1. There are two different points A, B and each point has different electric potential. Let a proton is on the initial position B and starts to move. There are two different ways to get to the position A. When the proton arrived at A, is the work done by the field is independent of the paths? (O , X) [5 pts]

2. Let's say there is an electric field that form closed loop. A charged particle moved around a closed path. When the particle arrived at the initial point, do you think the electric field will have worked to the particle? (O , X) [5 pts]

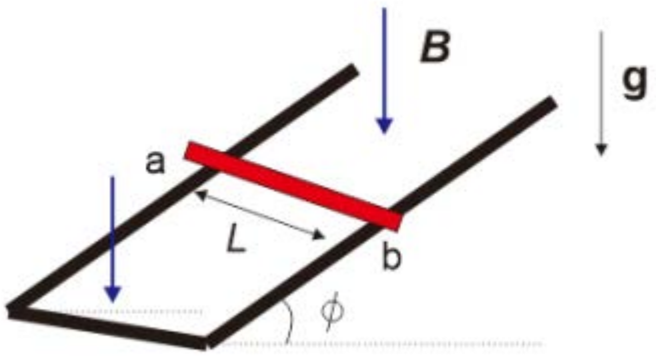
3. Electric field cannot form a closed loop. (O , X) [5 pts]

4. Let's think about an RLC circuit with AC voltage whose resonance frequency is ω_0. Relation between voltage-current phase difference and AC voltage frequency depends on resistance R. To make a sharp turn of phase difference near ω_0, what should we do to resistance R. [5 pts]

Reduce or remove the resistor

(1~4 부분점수 없음)

5. A metal bar with length L, mass m, and resistance R is placed on frictionless metal rails that are inclined at an angle ϕ above the horizontal. The rails have negligible resistance. There is a uniform magnetic field of magnitude **B** directed downward in figure below. The bar is released from rest and slides down the rails. Find the terminal speed of the bar.



Induced emf $\epsilon = -BLv\cos\phi$

Then the current flowing on the metal bar is $\mathrm{I} = \frac{\epsilon}{\mathrm{R}} = -\frac{BLv}{R}\cos\phi$ [+5 pt]

Lorentz force on the rod is exerted upward as $\mathrm{F} = \mathrm{IL} \times \mathrm{B}$.

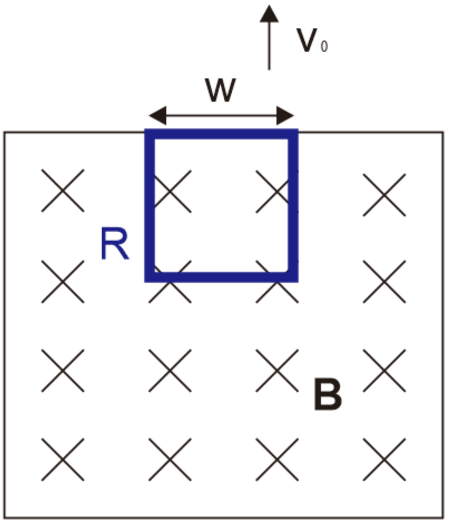
The velocity of metal bar increases until the Lorentz force balances with the gravity force

$\mathrm{ILB}\cos\phi = \mathrm{mg}\sin\phi$ [+5pt]

By substituting current, $\mathrm{mg}\sin\phi = \frac{\mathrm{B}^2L^2v_t}{R}\cos^2\phi$

The terminal velocity $\mathrm{v_t} = \frac{mgR}{B^2L^2}\tan\phi\sec\phi$ [+5 pt]

6. A rectangular loop of resistance R, mass m, width w is placed as figure below. Find the distance the loop travels when the initial velocity is v_0. (Ignore gravity and self-inductance)



$\epsilon = -\frac{d\Phi}{dt} = -Bwv$ [+3 pt]

Lorentz force exerted on the loop is

$$F = IwB = m\frac{dv}{dt}$$

$\frac{dv}{dt} = \frac{IwB}{m} = \frac{\epsilon}{R}\frac{wB}{m} = -\frac{(wB)^2}{mR}v$ [+5pt]

[Scoring for 2 cases]

1) If you assume the vertical length of loop is infinite,

Because $v(t=0) = v_0$

$v = v_0 \exp\left[-\frac{(wB)^2}{mR}t\right]$ [+3 pt]

Due to $z(t=0) = 0$,

$$z = \frac{mRv_0}{(wB)^2}\left(1 - \exp\left(-\frac{(wB)^2}{mR}t\right)\right)$$

The distance is $\frac{mRv_0}{(wB)^2}$ [+4pt]

or

2) If you assume the length L as the vertical distance of loop,

For $0 < z \le L$, $v = v_0 \exp\left[-\frac{(wB)^2}{mR}t\right]$ [+3 pt]

Due to $z(t=0) = 0$,

$$z = \frac{mRv_0}{(wB)^2}\left(1 - \exp\left(-\frac{(wB)^2}{mR}t\right)\right)$$

When the loop escapes the magnetic zone,

$$\mathrm{L} = \frac{\mathrm{mRv_0}}{(wB)^2}\left(1 - \exp\left(-\frac{(wB)^2}{mR} t_e\right)\right)$$

$$\mathrm{t_e} = -\frac{mR}{(wB)^2}\ln\left(1 - \frac{(wB)^2 L}{mRv_0}\right) \text{ [+1 pt]}$$

$$\mathrm{v_e} = v_o \exp\left[-\frac{(wB)^2}{mR} t_e\right] = v_o\left(1 - \frac{(wB)^2 L}{mR}\right) \text{ [1 pt]}$$

Therefore, $\mathrm{z(t)} = \mathrm{L} + \mathrm{v_e}(t - t_e) = L + v_0\left(1 - \frac{(wB)^2 L}{mR}\right)\left(t + \frac{mR}{(wB)^2}\ln\left(1 - \frac{(wB)^2 L}{mRv_0}\right)\right)$ for z>L [+2pt]

7. A series RLC circuit, driven with AC power of $V_{rms} = 120\,V$ at angular frequency $\omega = 100\,Hz$, contains $R = 13\,\Omega$ resistor, $L = 150\,mH$ inductor, $C = 1\,mF$ capacitor. What is the average power consumption in the circuit? [15 pts]

In the series RLC circuit driven with AC power, the impedance and phase difference between voltage and current $\phi = \phi_V - \phi_I$ is calculated as

$$Z = \sqrt{R^2 + (\omega L - 1/\omega C)^2} = \sqrt{13^2 + \left(100 \times 150 \times 10^{-3} - \frac{1}{100 \times 10^{-3}}\right)^2}\ \Omega = \sqrt{194}\ \Omega \cong 13.9\ \Omega \quad [4pt]$$

$$\phi = \tan^{-1}\left(\frac{\omega L - 1/\omega C}{R}\right) = \tan^{-1}\left(\frac{5}{13}\right) \cong 21^\circ$$

, respectively.

In textbook p.536, the time-average power in AC circuit is given as

$$\langle P \rangle = I_{rms} V_{rms} \cos\phi \quad [2pt]$$

, where I_{rms} and V_{rms} are the root mean square values of current and voltage.

Here, $\cos\phi$ can be calculated by

$$\cos\phi = \frac{R}{\sqrt{R^2 + (\omega L - 1/\omega C)^2}} = \frac{R}{Z} \quad [4pt]$$

Thus,

$$\langle P \rangle = I_{rms} V_{rms} \cos\phi = \frac{V_{rms}}{Z} \cdot V_{rms} \cdot \frac{R}{Z} = \frac{V_{rms}^2\, R}{Z^2} = \frac{(120\ \mathrm{V})^2 \times (13\ \Omega)}{194\ \Omega^2} \cong 965\ \mathrm{W} \quad [5pt]$$

[Evaluation criteria]

It allows to calculate $\cos\phi$ by using exact value of $\phi = \tan^{-1}\left(\frac{\omega L - 1/\omega C}{R}\right) = \tan^{-1}\left(\frac{5}{13}\right)$. [4pt]

Calculation mistake. [-2pt]

8. Consider a circuit with an inductor with self-inductance L and a capacitor with capacitance C. Initially, the capacitor is charge with charge q_0, and no current is flowing on the inductor. Using the energy conservation law, write down the second derivative equation for the charge at the capacitor. Get the charge $q(t)$ on the capacitor and the current $I(t)$ flowing on the inductor. [15 pts]

From the energy conservation, $\mathrm{U} = \frac{1}{2}\mathrm{LI}^2 + \frac{1}{2}\mathrm{CV}^2$. [4 pts]

$$0 = \frac{dU}{\mathrm{dt}} = \frac{d}{dt}\left(\frac{1}{2}LI^2 + \frac{1}{2}CV^2\right) = LI\frac{dI}{dt} + \mathrm{CV}\frac{\mathrm{dV}}{\mathrm{dT}}$$

With $\mathrm{V} = \mathrm{q}/\mathrm{C}$ and $\mathrm{I} = \mathrm{dq}/\mathrm{dt}$, we get the second derivative equation.

$$\mathrm{L}\frac{\mathrm{d}^2\mathrm{q}}{\mathrm{dt}^2} + \frac{\mathrm{q}}{\mathrm{C}} = 0 \text{ [5 pts]}$$

From initial conditions,

$$\mathrm{q(t)} = q_0 \cos \omega t \;\; [3\, pts]$$

$$\mathrm{I(t)} = -\omega \mathrm{q}_0 \sin \omega t \;\; \text{[3 pts]}$$

with $\omega = 1/\sqrt{\mathrm{LC}}$.

(Advanced Problems)

Q1) How much magnetic field (i.e. B-field) is generated in solenoid? (Suppose that solenoid is infinite and current is I). Let the current generation be AC in other words, $I(t) = I_0 \sin(\omega t)$. [1 pt]
Q2) How much electric field is generated on metal surface? (let area of solenoid be πR^2). Induced electric field accelerate electron in metal surface. Of course, it scatters with other electrons. But in here ignore about scattering. [4 pts]

Q3) Let one electron on surface in Electric field in above. Find out velocity of electron and what is relationship between velocity and B-field in solenoid. (First you have to find out angle with respect to time, and solve the newton equation using $\mathrm{F} = mR\frac{d^2\varphi}{dt^2}\hat{\varphi}$. This system does not change radius R, only depends on angle) [10 pts]

Q4) What happens when an electron with velocity $v(t)$ moves in resistance? What are the home appliances based on this principle? (No formula is needed for Q4.) [5 pts]

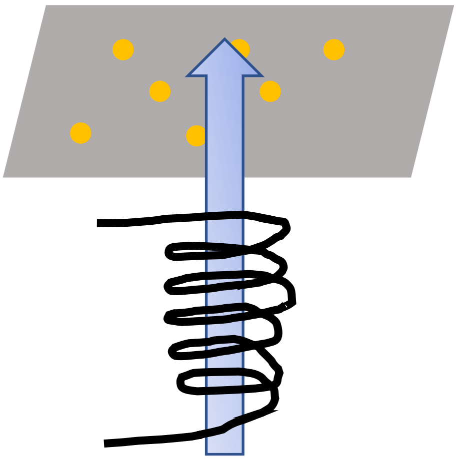


Fig. The schematic figure of system. Black is solenoid with length l and number of turns N. Blue arrow is B-field generated by solenoid. Orange dot is electron and gray surface is metal surface.

$$B = \frac{\mu_0 N}{l} I$$

Q1) 유도상관없이 답만 맞으면 1점

$$\oint \vec{E} \cdot dl = -\frac{d\phi_B}{dt} = -\pi R^2 \frac{\mu_0 N}{l} \omega I_0 \cos(\omega t)$$

$$\vec{E} = -R \frac{\mu_0 N}{2l} \omega I_0 \cos(\omega t)\, \hat{\varphi}$$

Q2) Faraday법칙을 쓰면 1점, E-field까지 맞추면 총 4점. (답 만 기입 0점)

그래서 총 4점

$$\vec{F} = \frac{R\mu_0 N}{2l}|e|\omega I_0 \cos(\omega t)\,\hat{\varphi}$$

$$mR\frac{d^2\varphi}{dt^2} = \frac{R\mu_0 N}{2l}|e|\omega I_0 \cos(\omega t)$$

$$\frac{d^2\varphi}{dt^2} = \frac{\mu_0 N}{2ml}|e|\omega I_0 \cos(\omega t)$$

$$\varphi(t) = -\frac{\mu_0 N}{2ml\omega}|e|I_0 \cos(\omega t)$$

$$v(t) = \frac{R\mu_0 N}{2ml}|e|I_0 \sin(\omega t) = \frac{R|e|}{2m}B(t)$$

angle solution 찾으면 5점, 속도 찾으면 3점, linear relationship 언급하면 2점

그래서 총 10점

making a Joule heating

열을 생성한다 등등의 유사 말도 전부 정답 취급 Joule heating에 5점. 보너스 문제라서 위에 다 틀려도 이것만 적어도 5점 제공.

추가적으로 달려있는 pdf로 채점

Prob) Q1) How much magnetic field (i.e. B-field) is generated in solenoid? (Suppose that solenoid is infinite and current is I). Let the current generation be AC in other words, $I(t) = I_0\sin(\omega t)$. (1pt) Q2) How much electric field is generated on metal surface? (let area of solenoid be πR^2). Induced electric field accelerate electron in metal surface. Of course, it scatters with other electrons. But in here ignore about scattering. (4pt) Q3) Let one electron on surface in Electric field in above. Find out velocity of electron and what is relationship between velocity and B-field in solenoid. (First you have to find out angle with respect to time, and solve the newton equation using $\mathrm{F} = mR\frac{d^2\varphi}{dt^2}\hat{\varphi}$. This system does not change radius R, only depends on angle) (10pt) Q4) What happens when an electron with velocity $v(t)$ moves in resistance? What are the home appliances based on this principle? (No formula is needed for Q4.) (5pt)

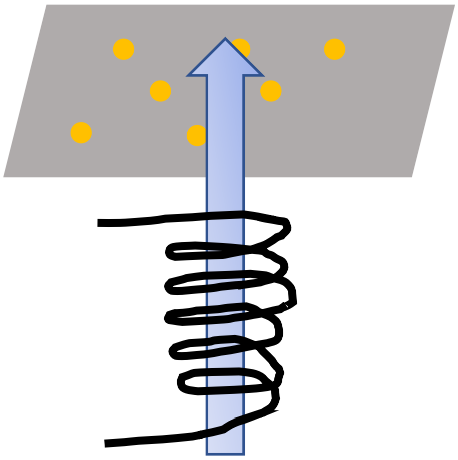


Fig. The schematic figure of system. Black is solenoid with length l and number of turns N. Blue arrow is B-field generated by solenoid. Orange dot is electron and gray surface is metal surface.

Sol)

$$B = \frac{\mu_0 N}{l} I$$

Q1) 유도상관없이 답만 맞으면 1점

1. $r > R$

$$\oint \vec{E}\cdot dl = E(2\pi r) = -\frac{d\phi_B}{dt} = -\pi R^2 \frac{\mu_0 N}{l}\omega I_0\cos(\omega t)$$

$$\vec{E} = -\frac{R^2}{2r}\frac{\mu_0 N}{2l}\omega I_0\cos(\omega t)\,\hat{\varphi}$$

2. $r < R$

$$\oint \vec{E}\cdot dl = E(2\pi r) = -\frac{d\phi_B}{dt} = -\pi r^2 \frac{\mu_0 N}{l}\omega I_0\cos(\omega t)$$

$$\vec{E} = -\frac{r}{2}\frac{\mu_0 N}{2l}\omega I_0 \cos(\omega t)\,\hat{\varphi}$$

Electric field at r=R,

$$\vec{E} = -\frac{R}{2}\frac{\mu_0 N}{2l}\omega I_0 \cos(\omega t)\,\hat{\varphi}$$

Q2) Faraday법칙 언급하면($\oint \vec{E}\cdot dl = -\frac{d\phi_B}{dt}$) 1점, E-field까지 맞추면 총 4점.(답 만 기입 0점) 그래서 총 4점

We are only interested in r=R

$$\vec{E} = -\frac{R}{2}\frac{\mu_0 N}{2l}\omega I_0 \cos(\omega t)\,\hat{\varphi}$$

$$\vec{F} = \frac{R\mu_0 N}{2l}|e|\omega I_0 \cos(\omega t)\,\hat{\varphi}$$

$$\varphi(t) = -\frac{\mu_0 N}{2ml\omega}|e| I_0 \cos(\omega t)$$

$$v(t) = \frac{R\mu_0 N}{2ml}|e| I_0 \sin(\omega t) = \frac{R|e|}{2m}B(t)$$

Q3) r=R에서 angle solution 찾으면 5점, 속도 찾으면 3점, linear relationship 언급하면 2점 그래서 총 10점

making a Joule heating

Q4) 열을 생성한다 등등의 유사 말도 전부 정답 취급 Joule heating에 5점. 보너스 문제라서 위에 다 틀려도 이것만 적어도 5점 제공.