Introduction to Computational Fluid Dynamics using OpenFOAM and Octave

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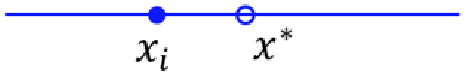
Kumaresh Selvakumar
(Session-6)

Instructions: Mon, Wed, Thu (3:30PM-4:30PM IST)
Query session: Sundays 8:30AM-9:00AM IST

Quick Recap

What Did We Discuss?

Taylor Series



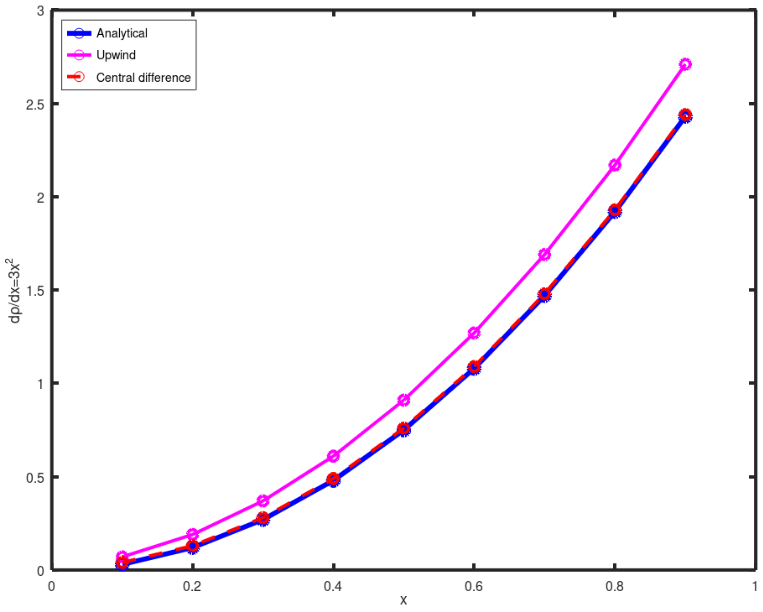
$$\rho(x^*) = \rho(x_i) + \frac{(x^* - x_i)}{1!}\left(\frac{d\rho}{dx}\right)_i + \frac{(x^* - x_i)^2}{2!}\left(\frac{d^2\rho}{dx^2}\right)_i + \frac{(x^* - x_i)^3}{3!}\left(\frac{d^3\rho}{dx^3}\right)_i + \cdots$$

$$\left(\frac{d\rho}{dx}\right)_i \approx \frac{\rho(x_{i+1}) - \rho(x_i)}{\Delta x_i}$$

First order upwind

$$\left(\frac{d\rho}{dx}\right)_i \approx \frac{\rho(x_{i+1}) - \rho(x_{i-1})}{2\Delta x_i}$$

Second order central difference



Current Session

Overview

- Introduction to Octave programming
- Stability Analysis

Introducing Octave

6_a_octave_introduction.m

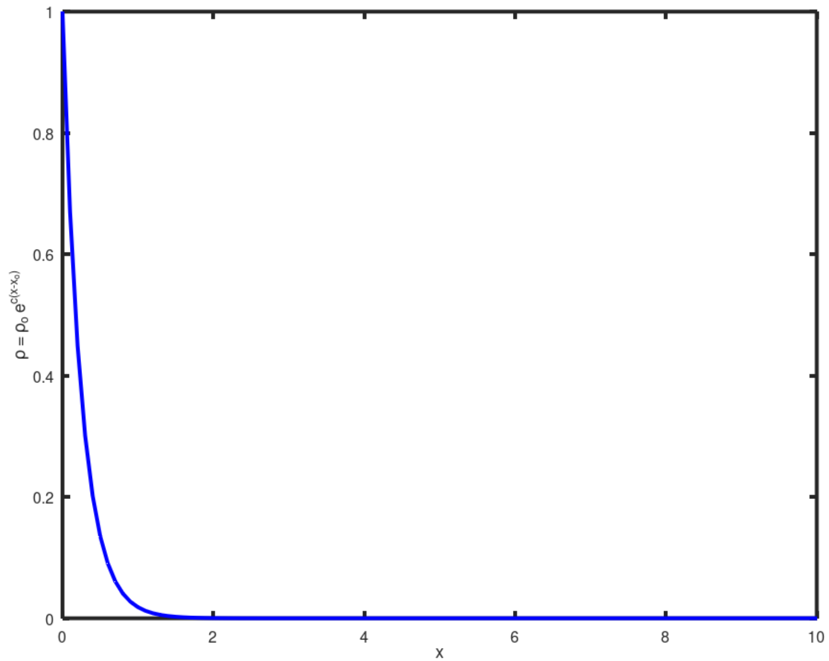
Numerical Stability

- Numerical approach should not magnify the error that appears in the solution.

$$\frac{d\rho}{dx} = -c\rho$$

$$\int_{\rho_o}^{\rho} \frac{d\rho}{\rho} = \int_{x_o}^{x} -c\,dx$$

$$\rho = \rho_o e^{-c(x - x_o)}$$



$$c = 4, x_o = 0, \rho_o = 1, x \in [0,10]$$

Numerical Stability

- Numerical discretization

$$\frac{d\rho}{dx} = -c\rho$$

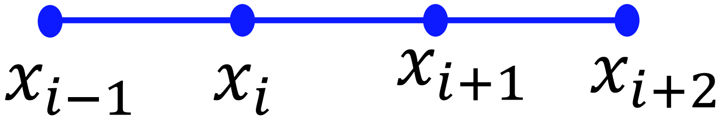
$$\frac{\rho_{i+1} - \rho_i}{\Delta x} = -c\rho_i$$

$$\rho_{i+1} = \rho_i(1 - c\Delta x)$$

Numerical Stability

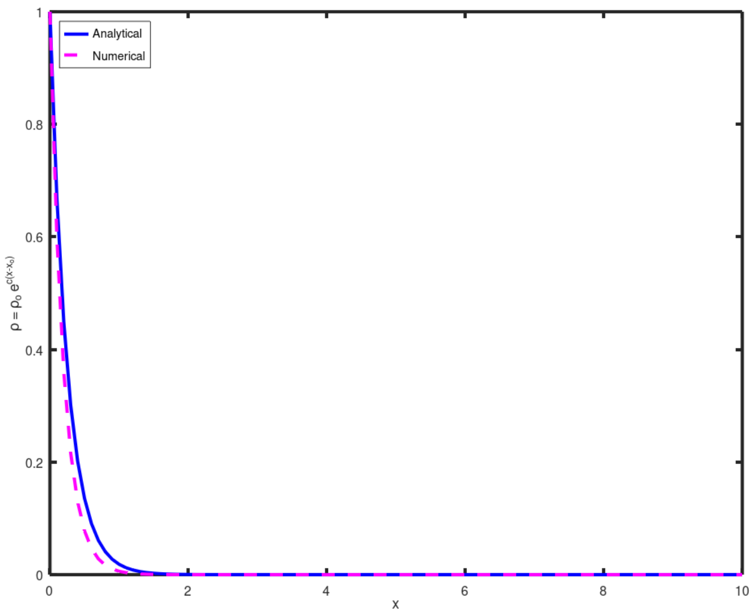
- Numerical discretization

$$\frac{d\rho}{dx} = -c\rho$$



$$\frac{\rho_{i+1} - \rho_i}{\Delta x} = -c\rho_i$$

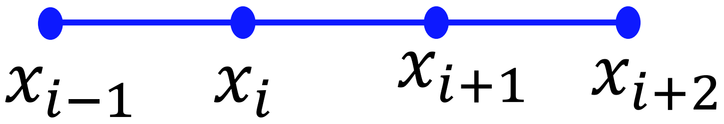
$$\rho_{i+1} = \rho_i(1 - c\Delta x)$$



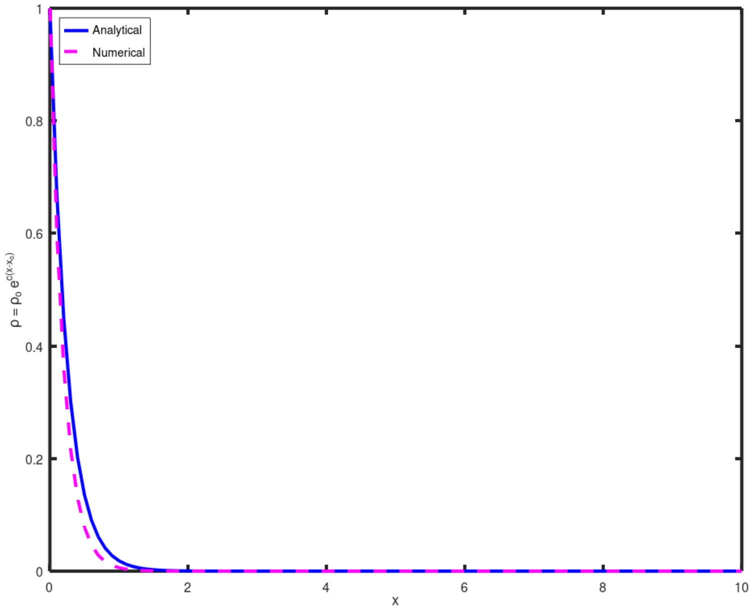
$c = 4, x_o = 0, \rho_o = 1, x \in [0,10], \Delta x = 0.1$

Numerical Stability

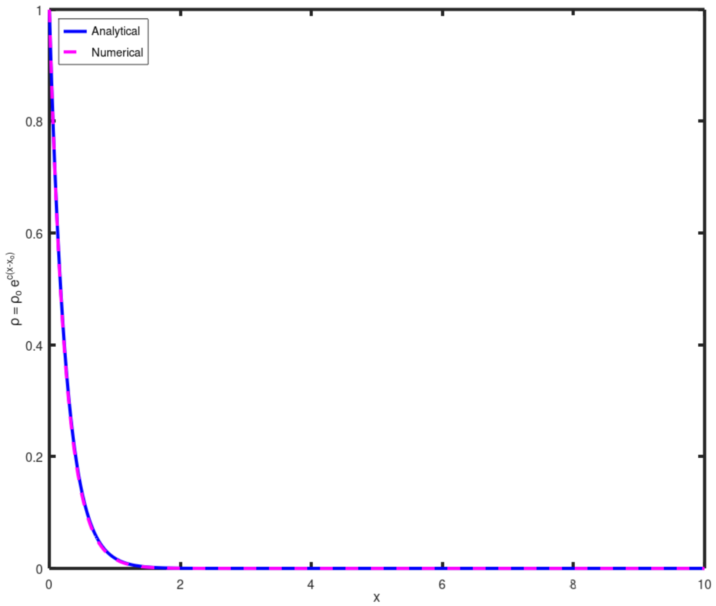
$$\frac{d\rho}{dx} = -c\rho$$



$$\rho_{i+1} = \rho_i(1 - c\Delta x)$$

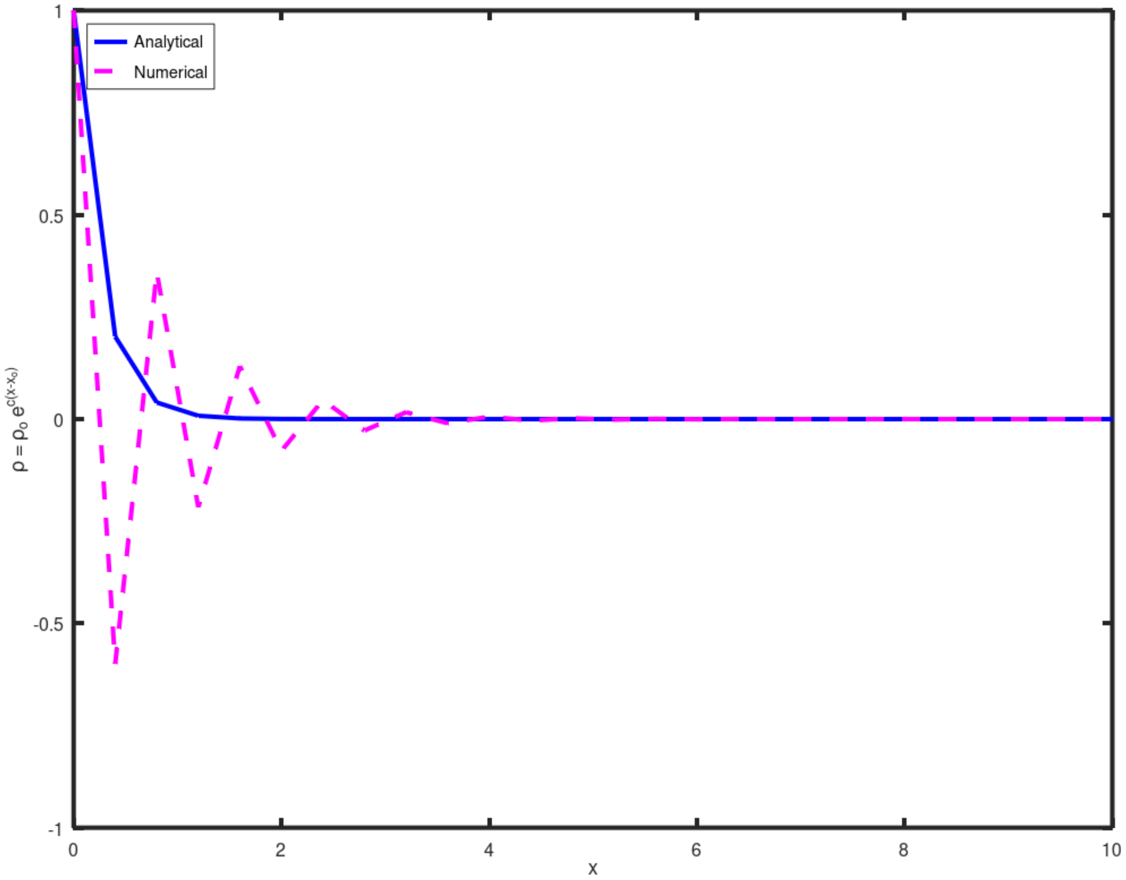


$c = 4, x_o = 0, \rho_o = 1, x \in [0,10], \Delta x = 0.1$



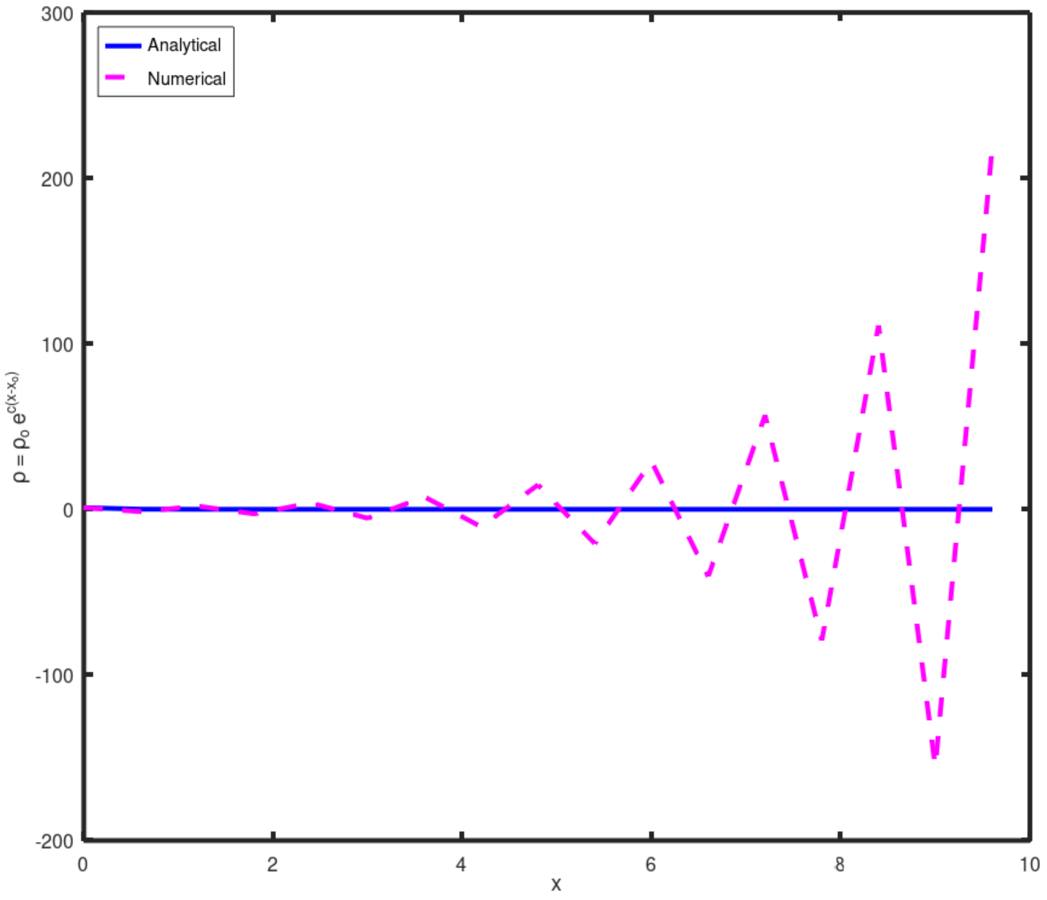
$c = 4, x_o = 0, \rho_o = 1, x \in [0,10], \Delta x = 0.01$

Numerical Stability



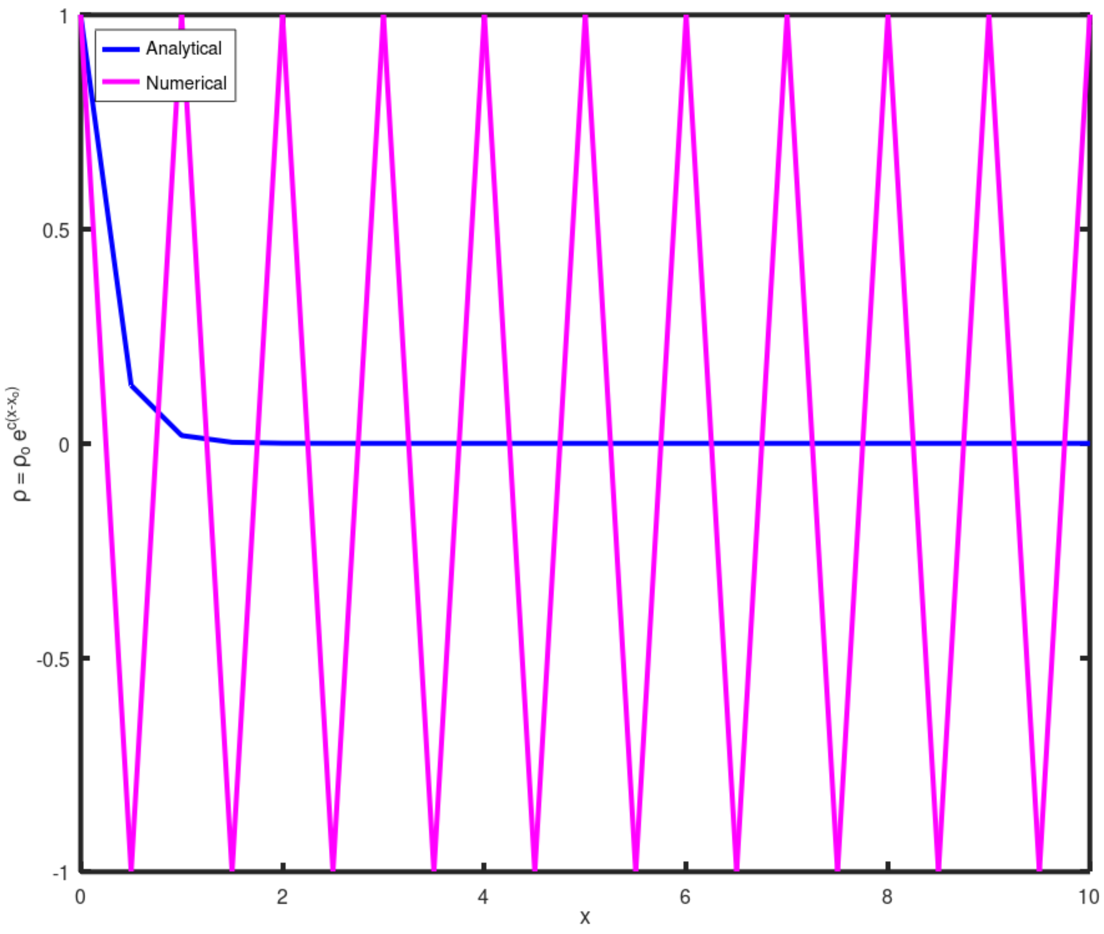
$c = 4, x_o = 0, \rho_o = 1, x \in [0,10], \Delta x = 0.4$

Numerical Stability



$c = 4, x_o = 0, \rho_o = 1, x \in [0,10], \Delta x = 0.6$

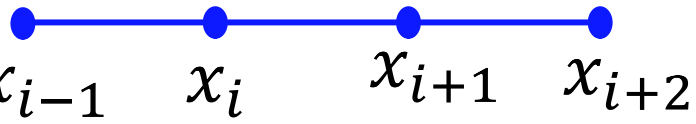
Numerical Stability



$c = 4, x_o = 0, \rho_o = 1, x \in [0,10], \Delta x = 0.5$

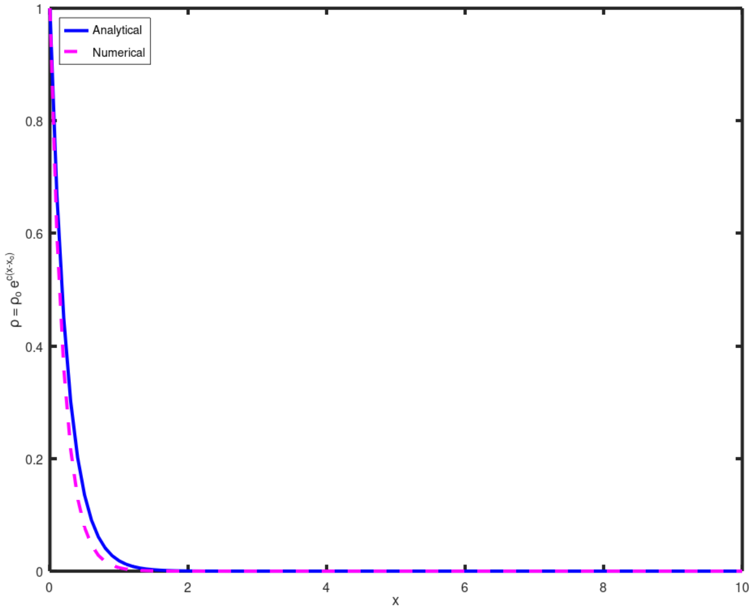
Numerical Stability

$$\frac{d\rho}{dx} = -c\rho$$

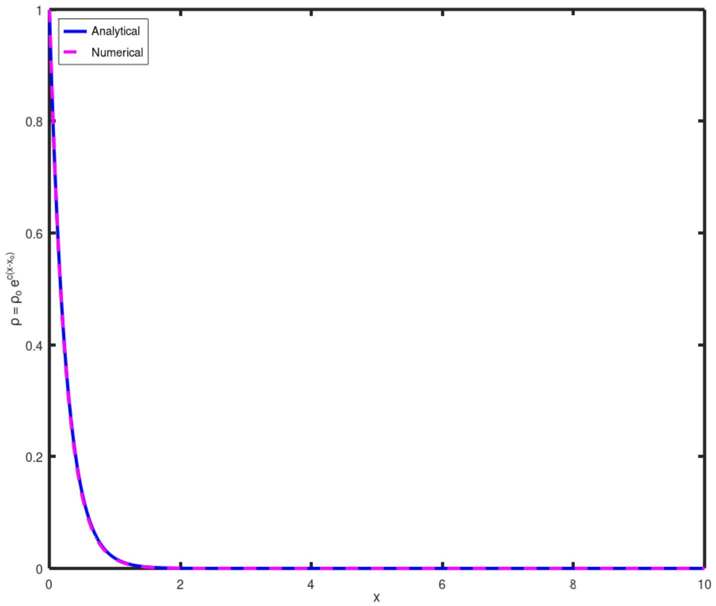


$$\left|\frac{\rho_{i+1}}{\rho_i}\right| < 1$$

$$\rho_{i+1} = \rho_i(1 - c\Delta x)$$



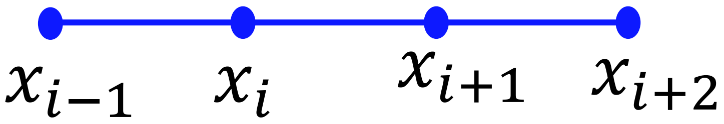
$c = 4, x_o = 0, \rho_o = 1, x \in [0,10], \Delta x = 0.1$



$c = 4, x_o = 0, \rho_o = 1, x \in [0,10], \Delta x = 0.01$

Numerical Stability

$$\frac{d\rho}{dx} = -c\rho$$



$$\left|\frac{\rho_{i+1}}{\rho_i}\right| < 1$$

$$\rho_{i+1} = \rho_i(1 - c\Delta x)$$

$$\left|\frac{\rho_{i+1}}{\rho_i}\right| = |(1 - c\Delta x)| < 1$$

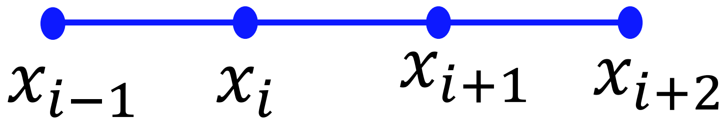
$$-1 < (1 - c\Delta x) < 1$$

$$0 < \Delta x < 2/c$$

$$\boxed{\Delta x < 2/c}$$

Numerical Stability

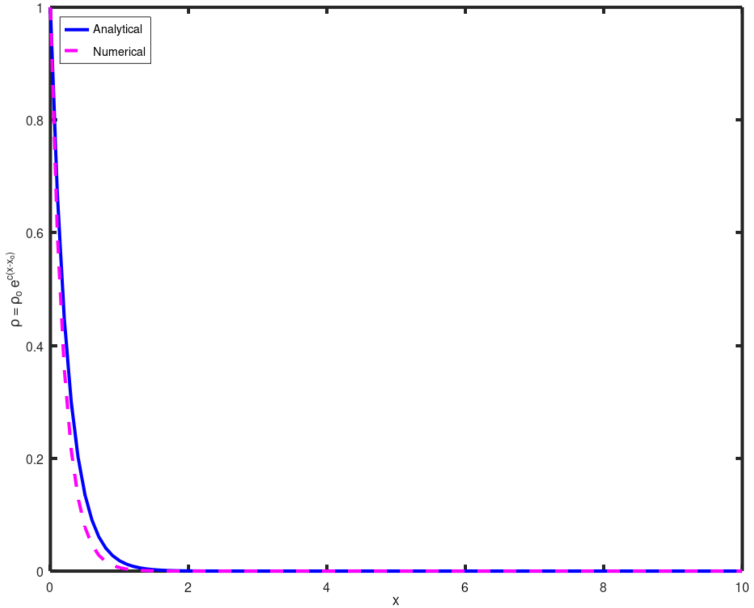
$$\frac{d\rho}{dx} = -c\rho$$



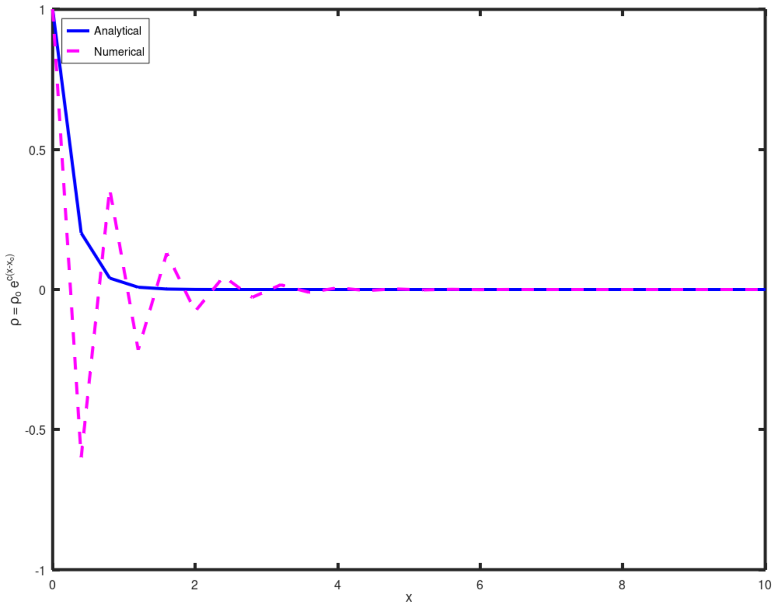
$$\Delta x < \frac{2}{c}$$

$$\frac{2}{c} = \frac{2}{4} = 0.5$$

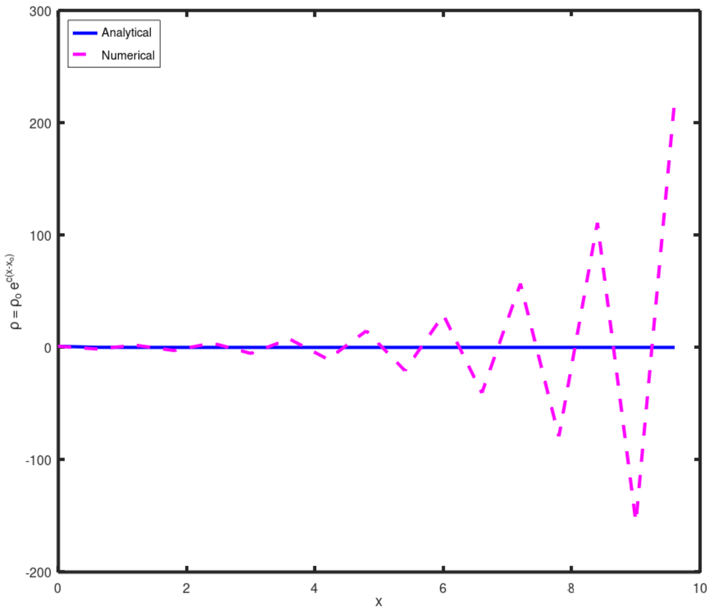
$$\rho_{i+1} = \rho_i(1 - c\Delta x)$$



$c = 4, x_o = 0, \rho_o = 1, x \in [0,10], \Delta x = 0.1$

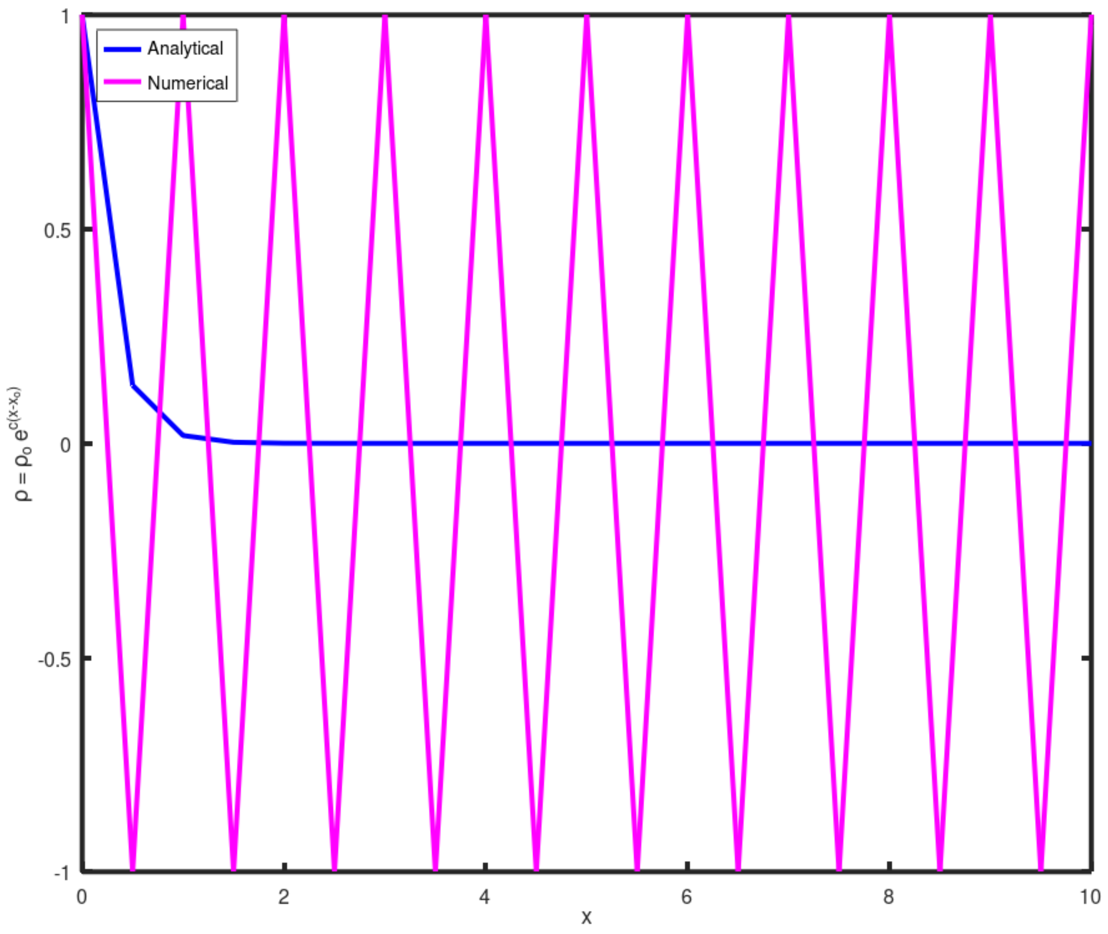


$c = 4, x_o = 0, \rho_o = 1, x \in [0,10], \Delta x = 0.4$



$c = 4, x_o = 0, \rho_o = 1, x \in [0,10], \Delta x = 0.6$

Numerical Stability



$c = 4, x_o = 0, \rho_o = 1, x \in [0,10], \Delta x = 0.5$

$$\Delta x < \frac{2}{c}$$

$$\frac{2}{c} = \frac{2}{4} = 0.5$$

Next Session

- Brief look at stability analysis again
- Introduction to C++ for OpenFOAM

Thank you