MATLAB assignment 4

Introduction to Linear Algebra (Week 4) **Fall, 2020**

1. *Symbolic Computation* in MATLAB

   (a) In this problem, we are doing several calculus problems by symbolic computation.

   i. Using the MATLAB commands `diff`, find f_{xx} which is the second derivative of the function f with respect to x where

   $$f(x,y) = \sin(x^2 y) + \cos(xy^2).$$

   ii. From the result of (i), using the MATLAB command `subs`, find $f_{xx}(1,2)$. In order to change a symbolic answer to a numeric one, use the MATLAB command `double` or `eval` which stands for double precision.

   iii. Using the MATLAB command `limit`, find the following limit

   $$\lim_{h\to 0} \frac{\cos(x+h) - \cos(x)}{h}.$$

   (b) In this problem, we plot the Taylor series expansion of the following function:

   $$f(x) = \frac{1}{5+\cos x}.$$

   i. Use the MATLAB command `taylor` to find the Taylor series expansion for $f(x)$ centered at $x = \frac{\pi}{2}$ up to the 10th order terms.
   [Remember that a constant term is 0th order term.]

   ii. Use the MATLAB commands `fplot` and `plot`(and also `hold on`) to plot the resulting polynomial of (i) for $0 \le x \le \pi$ together with the given function $f(x)$ in the same figure.

   (c) [HW problem] Let f and g be two functions given by

   $$f(x) = \arctan(x).$$

   Use the MATLAB command `int` to compute value of $\int_{-1}^{1} \left| f(x) - T_f^9(x) \right|^2 \mathrm{d}x$ where the $T_f^9(x)$ denotes 9th order Maclaurin expansion for $f(x)$.

2. *User Function file* in MATLAB

   (a) [HW problem] There are 2 ways to approximate e.

$$e = \sum_{k=0}^{\infty} \frac{1}{k!} \qquad \Longrightarrow \qquad e \approx \sum_{k=0}^{N_1} \frac{1}{k!}.$$

$$e = \lim_{k\to\infty} \left(1 + \frac{1}{k}\right)^k \qquad \Longrightarrow \qquad e \approx \left(1 + \frac{1}{N_2}\right)^{N_2}.$$

   Make a user function which find the smallest numbers N_1, N_2 which satisfy the $error < T$. where $error = |e - approximation|$. User function has scalar input(T) and 1×2 vector outputs (N_1, N_2).

   (b) There are 2 ways to approximate π.

$$\pi = 4\sum_{k=0}^{\infty} \frac{(-1)^k}{2k+1} \qquad \Longrightarrow \qquad \pi \approx 4\sum_{k=0}^{N_3} \frac{(-1)^k}{2k+1}.$$

$$\pi = 16\sum_{k=0}^{\infty} \frac{(-1)^k}{2k+1}\left(\frac{1}{5}\right)^{2k+1} - 4\sum_{k=0}^{\infty} \frac{(-1)^k}{2k+1}\left(\frac{1}{239}\right)^{2k+1}$$

$$\Longrightarrow \quad \pi \approx 16\sum_{k=0}^{N_4} \frac{(-1)^k}{2k+1}\left(\frac{1}{5}\right)^{2k+1} - 4\sum_{k=0}^{N_4} \frac{(-1)^k}{2k+1}\left(\frac{1}{239}\right)^{2k+1}.$$

   Make a user function which find the smallest numbers N_3, N_4 which satisfy the $error < T$. where $error = |\pi - approximation|$. User function has scalar input(T) and 1×2 vector outputs (N_3, N_4).

   (c) [HW problem] In this problem, we will make a user function file that compute the determinant of matrix A by using the co-factor expansion along the first column for the smaller matrices. For 2×2 matrix $A = (a_{ij})$, the determinant is $a_{11}a_{22} - a_{12}a_{21}$. You may use `for, while, if` statements simultaneously.

   - MATLAB indexing starts with 1. To get the elements of the matrix $A = (a_{ij})$, you can use the $(\cdot, \cdot)$. For example, if you want get a a_{13}, then type $A(1, 3)$. To get the size of matrix A, use the command `size`. For example, if A is 5×4 matrix, then the code `[m, n] = size(A)` gives m=5 and n=4.
   - By using the colon(:), you can easily get a i^{th} row vector by a code $A(i,\ :)$. Similarly $A(:,\ j)$ gives the j^{th} column vector.
   - You can get the sub-matrix by typing the $A(r_1 : r_2,\ c_1 : c_2)$. For example, given 10×10 matrix $A = (a_{ij})$, code $A(2 : 3,\ 4 : 6)$ givse the 2×3 sub-matrix which has the elements like

$$A(2 : 3,\ 4 : 6) = \begin{pmatrix} a_{24} & a_{25} & a_{26} \\ a_{34} & a_{35} & a_{36} \end{pmatrix}$$

Similarly, $A(5:6,\ [1:3, 8:9])$ gives the 2×5 sub-matrix which has the elements like

$$A(5:6,\ [1:3,8:9]) = \begin{pmatrix} a_{51} & a_{52} & a_{53} & a_{58} & a_{59} \\ a_{61} & a_{62} & a_{63} & a_{68} & a_{69} \end{pmatrix}$$

3. Read the section in the *'MATLAB basic (Lee, Jeon)'* that corresponds to the this week class and practice by your self.

Submission guide

- Download the guide code('assignment4_guide.zip') in the KLMS.
- Fill out missing parts to find the value of $\int_{-1}^{1} \left| f(z) - T_f^9(x) \right|^2 \mathrm{d}x$ which are in the problem 1-(c). Use **only symbolic computations** to get the values. Upload the completed code files, `HW2_1c_<id_number>.m`, to 'Homework box for MATLAB assignment 4' in the KLMS. (Like HW2_1c_20209999.m)
- Fill out missing parts to find the values of N_1, N_2 which are in the problem 2-(a). Use **for, while, if** statement to get the values. Upload the completed code files, `HW2_2a_<id_number>.m`, to 'Homework box for MATLAB assignment 4' in the KLMS. (Like HW2_2a_20209999.m)
- Fill out missing parts to find the value of determinant of matrix A which is in the problem 2-(c). Use **for, while, if** statement to get the values. Upload the completed code files, `HW2_2c_<id_number>.m`, to 'Homework box for MATLAB assignment 4' in the KLMS. (Like HW2_2c_20209999.m)
- Incorrect file names and incorrect file format will cause a disadvantage in the scoring process

Due date : Oct 8 (Thu) 23:59
Late submission will not be allowed.