Notice: Only the questions with the score written have been scored. Also, each small problems are given equal points. For example, if a problem is given 16 points, and there are two small problems for that problem, then each small problems are given 8 points.

1

(a) The probability of choosing each digit is $1/1000 = 0.001$. There are three digits 4, 9, 1, so the winning number is 491, 419, 149, 194, 941, 914 that are $P_{33} = 3 \times 2 \times 1 = 6$ combinations of the three digits. Hence the probability of your winning is $6 \times 1/1000 = 0.006$.

(b) There is only one digit 2, so the winning number is only 222 in any order. The probability of your winning is $1/1000 = 0.001$.

2

14 points

(a) The probability of at least including 2 women = P(2 women and 1 man) + (3 women and 0 man) $= \frac{\binom{4}{2}\binom{5}{1}}{\binom{9}{3}} + \frac{\binom{4}{3}\binom{5}{0}}{\binom{9}{3}} = \frac{17}{42} = 0.405$.

(b) The probability of neither being able to do the job $= \frac{\binom{3}{2}}{\binom{12}{2}} = \frac{1}{22} = 0.0455$

3

20 points

(a) $P(\bar{A}) = 1 - P(A) = 1 - \frac{1}{2} = \frac{1}{2}$
By $P(A|B) = P(AB)/P(B)$, we have
$P(AB) = P(A|B)P(B) = \frac{1}{3} \times \frac{1}{4} = \frac{1}{12}$
$P(A \cup B) = P(A) + P(B) - P(AB) = \frac{1}{2} + \frac{1}{4} - \frac{1}{12} = \frac{2}{3}$

(b) (Use a Venn Diagram)

$$P(A\bar{B}) = P(A) - P(AB) = \frac{1}{2} - \frac{1}{12} = \frac{5}{12}$$

$$P(\bar{A} \cup \bar{B}) = P((AB)^c) = 1 - \frac{1}{12} = \frac{11}{12}$$

4

11 points

B_i: production line i, $i = 1, 2, 3$
A: two failed out of five tests
$P(B_1) = P(B_2) = P(B_3) = 1/3$
$P(A|B_1) = 0.0729$, $P(A|B_2) = 0.2048$, $P(A|B_3) = 0.3087$
$P(B_3|A) = 0.5264$

5

11 points

Define the events: E: person is exposed to the flu, F: person gets the flu.
Consider two employees, one of who is inoculated and one not. The probability of interest is the probability that at least one contracts the flu. Consider the complement: P(at least one gets the flu) = 1 – P(neither employee gets the flu). For the inoculated employee: $= .8(.6) + 1(.4) = 0.88$. For the non-inoculated employee: $= .1(.6) + 1(.4) = 0.46$. So, P(at least one gets the flu) = 1 – $.88(.46) = 0.5952$

3.34 20 points

Define the events: E: the test indicates that the man has elevated PSA level, C: the man has the cancer.

(a)

$$\begin{aligned} P(C|E) &= \frac{P(E|C)P(C)}{P(E|C)P(C) + P(E|C^c)P(C^c)} \\ &= \frac{(.268)(.7)}{(.268)(.7) + (.135)(.3)} \\ &= .8224 \end{aligned}$$

(b)

$$\begin{aligned} P(C|E^c) &= \frac{P(E^c|C)P(C)}{P(E^c|C)P(C) + P(E^c|C^c)P(C^c)} \\ &= \frac{(.732)(.7)}{(.732)(.7) + (.865)(.3)} \\ &= .6638 \end{aligned}$$

When the physician initially believes there is a 30 percent chance the man has prostate cancer, we have following.

(a)

$$\begin{aligned} P(C|E) &= \frac{P(E|C)P(C)}{P(E|C)P(C) + P(E|C^c)P(C^c)} \\ &= \frac{(.268)(.3)}{(.268)(.3) + (.135)(.7)} \\ &= .4597 \end{aligned}$$

(b)

$$\begin{aligned} P(C|E^c) &= \frac{P(E^c|C)P(C)}{P(E^c|C)P(C) + P(E^c|C^c)P(C^c)} \\ &= \frac{(.732)(.3)}{(.732)(.3) + (.865)(.7)} \\ &= .2661 \end{aligned}$$

3.37 24 points

(a) $p_1p_2p_3 + p_4p_5 - p_1p_2p_3p_4p_5$

(b) $p_5(p_1p_2 + p_3p_4 - p_1p_2p_3p_4)$

(c) $(1 - p_3)(p_1p_4 + p_2p_5 - p_1p_2p_4p_5) + p_3(p_1 + p_2 - p_1p_2)(p_4 + p_5 - p_4p_5)$