

# Introduction to Computational Fluid Dynamics using OpenFOAM and Octave

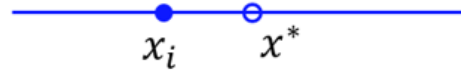
Dr. Lakshman Anumolu (Sr. Research Engineer)  
Kumaresh Selvakumar (PhD candidate)  
(Session-7)

*Instructions: Mon, Wed, Thu (5:30PM-6:30PM IST)  
Query session: Sundays 8AM-8:30AM IST*

# Quick Recap

# What Did We Discuss?

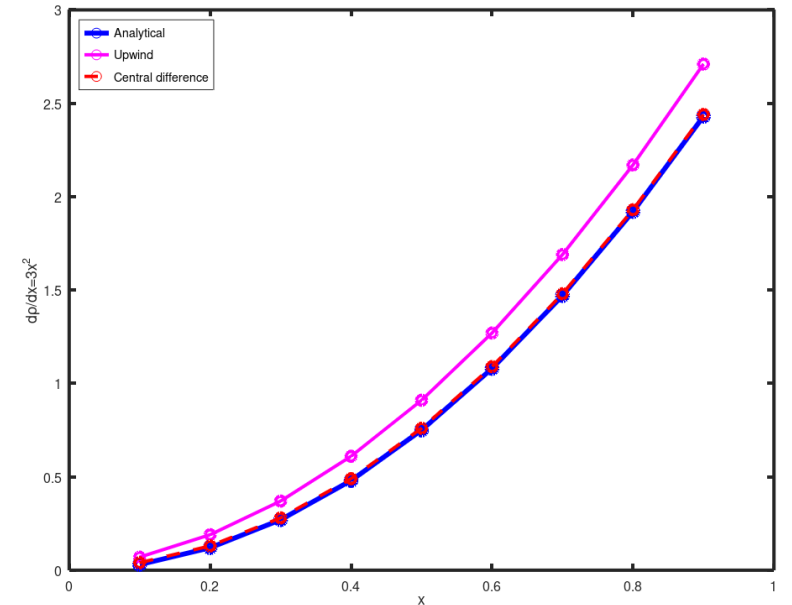
## Taylor Series



$$\rho(x^*) = \rho(x_i) + \frac{(x^* - x_i)}{1!} \left( \frac{d\rho}{dx} \right)_i + \frac{(x^* - x_i)^2}{2!} \left( \frac{d^2\rho}{dx^2} \right)_i + \frac{(x^* - x_i)^3}{3!} \left( \frac{d^3\rho}{dx^3} \right)_i + \dots$$



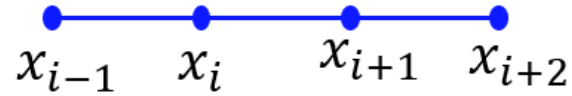
$$\left( \frac{d\rho}{dx} \right)_i \approx \frac{\rho(x_{i+1}) - \rho(x_i)}{\Delta x_i} \qquad \left( \frac{d\rho}{dx} \right)_i \approx \frac{\rho(x_{i+1}) - \rho(x_{i-1}))}{2\Delta x_i}$$



# What Did We Discuss?

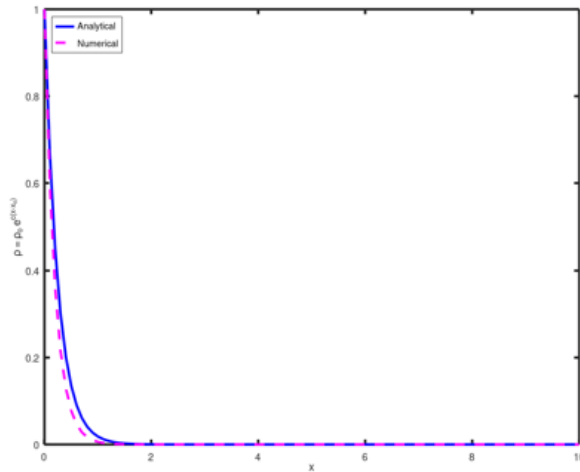
## Numerical Stability

$$\frac{d\rho}{dx} = -c\rho$$

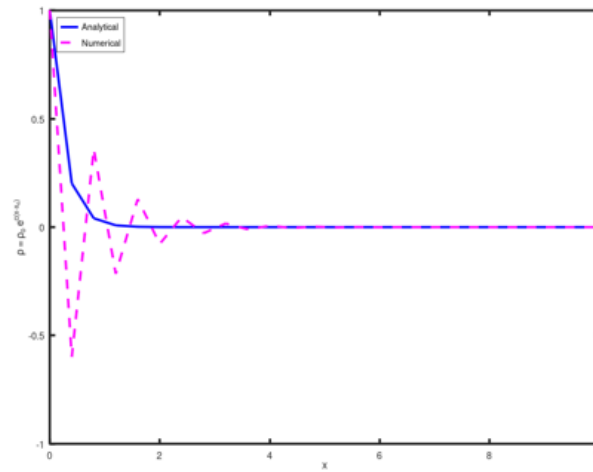


$$\rho_{i+1} = \rho_i(1 - c\Delta x)$$

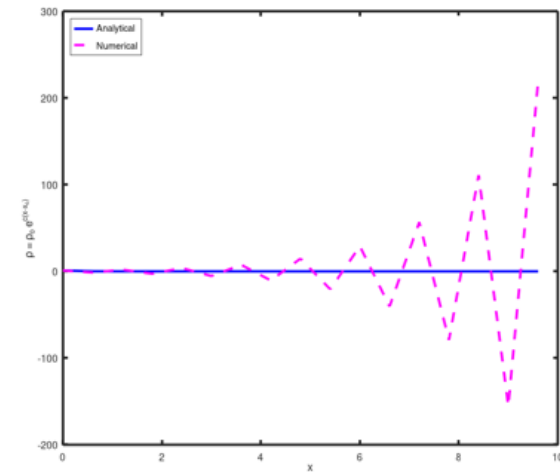
$$\Delta x < \frac{2}{c}$$
$$\frac{2}{c} = \frac{2}{4} = 0.5$$



$c = 4, x_0 = 0, \rho_0 = 1, x \in [0, 10], \Delta x = 0.1$



$c = 4, x_0 = 0, \rho_0 = 1, x \in [0, 10], \Delta x = 0.4$



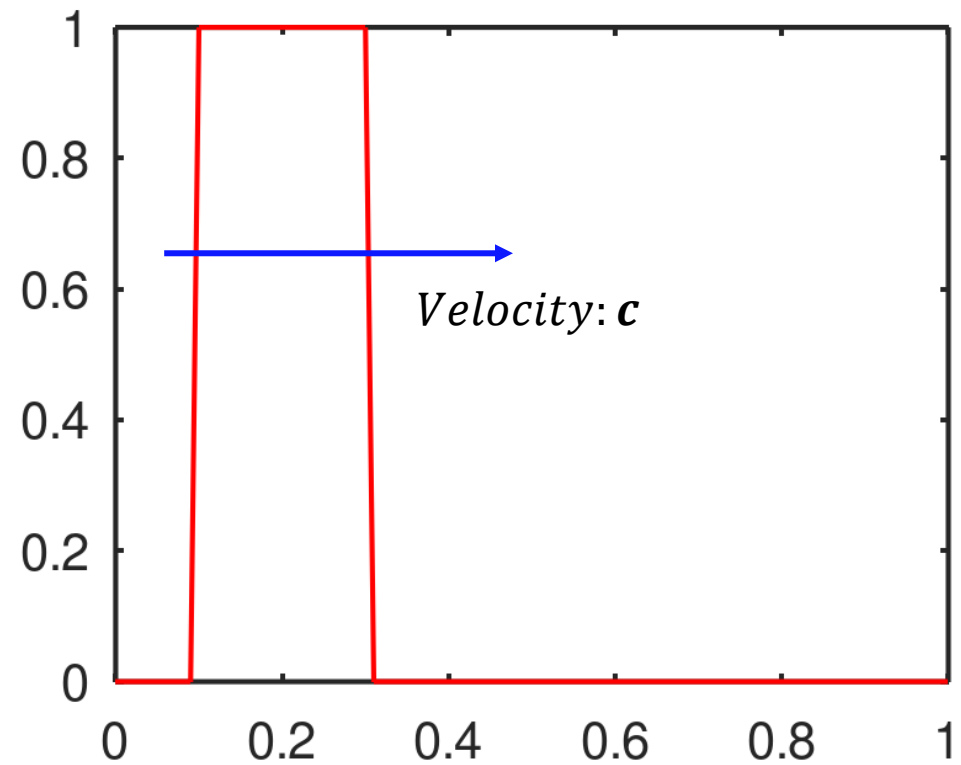
$c = 4, x_0 = 0, \rho_0 = 1, x \in [0, 10], \Delta x = 0.6$

Current Session

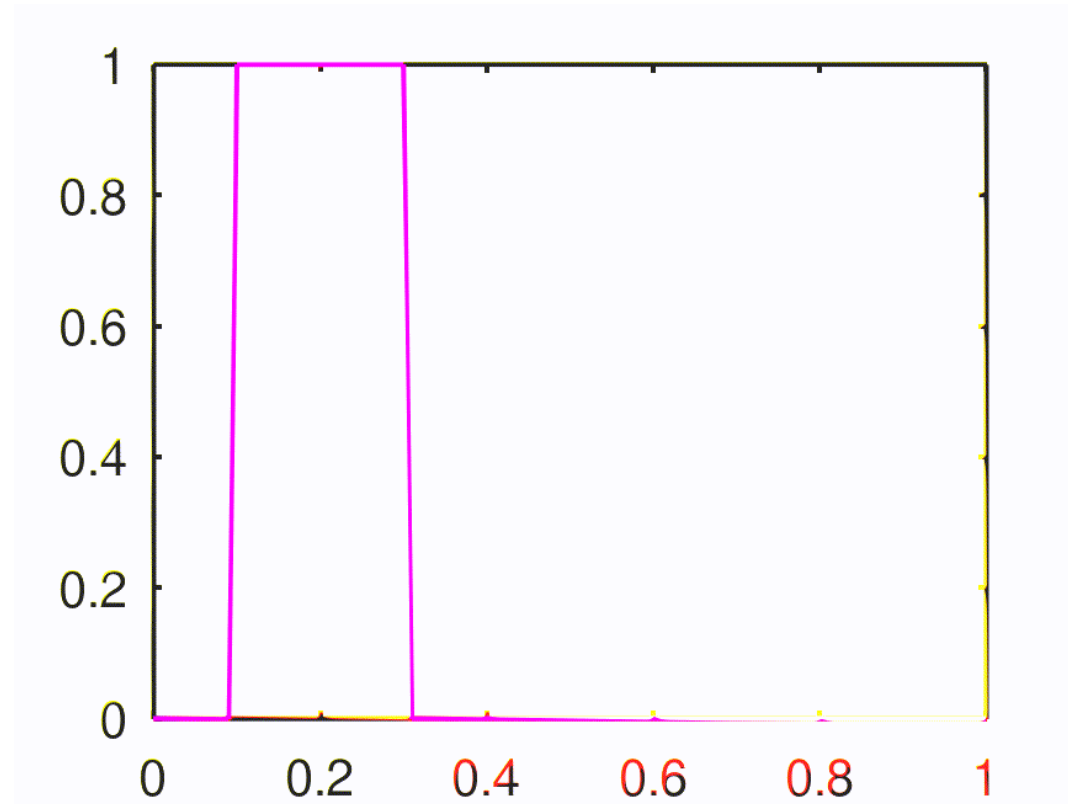
# Overview

- Stability Analysis (contd.)
- Introduction to C++ for OpenFOAM

# Numerical Stability



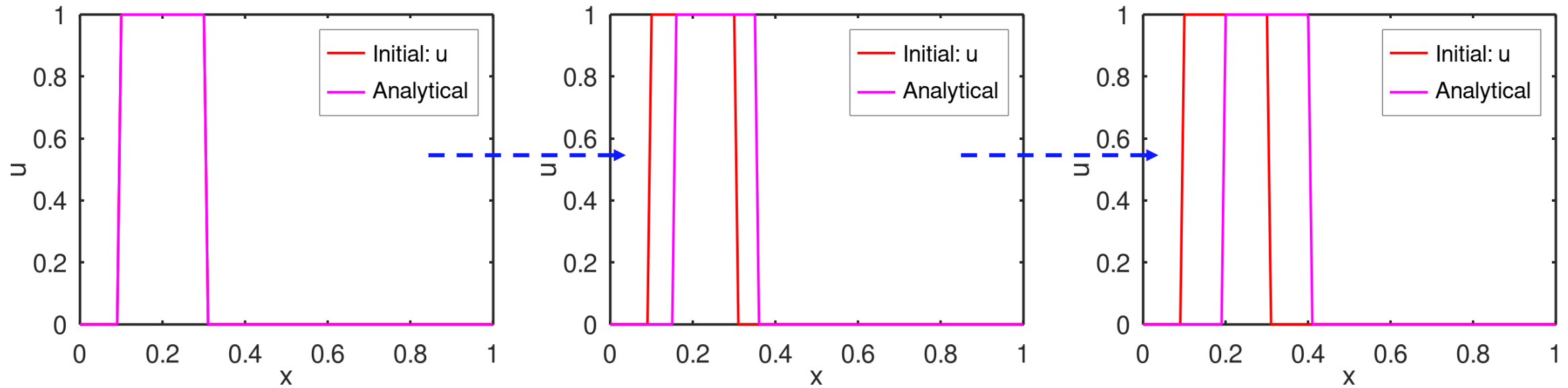
# Numerical Stability



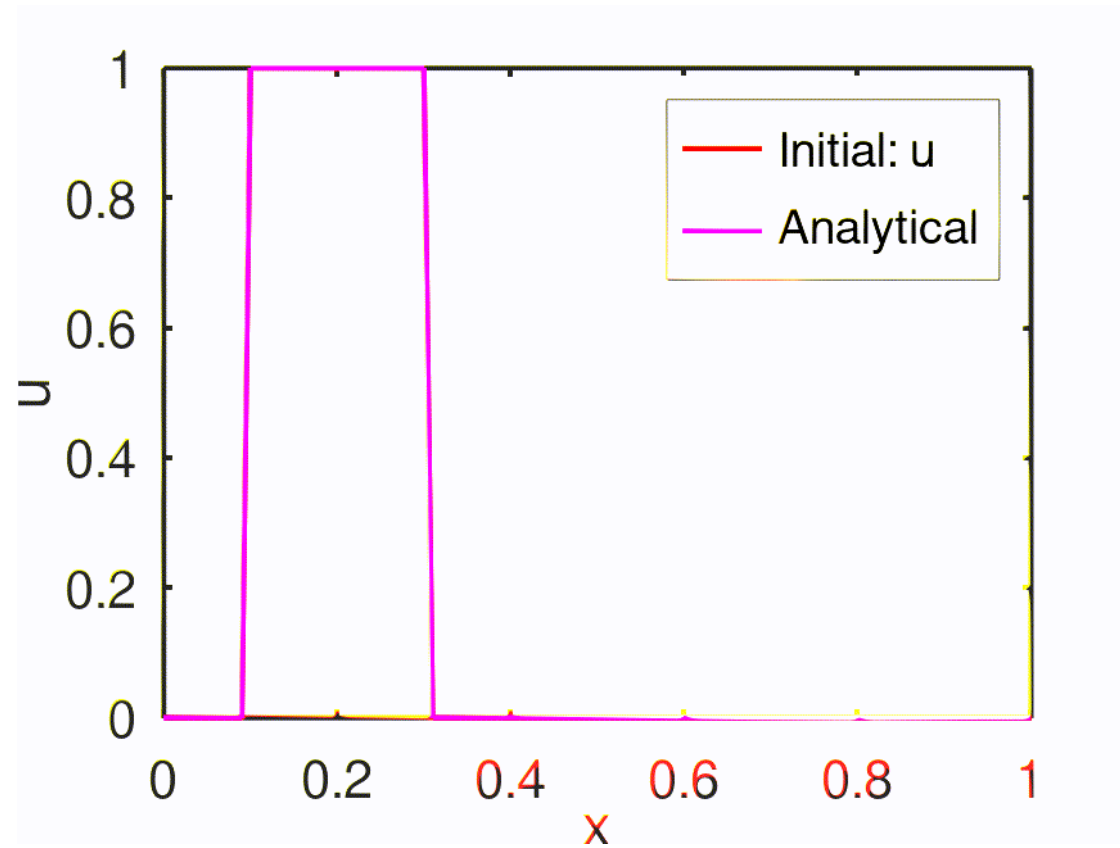


# Numerical Stability

$$\frac{\partial u}{\partial t} + c \frac{\partial u}{\partial x} = 0 \quad \leftarrow \text{Advection equation}$$



# Numerical Stability




# Numerical Stability

$$\frac{\partial u}{\partial t} + c \frac{\partial u}{\partial x} = 0$$

$$\frac{u_i^{n+1} - u_i^n}{\Delta t} + c \left( \frac{\partial u}{\partial x} \right)_i^n = 0$$




$$\left( \frac{d\rho}{dx} \right)_i \approx \frac{\rho(x_{i+1}) - \rho(x_i)}{\Delta x_i} \quad \left( \frac{d\rho}{dx} \right)_i \approx \frac{\rho(x_{i+1}) - \rho(x_{i-1}))}{2\Delta x_i}$$

$$u_i^{n+1} = u_i^n - c\Delta t \left( \frac{\partial u}{\partial x} \right)_i^n$$

Two arrows point from the derivative term in the equation above to the following approximations:

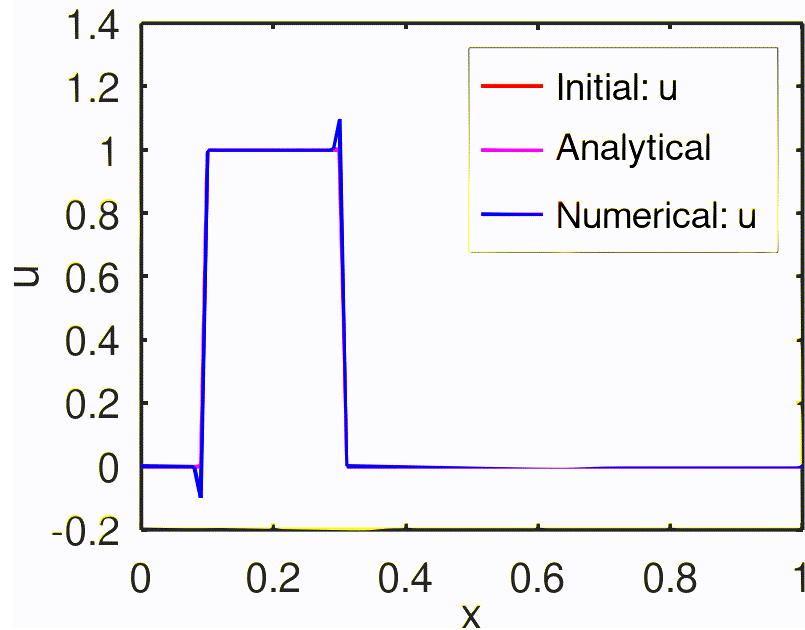
- Simple forward difference scheme:  $\left( \frac{\partial u}{\partial x} \right)_i^n \approx \frac{u_{i+1}^n - u_i^n}{\Delta x_i}$
- Central difference:  $\left( \frac{\partial u}{\partial x} \right)_i^n \approx \frac{u_{i+1}^n - u_{i-1}^n}{2\Delta x_i}$

# Numerical Stability

$$u_i^{n+1} = u_i^n - c\Delta t \left( \frac{\partial u}{\partial x} \right)_i^n$$

Two approximations for the spatial derivative are shown:

- Simple forward difference scheme:  $\left( \frac{\partial u}{\partial x} \right)_i^n \approx \frac{u_{i+1}^n - u_i^n}{\Delta x_i}$
- Central difference:  $\left( \frac{\partial u}{\partial x} \right)_i^n \approx \frac{u_{i+1}^n - u_{i-1}^n}{2\Delta x_i}$

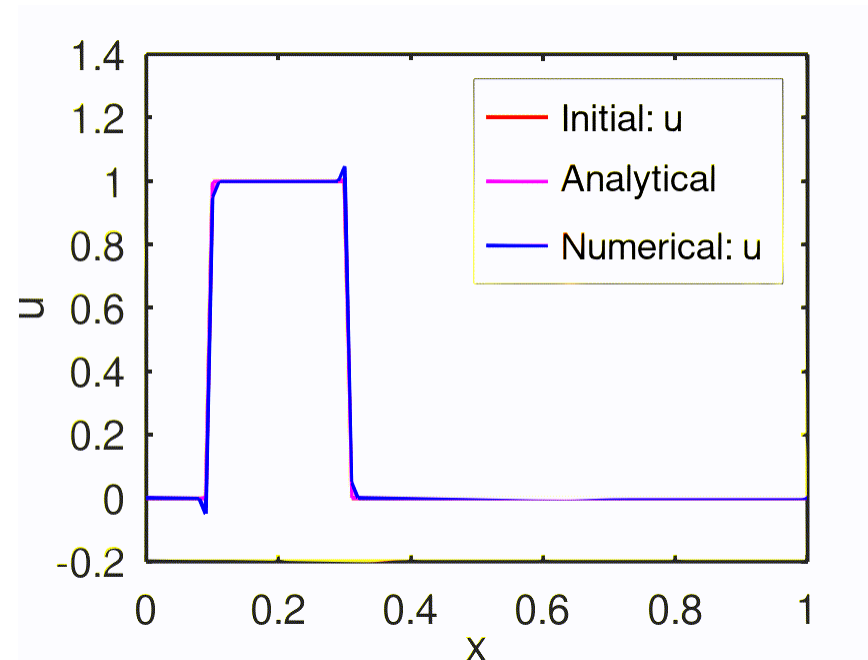


# Numerical Stability

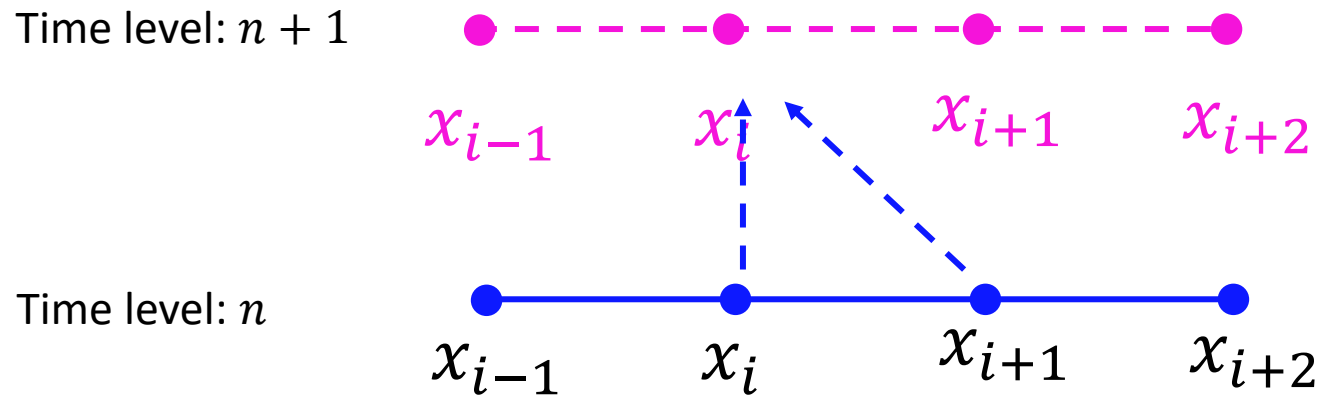
$$u_i^{n+1} = u_i^n - c\Delta t \left( \frac{\partial u}{\partial x} \right)_i^n$$

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- Simple forward difference scheme:  $\left( \frac{\partial u}{\partial x} \right)_i^n \approx \frac{u_{i+1}^n - u_i^n}{\Delta x_i}$
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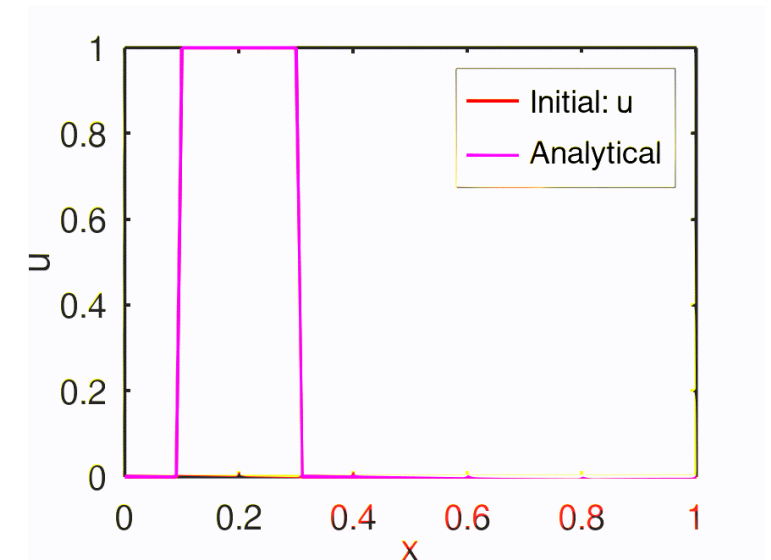
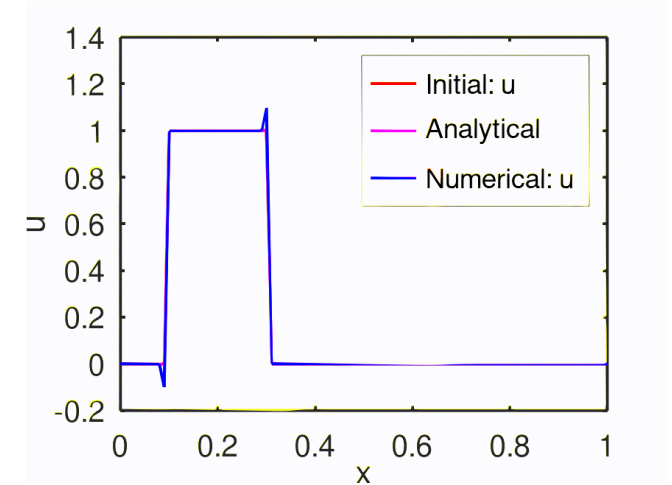


# Numerical Stability

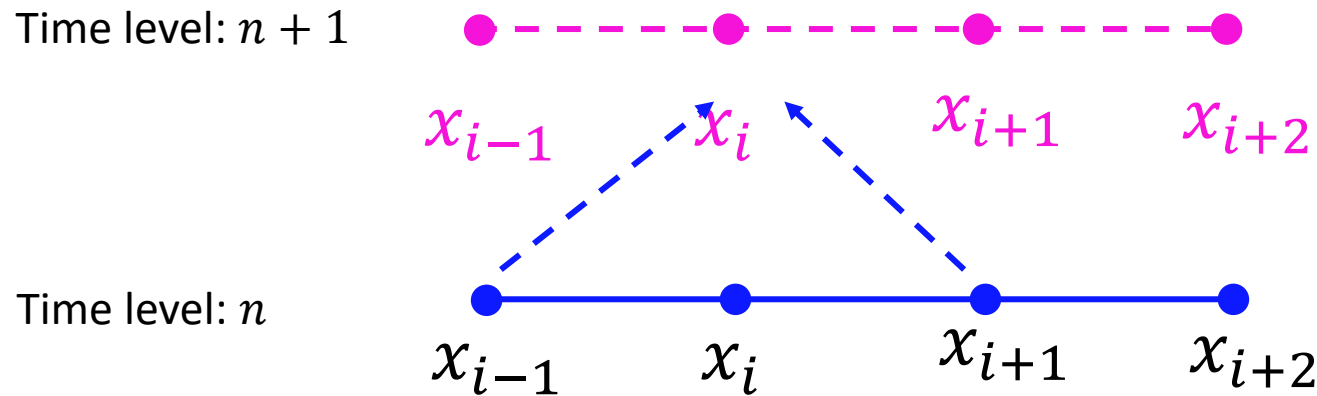


$$u_i^{n+1} = u_i^n - c\Delta t \left( \frac{\partial u}{\partial x} \right)_i^n$$

$\left( \frac{\partial u}{\partial x} \right)_i^n \approx \frac{u_{i+1}^n - u_i^n}{\Delta x_i}$  Simple forward difference scheme  
 $\left( \frac{\partial u}{\partial x} \right)_i^n \approx \frac{u_{i+1}^n - u_{i-1}^n}{2\Delta x_i}$  Central difference

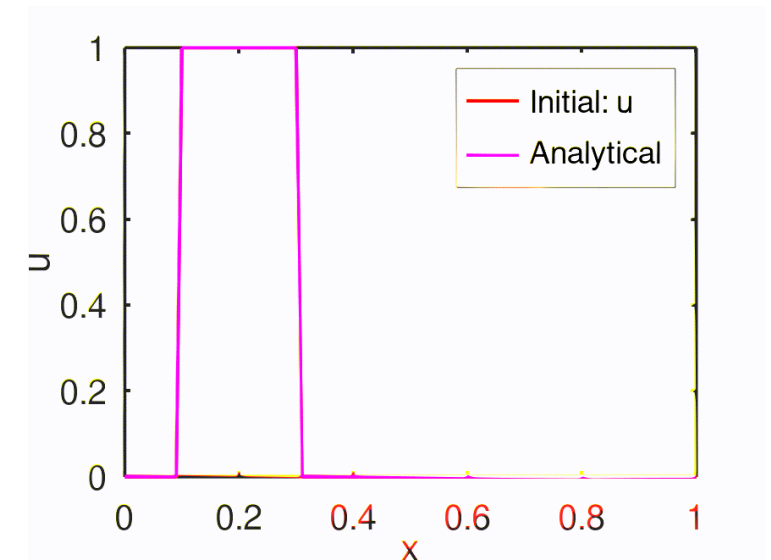
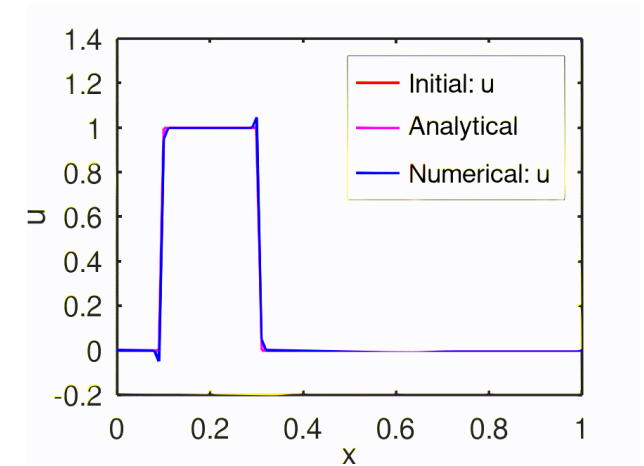


# Numerical Stability

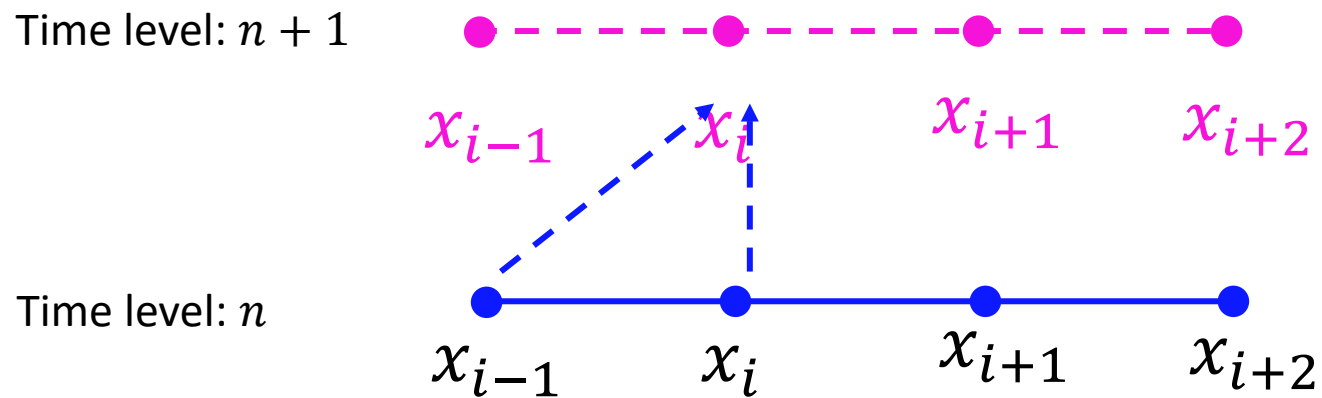


$$u_i^{n+1} = u_i^n - c\Delta t \left( \frac{\partial u}{\partial x} \right)_i^n$$

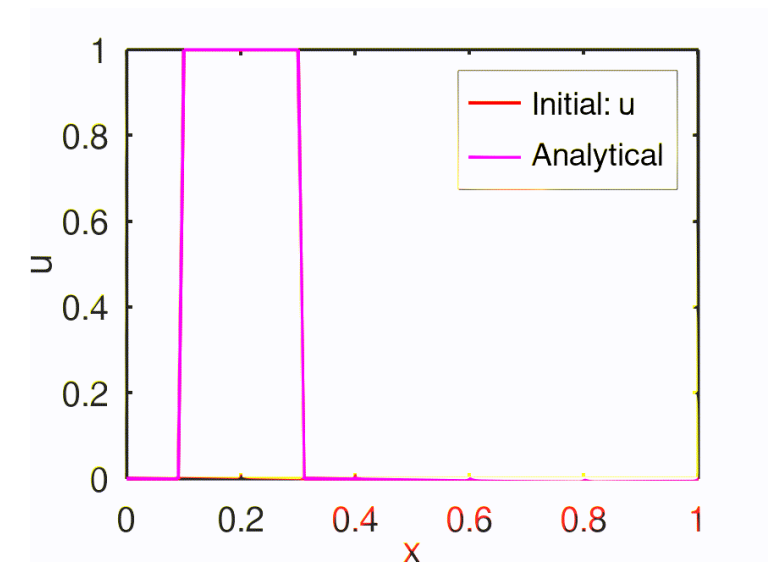
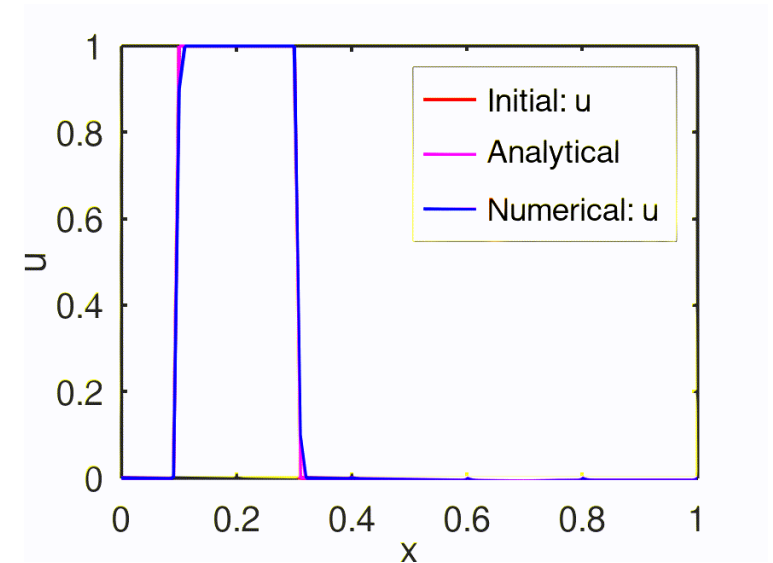
$\left( \frac{\partial u}{\partial x} \right)_i^n \approx \frac{u_{i+1}^n - u_i^n}{\Delta x_i}$  Simple forward difference scheme  
 $\left( \frac{\partial u}{\partial x} \right)_i^n \approx \frac{u_{i+1}^n - u_{i-1}^n}{2\Delta x_i}$  Central difference



# Numerical Stability



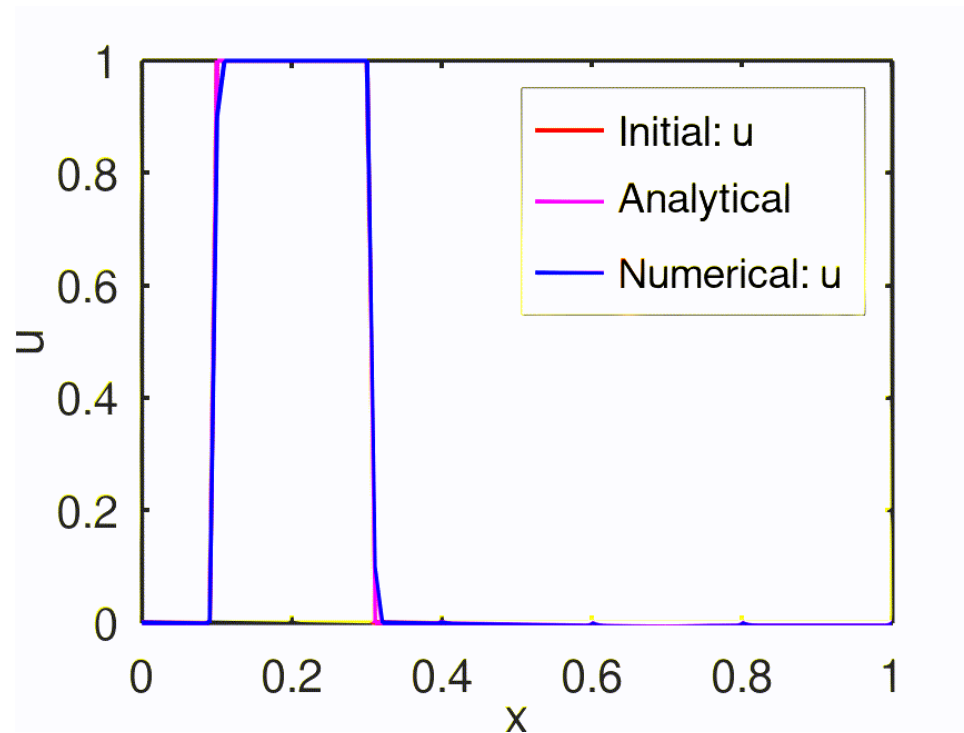
$$u_i^{n+1} = u_i^n - c\Delta t \left( \frac{\partial u}{\partial x} \right)_i^n \longrightarrow \left( \frac{\partial u}{\partial x} \right)_i^n \approx \frac{u_i^n - u_{i-1}^n}{\Delta x_i} \quad \text{Upwind}$$





# Numerical Stability

$$u_i^{n+1} = u_i^n - c\Delta t \left( \frac{\partial u}{\partial x} \right)_i^n \longrightarrow \left( \frac{\partial u}{\partial x} \right)_i^n \approx \frac{u_i^n - u_{i-1}^n}{\Delta x_i} \quad \text{Upwind}$$



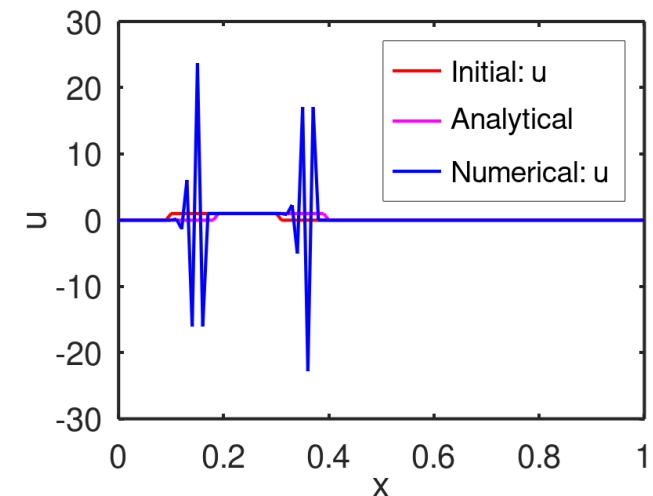
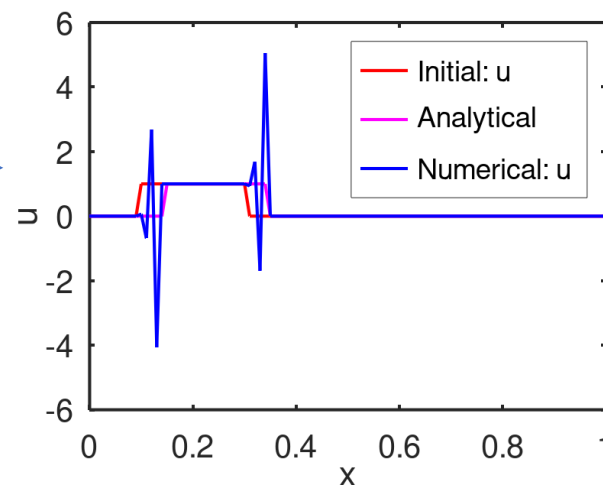
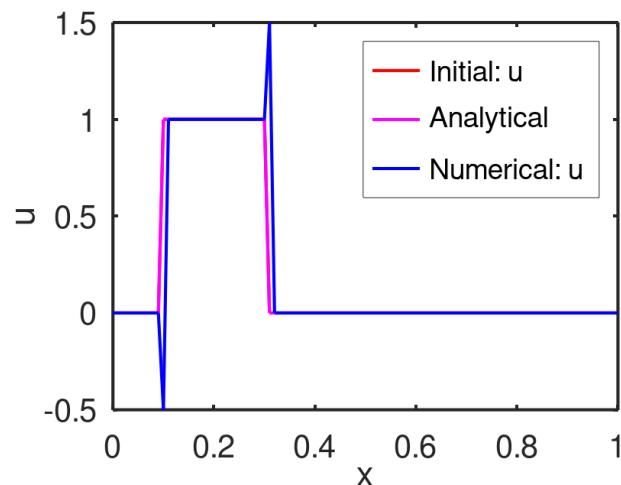
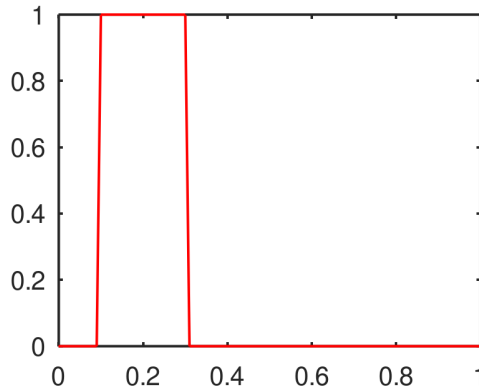
$$CFL: \frac{c\Delta t}{\Delta x}$$

**CFL = 0.1**

# Numerical Stability

$$u_i^{n+1} = u_i^n - c\Delta t \left( \frac{\partial u}{\partial x} \right)_i^n \longrightarrow \left( \frac{\partial u}{\partial x} \right)_i^n \approx \frac{u_i^n - u_{i-1}^n}{\Delta x_i} \quad \text{Upwind}$$

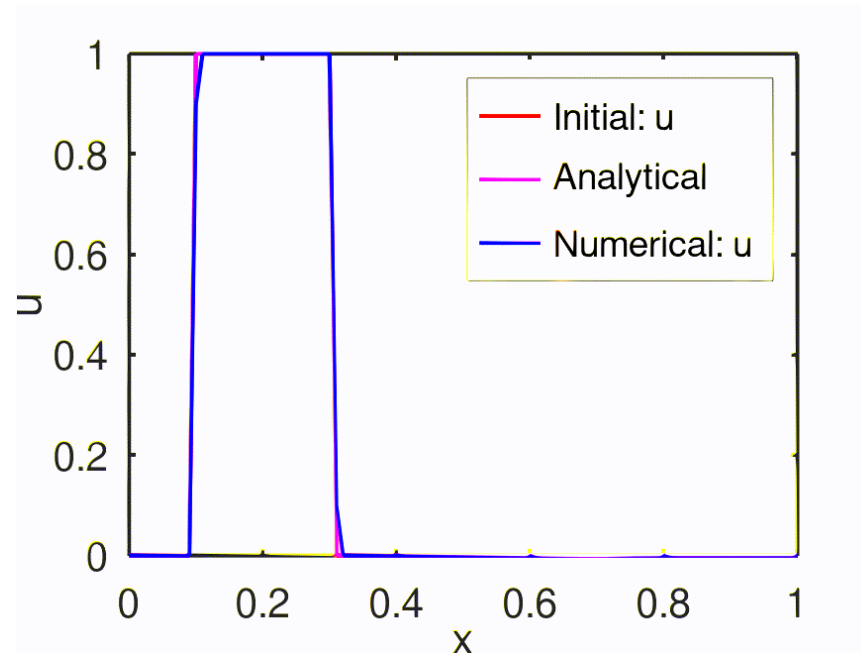
$$CFL: \frac{c\Delta t}{\Delta x}$$



**CFL = 1.5**

# What did we discuss?

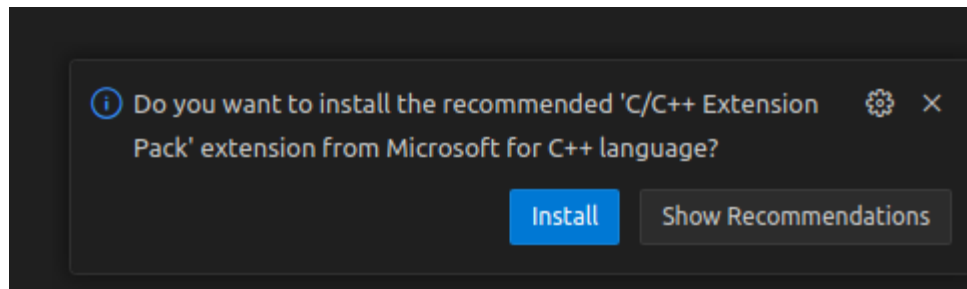
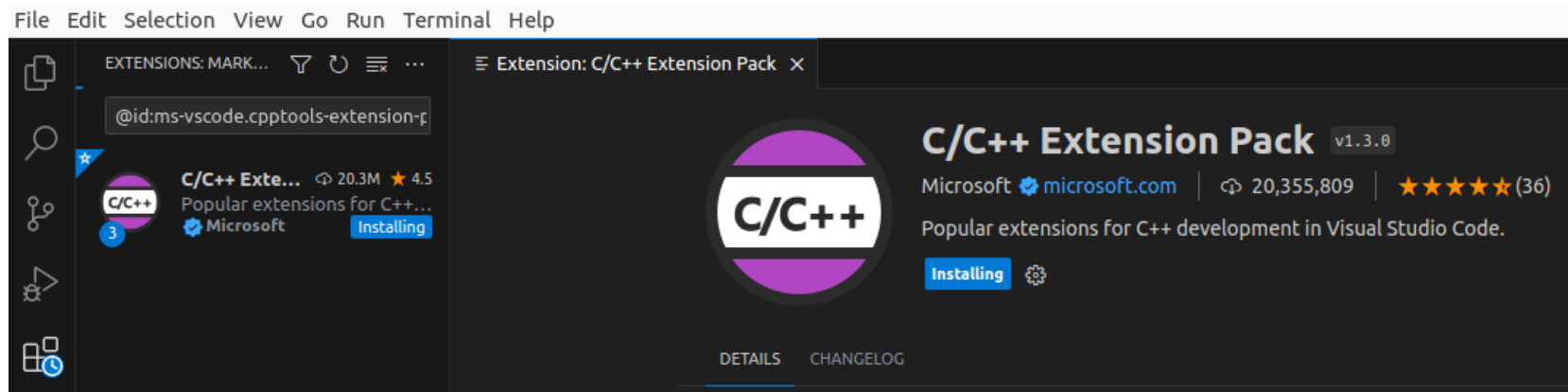
- Proper discrete approximations need to be chosen based on the velocity field
- CFL number is critical to ensure numerical stability



$$u_i^{n+1} = u_i^n - c\Delta t \left( \frac{\partial u}{\partial x} \right)_i^n \longrightarrow \left( \frac{\partial u}{\partial x} \right)_i^n \approx \frac{u_i^n - u_{i-1}^n}{\Delta x_i} \quad \text{Upwind}$$

# Introduction to C++ for OpenFOAM

*b6\_sample.cpp*



# Next Session

- Rate of convergence
- Introduction to C++ for OpenFOAM

Thank you