

Introduction to Computational Fluid Dynamics using OpenFOAM and Octave

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(Session-5)

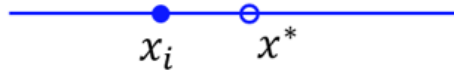
Instructions: Mon, Wed, Thu (3:30PM-4:30PM IST)

Query session: Sundays 8:30AM-9:00AM IST

Quick Recap

What Did We Discuss?

Taylor Series



$$\rho(x^*) = \rho(x_i) + \frac{(x^* - x_i)}{1!} \left(\frac{d\rho}{dx} \right)_i + \frac{(x^* - x_i)^2}{2!} \left(\frac{d^2\rho}{dx^2} \right)_i + \frac{(x^* - x_i)^3}{3!} \left(\frac{d^3\rho}{dx^3} \right)_i + \dots$$



$$\left(\frac{d\rho}{dx} \right)_i \approx \frac{\rho(x_{i+1}) - \rho(x_i)}{\Delta x_i}$$

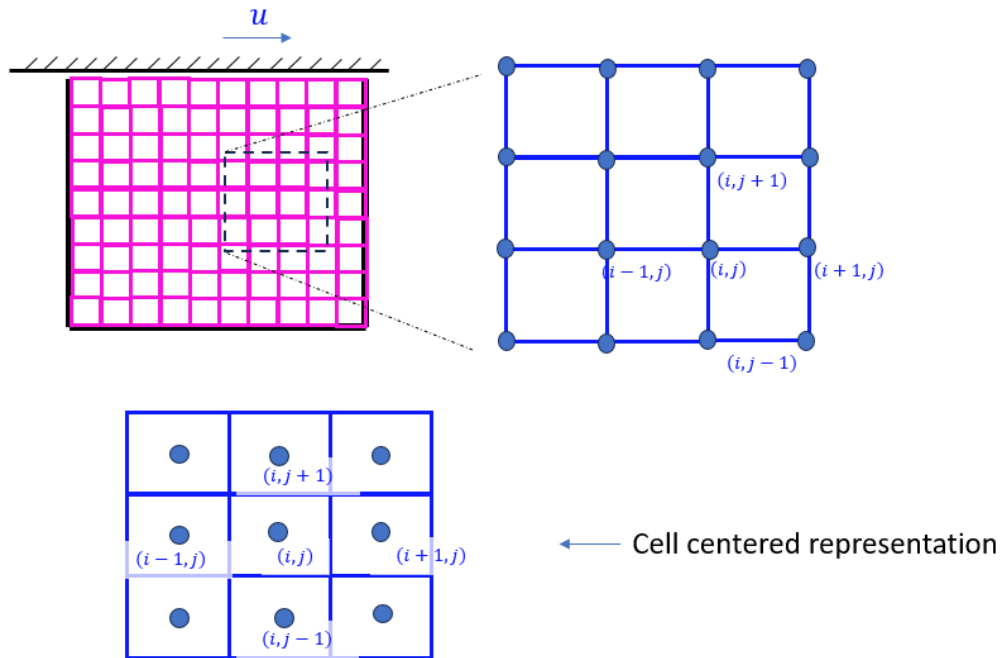
First order upwind

$$\left(\frac{d\rho}{dx} \right)_i \approx \frac{\rho(x_{i+1}) - \rho(x_{i-1}))}{2\Delta x_i}$$

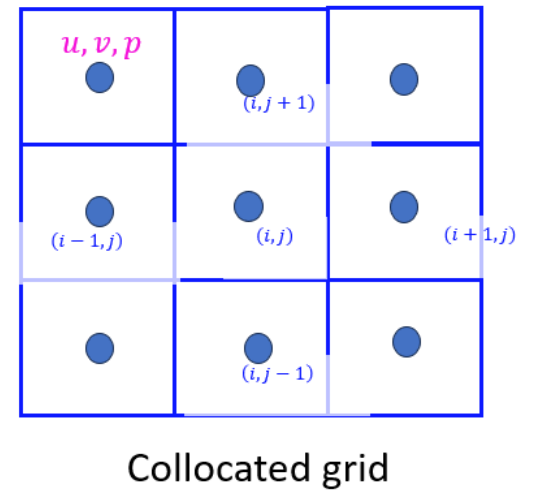
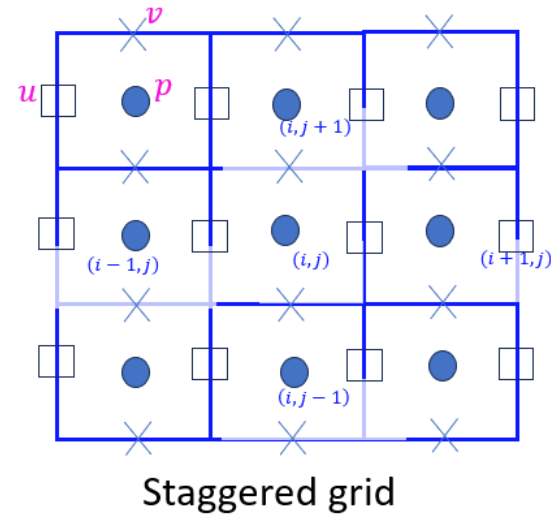
Second order central difference

What Did We Discuss?

Grid Layout



- Cell centered representation

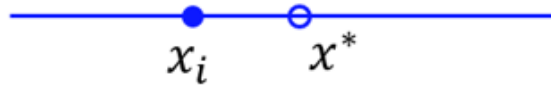


Current Session

Overview

- Taylor series analysis (exercise-2)
- Introduction to Octave programming
- Exercise using Octave

First Order Derivative Approximations



$$\rho(x^*) = \rho(x_i) + \frac{(x^* - x_i)}{1!} \left(\frac{d\rho}{dx} \right)_i + \frac{(x^* - x_i)^2}{2!} \left(\frac{d^2\rho}{dx^2} \right)_i + \frac{(x^* - x_i)^3}{3!} \left(\frac{d^3\rho}{dx^3} \right)_i + \dots$$



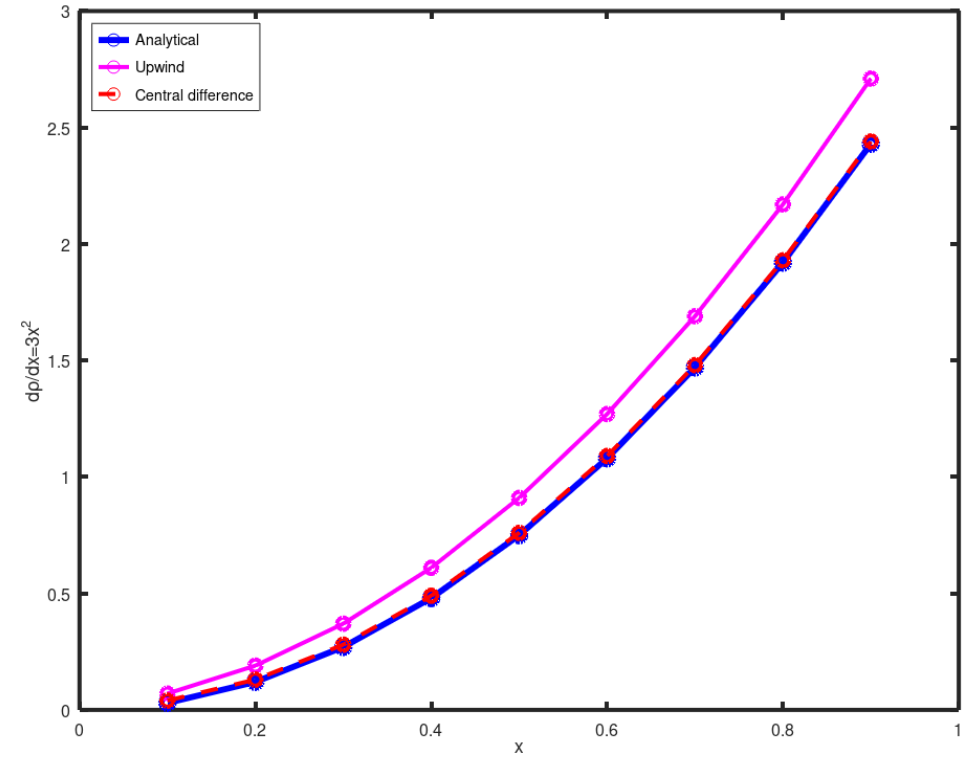
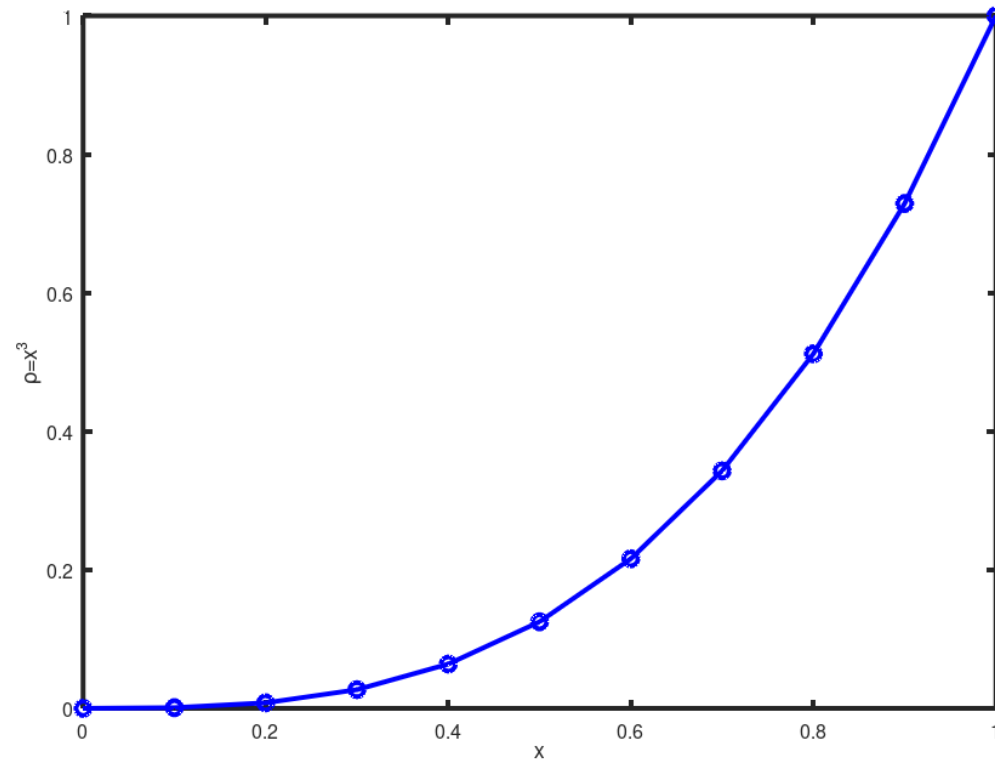
$$\left(\frac{d\rho}{dx} \right)_i \approx \frac{\rho(x_{i+1}) - \rho(x_i)}{\Delta x_i}$$

First order upwind

$$\left(\frac{d\rho}{dx} \right)_i \approx \frac{\rho(x_{i+1}) - \rho(x_{i-1}))}{2\Delta x_i}$$

Second order central difference

First Order Derivative Approximations

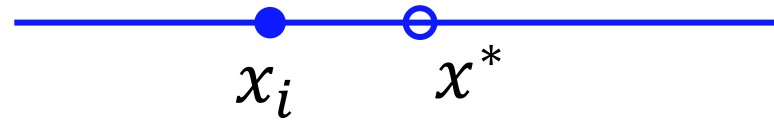


Exercise



- Derive expression for $\left(\frac{d^2\rho}{dx^2}\right)_i$
- What is the accuracy of the resultant expression?

Taylor Series



$$\rho(x^*) = \rho(x_i) + (x^* - x_i) \left(\frac{d\rho}{dx} \right)_i + \frac{(x^* - x_i)^2}{2!} \left(\frac{d^2\rho}{dx^2} \right)_i + \frac{(x^* - x_i)^3}{3!} \left(\frac{d^3\rho}{dx^3} \right)_i + \dots$$

Approximating Second Order Derivative

Derive expression for $\left(\frac{d^2\rho}{dx^2}\right)_i$



$$\rho(x_{i+1}) = \rho(x_i) + (x_{i+1} - x_i) \left(\frac{d\rho}{dx}\right)_i + \frac{(x_{i+1} - x_i)^2}{2} \left(\frac{d^2\rho}{dx^2}\right)_i + \frac{(x_{i+1} - x_i)^3}{6} \left(\frac{d^3\rho}{dx^3}\right)_i + \frac{(x_{i+1} - x_i)^4}{24} \left(\frac{d^4\rho}{dx^4}\right)_i + \dots$$

$$(1) \quad \rho(x_{i+1}) = \rho(x_i) + \Delta x_i \left(\frac{d\rho}{dx}\right)_i + \frac{\Delta x_i^2}{2} \left(\frac{d^2\rho}{dx^2}\right)_i + \frac{\Delta x_i^3}{6} \left(\frac{d^3\rho}{dx^3}\right)_i + \frac{\Delta x_i^4}{24} \left(\frac{d^4\rho}{dx^4}\right)_i + O(\Delta x_i^5)$$

$$(2) \quad \rho(x_{i-1}) = \rho(x_i) - \Delta x_i \left(\frac{d\rho}{dx}\right)_i + \frac{\Delta x_i^2}{2} \left(\frac{d^2\rho}{dx^2}\right)_i - \frac{\Delta x_i^3}{6} \left(\frac{d^3\rho}{dx^3}\right)_i + \frac{\Delta x_i^4}{24} \left(\frac{d^4\rho}{dx^4}\right)_i + O(\Delta x_i^5)$$

Approximating Second Order Derivative

$$(1) \quad \rho(x_{i+1}) = \rho(x_i) + \Delta x_i \left(\frac{d\rho}{dx} \right)_i + \frac{\Delta x_i^2}{2} \left(\frac{d^2\rho}{dx^2} \right)_i + \frac{\Delta x_i^3}{6} \left(\frac{d^3\rho}{dx^3} \right)_i + \frac{\Delta x_i^4}{24} \left(\frac{d^4\rho}{dx^4} \right)_i + O(\Delta x_i^5)$$

$$(2) \quad \rho(x_{i-1}) = \rho(x_i) - \Delta x_i \left(\frac{d\rho}{dx} \right)_i + \frac{\Delta x_i^2}{2} \left(\frac{d^2\rho}{dx^2} \right)_i - \frac{\Delta x_i^3}{6} \left(\frac{d^3\rho}{dx^3} \right)_i + \frac{\Delta x_i^4}{24} \left(\frac{d^4\rho}{dx^4} \right)_i + O(\Delta x_i^5)$$



Add (1) and (2)

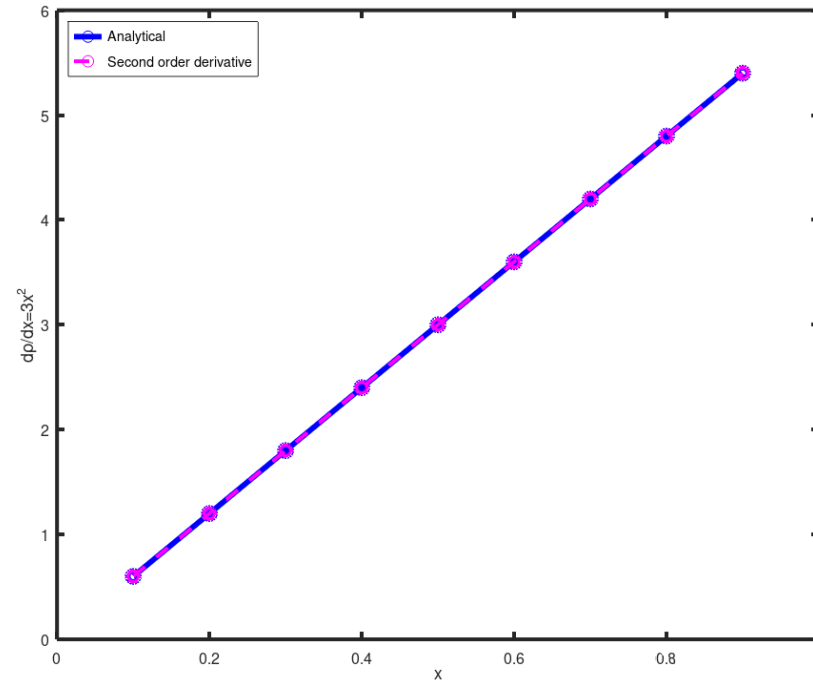
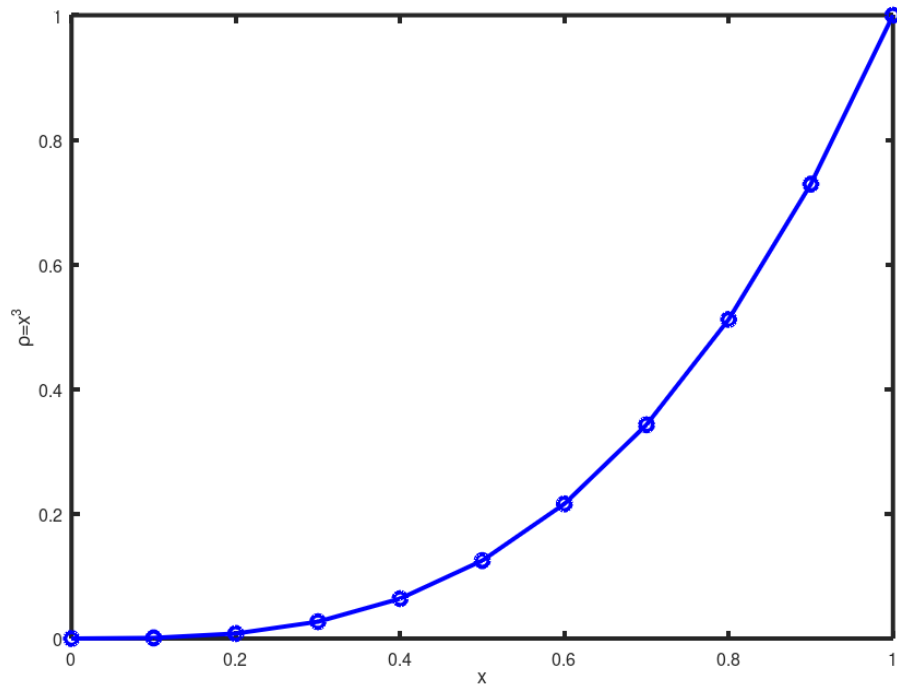
$$\rho(x_{i+1}) + \rho(x_{i-1}) = 2\rho(x_i) + \Delta x_i^2 \left(\frac{d^2\rho}{dx^2} \right)_i + \frac{\Delta x_i^4}{12} \left(\frac{d^4\rho}{dx^4} \right)_i + \dots$$

$$\left(\frac{d^2\rho}{dx^2} \right)_i = \frac{\rho(x_{i+1}) - 2\rho(x_i) + \rho(x_{i-1}))}{\Delta x_i^2} + \frac{\Delta x_i^4}{12\Delta x_i^2} \left(\frac{d^4\rho}{dx^4} \right)_i + \dots$$

$$\boxed{\left(\frac{d^2\rho}{dx^2} \right)_i = \frac{\rho(x_{i+1}) - 2\rho(x_i) + \rho(x_{i-1}))}{\Delta x_i^2} + O(\Delta x_i^2)}$$

Approximating Second Order Derivative

$$\left(\frac{d^2\rho}{dx^2}\right)_i = \frac{\rho(x_{i+1}) - 2\rho(x_i) + \rho(x_{i-1}))}{\Delta x_i^2} + o(\Delta x_i^2)$$



Introducing Octave

Introducing Octave

5_b_octave_introduction.m

Exercise



- Approximate $\left(\frac{d\rho}{dx}\right)_i$ using first order downwind scheme.
- Make a comparison plot between analytical and numerical results.

$$\left(\frac{d\rho}{dx}\right)_i = \frac{\rho(x_i) - \rho(x_{i-1})}{\Delta x_i} + O(\Delta x_i)$$

Update **5_c_exercise_downwind_scheme.m**

$$\left(\frac{d\rho}{dx}\right)_i = \frac{\rho(x_{i+1}) - \rho(x_i)}{\Delta x_i} + O(\Delta x_i) \quad \leftarrow \text{First order upwind scheme}$$

Thank you