

1. **(30 points) Basic Quantities:** A constant current of 3 A for 4 hours is required to charge an automobile battery. If the terminal voltage is $10 + \frac{t}{2}$ V, where t is in hours,

(a) (10 points) how much charge is transported as a result of the charging?

Let $T = 4\text{h} = 4 \times 3600 = 14,400$ (+1pts)

$$\begin{aligned} Q &= \int_0^T i dt \quad (+5\text{pts}) \\ &= \int_0^{14400} 3 dt = 3 \times 14400 = 43.2kC \quad (+4\text{pts}) \end{aligned}$$

If the answer is wrong, 2pts will be deducted.

(b) (10 points) how much energy is expended?

$$\begin{aligned} W &= \int pdt = \int_0^T vidt \quad (+5\text{pts}) \\ &= \int_0^{14400} 3 \times \left(10 + \frac{0.5t}{3600}\right) dt = 3 \times \left(10t + \frac{0.25t^2}{3600}\right) \Big|_0^{14400} \\ &= 3 \times (10 \times 14400 + 0.25 \times 4 \times 14400) = 475.2\text{kJ} \quad (+5\text{pts}) \end{aligned}$$

If the answer is wrong, 3pts will be deducted.

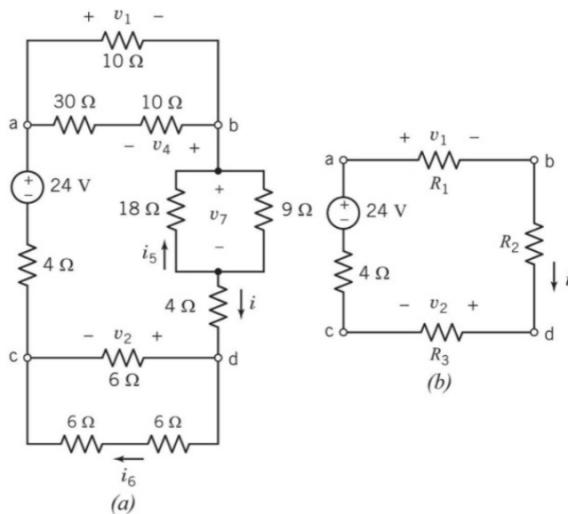
(c) (10 points) how much does the charging cost? Assume that electricity costs 9 cents/kWh.

$$W = 475.2\text{kJ} = 475.2\text{kWs} \quad (1\text{W} = 1\text{J/s}) \quad (+2\text{pts})$$

$$\text{cost} = \frac{475.2}{3600} \text{kWh} \times 9\text{cents} = 1.188\text{cents} \quad (+8\text{pts})$$

If the answer is wrong, 4pts will be deducted.

2. (30 points) **Resistive Circuits:** The circuit shown in figure (b) has been obtained from the circuit shown in figure (a) by replacing series and parallel combinations of resistances by equivalent resistances.



- (a) (10 points) Determine the values of the resistances R_1 , R_2 , and R_3 in figure (b) so that the circuit shown in figure (b) is equivalent to the circuit shown in figure (a).

$$R_1 = 4 \parallel 10 = 8$$

$$R_2 = 18 \parallel 9 + 4 = 10 \quad \dots 3 \text{ points each}$$

$$R_3 = 12 \parallel 6 = 4 \quad \dots \text{full points when all answers is right.}$$

- (b) (10 points) Determine the values of v_1 , v_2 , and i in figure (b).

$$V_1 = 24 \times \frac{R_1}{4 + R_1 + R_2 + R_3} = 24 \times \frac{8}{26} \approx 7.4$$

$$V_2 = 24 \times \frac{R_3}{4 + R_1 + R_2 + R_3} = 24 \times \frac{4}{26} \approx 3.7$$

$$i = \frac{24}{4 + R_1 + R_2 + R_3} = 24 \times \frac{1}{26} \approx 0.9$$

... 3 points each

*... 1 point deduction if sign error,
calculation error.*

... full points if all answer is right

- (c) (10 points) Determine the values of v_4 , i_5 , i_6 , and v_7 in figure (a).

$$V_4 = -V_1 \times \frac{10}{10 + 30} \approx -1.9$$

... 2.5 points each

$$i_5 = -i \times \frac{9}{18 + 9} \approx -0.3$$

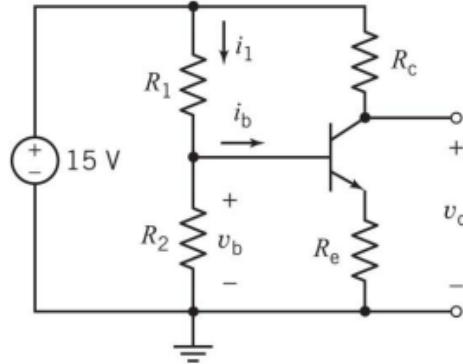
... 1 point deduction if sign error,

calculation error.

$$i_6 = i \times \frac{6}{6 + 12} \approx 0.3$$

$$V_1 = i \times 18 \parallel 9 \approx 5.4$$

3. (30 points) **Transistor Biasing:** The following figure shows a transistor amplifier. The values of R_1 and R_2 are to be selected. Resistances R_1 and R_2 are used to bias the transistor, that is, to create useful operating conditions. In this problem, we want to select R_1 and R_2 so that $v_b = 5$ V. We expect the value of i_b to be approximately $10 \mu\text{A}$. When $i_t \geq 10i_b$, it is customary to treat i_b as negligible, that is, to assume $i_b = 0$. In that case, R_1 and R_2 comprise a voltage divider.



- (a) (15 points) Determine the values of R_1 and R_2 so that $v_b = 5$ V, and the total power absorbed by R_1 and R_2 is 1.875 mW.

To insure that i_b is negligible we require

$$i_1 = \frac{15}{R_1 + R_2} \geq 10(10 \times 10^{-6})$$

so

$$R_1 + R_2 \leq 150 \text{ k}\Omega \quad (+2)$$

Next to cause $v_b = 5$ V we require

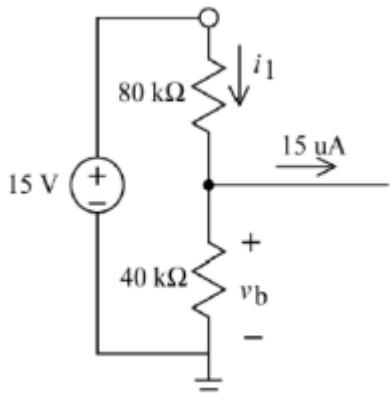
$$5 = v_b = \frac{R_2}{R_1 + R_2} 15 \quad \Rightarrow \quad R_1 = 2R_2 \quad (+3)$$

Total power absorbed by R_1 and R_2 is 1.875 mW,

$$\text{so} \quad 1.875 \text{ mW} = \frac{10^2}{R_1} + \frac{5^2}{R_2} = \frac{150}{2R_2} \quad (+5)$$

$$\rightarrow R_2 = 40k \text{ , } R_1 = 80k \quad (+5)$$

- (b) (15 points)** An inferior transistor could cause i_b to be larger than expected. Using the values of R_1 and R_2 obtained from (a), determine the value of v_b that would result from $i_b = 15 \mu\text{A}$.



$$\text{KVL gives } (80 \times 10^3) i_1 + v_b - 15 = 0 \quad (+5)$$

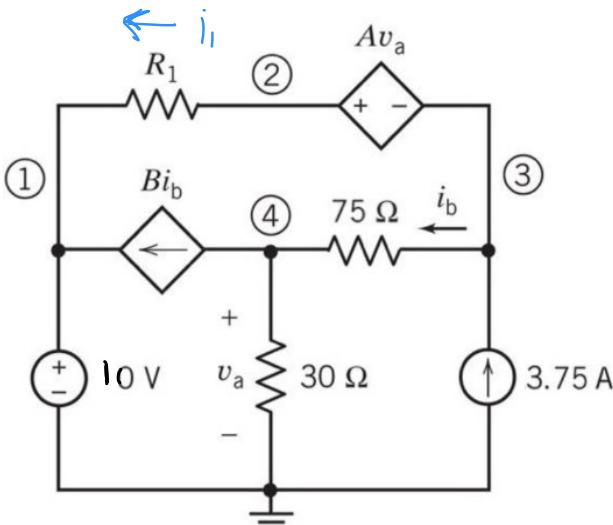
$$\text{KCL gives } i_1 = \frac{v_b}{40 \times 10^3} + 15 \times 10^{-6} \quad (+5)$$

$$\text{Therefore } (80 \times 10^3) \left(\frac{v_b}{40 \times 10^3} + 15 \times 10^{-6} \right) + v_b = 15$$

Finally

$$3v_b + 1.2 = 15 \quad \Rightarrow \quad v_b = \frac{13.8}{3} = 4.6 \text{ V} \quad (+5)$$

4. (30 points) Nodal Analysis: The voltages v_1 , v_2 , v_3 , and v_4 in the figure shown below are the node voltages corresponding to nodes 1, 2, 3, and 4. The values of these voltages are $v_1 = 10 \text{ V}$, $v_2 = 75 \text{ V}$, $v_3 = -15 \text{ V}$, and $v_4 = 22.5 \text{ V}$. Determine the values of the gains of the dependent sources, A and B , and the value of the resistance R_1 .



$$v_2 - v_3 = Av_a \quad 2$$

$$A = \frac{75 - (-15)}{22.5} = 4 \quad (v/v) \quad 4 \quad 5 \quad 1$$

$$v_a = 20.5 \text{ V}, \quad i_b = \frac{v_3 - v_4}{75} = \frac{-15 - 22.5}{75} = -0.5 \text{ A}$$

KCL in node 4

$$Bi_b - i_b + \frac{v_a}{30} = -0.5(B-1) + \frac{22.5}{30} = 0 \quad 7$$

$$B = 2.5(A/A) \quad 2$$

KCL in node 3

$$i_1 + i_b = 3.75 \rightarrow i_1 = 3.75 - (-0.5) = 4.25 \text{ (A)} \quad 7$$

$$R_1 = \frac{v_2 - v_1}{i_1} = \frac{75 - 10}{4.25} = 65 \times \frac{4}{17} = \frac{260}{17} \quad 2$$

$$= 15.3 \text{ (Ω)}$$

$$A : \text{식(4), 답(2)} \Rightarrow 6\text{점}$$

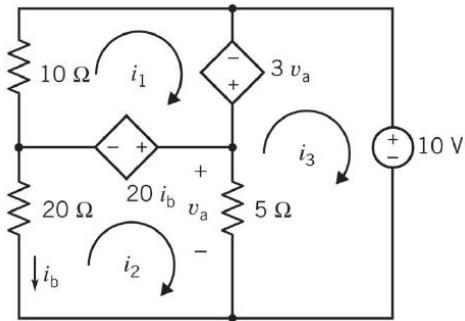
$$B : \text{식(7), 답(2)} \Rightarrow 9\text{점}$$

$$i_b : \underbrace{\text{식(5), 답(1)}}_{\text{안 적어도 } B, R \text{ 식 정답이면 정답}} \Rightarrow 6\text{점}$$

$$R : \text{식(7), 답(2)} \Rightarrow 9\text{점}$$

안 적어도 B, R 식 정답이면 정답

5. (30 Points) Loop Analysis: The currents i_1 , i_2 and i_3 are the mesh currents of the circuit shown below. Determine the values of i_1 , i_2 and i_3 .



Express v_a and i_b , the controlling voltage and current of the dependent sources, in terms of the mesh currents (**+3 Points each**)

$$v_a = 5(i_2 - i_3)$$

$$i_b = -i_2$$

Apply KVL to the meshes (**+5 Points each**)

$$-3v_a + 20i_b + 10i_1 = 0$$

$$-20i_b + v_a + 20i_2 = 0$$

$$10 - v_a + 3v_a = 0$$

These equations can be written as

$$-15(i_2 - i_3) + (-20i_2) + 10i_1 = 0$$

$$-(-20i_2) + 5(i_2 - i_3) + 20i_2 = 0$$

$$10 - 5(i_2 - i_3) + 15(i_2 - i_3) = 0$$

That is, (**+2 Points each**)

$$2i_1 - 7i_2 + 3i_3 = 0$$

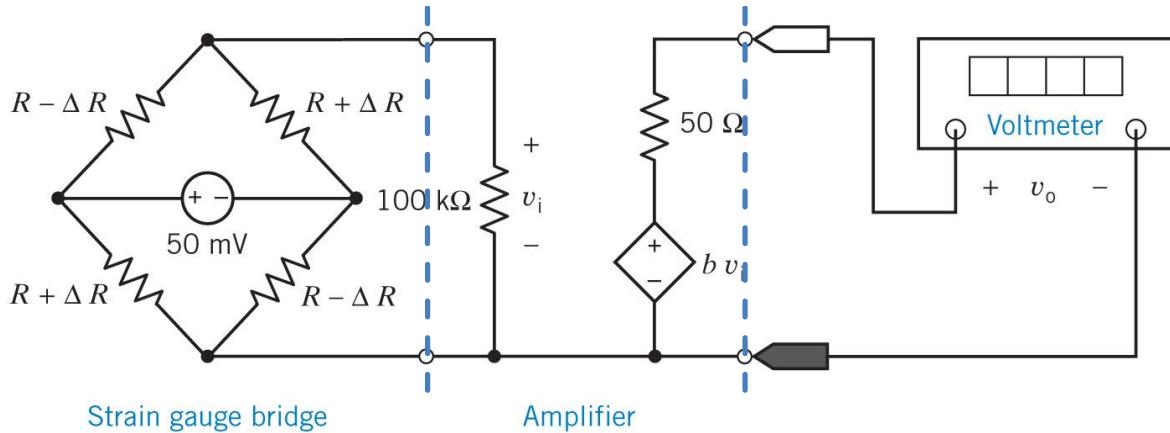
$$9i_2 - i_3 = 0$$

$$i_2 - i_3 = -1$$

Therefore, (**+1 Points each**)

$$i_1 = -1.25 \text{ A}, i_2 = 0.125 \text{ A}, \text{ and } i_3 = 1.125 \text{ A}.$$

6. (45 points) **Strain Gauge Bridge:** Strain gauges are transducers that measure mechanical strain. Electrically, the strain gauges are resistors. The strain causes a change in resistance that is proportional to the strain. The following figure shows four strain gauges connected in a configuration called a bridge.

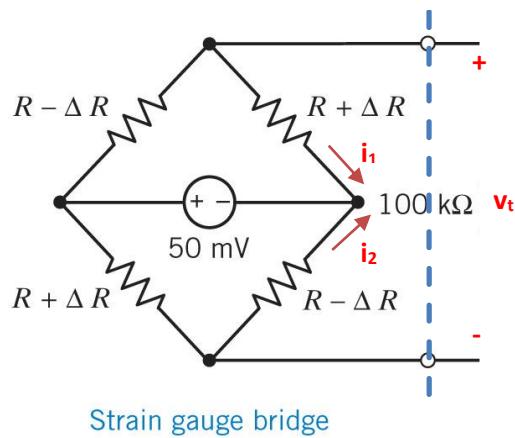


The bridge output is usually a small voltage. In this figure, an amplifier multiplies the bridge output, v_i , by a gain to obtain a larger voltage, v_o , which is displayed by the voltmeter.

A strain gauge is used to measure force. The strain gauges have been positioned so that the force will increase the resistance of two of the strain gauges while, at the same time, decreasing the resistance of the other two strain gauges.

The strain gauges used in the bridge have nominal resistances of $R = 120 \Omega$. (The nominal resistance is the resistance when the strain is zero.) This resistance is expected to increase or decrease by no more than 2Ω due to strain. This means that $-2 \Omega \leq \Delta R \leq +2 \Omega$. The output voltage v_o is required to vary from -10 V to $+10 \text{ V}$ as ΔR varies from -2Ω to $+2 \Omega$.

Obtain the Thevenin equivalent of the strain gauge bridge, and by using it, determine the amplifier gain b needed to cause v_o to be related to ΔR by $v_o = 5 \times \Delta R$. (Note that the voltmeter is ideal, and this it draws no current.)



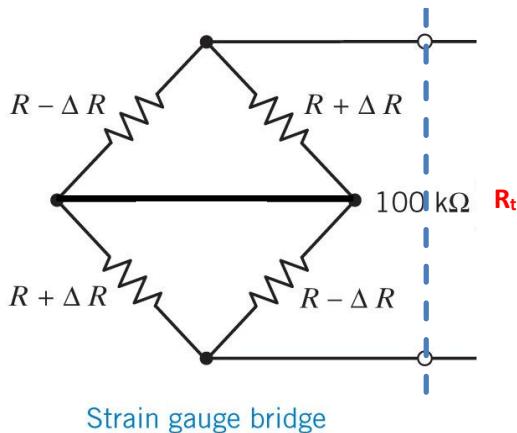
$$i_1 = \frac{50mV}{(R - \Delta R) + (R + \Delta R)} = \frac{50mV}{2R} \quad 5\text{pts}$$

$$i_2 = \frac{50mV}{(R - \Delta R) + (R + \Delta R)} = \frac{50mV}{2R} \quad 5\text{pts}$$

$$v_t = (R + \Delta R)i_1 - (R - \Delta R)i_2$$

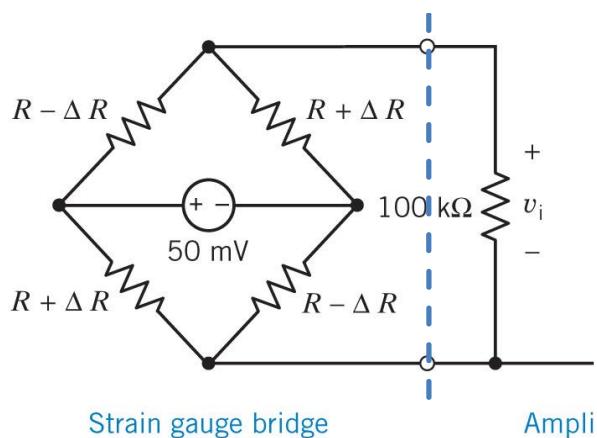
$$= (2\Delta R) \frac{50mV}{2R}$$

$$= \left(\frac{\Delta R}{R}\right) 50mV = (0.4167 \times 10^{-3})\Delta R \quad 5\text{pts}$$



$$R_t = \frac{(R - \Delta R)(R + \Delta R)}{(R - \Delta R) + (R + \Delta R)} + \frac{(R + \Delta R)(R - \Delta R)}{(R + \Delta R) + (R - \Delta R)}$$

$$= 2 \frac{R^2 - \Delta R^2}{2R} \approx R \quad 10\text{pt}$$



$$v_i = \frac{100k\Omega}{100k\Omega + R_t} v_t = 0.9988 v_t$$

$$= (0.4162 \times 10^{-3})\Delta R \quad 10\text{pt}$$

$$v_o = b v_i = b(0.4162 \times 10^{-3})\Delta R \quad 5\text{pts}$$

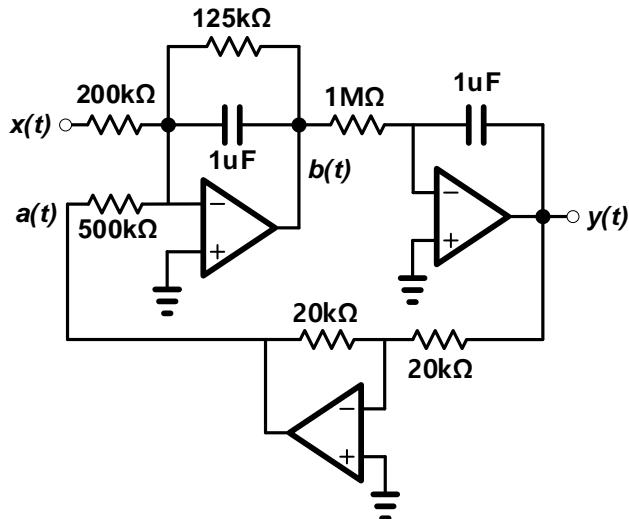
$$b(0.4162 \times 10^{-3})\Delta R = 5$$

$$b = 12,013 \quad 5\text{pts}$$

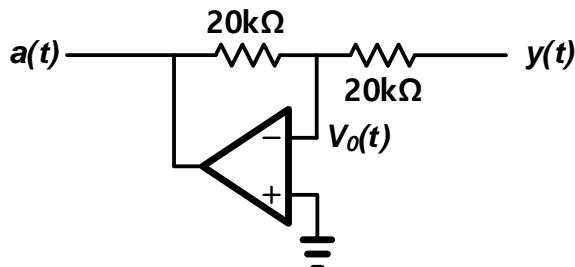
$$v_o = 4.9998 \Delta R$$

Problem 7 - 45points

$$a \frac{d^2}{dt^2} y(t) + b \frac{d}{dt} y(t) + c y(t) = dx(t)$$



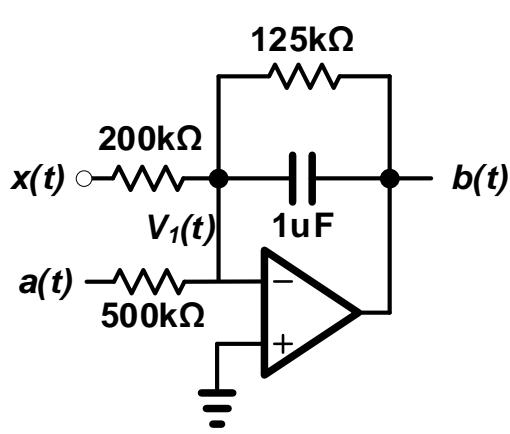
Step 1. – 10 pts (equation (5) + answer(5))



$$\frac{y(t) - V_0(t)}{20k} = \frac{V_0(t) - a(t)}{20k} \quad [+5]$$

$$V_0(t) = 0, \quad a(t) = -y(t) \quad [+5]$$

Step 2. – 15 pts (equation (10) + answer(5))



$$\frac{x(t) - V_1(t)}{200k} + \frac{a(t) - V_1(t)}{500k} =$$

$$\frac{V_1(t) - b(t)}{125k} + 1u \times \left(\frac{d}{dt}\right)(V_1(t) - b(t))$$

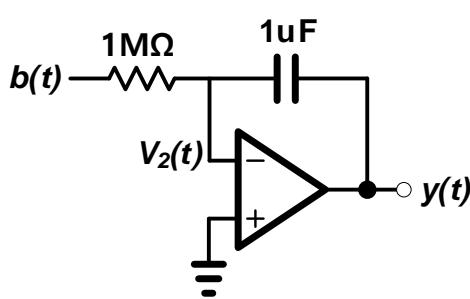
$$V_1(t) = 0$$

$$\frac{x(t)}{200k} + \frac{a(t)}{500k} =$$

$$-\frac{b(t)}{125k} + 1u \times \frac{d}{dt}(-b(t)) \quad [+5]$$

$$\text{By, step1, } \frac{x(t)}{200k} - \frac{y(t)}{500k} = -\frac{b(t)}{125k} + 1u \times \frac{d}{dt}(-b(t))$$

Step 3. – 15 pts (equation (10) + answer(5))



$$\frac{b(t) - V_2(t)}{1M} = 1u \times \frac{d}{dt} (V_2(t) - y(t)) \quad [+10]$$

$$V_2(t) = 0, \quad \frac{b(t)}{1M} = 1u \times \frac{d}{dt} (-y(t))$$

$$b(t) = \frac{d}{dt} (-y(t)) \quad [+5]$$

By step2 & step3,

$$\frac{x(t)}{200k} - \frac{y(t)}{500k} = -\frac{1}{125k} \cdot \frac{d}{dt} (-y(t)) + 1u \times \frac{d^2}{dt^2} (y(t))$$

Multiply 1M and arrange the equation

$$\frac{d^2}{dt^2} (y(t)) + 8 \frac{d}{dt} (y(t)) + 2y(t) = 5x(t)$$

So,

Answer) $a : b : c : d = 1 : 8 : 2 : 5 \quad [+5]$

If the ratio of 'a, b, c, and, d' is correct, it will consider as right answer.

Any calculation error from equation to answer 1 point will be deducted