We will solve for two systems:

$$\begin{cases} w[n] = x[n] - e^{-4\beta}x[n-4] \\ y[n] + e^{-2\beta}y[n-2] = w[n] \end{cases} \tag{1}$$

Inserting $x[n] = \delta[n]$ to the first system gives the impulse response

$$h_1[n] = \delta[n] - e^{-4\beta}\delta[n-4] \tag{6}$$

Next, the IR of the second system can be derived as

Homogeneous equation with inhomogeneous auxiliary condition

$$\begin{gathered} y[n] = Az^n \\ Az^n + e^{-2\beta}Az^{n-2} = 0 \rightarrow Az^n(1 + e^{-2\beta}z^{-2}) = 0 \\ z = \pm ie^{-\beta} \\ h_2[n] = \left[A_1(ie^{-\beta})^n + A_2(-ie^{-\beta})^n\right]u[n] \end{gathered} \tag{7}$$

Auxiliary condition:

$$\begin{gathered} h_2[0] = 1, h_2[1] = 0 \rightarrow A_1 + A_2 = 1,\ A_1 - A_2 = 0 \\ A_1 = 1/2, A_2 = 1/2 \\ h_2[n] = \frac{1 + (-1)^n}{2} i^n e^{-\beta n} u[n] \end{gathered} \tag{8}$$

Total impulse response:

$$\begin{aligned} h[n] = h_1[n] * h_2[n] &= \left(\delta[n] - e^{-4\beta}\delta[n-4]\right) * \left(\frac{1 + (-1)^n}{2} i^n e^{-\beta n} u[n]\right) && (9) \\ &= \left(\frac{1 + (-1)^n}{2} i^n e^{-\beta n} u[n]\right) - \left(e^{-4\beta} \frac{1 + (-1)^{(n-4)}}{2} i^{n-4} e^{-\beta(n-4)} u[n-4]\right) && (10) \\ &= \left(\frac{1 + (-1)^n}{2} i^n e^{-\beta n}\right) (u[n] - u[n-4]) && (11) \end{aligned}$$

The above result gives $h[0] = 1,\ h[1] = 0,\ h[2] = -e^{-2\beta}$ and $h[n] = 0$ for $n \geq 3$. Therefore, the IR becomes

$$h[n] = \delta[n] - e^{-2\beta}\delta[n-2] \tag{12}$$