EE210: Mid Term

Tuesday Oct. 21, 2014

9:00 a.m. – 11:45 a.m.

1. [10 points] Consider an infinite sequence of independent tosses of a coin that comes up heads with probability p.

   (a) (4 pts.) Let X be such that the first heads appears on the Xth toss. In other words, X is the number of tosses required to obtain a head. Compute (in terms of p) the expectation $E[X]$.

   (b) (3 pts.) Compute (in terms of p) the probability that exactly 5 of the first 10 tosses are heads.

   (c) (3 pts.) Compute (in terms of p) the probability that the 5th head appears on the 10th toss.

2. [10 points] Nightingale sends her resume to 1000 companies she finds on monster.com. Each company responds with probability 3/1000 (independently of what all the other companies do). Let R be the number of companies that respond.

   (a) (3 pts.) Compute $E[R]$.

   (b) (3 pts.) Compute Var[R].

   (c) (4 pts.) Use a Poisson random variable approximation to estimate the probability $P[R = 3]$.

3. [10 points] Compute how many:

   (a) (5 pts.) Quadruples (x_1, x_2, x_3, x_4) of non-negative integers with $x_1 + x_2 + x_3 + x_4 = 10$?

   (b) (5 pts.) Ways to divide 15 books into five groups of size 1, 2, 3, 4, and 5.

4. [10 points] A radio station gives a pair of concert tickets to the sixth caller who knows the birthday of the performer. For each person who calls, the probability is 0.75 of knowing the performer's birthday. All calls are independent.

(a) (3 pts.) What is the PMF of L, the number of calls necessary to find the winner?
(b) (3 pts.) What is the probability of finding the winner on the tenth call?
(c) (4 pts.) What is the probability that the station will need nine or more calls to find a winner?

5. [10 points] The number of buses that arrive at a bus stop in T minutes is a Poisson random variable B with expected value $T/5$.

(a) (2 pts.) What are the PMF of B, the number of buses that arrive in T minutes?
(b) (2 pts.) What is the probability that in two-minute interval, three buses will arrive?
(c) (3 pts.) What is the probability of no buses arriving in a 10-minute interval?
(d) (3 pts.) How much time should you allow so that with probability 0.99 at least one bus arrive?

6. [10 points] A source wishes to transmit data packets to a receiver over a radio link. The receiver uses error detection to identify packets that have been corrupted by radio noise. When a packet is received error-free, the receiver sends an acknowledgement (ACK) back to the source. When the receiver gets a packet with errors, a negative acknowledgement (NAK) message is sent back to the source. Each time the source receives a NAK, the packet is retransmitted. We assume that each packet transmission is independently corrupted by errors with probability q.

(a) (2 pts.) Find the PMF of X, the number of times that a packet is transmitted by the source.

(b) (3 pts.) Suppose each packet takes 1 millisecond to transmit and that the source waits an additional millisecond to receive the acknowledgement message (ACK or NAK) before transmitting. Let T equal the time required until the packet is successfully received. What is the relationship between T and X? What is the PMF of T?

(c) (5 pts.) In packet transmission, the time between successfully received packets is called the interarrival time, and the randomness in packet interarrival times is called *jitter*. In real-time packet communications, jitter is undesirable. One measure of jitter is the standard deviation of the packet interarrival time. Calculate the jitter σ_T. How large must the successful transmission probability q be to ensure that the jitter is less than 2 milliseconds?

7. [10 points] Select integrated circuits, test them in sequence until you find the first failure, and then stop. Let N be the number of tests. All tests are independent with probability of failure $p = 0.1$. Consider the condition $B = \{N \geq 20\}$. .

(a) (3 pts.) Find the PMF $P_N(n)$.

(b) (3 pts.) Find $P_{N|B}(n)$, the conditional PMF of N given that there have been 20 consecutive tests without a failure.

(c) (4 pts.) What is $E[N|B]$, the expected number of tests given that there have been 20 consecutive tests without a failure?

8. [10 points] The time between telephone calls at a telephone switch is an exponential random variable T with expected value 0.01. Given $T > 0.02$.

(a) (5 pts.) What is $E[T|T > 0.02]$, the conditional expected value of T?

(b) (5 pts.) What is Var[$T|T > 0.02$], the conditional variance of T?

9. [10 points] The uniform ($-r/2$, $r/2$) random variable X is processed by a b-bit uniform quantizer to produce the quantized output Y. Random variable X is rounded to the nearest quantizer level. With a b-bit quantizer, there are $n = 2^b$ quantization levels. The quantization step size $\Delta = r/n$, and Y takes on values in the set

$$Q_Y = \{y_i = \Delta/2 + i\,\Delta | i = -\frac{n}{2}, -\frac{n}{2} + 1, \ldots, \frac{n}{2} - 1\}.$$

(a) (3 pts.) Given the event B_i that $Y = y_i$, find the conditional PDF of X given B_i.
(b) (3 pts.) The difference $Z = X - Y$ is the quantization error or quantization "noise." Given event B_i and X is in the ith quantization interval, find the conditional PDF of Z.
(c) (4 pts.) Show that Z is a uniform random variable. Find the PDF, the expected value, and the variance of Z.

10. [10 points] When you make a phone call, the line is busy with probability 0.2 and no one answers with probability 0.3. The random variable X describes the conversation time (in minutes) of a phone call that is answered. X is an exponential random variable with $E[X] = 3$ minutes. Let the random variable W denote the conversation time (in seconds) of all calls ($W = 0$ when the line is busy or there is no answer.).

(a) (3 pts.) What is $F_W(w)$?
(b) (3 pts.) What is $f_W(w)$?
(c) (4 pts.) What are $E[W]$ and Var[W]?