

Introduction to Computational Fluid Dynamics using OpenFOAM and Octave

Lakshman Anumolu
Kumaresh Selvakumar
(Session-2)

Instructions: Mon, Wed, Thu (3:30PM-4:30PM IST)

Query session: Sundays 8:30AM-9:00AM IST

Overview

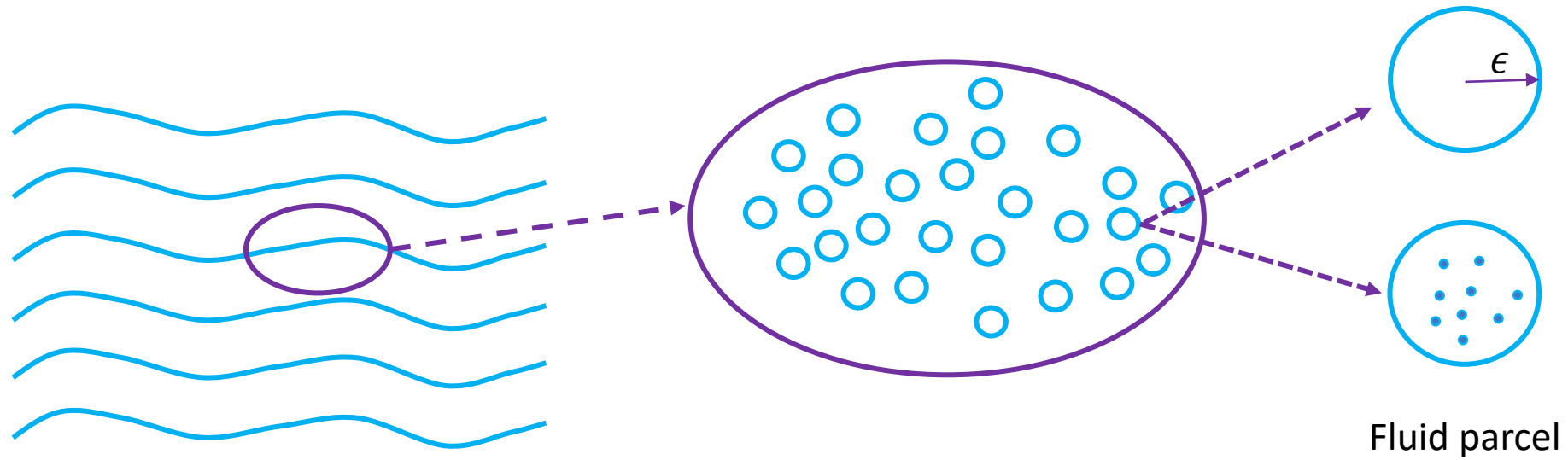
- Fluid Behavior & Mathematical Operators
- Lagrangian & Eulerian Frames
- Governing Equations

Reminder

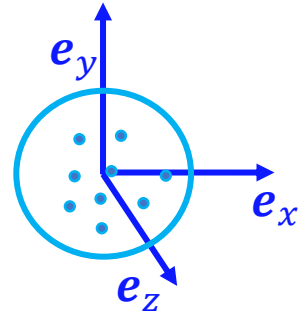
- Exercise-1
 - Github
 - Discussion forum:
 - <https://github.com/exaslate-courses/cfd-openfoam-b2/discussions>
- Operating System:
 - Ubuntu 22.04
- Softwares:
 - OpenFOAM v2306
 - Octave

Fluid Behavior & Mathematical Operators

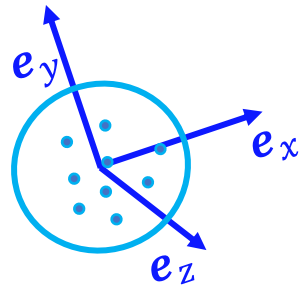
Fluid



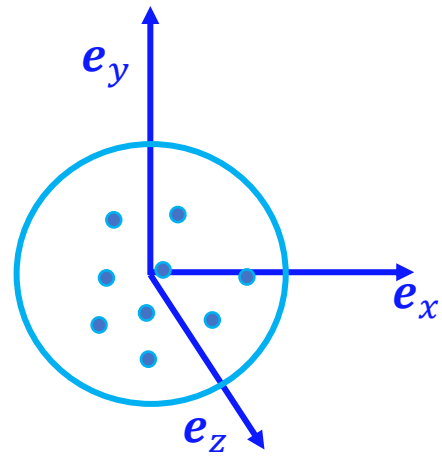
Fluid Behavior



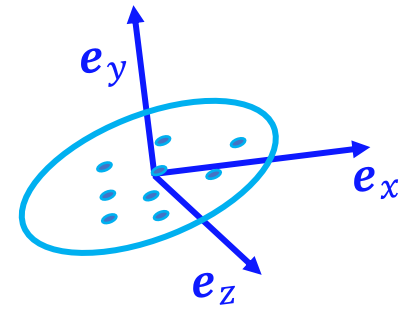
Fluid parcel



Rotation



Expansion



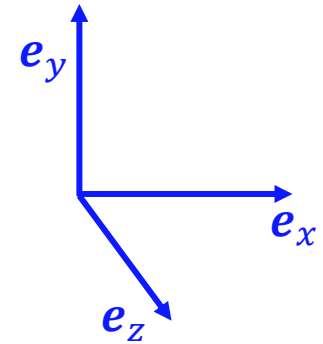
Deformation

Mathematical Operators

- Gradient

$$\nabla \rho = \left(\frac{\partial}{\partial x} \mathbf{e}_x + \frac{\partial}{\partial y} \mathbf{e}_y + \frac{\partial}{\partial z} \mathbf{e}_z \right) \rho = \left(\frac{\partial \rho}{\partial x} \mathbf{e}_x + \frac{\partial \rho}{\partial y} \mathbf{e}_y + \frac{\partial \rho}{\partial z} \mathbf{e}_z \right)$$

$$\nabla \mathbf{u} = \begin{bmatrix} \frac{\partial u}{\partial x} & \frac{\partial v}{\partial x} & \frac{\partial w}{\partial x} \\ \frac{\partial u}{\partial y} & \frac{\partial v}{\partial y} & \frac{\partial w}{\partial y} \\ \frac{\partial u}{\partial z} & \frac{\partial v}{\partial z} & \frac{\partial w}{\partial z} \end{bmatrix}$$



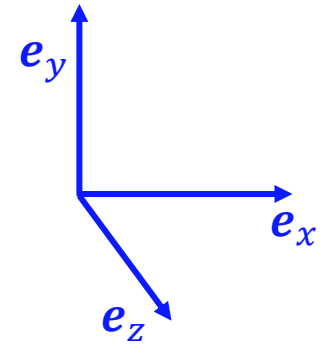
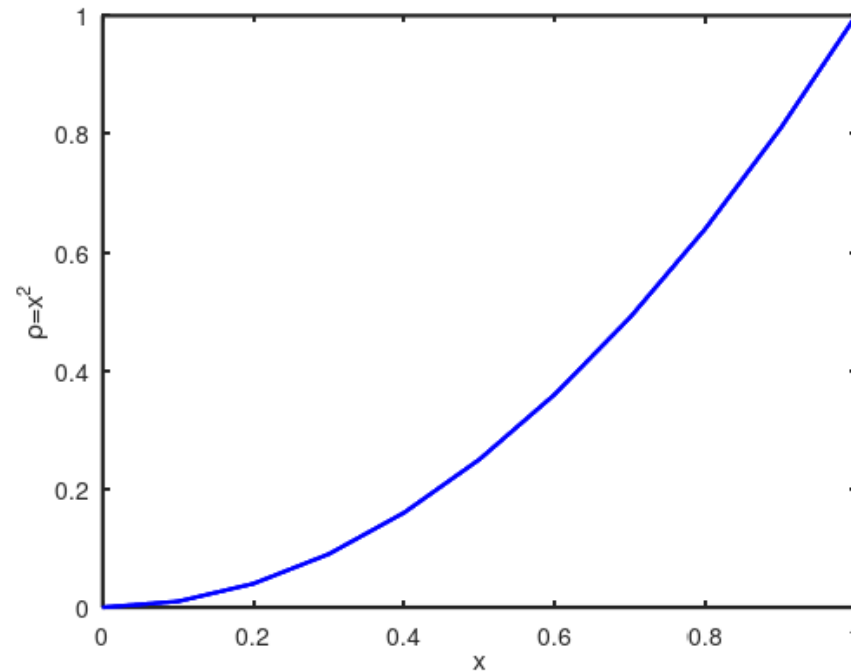
- Divergence

$$\nabla \cdot \mathbf{u} = \left(\frac{\partial}{\partial x} \mathbf{e}_x + \frac{\partial}{\partial y} \mathbf{e}_y + \frac{\partial}{\partial z} \mathbf{e}_z \right) (u \mathbf{e}_x + v \mathbf{e}_y + w \mathbf{e}_z) = \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} \right)$$

Gradient

$$\nabla \rho = \left(\frac{\partial}{\partial x} \mathbf{e}_x + \frac{\partial}{\partial y} \mathbf{e}_y + \frac{\partial}{\partial z} \mathbf{e}_z \right) \rho = \left(\frac{\partial \rho}{\partial x} \mathbf{e}_x + \frac{\partial \rho}{\partial y} \mathbf{e}_y + \frac{\partial \rho}{\partial z} \mathbf{e}_z \right)$$

$$\frac{\partial \rho}{\partial x} = \frac{d\rho}{dx} \text{ (in 1D)}$$



Gradient

$$\nabla \rho = \left(\frac{\partial}{\partial x} \mathbf{e}_x + \frac{\partial}{\partial y} \mathbf{e}_y + \frac{\partial}{\partial z} \mathbf{e}_z \right) \rho = \left(\frac{\partial \rho}{\partial x} \mathbf{e}_x + \frac{\partial \rho}{\partial y} \mathbf{e}_y + \frac{\partial \rho}{\partial z} \mathbf{e}_z \right)$$

- Numerical Approximation



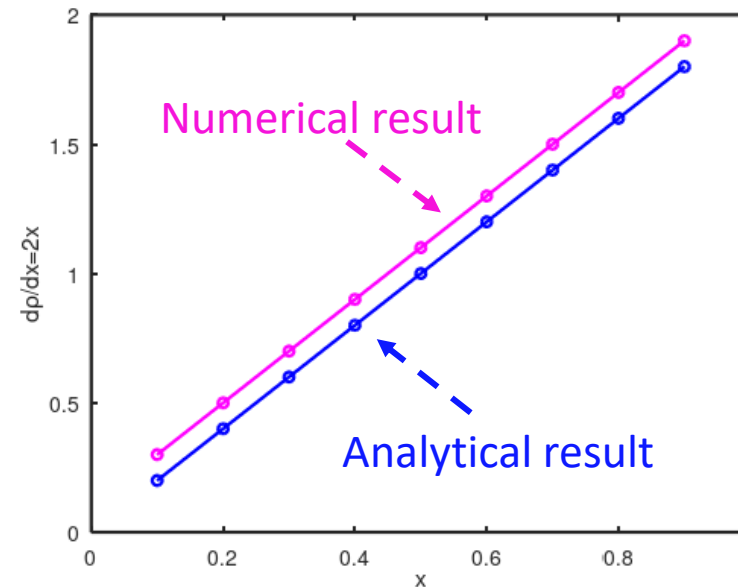
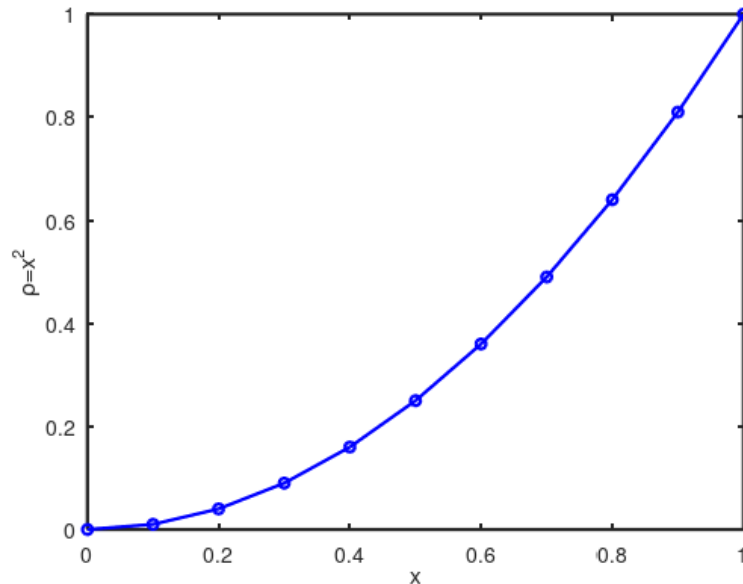
$$\left(\frac{\partial \rho}{\partial x} \right)_i = \frac{\rho(x_{i+1}) - \rho(x_i)}{\Delta x_i} + O(\Delta x_i)$$

Gradient

- Numerical Approximation



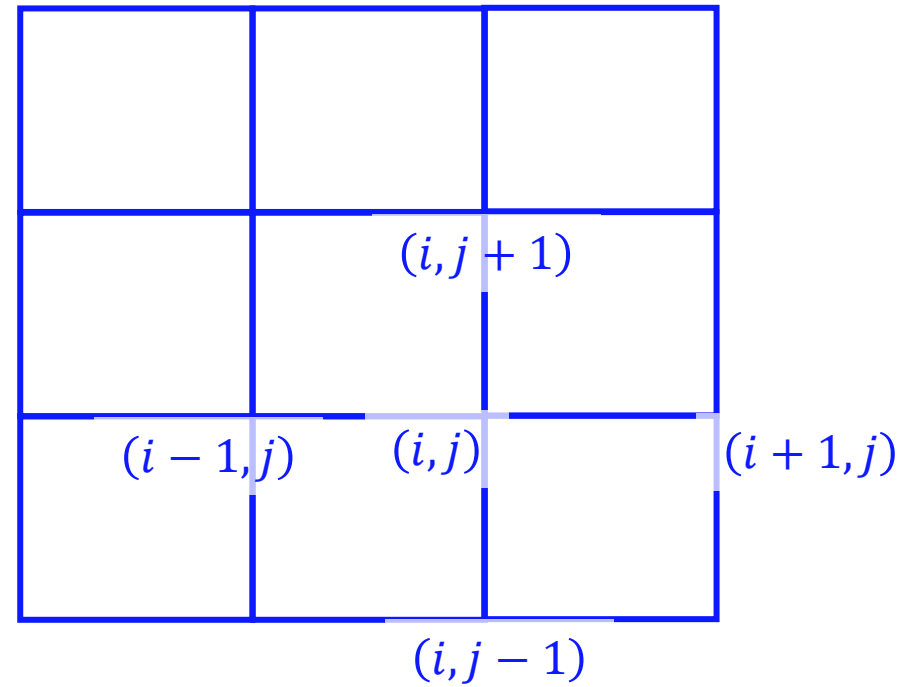
$$\left(\frac{\partial \rho}{\partial x}\right)_i = \frac{\rho(x_{i+1}) - \rho(x_i)}{\Delta x_i} + O(\Delta x_i)$$



Gradient

$$\nabla \rho = \left(\frac{\partial}{\partial x} \mathbf{e}_x + \frac{\partial}{\partial y} \mathbf{e}_y + \frac{\partial}{\partial z} \mathbf{e}_z \right) \rho = \left(\frac{\partial \rho}{\partial x} \mathbf{e}_x + \frac{\partial \rho}{\partial y} \mathbf{e}_y + \frac{\partial \rho}{\partial z} \mathbf{e}_z \right)$$

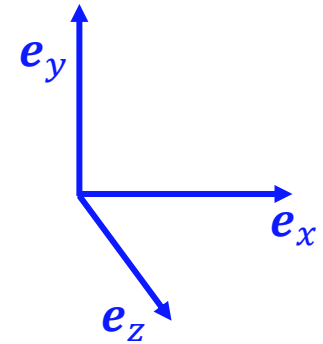
$$\nabla \mathbf{u} = \begin{bmatrix} \partial u / \partial x & \partial v / \partial x & \partial w / \partial x \\ \partial u / \partial y & \partial v / \partial y & \partial w / \partial y \\ \partial u / \partial z & \partial v / \partial z & \partial w / \partial z \end{bmatrix}$$



Mathematical Operations

- Divergence

$$\nabla \cdot \mathbf{u} = \left(\frac{\partial}{\partial x} \mathbf{e}_x + \frac{\partial}{\partial y} \mathbf{e}_y + \frac{\partial}{\partial z} \mathbf{e}_z \right) (u \mathbf{e}_x + v \mathbf{e}_y + w \mathbf{e}_z) = \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} \right)$$



- Numerical approximation

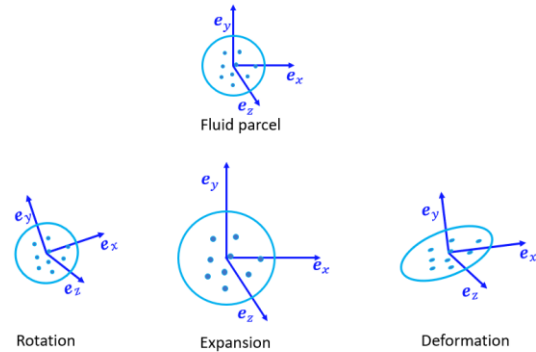
- Same as earlier

- Physical significance

- Positive value : Source or expansion of fluid volume
- Negative value: Sink
- Zero signifies incompressible nature or no change in volume

What Did We Discuss?

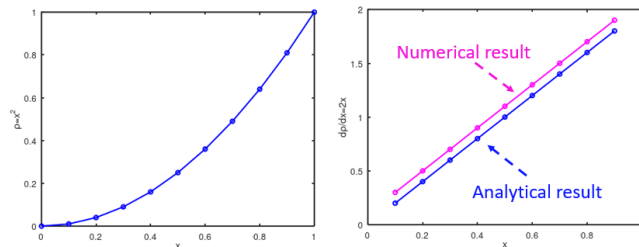
Fluid Behavior



Gradient

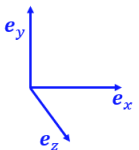
- Numerical Approximation

$$\begin{array}{ccccccc} \bullet & & \bullet & & \bullet & & \bullet \\ x_{i-1} & & x_i & & x_{i+1} & & x_{i+2} \end{array} \quad \left(\frac{\partial \rho}{\partial x} \right)_i = \frac{\rho(x_{i+1}) - \rho(x_i)}{\Delta x_i} + O(\Delta x_i)$$



Mathematical Operations

- Divergence

$$\nabla \cdot \mathbf{u} = \left(\frac{\partial}{\partial x} e_x + \frac{\partial}{\partial y} e_y + \frac{\partial}{\partial z} e_z \right) (u e_x + v e_y + w e_z) = \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} \right)$$


- Numerical approximation

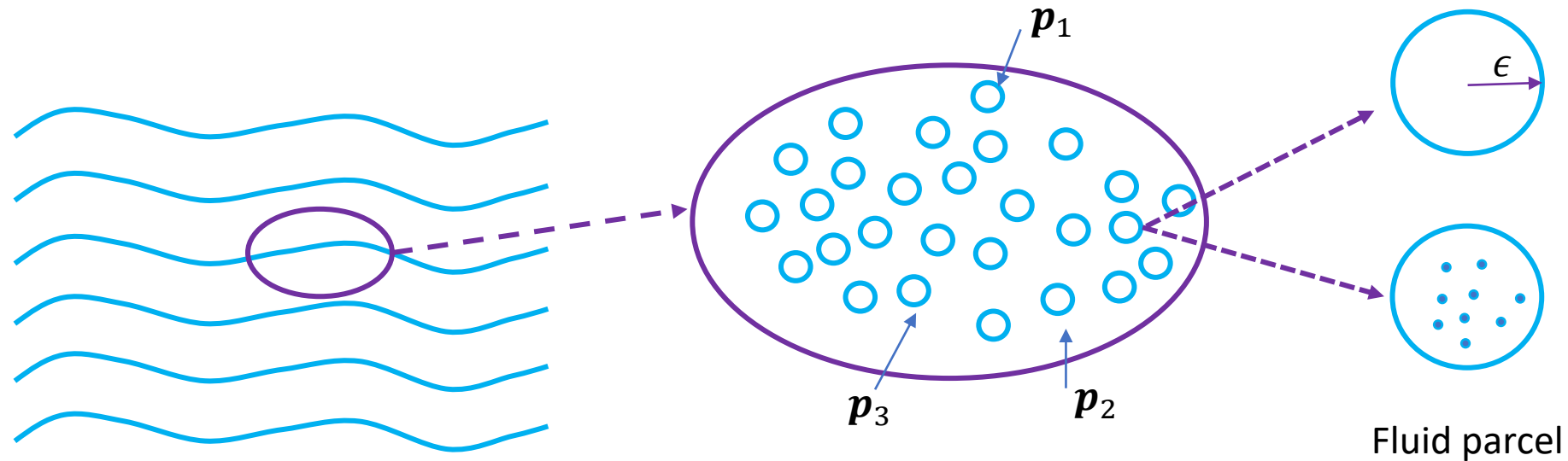
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Lagrangian & Eulerian Frameworks

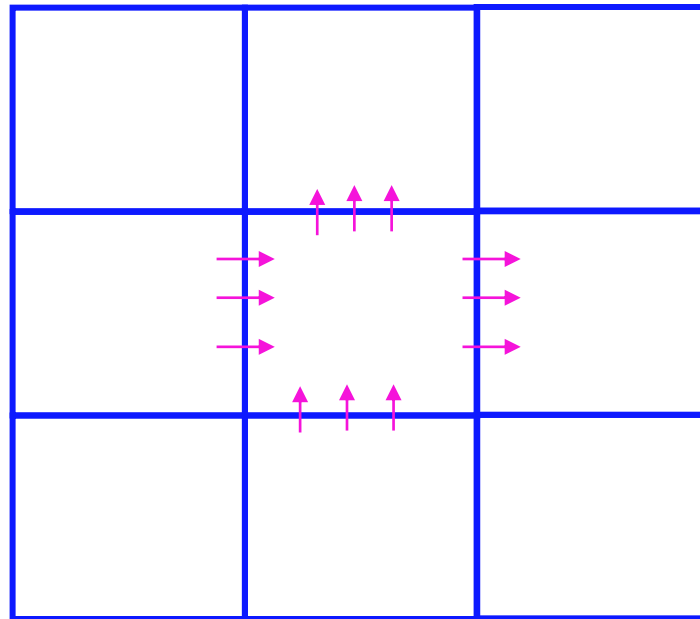
Lagrangian frame



- Follow a fluid parcel ($\mathbf{p}_1, \mathbf{p}_2, \dots$)
- Flow property at a location is obtained from the fluid parcel that happens to be at that location at that time
- Useful to derive conservation laws

Eulerian frame

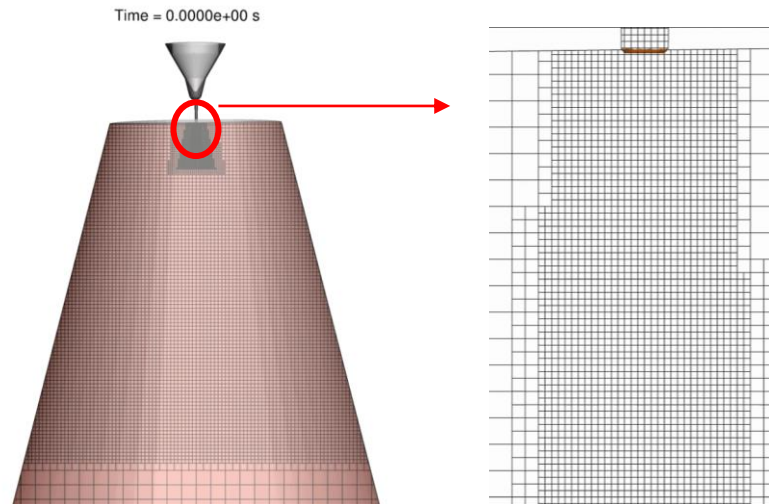
- Conservation laws are applied around a fixed “control volume” in space
- Flux of quantities through the boundary is used to estimate the flow variables
- Useful to take observations at fixed locations



Lagrangian-Eulerian [Material Derivative]

$$\frac{D\phi(\mathbf{X}(\mathbf{p}_i, t), t)}{Dt} = \frac{\partial\phi}{\partial t} + \frac{\partial\phi}{\partial x} \frac{DX}{Dt} + \frac{\partial\phi}{\partial y} \frac{DY}{Dt} + \frac{\partial\phi}{\partial z} \frac{DZ}{Dt}$$

$$\frac{D\phi}{Dt} = \frac{\partial\phi}{\partial t} + \mathbf{u} \cdot \nabla\phi$$



Governing Equations

Conservation Laws

- Conservation of mass

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{u}) = 0; \quad \nabla \cdot (\rho \mathbf{u}) = \frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} + \frac{\partial(\rho w)}{\partial z}$$

- Conservation of momentum

$$\frac{\partial \rho \mathbf{u}}{\partial t} + \nabla \cdot (\rho \mathbf{u} \mathbf{u}) = -\nabla p + \nabla \cdot \boldsymbol{\sigma} + \rho \mathbf{g};$$

- Scalar conservation law

$$\frac{\partial \rho \phi}{\partial t} + \nabla \cdot (\rho \mathbf{u} \phi) = \nabla \cdot (\Gamma \nabla \phi) + S_\phi$$

Integral Form – Differential Form

- Conservation of mass

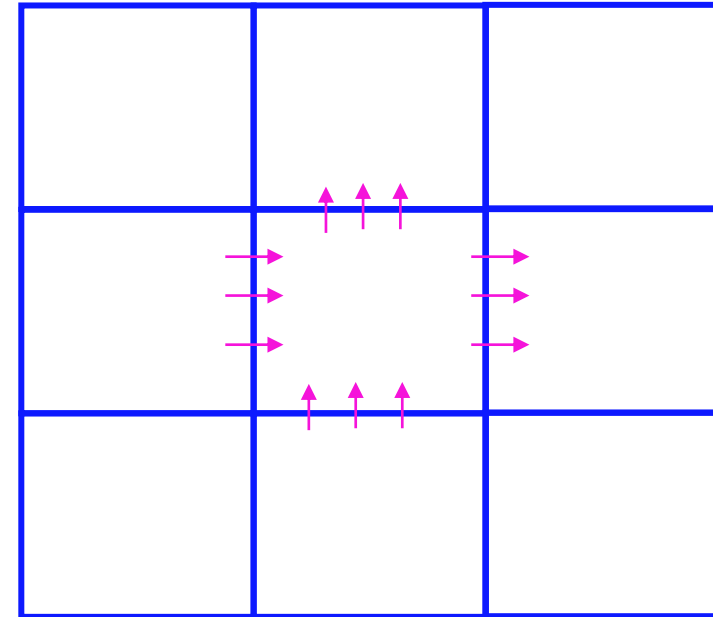
$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{u}) = 0$$

- Integrate over a control volume

$$\int_V \left(\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{u}) \right) dV = 0$$

$$\int_V \frac{\partial \rho}{\partial t} dV + \int_V \nabla \cdot (\rho \mathbf{u}) dV = 0$$

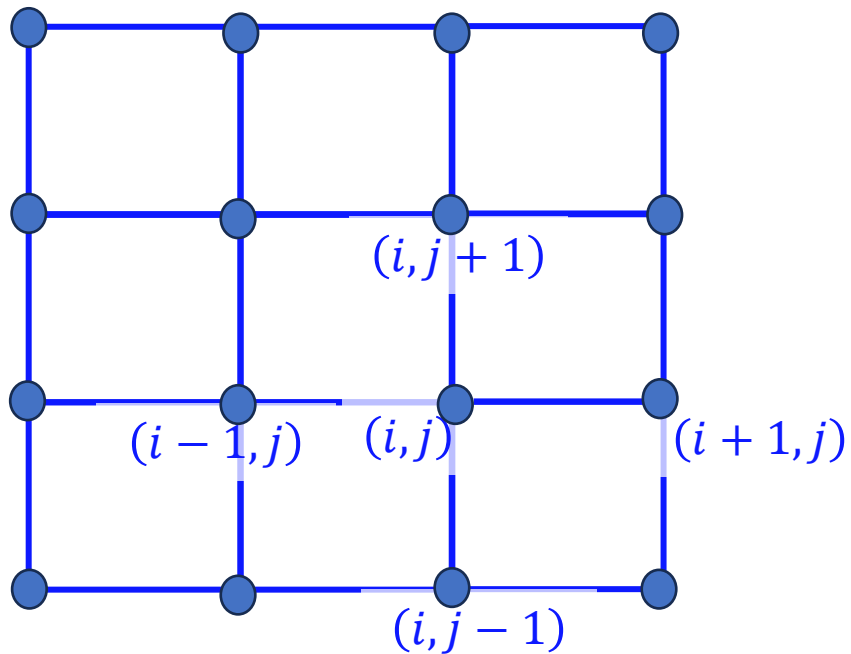
$$\frac{\partial}{\partial t} \int_V \rho dV + \oint_S \rho \mathbf{u} \cdot d\mathbf{S} = 0$$



Finite Difference – Finite Volume

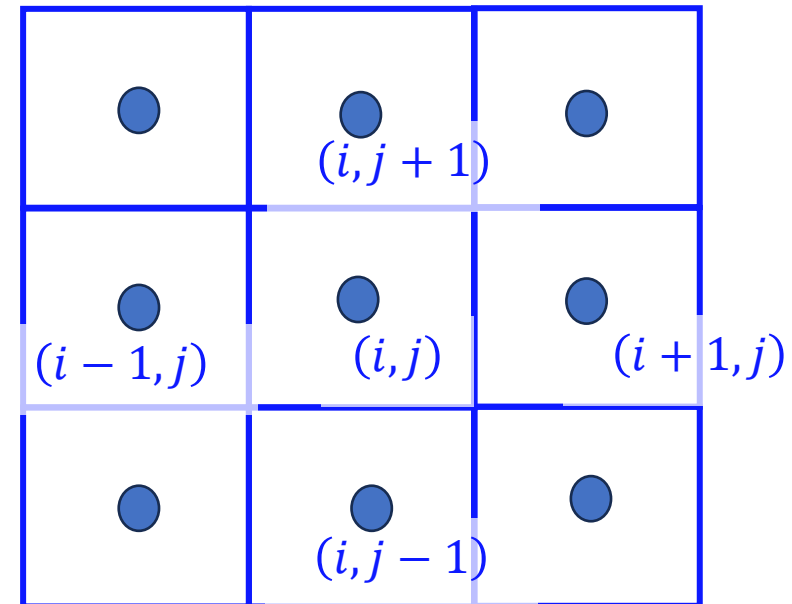
Differential form

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{u}) = 0$$



Integral form

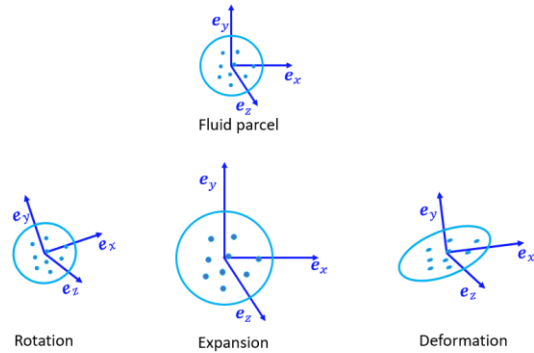
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Quick Recap

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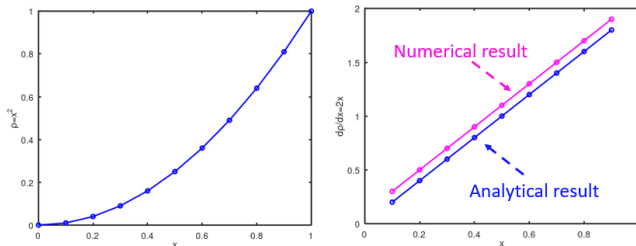
Fluid Behavior



Gradient

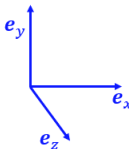
- Numerical Approximation

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Mathematical Operations

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- Numerical approximation

- Same as earlier

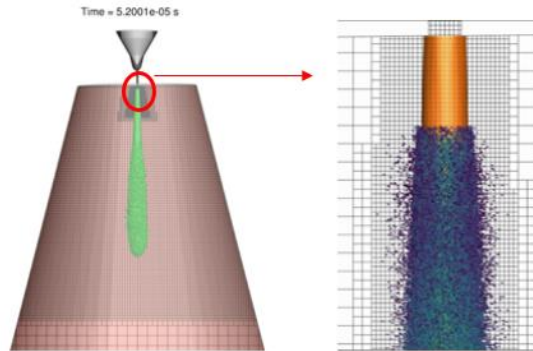
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What Did We Discuss?

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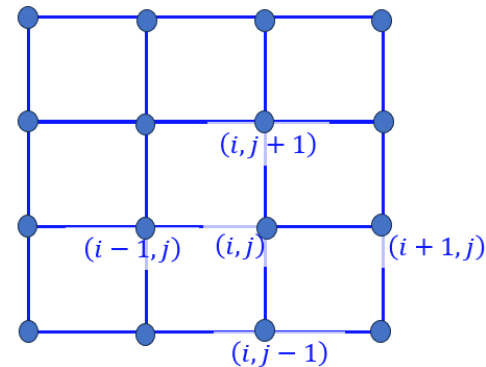
$$\frac{D\phi}{Dt} = \frac{\partial \phi}{\partial t} + \mathbf{u} \cdot \nabla \phi$$



Finite Difference – Finite Volume

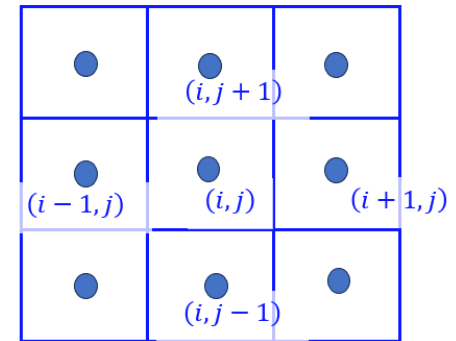
Differential form

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{u}) = 0$$



Integral form

$$\frac{\partial}{\partial t} \int_V \rho dV + \oint_S \rho \mathbf{u} \cdot d\mathbf{S} = 0$$



Next Session

- Reynolds Transport Theorem
- Gauss Divergence Theorem

Installations

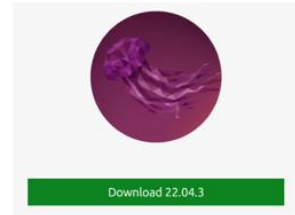
Required Applications

- Preconfiguration packages:
 - <https://1drv.ms/f/s!AqT2YEB97-1RgP8MtsMPqoOGsq4ddg?e=locXv0>
- List
 - Virtual Box [to create virtual machines]
 - Ubuntu 22.04 [OS to install OpenFOAM & Octave]
 - AnyDesk [For remote access]

Exercise-1

- Operating System:

- Ubuntu 22.04



- Softwares:

- OpenFOAM v2306



- Octave



- Create a github account:

- <https://github.com>
 - Discussion forum:
 - <https://github.com/exaslate-courses/cfd-openfoam-b2/discussions>

Test Octave

- Run numerical_derivative_first_order_approximation.m

Gradient

- Numerical Approximation

$$\begin{array}{ccccccc} & \bullet & & \bullet & & \bullet & & \bullet \\ & x_{i-1} & & x_i & & x_{i+1} & & x_{i+2} \end{array}$$

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