EE203B Digital System
Spring 2009

Prof. Youngsoo Shin

Midterm Exam
March 27, 2009

Name ________________________________

Student ID ________________________________

Notes:

- Turn-off your cell phone; do not use calculator or any other electronic devices.
- Up to 3-hours (10 am ~ 1 pm) are allocated for this exam, but you are allowed to hand in your solution sheet early and leave the room (at your own risk) when you are done.
- Write down (clearly) on this sheet.

Question	Max Points	Score
1	20	
2	10	
3	15	
4	15	
5	10	
6	40	
Total	110	

1 Prove that an unsigned nonzero binary integer x is a power of 2 if and only if the bit-wise AND of x and $x - 1$ is 0. [20 pts]

2 Left/right shift is used to double/halve unsigned binary integers. For example of 0101, left-shift yields 1010, which is 5*2=10; right-shift yields 0010, which is 5/2 = 2 (if we ignore the remainder). How can we use left/right shift to double/halve 1's and 2's complement numbers? [10 pts]

3 Using a cofactor and Shannon expansion theorem, which were addressed in the homework, show that each of the following holds. [15 pts]

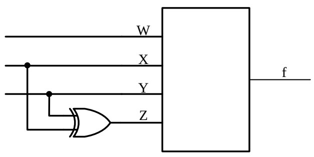
(a) $(f \cdot g)_x = f_x \cdot g_x$

(b) $f = (x' + f_x)(x + f_{x'})$

(c) $(f')_x = (f_x)'$

4 Assume that we have the following circuit with *f* given as minterm expansion:

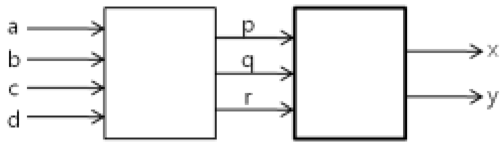
$$f(\mathrm{W,X,Y,Z}) = \sum \mathrm{m}(0,5,11)$$



(a) Using Q-M method, derive all prime implicants and indicate which are essential prime implicants. [10 pts]

(b) Use Petrick's method to find all minimum SOP forms. [5 pts]

5 In the following circuit, $p(a, b, c, d) = \sum m(1, 5, 11, 13)$, $q(a, b, c, d) = \sum m(3, 4, 7, 8, 10, 14, 15)$, and $r(a, b, c, d) = \sum m(2, 4, 7, 8, 9, 14)$. Derive a minimum two-level multi-output NAND-NAND network to realize $x(p, q, r) = \sum m(1, 4, 5)$ and $y(p, q, r) = \sum m(1, 2)$. [10 pts]



6 Answer the followings:

(a) When we derive minimum POS of *f*, we derive minimum SOP of *f′* and take its complement. Explain why this is true (i.e. complement of minimum SOP of *f′* is minimum POS of *f*). [5 pts]

(b) In 4-bit 2's complement number system, we use 2^4 to define a negative number (i.e. negative number of 3=0011 is 13=1101 because 3+13=16); in 1's complement, we use 2^4–1 to define a negative number. Can we use 2^4+1 and 2^4-2 to define a negative number? Explain what happens. [5 pts]

(c) There are two reasons why we use heuristic 2-level minimizer such as Espresso instead of exact minimizers. What are they? [5 pts]

(d) Explain the steps of Boolean minimization for 1-function in 2-level, multi-function in 2-level, 1-function in multi-level, and multi-function in multi-level. [5 pts]

(e) We are given an expression f, which we want to simplify in 2-level. Imagine that somehow we discovered all its essential prime implicants, and denote them as SOP f_1. In the expression f, we assume that all the minterms that belong to f_1 are now don't cares, and denote the modified expression by f_2. If we simplify f_2 and add it to f_1 (i.e. $f_1 + f_2$), it is a minimum SOP for f. If this is true, explain; otherwise, suggest a counter-example. [10 pts]

(f) Is an "irredundant and prime cover" (i.e. a cover, where all cubes are prime and irredundant) a "minimum cover"? Prove or show a counterexample. [10 pts]