Introduction to Computational Fluid Dynamics using OpenFOAM and Octave

Dr. Lakshman Anumolu (Sr. Research Engineer)
Kumaresh Selvakumar (PhD candidate)

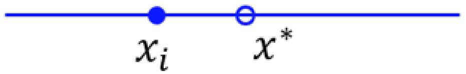
(Session-7)

Instructions: Wed, Fri (4:30-5:30PM IST), Sat (4PM-5PM IST)
Query sessions: Sundays 9:00AM-9:30AM IST

Quick Recap

What Did We Discuss?

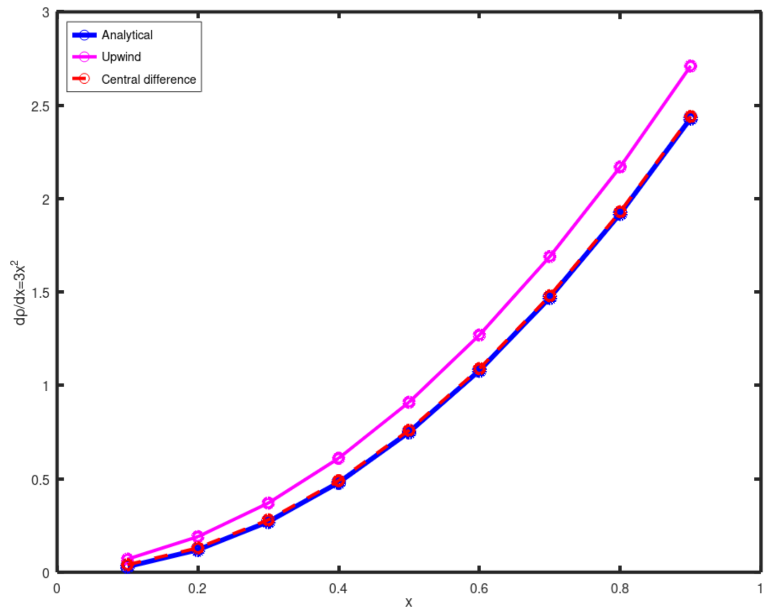
Taylor Series



$$\rho(x^*) = \rho(x_i) + \frac{(x^* - x_i)}{1!}\left(\frac{d\rho}{dx}\right)_i + \frac{(x^* - x_i)^2}{2!}\left(\frac{d^2\rho}{dx^2}\right)_i + \frac{(x^* - x_i)^3}{3!}\left(\frac{d^3\rho}{dx^3}\right)_i + \cdots$$

$$\left(\frac{d\rho}{dx}\right)_i \approx \frac{\rho(x_{i+1}) - \rho(x_i)}{\Delta x_i}$$

$$\left(\frac{d\rho}{dx}\right)_i \approx \frac{\rho(x_{i+1}) - \rho(x_{i-1})}{2\Delta x_i}$$



What Did We Discuss?

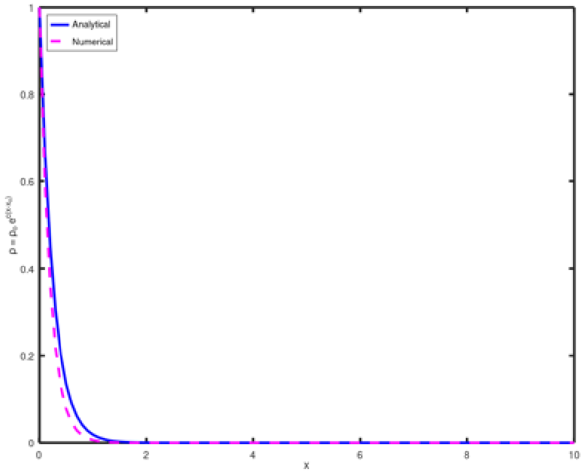
Numerical Stability

$$\frac{d\rho}{dx} = -c\rho$$

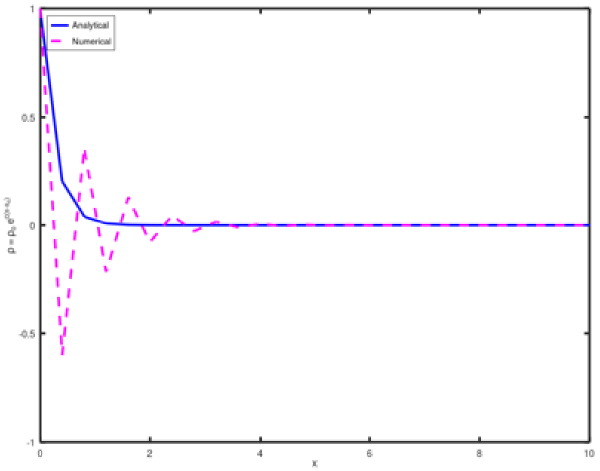
x_{i-1} x_i x_{i+1} x_{i+2}

$$\rho_{i+1} = \rho_i(1 - c\Delta x)$$

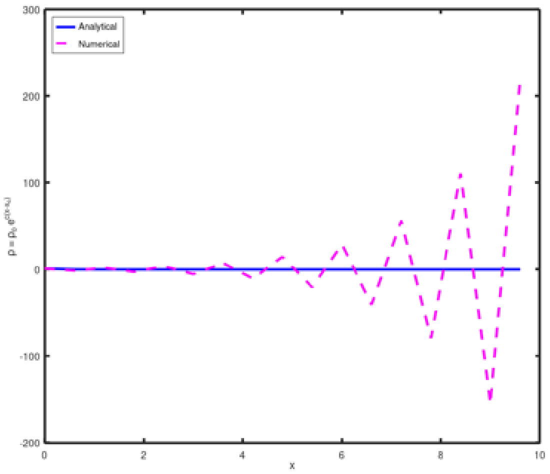
$$\Delta x < \frac{2}{c}$$
$$\frac{2}{c} = \frac{2}{4} = 0.5$$



$c = 4, x_o = 0, \rho_o = 1, x \in [0,10], \Delta x = 0.1$



$c = 4, x_o = 0, \rho_o = 1, x \in [0,10], \Delta x = 0.4$



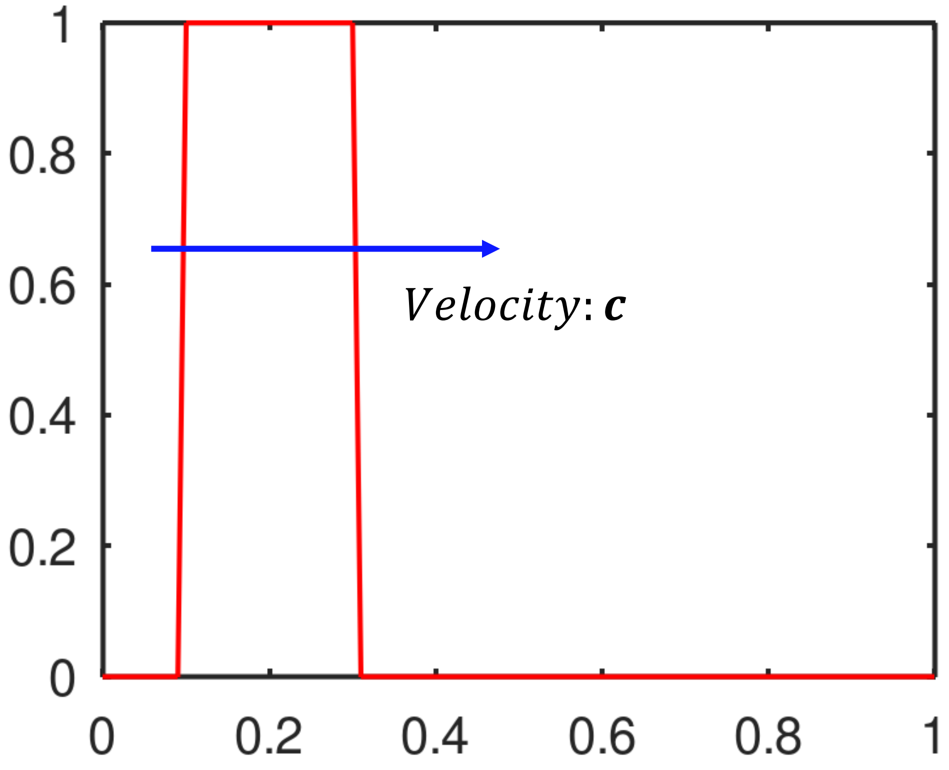
$c = 4, x_o = 0, \rho_o = 1, x \in [0,10], \Delta x = 0.6$

Current Session

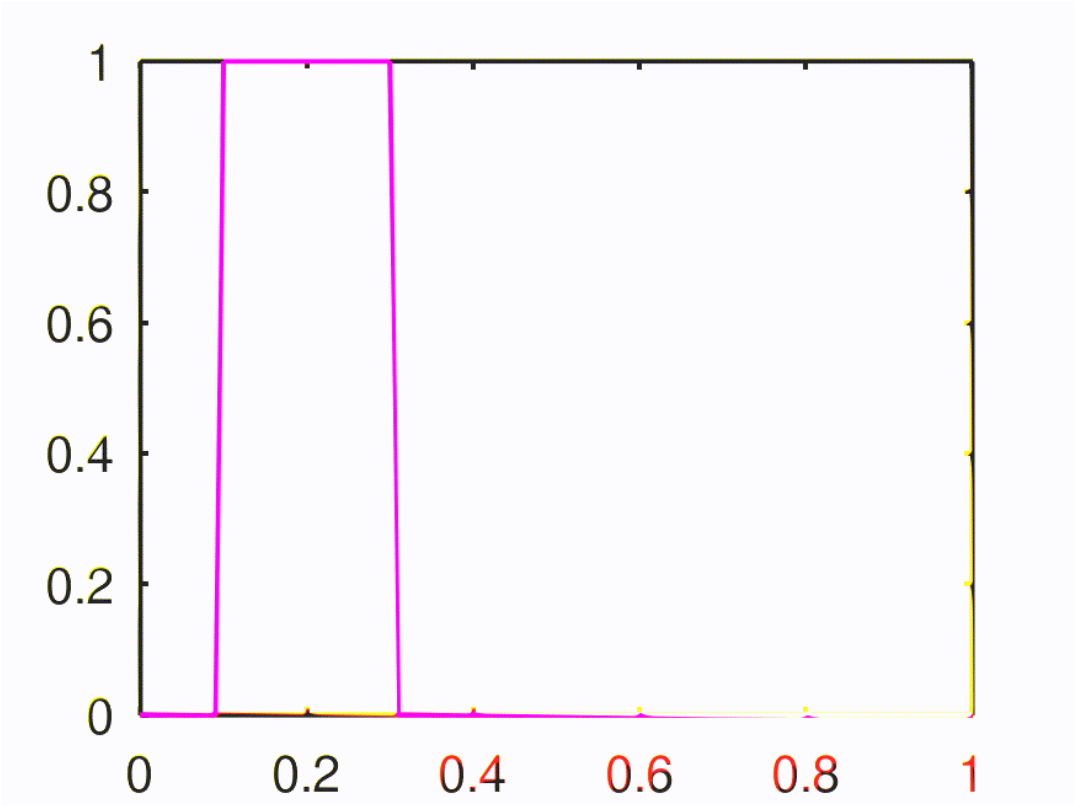
Overview

- Stability Analysis (contd.)
- Introduction to C++ for OpenFOAM

Numerical Stability

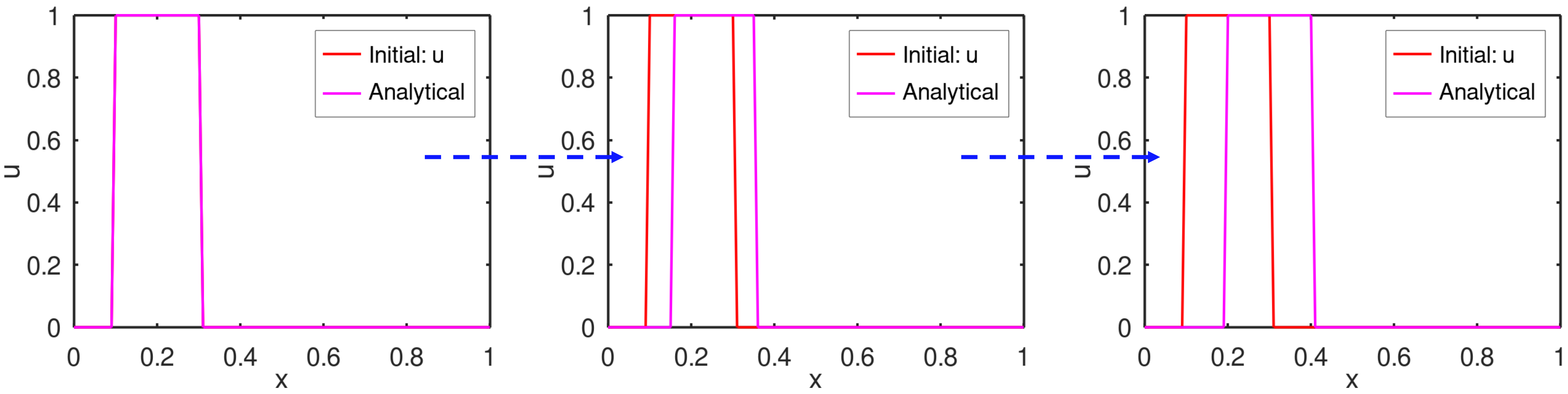


Numerical Stability

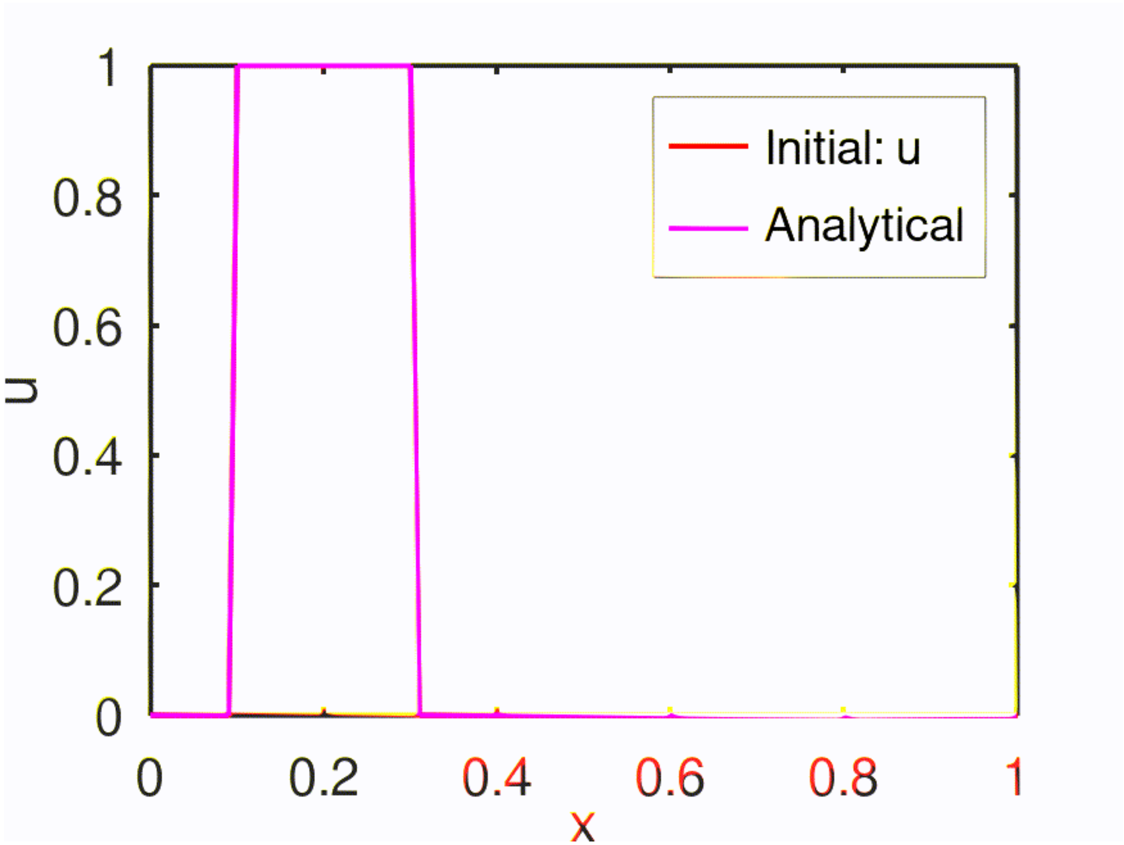


Numerical Stability

$$\frac{\partial u}{\partial t} + c\frac{\partial u}{\partial x} = 0 \longleftarrow$$ Advection equation



Numerical Stability



Numerical Stability

$$\frac{\partial u}{\partial t} + c\frac{\partial u}{\partial x} = 0$$

$$\frac{u_i^{n+1} - u_i^n}{\Delta t} + c\left(\frac{\partial u}{\partial x}\right)_i^n = 0$$

$$\left(\frac{d\rho}{dx}\right)_i \approx \frac{\rho(x_{i+1}) - \rho(x_i)}{\Delta x_i} \qquad \left(\frac{d\rho}{dx}\right)_i \approx \frac{\rho(x_{i+1}) - \rho(x_{i-1})}{2\Delta x_i}$$

$$u_i^{n+1} = u_i^n - c\Delta t\left(\frac{\partial u}{\partial x}\right)_i^n$$

$$\left(\frac{\partial u}{\partial x}\right)_i^n \approx \frac{u_{i+1}^n - u_i^n}{\Delta x_i}$$

Simple forward difference scheme

$$\left(\frac{\partial u}{\partial x}\right)_i^n \approx \frac{u_{i+1}^n - u_{i-1}^n}{2\Delta x_i}$$

Central difference

Numerical Stability

$$u_i^{n+1} = u_i^n - c\Delta t \left(\frac{\partial u}{\partial x}\right)_i^n$$

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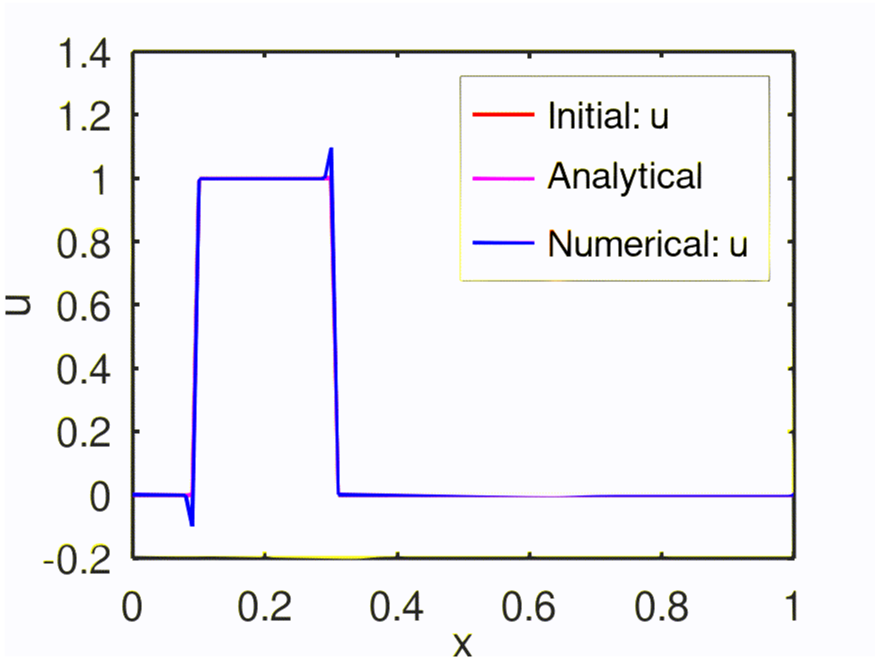
Central difference



Numerical Stability

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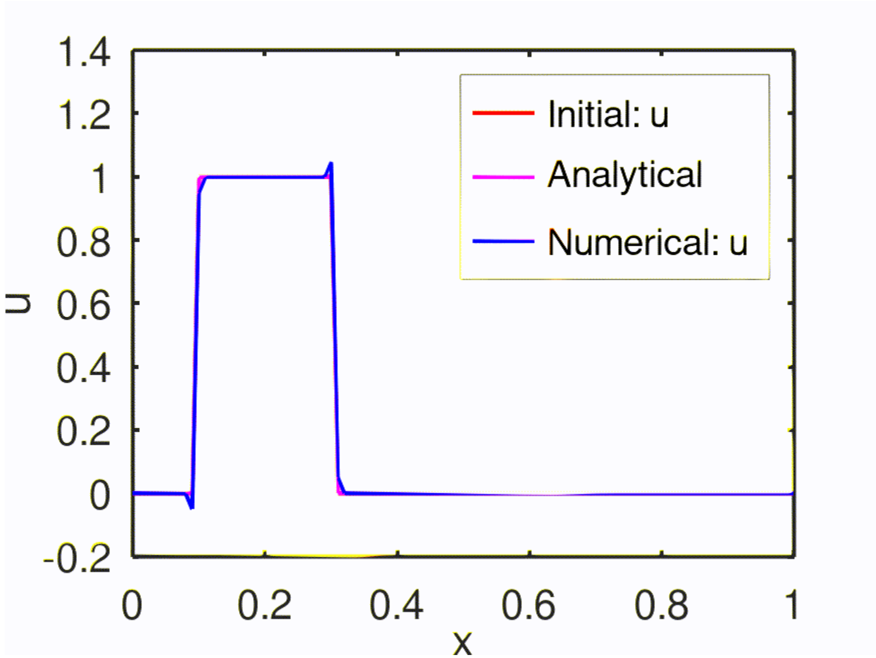
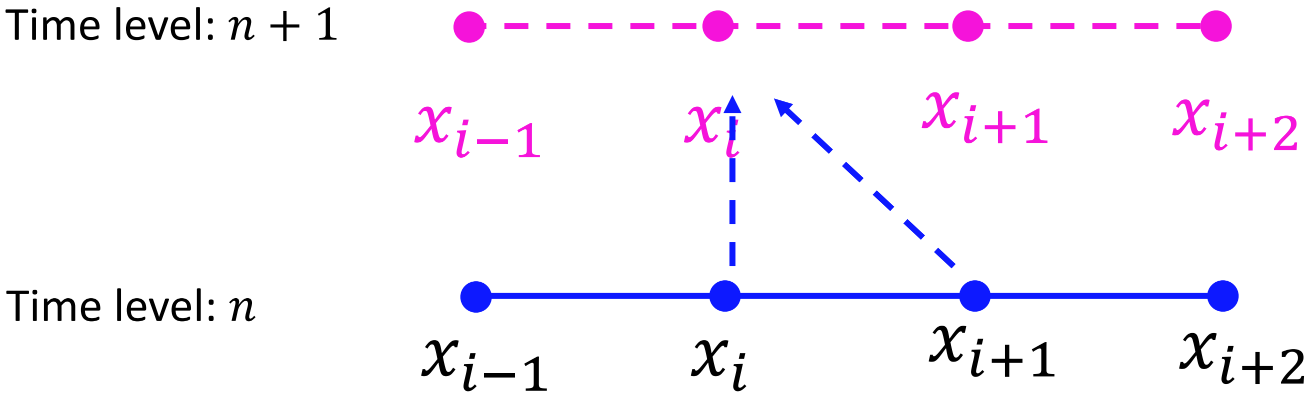
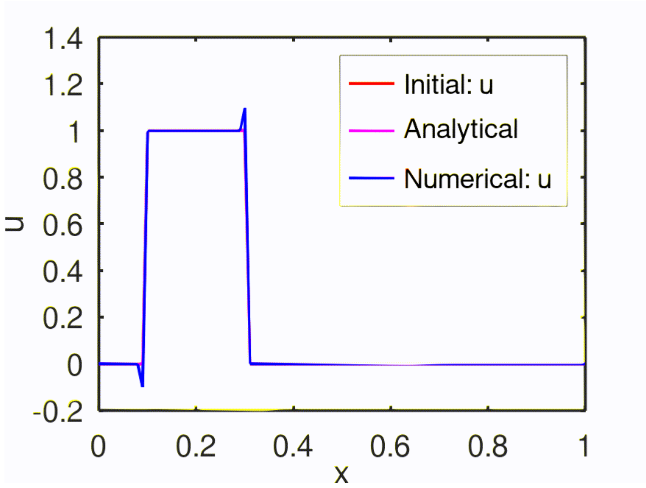
Central difference



Numerical Stability





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$$\left(\frac{\partial u}{\partial x}\right)_i^n \approx \frac{u_{i+1}^n - u_i^n}{\Delta x_i}$$ Simple forward difference scheme

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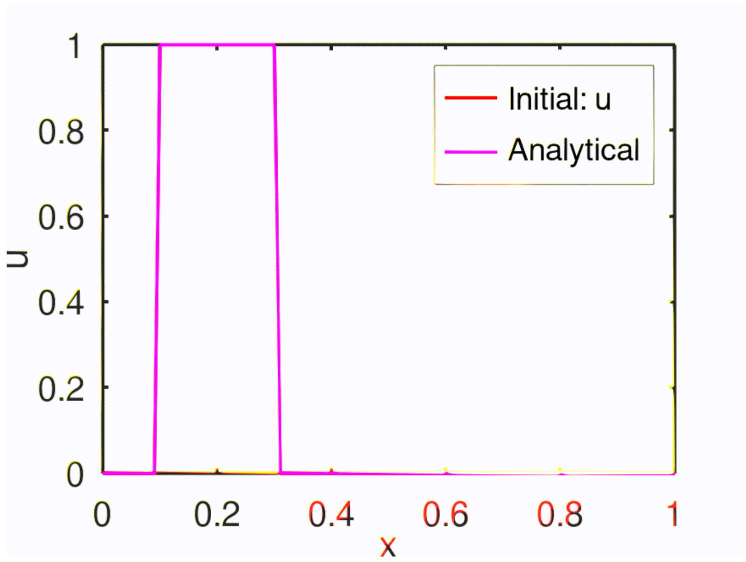
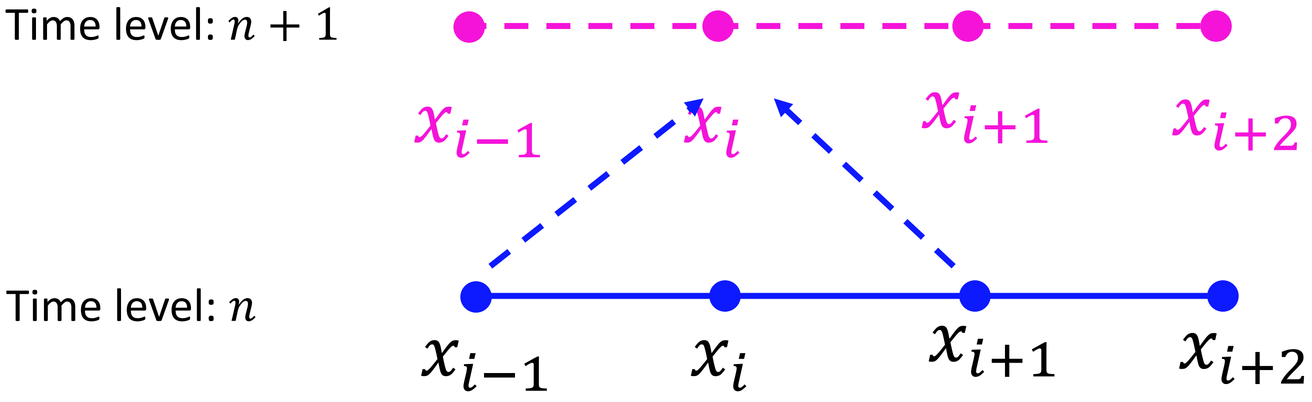
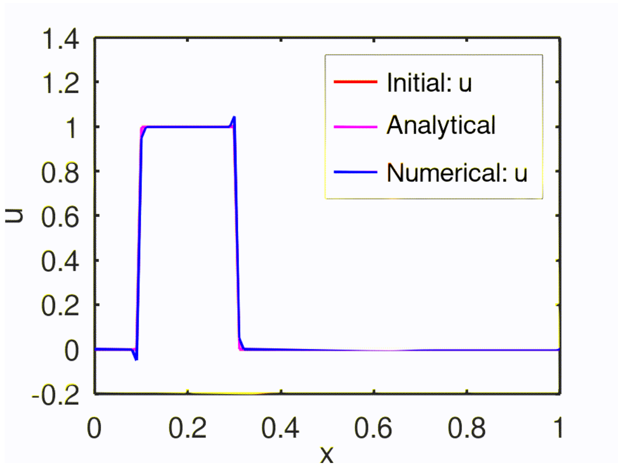
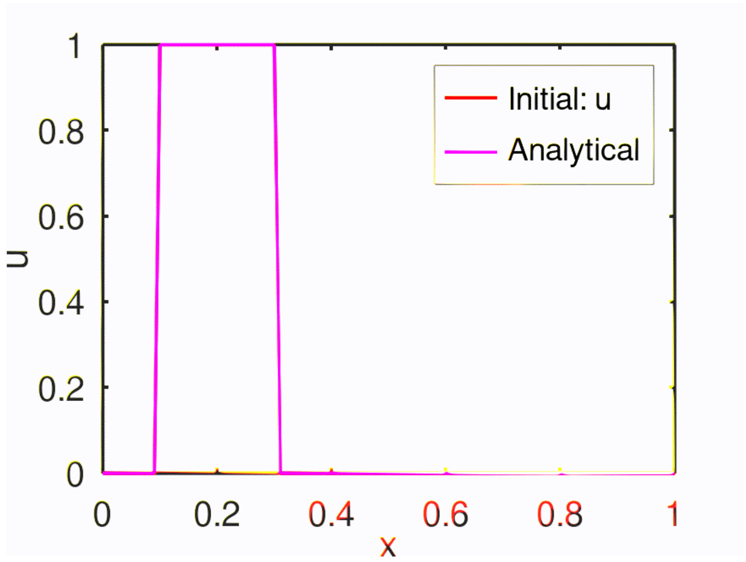


Numerical Stability







$$u_i^{n+1} = u_i^n - c\Delta t \left(\frac{\partial u}{\partial x}\right)_i^n$$

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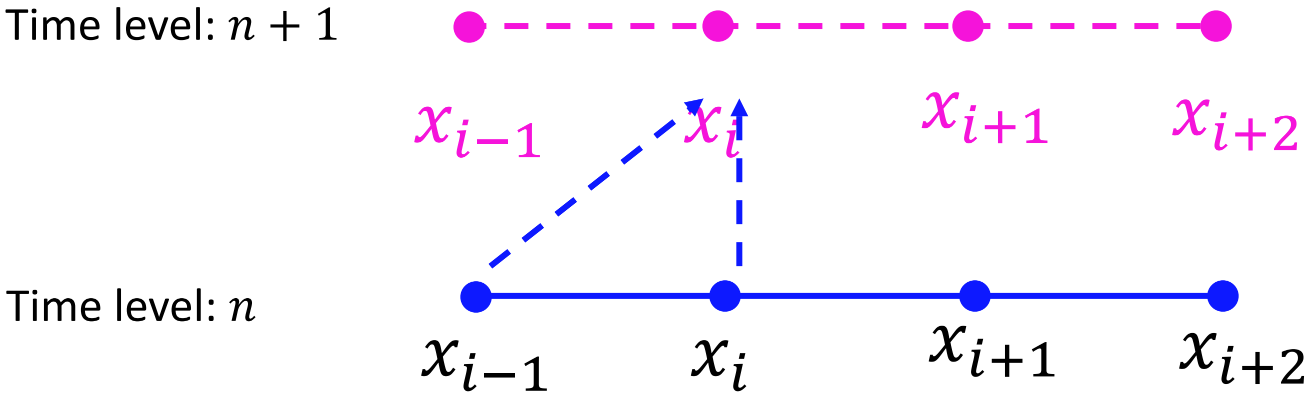
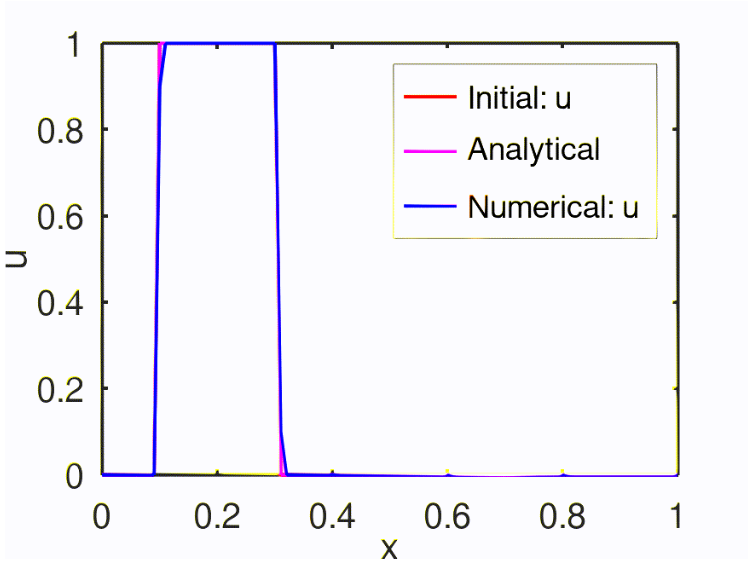
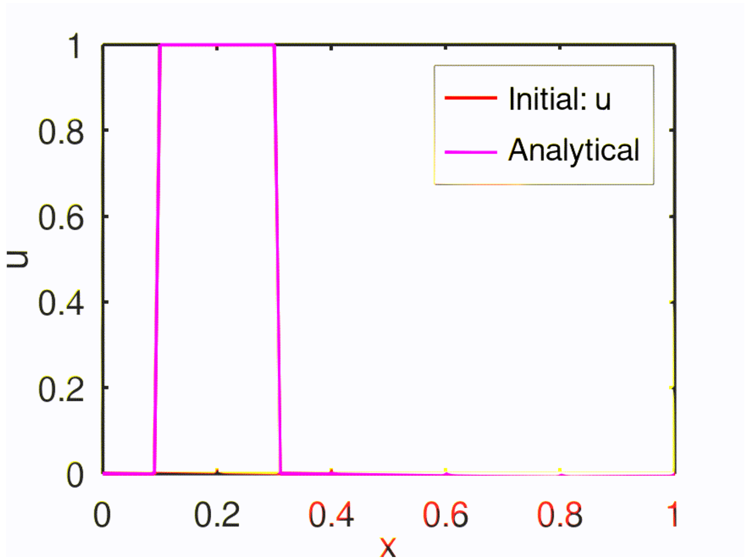
Numerical Stability







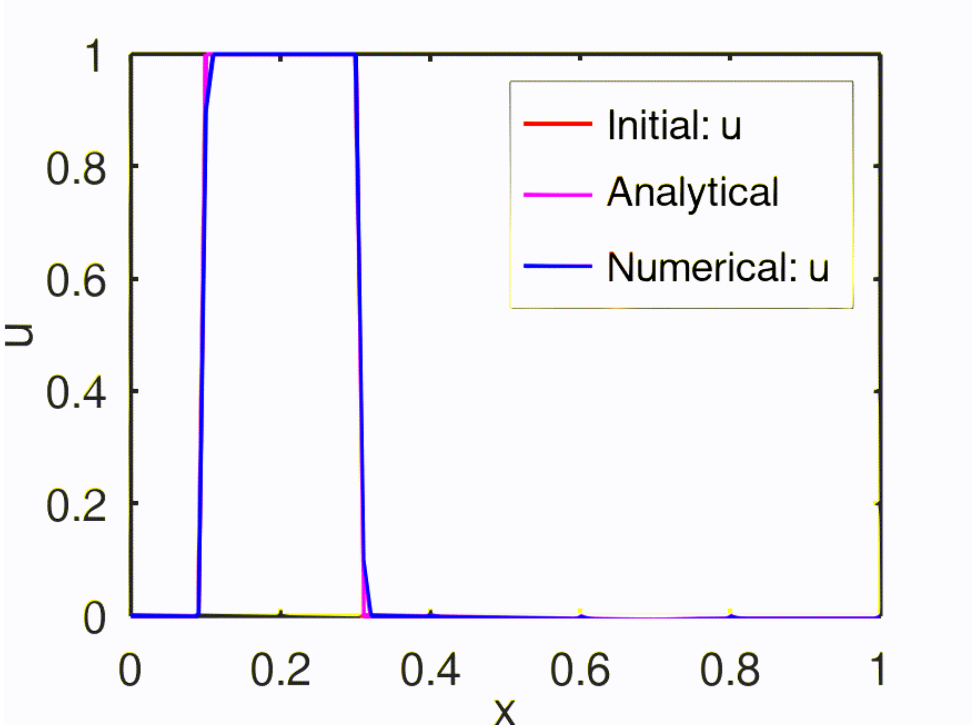
$$u_i^{n+1} = u_i^n - c\Delta t \left(\frac{\partial u}{\partial x}\right)_i^n \longrightarrow \left(\frac{\partial u}{\partial x}\right)_i^n \approx \frac{u_i^n - u_{i-1}^n}{\Delta x_i}$$

Upwind

Numerical Stability

$$u_i^{n+1} = u_i^n - c\Delta t \left(\frac{\partial u}{\partial x}\right)_i^n \longrightarrow \left(\frac{\partial u}{\partial x}\right)_i^n \approx \frac{u_i^n - u_{i-1}^n}{\Delta x_i}$$ Upwind

$$CFL: \frac{c\Delta t}{\Delta x}$$

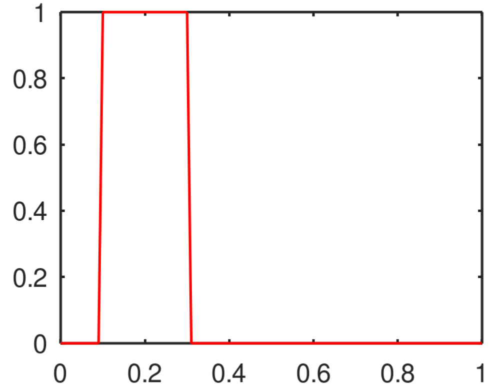


CFL = 0.1

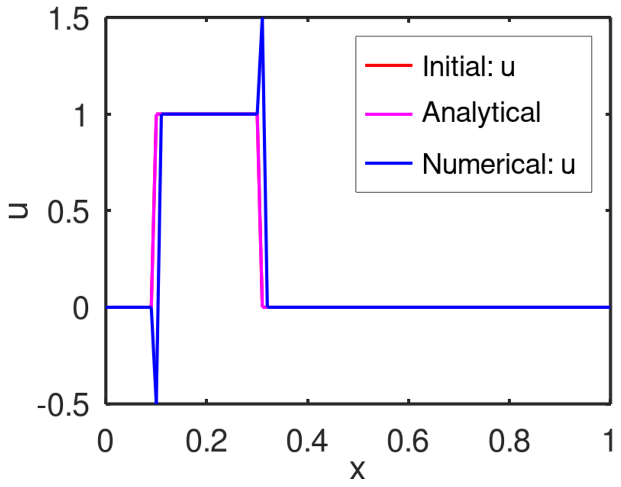
Numerical Stability

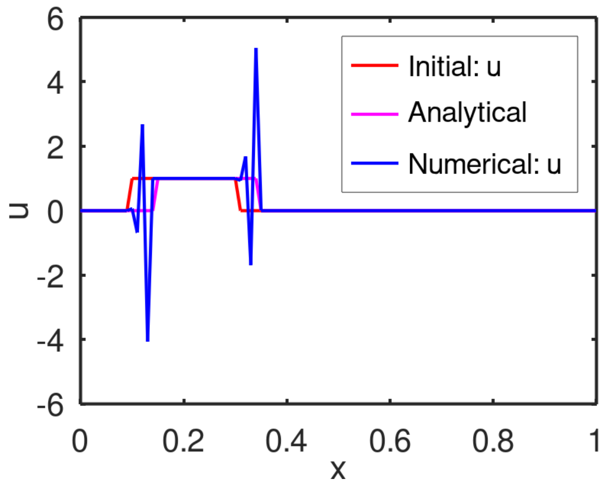
$$u_i^{n+1} = u_i^n - c\Delta t \left(\frac{\partial u}{\partial x}\right)_i^n \longrightarrow \left(\frac{\partial u}{\partial x}\right)_i^n \approx \frac{u_i^n - u_{i-1}^n}{\Delta x_i}$$

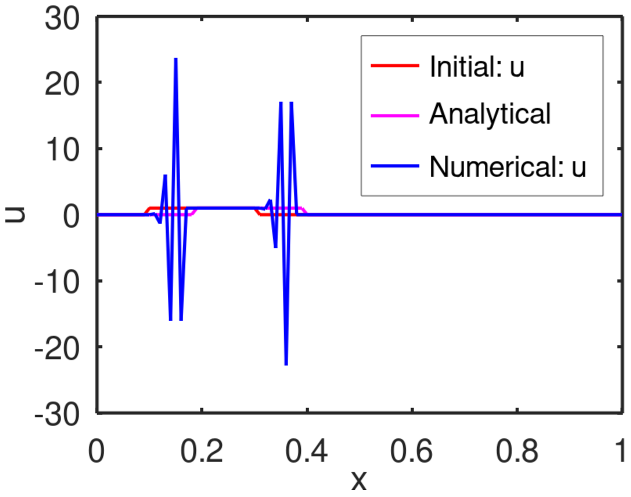
Upwind



$$CFL: \frac{c\Delta t}{\Delta x}$$



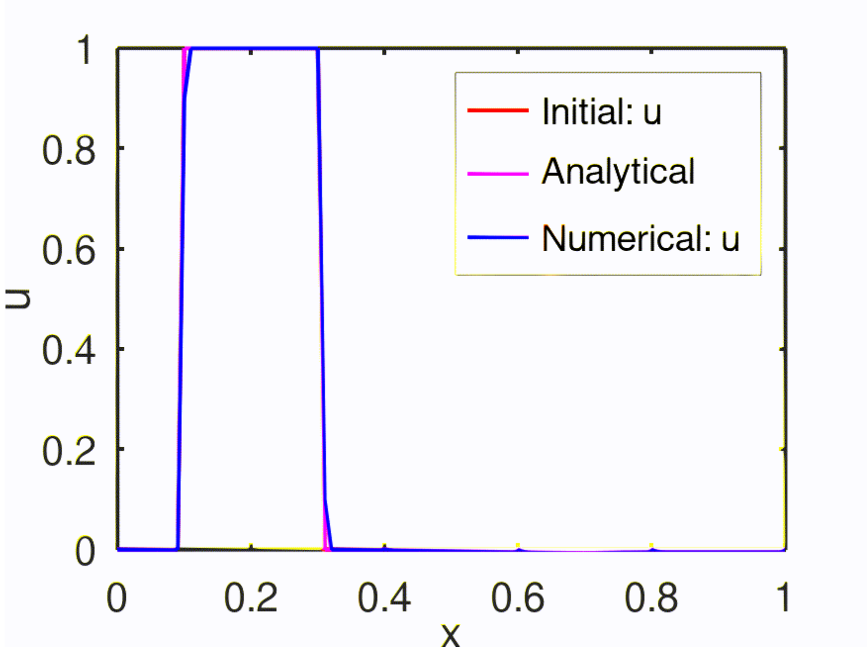




CFL = 1.5

What did we discuss?

- Proper discrete approximations need to be chosen based on the velocity field
- CFL number is critical to ensure numerical stability



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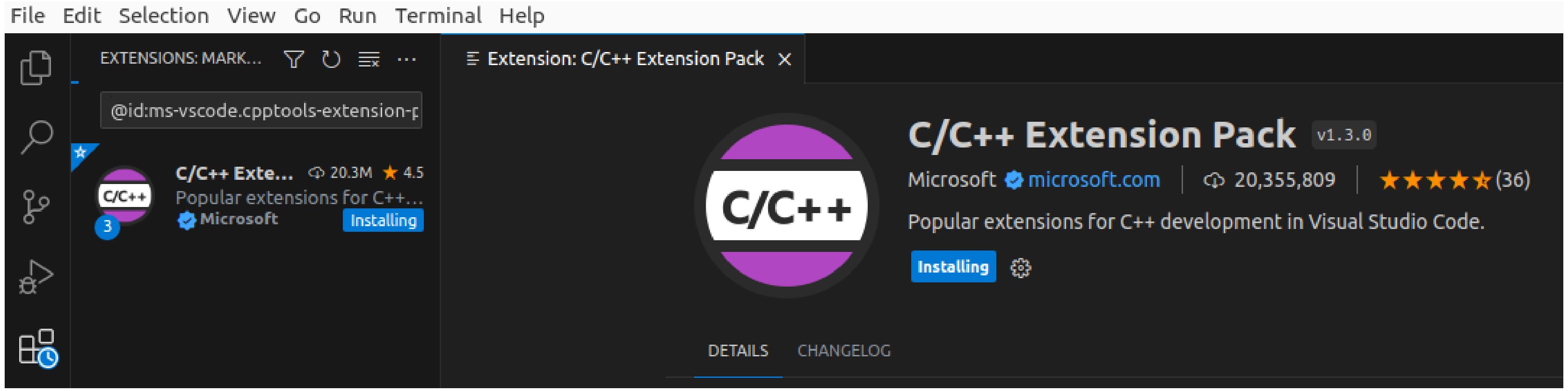
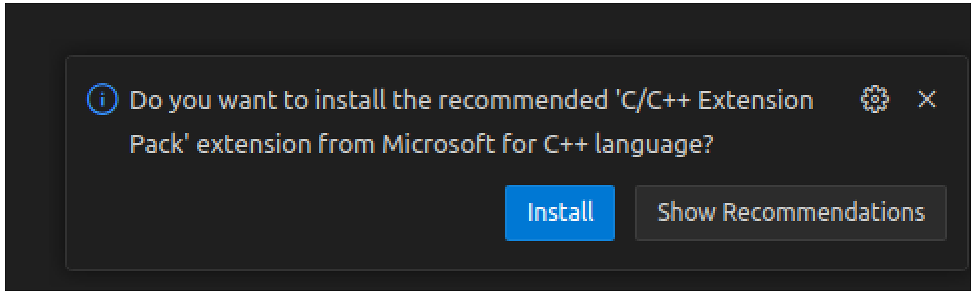
Introduction to C++ for OpenFOAM

b6_sample.cpp





Next Session

- Rate of convergence
- Introduction to C++ for OpenFOAM

Thank you