the nominal 24-V fan voltage.

From $V_F = 100 I_F$, each fan has a resistance of 100Ω.

Since the voltage across the sense resistor should be 2% of 24V, or 0.48V, the fan current, I_F, is calculated as

$$I_F = \frac{24 - 0.48}{100} = 235.2\text{mA}$$

and the required value of the sense resistor, R_{sense}, is given by

$$R_{sense} = \frac{0.48}{235.2\text{m}} = \underline{2.04\,\Omega}$$

(b) (30 points) Design an op-amp circuit that will produce an output voltage proportional to total air flow, in which 1 V corresponds to 50 CFM.

Since $F_{CFM} = 200 I_{fi}$ for a single fan, $I_{fi} = F_{CFM}/200$.

$V_{sense} = R_{sense} \times I_{fi} = 2.04 \times F_{CFM}/200 = 0.0102 \times F_{CFM}$.

$$V_o = V_{sense} \times -\left(\frac{R_4}{R_1} + \frac{R_4}{R_2} + \frac{R_4}{R_3}\right) \times -\left(\frac{R_6}{R_5}\right)$$

$$= \left(\frac{R_4}{R_1} + \frac{R_4}{R_2} + \frac{R_4}{R_3}\right) \times \left(\frac{R_6}{R_5}\right) \times 0.0102 \times F_{CFM}$$

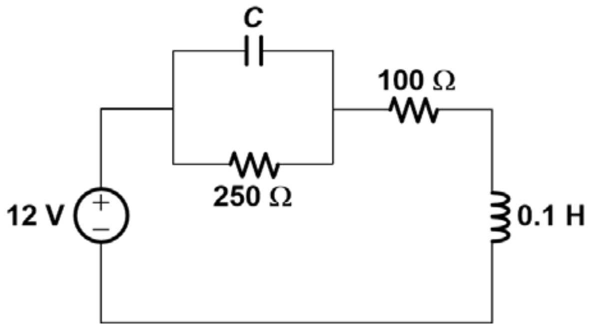
$$= \frac{1}{50} F_{CFM} \quad (\because 1V \text{ of } V_o \text{ corresponds to } 50 \text{ CFM})$$

Assuming $R_1 = R_2 = R_3$ and $R_5 = R_6$,

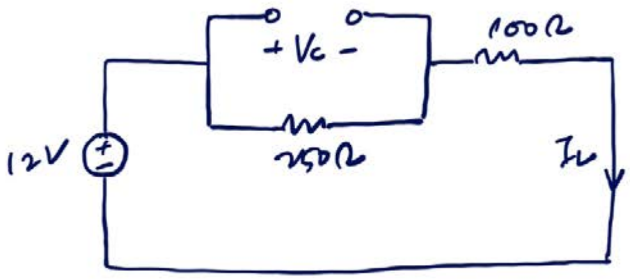
$$3 \times \frac{R_4}{R_1} \times 0.0102 = \frac{1}{50} \rightarrow R_4 = 0.65 R_1$$

Note that the resistors at the summer input are connected in parallel with the sense resistors. To ensure that all the fan current flows in the sense resistors, we select very large values for the op-amp resistors. We can choose $\underline{R_1 = R_2 = R_3 = 200k\Omega,\ R_4 = 130k\Omega,\ R_5 = R_6 = 200k\Omega}$.

10. **(20 points)** Find the value of *C* if the energy stored in the capacitor in Fig. 10 equals the energy stored in the inductor.



In steady state, the capacitor can be regarded as an open circuit and the inductor as a short circuit.



$V_c = 12 \times \frac{250}{250+100} = 8.57V$

$I_L = 12/(250+100) = 34.29\,mA$

Since the energy stored in the capacitor equals the energy stored in the inductor,

$\frac{1}{2} C \times (8.57)^2 = \frac{1}{2} \times 0.1 \times (34.29m)^2$

$\therefore C = \underline{1.6\,\mu F}$