Applied Computational Fluid Dynamics with OpenFOAM

Day – 6

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Quick Recap

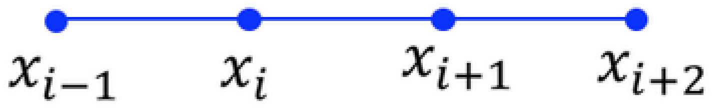
[Exercise-2] Solve using first order forward derivative scheme #3

kummi0402 started this conversation in General

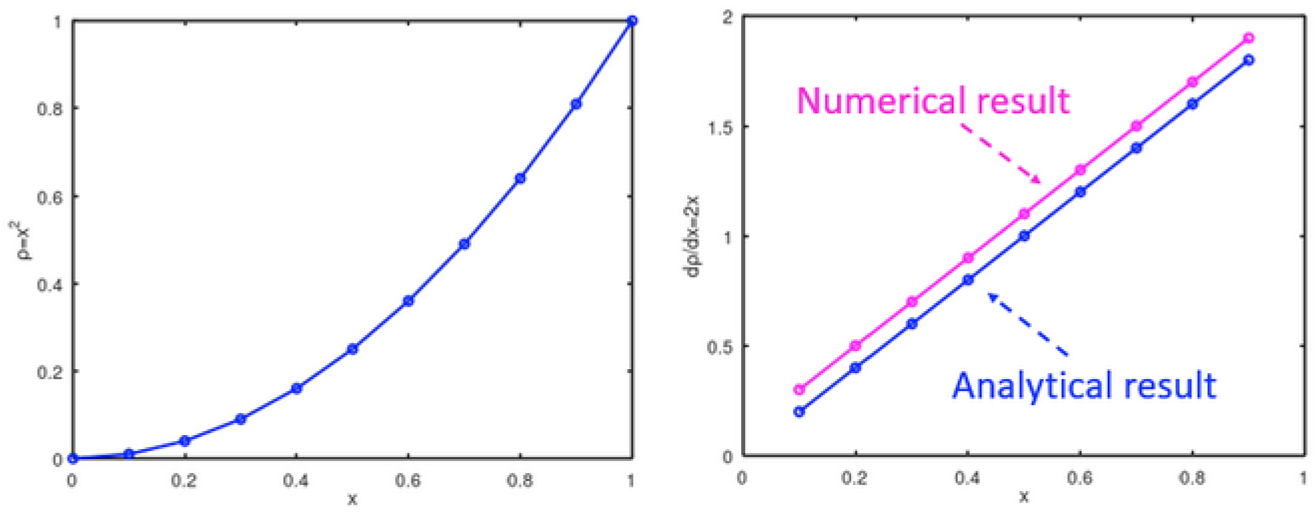
kummi0402 last week Maintainer edited

Make use of Octave code and plot for the forward first order derivative: when $\rho = x^2$ and x^3
OCTAVE code:
https://github.com/exaslate-learn/applied-cfd-using-openfoam-ksr-spring-2025/tree/main/day-2

Analytical and Numerical solutions



$$\left(\frac{\partial \rho}{\partial x}\right)_i = \frac{\rho(x_{i+1}) - \rho(x_i)}{\Delta x_i} + O(\Delta x_i)$$



Resolved in OCTAVE

1

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Quick Recap

[Exercise-3] Solve using first order backward and second order central difference schemes #5

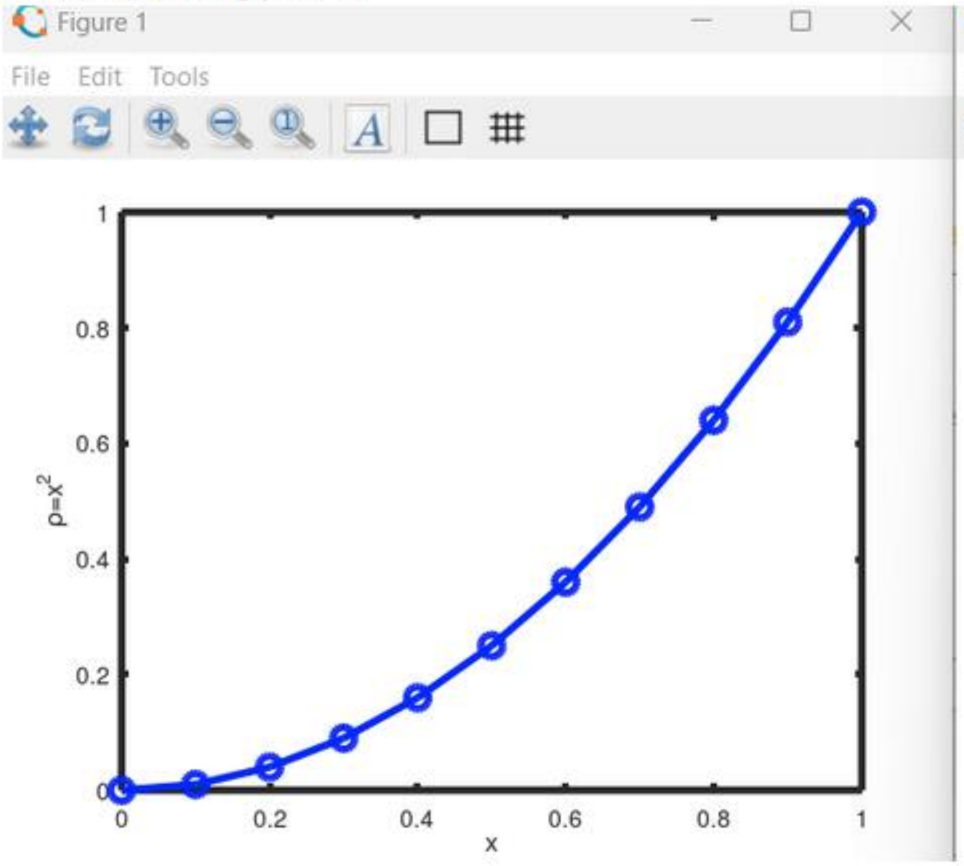
Edit

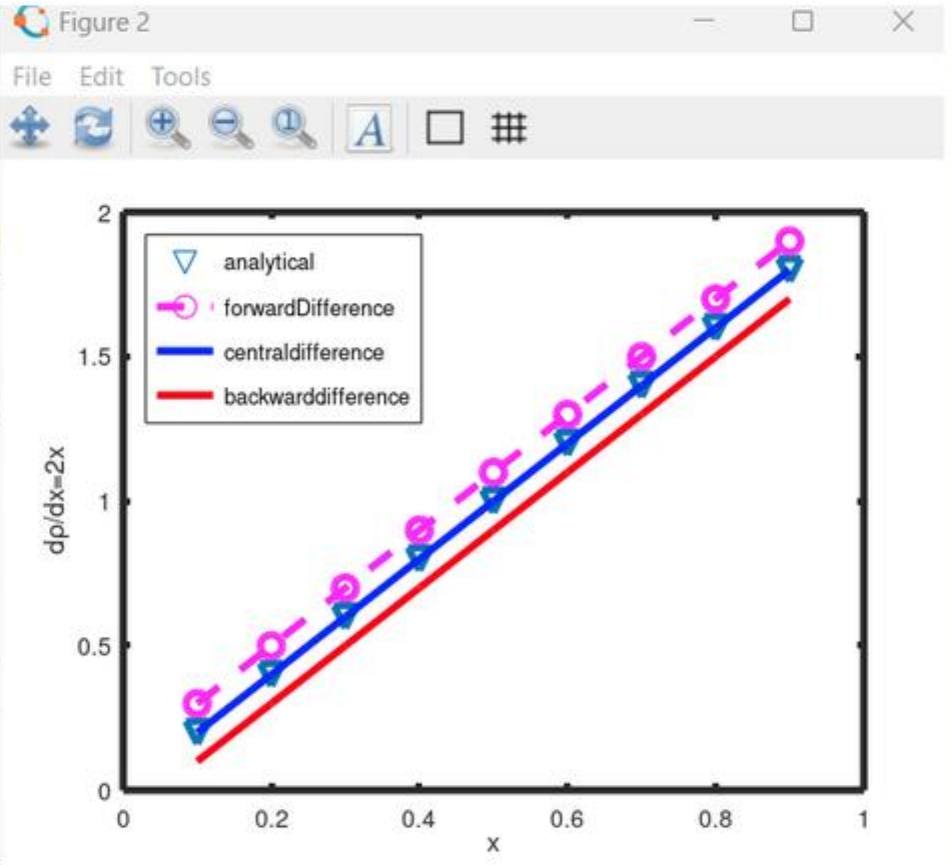
kummi0402 started this conversation in General

kummi0402 last week Maintainer

Make use of Octave code and plot for the backward and central difference schemes: when $\rho = x^2$ and x^3
Octave code: https://github.com/exaslate-learn/applied-cfd-using-openfoam-ksr-spring-2025/tree/main/day-3

For example, using $\rho = x^2$





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Quick Recap

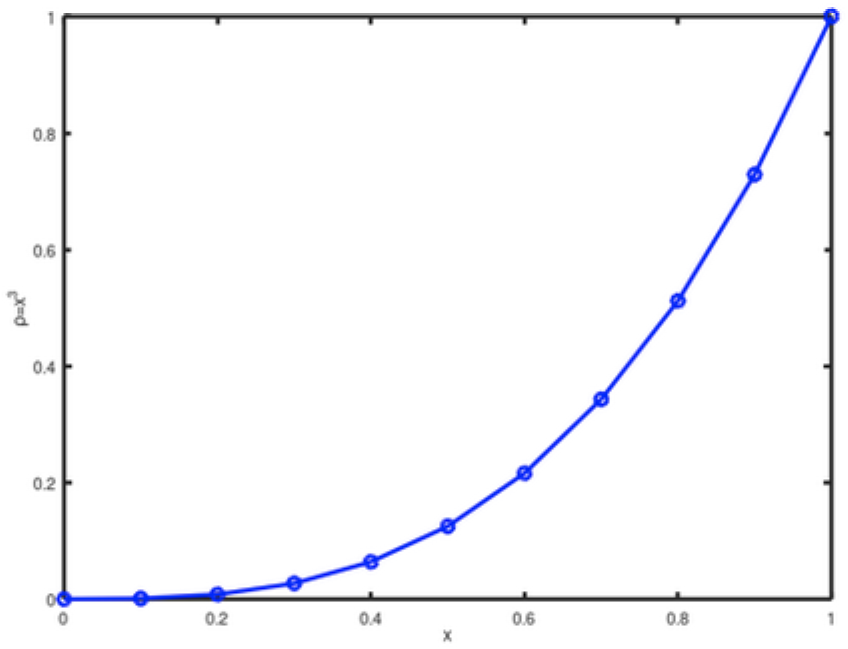
[Exercise-4] Estimate second order derivative using second order central difference scheme #6

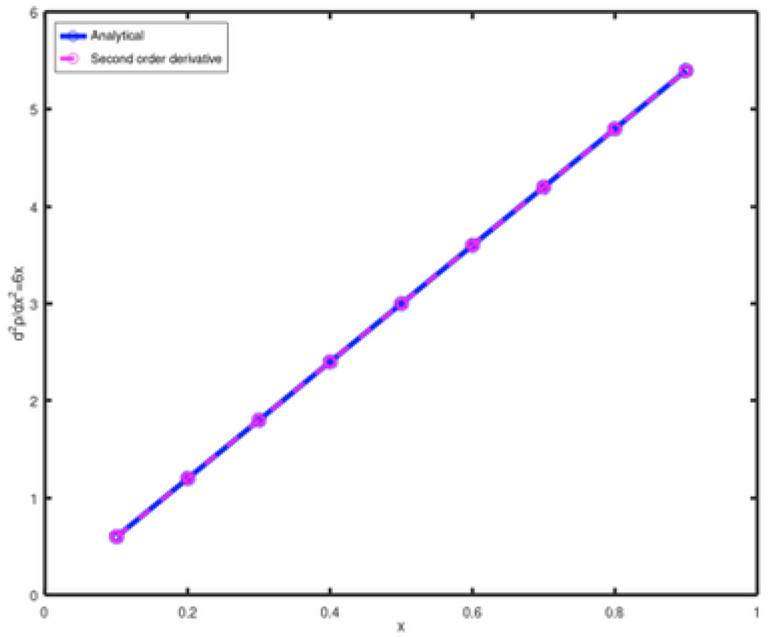
kummi0402 started this conversation in **General**

kummi0402 last week Maintainer

Approximating Second Order Derivative

$$\boxed{\left(\frac{d^2\rho}{dx^2}\right)_i = \frac{\rho(x_{i+1}) - 2\rho(x_i) + \rho(x_{i-1})}{\Delta x_i^2} + O(\Delta x_i^2)}$$





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Quick Recap

[Exercise-5] Advection equation and Stability analysis #7

kummi0402 started this conversation in General

kummi0402 5 days ago Maintainer

Exercise – 5 (i)

1. Solve the following advection equation analytically in octave

$$\frac{\partial u}{\partial t} + c\frac{\partial u}{\partial x} = 0$$

2. Upload in GitHub

Exercise – 5 (ii)

1. Solve the following advection equation numerically in octave

$$\frac{\partial u}{\partial t} + c\frac{\partial u}{\partial x} = 0$$

   a) Central difference with CFL = 0.1 (dx = 0.01, c = 0.01, dt = 0.1, t_final = 5)

2. Upwind scheme (backward difference) with CFL = 0.1 (dx = 0.01, dt = 0.1, t_final = 5). Change the "c" value between 0.01 and -0.01 and analyze the stability. Hint: Upwind scheme with c = -0.01 becomes unstable and act as downwind.
3. Downwind scheme (forward difference) with CFL = 0.1 (dx = 0.01, dt = 0.1, t_final = 5). Change the "c" value between 0.01 and -0.01 and analyze the stability. Hint: Downwind scheme with c = -0.01 becomes stable and act as upwind.
4. Examine CFL numbers. Analyse the upwind scheme with CFL = 0.1, 1.0, and 10. Analyze the stability. (dx = 0.01, c = 0.01, dt = 0.1, t_final = 5)
5. Upload in GitHub.

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Contents

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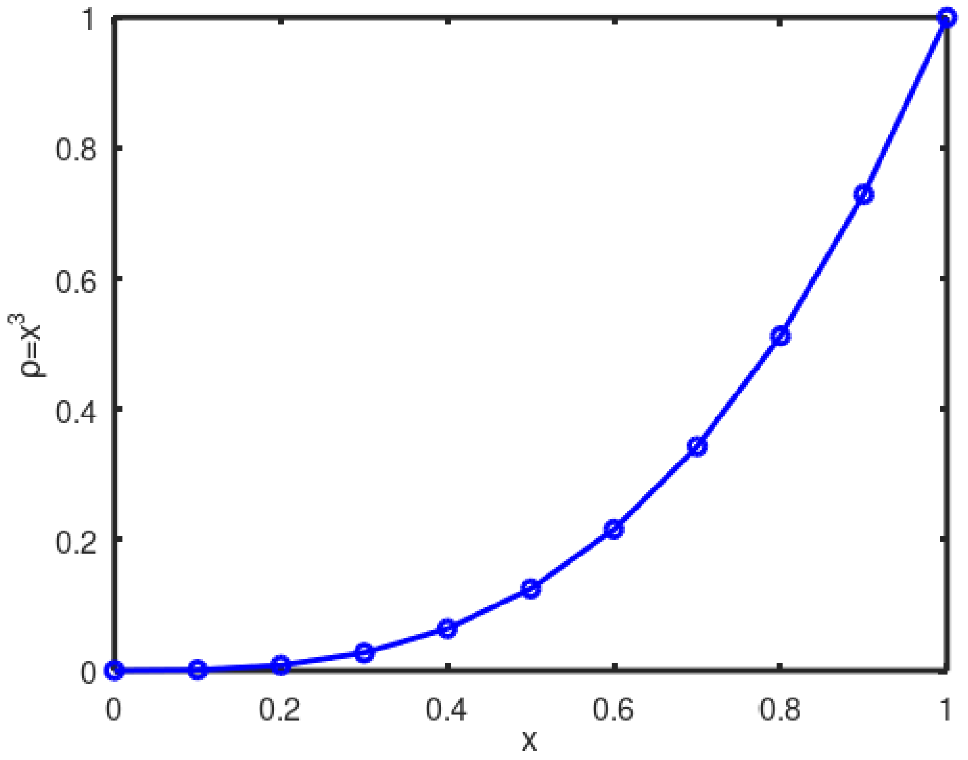
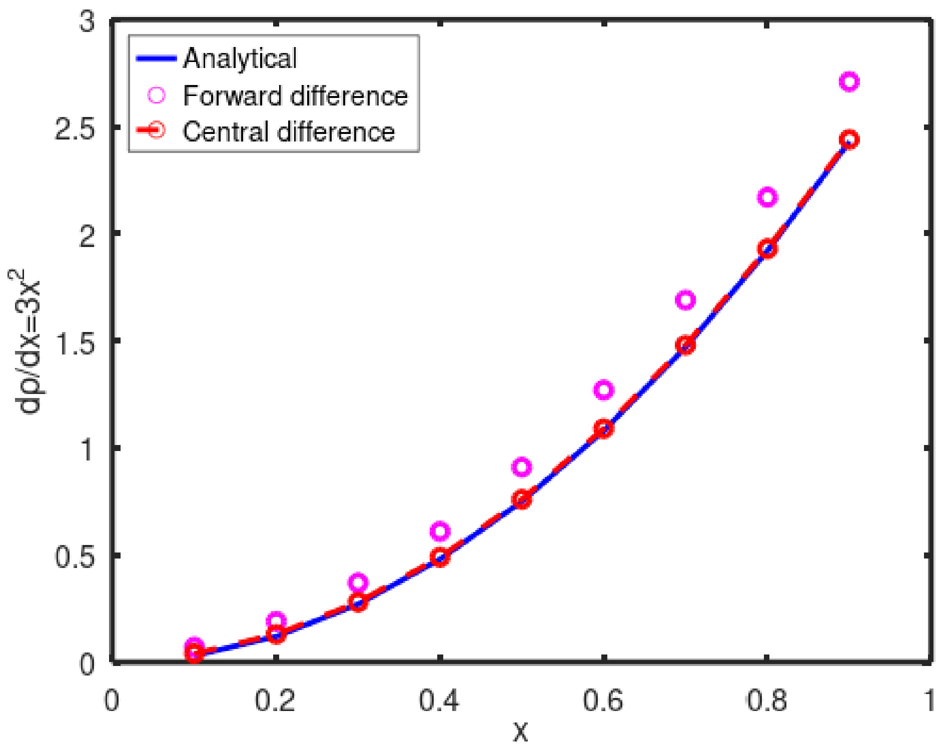
Rate of Convergence



$$\rho = x^3$$



$$\frac{\partial \rho}{\partial x} = 3x^2$$

First order forward difference approximation

$$\left(\frac{d\rho}{dx}\right)_i \approx \frac{\rho(x_{i+1}) - \rho(x_i)}{\Delta x_i} + O(\Delta x_i)$$

Second order central difference approximation

$$\left(\frac{d\rho}{dx}\right)_i = \frac{\rho(x_{i+1}) - \rho(x_{i-1})}{2\Delta x_i} + O\left(\Delta x_i^2\right)$$

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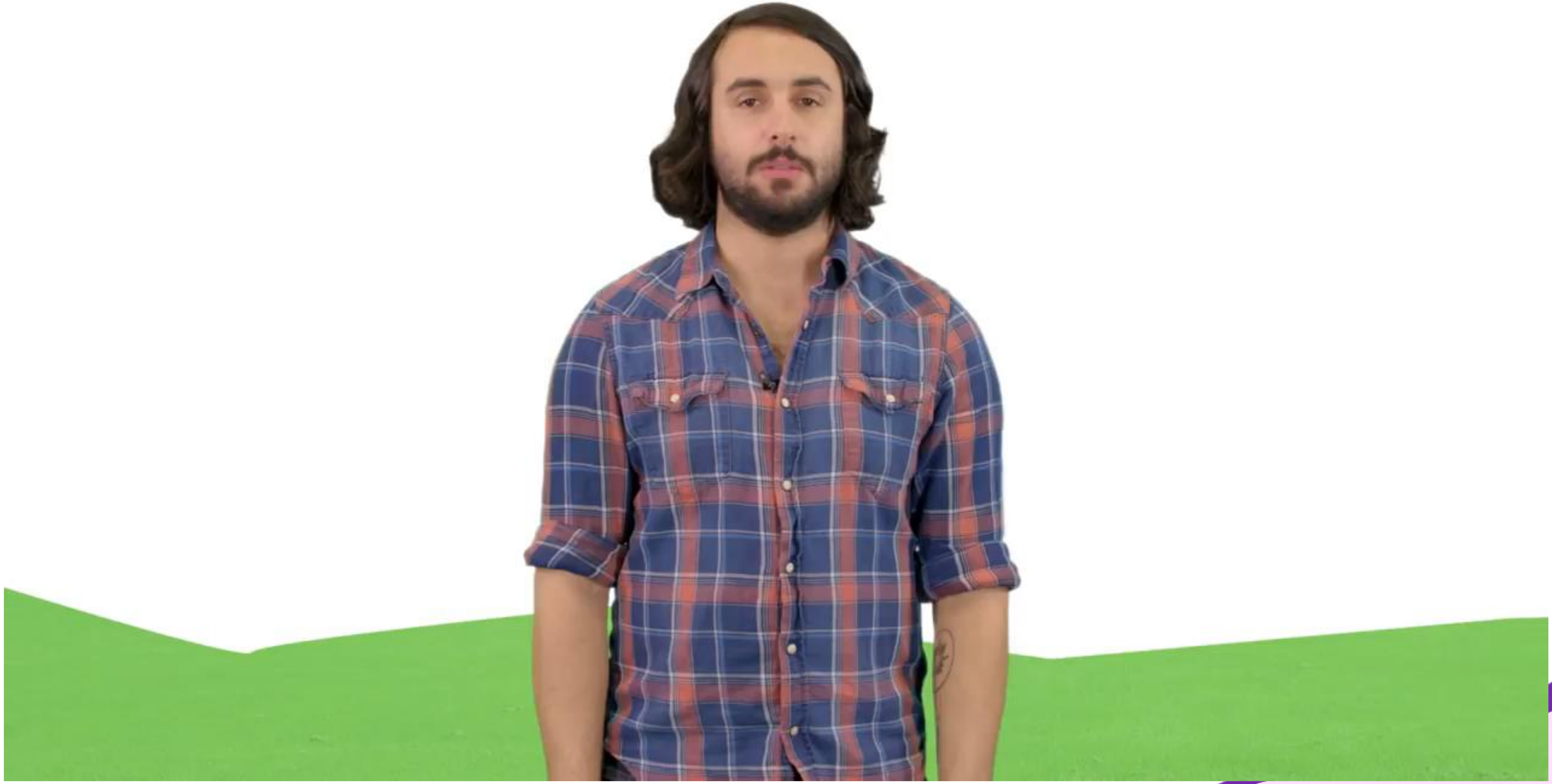
Higher Derivatives and Their Applications



https://www.youtube.com/watch?v=zUDxgehxQqs

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How to solve error and rate of convergence

Check the codes:

1. error_calculate.m ➔ Tabulate (in Excel) the results for forward and central difference method and compare with analytical solutions and post in GitHub.
2. get_rate_of_convergence.m ➔ Extract ROC values for first and second order values.

First order forward difference approximation

$$\left(\frac{d\rho}{dx}\right)_i \approx \frac{\rho(x_{i+1}) - \rho(x_i)}{\Delta x_i} + O(\Delta x_i)$$

Δx	Error	Rate of convergence
0.1	0.16	-
0.05	0.0775	1.04
0.025	0.038125	1.02

Second order central difference approximation

$$\left(\frac{d\rho}{dx}\right)_i = \frac{\rho(x_{i+1}) - \rho(x_{i-1})}{2\Delta x_i} + O\left(\Delta x_i^2\right)$$

Δx	Error	Rate of convergence
0.1	0.01	-
0.05	0.0025	2
0.025	0.000625	2

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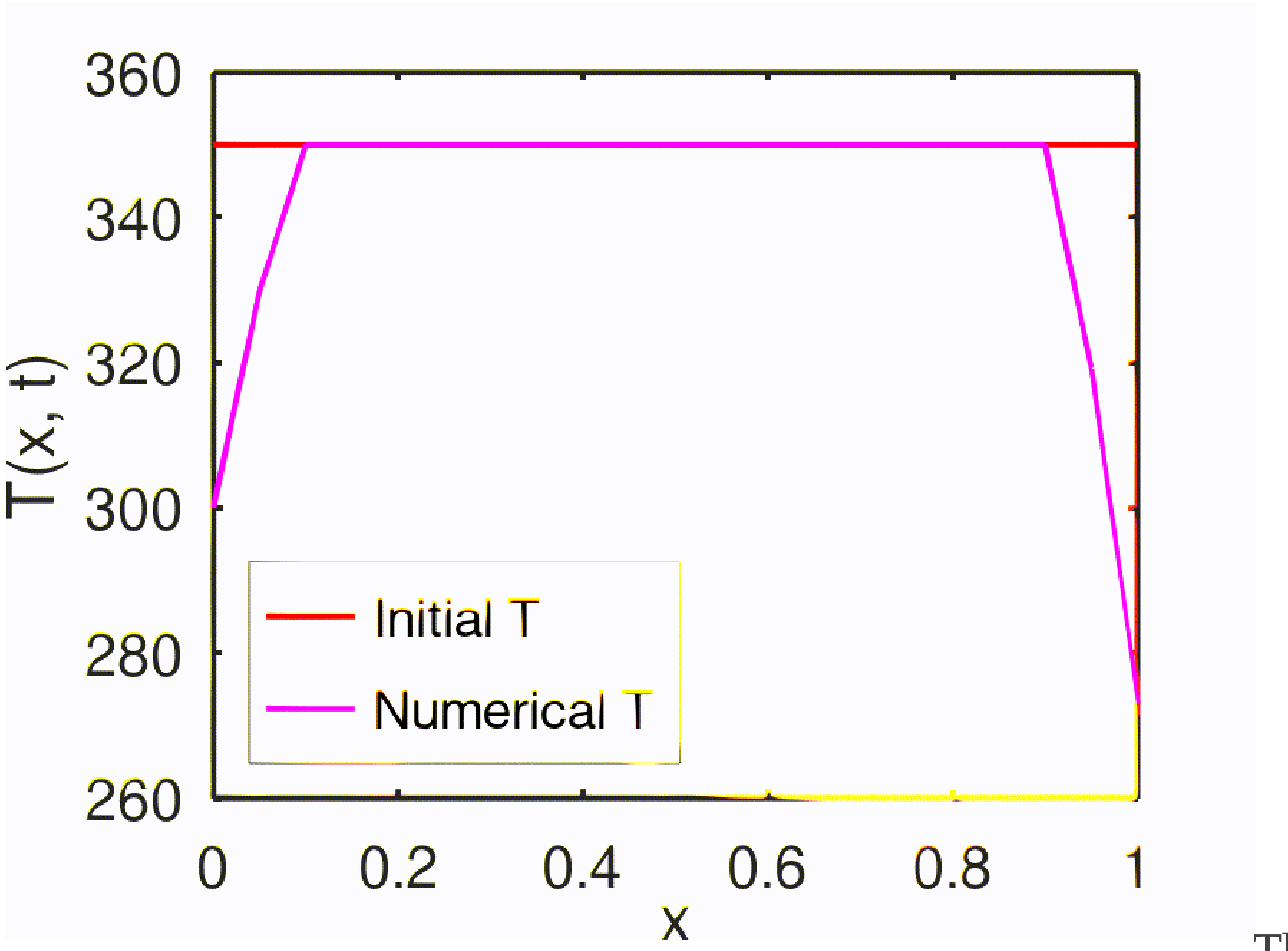
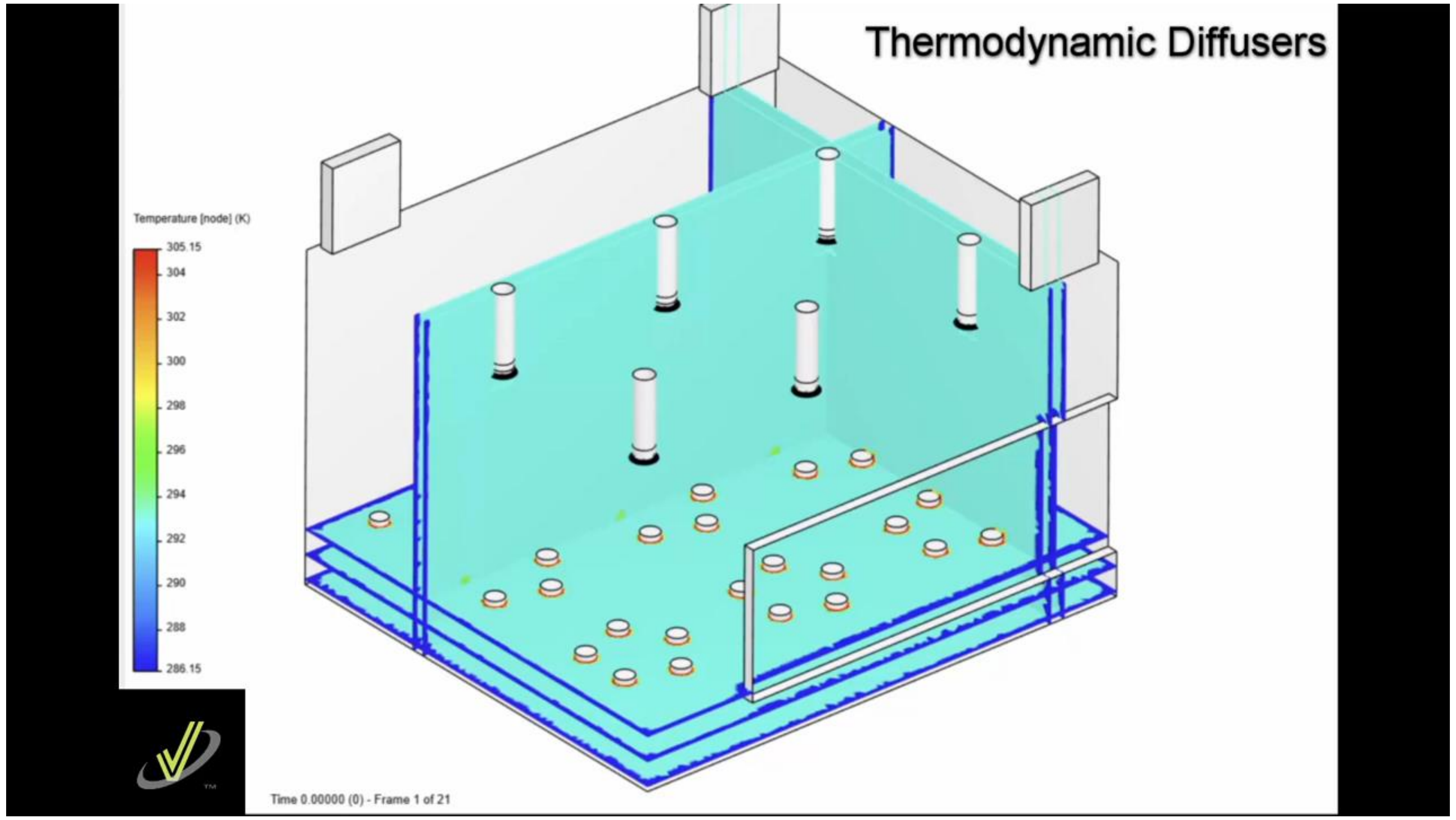
1. Fick's law describes the movement of particles from a region of high concentration to a region of lower concentration.
2. Diffusion generally represents the transport of a fluid property (momentum) due to fluctuating motions that are not captured by the bulk motion that is represented by the continuum velocity eqn.

Diffusion





Thermodynamic diffusers supply heated air with a downward jet, the difference in air density causes hot air to rise in the first minute after starting the system.

https://www.youtube.com/watch?v=Zwa0CA-CyAA
https://www.youtube.com/watch?v=VaZPVV6-VUY

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Numerical Solution to Diffusion Equation

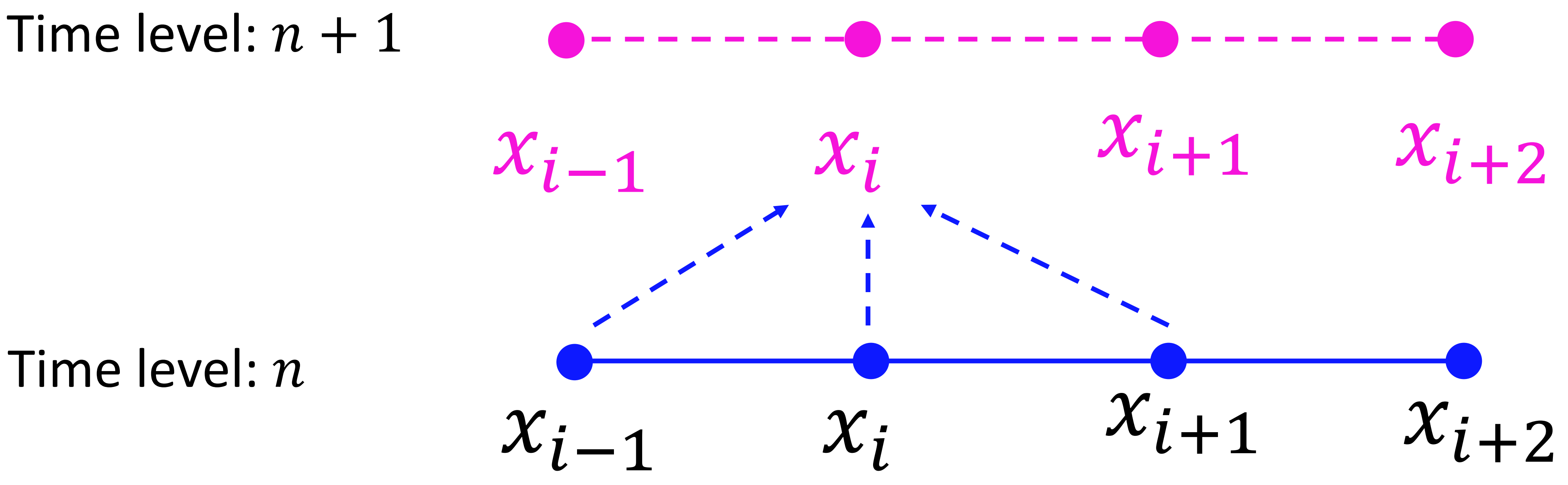
$$\boxed{\frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial x^2}}$$

$$\frac{T_i^{n+1} - T_i^n}{\Delta t} = \alpha \left(\frac{\partial^2 T}{\partial x^2}\right)_i^n$$

$$\boxed{\left(\frac{d^2 T}{dx^2}\right)_i = \frac{T(x_{i+1}) - 2T(x_i) + T(x_{i-1})}{\Delta x_i^2} + O(\Delta x_i^2)}$$

$$\frac{T_i^{n+1} - T_i^n}{\Delta t} = \alpha \frac{T_{i+1}^n - 2T_i^n + T_{i-1}^n}{\Delta x^2}$$

$$\boxed{T_i^{n+1} = T_i^n + \Delta t \alpha \frac{T_{i+1}^n - 2T_i^n + T_{i-1}^n}{\Delta x^2}}$$



Information is from both left and right end

$$u_i^{n+1} = u_i^n - c\Delta t \left(\frac{\partial u}{\partial x}\right)_i^n \longrightarrow \left(\frac{\partial u}{\partial x}\right)_i^n \approx \frac{u_{i+1}^n - u_{i-1}^n}{2\Delta x_i}$$

Central difference

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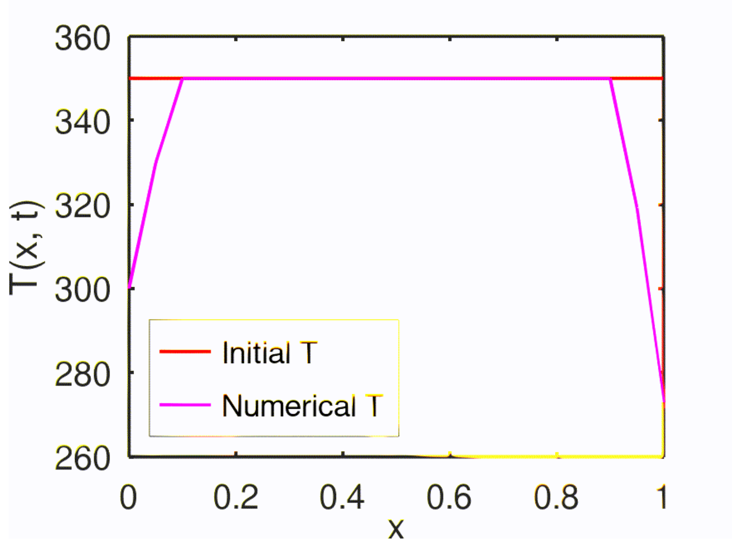
Exercise – 6 (Let's solve the Diffusion Equation)



$$T_i^{n+1} = T_i^n + \Delta t \alpha \frac{T_{i+1}^n - 2T_i^n + T_{i-1}^n}{\Delta x^2}$$

a7_solve_diffusion_sample.m

1. Resolve the diffusion equation with α = 1, dt = 0.001, dx = 0.05, Dirichlet boundaries (T_left = 300K , T_right = 273K), and write the above numerical solution to extract the results.
2. Change the right boundary condition from Dirichlet to Neumann. Explain about it in few words.
3. Analyze for different time steps (dt) 0.1 and 0.01 and give your comments. Hint: Von Neumann stability analysis.
4. Learning debug skills – fix breakpoints, run , and understand the codes. Explain about it in few words.

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What is your research topic related to CFD ?

Any research topic presentation for 10 minutes.

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THANK YOU