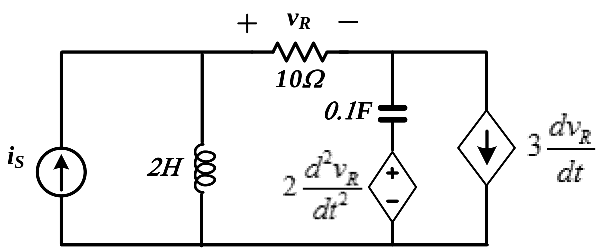
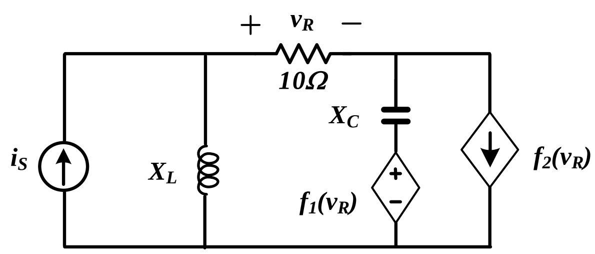
학번:________________ 이름:________________

Final Exam (2.5 hours)

1. Determine whether the following input, output pair is possible in a linear system. (ω>0) [5pts]
   (a) input = cos(ωt) , output = cos(ωt+θ) (**True**, False)
   (b) input = cos(ωt) , output = cos(2ωt) (True, **False**)
   (c) input = cos(ωt) , output = Acos(2ωt), where A is complex (True, **False**)
   (d) input = cos(ωt) , output = A_1cos(ωt) + A_2sin(ωt), where A_1,A_2 are real. (**True**, False)
   (e) input = δ(t) , output = exp(-t/τ) cos(ωt) u(t) (**True**, False)

2. Represent the impedance of the below circuit in complex domain when input current (i_S) has frequency of ω. [10pts]





(sol)

X_L= ___$j2\omega$___, X_C= ___$-10j/\omega$___, $f_1(v_R)$ = ___$-2\omega^2 v_R$___, $f_2(v_R)$ = ___$j3\omega v_R$___.

\

$$X_L = j\omega L = j2\omega$$

$$X_C = \frac{1}{j\omega C} = \frac{1}{j(0.1)\omega} = \frac{-10j}{\omega}$$

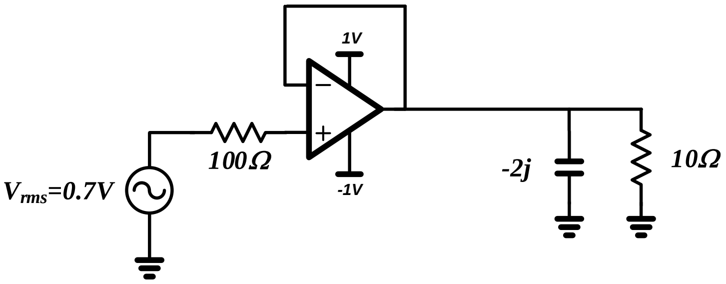
Let $v_R = V_R e^{j\omega t}$.

$$f_1(v_R) = 2\frac{d^2 v_R}{dt^2} = 2\frac{d^2(V_R e^{j\omega t})}{dt^2} = 2V_R\frac{d^2(e^{j\omega t})}{dt^2} = 2V_R(j\omega)^2 e^{j\omega t} = -2\omega^2 V_R e^{j\omega t} = -2\omega^2 v_R$$

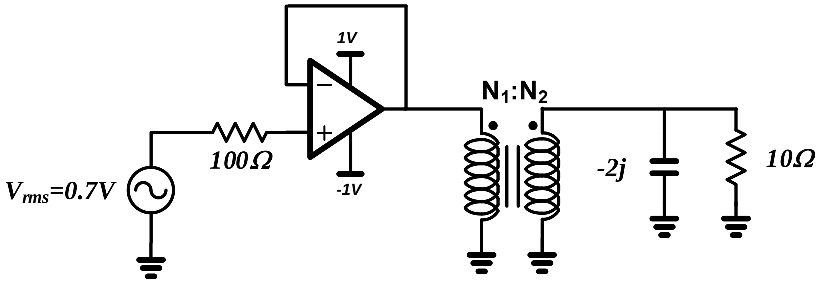
$$f_2(v_R) = 3\frac{dv_R}{dt} = 3\frac{d(V_R e^{j\omega t})}{dt} = 3V_R\frac{d(e^{j\omega t})}{dt} = 3V_R(j\omega)e^{j\omega t} = j3\omega V_R e^{j\omega t} = j3\omega v_R$$

3. Consider the circuits shown below [12pts]

(a) Consider the circuit shown below, where the output of the opamp is limited by its supply voltage. Please modify the circuit by adding passive device(s) such that the power delivered to the output load (R_L) is 10W. Specify the value of parameters of the passive device(s) that you added. [6pts]



Solution> Transformer

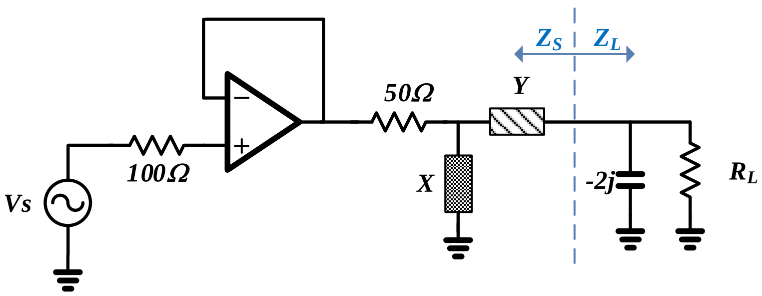


$$V_{out\_rms} = 10V$$

$$\frac{V_{out\_rms}}{V_{rms}} = \frac{10}{0.7} = \frac{N_2}{N_1} = \frac{100}{7}$$

(b) Please determine the value of the device R_L, X and Y in the below circuit such that maximum power is delivered to the resistive load, R_L. [6pts]

Solution>



[Maximum power delivery with complex load]

Maximum power delivery of R_L at maximum power delivery of Z_L.

Maximum power delivery condition:\

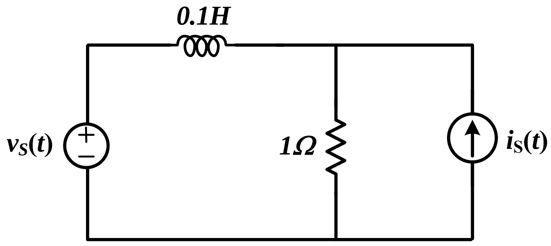
$$Z_S = Z_L^*$$

X=2j

Y=0

R_L=50Ω

4. Consider the circuit shown below. [20pts]



(a) Suppose $v_S(t) = \cos(10t)$ and $i_S(t) = \sqrt{2}\cos(10t+45^{\circ})$. Find the steady-state voltage across the resistor $v_R(t)$. Express your answer in time-domain. [4pts]

Sol>

$$\boldsymbol{V_R} = 1 \cdot \frac{1}{1+j1} + \sqrt{2}\angle 45^{\circ} \cdot \frac{j1}{1+j1} \cdot 1$$

$$= \frac{1-j1}{2} + j1$$

$$= \frac{\sqrt{2}}{2}\angle 45^{\circ}$$

$$\therefore\ v_R(t) = \frac{\sqrt{2}}{2}\cos(10t + 45^{\circ})$$

(b) What is the average power dissipated in the resistor R? [4pts]

Sol>

$$P_{avg} = \frac{1}{2}\left(\frac{\sqrt{2}}{2}\right)^2 \cdot 1 = \frac{1}{4}\ W$$

(c) Suppose $v_S(t) = \cos(10t)$ and $i_S(t) = \cos(20t)$. Find the steady-state voltage across the resistor $v_R(t)$. Express your answer in time-domain. [4pts]

Sol>

i) $\omega = 10$

$$\boldsymbol{V_R} = 1 \cdot \frac{1}{1+j1} = \frac{1-j1}{2} = \frac{\sqrt{2}}{2}\angle -45^{\circ}$$

ii) $\omega = 20$

$$\boldsymbol{V_R} = 1 \cdot \frac{j2}{1+j2} \cdot 1 = \frac{4+j2}{5} = \frac{2\sqrt{5}}{5}\angle \tan^{-1}\left(\frac{1}{2}\right)$$

$$\therefore\ v_R(t) = \frac{\sqrt{2}}{2}\cos\left(10t - 45^{\circ}\right) + \frac{2\sqrt{5}}{5}\cos\left(20t + \tan^{-1}\left(\frac{1}{2}\right)^{\circ}\right)$$

(d) Suppose voltage across a resistor is $A_1\cos(\omega_0 t)+A_2\cos(n\omega_0 t)$. What is the average power over T_0, where $T_0=2\pi/\omega_0$? [4pts]

Sol>

$$P_{avg} = \frac{1}{T_0}\int_0^{T_0}(A_1\cos(\omega_0 t) + A_2\cos(n\omega_0 t))^2\,dt$$

$$= \frac{1}{T_0}\int_0^{T_0}(A_1\cos(\omega_0 t))^2 + (A_2\cos(n\omega_0 t))^2 + 2A_1A_2\cos(\omega_0 t)\cos(n\omega_0 t)\,dt$$

$$= \frac{A_1^2}{2} + \frac{A_2^2}{2} + \frac{1}{T_0}\int_0^{T_0} 2A_1A_2\cos(\omega_0 t)\cos(n\omega_0 t)\,dt$$

$$= \frac{A_1^2}{2} + \frac{A_2^2}{2} + \frac{1}{T_0}\int_0^{T_0} A_1A_2(\cos((n+1)\,\omega_0 t) + \cos((n-1)\omega_0 t))\,dt$$

i) $n = 1$

$$P_{avg} = \frac{A_1^2}{2} + \frac{A_2^2}{2} + A_1A_2 = \frac{1}{2}(A_1 + A_2)^2$$

ii) $n \neq 1$

$$P_{avg} = \frac{A_1^2}{2} + \frac{A_2^2}{2}$$

(e) Suppose $v_S(t) = u(t)$ and $i_S(t)= \cos(10^3t+60^{O})$. Find the steady-state voltage across the resistor $v_R(t)$. Express your answer in time-domain. [4pts]

Sol>

i) $v_s(t) = u(t)$

steady state $\mathbf{v_R(t)} = 1$

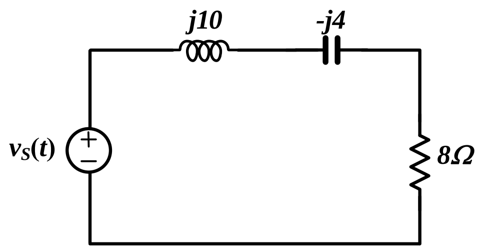
ii) $i_s(t) = \cos(10^3 t + 60°)$

$$\boldsymbol{V_R} = \frac{100\sqrt{100^2+1}}{1+100^2}\angle\tan^{-1}\left(\frac{1}{100}\right)° \cdot 1\angle 60°$$

$$= \frac{100\sqrt{100^2+1}}{1+100^2}\angle\left(60° + \tan^{-1}\left(\frac{1}{100}\right)°\right)$$

$$\therefore\ v_R(t) = 1 + \frac{100\sqrt{1+100^2}}{1+100^2}\cos\left(10^3 t + 60° + \tan^{-1}\left(\frac{1}{100}\right)°\right)$$

5. Consider the circuit shown below. [15pts]



(a) Suppose the voltage source has a frequency of 60Hz. What is the power factor (PF)? [3pts]

Sol)

Let $V_s = V_{eff}\angle\theta$

$$I_s = \frac{V_{eff}\angle\theta}{8+6j} = \frac{V_{eff}}{10}\angle(\theta-\varphi) \cdots\cdots\cdots [where\ \varphi = tan^{-1}\left(\frac{6}{8}\right)]$$

$$\Rightarrow I_{eff} = \frac{V_{eff}}{10}$$

$$P_{average} = I_{eff}^2 R = \frac{8V_{eff}^2}{100}$$

$$P_{apparent} = V_{eff}I_{eff} = \frac{V_{eff}^2}{10}$$

$$PF = \frac{P_{average}}{P_{apparent}} = \frac{8}{10}$$

(b) How does the PF change if frequency is increased from 60Hz to 120Hz? [2pts]

(i) PF increases (ii) **PF decreases** (iii) PF stays the same

>> The reactive load of this circuit increases from j6 to j18 as frequency change. Thus, the PF will decrease.

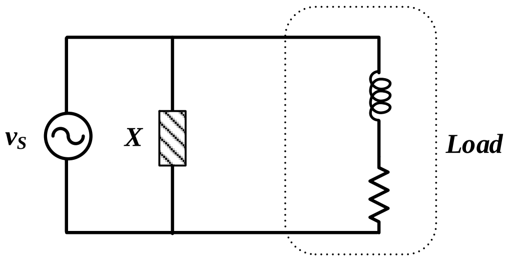
(c) Suppose the input is a square wave of 60Hz. Will the PF increase or decrease compared to (a)? Explain why. [4pts]

Sol) The PF can be defined as below,

$$PF = \frac{\Sigma P_{average}}{\Sigma P_{apparent}}$$

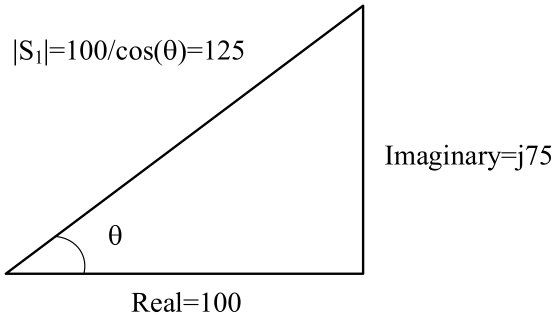
Since a square wave is made up of a sum of odd harmonics, the PF will decrease as a result of the problem (b).

(d) The load in the below circuit consumes 100Watts at a lagging PF of 0.8. The source voltage is 100 V rms at 100 rad/s. In order to increase the PF to 1, a corrective device (X) is added. Determine the type and value of X. [6pts]



Sol)

The complex power of original circuit must have a real part of 100W and an angle of θ, $\cos^{-1}(0.8)$. Hence,



$S_1 = 100 + \mathrm{j}75$

In order to achieve a PF of 1, the total complex power must become

$$S = S_1 + S_2 = 100\angle 0^{\circ} = 100 \text{ VA}$$

$\therefore S_2 = -j75 \text{ VA}$

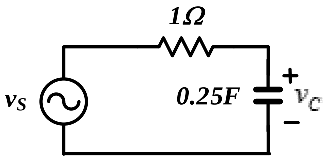
$$I_X^* = \frac{S_2}{V} = \frac{-j75}{100}\ A$$

$I_X = j0.75\ A$

$$\therefore \mathrm{X} = \frac{V}{I_X} = \frac{100}{j0.75} = -j133.33\ \Omega$$

This load can be provided by a 7.5mF capacitor.

6. Find out $v_C(t)$ of the below circuit for t>0, when $v_S(t) = e^{-3t}u(t)$ [5pts]



Sol1>

$$V_c(s) = \frac{\frac{4}{s}}{1+\frac{4}{s}} \frac{1}{s+3}$$

$$= \frac{4}{s+4} \frac{1}{s+3}$$

$$= \frac{-4}{s+4} + \frac{4}{s+3}$$

Taking the inverse Laplace transform, we find that

$\therefore v_c(t) = (-4\,e^{-4t} + 4\,e^{-3t})\,u(t)$

Sol2>

$v_c(0) = 0\,, v_c(\infty) = 0$

$t > 0$

$KVL: \frac{1}{4}\frac{dv_c}{dt} + v_c = v_s$

$\Rightarrow v_c(t) = A\,e^{-4t} + B\,e^{-3t} + C$

$\frac{1}{4}\frac{dv_c}{dt} + v_c = \frac{1}{4}B\,e^{-3t} + C = e^{-3t}$

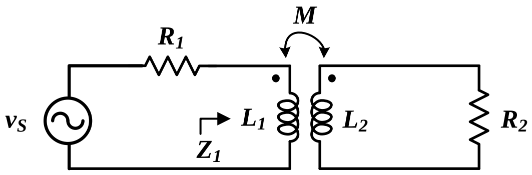
$\therefore B = 4, C = 0$

$v_c(0) = A + 4 = 0$

$\therefore A = -4$

$\therefore v_c(t) = (-4\,e^{-4t} + 4\,e^{-3t})\,u(t)$

7. [Mutual inductance] [10pts]



P_{SRC} is the power delivered from the source. $P_{R1,2}$ is the power dissipated in the resistors. k is the coupling coefficient. E_{SRC} is the energy delivered from the source. E_X is the energy stored or dissipated in device X.

(a) Suppose $v_s(t) = u(t)$ and k=1. Is $P_{SRC} = P_{R1} + P_{R2}$ at any given time instant? (Yes, **No**)

If your answer is "No", then please describe where the missing power is. [3pts]

Sol)

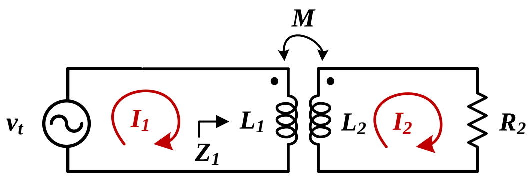
When an electric current is flowing in an inductor, there is energy stored in the magnetic field that is delivered from the source, but not dissipated in the resistors.

(b) Suppose $v_s(t) = u(t)$ and $k < 1$. Is $E_{SRC} = E_{R1} + E_{R2} + E_{L1} + E_{L2} + E_M$? (**Yes**, No)

If your answer is "No", then please describe where the energy is lost. [3pts]

(c) Suppose k<1. What is the resistance (NOT impedance) seen from Z_1? Assume L_1=L_2=L, R_1=R_2=R, ωL >> R. [4pts]

Sol)



$$V_t = jI_1\omega L - jI_2\omega M \quad \cdots \quad ①$$

$$V_R = I_2 R = -jI_2\omega L + jI_1\omega M$$

$$I_2 = \frac{j\omega M}{R+j\omega L} I_1 \quad \cdots \quad ②$$

$$V_t = I_1\left(j\omega L + \frac{\omega^2 M^2}{R+j\omega L}\right)$$

$$Z_1 = \frac{V_t}{I_1} = \frac{j\omega RL - \omega^2 L^2 + \omega^2 M^2}{R + j\omega L}$$

$$= \frac{j\omega RL - (1-k^2)\omega^2 L^2}{R + j\omega L} \times \frac{R - j\omega L}{R - j\omega L}$$

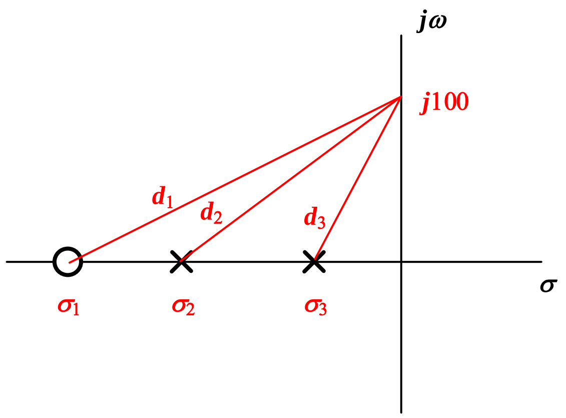
$$= \frac{j\omega R^2 L - (1-k^2)R\omega^2 L^2 + R\omega^2 L^2 + j(1-k^2)\omega^3 L^3}{R^2 + \omega^2 L^2}$$

$$Re\{Z_1\} = \frac{k^2 R\omega^2 L^2}{R^2 + \omega^2 L^2} \cong \frac{k^2 R\omega^2 L^2}{\omega^2 L^2} \quad \cdots \ (\because \omega L \gg R)$$

$$= k^2 R$$

8. Please draw and explain the meaning of magnitude and phase of frequency response of a transfer function H(s) in a complex-domain plot. [8pts]

(a) In the below pole-zero plot, explain graphically what the magnitude of H(s) is at ω=100Hz. [4pts]



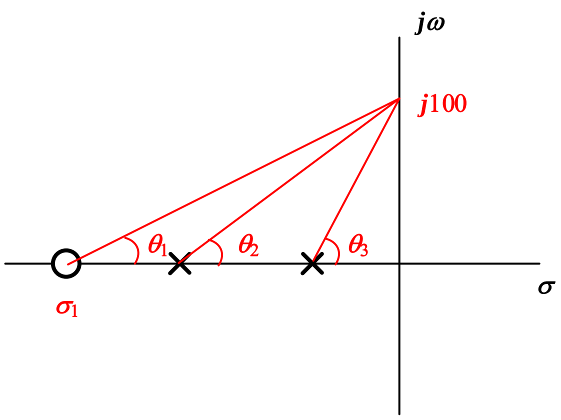
Sol) zero: σ_1 , poles: σ_2, σ_3

$$H(s) = \frac{(s+\sigma_1)}{(s+\sigma_2)(s+\sigma_3)} = \frac{|s+\sigma_1|\angle(s+\sigma_1)}{|s+\sigma_2|\angle(s+\sigma_2)|s+\sigma_3|\angle(s+\sigma_3)}$$

$$= \frac{|s+\sigma_1|}{|s+\sigma_2||s+\sigma_3|}(\angle(s+\sigma_1) - \angle(s+\sigma_2) - \angle(s+\sigma_3))$$

$$\therefore \left|H(s)|_{s=j100}\right| = \frac{|j100+\sigma_1|}{|j100+\sigma_2||j100+\sigma_3|} = \frac{d_1}{d_2 d_3}$$

(b) In the below pole-zero plot, explain graphically what the phase of H(s) is at ω=100Hz. [4pts]



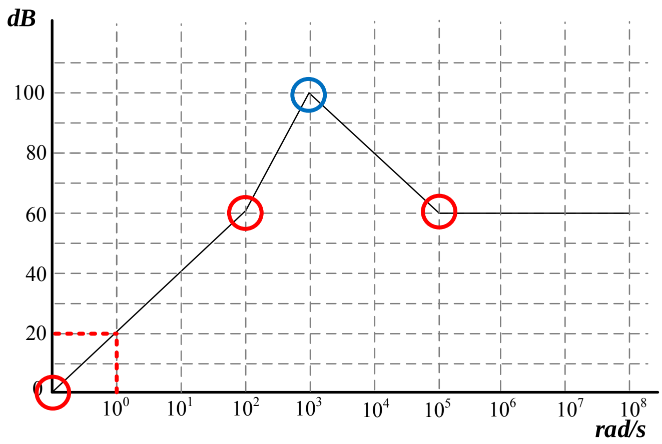
Sol) By using the result from (a),

$$H(s) = \frac{|s+\sigma_1|}{|s+\sigma_2||s+\sigma_3|}(\angle(s+\sigma_1) - \angle(s+\sigma_2) - \angle(s+\sigma_3)) .$$

$$\therefore \angle H(s)|_{s=j100} = \angle(s+\sigma_1) - \angle(s+\sigma_2) - \angle(s+\sigma_3) = \theta_1 - \theta_2 - \theta_3$$

9. [Frequency response] [8pts]

(a) What transfer function H(s) results in the below frequency response of |H(s)|? [4pts]



Sol)

Corner frequencies with positive gradient: 0, 10^2, 10^5, and negative gradient: 10^3

$$A_{db} = 20\log_{10}|H(jw)|$$

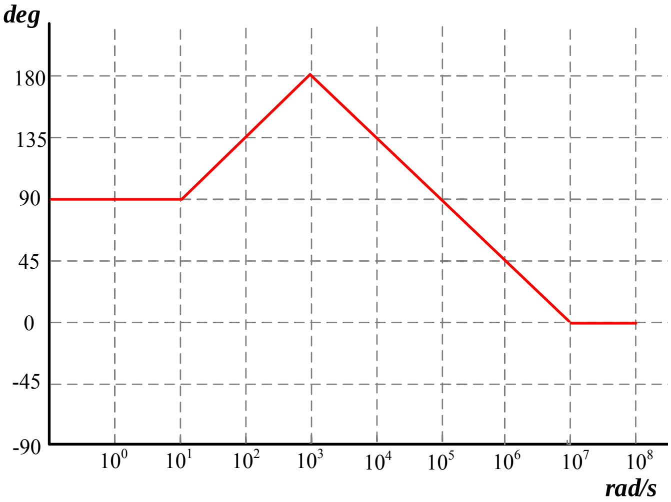
$$A_{db} = a_0 20 + a_1 20\log_{10}|jw| + a_2 20\log_{10}\left|1+\frac{jw}{10^2}\right| + a_3 20\log_{10}\left|1+\frac{jw}{10^5}\right| - a_4 20\log_{10}\left|1+\frac{jw}{10^3}\right|$$

$$a_0 = 1,\ a_1 = 1,\ a_2 = 1,\ a_3 = 1,\ a_4 = 3$$

$$\therefore H(s) = 10\frac{s(1+\frac{s}{10^2})(1+\frac{s}{10^5})}{(1+\frac{s}{10^3})^3} = 10^3\frac{s(10^2+s)(10^5+s)}{(10^3+s)^3}$$

(b) Draw the phase plot of H(s). [4pts]

$$H(s) = \frac{s(s+10^2)}{(s+10^4)(s+10^6)}$$



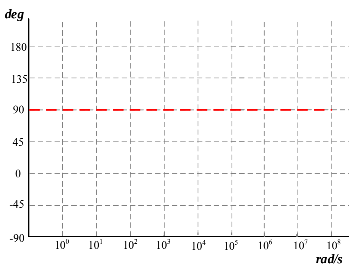
Sol)

$$H(jw) = \left.\frac{s(s+10^2)}{(s+10^4)(s+10^6)}\right|_{s=jw} = 10^{-8}\frac{|jw|\left|\frac{jw}{10^2}+1\right|}{\left|\frac{jw}{10^4}+1\right|\left|\frac{jw}{10^6}+1\right|}\angle(\phi_{jw}+\phi_{(\frac{jw}{10^2}+1)}-\phi_{(\frac{jw}{10^4}+1)}-\phi_{(\frac{jw}{10^6}+1)})$$

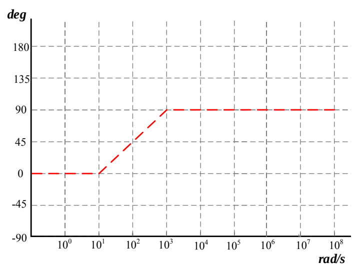
$$\therefore \theta(w) = \phi_{jw}+\phi_{(\frac{jw}{10^2}+1)}-\phi_{(\frac{jw}{10^4}+1)}-\phi_{(\frac{jw}{10^6}+1)}$$

$$\text{, where } \phi_{jw} = 90°,\ \phi_{(\frac{jw}{10^2}+1)} = \tan^{-1}(\frac{w}{10^2}),\ \phi_{(\frac{jw}{10^4}+1)} = \tan^{-1}(\frac{w}{10^4}),\ \phi_{(\frac{jw}{10^6}+1)} = \tan^{-1}(\frac{w}{10^6})$$

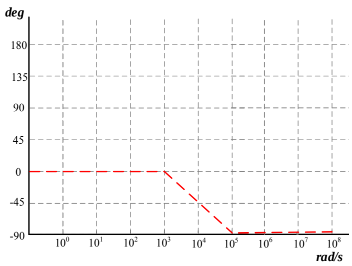
$$\phi_{jw} = 90°$$



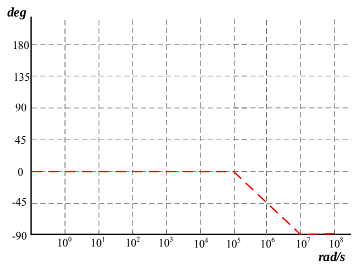
$$\phi_{(\frac{jw}{10^2}+1)} = \tan^{-1}(\frac{w}{10^2})$$



$$-\phi_{(\frac{jw}{10^4}+1)} = -\tan^{-1}(\frac{w}{10^4})$$

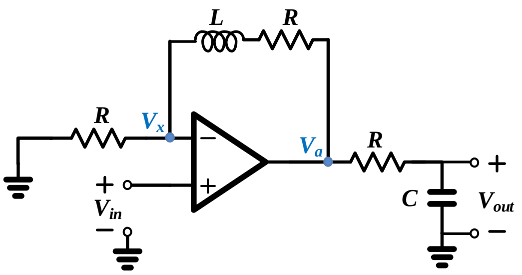


$$-\phi_{(\frac{jw}{10^6}+1)} = -\tan^{-1}(\frac{w}{10^6})$$



10. Consider the circuit shown below.

(a) Find the transfer function $H_0(s)$, from V_{in} to V_{out}. [4pts]



Solution>

$$H_0(s) = \frac{V_{out}}{V_{in}} = \frac{V_{out}}{V_x} = \frac{V_a}{V_x}\cdot\frac{V_{out}}{V_a}$$

V_x is virtual ground node with V_{in}.

KCL at node V_x : $\dfrac{V_x}{R} + \dfrac{V_x - V_a}{R+sL} = 0$

$$\frac{V_a}{V_x} = (R+sL)\cdot\left(\frac{1}{R} + \frac{1}{R+sL}\right) = 2 + s\frac{L}{R}$$

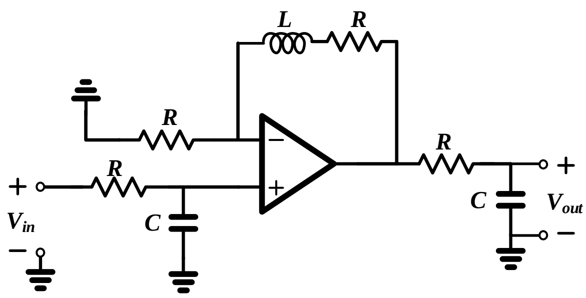
$$\frac{V_{out}}{V_a} = \frac{1}{1+sRC}$$

$$H_0(s) = \frac{V_{out}}{V_{in}} = \frac{V_{out}}{V_x} = \frac{V_a}{V_x}\cdot\frac{V_{out}}{V_a} = \left(2 + s\frac{L}{R}\right)\cdot\left(\frac{1}{1+sRC}\right) \quad \text{(1 pole, 1 zero system)}$$

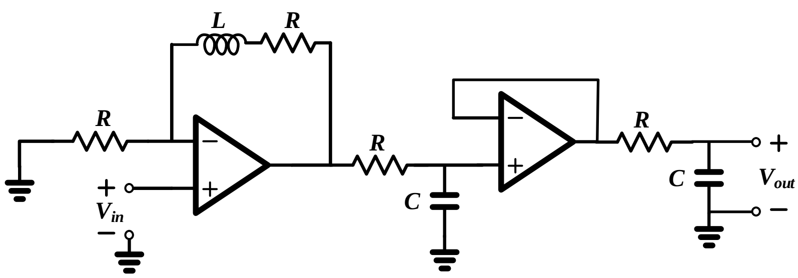
(b) Suppose we want a new transfer function, $H_{new}(s) = H_0(s) \times \frac{1}{(sRC+1)}$. Please design a circuit that provides the above function. You will receive more points if you minimize the number of components used. [3pts]

Solution> There are lots of variations. Simple example circuits are shown below.

Example 1> Simplest one.



Example 2>



Example 1 is better than Example 2, because the Opamp is much more expensive than the other passive blocks.