HW#5 Solution

E7.9 The primary mirror of a large telescope can have a diameter of 10 m and a mosaic of 36 hexagonal segments with the orientation of each segment actively controlled. Suppose this unity feedback system for the mirror segments has the loop transfer function

$$L(s) = G_c(s)G(s) = \frac{K}{s(s^2 + 2s + 5)}.$$

(a) Find the asymptotes and sketch them in the s-plane.
(b) Find the angle of departure from the complex poles.
(c) Determine the gain when two roots lie on the imaginary axis.
(d) Sketch the root locus.

(Ans)

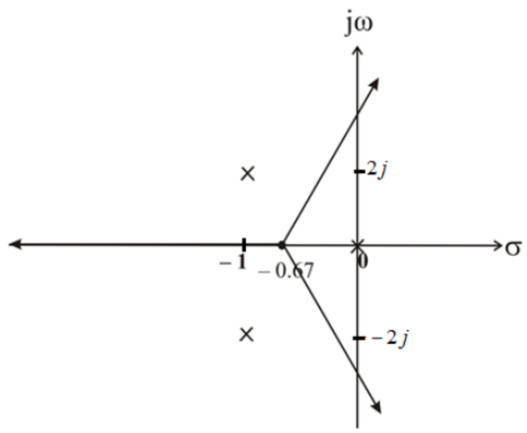
The characteristic equation is

$$1 + K\,\frac{1}{s(s^2 + 2s + 5)} = 0$$

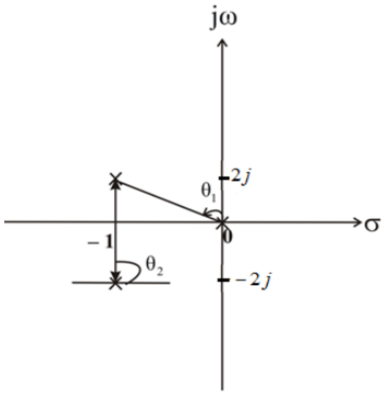
or

$$s^3 + 2s^2 + 5s + K = 0\,.$$

(a) The system has three poles at $s = 0$ and $-1 \pm j2$. The number of asymptotes in $n_p - n_z = 3$ centered at $\sigma_{cent} = -2/3$, and the angles are ϕ_{asymp} at $\pm 60°, 180°$.



(b) The angle of departure, θ_d, is $90° + \theta_d + 116.6° = 180°$, so $\theta_d = -26.6°$.



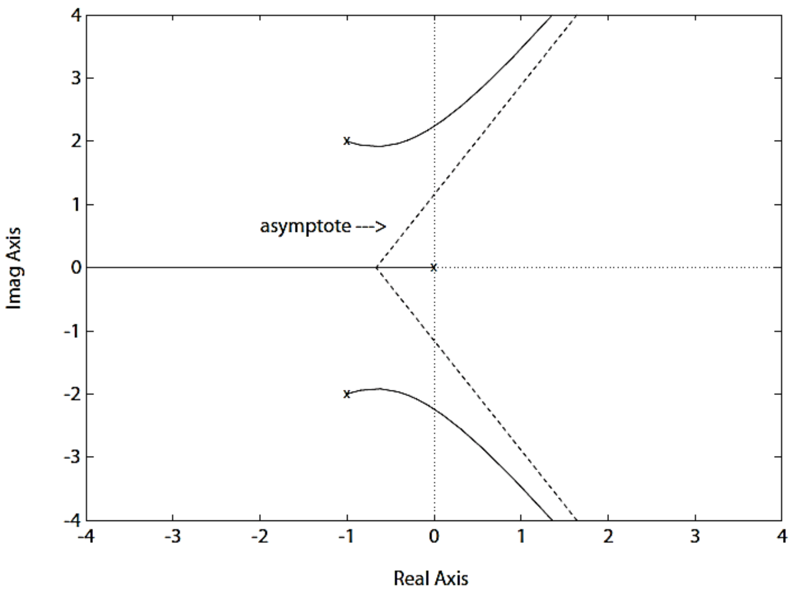
(c) The Routh array is

$$
\begin{array}{c|cc}
s^3 & 1 & 5 \\
s^2 & 2 & K \\
s & b & \\
s^0 & K &
\end{array}
$$

where $b = 5 - K/2$. So, when $K = 10$ the roots lie on the imaginary axis. The auxilary equation is

$$2s^2 + 10 = 0 \quad \text{which implies} \quad s_{1,2} = \pm j\sqrt{5}\,.$$

(d) The root locus is shown in below:



E7.13 A unity feedback system has a loop transfer function

$$L(s) = G_c(s)G(s) = \frac{4(s+z)}{s(s+1)(s+3)}.$$

(a) Draw the root locus as z varies from 0 to 100. (b) Using the root locus, estimate the percent overshoot and settling time (with a 2% criterion) of the system at $z = 0.6$, 2, and 4 for a step input. (c) Determine the actual overshoot and settling time at $z = 0.6$, 2, and 4.

(Ans)

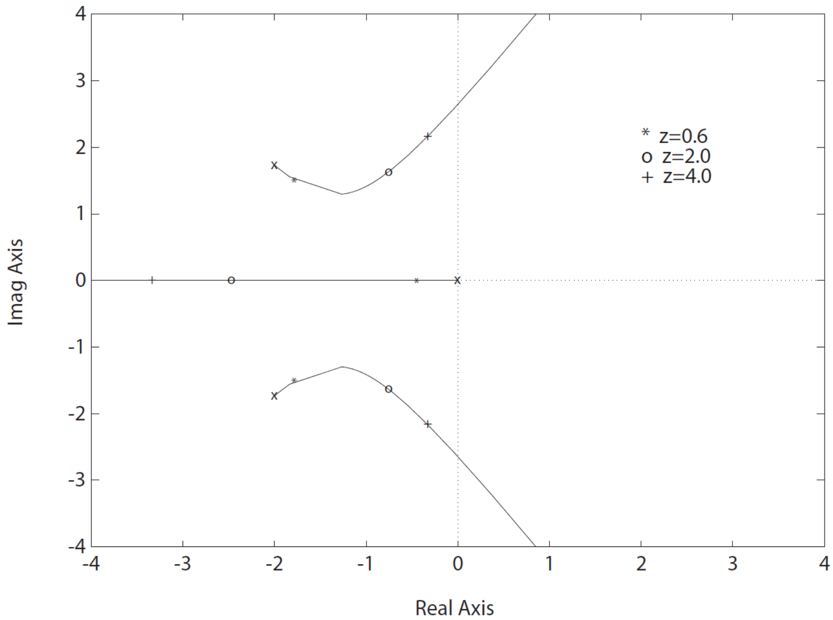
(a) The characteristic equation is

$$s(s+1)(s+3) + 4s + 4z = 0.$$

Rewriting with z as the parameter of interest yields

$$1 + z\frac{4}{s(s+1)(s+3)+4s} = 0.$$

The system has three poles at $s = 0$ and $-2 \pm j\sqrt{3}$. The number of asymptotes in $n_p - n_z = 3$ centered at $\sigma_A = -4/3$, and the angles are ϕ_A at $\pm 60°, 180°$. The root locus is shown in below:



(b) When $z = 0.6$, we have $\zeta = 0.76$ and $\omega_n = 2.33$. Therefore, the step response is

$$P.O. = 2.4\%, \quad T_s = 2.3\text{sec}$$

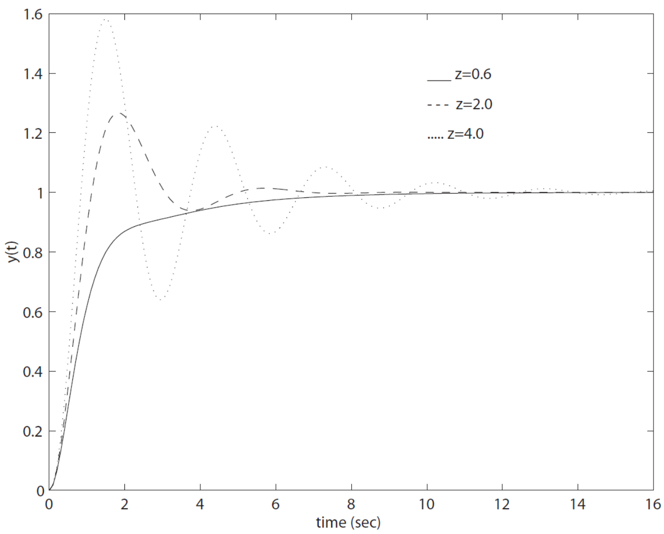
When $z = 2.0$, we have $\zeta = 0.42$ and $\omega_n = 1.79$. Therefore, the step response is

$$P.O. = 23\%, \quad T_s = 5.3\text{sec}$$

When $z = 4.0$, we have $\zeta = 0.15$ and $\omega_n = 2.19$. Therefore, the step response is

$$P.O. = 62\%, \quad T_s = 12\text{sec}$$

(c) The actual step responses are shown in below:



From the actual step responses, we can get actual overshoot and settling time as below:

When $z = 0.6$,

$$P.O. = \text{no overshoot}, \quad T_s = 6.5\text{sec}$$

When $z = 2.0$,

$$P.O. = 27\%, \quad T_s = 4.7\text{sec}$$

When $z = 4.0$,

$$P.O. = 58\%, \quad T_s = 11.8\text{sec.}$$

E7.28 Consider the feedback system in Figure E7.28. Obtain the negative gain root locus as $-\infty < K \leq 0$. For what values of K is the system stable?

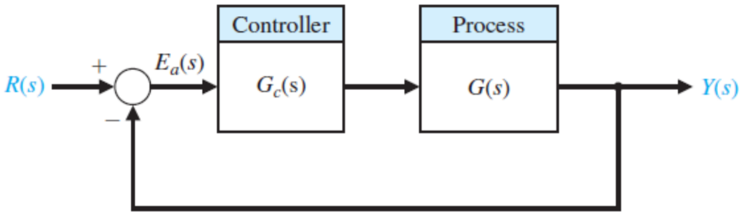


FIGURE E7.28 Feedback system for negative gain root locus.

$$G_c = K, \quad G(s) = \frac{s-1}{s(s^2+2s+2)}$$

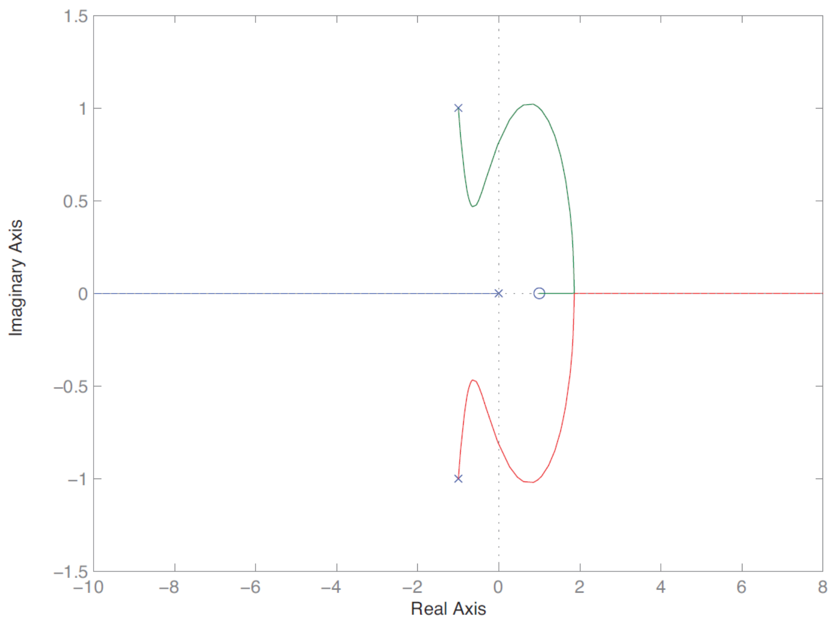
(Ans)

The characteristic equation is

$$1 + K\frac{s-1}{s(s^2+2s+2)} = 0.$$

The system has one zero at $s = 1$ and three poles at $s = 0$ and $-1 \pm j$. The number of asymptotes in $n_p - n_z = 2$ centered at $\sigma_A = -3/2$, and the angles are ϕ_A at $180°$.

The negative gain root locus is shown in below:



Determine the range of K for a stable system using R-H criterion.

$$s(s^2+2s+2)+K(s-1)=s^3+2s^2+(2+K)s-K=0$$

$$\begin{array}{c|cc} s^3 & 1 & 2+K \\ s^2 & 2 & -K \\ s^1 & b & 0 \\ s^0 & -K & \end{array}$$

$$b=-\frac{1}{2}(-K-4-2K)>0$$

$$-K>0$$

Therefore, the range of K for stable condition lies between

$$-\frac{4}{3}<K<0.$$

P7.33 The Bell-Boeing V-22 Osprey Tiltrotor is both an airplane and a helicopter. Its advantage is the ability to rotate its engines to 90° from a vertical position for takeoffs and landings as shown in Figure P7.33(a), and then to switch the engines to a horizontal position for cruising as an airplane [20]. The altitude control system in the helicopter mode is shown in Figure P7.33(b). (a) Determine the root locus as K varies and determine the range of K for a stable system. (b) For $K = 280$, find the actual $y(t)$ for a unit step input $r(t)$ and the percentage overshoot and settling time (with a 2% criterion). (c) When $K = 280$ and $r(t) = 0$, find $y(t)$ for a unit step disturbance, $T_d(s) = 1/s$.

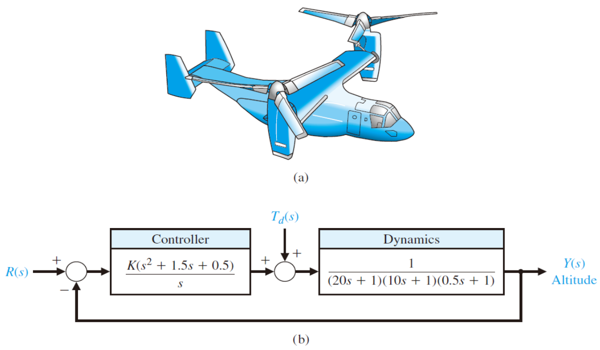


FIGURE P7.33 (a) Osprey Tiltrotor aircraft. (b) Its control system.

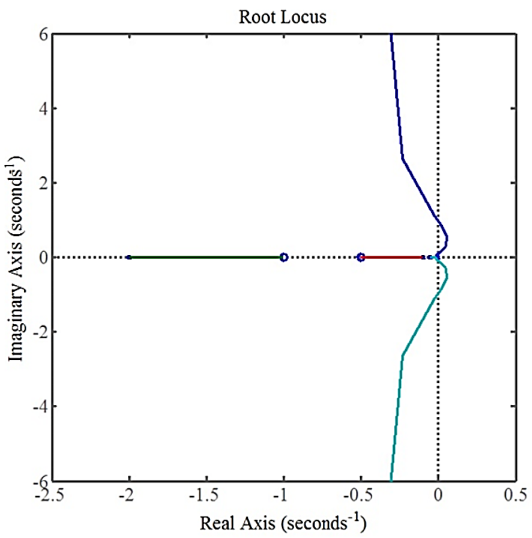
(Ans)

(a) $L(s) = \frac{K(s^2+1.5s+0.5)}{s} \times \frac{1}{(20s+1)(10s+1)(0.5s+1)}$

Characteristic equation is

$$1 + L(s) = 1 + \frac{K(s^2 + 1.5s + 0.5)}{s(20s + 1)(10s + 1)(0.5s + 1)}.$$

Plot root locus of the system as K varies



Determine the range of K for a stable system

$$s(20s+1)(10s+1)(0.5s+1)+K(s^2+1.5s+0.5)=0$$

$$\begin{array}{c|ccc}
s^4 & 100 & 30.5+K & 0.5K \\
s^3 & 215 & 1+1.5K & 0 \\
s^2 & \dfrac{215(30.5+K)-100(1+1.5K)}{215}=\varepsilon & 0.5K & \\
s & \dfrac{(1+1.5K)\varepsilon-(107.5K)}{\varepsilon} & & \\
s^0 & 0.5K & &
\end{array}$$

$$\frac{(215)(30.5+K)-100(1+1.5K)}{215}>0 \rightarrow K> \ -99.35 \cdots (1)$$

$$\frac{(1+1.5K)\left(\dfrac{(215)(30.5+K)-100(1+1.5K)}{215}\right)-107.5K}{\dfrac{(215)(30.5+K)-100(1+1.5K)}{215}}>0$$

$$\rightarrow K<0.48 \quad \text{or} \quad K>136.5 \cdots (2)$$

$$0.5K>0 \ \rightarrow K> \ 0 \cdots (3)$$

Therefore, the range of K for stable condition lies between

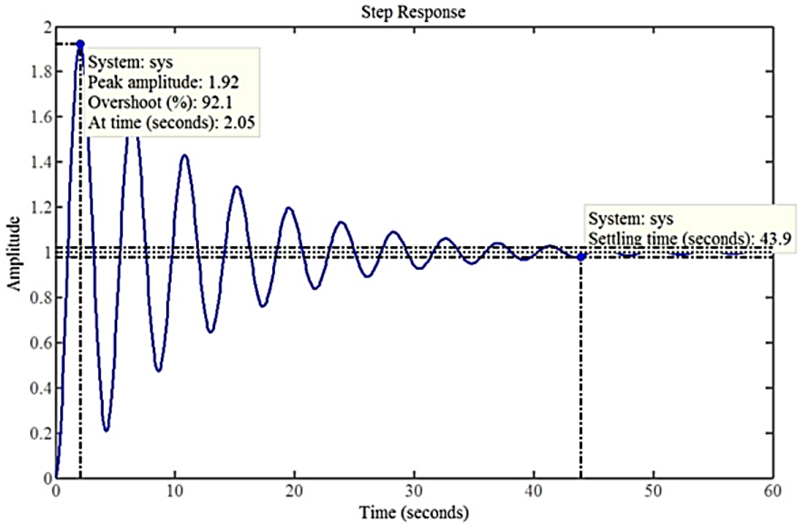
$$0<K<0.48 \quad \text{or} \quad K>136.5\,.$$

(b) When $K=280$

$$Y(s)=\frac{L(s)}{1+L(s)}R(s)$$

$$=\frac{\dfrac{K(s^2+1.5s+0.5)}{s(20s+1)(10s+1)(0.5s+1)}}{1+\dfrac{K(s^2+1.5s+0.5)}{s(20s+1)(10s+1)(0.5s+1)}}R(s)$$

$$=\frac{280s^2+420s+140}{100s^4+215s^3+310.5s^2+421s+140}R(s).$$



The step response has $P.O. = 92\%$ and $T_s = 43.9$ sec .

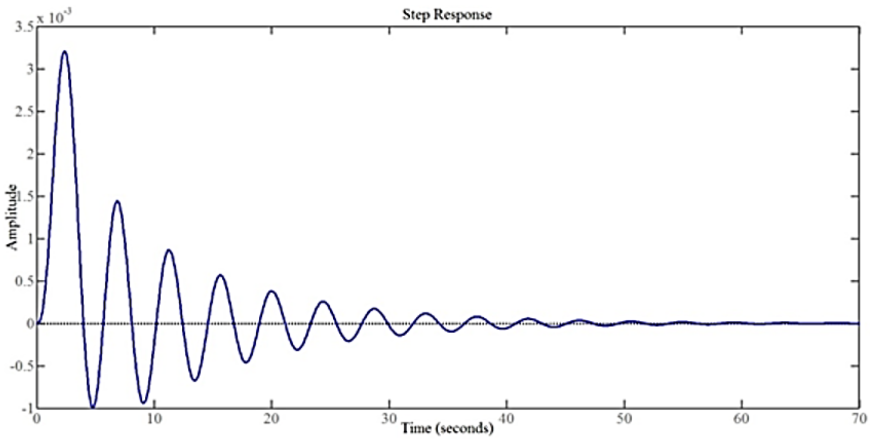
(c) When $r(t) = 0$

$$Y(s) = \frac{\frac{1}{(20s+1)(10s+1)(0.5s+1)}}{1 + \frac{K(s^2+1.5s+0.5)}{s(20s+1)(10s+1)(0.5s+1)}} T_d(s)\,.$$

When $K = 280$,

$$Y(s) = \frac{s}{100s^4 + 215s^3 + 310.5s^2 + 421s + 140} \times \frac{1}{s}\,.$$

Thus, plot of $y(t)$ for a unit step disturbance is



AP7.14 Consider the unity feedback control system shown in Figure AP7.14. Design a PID controller using Ziegler–Nichols methods. Determine the unit step response and the unit disturbance response. What is the maximum percent overshoot and settling time for the unit step input?

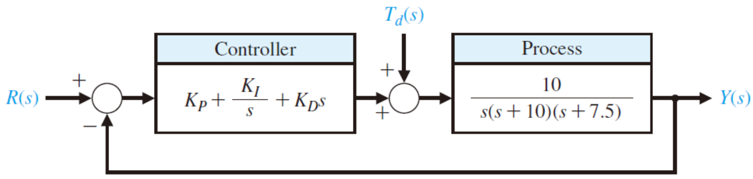


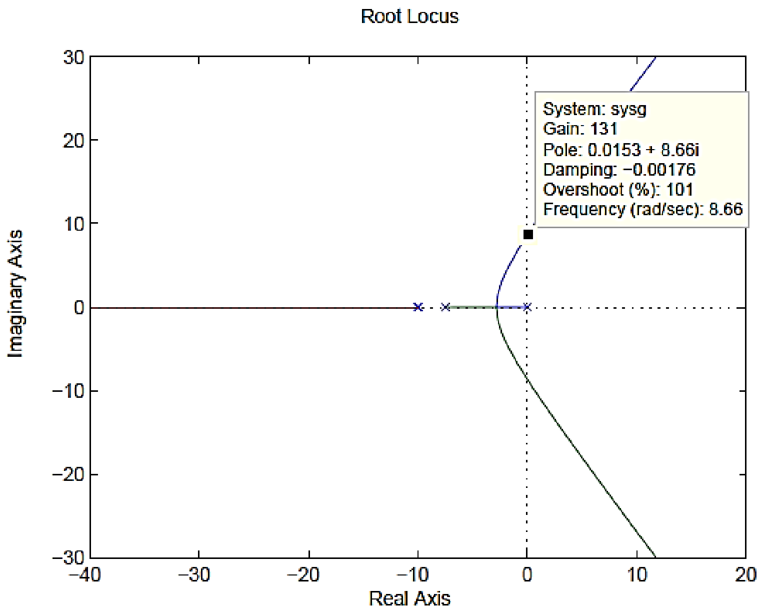
FIGURE AP7.14 Unity feedback loop with PID controller.

(Ans)

Set $K_I = K_D = 0$.

$$\frac{Y(s)}{R(s)} = \frac{K_P \dfrac{10}{s(s+10)(s+7.5)}}{1 + K_P \dfrac{10}{s(s+10)(s+7.5)}}.$$

The root locus of the uncompensated transfer function is



Characteristic equation is

$$s(s+10)(s+7.5) + 10K_P = 0$$

$$s^3 + 17.7s^2 + 75s + 10K_P = 0$$

The Routh array is

$$\begin{array}{c|cc} s^3 & 1 & 75 \\ s^2 & 17.5 & 10K_p \\ s & \dfrac{17.5\times 75 - 10K_P}{17.5} & 0 \\ s^0 & 10K_P & 0 \end{array}$$

The system to be stable when

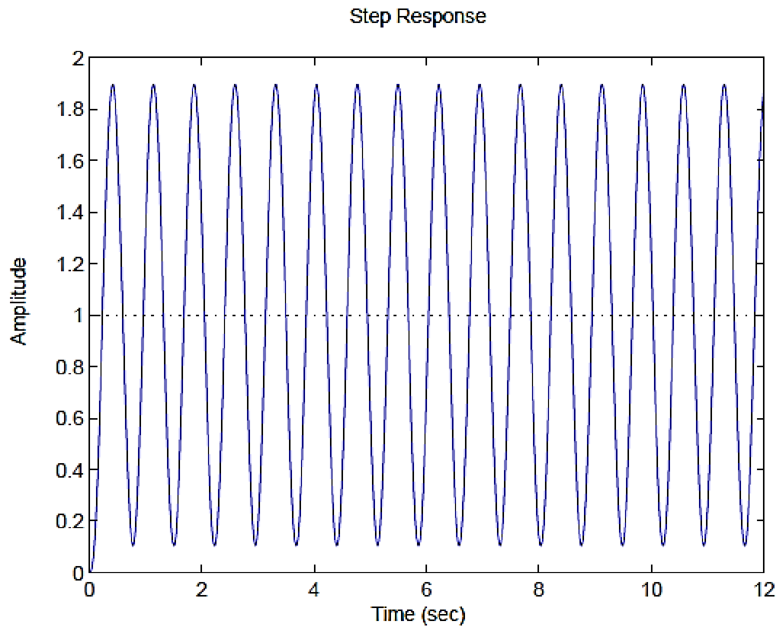
$$\frac{17.5\times 75 - 10K_P}{17.5} > 0 \text{ and } 10K_P > 0$$

$$\rightarrow 0 < K_P < 131.25\,.$$

Thus $K_u = 131.25$

$$Y(s) = \frac{1312.5}{s^3 + 17.7s^2 + 75s + 1312.5}R(s)\,.$$

The step response at the ultimate gain $K_u = 131$.



By analyzing the figure, we can determine $T_u = 0.72$ sec .

Thus,

$$K_P = 0.6K_u = 78.75$$

$$K_I = \frac{1.2K_u}{T_u} = 218.75$$

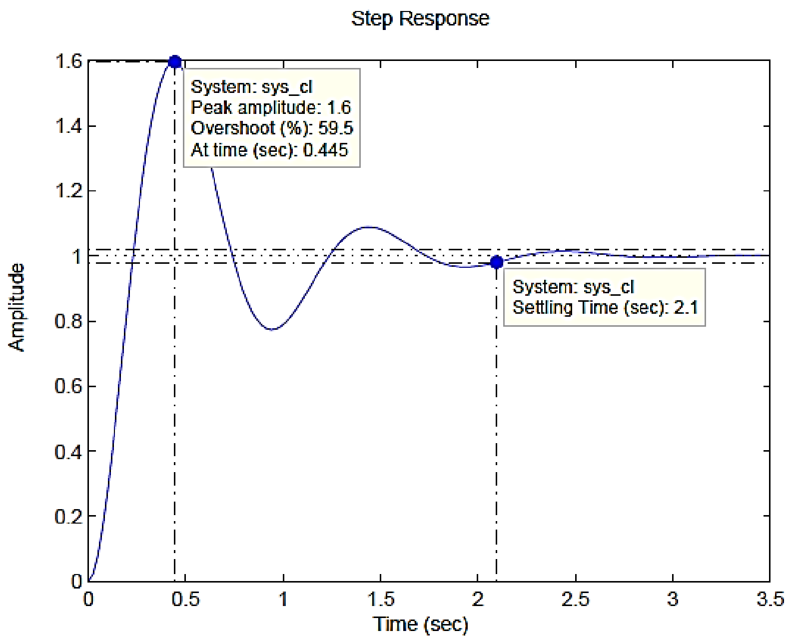
$$K_D = 0.6K_uT_u/8 = 7.0875$$

i) For unit step input

$$Y(s) = \frac{\dfrac{10(K_P + \frac{K_I}{s} + K_D s)}{s(s+10)(s+7.5)}}{1 + \dfrac{10(K_P + \frac{K_I}{s} + K_D s)}{s(s+10)(s+7.5)}} R(s)$$

$$= \frac{70.875s^2 + 787.5s + 2187.5}{s^4 + 17.5s^3 + (75 + 70.875)s^2 + 787.5s + 2187.5} \cdot \frac{1}{s}.$$

The step response with the Ziegler-Nichols tuned PID controller is



By analyzing the figure, $P.O. = 58.9\ \%$ and $T_s = 2.09$ sec .

ii) Also for unit disturbance response,

$$Y(s) = \frac{\frac{10}{s(s+10)(s+7.5)}}{1 + \frac{10(K_P + \frac{K_I}{s} + K_D s)}{s(s+10)(s+7.5)}} T_d(s)$$

$$= \frac{10s}{s^4 + 17.5s^3 + (75 + 70.875)s^2 + 787.5s + 2187.5} \cdot \frac{1}{s}.$$

The disturbance response with the Ziegler-Nichols tuned PID controller is