Introduction to Computational Fluid Dynamics using OpenFOAM and Octave

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Kumaresh Selvakumar
(Session-2)

Instructions: Mon, Wed, Thu (5:30PM-6:30PM IST)
Query session: Sundays 8AM-8:30AM IST

Overview

- Fluid Behavior & Mathematical Operators
- Lagrangian & Eulerian Frames
- Governing Equations

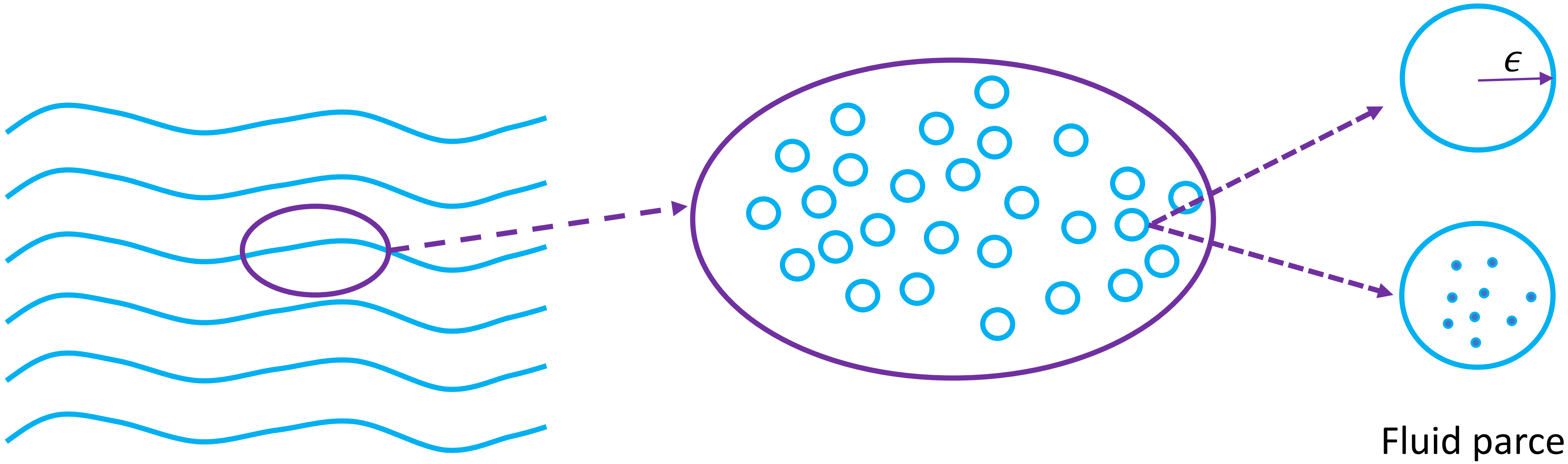
Reminder

- Exercise-1
  - Github
    - Discussion forum:
    - https://github.com/exaslate-courses/cfd-openfoam-b1/discussions

- Operating System:
  - Ubuntu 22.04
- Softwares:
  - OpenFOAM v2306
  - Octave

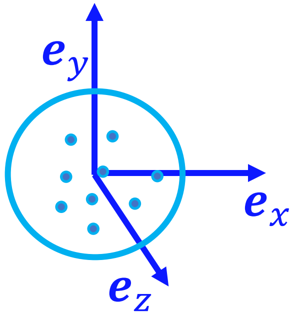
Fluid Behavior & Mathematical Operators

Fluid



Fluid parcel

Fluid Behavior



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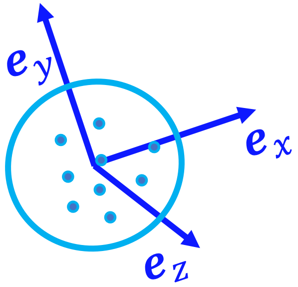
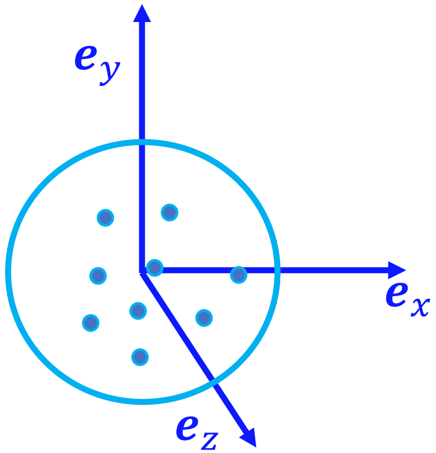
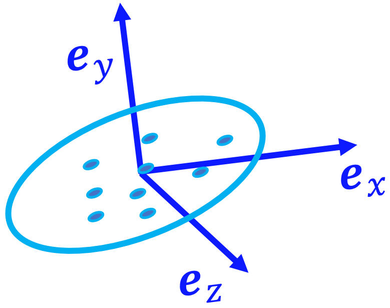


Rotation



Expansion

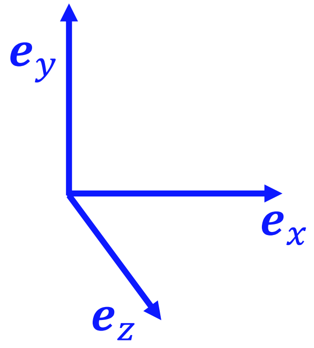


Deformation

Mathematical Operators

- Gradient

$$\nabla\rho = \left(\frac{\partial}{\partial x}\boldsymbol{e}_x + \frac{\partial}{\partial y}\boldsymbol{e}_y + \frac{\partial}{\partial z}\boldsymbol{e}_z\right)\rho = \left(\frac{\partial\rho}{\partial x}\boldsymbol{e}_x + \frac{\partial\rho}{\partial y}\boldsymbol{e}_y + \frac{\partial\rho}{\partial z}\boldsymbol{e}_z\right)$$



$$\nabla\boldsymbol{u} = \begin{bmatrix} {\partial u}/{\partial x} & {\partial v}/{\partial x} & {\partial w}/{\partial x} \\ {\partial u}/{\partial y} & {\partial v}/{\partial y} & {\partial w}/{\partial y} \\ {\partial u}/{\partial z} & {\partial v}/{\partial z} & {\partial w}/{\partial z} \end{bmatrix}$$

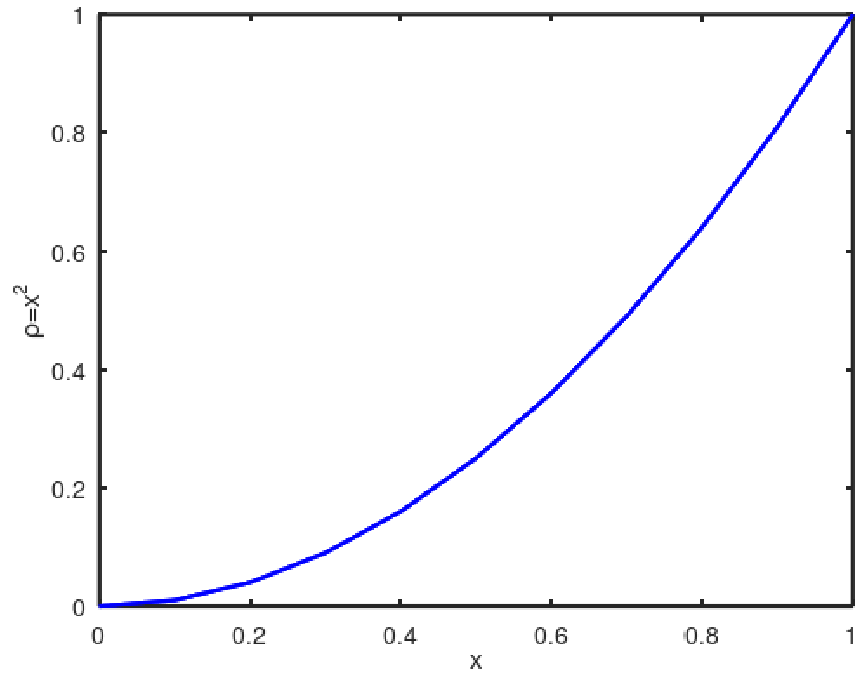
- Divergence

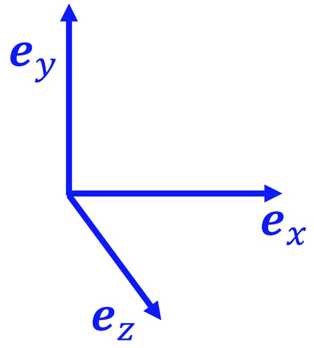
$$\nabla\cdot\boldsymbol{u} = \left(\frac{\partial}{\partial x}\boldsymbol{e}_x + \frac{\partial}{\partial y}\boldsymbol{e}_y + \frac{\partial}{\partial z}\boldsymbol{e}_z\right)\left(u\boldsymbol{e}_x + v\boldsymbol{e}_y + w\boldsymbol{e}_z\right) = \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right)$$

Gradient

$$\nabla\rho = \left(\frac{\partial}{\partial x}\boldsymbol{e}_x + \frac{\partial}{\partial y}\boldsymbol{e}_y + \frac{\partial}{\partial z}\boldsymbol{e}_z\right)\rho = \left(\frac{\partial\rho}{\partial x}\boldsymbol{e}_x + \frac{\partial\rho}{\partial y}\boldsymbol{e}_y + \frac{\partial\rho}{\partial z}\boldsymbol{e}_z\right)$$

$$\frac{\partial\rho}{\partial x} = \frac{d\rho}{dx}(in\ 1D)$$





Gradient

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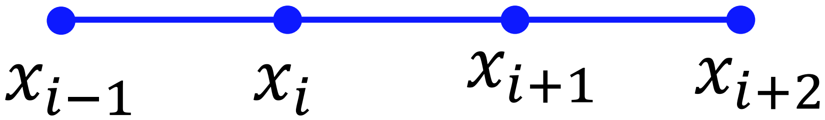
- Numerical Approximation

$$\left(\frac{\partial\rho}{\partial x}\right)_i = \frac{\rho(x_{i+1}) - \rho(x_i)}{\Delta x_i} + O(\Delta x_i)$$

Gradient

- Numerical Approximation



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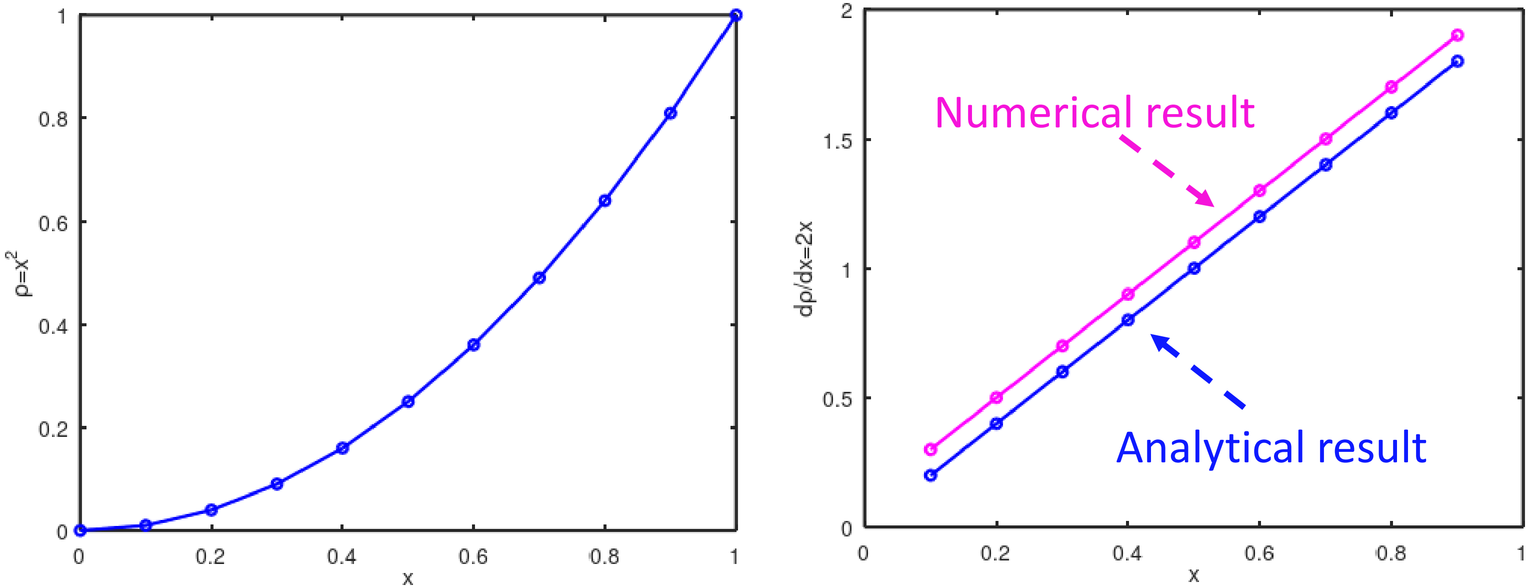

Gradient

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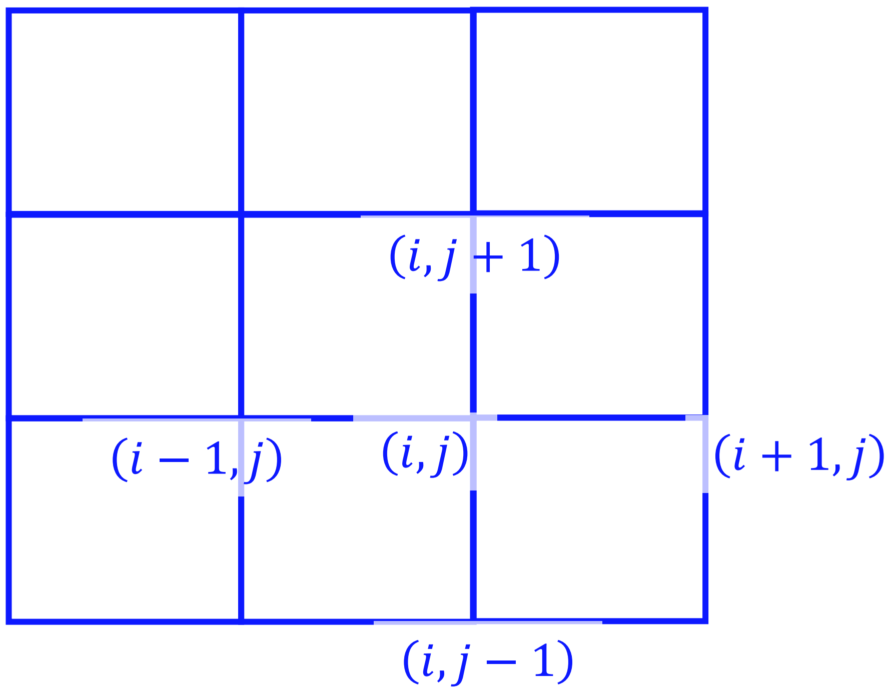
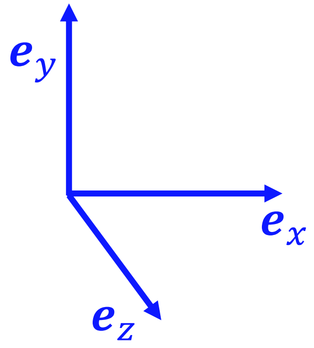


Mathematical Operations

- Divergence

$$\nabla \cdot \boldsymbol{u} = \left(\frac{\partial}{\partial x}\boldsymbol{e}_x + \frac{\partial}{\partial y}\boldsymbol{e}_y + \frac{\partial}{\partial z}\boldsymbol{e}_z\right)(u\boldsymbol{e}_x + v\boldsymbol{e}_y + w\boldsymbol{e}_z) = \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right)$$

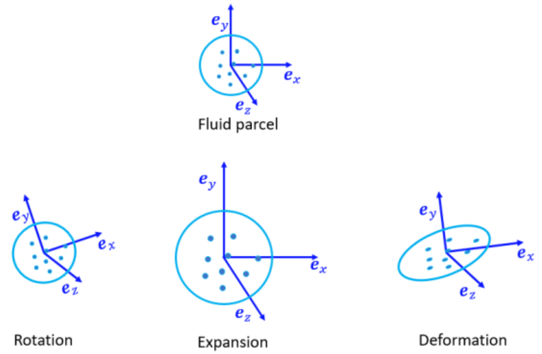


- Numerical approximation
  - Same as earlier

- Physical significance
  - Positive value : Source or expansion of fluid volume
  - Negative value: Sink
  - Zero signifies incompressible nature or no change in volume

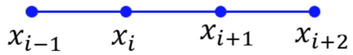
What Did We Discuss?

Fluid Behavior



Gradient

- Numerical Approximation



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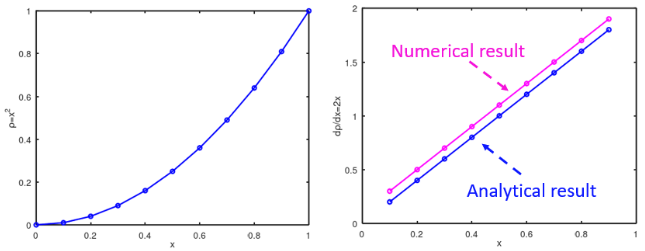

Mathematical Operations

- Divergence

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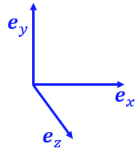

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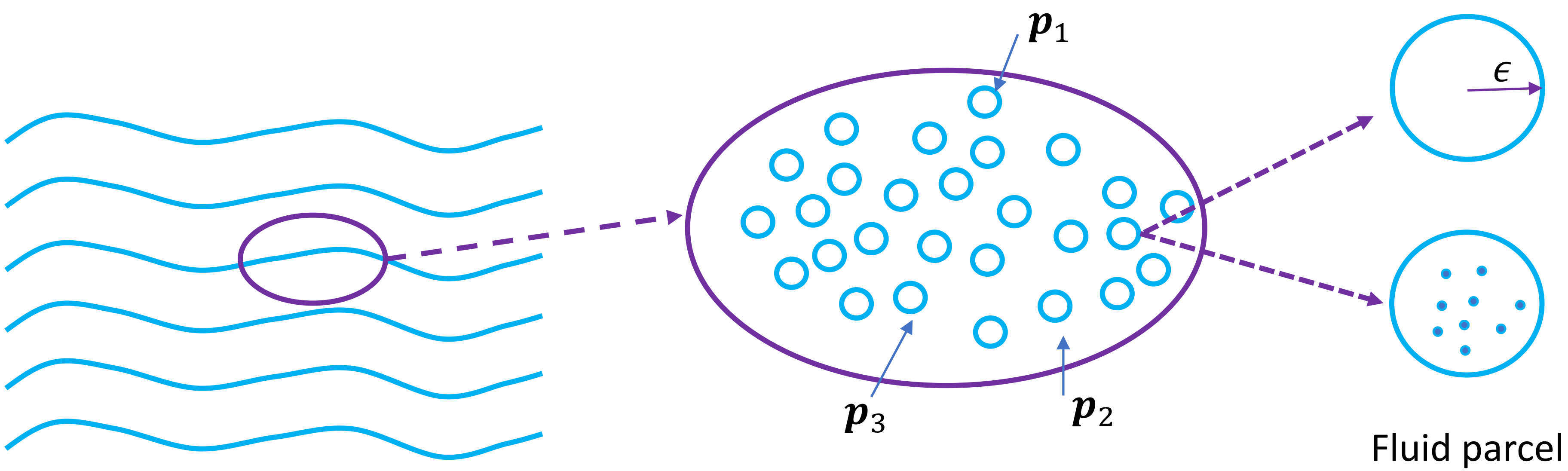
Lagrangian & Eulerian Frameworks

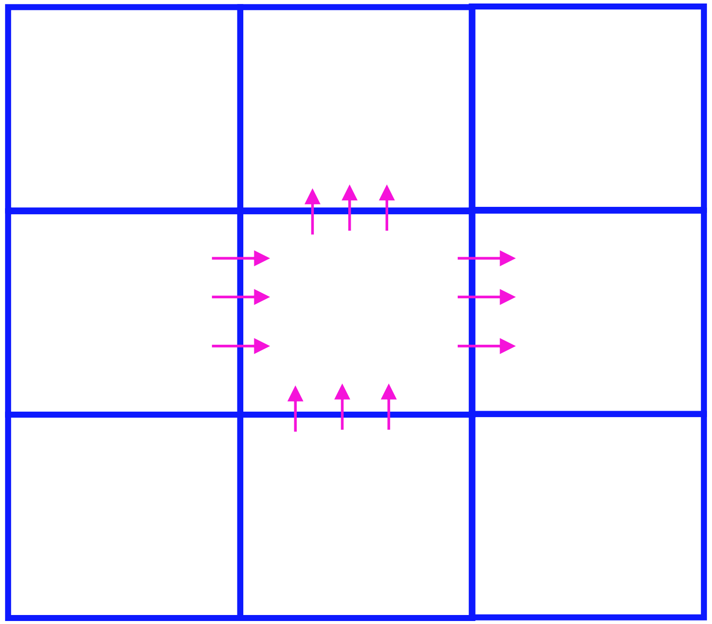
Lagrangian frame



- Follow a fluid parcel ($\boldsymbol{p}_1, \boldsymbol{p}_2$, …)
- Flow property at a location is obtained from the fluid parcel that happens to be at that location at that time
- Useful to derive conservation laws

Eulerian frame

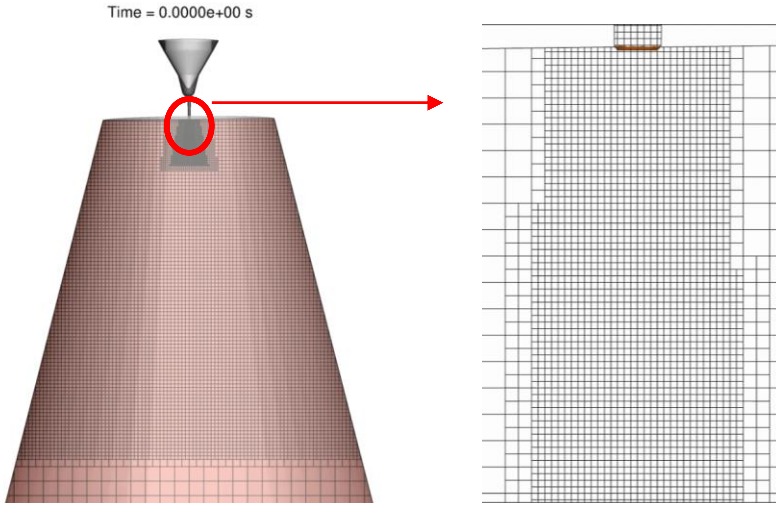
- Conservation laws are applied around a fixed "control volume" in space
- Flux of quantities through the boundary is used to estimate the flow variables
- Useful to take observations at fixed locations



Lagrangian-Eulerian [Material Derivative]

$$\frac{D\phi(\boldsymbol{X}(\boldsymbol{p}_i,t),t)}{Dt} = \frac{\partial\phi}{\partial t} + \frac{\partial\phi}{\partial x}\frac{DX}{Dt} + \frac{\partial\phi}{\partial y}\frac{DY}{Dt} + \frac{\partial\phi}{\partial z}\frac{DZ}{Dt}$$

$$\frac{D\phi}{Dt} = \frac{\partial\phi}{\partial t} + \boldsymbol{u}\cdot\nabla\phi$$



Governing Equations

Conservation Laws

- Conservation of mass

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \boldsymbol{u}) = 0; \quad \nabla \cdot (\rho \boldsymbol{u}) = \frac{\partial (\rho u)}{\partial x} + \frac{\partial (\rho v)}{\partial y} + \frac{\partial (\rho w)}{\partial z}$$

- Conservation of momentum

$$\frac{\partial \rho \boldsymbol{u}}{\partial t} + \nabla \cdot (\rho \boldsymbol{u}\boldsymbol{u}) = -\nabla p + \nabla \cdot \boldsymbol{\sigma} + \rho \boldsymbol{g};$$

- Scalar conservation law

$$\frac{\partial \rho \phi}{\partial t} + \nabla \cdot (\rho \boldsymbol{u} \phi) = \nabla \cdot (\Gamma \nabla \phi) + S_{\phi}$$

Integral Form – Differential Form

- Conservation of mass

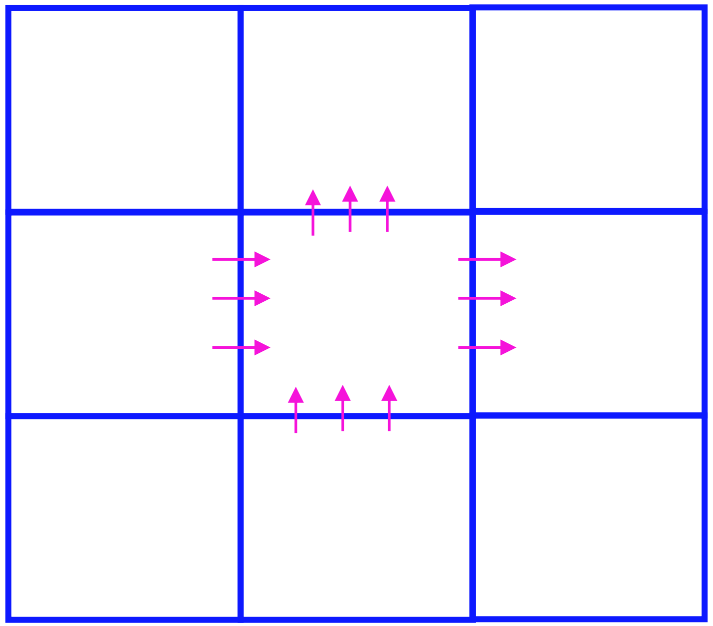
$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \boldsymbol{u}) = 0$$

- Integrate over a control volume

$$\int_V \left(\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \boldsymbol{u}) \right) dV = 0$$

$$\int_V \frac{\partial \rho}{\partial t} dV + \int_V \nabla \cdot (\rho \boldsymbol{u}) dV = 0$$

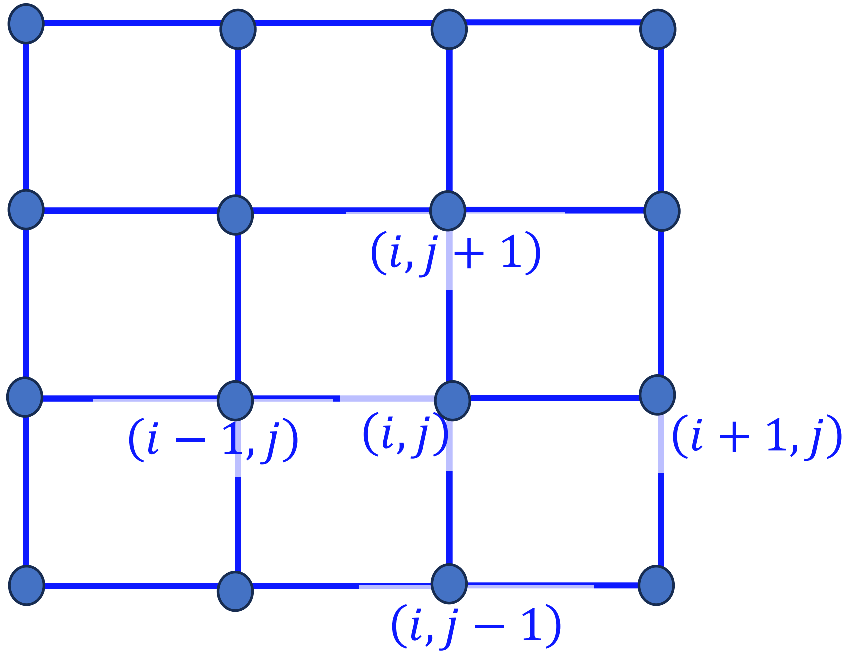
$$\frac{\partial}{\partial t} \int_V \rho dV + \oint_S \rho \boldsymbol{u} \cdot d\boldsymbol{S} = 0$$



Finite Difference – Finite Volume

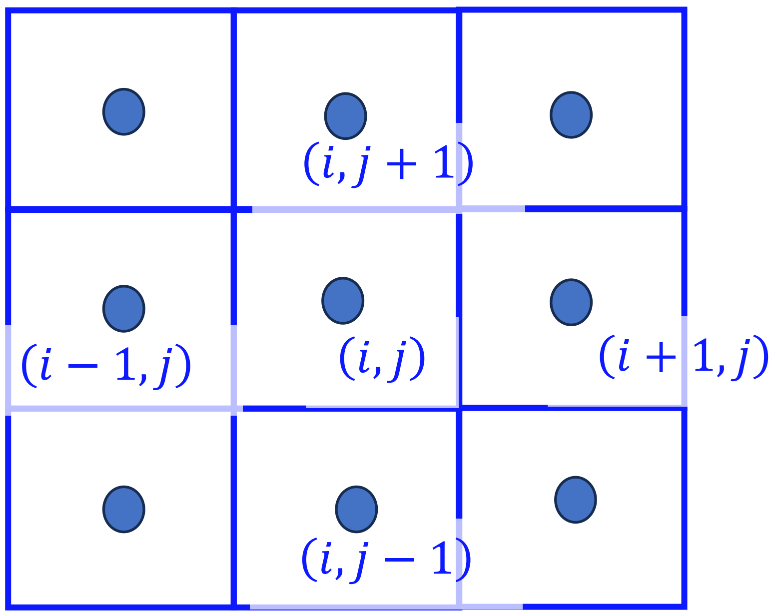
Differential form

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \boldsymbol{u}) = 0$$



Integral form

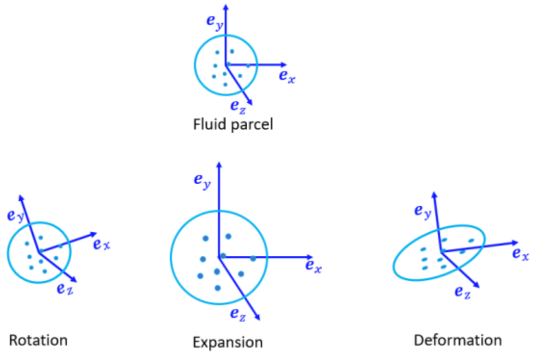
$$\frac{\partial}{\partial t} \int_V \rho dV + \oint_S \rho \boldsymbol{u} \cdot d\boldsymbol{S} = 0$$



Quick Recap

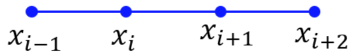
What Did We Discuss?

Fluid Behavior



Gradient

- Numerical Approximation



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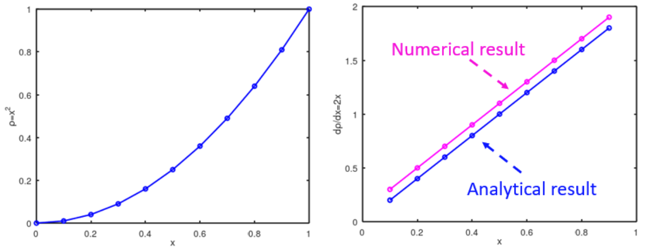

Mathematical Operations

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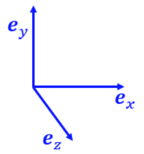

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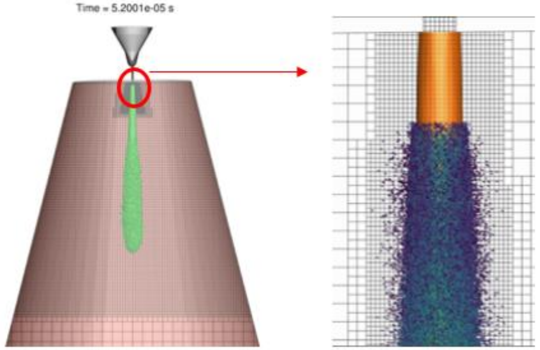

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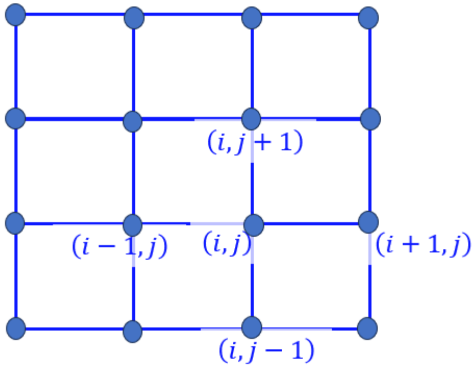

Integral form

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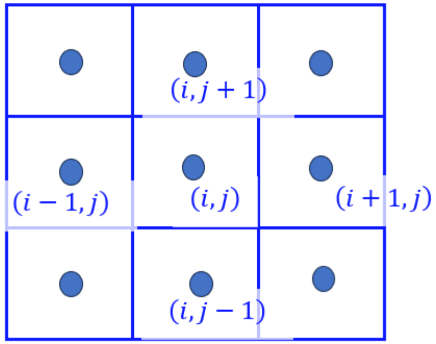


Next Session

- Reynolds Transport Theorem
- Gauss Divergence Theorem

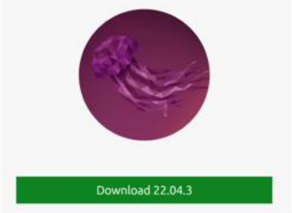
Installations

Required Applications

- Preconfiguration packages:
  - https://1drv.ms/f/s!AqT2YEB97-1RgP8MtsMPqoOGsq4ddg?e=IocXv0
- List
  - Virtual Box [to create virtual machines]
  - Ubuntu 22.04 [OS to install OpenFOAM & Octave]
  - AnyDesk [For remote access]

Exercise-1

- Operating System:
  - Ubuntu 22.04



- Softwares:
  - OpenFOAM v2306

Open∇FOAM®

OpenFOAM-v2306 openfoam

  - Octave

GNU Octave

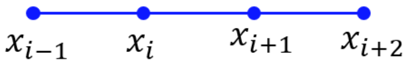
- Create a github account:
  - https://github.com
  - Discussion forum:
    - https://github.com/exaslate-courses/cfd-openfoam-b1/discussions

Test Octave

- Run numerical_derivative_first_order_approximation.m

Gradient

- Numerical Approximation



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