Applied Computational Fluid Dynamics Using OpenFOAM

Value Added Course
College/University: KSR
Spring 2025

ExaSlate
Develop ≡ Guide ≡ Collab

Contents

- Taylor series: Central Difference Scheme (2nd order)

- Taylor series: Backward Difference Scheme (1st order)

- Exercise – 5

Kumaresh Selvakumar
kumaresh@exaslate.com

$$\frac{\partial}{\partial t}(\rho u) + \nabla \bullet (\rho \mathrm{u} u) = \nabla \bullet (\mu \nabla u) - \boxed{\nabla p} + S_u$$
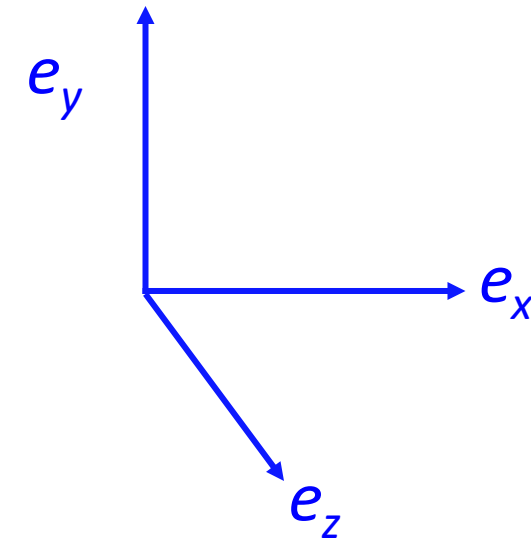
Mathematical operations

Gradient:

$$\nabla \rho = \left(\frac{\partial}{\partial x} \boldsymbol{e}_x + \frac{\partial}{\partial y} \boldsymbol{e}_y + \frac{\partial}{\partial z} \boldsymbol{e}_z \right) \rho = \left(\frac{\partial \rho}{\partial x} \boldsymbol{e}_x + \frac{\partial \rho}{\partial y} \boldsymbol{e}_y + \frac{\partial \rho}{\partial z} \boldsymbol{e}_z \right)$$

$$\nabla \boldsymbol{u} = \begin{bmatrix} \partial u / \partial x & \partial v / \partial x & \partial w / \partial x \\ \partial u / \partial y & \partial v / \partial y & \partial w / \partial y \\ \partial u / \partial z & \partial v / \partial z & \partial w / \partial z \end{bmatrix}$$

$$\frac{\partial \rho}{\partial x} = \frac{d \rho}{d x} (in\ 1D)$$

e_y

e_x

e_z

Kumaresh Selvakumar
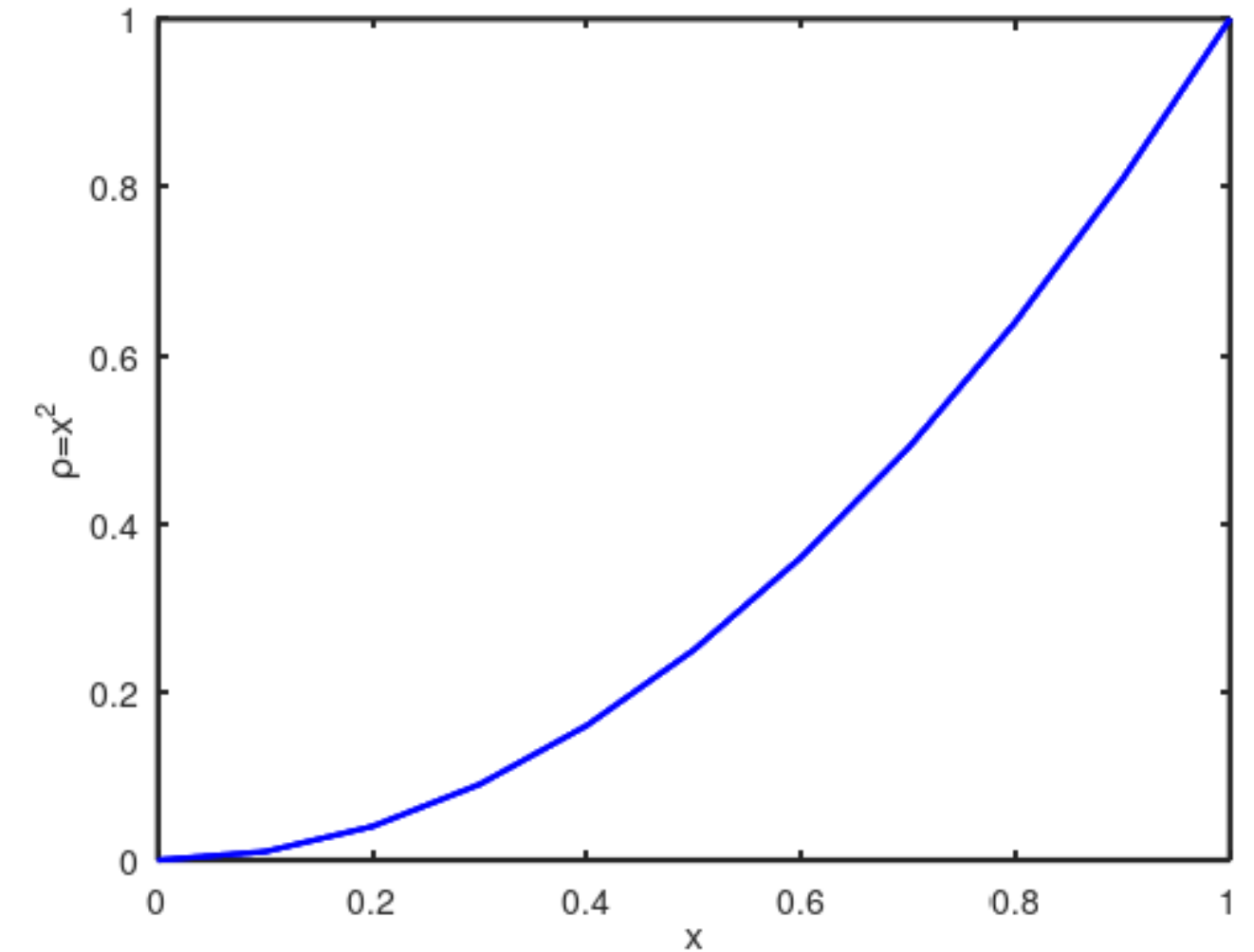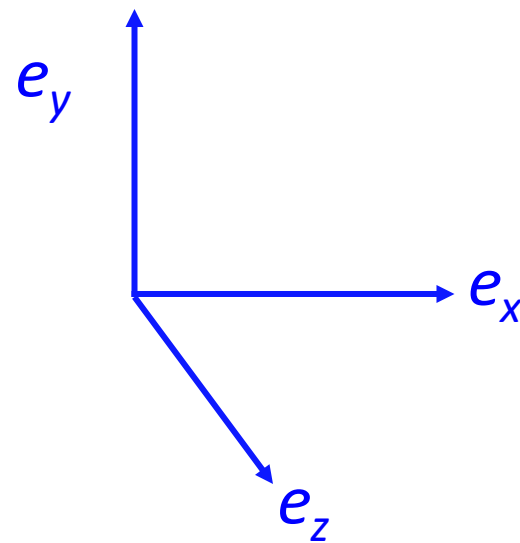kumaresh@exaslate.com

Mathematical operations

Gradient

$$\nabla\rho = \left(\frac{\partial}{\partial x}\boldsymbol{e}_x + \frac{\partial}{\partial y}\boldsymbol{e}_y + \frac{\partial}{\partial z}\boldsymbol{e}_z\right)\rho = \left(\frac{\partial\rho}{\partial x}\boldsymbol{e}_x + \frac{\partial\rho}{\partial y}\boldsymbol{e}_y + \frac{\partial\rho}{\partial z}\boldsymbol{e}_z\right)$$

$$\frac{\partial\rho}{\partial x} = \frac{d\rho}{dx}\,(in\ 1D)$$

e_y

e_x

e_z

Kumaresh Selvakumar
kumaresh@exaslate.com

Finite Difference Method (FDM)

$$\nabla \rho = \left(\frac{\partial}{\partial x} \boldsymbol{e}_x + \frac{\partial}{\partial y} \boldsymbol{e}_y + \frac{\partial}{\partial z} \boldsymbol{e}_z \right) \rho = \left(\frac{\partial \rho}{\partial x} \boldsymbol{e}_x + \frac{\partial \rho}{\partial y} \boldsymbol{e}_y + \frac{\partial \rho}{\partial z} \boldsymbol{e}_z \right)$$
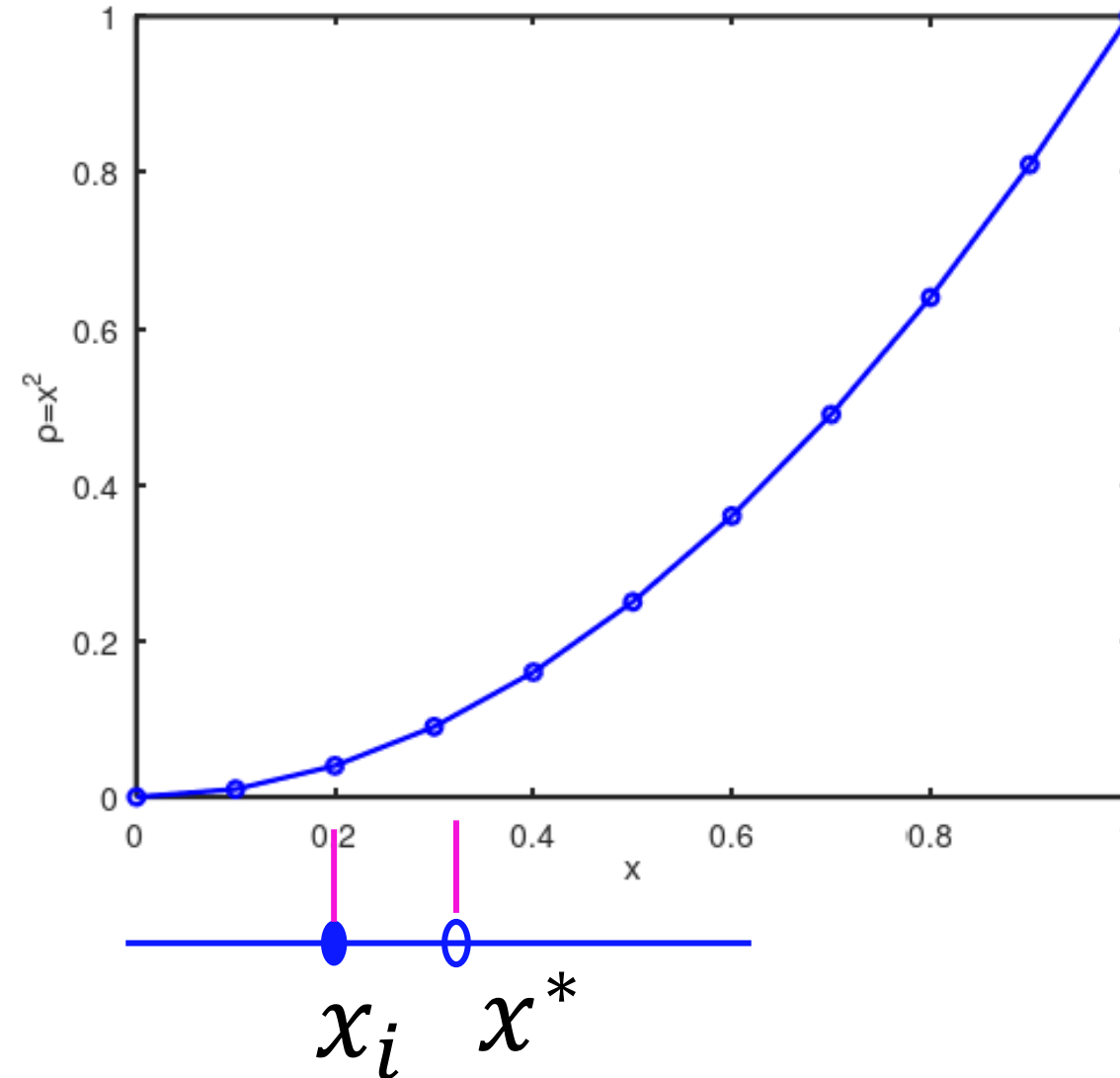
$$\left(\frac{\partial \rho}{\partial x} \right)_i = \frac{\rho(x_{i+1}) - \rho(x_i)}{\Delta x_i} + O(\Delta x_i)$$

Kumaresh Selvakumar
kumaresh@exaslate.com
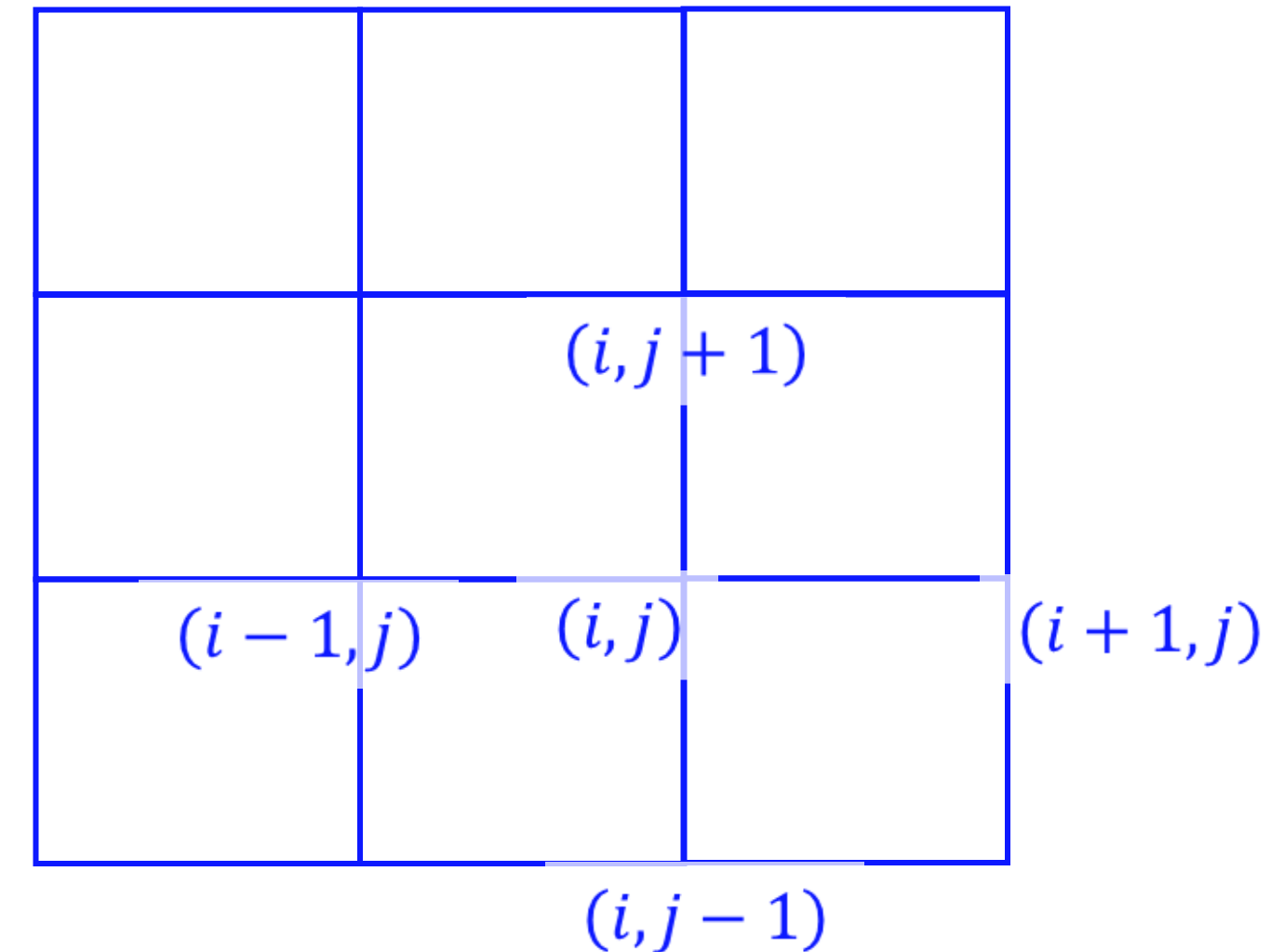
Taylor series expansion

$$\rho(x^*) = \rho(x_i) + (x^* - x_i)\left(\frac{d\rho}{dx}\right)_i + (x^* - x_i)^2\left(\frac{d^2\rho}{dx^2}\right)_i + (x^* - x_i)^3\left(\frac{d^3\rho}{dx^3}\right)_i + \cdots$$

Kumaresh Selvakumar

kumaresh@exaslate.com

Taylor series expansion

Kumaresh Selvakumar
kumaresh@exaslate.com

$$\rho(x_{i+1}) = \rho(x_i) + (x_{i+1} - x_i)\left(\frac{\partial \rho}{\partial x}\right)_i + (x_{i+1} - x_i)^2 \left(\frac{\partial^2 \rho}{\partial x^2}\right)_i + (x_{i+1} - x_i)^3 \left(\frac{\partial^3 \rho}{\partial x^3}\right)_i + \cdots$$

$$\rho(x_{i+1}) = \rho(x_i) + (x_{i+1} - x_i)\left(\frac{\partial \rho}{\partial x}\right)_i + O(\Delta x_i^2); \qquad \Delta x_i = (x_{i+1} - x_i)$$

$$\rho(x_{i+1}) = \rho(x_i) + \Delta x_i \left(\frac{\partial \rho}{\partial x}\right)_i + O(\Delta x_i^2)$$

Taylor series and FDM

Taylor series:

$$\rho(x_{i+1}) = \rho(x_i) + \Delta x_i \left(\frac{\partial \rho}{\partial x}\right)_i + O(\Delta x_i^2)$$

$$\left(\frac{\partial \rho}{\partial x}\right)_i = \frac{\rho(x_{i+1}) - \rho(x_i)}{\Delta x_i} + \frac{1}{\Delta x_i} O(\Delta x_i^2)$$

$$\left(\frac{\partial \rho}{\partial x}\right)_i = \frac{\rho(x_{i+1}) - \rho(x_i)}{\Delta x_i} + O(\Delta x_i)$$

$$\left(\frac{\partial \rho}{\partial x}\right)_i \approx \frac{\rho(x_{i+1}) - \rho(x_i)}{\Delta x_i}$$

First order forward difference scheme

Finite difference

$$\nabla \rho = \left(\frac{\partial}{\partial x}\boldsymbol{e}_x + \frac{\partial}{\partial y}\boldsymbol{e}_y + \frac{\partial}{\partial z}\boldsymbol{e}_z\right)\rho = \left(\frac{\partial \rho}{\partial x}\boldsymbol{e}_x + \frac{\partial \rho}{\partial y}\boldsymbol{e}_y + \frac{\partial \rho}{\partial z}\boldsymbol{e}_z\right)$$

$$\left(\frac{\partial \rho}{\partial x}\right)_i = \frac{\rho(x_{i+1}) - \rho(x_i)}{\Delta x_i} + O(\Delta x_i)$$

Kumaresh Selvakumar
kumaresh@exaslate.com

Taylor series expansion

Kumaresh Selvakumar
kumaresh@exaslate.com

$$\rho(x_{i+1}) = \rho(x_i) + (x_{i+1} - x_i)\left(\frac{d\rho}{dx}\right)_i + (x_{i+1} - x_i)^2 \left(\frac{d^2\rho}{dx^2}\right)_i + (x_{i+1} - x_i)^3 \left(\frac{d^3\rho}{dx^3}\right)_i + \cdots$$

$$(1) \quad \rho(x_{i+1}) = \rho(x_i) + \Delta x_i \left(\frac{d\rho}{dx}\right)_i + \Delta x_i^2 \left(\frac{d^2\rho}{dx^2}\right)_i + O(\Delta x_i^3)$$

$$(2) \quad \rho(x_{i-1}) = \rho(x_i) - \Delta x_i \left(\frac{d\rho}{dx}\right)_i + \Delta x_i^2 \left(\frac{d^2\rho}{dx^2}\right)_i + O(\Delta x_i^3)$$

Taylor series: Central Difference Scheme (2nd order)

(1) $\rho(x_{i+1}) = \rho(x_i) + \Delta x_i \left(\dfrac{d\rho}{dx}\right)_i + \Delta x_i^2 \left(\dfrac{d^2\rho}{dx^2}\right)_i + O(\Delta x_i^3)$

(2) $\rho(x_{i-1}) = \rho(x_i) - \Delta x_i \left(\dfrac{d\rho}{dx}\right)_i + \Delta x_i^2 \left(\dfrac{d^2\rho}{dx^2}\right)_i + O(\Delta x_i^3)$

$x_{i-1} \qquad x_i \qquad x_{i+1} \qquad x_{i+2}$

Subtract (2) from (1)

$$\rho(x_{i+1}) - \rho(x_{i-1}) = 2\Delta x_i \left(\frac{d\rho}{dx}\right)_i + O(\Delta x_i^3)$$

$$\boxed{\left(\frac{d\rho}{dx}\right)_i = \frac{\rho(x_{i+1}) - \rho(x_{i-1})}{2\Delta x_i} + O(\Delta x_i^2)}$$

Second order central difference scheme

Kumaresh Selvakumar
kumaresh@exaslate.com

Taylor series: Backward Difference Scheme (1st order)

$$\rho(x_{i-1}) = \rho(x_i) - \Delta x_i \left(\frac{d\rho}{dx}\right)_i + \Delta x_i^2 \left(\frac{d^2\rho}{dx^2}\right)_i + O(\Delta x_i^3)$$

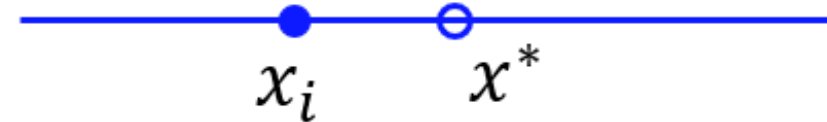$$\rho(x_{i-1}) = \rho(x_i) - \Delta x_i \left(\frac{d\rho}{dx}\right)_i + O(\Delta x_i^2)$$

$$\rho(x_{i-1}) = \rho(x_i) - \Delta x_i \left(\frac{d\rho}{dx}\right)_i + O(\Delta x_i^2)$$

$$\left(\frac{d\rho}{dx}\right)_i = \frac{\rho(x_i) - \rho(x_{i-1})}{\Delta x_i} + O(\Delta x_i)$$

$$\left(\frac{d\rho}{dx}\right)_i \approx \frac{\rho(x_i) - \rho(x_{i-1})}{\Delta x_i}$$

First order backward difference scheme

Kumaresh Selvakumar
kumaresh@exaslate.com

Taylor series: Summary

$$\rho(x^*) = \rho(x_i) + (x^* - x_i)\left(\frac{d\rho}{dx}\right)_i + (x^* - x_i)^2\left(\frac{d^2\rho}{dx^2}\right)_i + (x^* - x_i)^3\left(\frac{d^3\rho}{dx^3}\right)_i + \cdots$$

First order forward difference

$$\left(\frac{d\rho}{dx}\right)_i \approx \frac{\rho(x_{i+1}) - \rho(x_i)}{\Delta x_i}$$

Second order central difference

$$\left(\frac{d\rho}{dx}\right)_i \approx \frac{\rho(x_{i+1}) - \rho(x_{i-1})}{2\Delta x_i}$$

First order backward difference

$$\left(\frac{d\rho}{dx}\right)_i \approx \frac{\rho(x_i) - \rho(x_{i-1})}{\Delta x_i}$$

Kumaresh Selvakumar
kumaresh@exaslate.com

Exercise – 5

[Exercise-5] Solve using first order backward and second order central difference schemes #6
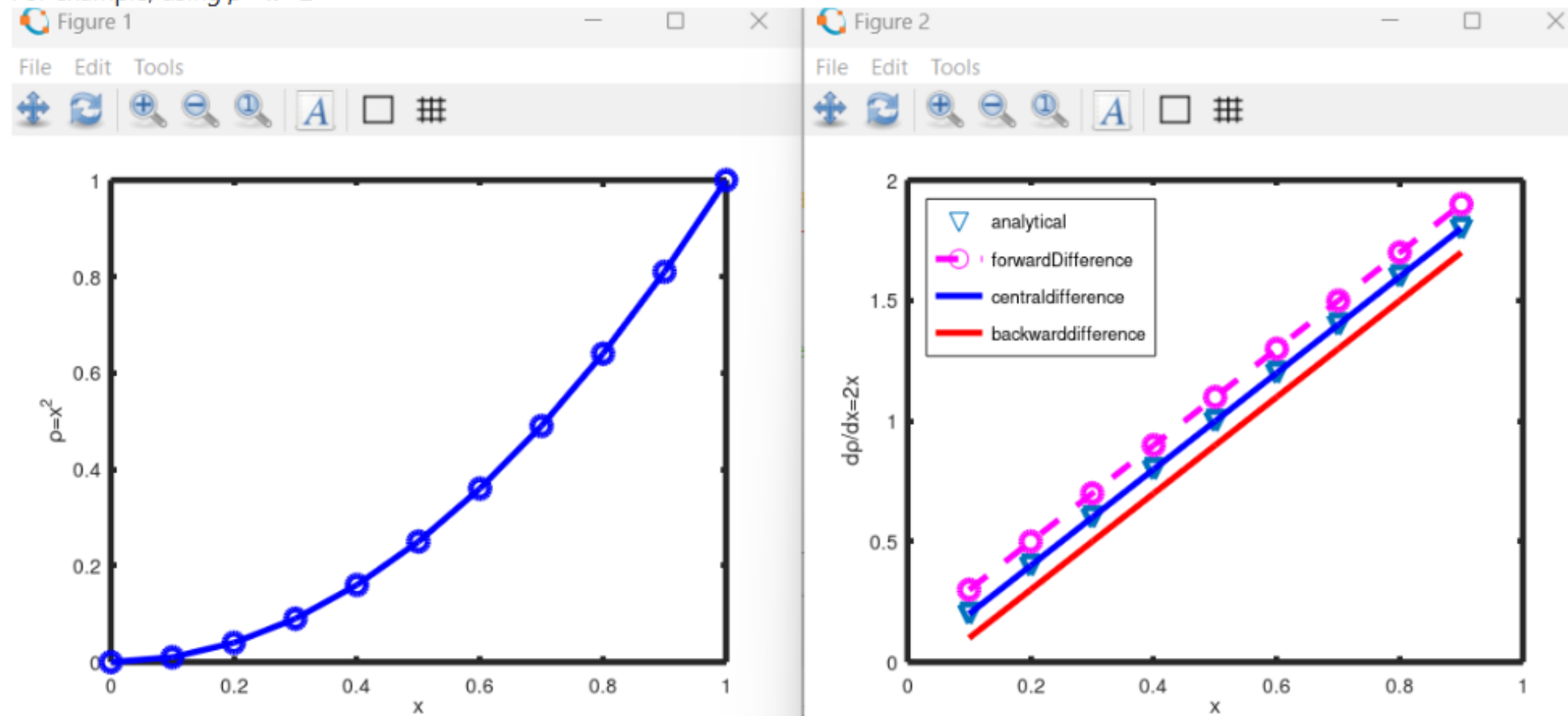
Edit

kummi0402 started this conversation in **General**

---

kummi0402 1 minute ago **Maintainer** ···

Make use of Octave code and plot for the backward and central difference schemes: when $\rho = x$^2 and x^3

Octave code: https://github.com/exaslate-learn/applied-cfd-using-openfoam-aec-2025/tree/main/DAY1-2

For example, using $\rho = x$^2

Category ⚙

💬 General

Labels ⚙

None yet

1 participant

Notifications

🔕 Unsubscribe

You're receiving notifications because you authored the thread.

🔒 Lock conversation
→ Transfer this discussion
📌 Pin discussion
📌 Pin discussion to General
⊙ Create issue from discussion
🗑 Delete discussion

Kumaresh Selvakumar
kumaresh@exaslate.com

THANK YOU