1. **(10 points)** A small, experimental space rocket uses a two-element circuit, as shown below, to control a jet valve from point of liftoff at $t = 0$ until expiration of the rocket after one minute. The energy that must be supplied to Element 2 (= jet valve controller) by Element 1 (= battery) for the one-minute period is 40 mJ.

It is known that $i(t) = De^{-t/60}$ mA for $t \geq 0$, and the voltage across Element 2 is $v(t) = Be^{-t/60}$ V for $t \geq 0$. The maximum magnitude of the current, D, is limited to 1 mA. Design the circuit by determining the values of the constants D and B, and describe the required battery voltage.

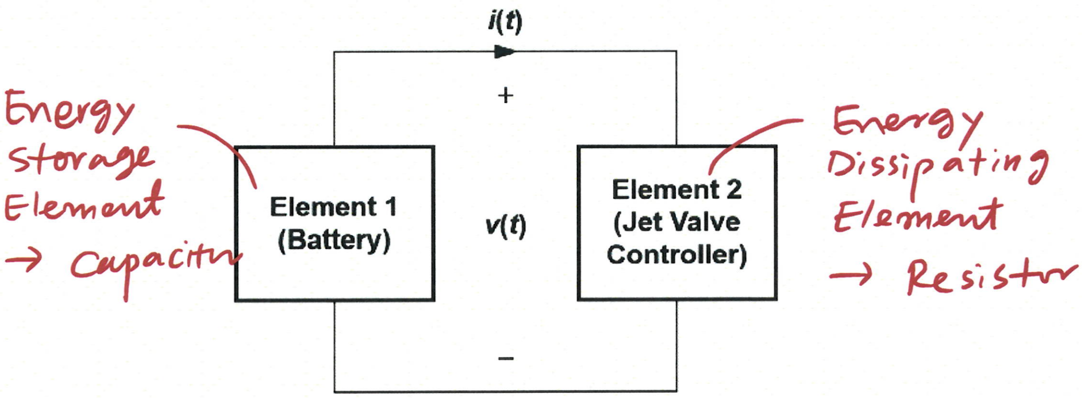
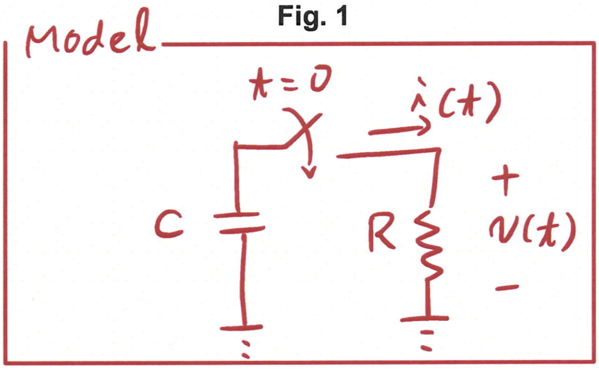


Fig. 1



$$p(t) = v(t)i(t) = DBe^{-t/30} \text{ mW}$$

$$w(t) = \int_0^t p(t)\,dt \rightarrow W = \int_0^{60} DBe^{-t/30} \times 10^{-3}\,dt$$

$$= -30DB \times 10^{-3} \times (e^{-2} - 1)$$

$$= 25.9\,DB \times 10^{-3} = 40 \text{ mJ}$$

$$\rightarrow DB = 1.54$$

$D \leq 1 \Rightarrow B \geq 1.54$ V.

We can choose the voltage of the battery higher than 1.54 V.

2. **(10 points)** Three light bulbs are connected to a 9-V battery as shown below.

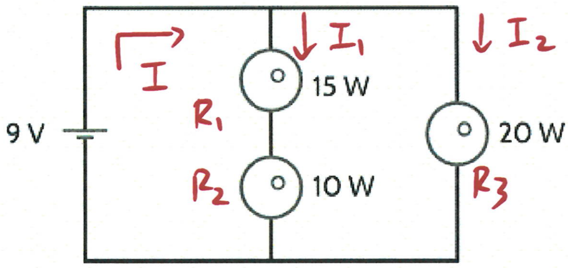


Fig. 2

(a) (4 points) Calculate the total current supplied by the battery.

power delivered by 9-V battery $= 9 \times I$.

power dissipated by bulbs $= 15 + 10 + 20$ [W]

$\Rightarrow \quad 9 \times I = 15 + 10 + 20 = 45$

$\therefore \quad I = 5A$.

(b) (3 points) Calculate the current through each bulb.

$I_1^2 (R_1 + R_2) = 15 + 10 = 25\,W$

$I_1 (R_1 + R_2) = 9\,V$

$\Rightarrow \quad I_1 = \frac{25}{9}\,A$.

$I_2^2 R_3 = 20W$

$I_2 R_3 = 9V$

$\Rightarrow \quad I_2 = \frac{20}{9}\,A$.

(c) (3 points) Calculate the resistance of each bulb.

$I_1^2 R_1 = 15W$

$\Rightarrow \quad R_1 = \frac{15}{(25/9)^2} = 1.944\Omega$

$I_1^2 R_2 = 10W$

$\Rightarrow \quad R_2 = \frac{10}{(25/9)^2} = 1.296\Omega$

$I_2^2 R_3 = 20W$

$\Rightarrow \quad R_3 = \frac{20}{(20/9)^2} = 4.05\Omega$

3. **(10 points)** Use nodal analysis to find the node voltages, V_1, V_2, V_3 and V_4, which are defined in the circuit shown below.

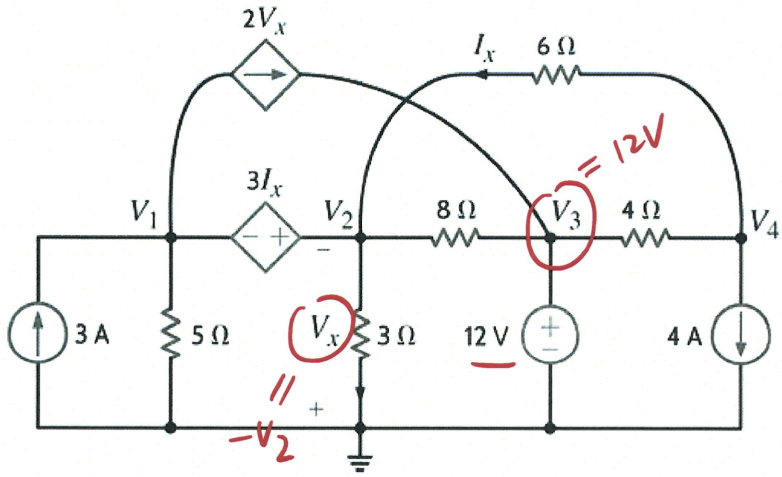


Fig. 3

①

$$V_2 - V_1 = 3I_x = 3\cdot\frac{V_4-V_2}{6} = \frac{1}{2}(V_4-V_2)$$

$$\rightarrow V_1 - \frac{3}{2}V_2 + \frac{1}{2}V_4 = 0$$

②

$$4 + \frac{V_4-V_2}{6} + \frac{V_4-V_3}{4} = 0$$

$$\rightarrow -\frac{1}{6}V_2 - \frac{1}{4}V_3 + \frac{5}{12}V_4 = -4 \rightarrow -\frac{1}{6}V_2 + \frac{5}{12}V_4 = -1$$

③

$$3 = \frac{1}{5}V_1 + 2V_x + \frac{1}{3}V_2 + \frac{V_2-V_3}{8} + \frac{V_2-V_4}{6}$$

$$\rightarrow \frac{1}{5}V_1 - \frac{33}{24}V_2 - \frac{1}{8}V_3 - \frac{1}{6}V_4 = 3$$

$$\rightarrow \frac{1}{5}V_1 - \frac{33}{24}V_2 - \frac{1}{6}V_4 = \frac{9}{2}$$

$$\Rightarrow \boxed{\begin{aligned} V_1 &= -3.05V \\ V_2 &= -3.27V \\ V_3 &= 12V \\ V_4 &= -3.71V \end{aligned}}$$

4. **(10 points)** Use loop analysis to find the power delivered by the independent 3-V source in the network shown below.

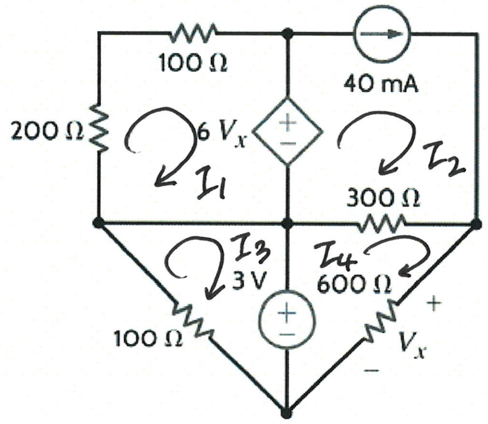


Fig. 4

$I_2 = 40mA$

KVL for loop 3:

$100 I_3 + 3 = 0 \longrightarrow I_3 = -30mA$ //

KVL for loop 4:

$300(I_4 - I_2) + 600 I_4 - 3 = 0$

$\rightarrow 900 I_4 = 3 + 300 I_2 = 3 + 300 \times 40m = 15$

$\rightarrow I_4 = 15/900 = 16.67mA$

Power delivered by the independent 3-V source

$P_{3V} = 3 \times (I_4 - I_3) = 3 \times (16.67m + 30m) = 140\ mW$ //

5. **(15 points)** There is an unknown linear network.

(a) (5 points) To characterize this network, we connect a test voltage source having the value of V_T and measure the current I_T flowing out of the network as shown below. When $V_T = 2$ V, I_T is measured to be 1.5 A. If V_T is changed to 4 V, I_T becomes 0.5 A. Derive the Thevenin equivalent circuit of the linear network.

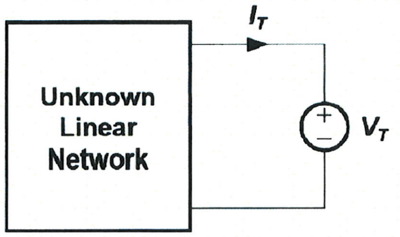
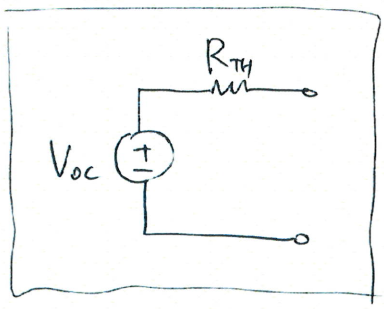


Fig. 5(a)



$$V_{oc} - 2V = R_{TH} \cdot \left(\frac{3}{2}\right)$$

$$V_{oc} - 4V = R_{TH} \cdot \left(\frac{1}{2}\right)$$

$$\therefore V_{oc} = 5V, \quad R_{TH} = 2\Omega$$

(b) (5 points) When a load resistance R_L is connected to the network characterized in (a) as shown below, write the expressions for V_{out}, I, P_{in} (= $I \times V_{in}$), P_{out} (= $I \times V_{out}$), and efficiency (= P_{out} / P_{in}) as a function of R_L.

$$P = I^2 \cdot R = \frac{V^2}{R}$$

$$V_{in} = I \cdot (R_{in} + R_L)$$

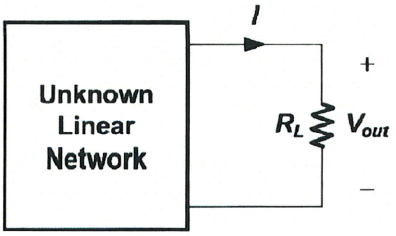
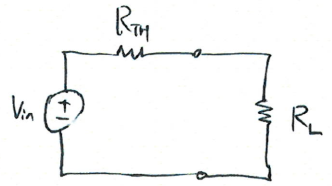


Fig. 5(b)

$\rightarrow$



$V_{in} = 5V$

$R_{TH} = 2\Omega$

i) $I = \dfrac{V_{in}}{R_{in} + R_L} = \dfrac{5}{R_L + 2}$

ii) $V_{out} = I \cdot R_L = \dfrac{5 \cdot R_L}{R_L + 2}$

iii) $P_{in} = I \cdot V_{in} = \dfrac{25}{R_L + 2}$

iv) $P_{out} = I \cdot V_{out} = \dfrac{25 \cdot R_L}{(R_L + 2)^2}$

V) $PE = \dfrac{P_{out}}{P_{in}} = \dfrac{R_L}{2 + R_L}$

(c) (5 points) Find the value of R_L when the maximum power is transferred to R_L. How much is that maximum power? How high is the efficiency when the maximum power is transferred?

$$\frac{\partial P_{out}}{\partial R_L} = \frac{25}{(R_L+2)^2} - \frac{50 \cdot R_L}{(R_L+2)^3} = 0 \rightarrow R_L = 2\Omega.$$

$\therefore$ $PE = 50\%$, $P_{out} = \dfrac{25}{8}$ Watt $= 3.125$ Watt.

6. **(15 points)** The input to the circuit in Fig. 5(a) is the voltage v_s. The output is the voltage v_o. The voltage v_b is used to adjust the relationship between the input and output. Design the circuit so that its input and output have the relationship specified by the graph shown in Fig. 5(b), by determining the values of R_1, R_2, R_3, R_4, R_5 and v_b appropriately.

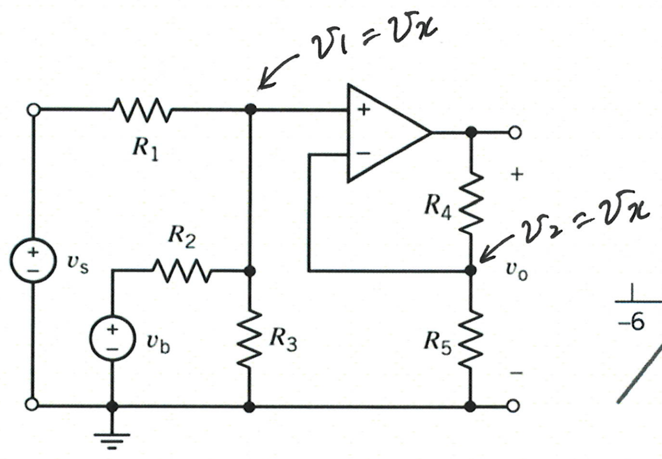


Fig. 5(a)

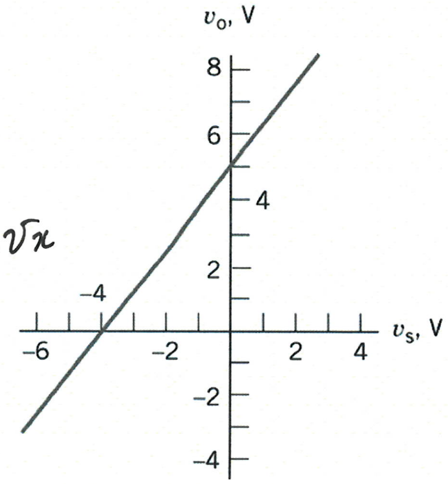


Fig. 5(b)

In Fig. 5(a), $v_1 = v_2 = v_x$ //

At node 1: $\dfrac{v_x - v_s}{R_1} + \dfrac{v_x - v_b}{R_2} + \dfrac{v_x}{R_3} = 0$

$$\rightarrow \frac{v_s}{R_1} = \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right) v_x - \frac{v_b}{R_2} \Rightarrow ①$$

At node 2: $v_x = \dfrac{R_5}{R_4 + R_5} v_o \rightarrow ②$

① & ②: $\dfrac{v_s}{R_1} = \left(\dfrac{R_1R_2 + R_2R_3 + R_3R_1}{R_1R_2R_3} \times \dfrac{R_5}{R_4+R_5}\right) v_o - \dfrac{v_b}{R_2}$

$$\Rightarrow v_o = \left(\frac{R_2R_3}{R_1R_2 + R_2R_3 + R_3R_1} \times \frac{R_4+R_5}{R_5}\right) v_s + \left(\frac{R_1R_3}{R_1R_2 + R_2R_3 + R_3R_1} \times \frac{R_4+R_5}{R_5}\right) v_b \rightarrow ③$$

From Fig. 5(b), $v_o = \dfrac{5}{4} v_s + 5 \rightarrow ④$

③ & ④: $\dfrac{R_2R_3}{R_1R_2 + R_2R_3 + R_3R_1} \times \dfrac{R_4+R_5}{R_5} = \dfrac{5}{4} \rightarrow ⑤$

$\dfrac{R_1R_3}{R_1R_2 + R_2R_3 + R_3R_1} \times \dfrac{R_4+R_5}{R_5} \times v_b = 5 \rightarrow ⑥$

Let $R_1 = R_2 = R_3 = 10k\Omega$ and $R_5 = 40k\Omega \rightarrow R_4 = 110k\Omega$, $v_b = 4V$ //

7. **(15 points)** Fig. 7(a) shows the op-amp with an input offset, which is modeled by the ideal op-amp having an offset voltage source connected to one of its two inputs. Assume that V_{offset} is 10 mV.

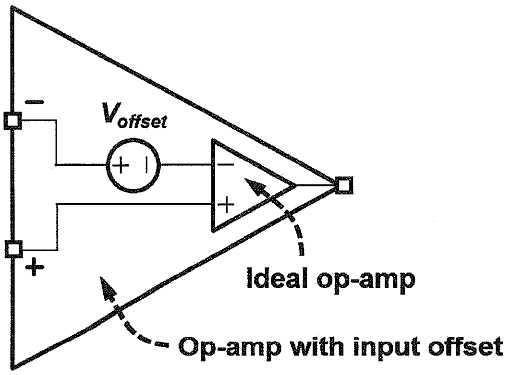


Fig. 7(a)

(a) (5 points) Using this op-amp with the input offset, a unity gain buffer is configured as shown below. If the input voltage V_{in} of 100 mV is applied, what is the value of the output voltage V_{out}?

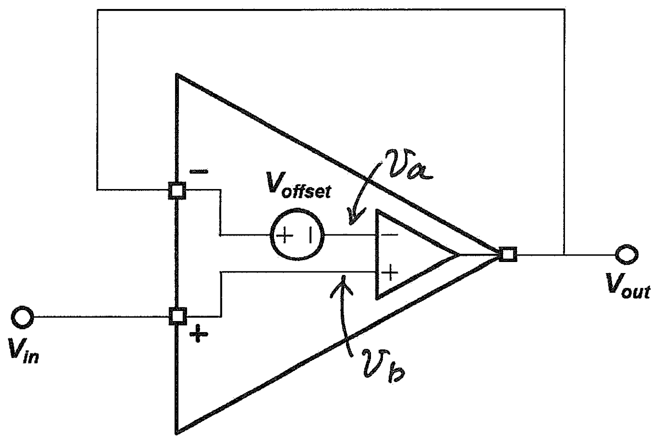


Fig. 7(b)

$$V_a = V_{out} - V_{offset}$$

$$V_b = V_{in}$$

$$V_a = V_b \longrightarrow V_{out} = V_{in} + V_{offset}$$

$$= 100m + 10m = 110mV_{//}$$

(b) (5 points) The unity gain buffer circuit is modified by adding some switches and a capacitor as shown below. In the beginning of circuit operation, SW_1 and SW_4 are open and SW_2 and SW_3 are closed. In this configuration, find the voltage V_C stored at the capacitor?

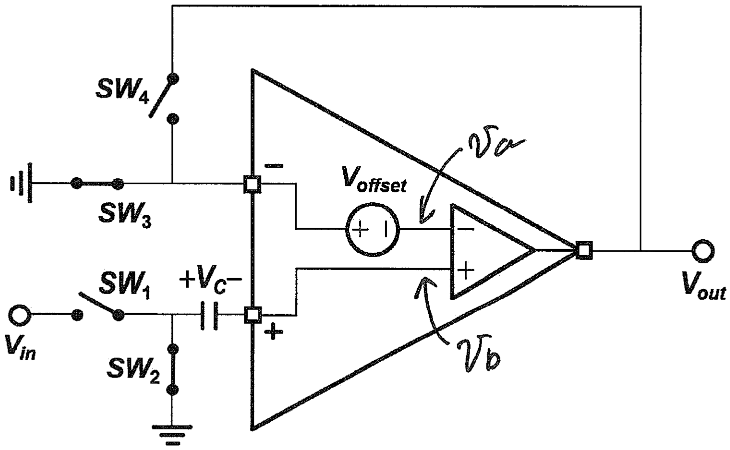


Fig. 7(c)

$V_a = 0 - V_{offset}$

$V_b = 0 - V_c$

$V_a = V_b \longrightarrow V_c = V_{offset} = 10mV$

(c) (5 points) Now the configuration of switches is changed. SW1 and SW4 are closed and SW2 and SW3 are open. With the stored voltage V_C maintained, if the input voltage V_{in} of 100 mV is applied, what is the value of the output voltage V_{out}?

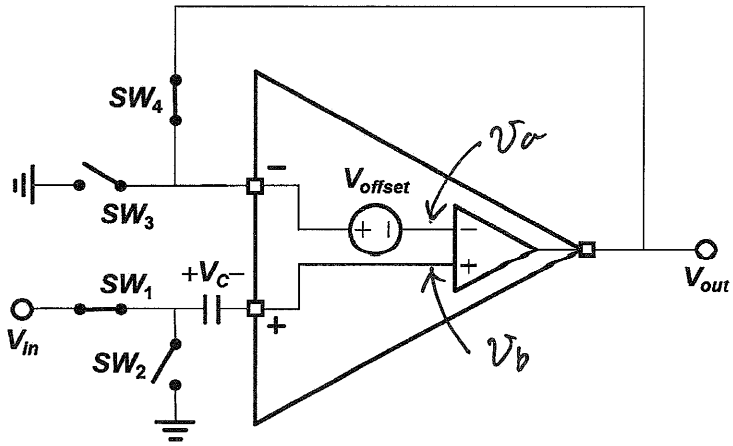


Fig. 7(d)

$V_a = V_{out} - V_{offset}$

$V_b = V_{in} - V_c$

$V_a = V_b \longrightarrow V_{out} = V_{in} - V_c + V_{offset}$

$= 100m - 10m + 10m = 100mV$

8. **(15 points)** A network of pre-charged capacitors is given as shown below.

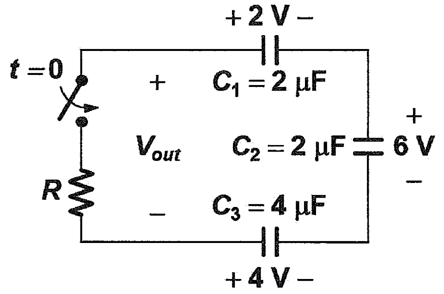


Fig. 8

(a) (5 points) Before $t = 0$, what is the value of V_{out}? How much energy is initially stored in the network?

$V_{out}(0-) = V_{C1}(0-) + V_{C2}(0-) + V_{C3}(0-) = 2 + 6 - 4 = 4V$

Initially stored energy:

$E(0-) = \frac{1}{2} C_1 V_{C1}(0-)^2 + \frac{1}{2} C_2 V_{C2}(0-)^2 + \frac{1}{2} C_3 V_{C3}(0-)^2$

$= \frac{1}{2} \times 2\mu \times 2^2 + \frac{1}{2} \times 2\mu \times 6^2 + \frac{1}{2} \times 4\mu \times (-4)^2 = 4\mu + 36\mu + 32\mu = 72\mu J$

(b) (5 points) At $t = 0$, the switch is closed and the network starts to discharge through R. After long enough time after the switch is closed, what value does V_{out} reach? At this steady state, find the voltage across each capacitor.

At $t = \infty$, no current flows in this network.

$\rightarrow V_{out}(\infty) = V_{C1}(\infty) + V_{C2}(\infty) + V_{C3}(\infty) = 0$ —①

Since all the capacitors are connected in series, their amount of charge change should be the same as Δq

$① \rightarrow \left(2 + \frac{\Delta q}{2\mu}\right) + \left(6 + \frac{\Delta q}{2\mu}\right) + \left(-4 + \frac{\Delta q}{4\mu}\right) = 0$

$\rightarrow \left(\frac{1}{2\mu} + \frac{1}{2\mu} + \frac{1}{4\mu}\right)\Delta q = -4 \rightarrow \Delta q = -\frac{16}{5}\mu C$

$\therefore V_{C1}(\infty) = 2 - \frac{16}{5}\mu \times \frac{1}{2\mu} = 0.4V$

$V_{C2}(\infty) = 6 - \frac{16}{5}\mu \times \frac{1}{2\mu} = 4.4V$

$V_{C3}(\infty) = -4 - \frac{16}{5}\mu \times \frac{1}{4\mu} = -4.8V$

(c) (5 points) At $t = \infty$, how much energy is left in the network? How much energy has been dissipated by discharging through R?

Total energy left in the network:

$$E(\infty) = \frac{1}{2}C_1 V_{C1}(\infty)^2 + \frac{1}{2}C_2 V_{C2}(\infty)^2 + \frac{1}{2}C_3 V_{C3}(\infty)^2$$

$$= \frac{1}{2}\times 2\mu \times (0.4)^2 + \frac{1}{2}\times 2\mu\times(4.4)^2 + \frac{1}{2}\times 4\mu\times(-4.8)^2$$

$$= 0.16\mu + 19.36\mu + 46.08\mu$$

$$= 65.6\ \mu J_{\,/\!/}$$

Energy dissipated by discharging through R:

$$E(0^-) - E(\infty) = 72\mu - 65.6\mu = 6.4\mu J_{\,/\!/}$$