EE202A Signals and Systems, Fall 2023
Oct 16, 2023
Time Limit: 09:00 - 11:45 Lecturer: Jung-Woo Choi

Score Table (for teacher use only)

Question:	1	2	3	4	Total
Points:	15	30	30	25	100
Score:					

This is a CLOSED-BOOK exam.
Please provide ALL DERIVATIONS and EXPLANATIONS with your answers.
Any communication with others during the exam will be regarded as a cheating case.
This exam contains 2 pages (including this cover page) and 4 questions.
[ST] indicates "Student-made Questions" (Questions for Questions)

1. (15 points) (T/F) Determine whether the following statements are true/false. Justify your answers.
   (a) (3 points) The system described by $y(t) = x(t-2) + 1$ is linear.
   (b) (3 points) The system described by $y(t) = \int_{-\infty}^{t} e^{(t-\tau)} x(\tau) d\tau$ is stable.
   (c) (3 points) The differential and convolution operators are commutative. In other words, the following always holds:
   $$\frac{d}{dt}\left(h(t) * g(t)\right) = \frac{dh(t)}{dt} * g(t) = h(t) * \frac{dg(t)}{dt}$$
   (d) (3 points) The system represented by $y(t) = \int_{-\infty}^{t} x(\tau) d\tau + 2$ is linear and time-invariant.
   (e) (3 points) The Fourier series coefficients of the signal $x[n] = \sum_{m=-\infty}^{\infty} (-1)^m \delta[n - mN]$ are $a_k = (1 - (-1)^k)/(2N)$.

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2. (30 points) The LTI system described by the following LCCDE satisfies the condition of initial rest.
   $$\frac{d^2 y(t)}{dt^2} + 2\frac{dy(t)}{dt} = x(t) \qquad (1)$$
   Find the impulse response of this system through the following steps.
   (a) (5 points) Find the impulse response of the LTI system satisfying the condition of initial rest and the following LCCDE.
   $$\frac{dw(t)}{dt} + 2w(t) = x(t) \qquad (2)$$
   (b) (5 points) Derive the LCCDE for the LTI inverse of the system defined by Eq. (2).
   (c) (10 points) Express $y(t)$ of Eq. (1) in terms of $w(t)$ of Eq. (2). From this relation, derive the impulse response $h(t)$ of Eq. (1).
   (Hint: if necessary, use the relation between unit impulse and unit step signal $u(t) = \int_{-\infty}^{t} \delta(\tau) d\tau$)
   (d) (5 points) Determine whether the system is stable or not. Justify your answer.

(e) (5 points) Find out the frequency response $H(j\omega)$ of this system. Hint: use the eigenfunction $x(t) = e^{j\omega t}$ to find out the response $y(t) = H(j\omega)x(t)$, and determine the eigenvalue $H(j\omega)$.

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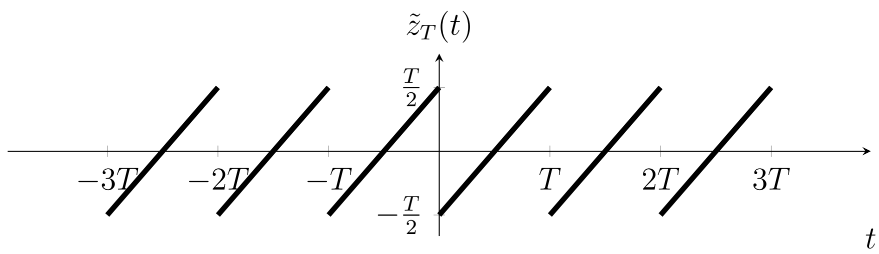
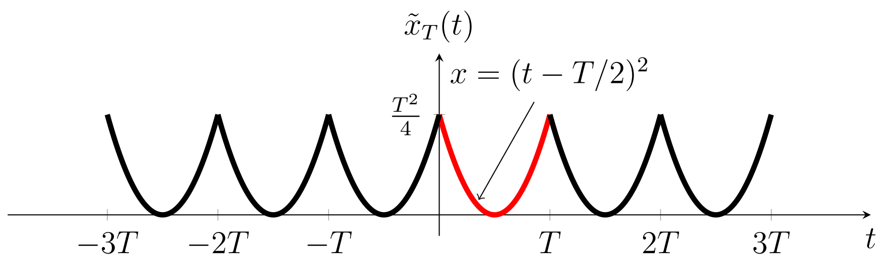
3. (30 points) [ST]

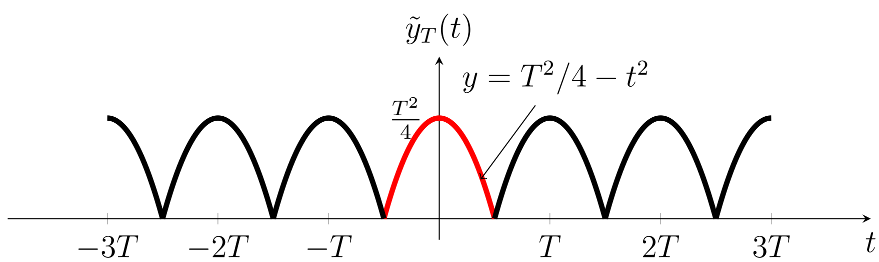
(a) (10 points) Determine the Fourier series coefficients of the following signal $\tilde{z}_T(t)$ with period T.



(b) (10 points) Derive the Fourier series coefficients of the following signal $\tilde{x}_T(t)$.



(c) (10 points) Derive the Fourier series coefficients of the following signal.



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4. (25 points) Let $\tilde{x}[n]$ be a periodic signal with period N and Fourier coefficients a_k. Assume that N is even.

(a) (5 points) Express the Fourier series coefficients b_k of $|\tilde{x}[n]|^2$ in terms of a_k.

(b) (5 points) If the coefficients a_k are real, is it guaranteed that the coefficients b_k are also real? Justify your answer.

(c) (5 points) Derive the Fourier Series coefficients of $\tilde{x}[n] - \tilde{x}[n - \frac{N}{2}]$.

(d) (10 points) Derive the Fourier Series coefficients of $\tilde{y}[n] = \begin{cases} \tilde{x}[n], & n \text{ even} \\ 0, & n \text{ odd} \end{cases}$. Assume that the fundamental period of $\tilde{y}[n]$ is still N.

[End of Problem]