Introduction to Computational Fluid Dynamics using OpenFOAM and Octave

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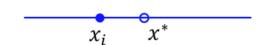
Instructions: Mon, Wed, Thu (5:30PM-6:30PM IST)

Query session: Sundays 8AM-8:30AM IST

Quick Recap

What Did We Discuss?

Taylor Series



$$\rho(x^*) = \rho(x_i) + \frac{(x^* - x_i)}{1!} \left(\frac{d\rho}{dx}\right)_i + \frac{(x^* - x_i)^2}{2!} \left(\frac{d^2\rho}{dx^2}\right)_i + \frac{(x^* - x_i)^3}{3!} \left(\frac{d^3\rho}{dx^3}\right)_i + \cdots$$



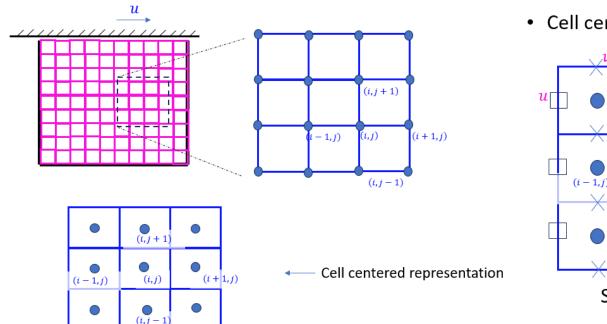
$$\left(\frac{d\rho}{dx}\right)_i \approx \frac{\rho(x_{i+1}) - \rho(x_i)}{\Delta x_i}$$

$$\left(\frac{d\rho}{dx}\right)_{i} \approx \frac{\rho(x_{i+1}) - \rho(x_{i-1})}{2\Delta x_{i}}$$

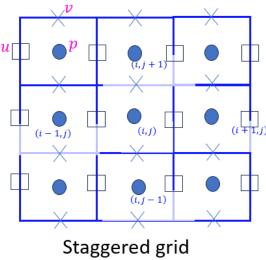
Second order central difference

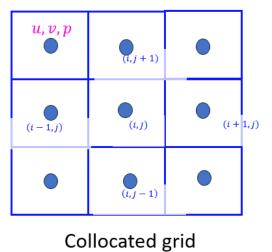
What Did We Discuss?

Grid Layout



• Cell centered representation



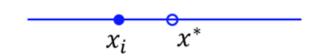


Current Session

Overview

- Taylor series analysis (exercise-2)
- Introduction to Octave programming
- Exercise using Octave

First Order Derivative Approximations



$$\rho(x^*) = \rho(x_i) + \frac{(x^* - x_i)}{1!} \left(\frac{d\rho}{dx}\right)_i + \frac{(x^* - x_i)^2}{2!} \left(\frac{d^2\rho}{dx^2}\right)_i + \frac{(x^* - x_i)^3}{3!} \left(\frac{d^3\rho}{dx^3}\right)_i + \cdots$$

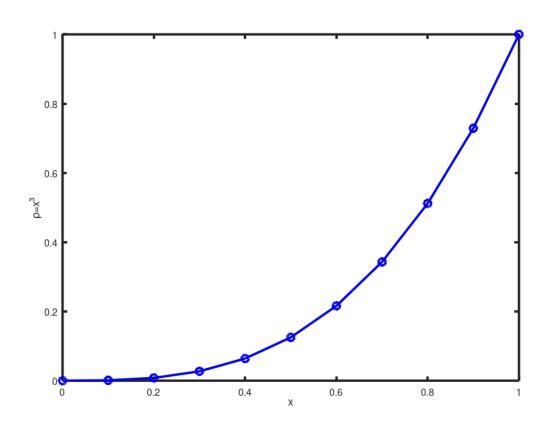


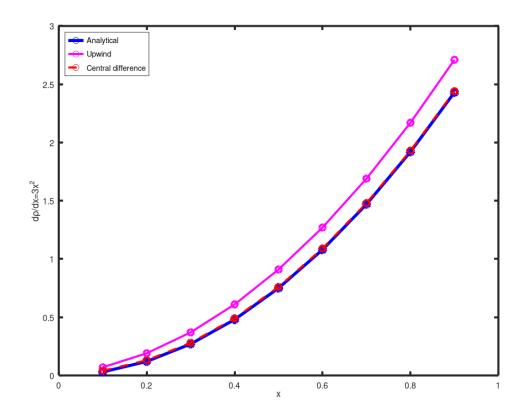
$$\left(\frac{d\rho}{dx}\right)_{i} \approx \frac{\rho(x_{i+1}) - \rho(x_{i})}{\Delta x_{i}} \qquad \left(\frac{d\rho}{dx}\right)_{i} \approx \frac{\rho(x_{i+1}) - \rho(x_{i-1})}{2\Delta x_{i}}$$

First order upwind

Second order central difference

First Order Derivative Approximations



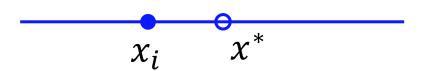


Exercise



- Derive expression for $\left(\frac{d^2\rho}{dx^2}\right)_i$
- What is the accuracy of the resultant expression?

Taylor Series



$$\rho(x^*) = \rho(x_i) + (x^* - x_i) \left(\frac{d\rho}{dx}\right)_i + \frac{(x^* - x_i)^2}{2!} \left(\frac{d^2\rho}{dx^2}\right)_i + \frac{(x^* - x_i)^3}{3!} \left(\frac{d^3\rho}{dx^3}\right)_i + \cdots$$

Approximating Second Order Derivative

Derive expression for $\left(\frac{d^2\rho}{dx^2}\right)_i$

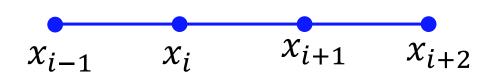
$$x_{i-1}$$
 x_i x_{i+1} x_{i+2}

$$\rho(x_{i+1}) = \rho(x_i) + (x_{i+1} - x_i) \left(\frac{d\rho}{dx}\right)_i + \frac{(x_{i+1} - x_i)^2}{2} \left(\frac{d^2\rho}{dx^2}\right)_i + \frac{(x_{i+1} - x_i)^3}{6} \left(\frac{d^3\rho}{dx^3}\right)_i + \frac{(x_{i+1} - x_i)^4}{24} \left(\frac{d^4\rho}{dx^4}\right)_i + \cdots$$

$$(1) \quad \rho(x_{i+1}) = \rho(x_i) + \Delta x_i \left(\frac{d\rho}{dx}\right)_i + \frac{\Delta x_i^2}{2} \left(\frac{d^2\rho}{dx^2}\right)_i + \frac{\Delta x_i^3}{6} \left(\frac{d^3\rho}{dx^3}\right)_i + \frac{\Delta x_i^4}{24} \left(\frac{d^4\rho}{dx^4}\right)_i + O(\Delta x_i^5)$$

$$(2) \quad \rho(x_{i-1}) = \rho(x_i) - \Delta x_i \left(\frac{d\rho}{dx}\right)_i + \frac{\Delta x_i^2}{2} \left(\frac{d^2\rho}{dx^2}\right)_i - \frac{\Delta x_i^3}{6} \left(\frac{d^3\rho}{dx^3}\right)_i + \frac{\Delta x_i^4}{24} \left(\frac{d^4\rho}{dx^4}\right)_i + O(\Delta x_i^5)$$

Approximating Second Order Derivative



Add (1) and (2)

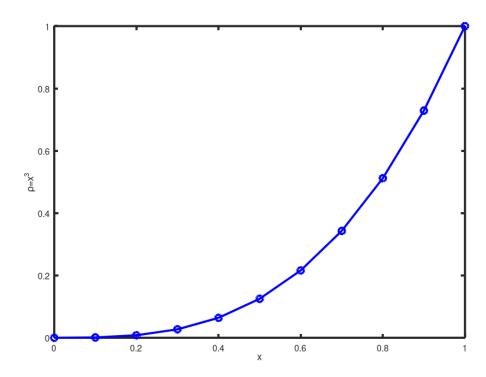
$$\rho(x_{i+1}) + \rho(x_{i-1}) = 2\rho(x_i) + \Delta x_i^2 \left(\frac{d^2 \rho}{dx^2}\right)_i + \frac{\Delta x_i^4}{12} \left(\frac{d^4 \rho}{dx^4}\right)_i + \cdots$$

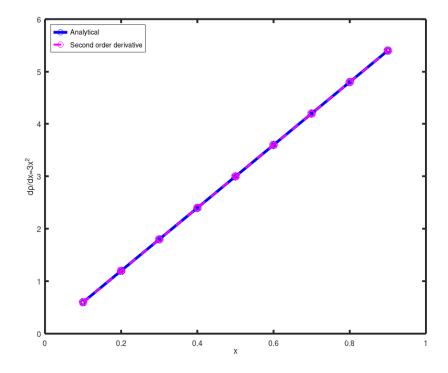
$$\left(\frac{d^{2}\rho}{dx^{2}}\right)_{i} = \frac{\rho(x_{i+1}) - 2\rho(x_{i}) + \rho(x_{i-1})}{\Delta x_{i}^{2}} + \frac{\Delta x_{i}^{4}}{12\Delta x_{i}^{2}} \left(\frac{d^{4}\rho}{dx^{4}}\right)_{i} + \cdots$$

$$\left(\frac{d^{2}\rho}{dx^{2}}\right)_{i} = \frac{\rho(x_{i+1}) - 2\rho(x_{i}) + \rho(x_{i-1})}{\Delta x_{i}^{2}} + O(\Delta x_{i}^{2})$$

Approximating Second Order Derivative

$$\left(\frac{d^2\rho}{dx^2}\right)_i = \frac{\rho(x_{i+1}) - 2\rho(x_i) + \rho(x_{i-1})}{\Delta x_i^2} + O\left(\Delta x_i^2\right)$$





Introducing Octave

Introducing Octave

5_b_octave_introduction.m

Exercise

$$x_{i-1}$$
 x_i x_{i+1} x_{i+2}

- Approximate $\left(\frac{d\rho}{dx}\right)_i$ using first order downwind scheme.
- Make a comparison plot between analytical and numerical results.

$$\left(\frac{d\rho}{dx}\right)_{i} = \frac{\rho(x_{i}) - \rho(x_{i-1})}{\Delta x_{i}} + O(\Delta x_{i})$$
 Update **5_c_exercise_downwind_scheme.m**

$$\left(\frac{d\rho}{dx}\right)_i = \frac{\rho(x_{i+1}) - \rho(x_i)}{\Delta x_i} + O(\Delta x_i) \quad \longleftarrow \quad \text{First order upwind scheme}$$

Thank you