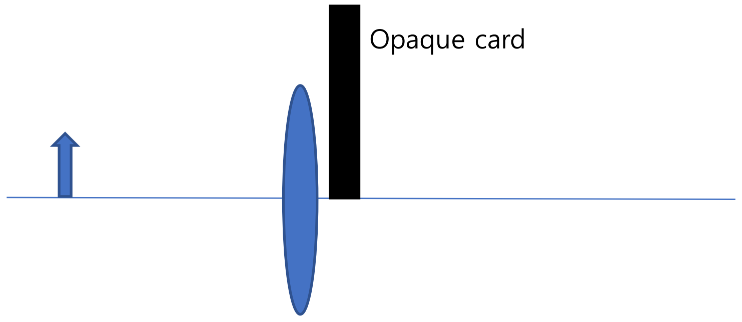
1. A converging lens in a vertical plane receives light from an object and forms an inverted image on a screen. An opaque card is then placed next to the lens, covering only the upper half of the lens. What happens to the image on the screen? [5 pts]



① The upper half of the image disappears.

② The lower half of the image disappears.

③ The entire image disappears.

④ The entire image is still visible, but is dimmer.

Sol) The entire image is visible, but only at half the intensity as compared to the case when there is no opaque card.

2. If John's face is 30.0 cm in front of a concave shaving mirror creating an upright image 1.50 times as large as the object, what is the mirror's focal length? [5 pts]

① 12.0 cm ② 20.0 cm ③ 70.0 cm ④ 90.0 cm

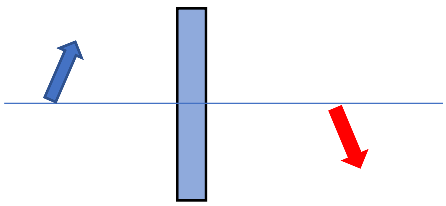
Sol) The image is upright, so the magnification is positive:

$$M = \frac{-q(\text{distance between the image and the lens})}{p(\text{distance between the object and the lens})}: \quad +1.50 = \frac{-q}{30.0\ cm} \rightarrow q = -45.0\ cm,$$

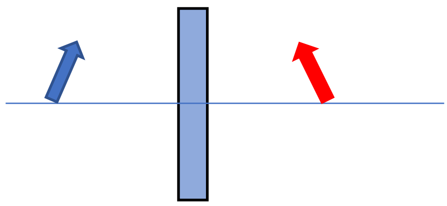
$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f}: \frac{1}{30.0\text{ cm}} + \frac{1}{-45.0\ cm} = \frac{1}{f} \rightarrow \quad f = 90.0\ cm$$

3. An object, represented by a blue arrow, is placed in front of a plane mirror. Which of the diagrams in the below figures correctly describes the image, represented by the red arrow? [5 pts]

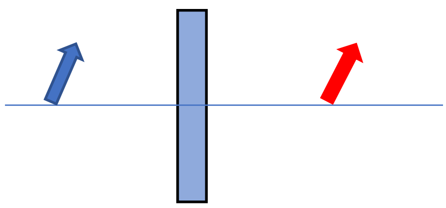
①



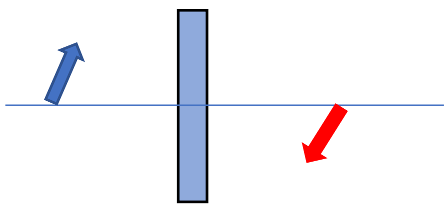
②



③

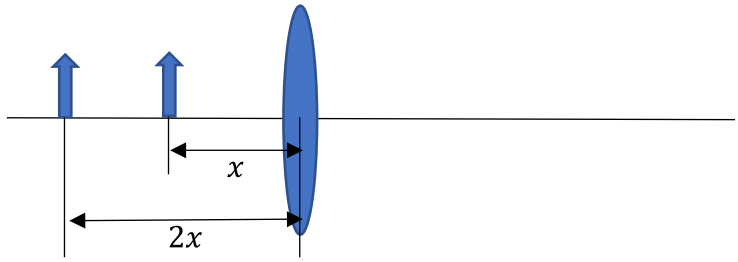


④



Sol) The image is upright, and corresponding parts of the object and image are the same distance from the mirror.

4. Suppose you want to use a converging lens to project the image of two arrows onto a screen. As shown in the below figure, one arrow is a distance x from the lens and the other is at $2x$. You adjust the screen so that the near arrow is in focus. [5 pts]



If you now want the far arrow to be in focus, you should move the screen toward the lens. (O/X)

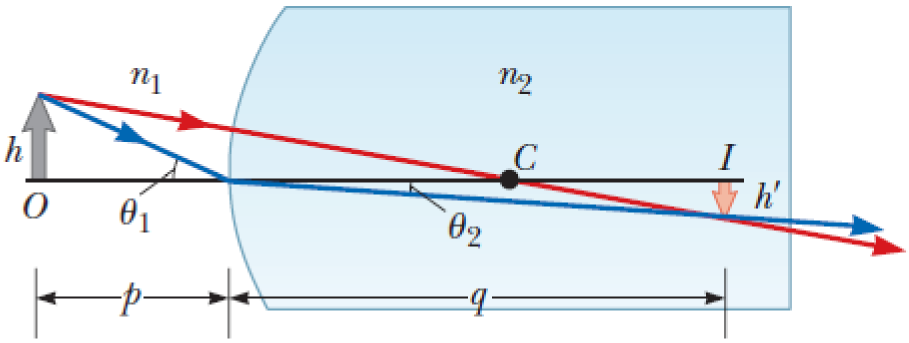
Sol) Lens equation: $\frac{1}{s}+\frac{1}{s\prime}=\frac{1}{f} \rightarrow \frac{1}{s\prime}=\frac{1}{f}-\frac{1}{s}=\frac{s-f}{fs}=\frac{1-f/s}{f} \rightarrow s'=\frac{f}{1-f/s}$

For $s=x$ case, $s_1'=\frac{f}{1-f/x}$.

For $s=2x$ case, $s_2'=\frac{f}{1-f/2x}$.

Since $s_1'>s_2'$, so if you want the far arrow to be in focus, you should move the screen toward the lens.

5. The below figure shows a curved surface separating a material with index of refraction n_2. The surface forms an image I of object O. The ray shown in red passes through the surface along a radial line. Its angles of incidence and refraction are both zero, so its direction does not change at the surface. For the ray shown in blue, the direction changes according to Snell's law, $n_1 \sin\theta_1 = n_2 \sin\theta_2$. For paraxial rays, we assume θ_1 and θ_2 are small, so we may write $n_1 \tan\theta_1 = n_2 \tan\theta_2$. The magnification is defined as $M = h'/h$. Prove that the magnification is given by $M = -n_1 q/n_2 p$. [15 pts]

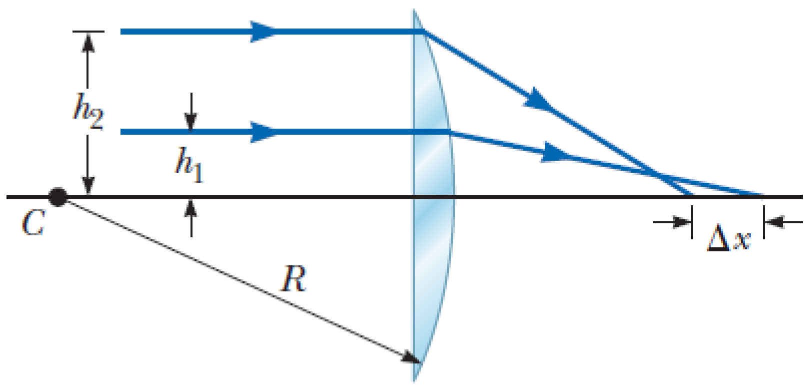


Sol) $\tan\theta_1 = \frac{h}{p}, \tan\theta_2 = \frac{-h'}{q}$ (+5pts)

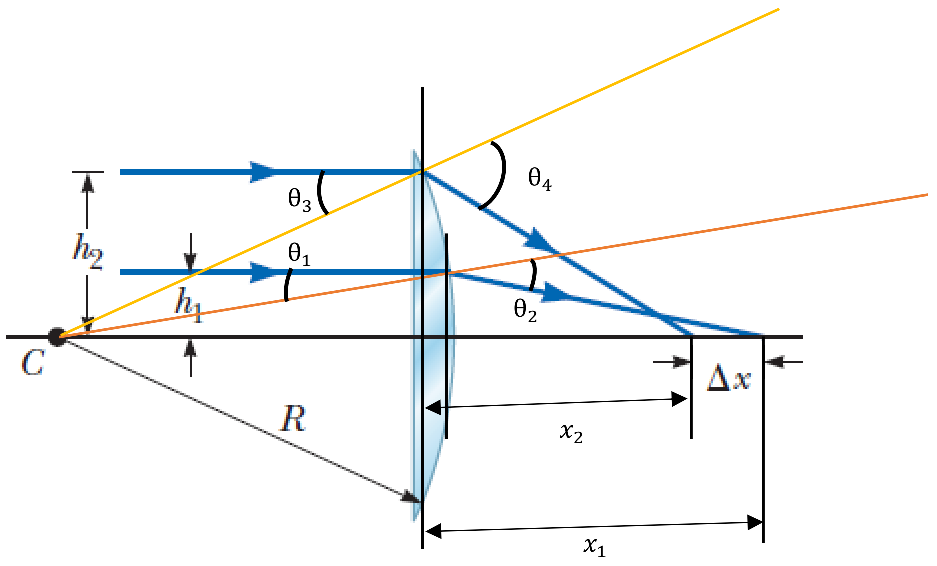
$n_1 \tan\theta_1 = n_2 \tan\theta_2$ → $n_1 \frac{h}{p} = -n_2 \frac{h'}{q}$. (+5pts)

➔ Magnification: $M = \frac{h'}{h} = -\frac{n_1 q}{n_2 p}$ (+5 pts)

6. Two rays traveling parallel to the principal axis strike a large plano-convex lens having a refractive index of 1.60. If the convex face is spherical, a ray near the edge does not pass through the focal point (spherical aberration occurs). Assume this face has a radius of curvature of $R = 20.0\ \mathrm{cm}$ and the two rays are at distances $h_1 = 0.500\ \mathrm{cm}$ and $h_2 = 12.0\ cm$ from the principal axis. Find difference Δx in the positions where each crosses the principal axis. (Calculate the final result rounded up to the first digit after the decimal point) [15 pts]



Sol)



$\sin\theta_1 = \frac{h_1}{R} = \frac{0.500\ cm}{20.0\ cm} = \frac{1}{40}$ ($\rightarrow$ $\tan\theta_1 = \frac{1}{\sqrt{1599}}$, $\cos\theta_1 = \frac{\sqrt{1599}}{40}$)

Then, $1.00\ \sin\theta_2 = 1.60 \sin\theta_1 = (1.60)\left(\frac{0.500\ cm}{20.0\ cm}\right) = \frac{1}{25}$ $\rightarrow$ $\tan\theta_2 = \frac{1}{\sqrt{624}}$

Focal length of the ray h_1 is

$$f_1 = \frac{h_1}{\tan(\theta_2 - \theta_1)}$$

$$= \frac{h_1}{(\tan\theta_2 - \tan\theta_1)/(1 + \tan\theta_1 \tan\theta_2)}$$

$$= \frac{h_1}{(\tan\theta_2 \,/\tan\theta_1 - 1)/(1/\tan\theta_1 + \tan\theta_2)}$$

$$= \frac{0.500\ cm}{(\sqrt{1599}/\sqrt{624} - 1)/(\sqrt{1599} + 1/\sqrt{624})}$$

$$= 0.5 \times \frac{\sqrt{1599} \times \sqrt{624} + 1}{\sqrt{1599} - \sqrt{624}}\ \mathrm{cm}$$

(+3pts)

$$x_1 = f_1 - R(1 - \cos\theta_1) = 0.5 \times \frac{\sqrt{1599} \times \sqrt{624} + 1}{\sqrt{1599} - \sqrt{624}}\ \mathrm{cm} - 20.0\ \mathrm{cm}\left[1 - \frac{\sqrt{1599}}{40}\right]$$

$$= 33.31\ cm$$

(+3pts)

$$\sin\theta_3 = \frac{h_2}{R} = \frac{12.0\ cm}{20.0\ cm} = \frac{3}{5}\ (\rightarrow \tan\theta_3 = \frac{3}{4}, \cos\theta_3 = \frac{4}{5})$$

Then,

$$1.00\ \sin\theta_4 = 1.60 \sin\theta_3 = (1.60)\left(\frac{12.0\ cm}{20.0\ cm}\right) = \frac{24}{25} \rightarrow \tan\theta_4 = \frac{24}{7}$$

$$f_2 = \frac{h_2}{\tan(\theta_4 - \theta_3)} = \frac{h_2}{(\tan\theta_4 - \tan\theta_3)/(1 + \tan\theta_3 \tan\theta_4)}$$

$$= \frac{h_2}{(\tan\theta_4 \,/\tan\theta_3 - 1)/(1/\tan\theta_3 + \tan\theta_4)}$$

$$= \frac{12.0\ cm}{(32/7 - 1)/(4/3 + 24/7)}$$

$$= 16.00\ cm$$

(+3pts)

$$x_2 = f_2 - R(1 - \cos\theta_3) = 16.00\ cm - 20.0\ cm[1 - \cos\theta_3] = 12.00\ \mathrm{cm}$$

(+3pts)

$$\varDelta x = x_1 - x_2 = 33.31\ cm - 12.00\ cm = 21.31\ cm \rightarrow \varDelta x = 21.3\ cm$$

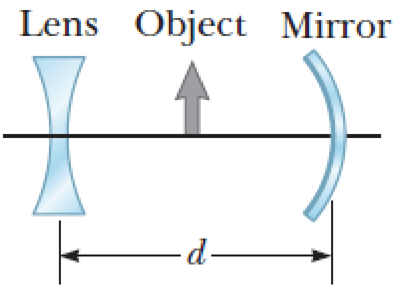
(+3pts)

7. The distance between the eyepiece and the objective lens in a certain compound microscope is $L_1 = 23\ cm$. The focal length of the eye piece is $f_1 = 2.5\ cm$ and that of the objective is $f_2 = 0.4\ \mathrm{cm}$. What is the overall magnification of the microscope? (Calculate the result up to 1's digit.)
[15 pts]

Sol) $M = -\left(\frac{L}{f_2}\right)\left(\frac{25\ cm}{f_1}\right)$ (+10pts)

$M = -\left(\frac{23\ cm}{0.4\ cm}\right)\left(\frac{25\ cm}{2.50\ cm}\right) = -575$ (+5pts)

8. The object in the below figure is midway between the lens and the mirror, which are separated by a distance d. The magnitude of the mirror's radius of curvature is $\frac{4}{5}d$, and the lens has a focal length of $-\frac{2}{3}d$. [15 pts]



(a) Considering only the light that leaves the object and travels first toward the mirror, locate the final image formed by this system.

Sol) Mirror equation: $\frac{1}{s_1}+\frac{1}{s_1'}=\frac{1}{f_1}=\frac{2}{\frac{4}{5}d}$ → $\frac{1}{s_1'}=\frac{1}{f_1}-\frac{1}{s_1}=\frac{5}{2d}-\frac{2}{d}=\frac{1}{2d}$

→$s_1'=2\text{d}$ (+2pts)

Lens equation: $\frac{1}{s_2}+\frac{1}{s_2'}=\frac{1}{f_2}=\frac{1}{-\frac{2}{3}d}$

→ $\frac{1}{s_2'}=\frac{1}{f_2}-\frac{1}{s_2}=\frac{1}{f_2}-\frac{1}{-(s_1'-d)}=-\frac{3}{2d}-\frac{1}{-d}=-\frac{1}{2d}$

→$s_2'=-2\text{d}$ (d to right of mirror) (+2pts)

(b) Is this image real or virtual?

Sol) Since $s_2'=-2\text{d}<0$, so the final image is virtual. (+3pts)

(c) Is it upright or inverted? And what is the overall magnification?

Sol) Calculate the overall magnification $M=M_1M_2$:

$M_1 = -\frac{s_1'}{s_1} = -\frac{2d}{\left(\frac{1}{2}d\right)} = -4$ (+2pts)

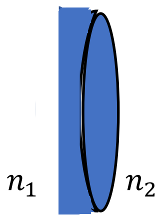
$M_2 = -\frac{s_2'}{s_2} = -\frac{-2d}{-d} = -2$ (+2pts)

$M = M_1 M_2 = 8$ (+3pts)

Since the overall magnification is positive, so the image is upright (+1pts)

9. Two lenses made of kinds of glass having different indices of refraction n_1 ($n_1 > 1$) and n_2 ($n_2 > 1$) are cemented together to form an optical doublet. Optical doublets are often used to correct chromatic aberrations in optical devices. The first lens of a certain doublet has index of refraction n_1, one flat side, and one concave side with a radius of curvature of magnitude R. The second lens has index of refraction n_2 and two convex sides with radii of curvature also of magnitude R. Show that the doublet can be modeled as a single thin lens with a focal length described by

$$\frac{1}{f} = \frac{2n_2 - n_1 - 1}{R}$$



Optical Doublet [20 pts]

Sol)

Using lens makers' equation,

The focal length of the first lens is

$\frac{1}{f_1} = (n_1 - 1)\left(\frac{1}{R_{1,1}} - \frac{1}{R_{1,2}}\right) = (n_1 - 1)\left(\frac{1}{\infty} - \frac{1}{R}\right) = \frac{1-n_1}{R}$. (+4pts)

The focal length of the second lens is

$\frac{1}{f_2} = (n_2 - 1)\left(\frac{1}{R_{2,1}} - \frac{1}{R_{2,2}}\right) = (n_2 - 1)\left(\frac{1}{+R} - \frac{1}{-R}\right) = \frac{2(n_2-1)}{R}$. (+4pts)

Let an object be placed at any distance p_1 large compared to the thickness of the doublet. The first lens forms an image according to

$$\frac{1}{p_1} + \frac{1}{q_1} = \frac{1}{f_1}$$

$$\frac{1}{q_1} = \frac{1}{f_1} - \frac{1}{p_1} = \frac{1-n_1}{R} - \frac{1}{p_1} \text{ (+4pts)}$$

This virtual ($q_1 < 0$) image (to the left of lens 1) is a real object for the second lens at distance $p_2 = -q_1$. For the second lens

$$\frac{1}{p_2} + \frac{1}{q_2} = \frac{1}{f_2}$$

$$\frac{1}{q_2} = \frac{1}{f_2} - \frac{1}{p_2} = \frac{2n_2 - 2}{R} + \frac{1}{q_1} = \frac{2n_2 - 2}{R} + \frac{1 - n_1}{R} - \frac{1}{p_1}$$

$$= \frac{2n_2 - n_1 - 1}{R} - \frac{1}{p_1} \text{ (+4pts)}$$

$$\rightarrow \frac{1}{p_1} + \frac{1}{q_2} = \frac{2n_2 - n_1 - 1}{R} \text{(+4pts)}$$

Thus, the doublet behaves like a single lens with $\frac{1}{f} = \frac{2n_2 - n_1 - 1}{R}$