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Problem 1. (20 points)
Suppose a bar-shaped semiconductor layer of width W, length L, and depth d, and assuming an arbitrary doping concentration Nd(x) variation with the depth x from the surface. Assume the electron mobility of μn(x).

(1) Express resistance R of the semiconductor along y direction. [10pt]

$$dR = \rho\frac{L}{Wdx} = \frac{L}{eW\mu_n(x)N_d(x)dx} \quad \ldots + 5pt$$

$$R = \frac{1}{\int_0^d \frac{eW}{L}\mu_n(x)N_d(x)dx} \quad \ldots + 5pt$$

(2) Express resistance R of the semiconductor along x direction. [10 pts]

$$dR = \rho\frac{dx}{WL} = \frac{dx}{eWL\mu_n(x)N_d(x)} \quad \ldots + 5pt$$

$$R = \int_0^d \frac{1}{eWL\mu_n(x)N_d(x)}dx \quad \ldots + 5pt$$

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Problem 2. (20 points)
Let the electron and hole mobility of a semiconductor be μn and μp, and the electron concentration be n0. [20 pts]

(1) Consider the conductivity of the semiconductor as a function of n0. What is the achievable minimum conductivity σ_{min}? [10 pts]

$$\sigma = e\mu_n n_0 + e\mu_p p_0$$

$$p_0 = \frac{n_i^2}{n_0}$$

Then,

$$\sigma = e\mu_n n_0 + \frac{e\mu_p n_i^2}{n_0} \ldots (1)$$

n0 에 대한 식을 세웠는지 +3

For σ_{min},

$$\frac{d\sigma}{dn_0} = 0$$

$$\frac{d\sigma}{dn_0} = e\mu_n - \frac{e\mu_p n_i^2}{n_0^2} = 0$$

최소조건에 대한 미분방정식 성립 여부 +3

Then,

$$e\mu_n = \frac{e\mu_p n_i^2}{n_0^2}$$

Therefore,

$$n_0 = \sqrt{\frac{e\mu_p n_i^2}{e\mu_n}} = n_i\sqrt{\frac{\mu_p}{\mu_n}}$$

Substituting upper relationship to the equation (1),

$$\sigma_{min} = 2en_i\sqrt{\mu_n\mu_p}$$

+4

(2) Find the relationship between n0 and ni at σ_{min}. [10 pts]

$$n_0 = \sqrt{\frac{e\mu_p n_i^2}{e\mu_n}} = n_i\sqrt{\frac{\mu_p}{\mu_n}}$$

+10

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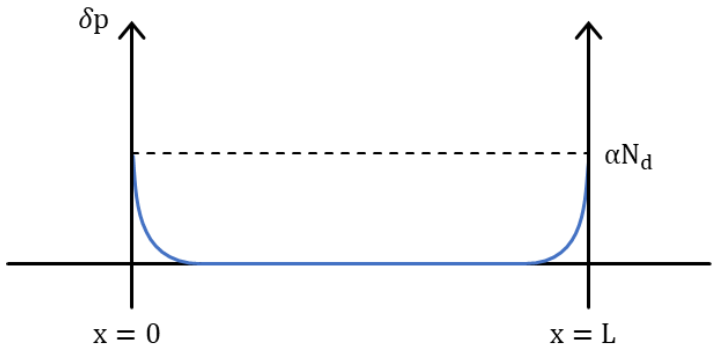
Problem 3. (20 points)

Let the electron and hole mobility of a semiconductor be μn and μp, and the electron concentration be n0. [20 pts]

(1) Draw the excess hole distribution along the x-axis. A rough graph is sufficient [5 pts]

(2) Find the expression δp over position x. [15 pts]

(1) – 5pt



(2) – 15pt

$$\delta p(x) = \frac{\alpha N_d}{\left(e^{-L/L_p}+1\right)} \left(e^{-x/L_p} + e^{(x-L)/L_p}\right)(0 \leq x \leq L),$$

where the $L_p = \sqrt{D_p \tau_{p0}}$

L_p 에 대한 설명이 없어도 무관 (diffusion cofficient 와 carrier life 이 주어지지 않아서)

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Problem 4. (20 points)

Consider an n-type semiconductor at T = 300 K with carrier concentrations of $n_0 = 10^{14}$ cm^{-3} and $n_i = 10^{10}$ cm^{-3}. Assume that the excess carrier lifetime is $\tau_{no} = 1$ μs and the diffusion constant $D_n = 25$ cm^2/s. In nonequilibrium, assume that the excess carrier concentration is $\delta n = 10^{12}$ cm^{-3}.

기존 시험의 문제 δn 조건을 따른 풀이 – 시험 중 지시사항에 의해 δp에 대한 풀이도 정답으로 인정해줄 것

(1) – 3pt

$$R = \frac{\delta n}{\tau_n} = \frac{10^{12}\mathrm{cm}^{-3}}{1 \times 10^{-6}\mathrm{s}} = 10^{18}\mathrm{cm}^{-3}/\mathrm{s}$$

(2) – 3pt

$$p_0 = \frac{n_i^2}{n_0} = \frac{(10^{10})^2}{10^{14}} = 10^6\mathrm{cm}^{-3}$$

$$\begin{aligned} E_{Fi} - E_{Fp} &= kTln\left(\frac{p_0 + \delta p}{n_i}\right) \\ &= 0.025\ln\left(\frac{10^6 + 10^{12}}{10^{10}}\right) \\ &= 0.1151\ \mathrm{eV} \end{aligned}$$

$$\begin{aligned} E_{Fn} - E_{Fi} &= kTln\left(\frac{n_0 + \delta n}{n_i}\right) \\ &= 0.025\ln\left(\frac{10^{14} + 10^{12}}{10^{10}}\right) \\ &= 0.2305\ \mathrm{eV} \end{aligned}$$

(3) – 4pt

$$L_n = \sqrt{D_n \tau_{n0}} = \sqrt{25 \times 10^{-6}} = 5 \times 10^{-3}\mathrm{cm}$$

(4) – 5pt

$$p = p_0 + \delta p \times \left(1 - \frac{1}{10^{-6}\mathrm{m}} x\right)$$
$$n = n_0 + \delta n \times (1 - \frac{1}{10^{-6}\mathrm{m}} x)$$

Therefore,

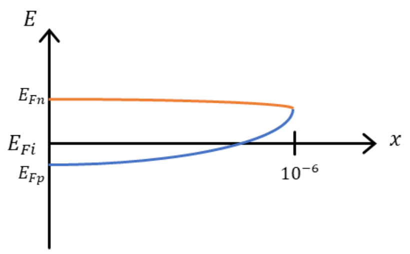
$$\mathrm{E_{Fi}} - \mathrm{E_{Fp}} = kTln\left(\frac{\mathrm{p_0} + \delta p\left(1 - \frac{1}{10^{-6}\mathrm{m}} x\right)}{n_i}\right)$$
$$\mathrm{E_{Fn}} - \mathrm{E_{Fi}} = kTln\left(\frac{\mathrm{n_0} + \delta n\left(1 - \frac{1}{10^{-6}\mathrm{m}} x\right)}{n_i}\right)$$

$x = 0$,

$$\mathrm{E_{Fi}} - \mathrm{E_{Fp}} = 0.1151 \text{ eV}$$
$$\mathrm{E_{Fn}} - \mathrm{E_{Fi}} = 0.2305 \text{ eV}$$

$x = 10^{-6}\mathrm{m}$,

$$\mathrm{E_{Fi}} - \mathrm{E_{Fp}} = -0.2302 \text{ eV}$$
$$\mathrm{E_{Fn}} - \mathrm{E_{Fi}} = 0.2302 \text{ eV}$$



(5) – 5pt

$$\delta\mathrm{n}(x) = \begin{cases} \delta\mathrm{n}(0) \times \mathrm{e}^{-x/L_n} & (x \geq 0) \\ \delta\mathrm{n}(0) \times \mathrm{e}^{x/L_n} & (x < 0) \end{cases}$$

$$\begin{aligned} \delta\mathrm{n}(\mathrm{x} = 10\mu m) &= \delta\mathrm{n}(0) \times (1 \times 10^{-3} cm) \\ &= 10^{12} \times \mathrm{e}^{-1\times10^{-3}/5\times10^{-3}} \\ &= 8.18 \times 10^{11}\mathrm{cm}^{-3} \end{aligned}$$

$$\begin{aligned} \delta\mathrm{n}(\mathrm{x} = -50\mu m) &= \delta\mathrm{n}(0) \times (-5 \times 10^{-3} cm) \\ &= 10^{12} \times \mathrm{e}^{-5\times10^{-3}/5\times10^{-3}} \\ &= 3.67 \times 10^{11}\mathrm{cm}^{-3} \end{aligned}$$

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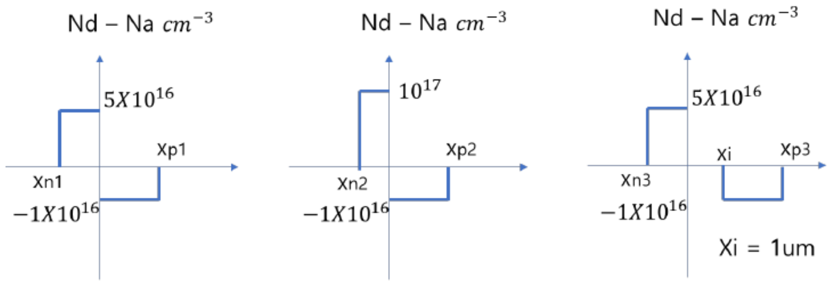
Problem 5. 20 points



(a) (b) (c)

(sol) Assume $n_i = 1.5 \times 10^{10}\ cm^{-3}$

As x_p~0.3μm in (b) which is the largest depletion region length occurred due to the junction, we can assume that the depletion length occurred at pin junction of n-, p type region is far smaller than (a), (b) case in that the intrinsic layer is about 1μm

(1) x_n: (a) > (b) > (c)

> Qualitative explanation for p-i-n : Much of the built-in voltage is dropped across the region, so that the depletion regions in the n- and p-type material need not extend as far as in a p-n junction. Therefore, depletion region across of the n-type region of p-i-n is the smallest.

> Quantitative analysis for (a) : (b)

: $x_n = \sqrt{\frac{2\epsilon_s V_{bi}}{e}\left(\frac{N_a}{N_d}\right)\left(\frac{1}{N_a+N_d}\right)}$

Similar built-in potential, $\frac{N_a}{N_d} \rightarrow 2:1$, $\frac{1}{N_a+N_d} \rightarrow 11:6$

Therefore, (a) > (b).

(2) x_p: (b) > (a) > (c)

> Qualitative explanation for p-i-n : Much of the built-in voltage is dropped across the region, so that the depletion regions in the n- and p-type material need not extend as far as in a p-n junction. Therefore, depletion region across of the p-type region of p-i-n is the smallest.

각 소문제당

(1) 3pt
(2) 3pt
(3) 4pt
(4) 5pt
(5) 5pt

부등호 정답:

(1) 2pt
(2) 2pt
(3) 3pt
(4) 4pt
(5) 4pt

(부분점수 x)

풀이: 1point
(대소비교 근거가 합리적이면 1point, 꼭 모든 값을 정량적으로 구할 필요는 없음)

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> Quantitative analysis for (a) : (b)

: $x_p = \sqrt{\frac{2\epsilon_s V_{bi}}{e}\left(\frac{N_d}{N_a}\right)\left(\frac{1}{N_a+N_d}\right)}$

Similar built-in potential (a < b) , $\frac{N_a}{N_d} \to 1:2$, $\frac{1}{N_a+N_d} \to 11:6$

Therefore, (a) < (b).

(3) E_{max}: (b) > (a) > (c)

> Qualitative explanation for p-i-n : Much of the built-in voltage is dropped across the region, so that the maximum field, which increases linearly with the extent of the space charge region in the doped material is therefore, reduced. Therefore, maximum electric field is smallest in p-i-n structure.

(Longer depletion region induces smaller maximum field because of the same built-in potentials)

> Quantitative analysis for (a) : (b)

$$E_{max} = \sqrt{\frac{2eV_{bi}}{\epsilon_s}\left(\frac{N_aN_d}{N_a+N_d}\right)}$$

Red -> 0.74: 0.76 / Blue -> 11:12 : a < b

(4) Built-in potentials: (b) > (a) = (c)

Apply $V_{bi} = \frac{kT}{e}\ln\left(\frac{N_dN_a}{n_i^2}\right)$

N_d of (b) > (a) = (c) and other parameters are the same.

So, built-in potentials: b > a =c

(5) Junction capacitance: (b) > (a) > (c)

$C' = \frac{\epsilon_s}{W}$ which is inversely proportional to depletion width.

W (depletion region): c > a > b

Depletion layer is biggest at p-i-n region.

> Quantitative analysis for (a) : (b)

: $W = \sqrt{\frac{2\epsilon_s V_{bi}}{e}\left(\frac{N_a+N_d}{N_aN_d}\right)}$

Red -> 0.74: 0.76 / Blue -> 12:11 : a > b

각 소문제당

(1) 3pt
(2) 3pt
(3) 4pt
(4) 5pt
(5) 5pt

부등호 정답:

(1) 2pt
(2) 2pt
(3) 3pt
(4) 4pt
(5) 4pt

(부분점수 x)

풀이: 1point
(대소비교 근거가 합리적이면 1point, 꼭 모든 값을 정량적으로 구할 필요는 없음)

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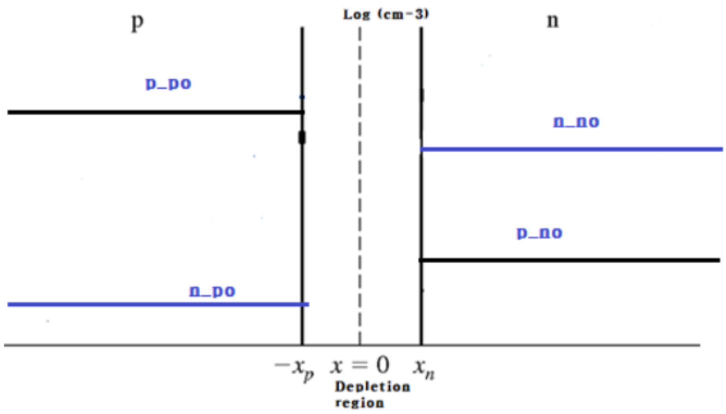
Problem 6. 20 points

1) In equilibrium, find minority carrier densities in each region and draw the majority and minority carrier density distributions in both regions in a semi-log plane shown below. [3 pts]



$$n_{p0} = 2250\ cm^{-3}, p_{n0} = 22500\ cm^{-3}$$

2) Derive injection law at the pn junction boundary with forward bias v_a from the built-in potential barrier equation. [3 pts]

$$V_{bi} = V_T \ln\left(\frac{N_a N_d}{n_i^2}\right)$$

$$\frac{n_i^2}{N_a N_d} = \frac{n_i^2}{n_{n0} \times \frac{n_i^2}{n_{p0}}} = \frac{n_{po}}{n_{n0}} = \exp\left(-\frac{eV_{bi}}{kT}\right)$$

$$n_{p0} = n_{n0} exp\left(-\frac{eV_{bi}}{kT}\right)$$

Similarly, $p_{p0} = p_{n0} \exp\left(\frac{eV_{bi}}{kT}\right)$

The potential barrier V_{bi} can be replaced by (V_{bi} - V_a) when the junction is forward biased.

$$n_p = n_{n0} exp\left(-\frac{e(V_{bi} - V_a)}{kT}\right) = n_{n0} exp\left(-\frac{eV_{bi}}{kT}\right) exp\left(\frac{eV_a}{kT}\right)$$

$$\boldsymbol{n_p = n_{p0} exp\left(\frac{eV_a}{kT}\right)}$$

$$\boldsymbol{p_n = p_{n0} \exp\left(\frac{eV_a}{kT}\right)}$$

1)
n_{p0}, p_{n0} 값 당 1 pts
그림 1 pts

2)
Boundary 에서의 최종 관계식 도출 3 pts (hole, electron 중 하나에 대해서만 구했어도 3 pts)

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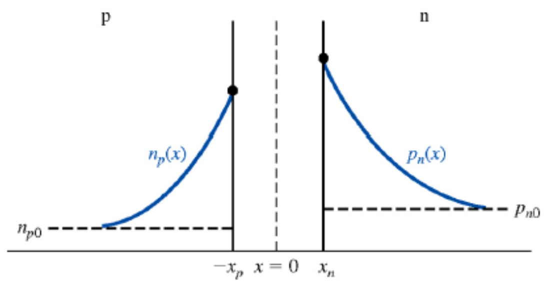
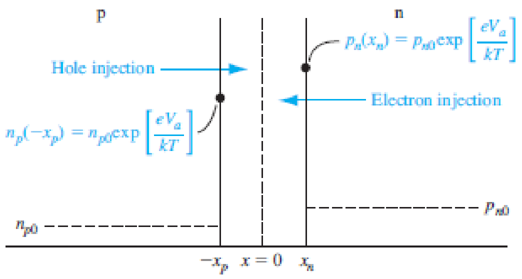
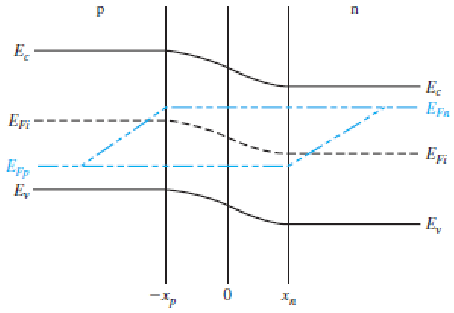
3) Draw the majority and minority carrier density distributions in both regions in the semi-log plane. Identify and explain the minority carrier injections across the depletion region. [4 pts]



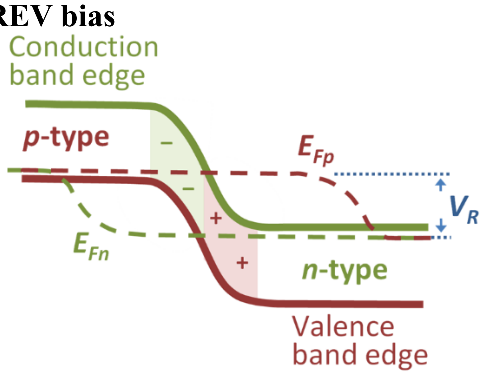


4) Schematically draw the energy band diagrams under the forward-bias and reverse-bias conditions. On top of the three energy band diagrams, schematically draw the quasi-Fermi level distributions. [5 pts]

- **FWD bias**



- **REV bias**



3)
그림 2pts
Hole 과 electron 의 주입에 대해 표시/설명한 경우 2 pts

4)
Band diagram for FWD bias: +1 pt
Band diagram for REV bias: +1 pt

Quasi-fermi level distribution for FWD bias: +1.5 pts
Quasi-fermi level distribution for REV bias: +1.5 pts

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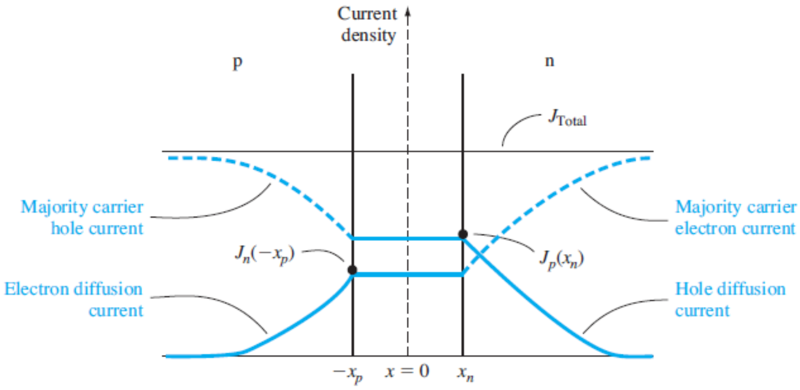
5) For the forward-bias condition, schematically draw the distributions of the electron and hole currents. Indicate the locations where the minority carrier diffusion currents are maximum. [5 pts]



Minority electron diffusion current is maximum at -x_p.
Minority hole diffusion current is maximum at x_n.