1. Diffraction occurs because, according to Huygens' principle, each point on a wavefront acts as a source of spherical waves, causing light to bend as it encounters sharp edges [O , X] [5 pts]

Solution)

1. [O]

2. According to Rayleigh criterion, if the circular aperture diameter D becomes smaller, minimum angular separation that can be resolved with light of wavelength λ becomes smaller [O , X] [5 pts]

Solution)

2. [X]

$$\theta_{\min} = \frac{1.22\lambda}{D}$$

If D becomes smaller, minimum angular separation that can be resolved with light of wavelength λ gets bigger

3. Consider a double-slit experiment. Which of the following do change when the slit is replaced by a multiple slit, or a diffraction grating ? Select all. [5pts]

Here, the centers of the two slits are identical and the grating is evenly spaced with the same distance with that of the double slit.

(a) Locations of the maxima on the screen where constructive interference occurs.

(b) Locations of the minima on the screen.

(c) The average intensity at the maxima on the screen where the light interferes constructively.

(d) The largest wavelength that can be discerned from 0.05nm difference by the first-order spectrum.

(e) All of the above. (* If this is the case, select (e) only. You need not choose a,b,c,d,e.)

Solution)

3. (b), (c), (d)

4. Consider a single silt experiment with a laser light. Select all the ways to enlarge the pattern.[5pts]

(a) Use a slit of larger width so that light can pass through a larger area.

(b) Use a laser of a shorter wavelength.

(c) Reduce the slit-to-screen distance.

(d) Fill the system with the gas of a higher refractive index.

(e) None of the above.

Solution)

4. (e)

5. A diffraction grating has $1.26 \text{ X } 10^4$ rulings uniformly spaced over width w = 25.4 mm. It is illuminated at normal incidence by yellow light from a sodium vapor lamp. This light contains two closely spaced emission lines (known as the sodium doublet) of wavelengths 589.00 nm and 589.59 nm. **At what angle** does the first order maximum occur (on either side of the center of the diffraction pattern) for the wavelength of 589.00 nm? [15 pts]

Solution)

The maxima produced by the diffraction grating can be determined with equation (32.1a) in the textbook.

$$\mathrm{d}\sin(\theta) = m\lambda$$

[5 pts]

Then, the grating spacing d using equation (32.4) is

$$\mathrm{d} = \frac{\mathrm{w}}{\mathrm{N}} = \frac{25.4 \times 10^{-3}\, m}{1.26 \times 10^{4}} = 2.016 \times 10^{-6}\, m = 2016\, nm$$

[5 pts]

The first-order maximum corresponds to m=1. Substituting these values for d and m into the equation above,

$$\theta = \sin^{-1}\frac{m\lambda}{d} = \sin^{-1}\frac{(1)(589.00\, nm)}{2016\, nm} = 16.99° \cong 17°$$

[5 pts]

No partial points.

6. Monolayer transition metal dichalcogenides (TMDCs) are atomically thin two-dimensional semiconductors, which are promising materials for the future industrial revolution. It is too thin to identify with optical microscope without proper substrate. However, by illuminating laser, we can find some peaks using grating originated from the molecular vibration, called Raman spectroscopy. When we illuminate one of the TMDCs, MoS2, with 532 nm laser, two Raman peaks occurs at $383\ \mathrm{cm}^{-1}$ and $408\ \mathrm{cm}^{-1}$. What is **the least number of rulings (*N*)** a grating should have to be able to resolve the peaks in the first order? (cm^{-1} is the unit of momentum shift of molecular vibration mode. You can convert this value to wavelength using the equation below)

$$\Delta\nu(\mathrm{cm}^{-1}) = \left(\frac{1}{\lambda_{laser}(\mathrm{nm})} - \frac{1}{\lambda_{raman\ peak}(\mathrm{nm})}\right) \times \frac{(10^7\ \mathrm{nm})}{(\mathrm{cm})}$$

Solution)

Using the given equation, two peaks are converted as

$$\lambda_{raman\ peak} = \frac{1}{\left(\frac{1}{532} - \frac{383}{10^7}\right)} \cong 543.07\ \mathrm{nm} \quad / \quad \frac{1}{\left(\frac{1}{532} - \frac{408}{10^7}\right)} \cong 543.80\ \mathrm{nm}$$

[8 pts]

Following the resolving power equation (32.5), we can find N

$$\mathrm{R} = \frac{\lambda}{\Delta\lambda} = m\mathrm{N} \rightarrow \mathrm{N} = \frac{543.07\ \text{or}\ 543.80\ \text{or}\ 543.43}{543.80 - 543.07} \cong 736$$

735 or 736 or 737 can all be answers

[7 pts]

No partial points.

7. A thin film of refractive index 1.33 and thickness 5000 Å is exposed to white light. What wavelengths in the visible region are reflected? [15 pts]

Solution)

According to eq (32.7) in textbook, the condition for a constructive interference at thin film is

$$2nd = (m + 1/2)\lambda$$

[5 pts]

Where n is the refractive index of thin film, m is non-negative integer, λ is wavelength of a light.

Substituting n = 1.33, d = 5·10-7 [m],

$$\lambda = (2.66 \cdot 10\text{-}10)/(2m+1)\ [\mathrm{m}]$$

[5 pts]

For non-negative integer m, only m=2 fits in the range of visible light wavelength.

$$\lambda = 532\ [\mathrm{nm}]$$

[5 pts]

8. We perform single-slit experiments using two single slits with different slit sizes a1 = b and a2 = 3b. For the slit with size 3b to have the same angular position for their first destructive interference from the center with the slit with size b, what should be the ratio of wavelength, ($\lambda 1/\lambda 2$), used in each experiment? [15 pts]

Solution)

According to eq (32.8) in textbook, the condition for a destructive interference in signle-slit diffraction is

$$a \sin\boldsymbol{\theta} = m\boldsymbol{\lambda} \quad (m = \text{nonzero integer})$$

[5 pts]

For the first destructive interference pattern, m = 1, and for the slit with size 3b to have the first destructive interference at the same angular position,

$$\boldsymbol{\lambda}_2 = 3b \sin \boldsymbol{\theta}_1 = 3\, \boldsymbol{\lambda}_1$$

[5 pts]

Thus,

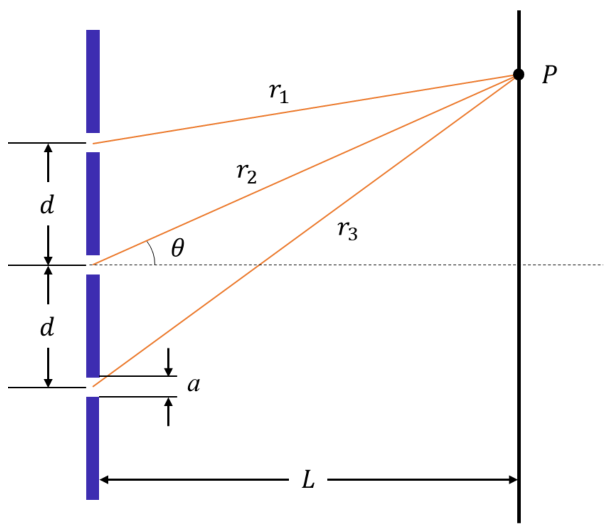
$$(\boldsymbol{\lambda}_1/\boldsymbol{\lambda}_2) = 3$$

[5 pts]

9. A monochromatic coherent source of light with wavelength λ passes through three parallel slits as shown in the figure below. The waves have same amplitude E_0 and angular frequency ω. Assume $L \gg d \gg \lambda$ and the angle between the three rays and horizontal line is identical to θ.

(a) Obtain the intensity I of triple-slit diffraction as a function of θ where I_0 is the intensity at the central maximum of the pattern ($\theta = 0$). λ, a, d should be included in the equation. You can use equations in the textbook (R. Wolfson's Essential University Physics) without prove. (14 pt)

(b) When $d = 5000\lambda$ and $a = \lambda$, plot the intensities of diffraction pattern as a function of angular position θ between – 90° and 90°, that is, x-axis being the angular position θ and y-axis being the intensity of the interference image. (6 pt)



Sol)

(a)

First calculate interference contribution.

Let the three waves emerging from the slits be

$$E_1 = E_0 \sin\omega t,\ \ E_2 = E_0 \sin(\omega t + \phi),\ \ E_3 = E_0 \sin(\omega t + 2\phi)$$

+3 pt (+1 pt each)

Using $\sin\alpha + \sin\beta = 2\cos\left(\frac{\alpha-\beta}{2}\right)\sin\left(\frac{\alpha+\beta}{2}\right)$

$$E_1 + E_3 = E_0[\sin\omega t + \sin(\omega t + 2\phi)] = 2E_0 \cos\phi \sin(\omega t + \phi)$$

$$E = E_1 + E_2 + E_3 = 2E_0 \cos\phi \sin(\omega t + \phi) + E_0 \sin(\omega t + \phi)$$
$$= E_0(1 + 2\cos\phi)\sin(\omega t + \phi)$$

where $\phi = 2\pi d \sin\theta / \lambda$

+3 pt, each time of calculation mistake -1 pt

The intensity

$$I \propto \langle E^2 \rangle = E_0^2 (1 + 2\cos\phi)^2 \langle \sin^2(\omega t + \phi) \rangle = \frac{E_0^2}{2}(1 + 2\cos\phi)^2$$

$I \propto \langle E^2 \rangle$ equation +2 pt, calculation +1 pt

The maximum intensity I_0 is when $\cos\phi = 1$.

$$\frac{I}{I_0} = \frac{(1 + 2\cos\phi)^2}{9}$$

$$I = \frac{I_0}{9}(1 + 2\cos\phi)^2 = \frac{I_0}{9}\left[1 + 2\cos\left(\frac{2\pi d \sin\theta}{\lambda}\right)\right]^2$$

+1 pt

For the diffraction contribution, from textbook (32.10)

$$I = I_0 \left[\frac{\sin\left(\frac{\phi'}{2}\right)}{\frac{\phi'}{2}}\right]^2$$

where $\phi' = \frac{2\pi}{\lambda} a \sin\theta$

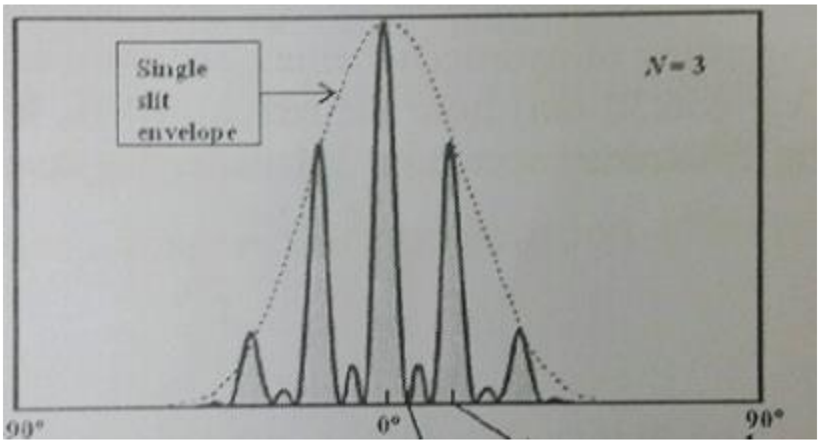
+2 pt

Overall, the intensity is

$$I = \frac{I_0}{9}\left[1 + 2\cos\left(\frac{2\pi d \sin\theta}{\lambda}\right)\right]^2 \left[\frac{\sin\left(\frac{\pi}{\lambda} a \sin\theta\right)}{\frac{\pi}{\lambda} a \sin\theta}\right]^2$$

+2 pt

(b)



+6 pt

부분점수

Case1. primary maximum 사이에 minimum이 두 개 있지만 그래프의 envelope가 감소하지 않는 경우

+3 pt

Case2. 그래프의 모양이 틀렸지만 envelope가 원점에서 멀어질수록 감소하는 경우

+1 pt

Intensity 그래프에서 minimum 포인트가 x축에 닿아 있지 않는 경우 0점

x축 없이 그래프를 그린 경우 0점

그래프를 반만 그린 경우 (e.g. $\theta = 0 \sim 90°$) 0점