EE201 Circuit Theory (Spring 2016)
Mid-Term Exam.

(Total: 210 Points / 9 Problems)

Student ID Number:

Name:

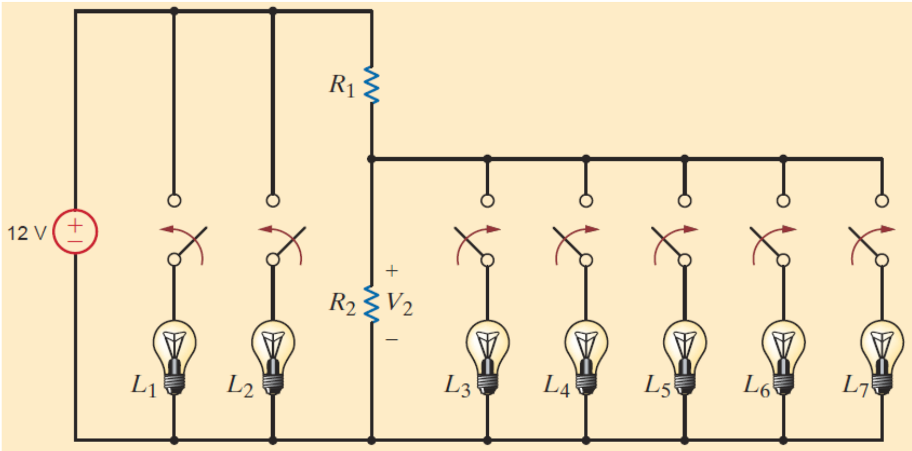
1. **The charge entering the positive terminal of an element is given by the expression $q(t) = -12e^{-2t}$ mC. The power delivered to the element is $p(t) = 2.4e^{-3t}$ W. Compute the current in the element, the voltage across the element, and the energy delivered to the element in the time interval $0 < t < 100$ ms. (20 points)**

$$i(t) = \frac{dq(t)}{dt} = \frac{d(-12e^{-2t})}{dt} = (-2) \times (-12e^{-2t}) = 24e^{-2t}\ \text{mA}$$

$$v(t) = \frac{p(t)}{i(t)} = \frac{(2.4e^{-3t}\ \text{W})}{(24e^{-2t}\ \text{mA})} = 100e^{-t}\ \text{V}$$

$$w(100\ \text{ms}) = \int_0^{100\ \text{ms}} p(t)dt = \int_0^{100\ \text{ms}} 2.4e^{-3t}dt = -\frac{2.4}{3}e^{-3t}\Big|_0^{100\ \text{ms}} = -\frac{2.4}{3}\left[e^{-300\times10^{-3}} - e^{0}\right] = 207\ \text{mJ}$$

2. **An electronic hobbyist who has built his own stereo amplifier wants to add a back-lit display panel to his creation for that professional look. His panel design requires seven light bulbs – two (L_1 and L_2) operate at 12 V / 15 mA and five (L_3 to L_7) at 9 V / 5 mA. Luckily, his stereo design already has a quality 12-V dc supply; however, there is no 9-V supply. Rather than building a new dc power supply, let us use the inexpensive circuit shown below to design a 12-V to 9-V converter with the restriction that the variation in V_2 be no more than ±5%. In particular, we must determine the necessary values of R_1 and R_2. (30 points)**

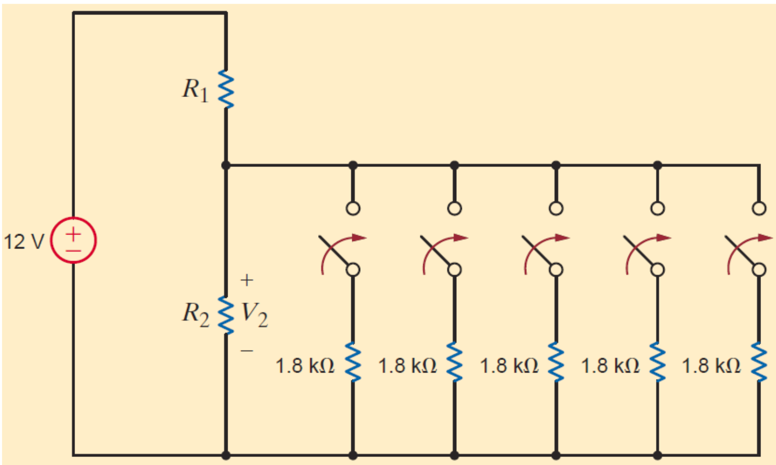


First, lamps L_1 and L_2 have no effect on V_2.

Second, when lamps L_3–L_7 are on, each of them has an equivalent resistance of

$$R_{eq} = \frac{9\ \text{V}}{5\ \text{mA}} = 1.8\ \text{k}\Omega$$

Thus, the requisite model circuit for this design is shown in the following figure.



The voltage V_2 will be at its maximum value of 9 + 5% when L_3–L_7 are all off.

In this case, R_1 and R_2 are in series, and V_2 can be expressed by simple voltage division as

$$V_2 = 9.45 = 12\left[\frac{R_2}{R_1 + R_2}\right]$$

Rearranging the equation yields

$$\frac{R_1}{R_2} = 0.27$$

A second expression involving R_1 and R_2 can be developed by considering the case when L_3–L_7 are all on, which causes V_2 to reach its minimum value of 9 – 5%, or 8.55 V.

Now, the effective resistance of the lamps is five 1.8-kΩ in parallel, or 360 Ω.

The corresponding expression for V_2 is

$$V_2 = 8.55 = 12\left[\frac{R_2 \mid\mid 360}{R_1 + (R_2 \mid\mid 360)}\right]$$

which can be rewritten in the form

$$\frac{\frac{360R_1}{R_2} + 360 + R_1}{360} = \frac{12}{8.55} = 1.4$$

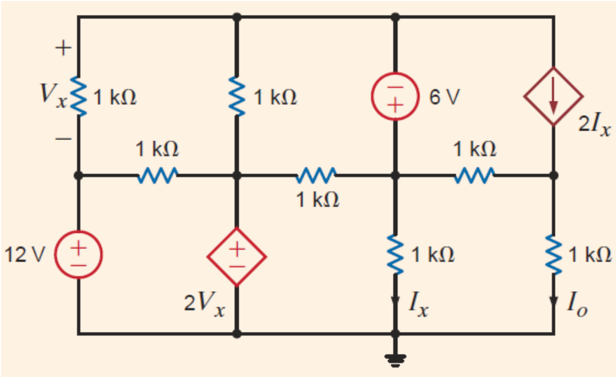
Substituting the value determined from R_1/R_2 into the preceding equation yields

$$R_1 = 360[1.4 - 1 - 0.27] = 46.8\ \Omega$$

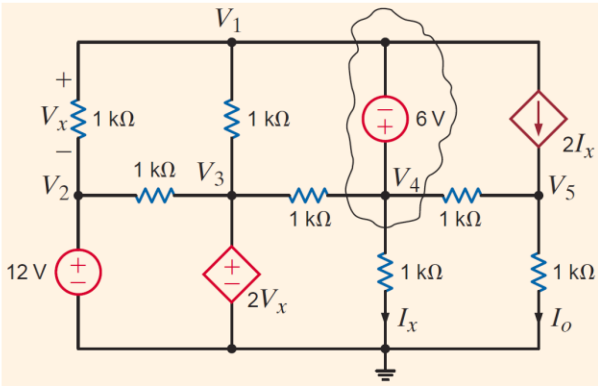
and

$$R_2 = 173.3\ \Omega$$

3. Find I_o in the network shown below using nodal analysis. (20 points)



The network is redrawn in the following figure to label the nodes and identify the supernode.



Since the network has six nodes, five linear independent equations are needed to determine the unknown node voltages.

The equation for the supernode are

$$V_1 - V_4 = -6$$

$$\frac{V_1 - V_2}{1\text{k}} + \frac{V_1 - V_3}{1\text{k}} + 2I_x + \frac{V_4 - V_3}{1\text{k}} + \frac{V_4}{1\text{k}} + \frac{V_4 - V_5}{1\text{k}} = 0$$

The three remaining equations are

$$V_2 = 12$$

$$V_3 = 2V_x$$

$$\frac{V_5 - V_4}{1\text{k}} + \frac{V_5}{1\text{k}} = 2I_x$$

The equations for the control parameters are

$$V_x = V_1 - 12$$

$$I_x = \frac{V_4}{1\text{k}}$$

Combining these equations yields the following set of equations:

$$-2V_1 + 5V_4 - V_5 = -36$$

$$V_1 - V_4 = -6$$

$$-3V_4 + 2V_5 = 0$$

Solving these equations yields

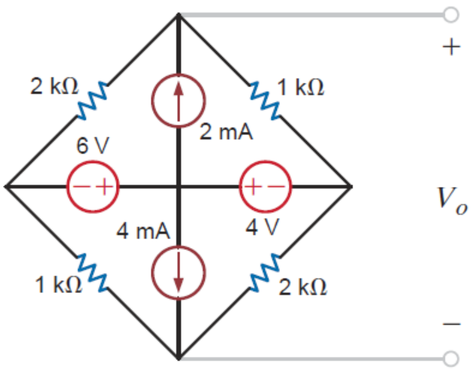
$$V_1 = -38 \text{ V}$$

$$V_4 = -32 \text{ V}$$

$$V_5 = -48 \text{ V}$$

Then, since $V_3 = 2V_x$, $V_3 = -100$ V. I_o is -48 mA.

4. Find V_o in the network shown below using loop analysis. (20 points)



Let us assign the loops as follows.

Loop 1 (I_1): 2 kΩ – 2 mA – 6 V

Loop 2 (I_2): 1 kΩ – 6 V – 4 mA

Loop 3 (I_3): 2 kΩ – 1 kΩ – 4 V – 6 V

Loop 4 (I_4): 1 kΩ – 6 V – 4 V – 2 kΩ

For two current sources, we can write

$$I_1 = -2 \text{ mA}$$

$$I_2 = 4 \text{ mA}$$

Applying KVL for loop 2 yields

$$2\text{k}\,(I_1 + I_3) + 1\text{k}\,I_3 - 4 + 6 = 0$$

Writing a KVL equation for loop 4 yields

$$1\text{k}\,(I_2 + I_4) - 6 + 4 + 2\text{k}\,I_4 = 0$$

We can rewrite these KVL equations as follows.

$$2\text{k}\,I_1 + 3\text{k}\,I_3 = -2$$

$$1\text{k}\,I_2 + 3\text{k}\,I_4 = 2$$

Substituting the values of I_1 and I_2 into these equations yields

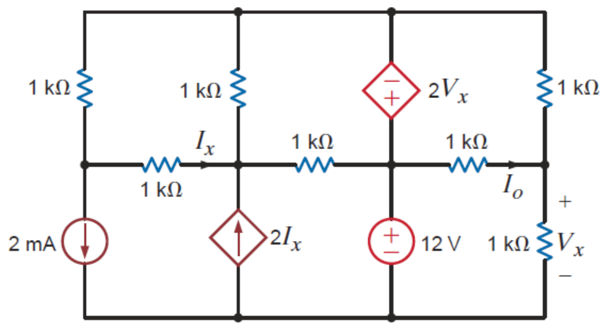
$$I_3 = \frac{2}{3} \text{ mA}$$

$$I_4 = -\frac{2}{3} \text{ mA}$$

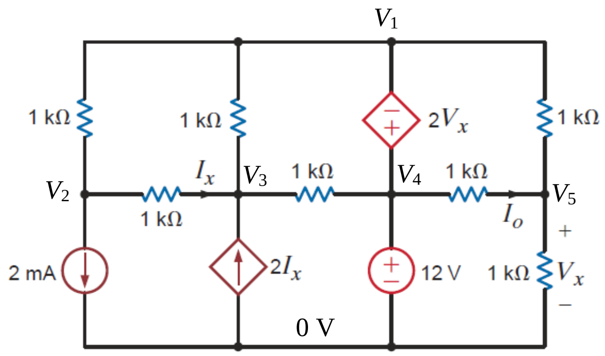
Therefore,

$$V_o = 1\text{k}\,I_3 + 2\text{k}\,I_4 = \frac{2}{3} - \frac{4}{3} = -\frac{2}{3} \text{ V}$$

5. **Use both nodal and loop analyses to determine I_o in the circuit shown below. (30 points)**



First, let us use nodal analysis to find I_o.



Since there are two voltage sources, we can write the following equations.

$$V_4 = 12$$

$$V_1 = 12 - 2V_x$$

For the dependent voltage source, we can write the following equation for the control parameter.

$$V_x = V_5$$

Applying KCL to the node labeled with V_5 yields

$$\frac{V_x - 12}{1\text{k}} + \frac{V_x - 12 + 2V_x}{1\text{k}} + \frac{V_x}{1\text{k}} = 0$$

We can rewrite this equation as follows.

$$5V_x = 24$$

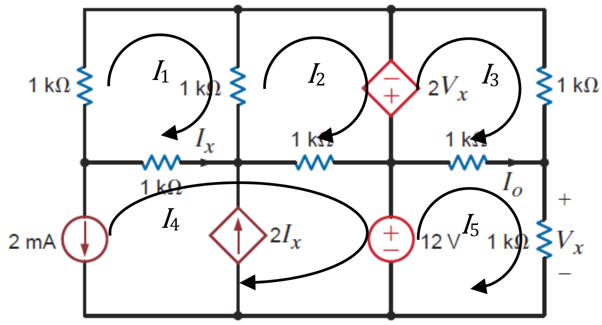
Therefore,

$$V_5 = V_x = 4.8$$

and

$$I_o = \frac{V_4 - V_5}{1\text{k}} = \frac{12 - 4.8}{1\text{k}} = 7.2\text{ mA}$$

Next, let us use loop analysis to solve the circuit.



For the dependent voltage source,

$$V_x = 1\text{k}\, I_5$$

Applying KVL to the loop 3 yields

$$2\text{k}\, I_5 + 1\text{k}\, I_3 + 1\text{k}\,(I_3 - I_5) = 0$$

We can rewrite this equation as follows.

$$2\text{k}\, I_3 + 1\text{k}\, I_5 = 0$$

Applying KVL to the loop 5 yields

$$-12 + 1\text{k}\,(I_5 - I_3) + 1\text{k}\, I_5 = 0$$

We can rewrite this equation as follows.

$$-1\text{k}\, I_3 + \ 2\text{k}\, I_5 = 12$$

By solving these equations together, we can obtain the following results.

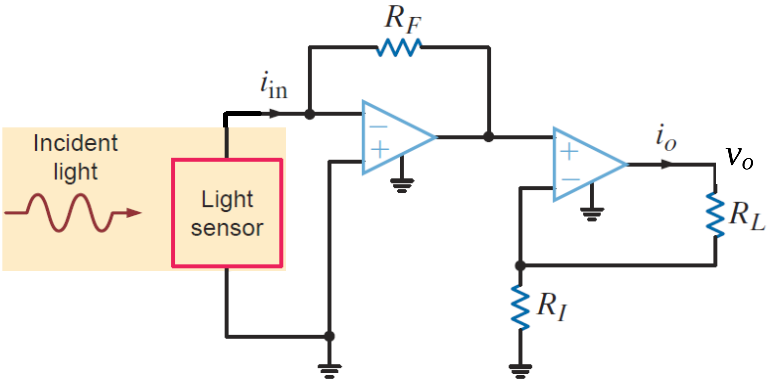
$$I_3 = -2.4 \text{ mA}$$

$$I_5 = 4.8 \text{ mA}$$

Therefore,

$$I_o = I_5 - I_3 = 7.2 \text{ mA}$$

6. **In a light meter, a sensor produces a current proportional to the intensity of the incident radiation. We wish to obtain a voltage proportional to the light's intensity using the circuit below. Find the input/output relationship (= v_o/i_{in}) for this current-to-voltage converting amplifier. (20 points)**



The node voltage v_1 at the positive input of the second op-amp is given by

$$v_1 = 0 - i_{in}R_F = -i_{in}R_F$$

Hence, the output voltage v_o is expressed as

$$v_o = v_o\left(1 + \frac{R_L}{R_I}\right) = -i_{in}R_F\left(1 + \frac{R_L}{R_I}\right)$$

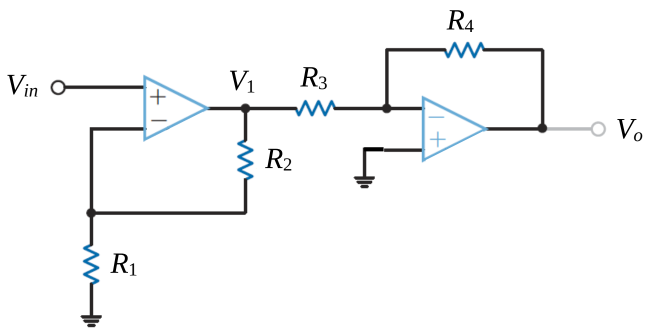
Therefore,

$$\frac{v_o}{i_{in}} = -R_F\left(1 + \frac{R_L}{R_I}\right)$$

7. Design a two-stage op-amp network that has a gain of –50,000 while drawing no current into its input terminal. Use no resistors smaller than 1 kΩ. (20 points)

Since no current should be drawn into its input terminal, we can employ a non-inverting amplifier configuration as the first stage of the two-stage circuit.

Then, an inverting amplifier can follow as the second stage to provide the negative voltage gain.



The voltage gain of this circuit is given by

$$\frac{V_o}{V_{in}} = -\left(1 + \frac{R_2}{R_1}\right)\frac{R_4}{R_3}$$

If we assign a gain of 500 to the first stage and the remaining gain of -100 to the second stage,

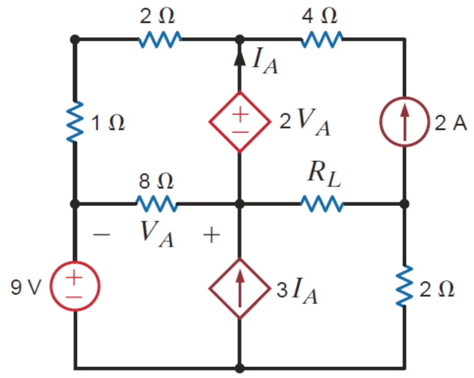
$$\frac{R_2}{R_1} = 499$$

and

$$\frac{R_4}{R_3} = 100$$

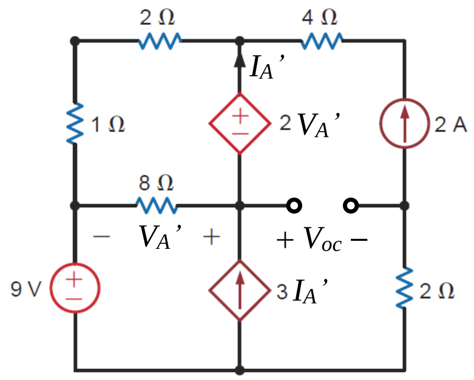
If we choose $R_1 = R_3 = 2\ \text{k}\Omega$, $R_2 = 998\ \text{k}\Omega$ and $R_4 = 200\ \text{k}\Omega$.

8. **In the circuit shown below, find the value of R_L for maximum power transfer and the maximum power that can be dissipated in R_L. (30 points)**



Let us derive the Thévenin equivalent circuit seen from the two terminals of R_L.

To do so, we need to find V_{oc} from the circuit shown below.



We can write

$$V_A' = 2I_A' \times 8 = 16I_A'$$

Applying KVL to the upper left mesh yields

$$-2V_A' + (I_A' + 2) \times 2 + (I_A' + 2) \times 1 - V_A' = 0$$

This equation can be rewritten as

$$-32I_A' + 2I_A' + 4 + I_A' + 2 - 16I_A' = 0$$

Therefore,

$$I_A' = \frac{6}{45} = 0.13$$

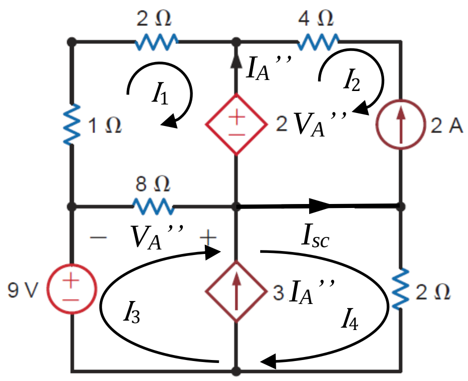
and

$$V_A' = 2.13$$

By applying KVL to the lower half loop, we can write

$$V_{oc} = 2 \times 2 + 9 + V_A' = 15.13 \text{ V}$$

Now, let us find I_{sc} from the following circuit.



We can write

$$I_2 = -2$$
$$V_A'' = 8 \times (I_1 - I_3)$$
$$I_A'' = I_2 - I_1 = -2 - I_1$$
$$3I_A'' = I_4 - I_3 = -6 - 3I_1$$

The last equation can be rearranged to obtain

$$3I_1 - I_3 + I_4 = -6 \qquad (eq. 1)$$

Applying KVL to the mesh 1 yields

$$3 \times I_1 + 16 \times (I_1 - I_3) + 8 \times (I_1 - I_3) = 0$$

This equation can be rewritten as

$$27I_1 - 24I_3 = 0$$

Therefore,

$$I_1 = \frac{8}{9} I_3 \qquad (eq. 2)$$

Applying KVL to the supermesh yields

$$-9 - 8 \times (I_1 - I_3) + 2 \times I_4 = 0$$

We can rewrite this equation as follows.

$$8I_1 - 8I_3 - 2I_4 = -9 \qquad (eq. 3)$$

By solving the equations (1) to (3) together, we can obtain

$$I_4 = 8.32 \text{ A}$$

Therefore,

$$I_{sc} = I_4 - I_2 = 10.32 \text{ A}$$

From V_{oc} and I_{sc} obtained, we can calculate R_{Th}.

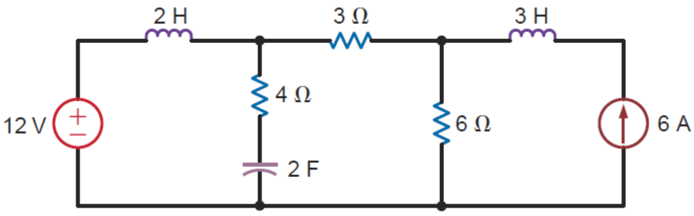
$$R_{Th} = \frac{V_{oc}}{I_{sc}} = 1.47\ \Omega$$

Therefore, the maximum power is transferred when R_L is 1.47 Ω.

The maximum power transferred is calculated by

$$\frac{V_{oc}^2}{4R_{Th}} = \frac{(15.13)^2}{4 \times 1.47} = 38.93 \text{ W}$$

9. **Given the network shown below, find the power dissipated in the 3-Ω resistor and the energy stored in the capacitor. (20 points)**



Since the circuit has only dc sources, the capacitor can be replaced with open circuit and the inductors can be regarded as short circuit.

Let us determine the current flowing through the 3-Ω resistor, I_R by using superposition technique.

$$I_R = 12\left(\frac{1}{3+6}\right) - 6\left(\frac{6}{3+6}\right) = -\frac{24}{9} = -2.67\ \mathrm{A}$$

Therefore, the power dissipated in the 3-Ω resistor is given by

$$P_{3\Omega} = 3 \times (-2.67)^2 = 21.3\ \mathrm{W}$$

For the energy stored in the capacitor, we can find that the voltage across the capacitor is 12 V .

Therefore, the energy stored in the capacitor can be calculated as

$$W_{2\mathrm{F}} = 0.5 \times 2 \times (12)^2 = 144\ \mathrm{J}$$