Applied Computational Fluid Dynamics with OpenFOAM

Day - 3

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Contents

- Taylor series: Central Difference Scheme (2nd order)
- Taylor series: Backward Difference Scheme (1st order)
- Exercise – 3

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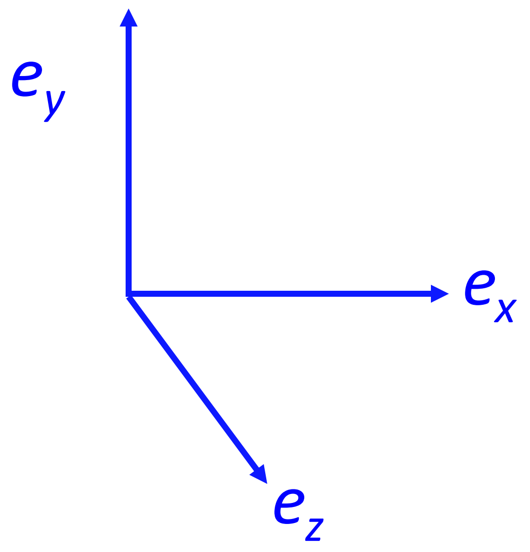
$$\frac{\partial}{\partial t}(\rho u)+\nabla\bullet(\rho \mathrm{u} u)=\nabla\bullet(\mu\,\nabla u)-\nabla p+S_u$$

Mathematical operations

Gradient:

$$\nabla\rho=\left(\frac{\partial}{\partial x}\boldsymbol{e}_x+\frac{\partial}{\partial y}\boldsymbol{e}_y+\frac{\partial}{\partial z}\boldsymbol{e}_z\right)\rho=\left(\frac{\partial\rho}{\partial x}\boldsymbol{e}_x+\frac{\partial\rho}{\partial y}\boldsymbol{e}_y+\frac{\partial\rho}{\partial z}\boldsymbol{e}_z\right)$$



$$\nabla\boldsymbol{u}=\begin{bmatrix}\partial u/\partial x & \partial v/\partial x & \partial w/\partial x\\ \partial u/\partial y & \partial v/\partial y & \partial w/\partial y\\ \partial u/\partial z & \partial v/\partial z & \partial w/\partial z\end{bmatrix}$$

$$\frac{\partial\rho}{\partial x}=\frac{d\rho}{dx}\,(in\ 1D)$$

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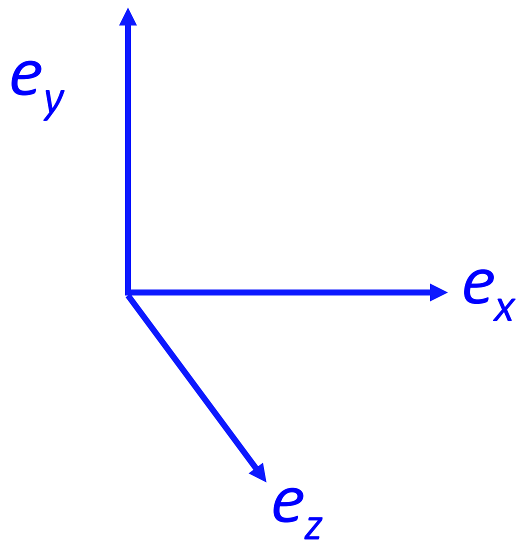
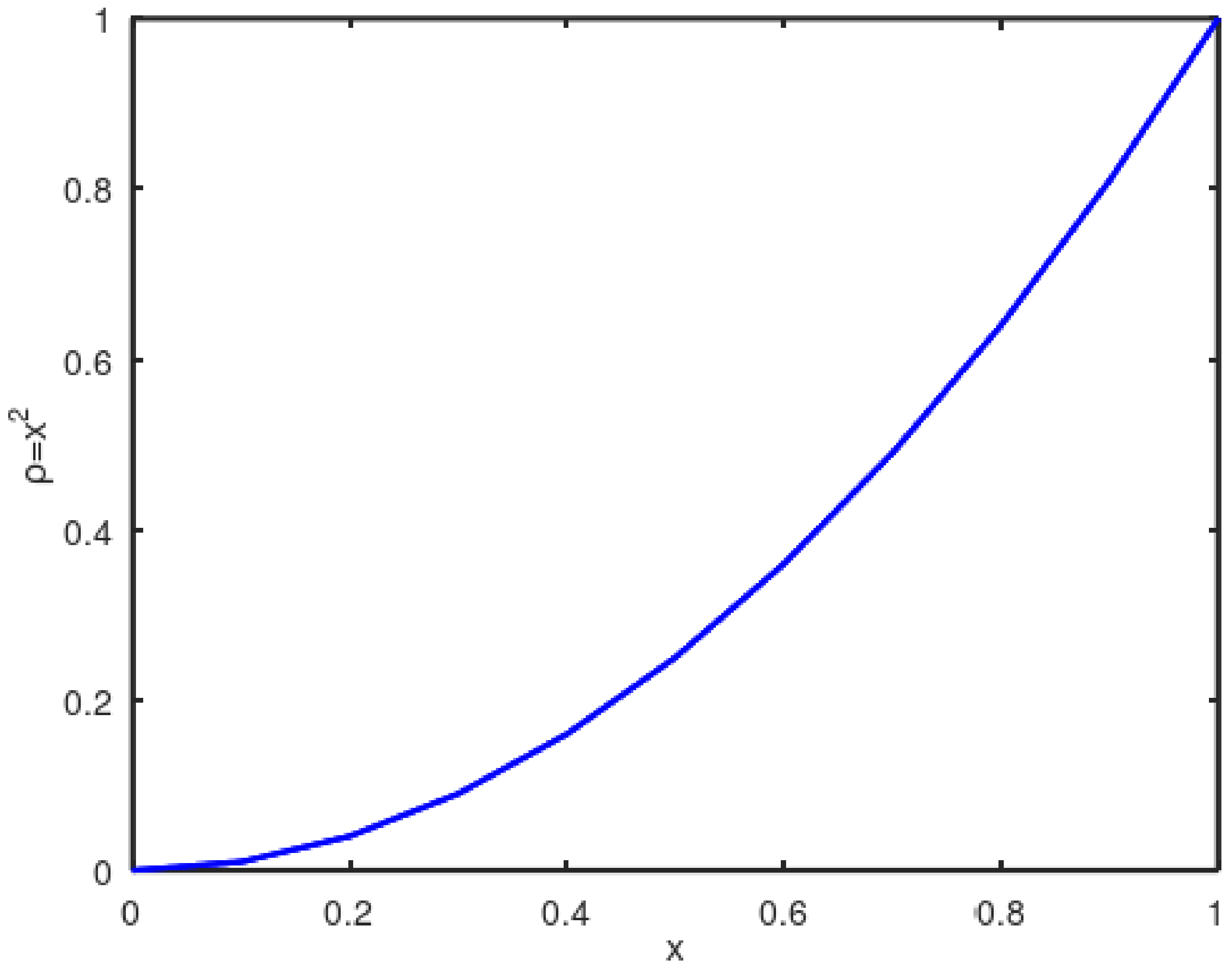
Mathematical operations

Gradient

$$\nabla\rho = \left(\frac{\partial}{\partial x}\boldsymbol{e}_x + \frac{\partial}{\partial y}\boldsymbol{e}_y + \frac{\partial}{\partial z}\boldsymbol{e}_z\right)\rho = \left(\frac{\partial\rho}{\partial x}\boldsymbol{e}_x + \frac{\partial\rho}{\partial y}\boldsymbol{e}_y + \frac{\partial\rho}{\partial z}\boldsymbol{e}_z\right)$$

$$\frac{\partial\rho}{\partial x} = \frac{d\rho}{dx}(in\ 1D)$$





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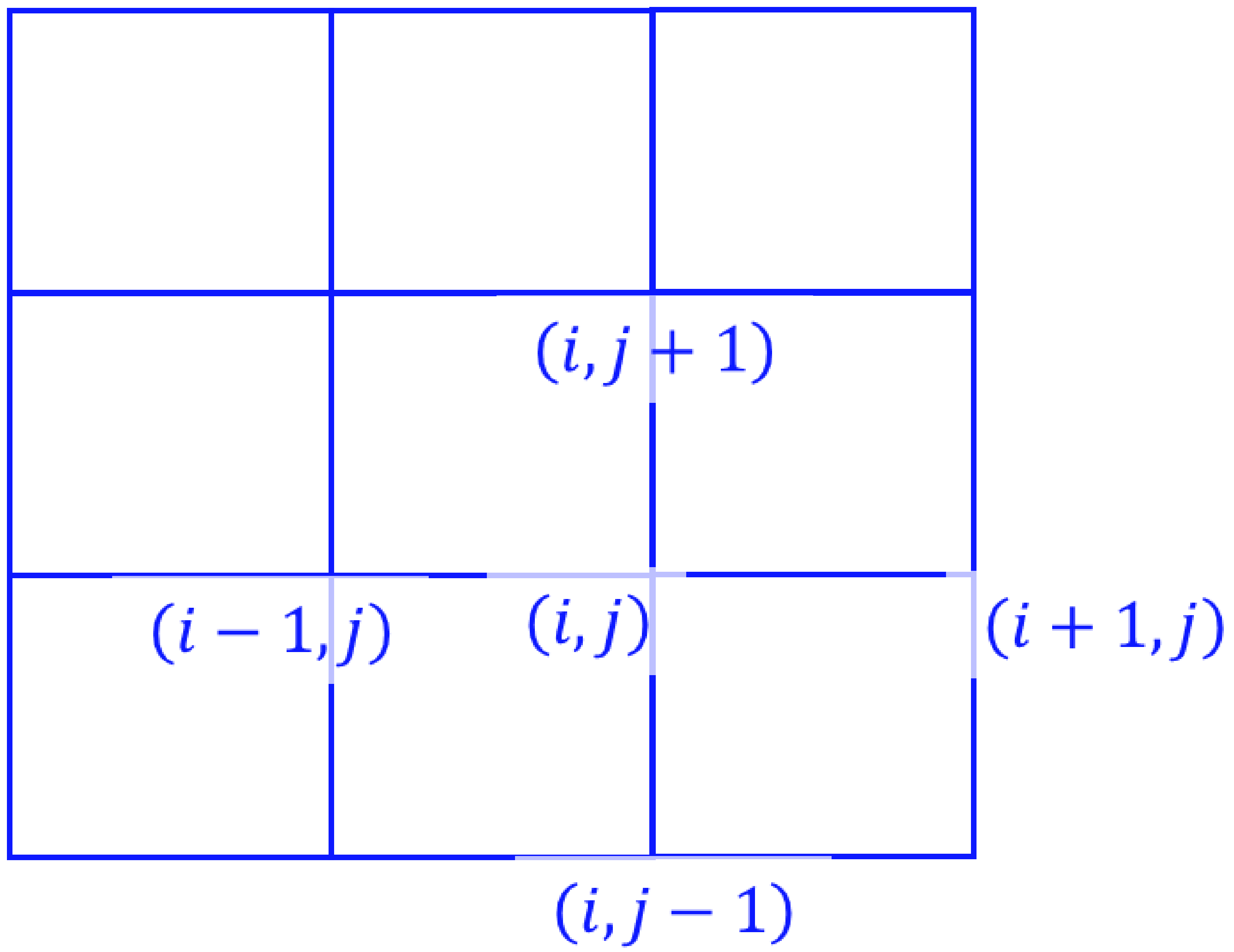
Finite Difference Method (FDM)

$$\nabla\rho = \left(\frac{\partial}{\partial x}\boldsymbol{e}_x + \frac{\partial}{\partial y}\boldsymbol{e}_y + \frac{\partial}{\partial z}\boldsymbol{e}_z\right)\rho = \left(\frac{\partial\rho}{\partial x}\boldsymbol{e}_x + \frac{\partial\rho}{\partial y}\boldsymbol{e}_y + \frac{\partial\rho}{\partial z}\boldsymbol{e}_z\right)$$

$$\left(\frac{\partial\rho}{\partial x}\right)_i = \frac{\rho(x_{i+1}) - \rho(x_i)}{\Delta x_i} + O(\Delta x_i)$$



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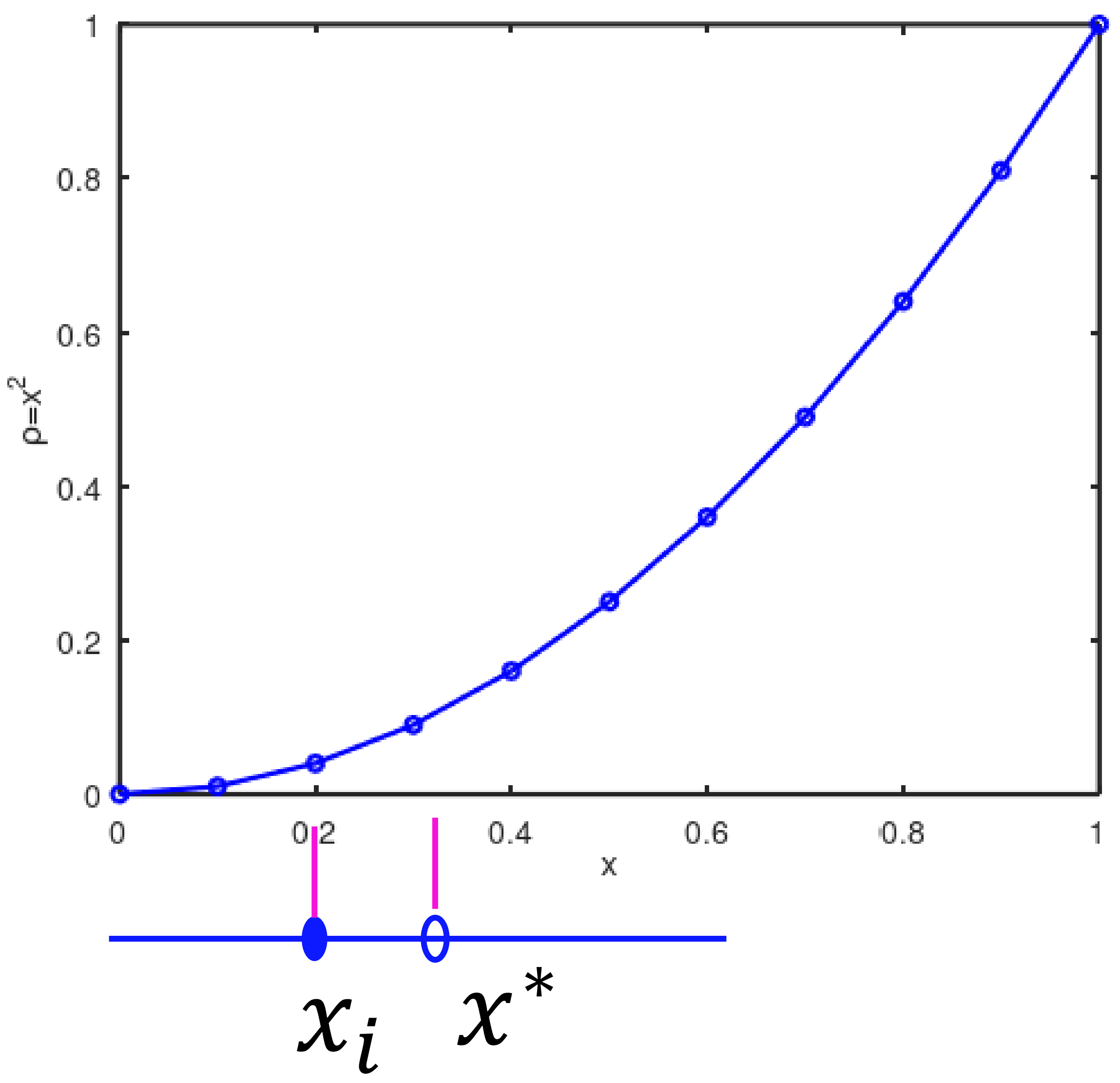
Taylor series expansion



$$\rho(x^*) = \rho(x_i) + (x^* - x_i)\left(\frac{d\rho}{dx}\right)_i + (x^* - x_i)^2\left(\frac{d^2\rho}{dx^2}\right)_i + (x^* - x_i)^3\left(\frac{d^3\rho}{dx^3}\right)_i + \cdots$$

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Taylor series expansion

$$\rho(x_{i+1}) = \rho(x_i) + (x_{i+1} - x_i)\left(\frac{\partial \rho}{\partial x}\right)_i + (x_{i+1} - x_i)^2\left(\frac{\partial^2 \rho}{\partial x^2}\right)_i + (x_{i+1} - x_i)^3\left(\frac{\partial^3 \rho}{\partial x^3}\right)_i + \cdots$$

$$\rho(x_{i+1}) = \rho(x_i) + (x_{i+1} - x_i)\left(\frac{\partial \rho}{\partial x}\right)_i + O\left(\Delta x_i^2\right); \qquad \Delta x_i = (x_{i+1} - x_i)$$

$$\rho(x_{i+1}) = \rho(x_i) + \Delta x_i\left(\frac{\partial \rho}{\partial x}\right)_i + O\left(\Delta x_i^2\right)$$

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Taylor series and FDM

Taylor series:

$$\rho(x_{i+1}) = \rho(x_i) + \Delta x_i \left(\frac{\partial \rho}{\partial x}\right)_i + O(\Delta x_i^2)$$

$$\left(\frac{\partial \rho}{\partial x}\right)_i = \frac{\rho(x_{i+1}) - \rho(x_i)}{\Delta x_i} + \frac{1}{\Delta x_i} O(\Delta x_i^2)$$

$$\left(\frac{\partial \rho}{\partial x}\right)_i = \frac{\rho(x_{i+1}) - \rho(x_i)}{\Delta x_i} + O(\Delta x_i)$$

$$\left(\frac{\partial \rho}{\partial x}\right)_i \approx \frac{\rho(x_{i+1}) - \rho(x_i)}{\Delta x_i}$$

First order forward difference scheme

Finite difference

$$\nabla \rho = \left(\frac{\partial}{\partial x}\boldsymbol{e}_x + \frac{\partial}{\partial y}\boldsymbol{e}_y + \frac{\partial}{\partial z}\boldsymbol{e}_z\right)\rho = \left(\frac{\partial \rho}{\partial x}\boldsymbol{e}_x + \frac{\partial \rho}{\partial y}\boldsymbol{e}_y + \frac{\partial \rho}{\partial z}\boldsymbol{e}_z\right)$$

$$\left(\frac{\partial \rho}{\partial x}\right)_i = \frac{\rho(x_{i+1}) - \rho(x_i)}{\Delta x_i} + O(\Delta x_i)$$

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Taylor series expansion

$$\rho(x_{i+1}) = \rho(x_i) + (x_{i+1} - x_i)\left(\frac{d\rho}{dx}\right)_i + (x_{i+1} - x_i)^2\left(\frac{d^2\rho}{dx^2}\right)_i + (x_{i+1} - x_i)^3\left(\frac{d^3\rho}{dx^3}\right)_i + \cdots$$

$$(1)\quad \rho(x_{i+1}) = \rho(x_i) + \Delta x_i\left(\frac{d\rho}{dx}\right)_i + \Delta x_i^2\left(\frac{d^2\rho}{dx^2}\right)_i + O\left(\Delta x_i^3\right)$$

$$(2)\quad \rho(x_{i-1}) = \rho(x_i) - \Delta x_i\left(\frac{d\rho}{dx}\right)_i + \Delta x_i^2\left(\frac{d^2\rho}{dx^2}\right)_i + O\left(\Delta x_i^3\right)$$

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Taylor series: Central Difference Scheme (2nd order)

$$(1)\quad \rho(x_{i+1}) = \rho(x_i) + \Delta x_i \left(\frac{d\rho}{dx}\right)_i + \Delta x_i^2 \left(\frac{d^2\rho}{dx^2}\right)_i + O(\Delta x_i^3)$$

$$(2)\quad \rho(x_{i-1}) = \rho(x_i) - \Delta x_i \left(\frac{d\rho}{dx}\right)_i + \Delta x_i^2 \left(\frac{d^2\rho}{dx^2}\right)_i + O(\Delta x_i^3)$$

Subtract (2) from (1)

$$\rho(x_{i+1}) - \rho(x_{i-1}) = 2\Delta x_i \left(\frac{d\rho}{dx}\right)_i + O(\Delta x_i^3)$$

$$\boxed{\left(\frac{d\rho}{dx}\right)_i = \frac{\rho(x_{i+1}) - \rho(x_{i-1})}{2\Delta x_i} + O(\Delta x_i^2)}$$

Second order central difference scheme

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Taylor series: Backward Difference Scheme (1st order)

$$\rho(x_{i-1}) = \rho(x_i) - \Delta x_i \left(\frac{d\rho}{dx}\right)_i + \Delta x_i^2 \left(\frac{d^2\rho}{dx^2}\right)_i + O(\Delta x_i^3)$$

$$\rho(x_{i-1}) = \rho(x_i) - \Delta x_i \left(\frac{d\rho}{dx}\right)_i + O\,(\Delta x_i^2)$$

$$\rho(x_{i-1}) = \rho(x_i) - \Delta x_i \left(\frac{d\rho}{dx}\right)_i + O\,(\Delta x_i^2)$$

$$\left(\frac{d\rho}{dx}\right)_i = \frac{\rho(x_i) - \rho(x_{i-1})}{\Delta x_i} + O\,(\Delta x_i)$$

$$\left(\frac{d\rho}{dx}\right)_i \approx \frac{\rho(x_i) - \rho(x_{i-1})}{\Delta x_i}$$

First order backward difference scheme

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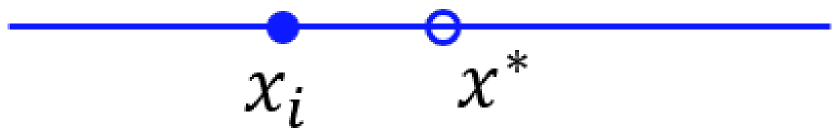
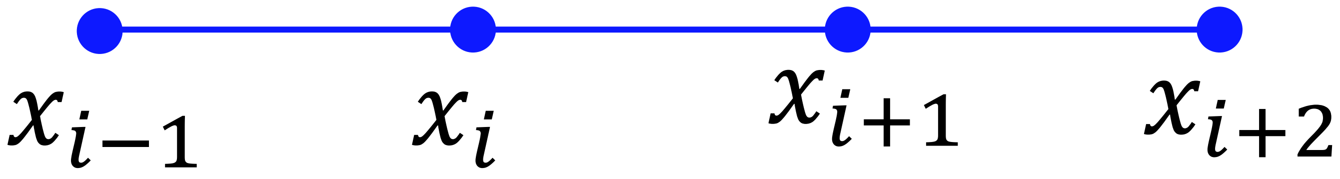
Taylor series: Summary



$$\rho(x^*) = \rho(x_i) + (x^* - x_i)\left(\frac{d\rho}{dx}\right)_i + (x^* - x_i)^2\left(\frac{d^2\rho}{dx^2}\right)_i + (x^* - x_i)^3\left(\frac{d^3\rho}{dx^3}\right)_i + \cdots$$



First order forward difference

$$\left(\frac{d\rho}{dx}\right)_i \approx \frac{\rho(x_{i+1}) - \rho(x_i)}{\Delta x_i}$$

Second order central difference

$$\left(\frac{d\rho}{dx}\right)_i \approx \frac{\rho(x_{i+1}) - \rho(x_{i-1})}{2\Delta x_i}$$

First order backward difference

$$\left(\frac{d\rho}{dx}\right)_i \approx \frac{\rho(x_i) - \rho(x_{i-1})}{\Delta x_i}$$

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Exercise – 3

[Exercise-3] Solve using first order backward and second order central difference schemes #5

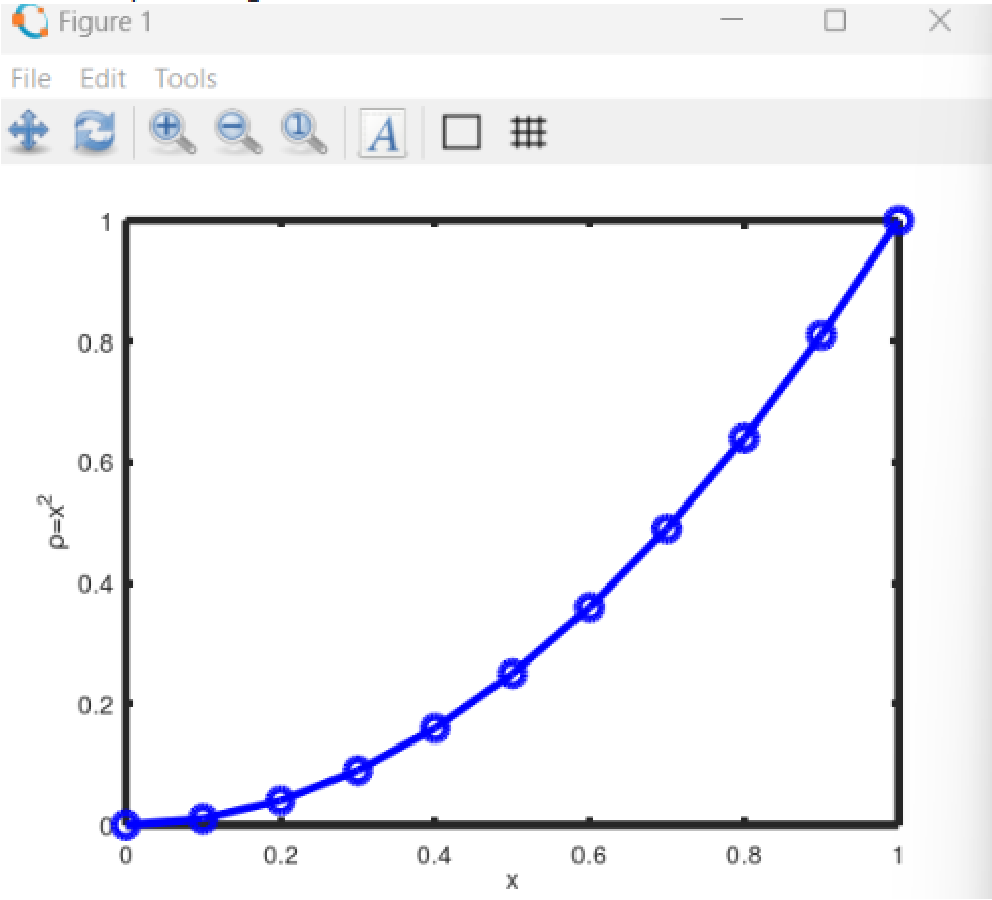
Edit

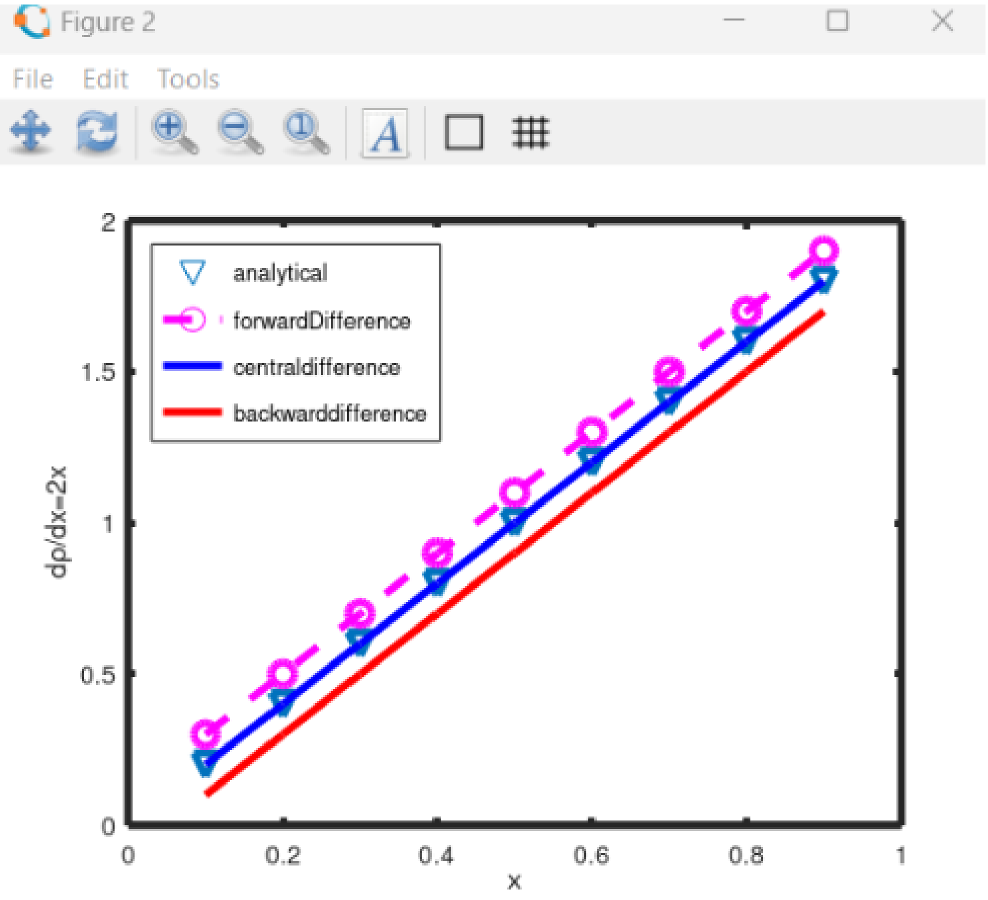
kummi0402 started this conversation in General

kummi0402 now Maintainer

Make use of Octave code and plot for the backward and central difference schemes: when $\rho = x^2$ and x^3
Octave code: https://github.com/exaslate-learn/applied-cfd-using-openfoam-ksr-spring-2025/tree/main/day-3

For example, using $\rho = x^2$





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THANK YOU