Introduction to Computational Fluid Dynamics using OpenFOAM and Octave

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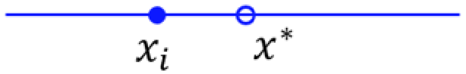
Kumaresh Selvakumar
(Session-5)

Instructions: Wed, Fri (4:30-5:30PM IST), Sat (4PM-5PM IST)
Query sessions: Sundays 9:00AM-9:30AM IST

Quick Recap

What Did We Discuss?

Taylor Series



$$\rho(x^*) = \rho(x_i) + \frac{(x^* - x_i)}{1!}\left(\frac{d\rho}{dx}\right)_i + \frac{(x^* - x_i)^2}{2!}\left(\frac{d^2\rho}{dx^2}\right)_i + \frac{(x^* - x_i)^3}{3!}\left(\frac{d^3\rho}{dx^3}\right)_i + \cdots$$

$$\left(\frac{d\rho}{dx}\right)_i \approx \frac{\rho(x_{i+1}) - \rho(x_i)}{\Delta x_i}$$

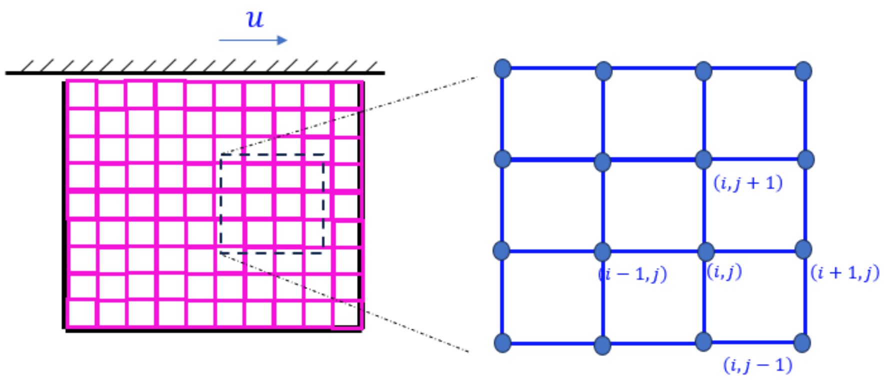
First order upwind

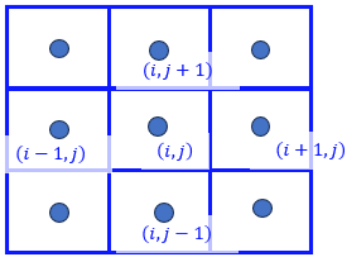
$$\left(\frac{d\rho}{dx}\right)_i \approx \frac{\rho(x_{i+1}) - \rho(x_{i-1})}{2\Delta x_i}$$

Second order central difference

What Did We Discuss?

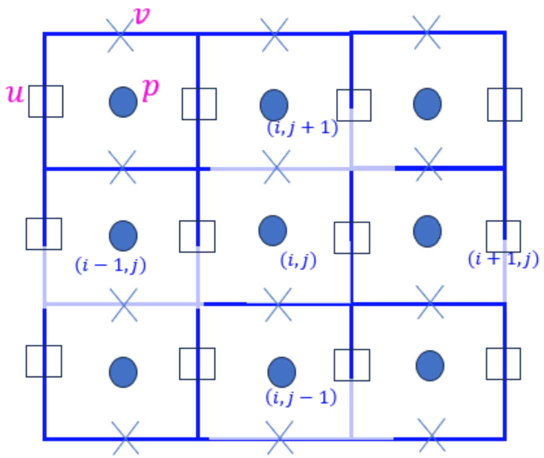
Grid Layout



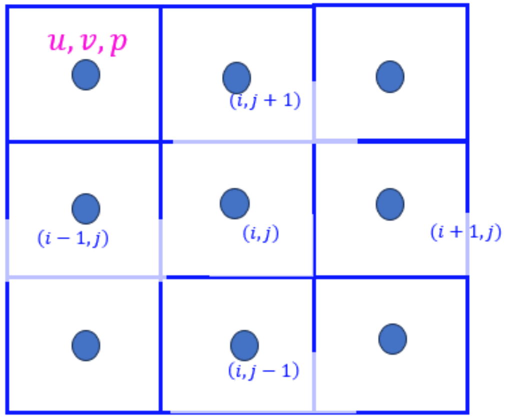


← Cell centered representation

- Cell centered representation



Staggered grid



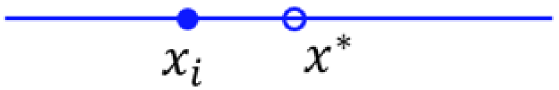
Collocated grid

Current Session

Overview

- Taylor series analysis (exercise-2)
- Introduction to Octave programming
- Exercise using Octave

First Order Derivative Approximations



$$\rho(x^*) = \rho(x_i) + \frac{(x^* - x_i)}{1!}\left(\frac{d\rho}{dx}\right)_i + \frac{(x^* - x_i)^2}{2!}\left(\frac{d^2\rho}{dx^2}\right)_i + \frac{(x^* - x_i)^3}{3!}\left(\frac{d^3\rho}{dx^3}\right)_i + \cdots$$

$$\left(\frac{d\rho}{dx}\right)_i \approx \frac{\rho(x_{i+1}) - \rho(x_i)}{\Delta x_i}$$

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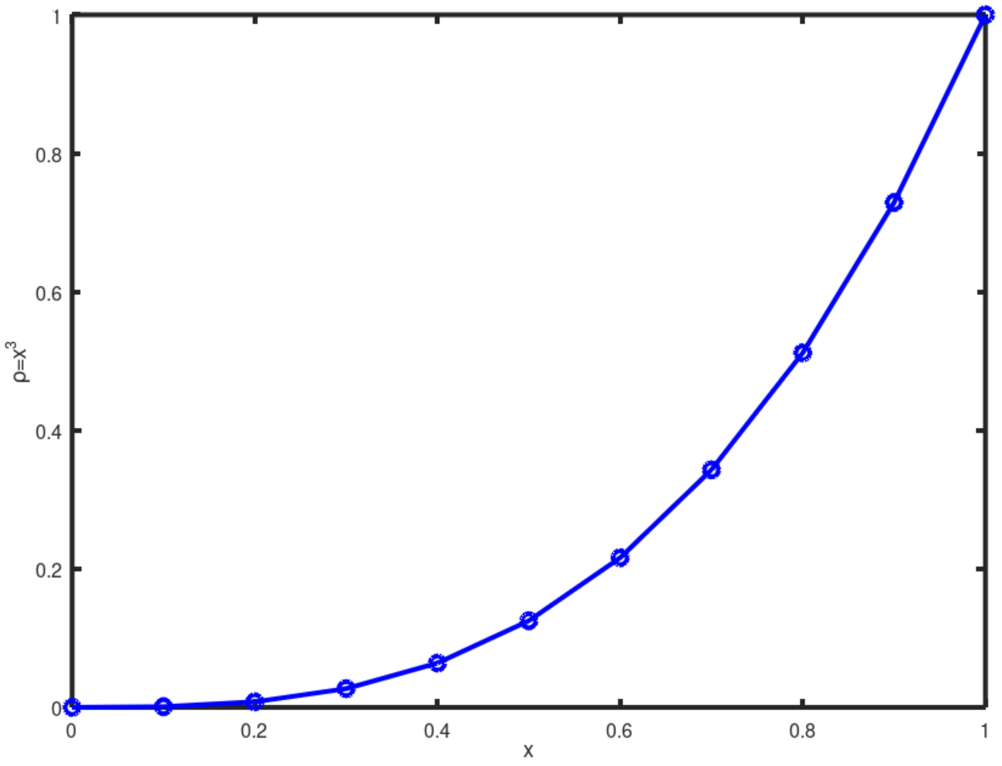
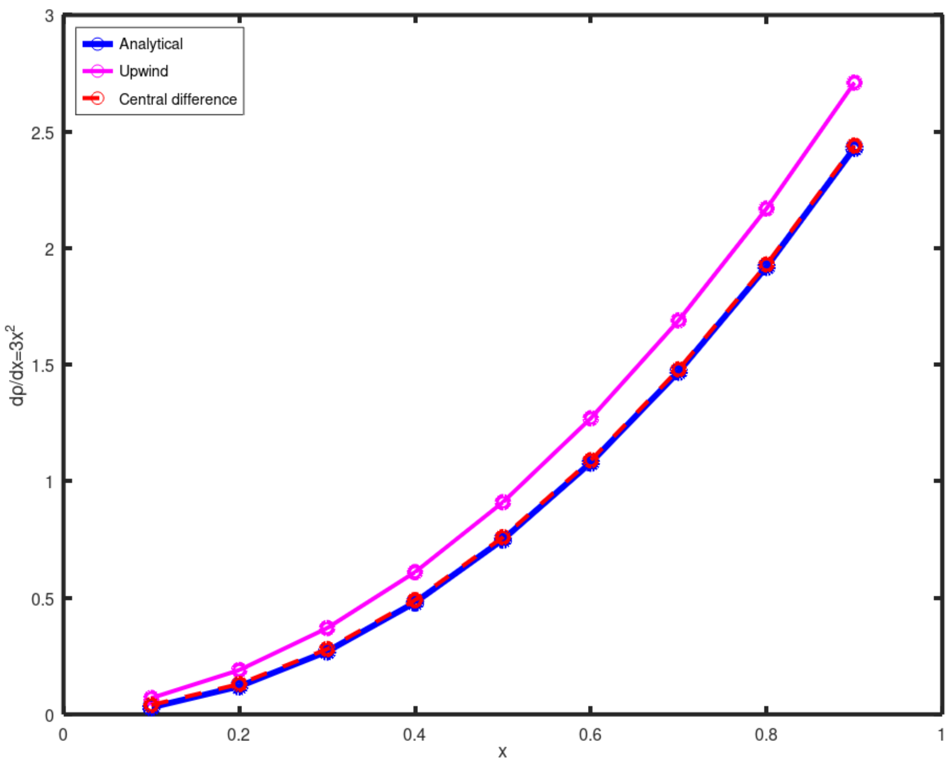
Second order central difference

First Order Derivative Approximations

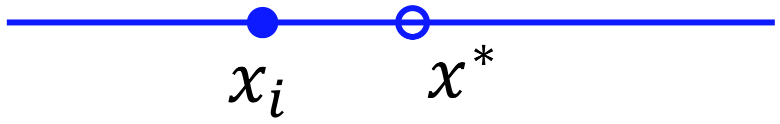




Exercise

- Derive expression for $\left(\frac{d^2\rho}{dx^2}\right)_i$
- What is the accuracy of the resultant expression?

Taylor Series



$$\rho(x^*) = \rho(x_i) + (x^* - x_i)\left(\frac{d\rho}{dx}\right)_i + \frac{(x^* - x_i)^2}{2!}\left(\frac{d^2\rho}{dx^2}\right)_i + \frac{(x^* - x_i)^3}{3!}\left(\frac{d^3\rho}{dx^3}\right)_i + \cdots$$

Approximating Second Order Derivative

Derive expression for $\left(\frac{d^2\rho}{dx^2}\right)_i$

$$\rho(x_{i+1}) = \rho(x_i) + (x_{i+1} - x_i)\left(\frac{d\rho}{dx}\right)_i + \frac{(x_{i+1} - x_i)^2}{2}\left(\frac{d^2\rho}{dx^2}\right)_i + \frac{(x_{i+1} - x_i)^3}{6}\left(\frac{d^3\rho}{dx^3}\right)_i + \frac{(x_{i+1} - x_i)^4}{24}\left(\frac{d^4\rho}{dx^4}\right)_i + \cdots$$

$$(1)\quad \rho(x_{i+1}) = \rho(x_i) + \Delta x_i\left(\frac{d\rho}{dx}\right)_i + \frac{\Delta x_i^2}{2}\left(\frac{d^2\rho}{dx^2}\right)_i + \frac{\Delta x_i^3}{6}\left(\frac{d^3\rho}{dx^3}\right)_i + \frac{\Delta x_i^4}{24}\left(\frac{d^4\rho}{dx^4}\right)_i + O\left(\Delta x_i^5\right)$$

$$(2)\quad \rho(x_{i-1}) = \rho(x_i) - \Delta x_i\left(\frac{d\rho}{dx}\right)_i + \frac{\Delta x_i^2}{2}\left(\frac{d^2\rho}{dx^2}\right)_i - \frac{\Delta x_i^3}{6}\left(\frac{d^3\rho}{dx^3}\right)_i + \frac{\Delta x_i^4}{24}\left(\frac{d^4\rho}{dx^4}\right)_i + O\left(\Delta x_i^5\right)$$

Approximating Second Order Derivative

(1) $\rho(x_{i+1}) = \rho(x_i) + \Delta x_i \left(\frac{d\rho}{dx}\right)_i + \frac{\Delta x_i^2}{2}\left(\frac{d^2\rho}{dx^2}\right)_i + \frac{\Delta x_i^3}{6}\left(\frac{d^3\rho}{dx^3}\right)_i + \frac{\Delta x_i^4}{24}\left(\frac{d^4\rho}{dx^4}\right)_i + O(\Delta x_i^5)$

(2) $\rho(x_{i-1}) = \rho(x_i) - \Delta x_i \left(\frac{d\rho}{dx}\right)_i + \frac{\Delta x_i^2}{2}\left(\frac{d^2\rho}{dx^2}\right)_i - \frac{\Delta x_i^3}{6}\left(\frac{d^3\rho}{dx^3}\right)_i + \frac{\Delta x_i^4}{24}\left(\frac{d^4\rho}{dx^4}\right)_i + O(\Delta x_i^5)$

Add (1) and (2)

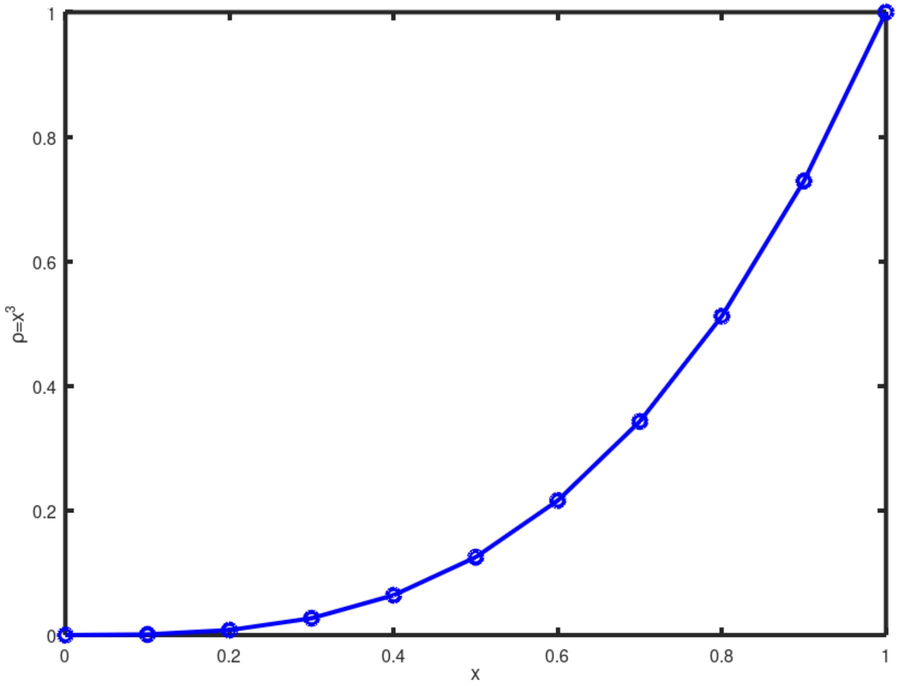
$$\rho(x_{i+1}) + \rho(x_{i-1}) = 2\rho(x_i) + \Delta x_i^2\left(\frac{d^2\rho}{dx^2}\right)_i + \frac{\Delta x_i^4}{12}\left(\frac{d^4\rho}{dx^4}\right)_i + \cdots$$

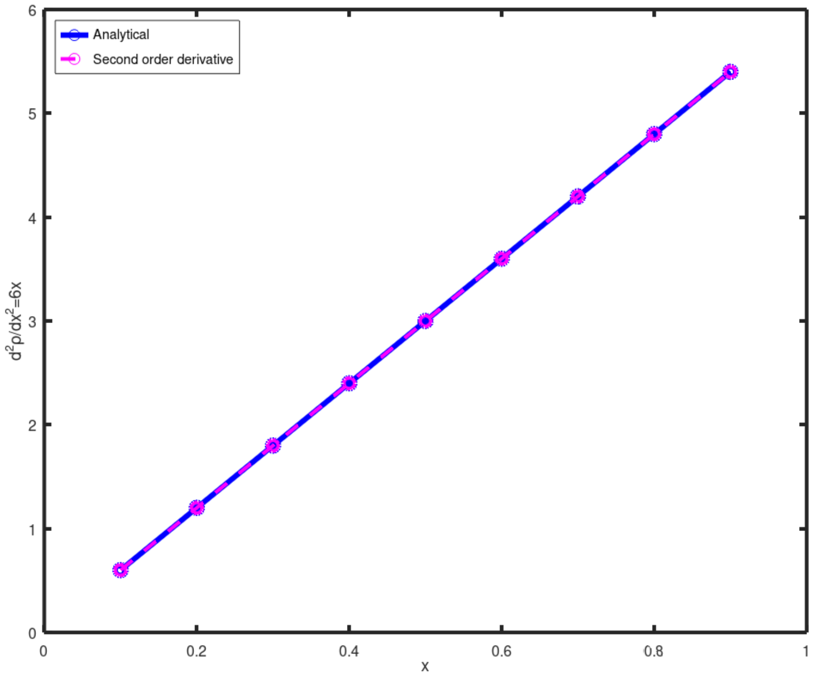
$$\left(\frac{d^2\rho}{dx^2}\right)_i = \frac{\rho(x_{i+1}) - 2\rho(x_i) + \rho(x_{i-1})}{\Delta x_i^2} + \frac{\Delta x_i^4}{12\Delta x_i^2}\left(\frac{d^4\rho}{dx^4}\right)_i + \cdots$$

$$\boxed{\left(\frac{d^2\rho}{dx^2}\right)_i = \frac{\rho(x_{i+1}) - 2\rho(x_i) + \rho(x_{i-1})}{\Delta x_i^2} + O(\Delta x_i^2)}$$

Approximating Second Order Derivative

$$\boxed{\left(\frac{d^2\rho}{dx^2}\right)_i = \frac{\rho(x_{i+1}) - 2\rho(x_i) + \rho(x_{i-1})}{\Delta x_i^2} + O\left(\Delta x_i^2\right)}$$





Introducing Octave

Introducing Octave

5_b_octave_introduction.m

Exercise

- Approximate $\left(\frac{d\rho}{dx}\right)_i$ using first order downwind scheme.
- Make a comparison plot between analytical and numerical results.

$$\boxed{\left(\frac{d\rho}{dx}\right)_i = \frac{\rho(x_i) - \rho(x_{i-1})}{\Delta x_i} + O(\Delta x_i)}$$

Update ***5_c_exercise_downwind_scheme.m***

$$\left(\frac{d\rho}{dx}\right)_i = \frac{\rho(x_{i+1}) - \rho(x_i)}{\Delta x_i} + O(\Delta x_i) \longleftarrow$$ First order upwind scheme

Thank you