Mid-Term Exam (3 hours)

1. **[Linearity & Electrical elements] [12pts]**

(a) Suppose the input and the output of a resistor are defined as voltage across the device and temperature of the device, respectively. Is this system linear? Assume that the temperature is proportional to power dissipated in the resistor. [2pts]

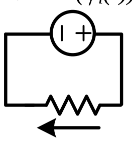
Sol)

No, it isn't.

$$T \propto P = \frac{V^2}{R}$$

(b) Suppose the input and output of the below circuits are the phase of the voltage and current, respectively. (i.e., input = $\phi_i(t)$, output = $\phi_o(t)$) Is this system linear? Assume R=1Ω, C=1F. [4pts]

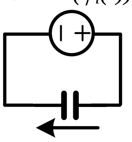
$V=sin(\phi_i(t))$



$I=sin(\phi_o(t))$

Linear? (**Yes**, No)

$V=sin(\phi_i(t))$



$I=sin(\phi_o(t))$

Linear? (Yes, **No**)

Sol)

Resistor

$$V = RI = I$$

$$sin(\phi_i(t)) = sin(\phi_o(t))$$

$$\phi_i(t) = \phi_o(t)$$

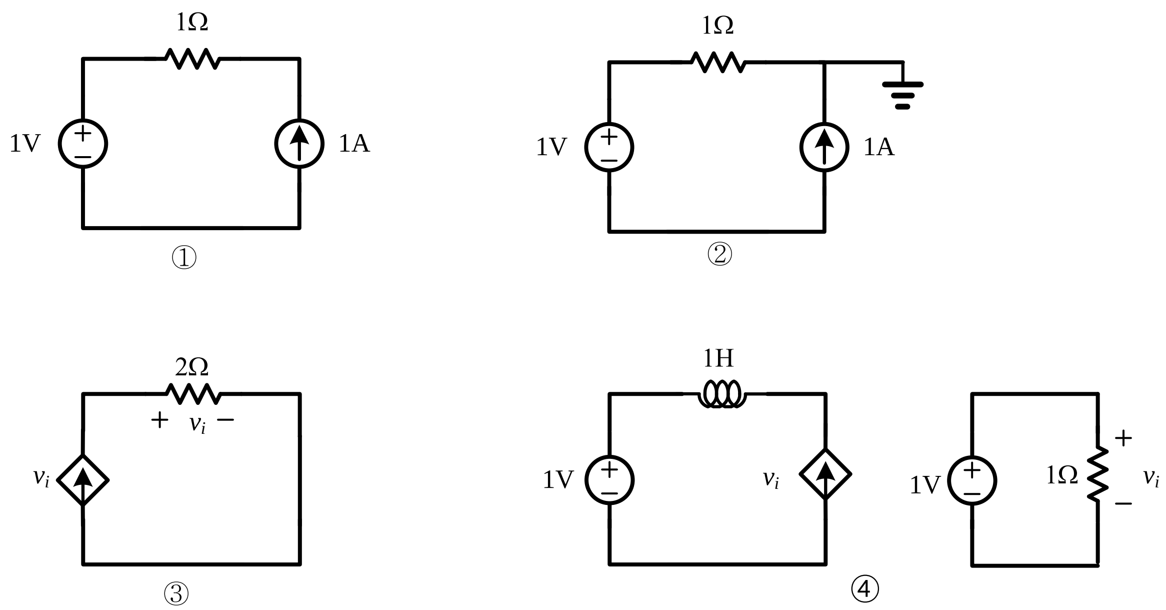
Capacitor

$$I = C\frac{dV}{dt} = \frac{dV}{dt}$$

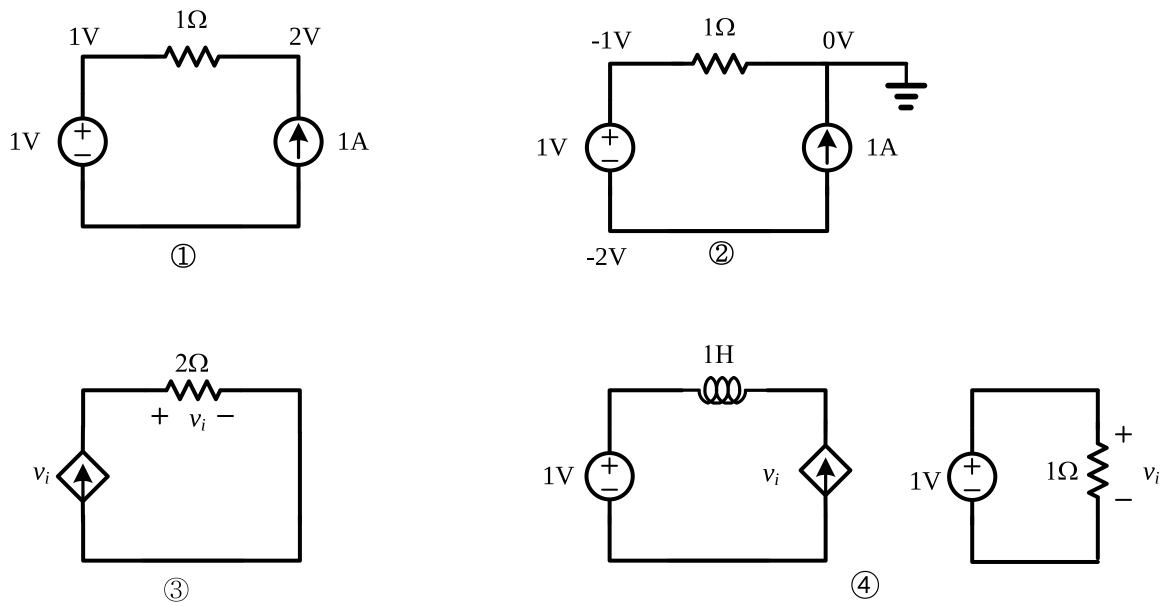
$$sin(\phi_i(t)) = \frac{d\,sin(\phi_o(t))}{dt} = \phi_o{}'(t)\,cos(\phi_o(t))$$

$\phi_o(t)$ and $\phi_i(t)$ are not in linear relationship.

(c) Please choose the circuits that are physically possible($0 < v_i < \infty$). [6pts]



Sol)



① See above figure

② See above figure

③ $v_i = 2v_i,\ v_i = 0$

④ $v_i = 1V$

Answer: (①, ②, ④ **)**

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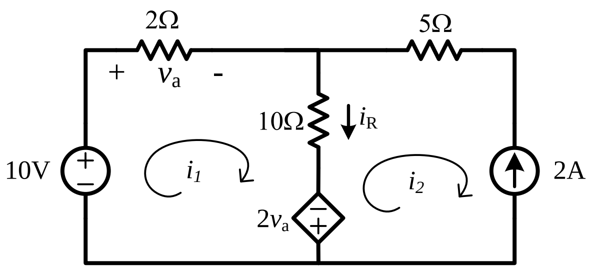
2. Analyze the circuit shown below. [9pts]



(a) Write down two equations necessary to analyze this circuit. [6pts]

Sol)

$$-10 + 2i_1 + 10(i_1 - i_2) - 2 \cdot 2i_1 = 0$$

$$i_2 = -2A$$

(b) Find i_R and the mesh currents i_1 and i_2. [3pts]

Sol)

From (a), $i_1 = -1.25A, i_2 = -2A.$ Thus, $i_R = 0.75A.$

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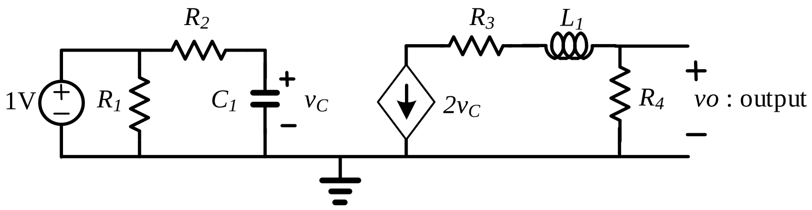
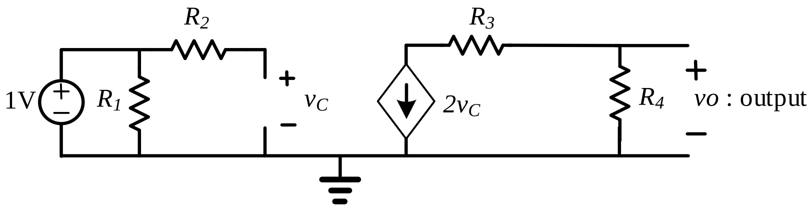
3. [Thevenin & Norton] [8pts]



(a) Find the Thevenin equivalent of the above circuit seen from the output at DC. [5pts]

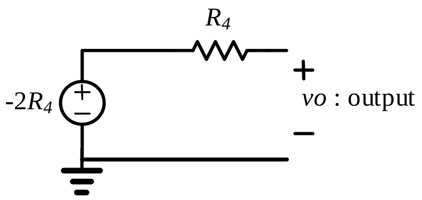
Sol)



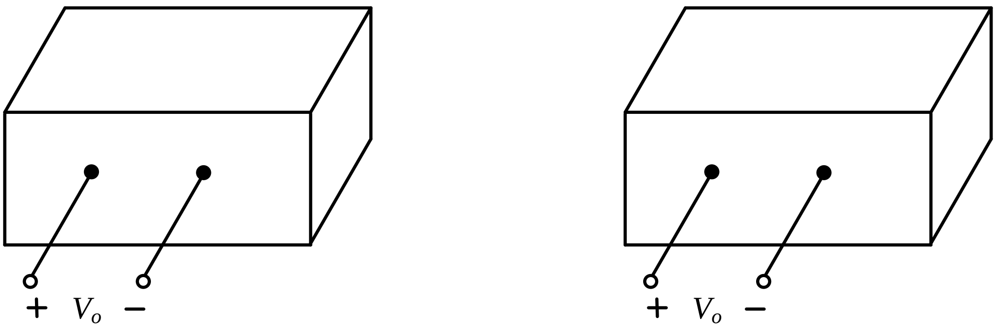
$v_c = 1\text{V}$

$v_O = v_{th} = -2v_C R_4 = -2R_4$

$R_{th} = R_4$



(b) Suppose there are two boxes with two electrical outputs. One has a Thevenin equivalent circuit of the above circuit, and the other has the Norton equivalent. Please describe how you can distinguish Norton and Thevenin between these two boxes. [3pts]



Sol)

Solution 1

It cannot be distinguished electrically.

Solution 2

It can be distinguished by measuring temperature when the output is open since current always flows in the Norton equivalent circuit.

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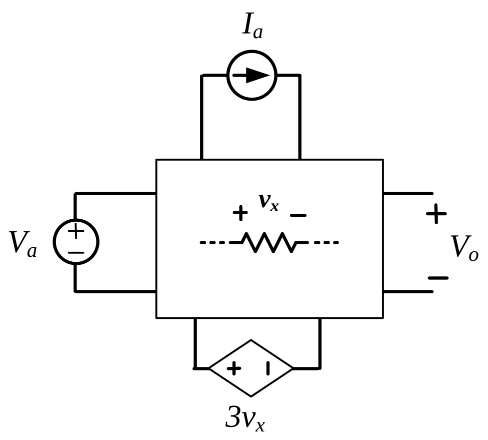
4. Please consider the circuit shown below. The rectangular box consists of resistors, capacitors and inductors only. Note that a dependent source is affected by one of the resistor in the box. [9pts]



(a) Suppose that $V_O = 0.5\text{V}$ when $(V_a, I_a) = (2\text{V}, 3\text{A})$ and $V_O = 1\text{V}$ when $(V_a, I_a) = (3\text{V}, 4\text{A})$, respectively. What is the value of V_O when $(V_a, I_a) = (1\text{V}, 2\text{A})$? If it is not solvable, explain why. [5pts]

Sol)

This system is linear, so the independent variables can form a linear combination.

$$V_O = a_1 \cdot V_a + a_2 \cdot I_a$$

Using superposition principle, we can manipulate above equation with the given conditions to get a_1 and a_2 which are 1 and -0.5, repectively.

$$\therefore V_o = 1 \cdot 1 - 0.5 \cdot 2 = 0$$

(b) Suppose the dependent source is changed to $3v_x^2$. Repeat (a). If it is not solvable, explain why. [4pts]

Sol)

With the dependent source, it is not solvable with given condition because this system is not linear.

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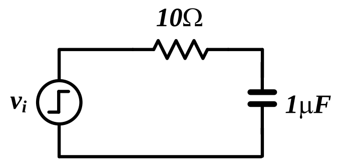
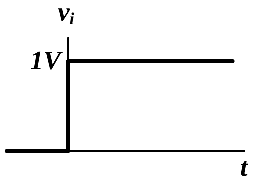
5. Please consider the circuit shown below. Assume that the circuit reaches steady-state when calculating energy. [12pts]

For (a) ~ (c), consider the circuit shown below. Assume that the initial charge in the capacitor is zero.





(a) What is the energy stored in the capacitor? [2pts]

Sol)

$$v_c(t) = \Delta v_i\left(1 - e^{-\frac{t}{RC}}\right) + v_c(0^+), i(t) = \frac{\Delta v_i}{R}e^{-\frac{t}{RC}} \text{ and } v_c(0^+) = 0V.$$

$$E_C = \frac{1}{2}Cv_c^2(\infty) = 0.5\ \mu J$$

(b) What is the energy drawn from the source? [2pts]

Sol)

$$E_V = \int_0^\infty p(t)\,dt = \int_0^\infty v_i(t)\cdot i(t)dt = v_i(0^+)\int_0^\infty i(t)dt = \frac{\Delta v_i^2}{R}\int_0^\infty e^{-\frac{t}{RC}}\,dt = C\Delta v_i^2 = 1\ \mu J$$

(c) What is the energy absorbed (dissipated) in the resistor? [2pts]

Sol.1)

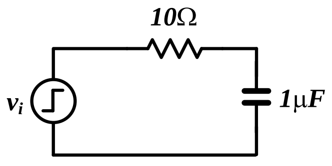
$E_R = E_V - E_C = 0.5\ \mu J$

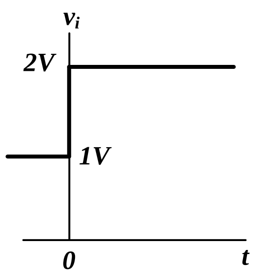
Sol.2)

$$v_R(t) = v_i(t) - v_C(t) = \Delta v_i - \Delta v_i\left(1 - e^{-\frac{t}{RC}}\right) = \Delta v_i \cdot e^{-\frac{t}{RC}}$$

$$E_R = \int_0^\infty p(t)\,dt = \int_0^\infty v_R(t)\cdot i(t)dt = \frac{\Delta v_i^2}{R}\int_0^\infty e^{-\frac{2t}{RC}}\,dt = \frac{1}{2}C\Delta v_i^2 = 0.5\ \mu J$$

For (d) ~ (e), consider the circuit shown below. Assume that the initial voltage across the capacitor is 1V.





(d) What is the energy stored in the capacitor? [2pts]

Sol)

$$v_c(t) = \Delta v_i\left(1 - e^{-\frac{t}{RC}}\right) + v_c(0^+), i(t) = \frac{\Delta v_i}{R}e^{-\frac{t}{RC}} \text{ and } v_c(0^+) = 1V.$$

$$E_C = \tfrac{1}{2}Cv_c^2(\infty) = 2\ \mu J.$$

(e) What is the energy drawn from the source? [2pts]

Sol)

$$E_V = \int_0^\infty p\,(t)dt = v_i(0^+)\int_0^\infty i(t)dt = \frac{v_i(0^+)\cdot\Delta v_i}{R}\int_0^\infty e^{-\frac{t}{RC}}\,dt = \ C v_i(0^+)\cdot\Delta v_i = 2\ \mu J$$

(f) What is the energy absorbed (dissipated) in the resistor? [2pts]

Sol)

$$v_R(t) = v_i(t) - v_C(t) = v_i(0^+) - \left[v_c(0^+) + \Delta v_i\left(1 - e^{-\frac{t}{RC}}\right)\right] = e^{-\frac{t}{RC}}$$

$$E_R = \int_0^\infty p(t)\,dt = \int_0^\infty i(t)\cdot v_R(t)dt = \frac{1}{R}\int_0^\infty e^{-\frac{2t}{RC}}\,dt = \frac{1}{2}C = 0.5\ \mu J$$

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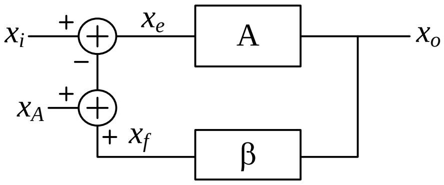
6. **[Opamp and Feedback] [13pts]**

(a) Consider the block diagram below. [3pts]



i. Express x_e in terms of x_i, x_A, A and β. [2pts]

Sol)

$x_e = x_i - (x_A + x_f)$

$= x_i - (x_A + \beta x_o)$

$= x_i - (x_A + A\beta x_e)$

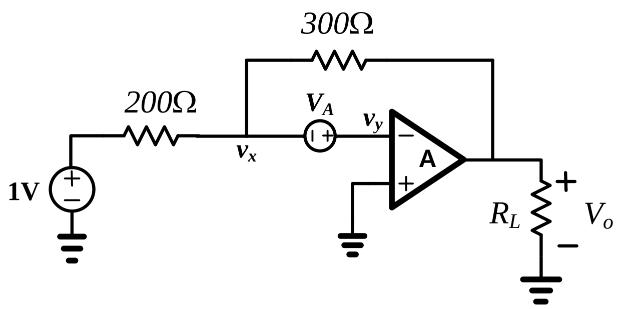
$\Rightarrow x_e = (x_i - x_A)/(1 + A\beta)$

ii. What happens to x_e as A goes to infinity? [1pt]

Sol)

x_e goes to zero.

For (b) ~ (e), consider the circuit shown below. Assume that the opamp is ideal. (i.e. A = ∞)



(b) Suppose V_A=0 and R_L = 100Ω. What is V_O? [3pts]

Sol)

$V_O = \left(-\frac{300\Omega}{200\Omega}\right) \cdot 1V = -1.5V.$

(c) Suppose $V_A \neq 0$ and R_L = 100Ω. [5pts]

i. Which of the following is true? [2pts]

① v_x=0 **② v_y=0** ③ none of the above.

ii. Express V_O in terms of V_A. [3pts]

Sol)

$$\frac{-V_A - 1}{200} = \frac{V_o + V_A}{300}$$

$$\Rightarrow V_o = -V_A\left(1 + \frac{3}{2}\right) - \frac{3}{2}$$

(d) What is the value of R_L that results in maximum power transfer? Assume V_A = 0. [2pts]

① 200Ω ② 300Ω ③ 120Ω ④ 500Ω **⑤ There is no specific value that results in maximum power transfer.**

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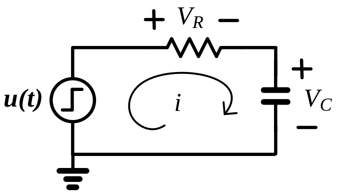
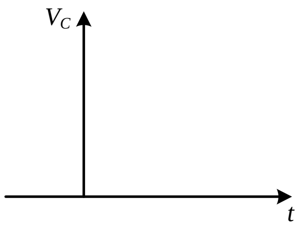
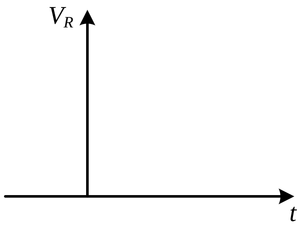
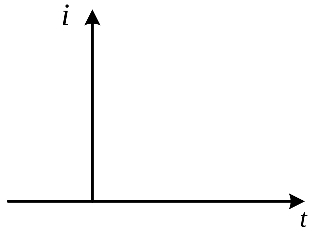
7. [RC Circuit] [16pts]



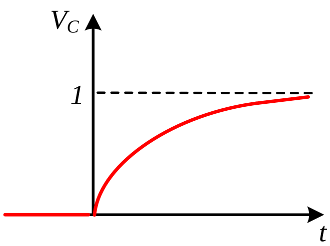
(a) Draw the voltage V_C, V_R and the current i when step input is applied for the above circuit. Assume that the initial charge in the capacitor is zero. Draw the answer both when t>0 and t<0. [6pts]

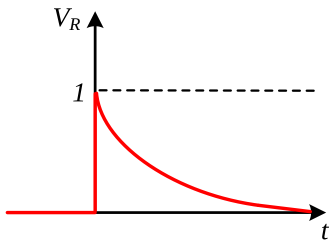


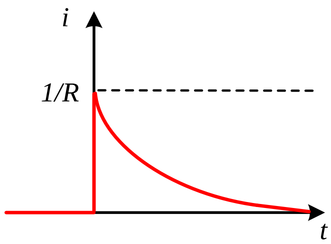




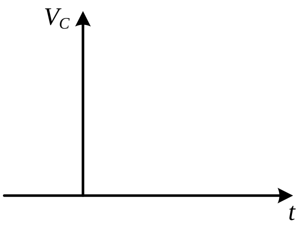
Sol)



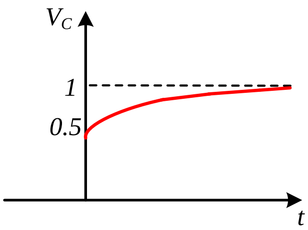




(b) Suppose the initial charge is 0.5V. Draw the answer when t>0 [2pts]



Sol)



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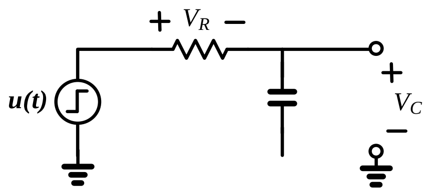
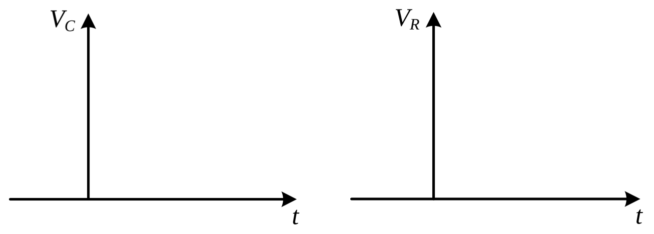
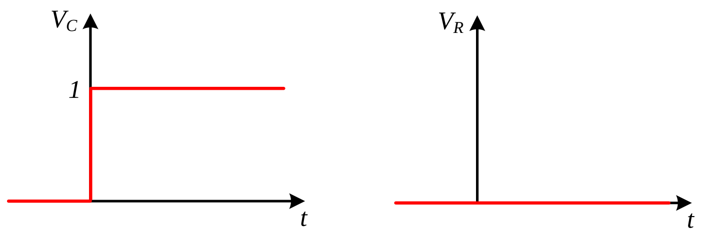
Suppose the circuit is cut and the ground is removed from the capacitor, as shown below.



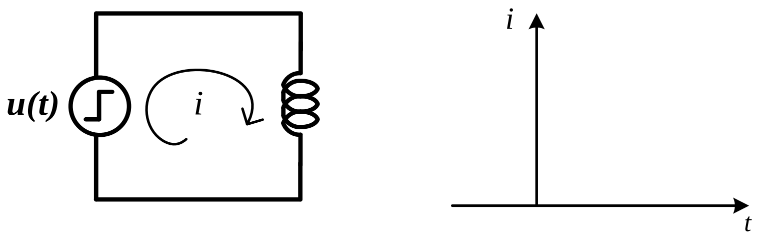
(c) Repeat (a) for the circuit shown in right. [6pts]



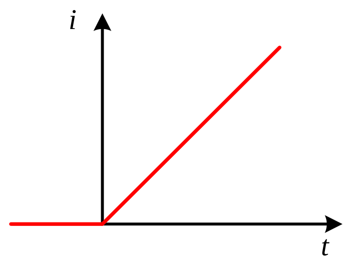
Sol)



(d) Draw the output current *i* in the below figure. Assume that initial current is zero. [2pts]



Sol)



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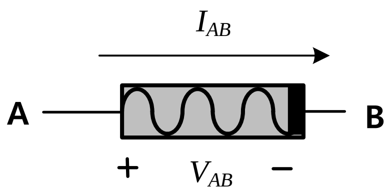
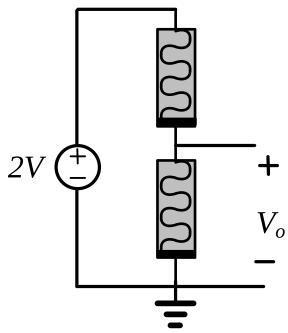
8. Consider the following 2-terminal device which has the following characteristics. [13pts]



$$I_{AB} = I_a\left(e^{k \cdot V_{AB}} - 1\right)$$

(a) What is the output voltage, V_O, of the below circuit? Assume I_a=10^{-6} (A) and k =0.5(V^{-1}) [3pts]

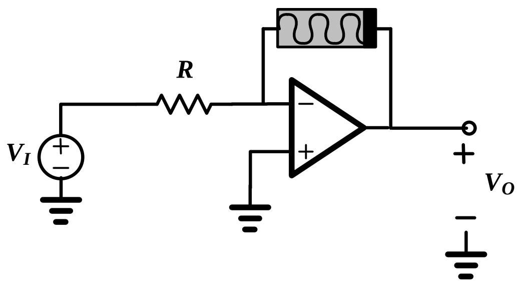


Sol)

The amount of current that flows through two elements is the same, so the voltage drop across the two elements should be the same.

$$\therefore V_O = \frac{2V}{2} = 1V$$

(b) Analyze the circuit shown below. Assume that the opamp is ideal. [5pts]



i. Is the circuit in a positive feedback or negative feedback? If the circuit does not operate properly, please fix the circuit so that it is stable. [1pt]

Sol)

It is a negative feedback. It operates properly.

ii. Write down the equation for KCL. Express your answer in terms of V_O, V_I, R, k and I_a. [2pts]

Sol)

$$\frac{0-V_I}{R} + I_{AB} = -\frac{V_I}{R} + I_a\left(e^{k\cdot(0-V_O)} - 1\right) = 0 \quad \Rightarrow \quad -\frac{V_I}{R} + I_a\left(e^{k\cdot(-V_O)} - 1\right) = 0$$

iii. Express V_O in terms of V_I, R, k and I_a. [2pts]

Sol)

$$e^{k\cdot(-V_O)} = \frac{V_I}{R \cdot I_a} + 1$$

$$V_O = -\frac{1}{k}\ln\left(\frac{V_I}{R \cdot I_a} + 1\right)$$

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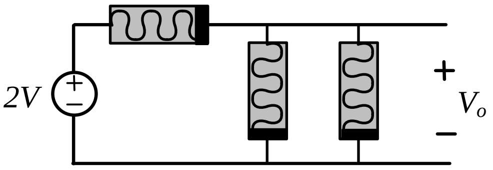
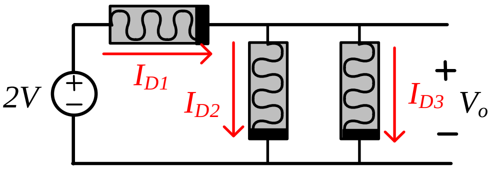
(c) What is the approximate output voltage, V_O? In order to calculate the approximate V_O, please assume that $I_{AB} \cong I_a e^{k \cdot V_{AB}}$.

Assume $I_a = 10^{-6}$ (A) and $k = 0.5(V^{-1})$ [5pts]



Sol)

Using KCL,



$I_{D2} = I_{D3}$

$I_{D1} = I_{D2} + I_{D3} = 2I_{D2}$

$I_a e^{k\cdot(2-V_o)} = 2I_a e^{k\cdot(V_o)}$

$k \cdot (2 - V_o) = k \cdot V_o + ln2$

$$2V_o = 2 - \frac{ln2}{k} = 2 - \frac{ln2}{0.5}$$

$V_o = 1 - ln2$

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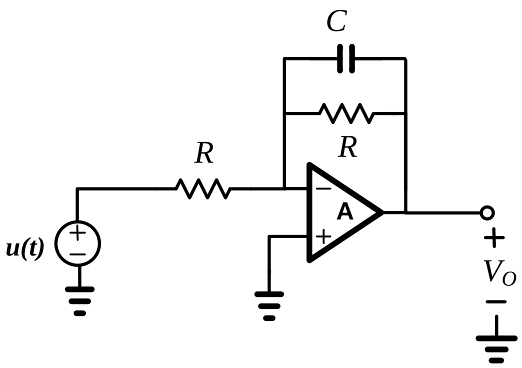
9. [RC Circuit] [8pts]



(a) What is the time-constant of the above circuit? [3pts]

Sol)

Resistance seen from the two terminals of the capacitor is R. Thus, $\tau = RC$.

(b) Draw a circuit that exhibits the same output voltage as the above circuit using a capacitor, a resistor, and a unit-step voltage source. [5pts]

Sol)