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2.80

The condition is $A \subset B, P(A) > 0, P(B) > 0$

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{P(A)}{P(B)} \neq P(A)$$

Thus, $P(A)P(B) \neq P(A \cap B)$

$So, A\ and\ B\ is\ not\ independent, unless\ P(B) = 1.$

2.88

$P(A) = 0.6, P(B) = 0.3$

a)

$P(A \cap B) \leq P(A), P(A \cap B) \leq P(B)$

Thus, $P(A \cap B) = 0.1$ is possible value.

b)

0, Disjoint

c)

$P(A \cap B) \leq P(A), P(A \cap B) \leq P(B)$

However, if $P(A \cap B) = 0.7$, $it\ is\ bigger\ than\ P(B)$.

Thus, it has a contradiction.

d)

$P(A \cap B) \leq \min(P(A), P(B))$

Thus, maximum value of $P(A \cap B)$ is 0.3

2.110

Defective한 Case가 아니려면, I + 오류가 아닌 경우 와 II + 오류가 아닌 경우가 존재한다.

Thus, $0.4 \times 0.92 + 0.6 \times 0.9 = 0.908$

2.114

let $T = \{\text{detects Truth}\}, L = \{detects\ lie\}$

Then the sample space is TT, TL, LT, LL

a) $P(LL) = 0.95 \times 0.1 = 0.095$ b) $P(LT) = 0.95 \times 0.9 = 0.885$

c) $P(TL) = 0.05 \times 0.1 = 0.005$ $d)1 - 0.05 \times 0.9 = 0.955$

2.134

In case A, 70%를 차지하고, 20%의 failure가 존재한다.

In case B, 30%를 차지하고, 10%의 failure가 존재한다.

F를 failure한 case로 취급하면, $P(F|A) = 0.2, P(A) = 0.7, P(F|B) = 0.1, P(B) = 0.3$

$$P(A|F) = \frac{P(F|A)P(A)}{P(F|A)P(A) + P(F|B)P(B)} = \frac{0.14}{0.17} = \frac{14}{17} \quad \because Bayes'\ Theorem$$

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2.172

$\mathrm{P(A|B)} + \mathrm{P}(\bar{A}|\mathrm{B}) = 1$ is True.
a) $\mathrm{P(A|B)} + \mathrm{P}(\bar{A}|\bar{B}) = 1$
$\Rightarrow \mathrm{P(A|B)} = 1 - \mathrm{P}(\bar{A}|\mathrm{B})$
We know that, $\mathrm{P(A|B)} = \dfrac{\mathrm{P(A \cap B)}}{\mathrm{P(B)}}$
$\Rightarrow \mathrm{P}(\bar{A}|\bar{B}) = 1 - \dfrac{\mathrm{P(A \cap B)}}{\mathrm{P(B)}} = \dfrac{\mathrm{P(B) - P(A \cap B)}}{P(B)}$
$\Rightarrow \mathrm{P}(\bar{A}|\bar{B}) = \dfrac{P(\bar{A} \cap B)}{P(B)} = P(\bar{A}|B)$
However, $\mathrm{P}(\bar{A}|\bar{B}) \neq P(\bar{A}|B)$ Thus, a contradiction arises from this. Hence a) is False
In the same way, b) is also False. Only c) is the true choice..

2.175

$\mathrm{P(A)} = \frac{1}{2}, P(B) = \frac{1}{2}, P(C) = \frac{1}{2}$ 이다.
$\mathrm{P(A \cap B)} = \frac{1}{4} = P(A) \times P(B)$
$\mathrm{P(A \cap C)} = \frac{1}{4} = P(A) \times P(C)$
$\mathrm{P(B \cap C)} = \frac{1}{4} = P(B) \times P(C)$
$\mathrm{P(A \cap B \cap C)} = \frac{1}{4} \neq P(A) \times P(B) \times P(C) = \frac{1}{8}$
Thus, A, B, C isn't mutually independent.

3.6

probability distribution을 table로 표현하면 다음과 같다.

a)

y	1	2	3	4	5
p(y)	0	0.1	0.2	0.3	0.4

b)

y	3	4	5	6	7	8	9
p(y)	0.1	0.1	0.2	0.2	0.2	0.1	0.1

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3.11

$Probability\ of\ \mathrm{O}^{+}\ is \frac{1}{3}\ and\ probability\ of\ O^{-} is \frac{1}{15}.$
따라서 주어진 조건에 따라 probability distribution을 table로 표현하면 다음과 같다.

X	0	1	2	3
p(x)	8/27	12/27	6/27	1/27

Y	0	1	2	3
p(y)	2744/3375	588/3375	42/3375	1/3375

The probability of blood type O is $\frac{1}{3}+\frac{1}{15}=\frac{2}{5}$
따라서, O형일 확률에 대한 probability distribution을 표현하면 다음과 같다.

X+Y	0	1	2	3
p(x+y)	27/125	54/125	36/125	8/125

3.26

The probability of meeting one person $\rightarrow \frac{1}{3}$, $\quad Earn\ 0\$ \rightarrow \frac{9}{10}$
The probability of meeting two people $\rightarrow \frac{2}{3}$, $\quad$ Earn 50000\$ $\rightarrow \frac{1}{10}$
Thus, let's think of it as money that makes a random variable y.
Then the probability distribution is following:

$$\mathrm{p}(0)=\frac{1}{3}\times\frac{9}{10}+\frac{2}{3}\times\frac{9}{10}\times\frac{9}{10}=\frac{252}{300}$$

$$\mathrm{p}(50000)=\frac{1}{3}\times\frac{1}{10}+\frac{2}{3}\times\frac{9}{10}\times\frac{1}{10}\times 2=\frac{46}{300}$$

$$p(100000)=\frac{2}{3}\times\frac{1}{10}\times\frac{1}{10}=\frac{2}{300}$$

And mean and standard deviation is following:

$$E(Y)=0\times\frac{252}{300}+50000\times\frac{46}{300}+100000\times\frac{2}{300}=\frac{25000}{3}\ (\$)$$

$$\mathrm{V(Y)}=E(Y^2)-\{E(Y)\}^2=380561111$$

$$\sigma=19507.98\ (\$)$$

3.29

Following the above steps:

$$\sum_{k=1}^{\infty}P(Y\geq k)=\sum_{k=1}^{\infty}\sum_{j=k}^{\infty}P(Y=k)=\sum_{k=1}^{\infty}\sum_{j=k}^{\infty}p(j)=\sum_{j=1}^{\infty}\sum_{k=1}^{j}p(j)=\sum_{j=1}^{\infty}jp(j)=\sum_{y=1}^{\infty}yp(y)=E(Y)$$

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3.33

a)

$\mathrm{E(aY + b)} = E(aY) + E(b) = \sum ayp(y) + E(b) = aE(Y) + b = a\mu + b$

b)

$\mathrm{V(aY + b)} = E((aY + b)^2) - \{E(aY + b)\}^2 = E(a^2Y^2 + 2abY + b^2) - (\mathrm{a\mu + b})^2$
$= a^2E(Y^2) + 2abE(Y) + E(b^2) - a^2\mu^2 - 2ab\mu - b^2$
$= a^2E(Y^2) + 2ab\mu + b^2 - a^2\mu^2 - 2ab\mu - b^2 = a^2(E(Y^2) - \{E(Y)\}^2) = a^2V(Y)$

3.34

Probability distribution의 table은 다음과 같다.

y	0	1	2
p(y)	0.1	0.5	0.4

Based on the data, the mean is as follows.
$\mathrm{E(10Y)} = 10E(Y) = 10(1 \times 0.5 + 2 \times 0.4) = 13\ (\$)$
And the variance is as follows.
$\mathrm{V(10Y)} = 100V(Y) = 100(E(Y^2) - \{E(Y)\}^2) = 100(1 \times 0.5 + 4 \times 0.4 - (1.3)^2) = 100 \times 0.41 = 41$