7. Heat Engine Cycles

INTRODUCTION

A heat engine is a device that does work by extracting thermal energy from a hot reservoir and exhausting thermal energy to a cold reservoir. In this experiment, the heat engine consists of air inside a cylinder that expands when an attached can is immersed in hot water. The expanding air pushes on a piston and does work by lifting a weight. The heat engine cycle is completed by immersing the can in cold water, which returns the air pressure and volume to the starting values.

THEORY

The theoretical maximum efficiency of a heat engine depends only on the temperature of the hot reservoir, T_H, and the temperature of the cold reservoir, T_C. The maximum efficiency is given by

$$e = \left(1 - \frac{T_C}{T_H}\right) \times 100\% . \tag{1}$$

The actual efficiency is defined as

$$e = \frac{W}{Q_H} \times 100\% , \tag{2}$$

where W is the work done by the heat engine on its environment and Q_H is the heat extracted from the hot reservoir.

At the beginning of the cycle, the air is held at a constant temperature while a weight is placed on top of the piston. Work is done on the gas, and heat is exhausted to the cold reservoir. The internal energy of the gas ($\Delta U = n\,C_V\,\Delta T$) does not change, as the temperature does not change. According to the First Law of Thermodynamics, $\Delta U = Q - W$, where Q is the heat added to the gas and W is the work done by the gas.

In the second part of the cycle, heat is added to the gas, causing the gas to expand, pushing the piston up, and doing work by lifting the weight. This process takes place at constant pressure (atmospheric pressure) because the piston is free to move. For an isobaric process, the heat added to the gas is $Q_P = nC_P\Delta T$, where n is the number of moles of gas in the container, C_P is the molar heat capacity for constant pressure, and ΔT is the change in temperature. The work done by the gas is found using the First Law of Thermodynamics, $W = Q - \Delta U$, where Q is the heat added to the gas and ΔU is the internal energy of the gas, given by $\Delta U = nC_V \Delta T$, where C_V is the molar heat capacity for a constant volume. As air consists mostly of diatomic molecules, $C_V = 5R/2$ and $C_P = 7R/2$.

In the third part of the cycle, the weight is lifted off the piston while the gas is held at the hotter temperature. Heat is added to the gas and the gas expands, doing work. During this isothermal process, the work done is given by

$$W = nRT \ln\left(\frac{V_f}{Vi}\right), \tag{3}$$

where V_i is the initial volume at the beginning of the isothermal process and V_f is the final volume at the end of the isothermal process. Given that the change in internal energy is zero for an isothermal process, the First Law of Thermodynamics shows that the heat added to the gas is equal to the work done by the gas:

$$\Delta U = Q - W = 0, \tag{4}$$

In the final part of the cycle, heat is exhausted from the gas to the cold reservoir, returning the piston to its original position. This process is isobaric and the same equations apply as in the second part of the cycle.

EXPERIMENTAL SETUP

1. Put the rod in the rod stand. Attach the Heat Engine to the rod by sliding the rod clamp of the Heat Engine onto the rod. The Heat Engine should be oriented with the piston end up and the Heat Engine should be positioned close to the bottom of the rod stand (see *Fig. 1*).

2. Attach the Rotary Motion Sensor to the top of the rod stand and align the medium groove of the pulley of the Rotary Motion Sensor in such a way that a string coming from the center of the piston platform of the Heat Engine will pass over the pulley.

3. Thread one end of a piece of string through the hole in the top of the piston platform and tie that end of the string to the shaft of the piston under the piston platform. See *Fig. 2*. Pass the other end of the string over the medium step of the Rotary Motion

Sensor pulley and attach a mass hanger and masses totaling 35 grams. This mass acts as a counterweight for the piston.

4. Position the piston 2 or 3 cm from the bottom of the cylinder and attach the tube from the can to one port on the Heat Engine. Attach the tube from the pressure sensor to the other port on the Heat Engine.

Fig. 1. Experimental setup.

Fig. 2. Attaching the string to the piston.

5. Connect the Pressure Sensor to Channel A, the two Temperature Sensors to Channels B and C, and the Rotary Motion Sensor to Channels 1 and 2 on the computer interface.

6. Put hot water (about 80° C) into one of the plastic containers (about half full). Put ice water in the other plastic container. The large (about 3 liters) containers keep the hot and cold temperatures constant during the heat engine cycle.

7. Place one temperature sensor in the hot water and place the other temperature sensor in the cold water. Here, the temperature sensors are labeled hot and cold in the software program; hence, it is necessary to pay close attention to which sensor is put into the hot water and which is put into the cold water.

SOFTWARE SETUP

1. Run “DataStudio”.

2. Load the file labeled "HeatEngineCycle". This file is set up to record a graph of the air pressure inside the cylinder versus the volume (i.e., a P-V diagram).

PROCEDURE

1. Perform the following cycle without hesitating between steps. It may be necessary to practice a few times before recording a data run. Start with the can in the cold water. This starting point is termed point *A*. Record the height of the bottom of the piston. Start recording data on the computer.
   - $A \to B$: Place a 200g mass on the platform.
   - $B \to C$: Move the can from the cold bath to the hot bath.
   - $C \to D$: Remove the 200g mass from the platform.
   - $D \to A$: Move the can from the hot bath to the cold bath.

2. Print a graph of the cycle. Label the four corners of the graph as *A*, *B*, *C*, and *D*. Identify the temperatures at points *A*, *B*, *C*, and *D*. Put arrows on the cycle to show the direction of the process.

3. Identify the types of processes (i.e., isothermal, etc.) and the actual physical performance (put mass on, put in hot bath, etc.) for *A* to *B*, *B* to *C*, *C* to *D*, and *D* to *A*.

4. Identify and label the two processes in which heat is added to the gas.

5. Calculate the ideal (maximum) efficiency for a heat engine operating between the two temperatures using *Eq.* (1).

6. Calculate Q_H, the heat added to the gas by the hot reservoir during the isobaric expansion from *B* to *C* and the isothermal expansion from *C* to *D*. The following will need to be calculated:

   (a) The initial volume, V_A, is unknown; however, this can be calculated by measuring the volume of the can and adding the initial volume of air in the cylinder. The volume in the tubes is ignored.

Calculate V_D using an isobar and the Ideal Gas Law: $\frac{V_A}{T_A} = \frac{V_D}{T_D}$.

$$V = (\pi r^2 h)_{can} + (A h_0)_{cylinder},$$

where A is the cross-sectional area of the piston.

(b) Calculate V_C using an Isotherm and the Ideal Gas Law: $P_C V_C = P_D V_D$.

(c) Calculate $Q_{C \to D}$. For an isotherm, $Q = nRT \ln(V_f / V_i)$. Because $PV = nRT$,

$Q_{C \to D} = P_D V_D \ln(V_D / V_C)$.

※ Remember that Absolute P = (Gauge P) + (Atmospheric P).

(d) Calculate $Q_{B \to C}$. For an isobar, $Q = nC_p \Delta T$. Because air is a diatomic gas

$C_p = 7R/2$ and $nR = PV/T$, and so $Q_{B \to C} = \frac{7}{2} \frac{P_D V_D}{T_D} (T_C - T_B)$.

(e) Calculate $Q_H = Q_{B \to C} + Q_{C \to D}$.

7. Calculate the work done by the gas by measuring the area inside the curve.

8. Calculate the efficiency $e =$ work done by the gas/heat extracted from the hot reservoir ($e = W / Q_H \times 100\%$). How does this compare with the ideal efficiency from Part 5?

9. Calculate the actual work done on the 200g mass using $W = mgh$. Be careful to use only the change in height of the mass. How does this compare to the work done by the gas from part 7? Does the gas do any work other than lifting the 200g mass?

10. Mix some of the ice water with the hot water and vice versa so that the two reservoirs are closer to the same temperature. Perform the cycle again. How high is the weight lifted now? What is the theoretical efficiency using the new temperatures?