On intrinsic equivalences of the finite helical axis, the instantaneous helical axis, and the SARA approach. A mathematical perspective.

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2nd Revision

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Word count: 3939

Short title: De-mystifying helical axes: on equivalences of FHA, IHA, and SARA

Keywords: Functional gait analysis, Joint axes, Joint centres, Helical axis, SARA, SCoRE

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Abstract

Accurate determination of joint axes is essential for understanding musculoskeletal function. Whilst numerous algorithms to compute such axes exist, the conditions under which each of the methods performs best remain largely unknown. Typically, algorithms are evaluated for specific conditions only limiting the external validity of conclusions regarding their performance. We derive exact mathematical relationships between three commonly used algorithms for computing joint axes from motion data: finite helical axes (FHA), instantaneous helical axes (IHA) and SARA (symmetrical axis of rotation approach), including relationships for an extension to the mean helical axes methods that facilitate determining joint centres and axes. Through the derivation of a sound mathematical framework to objectively compare the algorithms we demonstrate that the FHA and SARA approach are equivalent for the analysis of two time frames. Moreover, we show that the position of a helical axis derived from the IHA using positional data is affected by a systematic error perpendicular to the true axis direction, whereas the axis direction is identical to those computed with either the FHA or SARA approach (true direction). Finally, with an appropriate choice of weighting factors the mean FHA (MFHA) method is equivalent to the Symmetrical Centre of Rotation Estimation (SCoRE) algorithm for determination of a Centre of Rotation (CoR), and similarly, equivalent to the SARA algorithm for determination of an Axis of Rotation (AoR). The deep understanding of the equivalences between methods presented here enables readers to choose numerically efficient, robust methods for determining AoRs and CoRs with confidence.

Introduction

Accurate determination of joint kinematics is of great importance for understanding musculoskeletal function, where the former are often expressed based on the identification of centres and axes of rotation (Akbari Shandiz et al., 2016; Boeth et al., 2013; Bonny et al., 2017; Ellingson and Nuckley, 2015; Heller et al., 2011; Hooke et al., 2015; Meireles et al., 2017; Michaud et al., 2016; Monnet et al., 2007; Most et al., 2004; Piazza and Cavanagh, 2000; Sauret et al., 2016; Trepczynski et al., 2014; Trepczynski et al., 2012). The general concept of describing joint kinematics using screw theory is well established (Corke, 2017; Hunt, 1990; Reuleaux, 1875) and a number of approaches for determining an axis of rotation have been described in the literature (Cerveri et al., 2005; Ehrig et al., 2007; Gamage and Lasenby, 2002; Halvorsen et al., 1999; Schwartz and Rozumalski, 2005). However, the helical axes methods which include e.g. the Finite Helical Axis (FHA) and the Instantaneous Helical Axis (IHA) appear to be among the most often utilised approaches and the importance of the helical axes methods more generally is reflected in a wide spectrum of application areas including analyses of the movement of the hip (Besier et al., 2003; Camomilla et al., 2006), the intact and replaced knee (Akbari Shandiz et al., 2016; Besier et al., 2003; Colle et al., 2012; Colle et al., 2016; De Rosario et al., 2017), the talocrural and subtalar joints of the foot (Beimers et al., 2008; Ferraresi et al., 2017; Lewis et al., 2006; Sheehan, 2010), the shoulder (Amabile et al., 2016; De Rosario et al., 2014; Lempereur et al., 2010; Monnet et al., 2007; Nikooyan et al., 2011; Wu et al., 2005), the elbow (Stokdijk et al., 1999; Veeger and Yu, 1996), the joints of the wrist and hand (Goislard de Monsabert et al., 2014; Hooke et al., 2015; Pfaeffle et al., 2005; Tay et al., 2010), as well as the analysis of the motion of the spine (Aiyangar et al., 2017; Anderst et al., 2013; Beyer et al., 2015), the assessment of the temporomandibular joint (Hayashi et al., 2009), and the movement of teeth (Hayashi et al., 2007). Whilst the IHA has a physical meaning and the FHA is considered to be a theoretical axis, they are related yet computationally different approaches based on the helical axis concept, where the motion of a segment relative to another at a specific time frame is described by a rotation around a line in space and a translation along this line. The use of this concept in biomechanics dates back to the nineteen seventies, see e.g. (Soudan et al. (1979), but has acquired considerable popularity through a series of seminal papers by Woltring and co-workers (Woltring and Huiskes, 1985; Woltring et al., 1983; Woltring, 1990; Woltring et al., 1987; Woltring et al., 1985; Woltring et al., 1994), and Spoor and Veldpaus (1980). Since then, both types of helical axes were studied in many publications and are now routinely used in motion analysis. Both helical axis methods were furthermore used to estimate joint centre and joint axis positions. Here, the centre is often defined as the point which is nearest to all elements of a set of helical axes while the axis is defined as the line which is most parallel to all helical axes, although they may also be derived from the optimisation of a cost function. More recently, an alternative methodology for determining an axis of rotation from motion data - SARA, Symmetrical Axis of Rotation Approach - was developed by Ehrig and co-workers (2007) through a different mathematical approach to identify the axis based on singular value decomposition of a set of rotation matrices.

A number of studies have attempted to compare the different algorithms, however, there is no general agreement about the conditions under which each of the methods would perform best or when a method is likely to produce less accurate results compared to its competitors. Whilst some theoretical analyses exist in the literature (De Rosario et al., 2017; De Rosario et al., 2014), we are not aware of any work presenting in detail a sound mathematical analysis of the respective algorithms and their intrinsic relationships. Because comparisons are typically performed for specific, selected data sets only, the external validity of any conclusion regarding the performance of the methods based on experimental data remains inherently limited (Colle et al., 2016; Gastaldi et al., 2015; Monnet et al., 2007).

Therefore, we derive the exact mathematical relationships between three commonly used algorithms for computing joint axes from motion data to provide a sound mathematical framework to objectively evaluate the relative strengths and weaknesses of the respective algorithms.

Mathematical Preliminaries

For the ease of presentation we assume that one segment of the joint is stationary. Under these conditions it is possible to describe the movement of the non-stationary segment by time (τ) dependent rigid body transformations, i.e. rotations $R(\tau)$ and translations $t(\tau)$ that transform local coordinates defined on the moving segment into global coordinates. In practice, only discrete values R_i and t_i will be known.

Rotation matrices are typically interpreted as orthogonal 3×3 matrices with a determinant equal to one. For clarity of the derivations here it is advantageous to express rotations directly in terms of the rotation axis unit vector k, defined in the stationary system, and the angle of rotation ϑ around that axis using the Rodrigues formula (Shabana, 2005) in the form:

$$R = \cos\vartheta \; I + \sin\vartheta \; \hat{k} + \; (1 - \cos\vartheta) \; kk^T \tag{1}$$

where I is the 3×3 identity matrix and $\hat{k}$ is the cross-product matrix, i.e.:

$$\hat{k} = \begin{pmatrix} 0 & -k_3 & k_2 \\ k_3 & 0 & -k_1 \\ -k_2 & k_1 & 0 \end{pmatrix}. \tag{2}$$

where $\hat{k}v = k \times v$ holds for any vector v. With these definitions one may easily check that the following relationships, which will be repeatedly used in the following, also hold true:

$$\hat{k}k = 0, \quad kk^T kk^T = kk^T, \quad \hat{k}\hat{k} + I = kk^T . \tag{3}$$

The angle ϑ and the vector k can be determined from R for $\vartheta \neq 0, \pi$ using the relations:

$$\vartheta = \operatorname{acos}\left(\frac{\operatorname{tr}(R) - 1}{2}\right), \quad k = \frac{1}{2\sin\vartheta}(r_{32} - r_{23}, r_{13} - r_{31}, r_{21} - r_{12}) \tag{4}$$

where $\operatorname{tr}(R)$ is the trace of the matrix R.

Helical Axes

The determination of a helical axis requires information about the moving segment at two discrete instances of the motion (time frames), usually given by two rotations R_0, R_1, which, together with the associated translation vectors t_0, t_1, set up the transformations from local to global coordinates. In the following we will construct a continuous interpolation between these two time frames. For this purpose we apply the Rodrigues formula from equation (1) to the transformation $R_1 R_0^T$ which describes the rotational part of the transformation from the segment position in time frame 0 to those in time frame 1 and obtain:

$$R_1 R_0^T = \cos\vartheta \; I + \sin\vartheta \, \hat{k} + (1 - \cos\vartheta)\, kk^T , \tag{5}$$

with the axis unit vector k and the angle of rotation ϑ, i.e. $R_1 R_0^T k = k$, holds, which defines k as the rotation axis of $R_1 R_0^T$.

We further define a parameterized interpolation of $R(\tau)$ between R_0 and R_1 over a unit time step by:

$$R(\tau) = \left(\cos(\tau\vartheta)\ I + \sin(\tau\vartheta)\ \hat{k} + (1 - \cos(\tau\vartheta))\ kk^T\right) R_0\ ,\quad 0 \leq \tau \leq 1\ . \qquad (6)$$

In this notation, we have $R(0) = R_0$ and $R(1) = R_1$ [1]. It is of note that $R(\tau)$ in (6) is not an arbitrary parametrization but is in fact the parameterization of the shortest geodesic path in $SO(3)$, the Lie-group of orthogonal transformations in $\mathbb{R}^3$, from R_0 to R_1. We use a time parameterization for the translation $t(\tau), 0 \leq \tau \leq 1,$ between $t_0 = t(0)$ and $t_1 = t(1)$. We further consider that each point on the local axis is transformed from local coordinates c_{L} of the moving segment to a point c_{G} in the global representation of the helical axis by the equation:

$$R(\tau)\ c_{\mathrm{L}} + t(\tau) = c_{\mathrm{G}}\ , \qquad (7)$$

(Ehrig et al., 2006). To obtain a similar parameterization for $t(\tau)$ as for $R(\tau)$ in equation (6) we decompose $t(\tau)$ in components perpendicular to the axis vector k and parallel to k:

$$t(\tau) = (I - kk^T)\ t(\tau) + kk^T\ t(\tau)\ . \qquad (8)$$

Using equation (7) the first term in the right hand side (r.h.s) of this equation can be rewritten as:

$$\begin{aligned}(I - kk^T)\ t(\tau) &= (I - kk^T)(c_{\mathrm{G}} - R(\tau)c_{\mathrm{L}}) \\ &= (I - kk^T)\left(c_{\mathrm{G}} - R(\tau)\ R_0^T(c_{\mathrm{G}} - t_0)\right). \qquad (9)\end{aligned}$$

This term describes the pure rotational part of the helical motion. For the second term, which captures the translation parallel to k, we assume a constant speed along the axis, i.e.:

[1] Equation (6) is most often written as $R(\tau) = \exp(\tau \log(R_1 R_0^T))\ R_0$, where exp and log are the exponential and logarithmic map, respectively. The alternative formulation here facilitates the following derivations.

$$kk^T\, t(\tau) = kk^T\big(t_0 + \tau(t_1 - t_0)\big)\ . \tag{10}$$

Taken together, (9) and (10) yield:

$$t(\tau) = (I - kk^T)\big(c_\mathrm{G} - \ R(\tau)\, R_0^T(c_\mathrm{G} - t_0)\big) + kk^T\big(t_0 + \tau(t_1 - t_0)\big) \tag{11}$$

$$= (I - kk^T)c_\mathrm{G} - \Big((I - kk^T)\cos(\tau\vartheta)\ + \sin(\tau\vartheta)\, \hat{k}\Big)\,(c_\mathrm{G} - t_0) + kk^T\big(t_0 + \tau(t_1 - t_0)\big)$$

The interpolation schemes for the rotational part R of the motion given in (6) and for the translational part t given in (11) are the formalization of the implicit assumption behind all methods for the determination of helical axes, namely that the motion between two time frames is continuous, and can in fact be described as a rotation around an axis together with a translation parallel to this axis. If these assumptions do not hold, the computed helical axis will not reflect the true kinematics of the joint.

In the following, we will examine whether three commonly used algorithms for determining an axis of rotation, namely the Symmetrical Axis of Rotation Approach - SARA (Ehrig et al., 2007), the Finite Helical Axis approach - FHA (Spoor and Veldpaus, 1980; Woltring et al., 1983; Woltring et al., 1985), and the Instantaneous Helical Axis – IHA (Woltring and Huiskes, 1985; Woltring, 1990; Woltring et al., 1987; Woltring et al., 1994) can correctly identify the direction of the axis vector k and its position c_G from two measurements $(R_0, t_0), (R_1, t_1)$.

The Symmetrical Axis of Rotation Approach - SARA

The Symmetrical Axis of Rotation Approach - SARA (Ehrig et al., 2007) determines axis parameters by solving a linear least squares system. The problem that is solved using SARA for the case in which the motion between two time frames is considered and only one segment is moving can be derived from the mathematical definition of a axis joint, equation (7), as:

$$\begin{pmatrix} R_0 & -I \\ R_1 & -I \end{pmatrix} \begin{pmatrix} c_{\mathrm{L}} \\ c_{\mathrm{G}} \end{pmatrix} = \begin{pmatrix} -t_0 \\ -t_1 \end{pmatrix} . \tag{12}$$

This system may be solved directly w.r.t. c_{G} by elimination of c_{L} which yields

$$(R_1 R_0^T - I) c_{\mathrm{G}} = -t_1 + R_1 R_0^T t_0 \, . \tag{13}$$

The matrix $R_1 R_0^T - I$ has rank 2, therefore the 3x3 system is underdetermined, and the minimum norm solution for c_{G} can be computed through a SVD decomposition as follows: $R_1 R_0^T - I = U \Sigma V^T$, where U and V are the matrices of the left and right singular vectors, and Σ is the diagonal matrix of the singular values. The smallest of the 3 singular values, σ_3, is equal to 0 (a consequence of the matrix $R_1 R_0^T - I$ having rank 2), a condition than can be described by: $R_1 R_0^T v_3 - v_3 = U \Sigma V^T v_3 = 0$. Thus, one obtains $R_1 R_0^T v_3 = v_3$. The right singular vector v_3 associated to the singular value σ_3 is thus identified as the axis unit vector k of $R_1 R_0^T$. Thus the SARA algorithm correctly reproduces the axis vector and its position; the accuracy of the direction of the axis vector depends only on the accuracy of the rotations R_0, R_1, but not on the accuracy of the translation vectors.

The Finite Helical Axis - FHA

The Finite Helical Axis (FHA) approach was developed by Spoor and Veldpaus (1980) and Woltring and co-workers (Woltring et al., 1983; Woltring et al., 1985) for computing the helical axis from two time frame using geometrical considerations. As we will show, the FHA and SARA are closely related and therefore we base the derivation of the FHA on the SARA approach. As a first step, we combine two equations of type (13), one each for $R_0 R_1^T$ and $R_1 R_0^T$,

$$(I - R_1 R_0^T) c_G = t_1 - R_1 R_0^T t_0, \qquad (I - R_0 R_1^T) c_G = t_0 - R_0 R_1^T t_1\,, \tag{14}$$

as follows:

$$(2I - R_1 R_0^T - R_0 R_1^T) c_G = t_0 + t_1 - R_0 R_1^T t_1 - R_1 R_0^T t_0\,. \tag{15}$$

After replacing the matrix $R_1 R_0^T$ by the Rodrigues representation (5) the matrix on the left hand side of this term can be re-written as:

$$2I - R_0 R_1^T - R_1 R_0^T = 2(1 - \cos\vartheta)(I - kk^T)\,. \tag{16}$$

For the norm-minimal point $\tilde{c}_G$ on the axis c_G the following conditions holds: $k^T \tilde{c}_G = 0$, and equation (15) now reads:

$$2(1 - \cos\vartheta)\tilde{c}_G = t_0 + t_1 - R_0 R_1^T t_1 - R_1 R_0^T t_0\,. \tag{17}$$

Using relation (5) again for the r.h.s. of (17) we obtain the following closed solution for c_G:

$$\tilde{c}_G = \frac{(I - kk^T)(1 - \cos\vartheta)(t_0 + t_1) + \sin\vartheta\,\hat{k}(t_1 - t_0)}{2(1 - \cos\vartheta)}$$

$$= (\mathrm{I} - \mathrm{kk}^\mathrm{T})\frac{(\mathrm{t}_0 + \mathrm{t}_1)}{2} + \frac{\mathrm{k} \times (\mathrm{t}_1 - \mathrm{t}_0)}{2\tan\left(\frac{\vartheta}{2}\right)} \tag{18}$$

This latter expression is exactly the original definition of the FHA axis position as it appears in the papers of Woltring, e.g. (Woltring et al., 1985). Moreover, the definition of k and ϑ used in the current manuscript (equations (4)) is identical to the definition of these parameters by Woltring and co-workers (1985). We have therefore proven the following statement:

Statement 1: The FHA algorithm and the SARA approach for two time frames are equivalent.

Since both algorithms are mathematically equivalent they are affected by any errors in the input data in exactly the same way.

The Instantaneous Helical Axis – IHA

The time dependent representation of any point x defined in a local coordinate system, with respect to a global coordinate system, is given by $y(\tau) = R(\tau)x + t(\tau)$. Following the original definition in (Woltring and Huiskes, 1985; Woltring, 1990; Woltring et al., 1987; Woltring et al., 1994) the IHA at time point τ is defined as the set of points with minimum velocity $|\dot{y}(\tau)|$ which yields the following time dependent representation for the position of the IHA:

$$c_{\mathrm{G,IHA}}(\tau) = t(\tau) + \omega(\tau) \times \frac{\dot{t}(\tau)}{\omega^T(\tau)\,\omega(\tau)}\ , \tag{19}$$

with the direction vector of the angular velocity $\omega(\tau)$. Note that $c_{\mathrm{G,IHA}}(\tau)$ is not necessarily the point on the helical axis with minimum norm. We refrain from presenting the details of the derivation of equation (19) and refer the interested reader to the detailed revision of Woltring's original derivation provided by Reichl and Auzinger (2012). The derivation shows that the cross-product matrix $\widehat{\omega}$ associated to the vector ω can be written as $\widehat{\omega} = \dot{R}R^T$ and the computation of the IHA requires the time derivatives of R and t. These derivatives are usually estimated by discretization, a process for which different suggestions exist in the literature (Reichl and Auzinger, 2012; Woltring et al., 1994). Here, using the representations (6) and (11) for $R(\tau)$ and $t(\tau)$, we obtain exact expressions for the derivatives w.r.t. the time τ i.e.

$$\dot{R}(\tau) = \vartheta\left(-\sin(\tau\vartheta)\ I + \cos(\tau\vartheta)\,\hat{k} + \sin(\tau\vartheta)\,kk^T\right) R_0\ , \tag{20}$$

$$\dot{t}(\tau) = \vartheta\left[\sin(\tau\vartheta)\,(I - kk^T) - \cos(\tau\vartheta)\,\hat{k}\right](c_{\mathrm{G}} - t_0) + kk^T(t_1 - t_0\,) \tag{21}$$

Using the relationships between k and $\hat{k}$, equations (3), yields the simple equation, see derivation S1 in the supplementary material,

$$\hat{\omega} = \dot{R}(\tau)R^T(\tau) = \vartheta\hat{k}. \quad (22)$$

Thus:

$$\omega = \vartheta k. \quad (23)$$

As expected, the unit direction vector of the instantaneous helical axis is identical to the direction k of the rotation axis of the matrix $R_1 R_0^T$, and, of course, also the finite helical axis.

In order to determine the position of the axis, we insert the terms for $t(\tau)$, equation (11), ω, equation (23), $\dot{t}(\tau)$, equation (21), $\dot{R}(t)$, equation(20), into equation (19) and obtain after some calculations (for details see derivation S2 in the supplementary material) the following relationship: $c_{\mathrm{G,IHA}}(\tau) = (I - kk^T)c_{\mathrm{G}} + kk^T t_0$. Moreover, for the minimum norm point on the axis we obtain $\tilde{c}_{\mathrm{G,IHA}} = (I - kk^T)c_G = \tilde{c}_{\mathrm{G}}$. Thus, also the axis position is correctly reproduced by the *analytical* IHA approach.

In practice, however, equation (21) for the time derivative $\dot{t}(\tau)$ cannot be used since c_{G} is not previously known. When positional rather than velocity data serves as input, $\dot{t}(\tau)$ is thus usually estimated by time discretisation of $t(\tau)$ using finite differences or by analytic derivatives of spline interpolants for $t(\tau)$ (Woltring and Huiskes, 1985; Woltring, 1990; Woltring et al., 1994). To further explore this case we analyse the result for the simplest approach, a linear interpolation between t_0 and t_1, even though we would certainly recommend using such methodology in practice only if the angle ϑ is very small. With a linear interpolation $t(\tau) = t_0 + \tau(t_1 - t_0)$, we obtain $\dot{t}(\tau) = t_1 - t_0$. For a particular point $c_{\mathrm{G,IHA}}$ on the helical axis we then obtain, using equations (19) and (23), and for the mean of t_0 and t_1, i.e. for $(t_0 + t_1)/2$, (c.f. Woltring et al., 1985):

$$c_{\mathrm{G,IHA}} = \frac{t_0 + t_1}{2} + k \times \frac{t_1 - t_0}{\vartheta}, \tag{24}$$

The unique point $\tilde{c}_{\mathrm{G,IHA}}$ on the helical axis with minimum norm is then given by:

$$\tilde{c}_{\mathrm{G,IHA}} = (I - kk^T)\,\frac{t_0 + t_1}{2} + k \times \frac{t_1 - t_0}{\vartheta}\,. \tag{25}$$

Equation (25) is very similar to equation (18) obtained for the FHA, only the denominators in the second terms differ: whilst this denominator is here ϑ, it is $2\tan\left(\frac{\vartheta}{2}\right)$ for the FHA. However, the numerical differences between ϑ and $2\tan\left(\frac{\vartheta}{2}\right)$ are small only for very small angles. Thus, compared to the FHA, the position of the IHA is affected by an error perpendicular to the axis direction when using positional data as input. The reason for this error is the interpolation for $t(\tau)$, which approximates the circular arc around the helical axis, here using a straight line. The error may be substantially reduced if more accurate approximations for $\dot{t}(\tau)$ were used and if this particular source of error was completely eliminated, e.g. when using accurate measurement data of velocities as input, the results would be identical to those of the FHA approach. We should note that additional errors which may be introduced by numerical approximations of $\dot{R}(\tau)$ are not considered here. We summarize these considerations by the following statement:

Statement 2: The analytical solution of the IHA approach for two time frames is equivalent with the FHA or SARA approach. If the derivatives of t are calculated by numerical discretization because positional data are used as input the IHA algorithm yields a helical axis position which may be affected by an error perpendicular to the true axis direction, i.e. an error in the axis position. The axis direction k is identical to those computed with the FHA or SARA approach, i.e. the true axis direction, as long as the direction vector of the angular velocity $\omega = \vartheta k$ is known exactly, but additional errors in both axis position and direction may be introduced by numerical approximation of $\dot{R}$.

If the velocity $\dot{t}$ and the angular velocity ω can be measured accurately, e.g. by inertial measurement units (IMUs), the determination of the axis parameters will not suffer from discretization errors. Using equation (19) one will therefore obtain the same result as with the FHA or SARA methodologies.

Mean Finite and Mean Instantaneous Helical axes and their Relation to SCoRE and SARA

A further extension and important application of the helical axis methodology is the computation of a joint centre as the point with minimum root mean square distance from a set of helical axes, c.f. (Camomilla et al., 2006; De Rosario et al., 2014; Woltring, 1990), an approach that has therefore been referred to as *mean helical axis* (MHA) approach. As a next step, we will therefore analyze the relationship between the centre computed using either a helical axes approach or the Symmetric Centre of Rotation Estimation (SCoRE) method (Ehrig et al., 2006). We firstly consider a set of finite helical axes, and thus denote the resulting algorithm as mean *finite* helical axis approach, MHFA; this approach for locating centres of rotation is sometimes also called the helical pivot method (Camomilla et al., 2006; De Rosario et al., 2017; De Rosario et al., 2014; Woltring, 1990).

Given a set of n time frames, the first step for determining the MFHA, is to compute the FHA positions c_{ij} with minimal norm and the direction vectors k_{ij} for all possible combinations $i, j = 1, \dots, n, i < j$. Since the determination of the FHA is sensitive to measurement errors in particular if the FHA is determined from small ranges of motion (c.f. equation (18)), empirical weighting terms w_{ij} are typically used which depend on the angle ϑ_{ij} in a manner that $w_{ij} \to 0$ for $\vartheta_{ij} \to 0$ and thereby minimize the influence of less optimally defined FHAs. The minimization problem defines the joint centre c_{MFHA} and is then given by:

$$c_{\mathrm{MFHA}} = \underset{c\in\mathbb{R}^3}{\arg\min} \sum_{i,j=1,i<j}^{n} w_{ij} \left\{ (c - c_{ij}) - \left[(c - c_{ij})^{T} k_{ij} \right] k_{ij} \right\}^2 . \tag{26}$$

see e.g. (Woltring, 1990; Stokdijk et al., 1999; Camomilla et al., 2006). Using $k_{ij}^T c_{ij} = 0$ it can be shown that the minimizer of the functional in (26) is the least squares solution of the linear system:

$$\sum_{i,j=1,i<j}^{n} w_{ij}\left(I - k_{ij}k_{ij}^T\right) c_{\mathrm{MHFA}} = \sum_{i,j=1,i<j}^{n} w_{ij} c_{ij} \tag{27}$$

As shown in supplementary material S3 c_{MHFA} is identical to the closed form solution of the SCoRE approach for the determination of the centre of rotation of a ball joint if one defines the weighting factors as $w_{ij} = \sin^2\left(\frac{\vartheta_{ij}}{2}\right)$. We have thus proven the following generalization of the statement 1:

Statement 3: Using weighting factors $w_{ij} = \sin^2\left(\frac{\vartheta_{ij}}{2}\right)$ the mean finite helical axis (MFHA) method as an approach for the joint centre determination is equivalent to the SCoRE algorithm.

Yet, any definition of the weighting terms differing from the setting $w_{ij} = \sin^2\left(\frac{\vartheta_{ij}}{2}\right)$ chosen above leads to a system of equations which is very different from the SCoRE approach, and therefore also results in a different solution. Woltring and co-workers (1985) proposed an empirical weighting factor $w_{ij} = \sin\left(\frac{\vartheta_{ij}}{2}\right)$. Whilst their algorithm seems to be widely used in the literature (Camomilla et al., 2006; Tay et al., 2010), only the weighting $w_{ij} = \sin^2\left(\frac{\vartheta_{ij}}{2}\right)$ leads to a solution which optimally fits to the set of equations defining a joint centre, i.e. $R_i c_G + t_i = R_j c_G + t_j$ for all possible combinations $i, j = 1, \ldots, n, i < j$. Moreover, detailed tests in simulations (Ehrig et al., 2006) and experimental validation (Monnet et al., 2007) strongly suggest that SCoRE yields noticeably more precise results especially for small ranges of motion.

Mean Finite Helical Axes and their Relation to SARA

Given a set of helical axes, following again Woltring (1990), one may further define an optimal direction vector k through c_{MHFA}, which is as "parallel as possible" to all helical axes. Together with the specification of the position of a joint centre c_{MHFA} this approach then also provides a representation of an axis of rotation. As a next step, we will therefore analyze the relationship between such an optimal helical axis and an axis computed using the SARA method. In order to derive an axis of rotation satisfying such criteria, we consider the minimization of the following expression over all vectors k with $|k| = 1$:

$$k^T \sum_{i,j=1,i<j}^{n} w_{ij}\left(I - k_{ij}k_{ij}^T\right)\ k\ . \tag{28}$$

Expression (28) provides an implementation of the idea to define an optimal direction vector as described above; the more parallel k is to all k_{ij} the smaller the whole term will be. A detailed discussion of the expression (28) may also be found in Woltring (1990). Defining the weighting factors in (28) again as $w_{ij} = 1 - \cos\vartheta_{ij}$ and further proceeding in the same way as described in the supplementary material (S3) it follows that the determination of the optimal direction vector k requires to minimize:

$$k^T \left(n(n-1)I_3 - \sum_{i,j=1,i \neq j}^{n} R_i R_j^T \right) k. \tag{29}$$

This expression is minimal if k is chosen as the eigenvector associated to the smallest eigenvalue of the symmetric matrix given in brackets or, correspondingly, the eigenvector associated to the largest eigenvalue of $\sum_{i,j=1}^{n} R_i R_j^T$. Together with c_{MHFA} this k then defines the unit rotation vector of the mean finite helical axis. This approach is exactly the SARA algorithm as described by Ehrig and co-workers (2007, supplemental material). We summarize these findings in the following statement:

Statement 4: With the weighting factors chosen to be $w_{ij} = \sin^2\left(\frac{\vartheta_{ij}}{2}\right)$ the mean finite helical axis method as an approach for determination of an axis of rotation is equivalent to the SARA algorithm.

For any other choice of values for the weighting factors one may also compute estimates for the helical axis direction. The results, however, will be significantly different, and the error increases with a decreasing range of motion.

Mean Instantaneous Helical Axes and their Relation to SARA

Mean *instantaneous* helical (MIHA) axes were also discussed in the literature; see e.g. (De Rosario et al., 2017; De Rosario et al., 2014; Monnet et al., 2007; Stokdijk et al., 1999; Veeger and Yu, 1996; Woltring, 1990). Whilst the IHA approach reproduces the exact direction k_{ij} of the helical axis for each pair of frames, the positions of the axes derived in this manner are error-prone (statement 2), especially for larger rotation angles. It is therefore difficult to estimate the error in the computational results of the position of a joint centre or axis by an approach similar to equation (26) or (28). However, within the derivation of the equivalence between SARA and mean helical axes the specifics of how these axes were determined, i.e. whether they are FHA, IHA axes based on measurements of positions or velocities, or any other method, was not used in any way. Thus, also for computation of the mean IHA the weighting $w_{ij} = \sin^2\left(\frac{\vartheta_{ij}}{2}\right)$ is optimal since this is the only weighting which is consistent with the definition of a joint centre, equation (26), or a mean helical axis, equation (28). Since the inexact positions are not involved in the computation of the mean helical axis direction, see equation (27), the computed mean axis direction is identical to the corresponding result of the MFHA or SARA as long as errors arise from discretizations of $\dot{t}$ only.

Discussion

This study sought to derive exact mathematical relationships between three commonly used algorithms for computing joint axes from motion data. Here we have shown that the SARA/SCoRE methods are numerically equivalent to FHA approaches when computing an axis for two frames. This equivalence holds also more generally if the axes are computed from more frames i.e. the MFHA is used, as long as the weighting factors are appropriately chosen. If such weighting factors are sub-optimally chosen, e.g. using those originally proposed by Woltring and co-workers (1985), a small yet noticeable difference in the Axis of Rotation (AoR) parameters can be observed (Ehrig et al. 2007, Figures 3 and 4). The same equivalence of helical axes and SARA does not generally hold for the IHA approach. Whilst an IHA method based on velocities is mathematically equivalent to SARA and therefore also to FHA, care needs to be taken if position data serves as input to the method. Fed with such position data the IHA requires the calculation of time derivatives of the rotations and translations which can only be approximated numerically. Under these conditions IHA derived axes may suffer from reduced accuracy. If the IHA could be computed without any errors that would typically result from the time discretization of $\dot{t}$ and $\dot{R}$ identical results to those from the FHA approach could be obtained. Since it is next to impossible to completely eliminate any such errors when using positional data as input, IHA axes derived under such conditions may suffer from errors in the axis position due to the discretisation of $\dot{t}$ and from errors in both the axis position and direction due to the discretisation of $\dot{R}$. Using highly accurate direct measurement of velocities which may be possible using e.g. inertial measurement units (IMUs) such problems will likely be substantially reduced and may result in an increased appreciation of the IHA approach in future.

The FHA and IHA approaches were already compared by Woltring and co-workers (1994) who suggested that both methods are less accurate for small angles. At the time, tools for the direct measurement of angles or velocities were not available and the IHA approach was based on positional data. Under these conditions, according to the analyses of Wachowski and co-workers (2010), IHA parameters can be reliably computed only if the frames R_0, t_0 and R_1, t_1 are sufficiently close, i.e. "differentially close-by". However, accurate determination of these rotations and translations is known to be difficult for small angular velocities ω (Stokdijk et al., 1999), requiring angular velocities and therefore also ϑ to remain above a critical lower limit which is in direct conflict with the above requirement for frames to be "differentially close-by". This issue is further amplified if non-optimal weighting factors (e.g. $w_{ij} = 1$) are used for the calculation of the IHA. Our analyses suggest that the FHA, as long as the weightings are appropriately chosen, and SARA provide the best approach to compute an AoR for small angles when using positional data. When accurate measurements of linear and angular velocities are not available or when using positional data as input the use of the IHA is therefore not recommended.

Although methods for computing joint axes have been compared in a number of experimental studies, comparisons specifically between FHA, IHA or SARA are only available from a small number of studies (Colle et al., 2016; De Rosario et al., 2014). The study by De Rosario and co-workers (2014) provided evidence that an improvement in axis precision can be obtained by the specific choice of weightings when using the mean IHA, but the chosen weightings differed from the optimal ones and the authors did not evaluate the FHA. The study by Colle and co-workers (2016) compared implementations of a mean FHA and SARA approach and found differences in the angles between the anatomical transepicondylar axis (TEA) and the axes computed from either algorithm. Whilst one may fundamentally challenge the use of such models to describe the motion of the knee joint more generally, in the context of this manuscript it is most relevant that details regarding the helical axes (FHA vs. IHA, weightings) are not provided, however, and it remains unclear whether the observed differences resulted from the use of an IHA approach and the related limitations, from the use of sub-optimal weightings with the FHA, or their combination. In view of the theoretical derivation of the conditions for equivalence between helical axis methods and SARA presented here, recommendations on the use of either method for the computation of an AoR should be critically re-considered. Whilst the analyses presented here focused on deep theoretical analyses only with the aim to eliminate the need for data driven comparisons for seemingly different methods that are in fact intrinsically equivalent, this does not remove the need for more detailed analyses (De Rosario et al., 2017; De Rosario et al., 2013; De Rosario et al., 2014) of the behavior of methods that are truly distinct. Whilst it remains challenging to obtain a comprehensive understanding of the joint function, recent studies utilising the SARA method suggest that this approach is sufficiently sensitive to capture the load dependency of a functional AoR (Meireles et al., 2017). The deep understanding of the conditions of mathematical equivalence between the SARA/SCoRE approaches and the helical axis methods derived here enables users of either approach to focus on addressing such important questions on the load and activity dependency of joint function including the influence of soft tissue artefact (De Rosario et al., 2017; De Rosario et al., 2013) in the future and eliminates the need to consider the effect of the chosen analysis method when doing so. Such ability to focus on a more comprehensive understanding of the dynamic function of joints is increasingly important as emerging imaging and sensor technology are offering more detailed studies of kinematics than ever before.

Conclusion

Through the derivation of a sound mathematical framework to objectively compare the algorithms we could demonstrate that the FHA algorithm and the SARA approach are equivalent for the analysis of two time frames. For the IHA the same statements holds only if its implementation is based on direct measurements of linear and angular velocities. If on the other hand an IHA algorithm is based on the use of marker positions, the helical axis position may be affected by a systematic error perpendicular to the true axis direction, whereas the axis direction is identical to those computed with the FHA or SARA approach, i.e. the true axis direction. Finally, with an appropriate choice of weighting factors the MFHA and the MIHA (under the conditions described above) methods are equivalent to the SCoRE algorithm for determination of a CoR, and similarly, equivalent to the SARA algorithm for determination of an AoR.

Acknowledgements

We acknowledge the support of the research provided by the German Research Foundation (DFG EH-422/1-1), the Engineering and Physical Science Research Council (EP/K034847/1), and Arthritis Research UK (grant reference: 20194). The funders had no role in study design, data collection and analysis, decision to publish, or preparation of the manuscript.

References

Aiyangar, A., Zheng, L., Anderst, W., Zhang, X., 2017. Instantaneous centers of rotation for lumbar segmental extension in vivo. J Biomech 52, 113-121.

Akbari Shandiz, M., Boulos, P., Saevarsson, S.K., Yoo, S., Miller, S., Anglin, C., 2016. Changes in knee kinematics following total knee arthroplasty. Proc Inst Mech Eng H 230, 265-278.

Amabile, C., Bull, A.M.J., Kedgley, A.E., 2016. The centre of rotation of the shoulder complex and the effect of normalisation. J Biomech 49, 1938-1943.

Anderst, W., Baillargeon, E., Donaldson, W., Lee, J., Kang, J., 2013. Motion path of the instant center of rotation in the cervical spine during in vivo dynamic flexion-extension: implications for artificial disc design and evaluation of motion quality after arthrodesis. Spine 38, E594-601.

Beimers, L., Tuijthof, G.J., Blankevoort, L., Jonges, R., Maas, M., van Dijk, C.N., 2008. In-vivo range of motion of the subtalar joint using computed tomography. J Biomech 41, 1390-1397.

Besier, T.F., Sturnieks, D.L., Alderson, J.A., Lloyd, D.G., 2003. Repeatability of gait data using a functional hip joint centre and a mean helical knee axis. J Biomech 36, 1159-1168.

Beyer, B., Sholukha, V., Salvia, P., Rooze, M., Feipel, V., Van Sint Jan, S., 2015. Effect of anatomical landmark perturbation on mean helical axis parameters of in vivo upper costovertebral joints. J Biomech 48, 534-538.

Boeth, H., Duda, G.N., Heller, M.O., Ehrig, R.M., Doyscher, R., Jung, T., Moewis, P., Scheffler, S., Taylor, W.R., 2013. Anterior cruciate ligament-deficient patients with passive knee joint laxity have a decreased range of anterior-posterior motion during active movements. Am J Sports Med 41, 1051-1057.

Bonny, D.P., Howell, S.M., Hull, M.L., 2017. Changes in the rotational axes of the tibiofemoral joint caused by resection of the anterior cruciate ligament. J Orthop Res 35, 886-893.

Camomilla, V., Cereatti, A., Vannozzi, G., Cappozzo, A., 2006. An optimized protocol for hip joint centre determination using the functional method. J Biomech 39, 1096-1106.

Cerveri, P., Lopomo, N., Pedotti, A., Ferrigno, G., 2005. Derivation of centers and axes of rotation for wrist and fingers in a hand kinematic model: methods and reliability results. Ann Biomed Eng 33, 402-412.

Colle, F., Bignozzi, S., Lopomo, N., Zaffagnini, S., Sun, L., Marcacci, M., 2012. Knee functional flexion axis in osteoarthritic patients: comparison in vivo with transepicondylar axis using a navigation system. Knee surgery, sports traumatology, arthroscopy : official journal of the ESSKA 20, 552-558.

Colle, F., Lopomo, N., Visani, A., Zaffagnini, S., Marcacci, M., 2016. Comparison of three formal methods used to estimate the functional axis of rotation: an extensive in-vivo analysis performed on the knee joint. Comput Methods Biomech Biomed Engin 19, 484-492.

Corke, P., 2017. Robotics, Vision and Control. Fundamental Algorithms In MATLAB® Second, Completely Revised, Extended And Updated Edition. ed. Springer International Publishing.

De Rosario, H., Page, A., Besa, A., 2017. Analytical study of the effects of soft tissue artefacts on functional techniques to define axes of rotation. J Biomech 62, 60-67.

De Rosario, H., Page, Á., Besa, A., Valera, Á., 2013. Propagation of soft tissue artifacts to the center of rotation: A model for the correction of functional calibration techniques. J Biomech 46, 2619-2625.

De Rosario, H., Page, Á., Mata, V., 2014. Point of optimal kinematic error: Improvement of the instantaneous helical pivot method for locating centers of rotation. J Biomech 47, 1742-1747.

Ehrig, R.M., Taylor, W.R., Duda, G.N., Heller, M.O., 2006. A survey of formal methods for determining the centre of rotation of ball joints. J Biomech 39, 2798-2809.

Ehrig, R.M., Taylor, W.R., Duda, G.N., Heller, M.O., 2007. A survey of formal methods for determining functional joint axes. J Biomech 40, 2150-2157.

Ellingson, A.M., Nuckley, D.J., 2015. Altered helical axis patterns of the lumbar spine indicate increased instability with disc degeneration. J Biomech 48, 361-369.

Ferraresi, C., De Benedictis, C., Franco, W., Maffiodo, D., Leardini, A., 2017. In-vivo analysis of ankle joint movement for patient-specific kinematic characterization. Proc Inst Mech Eng H 231, 831-838.

Gamage, S.S.H.U., Lasenby, J., 2002. New least squares solutions for estimating the average centre of rotation and the axis of rotation. J Biomech 35, 87-93.

Gastaldi, L., Lisco, G., Pastorelli, S., 2015. Evaluation of functional methods for human movement modelling. Acta Bioeng Biomech 17, 32-38.

Goislard de Monsabert, B., Visser, J.M., Vigouroux, L., Van der Helm, F.C., Veeger, H.E., 2014. Comparison of three local frame definitions for the kinematic analysis of the fingers and the wrist. J Biomech 47, 2590-2597.

Halvorsen, K., Lesser, M., Lundberg, A., 1999. A new method for estimating the axis of rotation and the center of rotation. J Biomech 32, 1221-1227.

Hayashi, K., Reich, B., DeLong, R., Lee, S.P., Mizoguchi, I., 2009. A novel statistical model for mandibular helical axis analysis. Journal of oral rehabilitation 36, 102-109.

Hayashi, K., Uechi, J., Lee, S.P., Mizoguchi, I., 2007. Three-dimensional analysis of orthodontic tooth movement based on XYZ and finite helical axis systems. European journal of orthodontics 29, 589-595.

Heller, M.O., Kratzenstein, S., Ehrig, R.M., Wassilew, G., Duda, G.N., Taylor, W.R., 2011. The weighted optimal common shape technique improves identification of the hip joint center of rotation in vivo. J Orthop Res 29, 1470-1475.

Hooke, A.W., Pettersson, K., Sagerfors, M., An, K.N., Rizzo, M., 2015. An anatomic and kinematic analysis of a new total wrist arthroplasty design. Journal of wrist surgery 4, 121-127.

Hunt, K.H., 1990. Kinematic Geometry of Mechanisms. Oxford University Press, New York.

Lempereur, M., Leboeuf, F., Brochard, S., Rousset, J., Burdin, V., Rémy-Néris, O., 2010. In vivo estimation of the glenohumeral joint centre by functional methods: Accuracy and repeatability assessment. J Biomech 43, 370-374.

Lewis, G.S., Sommer, H.J., 3rd, Piazza, S.J., 2006. In vitro assessment of a motion-based optimization method for locating the talocrural and subtalar joint axes. J Biomech Eng 128, 596-603.

Meireles, S., De Groote, F., Van Rossom, S., Verschueren, S., Jonkers, I., 2017. Differences in knee adduction moment between healthy subjects and patients with osteoarthritis depend on the knee axis definition. Gait Posture 53, 104-109.

Michaud, B., Jackson, M., Arndt, A., Lundberg, A., Begon, M., 2016. Determining in vivo sternoclavicular, acromioclavicular and glenohumeral joint centre locations from skin markers, CT-scans and intracortical pins: A comparison study. Med Eng Phys 38, 290-296.

Monnet, T., Desailly, E., Begon, M., Vallée, C., Lacouture, P., 2007. Comparison of the SCoRE and HA methods for locating in vivo the glenohumeral joint centre. J Biomech 40, 3487-3492.

Most, E., Axe, J., Rubash, H., Li, G., 2004. Sensitivity of the knee joint kinematics calculation to selection of flexion axes. J Biomech 37, 1743-1748.

Nikooyan, A.A., van der Helm, F.C., Westerhoff, P., Graichen, F., Bergmann, G., Veeger, H.E., 2011. Comparison of two methods for in vivo estimation of the glenohumeral joint rotation center (GH-JRC) of the patients with shoulder hemiarthroplasty. PLoS One 6, e18488.

Pfaeffle, J., Blankenhorn, B., Stabile, K., Imbriglia, J., Goitz, R., Robertson, D., 2005. Development and validation of a computed tomography-based methodology to measure carpal kinematics. J Biomech Eng 127, 541-548.

Piazza, S.J., Cavanagh, P.R., 2000. Measurement of the screw-home motion of the knee is sensitive to errors in axis alignment. J Biomech 33, 1029-1034.

Reichl, I., Auzinger, W., 2012. Identifying Tibio-Femoral Joint Kinematics: Individual Adjustment versus Numerical Robustness. IFAC Proceedings Volumes 45, 819-824.

Reuleaux, F., 1875. Theoretische Kinemataik. Grundzüge einer Theorie des Maschinenwesens. Vieweg und Sohn, Braunschweig.

Sauret, C., Pillet, H., Skalli, W., Sangeux, M., 2016. On the use of knee functional calibration to determine the medio-lateral axis of the femur in gait analysis: Comparison with EOS biplanar radiographs as reference. Gait Posture 50, 180-184.

Schwartz, M.H., Rozumalski, A., 2005. A new method for estimating joint parameters from motion data. J Biomech 38, 107-116.

Shabana, A.A., 2005. Dynamics of Multibody Systems, 3 ed. Cambridge University Press, Cambridge.

Sheehan, F.T., 2010. The instantaneous helical axis of the subtalar and talocrural joints: a non-invasive in vivo dynamic study. J Foot Ankle Res 3, 13.

Soudan, K., Van Audekercke, R., Martens, M., 1979. Methods, difficulties and inaccuracies in the study of human joint kinematics and pathokinematics by the instant axis concept. Example: The knee joint. J Biomech 12, 27-33.

Spoor, C.W., Veldpaus, F.E., 1980. Rigid body motion calculated from spatial co-ordinates of markers. J Biomech 13, 391-393.

Stokdijk, M., Meskers, C.G., Veeger, H.E., de Boer, Y.A., Rozing, P.M., 1999. Determination of the optimal elbow axis for evaluation of placement of prostheses. Clin Biomech 14, 177-184.

Tay, S.C., van Riet, R., Kazunari, T., Amrami, K.K., An, K.N., Berger, R.A., 2010. In-vivo kinematic analysis of forearm rotation using helical axis analysis. Clin Biomech 25, 655-659.

Trepczynski, A., Kutzner, I., Bergmann, G., Taylor, W.R., Heller, M.O., 2014. Modulation of the relationship between external knee adduction moments and medial joint contact forces across subjects and activities. Arthritis Rheumatol 66, 1218-1227.

Trepczynski, A., Kutzner, I., Kornaropoulos, E., Taylor, W.R., Duda, G.N., Bergmann, G., Heller, M.O., 2012. Patellofemoral joint contact forces during activities with high knee flexion. J Orthop Res 30, 408-415.

Veeger, H.E.J., Yu, B., Year Orientation of axes in the elbow and forearm for biomechanical modelling. In Proceedings of the 1996 Fifteenth Southern Biomedical Engineering Conference.

Wachowski, M.M., Hawellek, T., Hubert, J., Lehmann, A., Mansour, M., Dumont, C., Dorner, J., Raab, B.W., Kubein-Meesenburg, D., Nagerl, H., 2010. Migration of the instantaneous axis of motion during axial rotation in lumbar segments and role of the zygapophysial joints. Acta Bioeng Biomech 12, 39-47.

Woltring, H., Huiskes, R., 1985. A statistically motivated approach to instantaneous helical axis estimation from noisy, sampled landmark coordinates. Human Kinetics Publ, 274-279.

Woltring, H., Huiskes, R., De Lange, A., Year Measurement error influence on helical axis accuracy in the description of 3d finite joint movement in biomechanics. In Woo, SL, Mates, RE (Etds.), Biomechanics symposium AMD.

Woltring, H.J., 1990. Estimation of the trajectory of the instantaneous centre of rotation in planar biokinematics. J Biomech 23, 1273-1274.

Woltring, H.J., de Lange, A., Kauer, J.M.G., Huiskes, R., 1987. Instantaneous Helical Axis Estimation Via Natural, Cross-Validated Splines, in: Bergmann, G., Kölbel, R., Rohlmann, A. (Eds.), Biomechanics: Basic and Applied Research: Selected Proceedings of the Fifth Meeting of the European Society of Biomechanics, September 8–10, 1986, Berlin, F.R.G. Springer Netherlands, Dordrecht, pp. 121-128.

Woltring, H.J., Huiskes, R., de Lange, A., Veldpaus, F.E., 1985. Finite centroid and helical axis estimation from noisy landmark measurements in the study of human joint kinematics. J Biomech 18, 379-389.

Woltring, H.J., Long, K., Osterbauer, P.J., Fuhr, A.W., 1994. Instantaneous helical axis estimation from 3-D video data in neck kinematics for whiplash diagnostics. J Biomech 27, 1415-1432.

Wu, G., van der Helm, F.C.T., Veeger, H.E.J., Makhsous, M., Van Roy, P., Anglin, C., Nagels, J., Karduna, A.R., McQuade, K., Wang, X.G., Werner, F.W., Buchholz, B., 2005. ISB recommendation on definitions of joint coordinate systems of various joints for the reporting of human joint motion - Part II: shoulder, elbow, wrist and hand. J Biomech 38, 981-992.