Applied Computational Fluid Dynamics with OpenFOAM

Day - 5

ExaSlate

Develop = Guide = Collab

ExaSlate
Develop = Guide = Collab

Quick Recap

[Exercise-2] Solve using first order forward derivative scheme #3

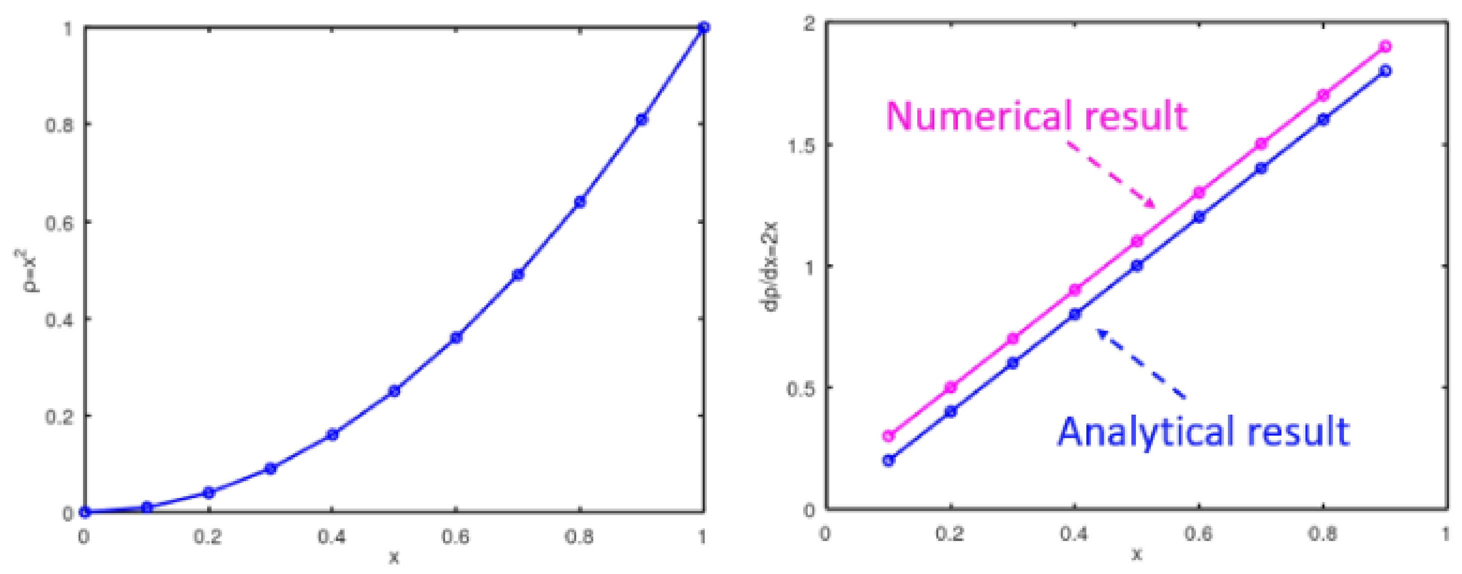
kummi0402 started this conversation in **General**

kummi0402 3 days ago Maintainer edited

Make use of Octave code and plot for the forward first order derivative: when $\rho = x$^2 and x^3
OCTAVE code:
https://github.com/exaslate-learn/applied-cfd-using-openfoam-ksr-spring-2025/tree/main/day-2

Analytical and Numerical solutions

$$\left(\frac{\partial \rho}{\partial x}\right)_i = \frac{\rho(x_{i+1}) - \rho(x_i)}{\Delta x_i} + O(\Delta x_i)$$



Resolved in OCTAVE

↑ 1

Kumaresh Selvakumar
kumaresh@exaslate.com

ExaSlate
Develop = Guide = Collab

Quick Recap

[Exercise-3] Solve using first order backward and second order central difference schemes #5

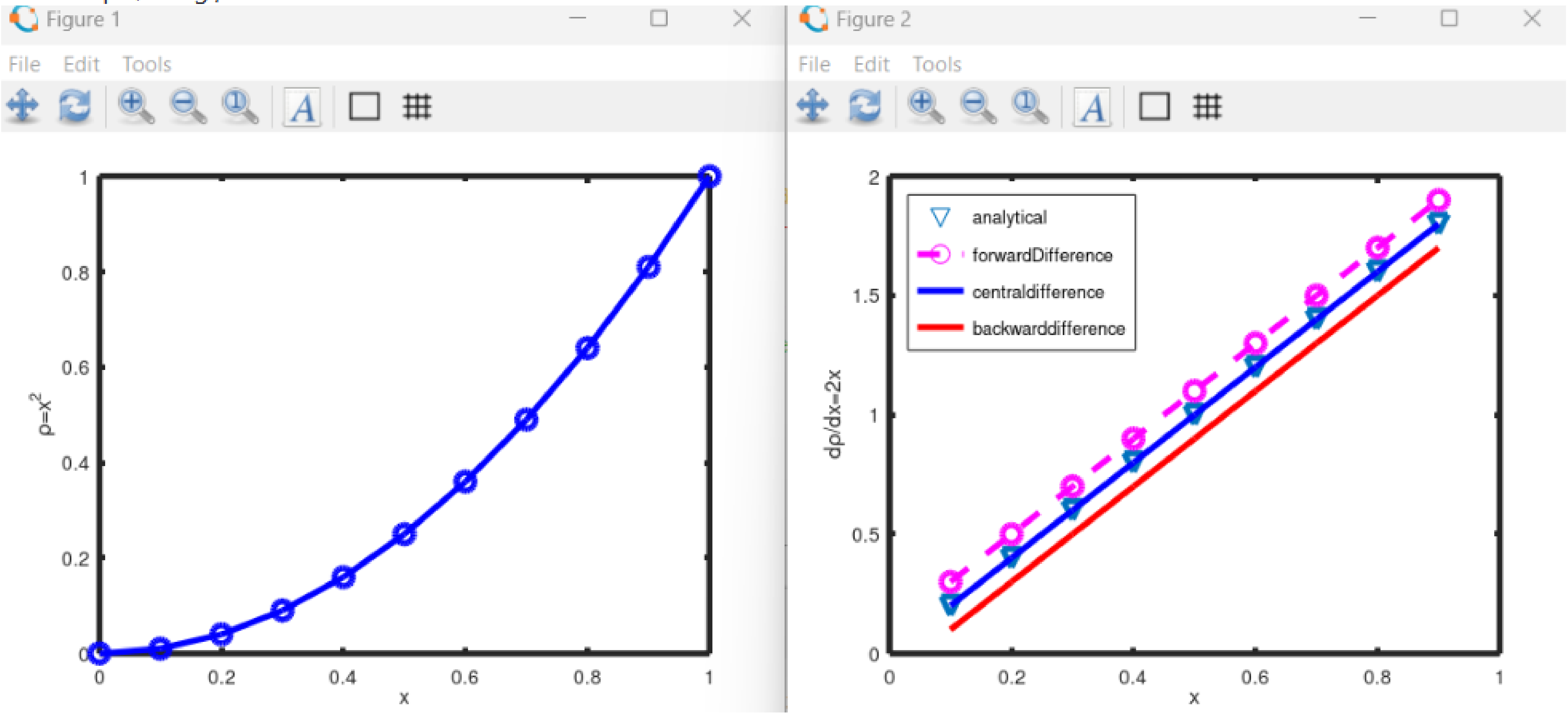
Edit

kummi0402 started this conversation in General

kummi0402 2 days ago Maintainer

Make use of Octave code and plot for the backward and central difference schemes: when $\rho = x^2$ and x^3
Octave code: https://github.com/exaslate-learn/applied-cfd-using-openfoam-ksr-spring-2025/tree/main/day-3

For example, using $\rho = x^2$



↑ 1

Category
General

Labels
None yet

1 participant

Notifications
Unsubscribe
You're receiving notifications because you authored the thread.

Lock conversation
Transfer this discussion
Pin discussion
Pin discussion to General
Create issue from discussion
Delete discussion

Kumaresh Selvakumar
kumaresh@exaslate.com

Quick Recap

ExaSlate
Develop = Guide = Collab

[Exercise-4] Estimate second order derivative using second order central difference scheme #6

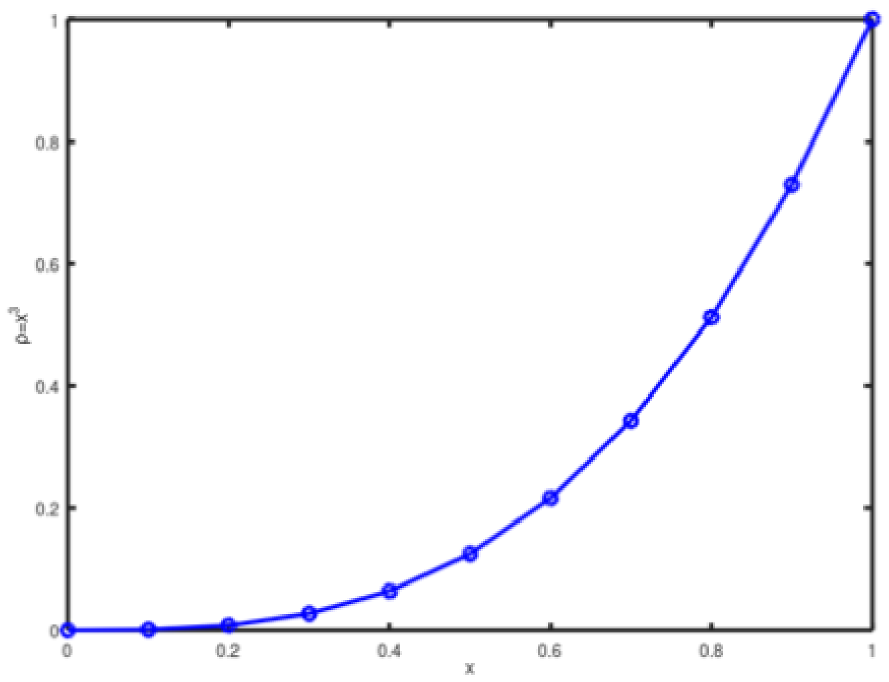
Edit

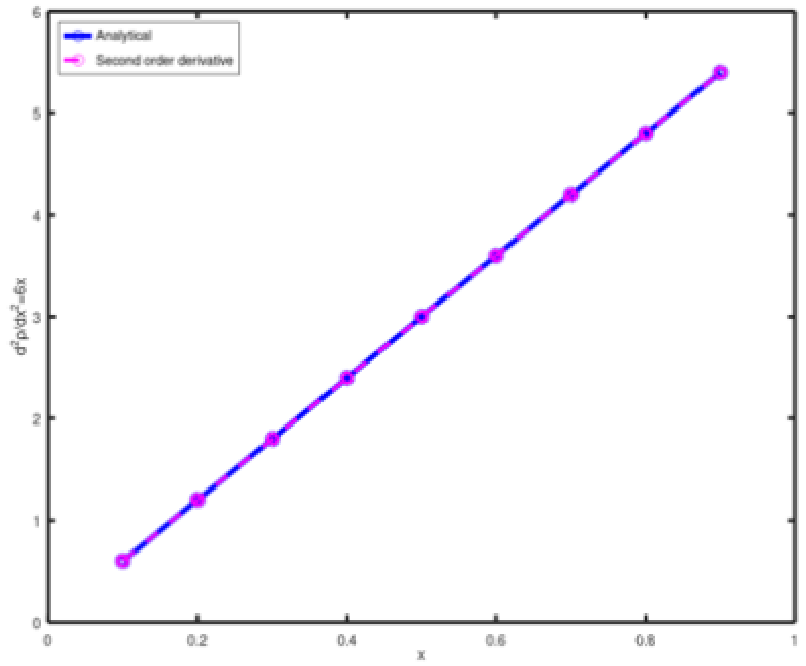
kummi0402 started this conversation in General

kummi0402 yesterday Maintainer

Approximating Second Order Derivative

$$\boxed{\left(\frac{d^2\rho}{dx^2}\right)_i = \frac{\rho(x_{i+1}) - 2\rho(x_i) + \rho(x_{i-1})}{\Delta x_i^2} + O(\Delta x_i^2)}$$





↑ 1

Category

General

Labels

None yet

2 participants

Notifications

Unsubscribe

You're receiving notifications because you authored the thread.

Lock conversation

Transfer this discussion

Pin discussion

Pin discussion to General

Create issue from discussion

Delete discussion

Kumaresh Selvakumar
kumaresh@exaslate.com

Contents

- Numerical stability
- Advection equation
- Exercise – 5 (i) and (ii)

Kumaresh Selvakumar
kumaresh@exaslate.com

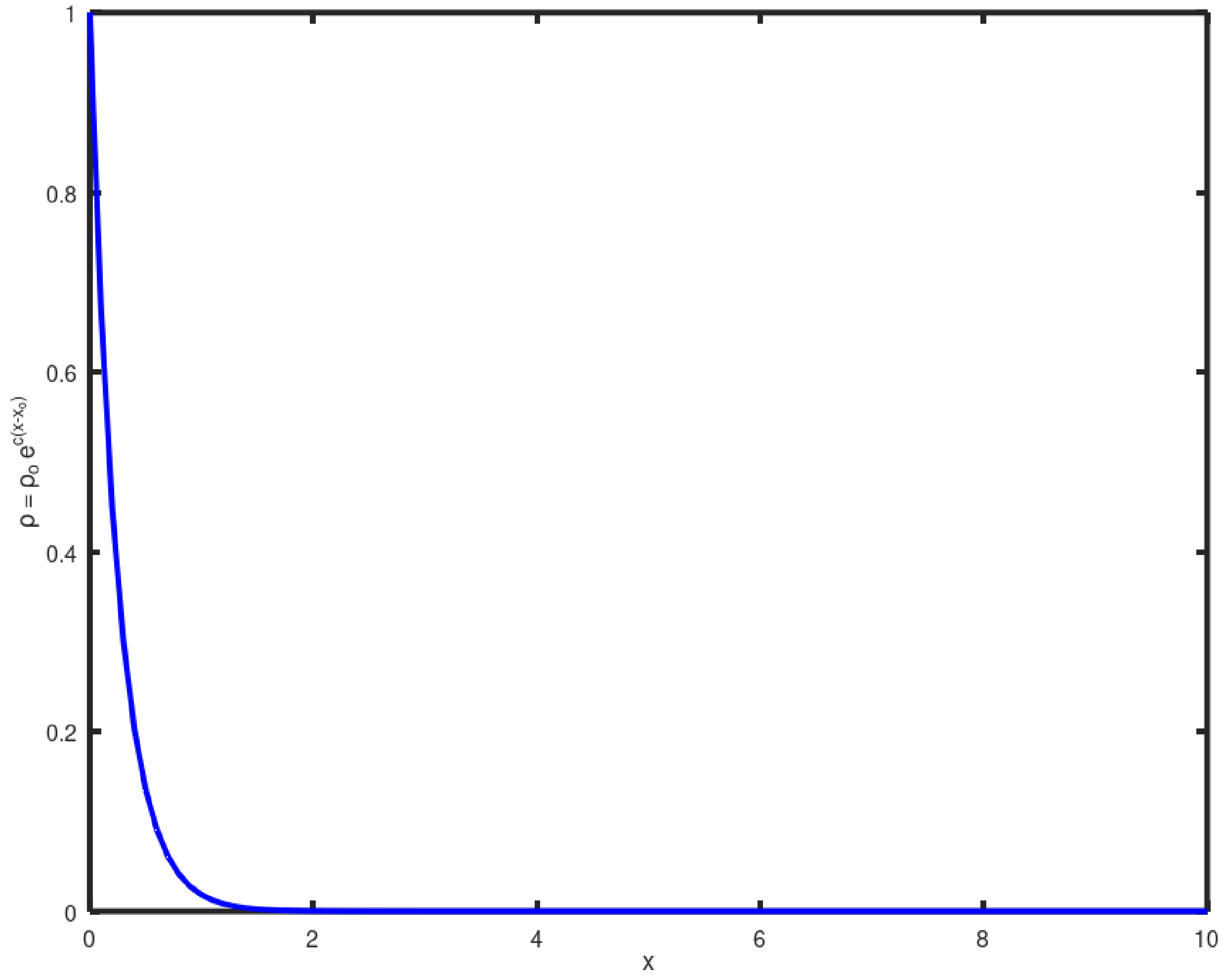
Numerical Stability

- Numerical approach should not magnify the error that appears in the solution.

$$\frac{d\rho}{dx} = -c\rho$$

$$\int_{\rho_o}^{\rho} \frac{d\rho}{\rho} = \int_{x_o}^{x} -c dx$$

Analytical $\boxed{\rho = \rho_o e^{-c(x - x_o)}}$



$c = 4, x_o = 0, \rho_o = 1, x \in [0,10]$

Kumaresh Selvakumar
kumaresh@exaslate.com

Numerical Stability

- Numerical discretization

$$\frac{d\rho}{dx} = -c\rho$$

$$\frac{\rho_{i+1} - \rho_i}{\Delta x} = -c\rho_i$$

Numerical $\boxed{\rho_{i+1} = \rho_i(1 - c\Delta x)}$

Kumaresh Selvakumar
kumaresh@exaslate.com

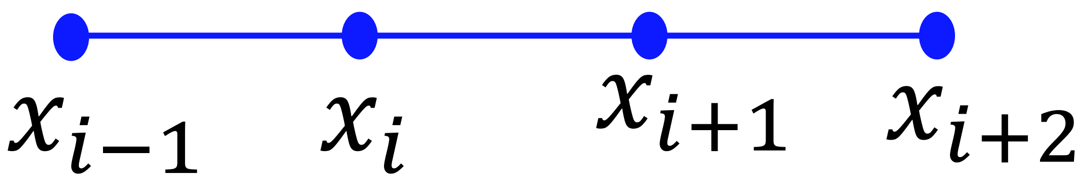
Numerical Stability

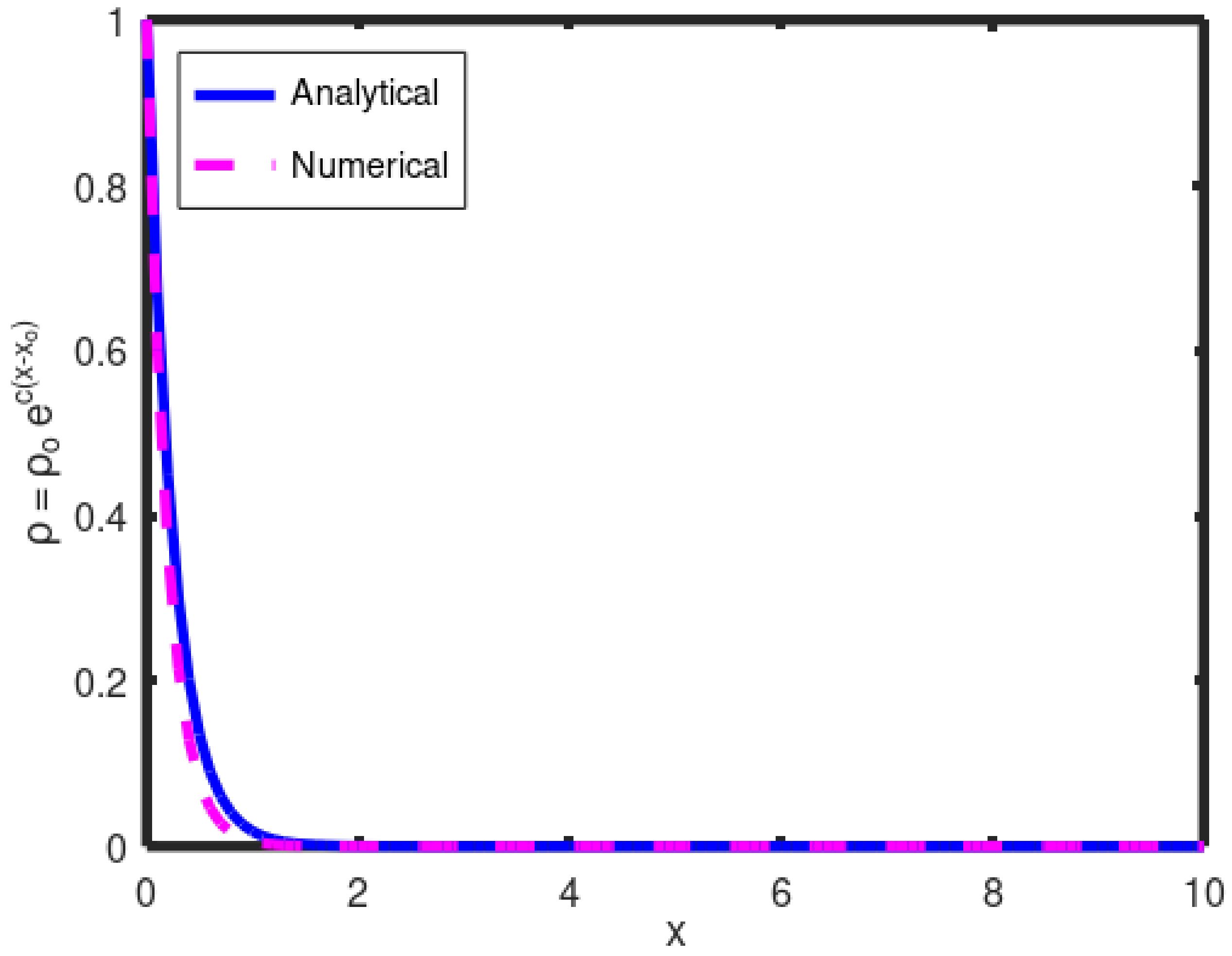
- Numerical discretization

$$\boxed{\frac{d\rho}{dx} = -c\rho}$$

$$\frac{\rho_{i+1} - \rho_i}{\Delta x} = -c\rho_i$$

$$\rho_{i+1} = \rho_i(1 - c\Delta x)$$





$c = 4, x_o = 0, \rho_o = 1, x \in [0,10], \Delta x = 0.1$

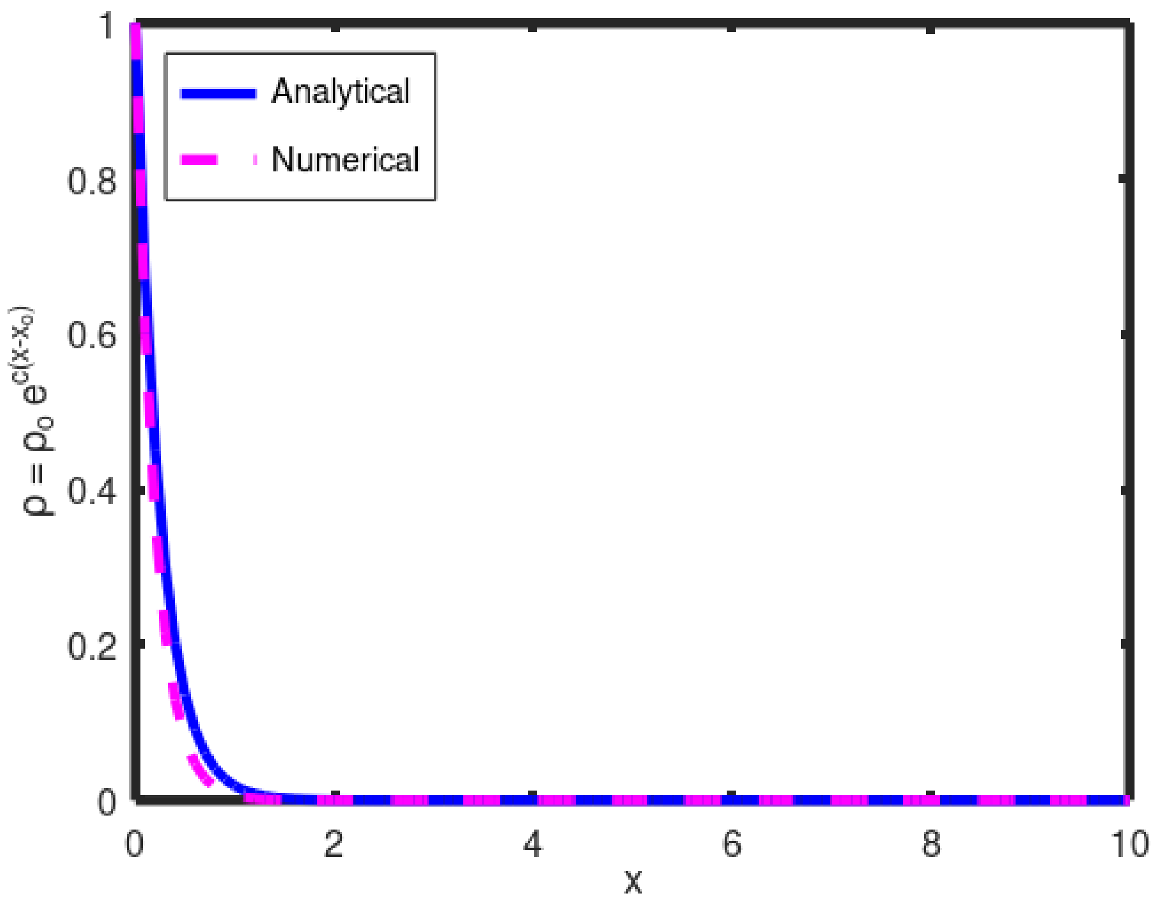
Kumaresh Selvakumar
kumaresh@exaslate.com

Numerical Stability

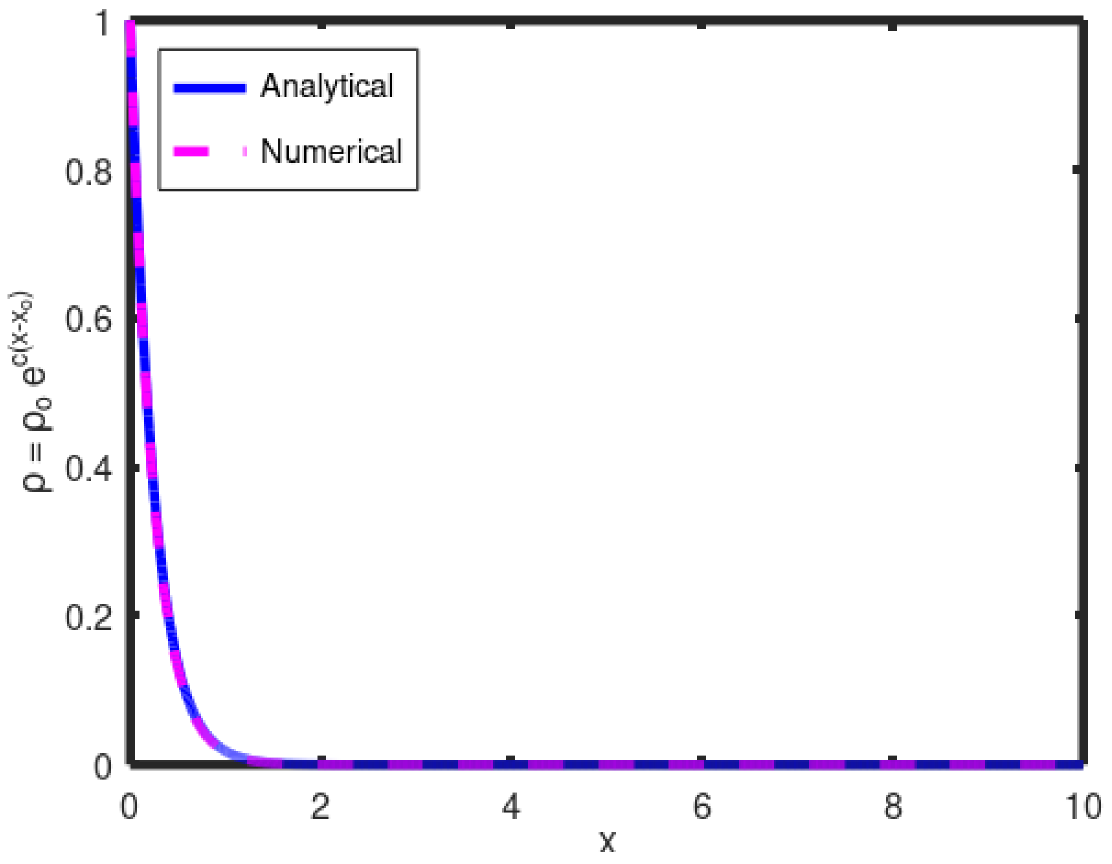
$$\boxed{\frac{d\rho}{dx} = -c\rho}$$

$$\boxed{\rho_{i+1} = \rho_i(1 - c\Delta x)}$$

x_{i-1} x_i x_{i+1} x_{i+2}



$c = 4, x_o = 0, \rho_o = 1, x \in [0,10],$
$\Delta x = 0.1$



$c = 4, x_o = 0, \rho_o = 1, x \in [0,10],$
$\Delta x = 0.01$

Stability Condition

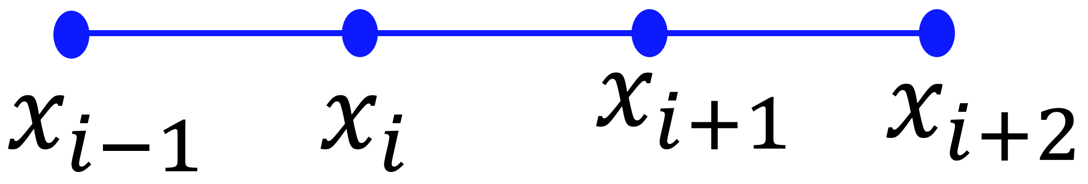
$$\left|\frac{\rho_{i+1}}{\rho_i}\right| < 1$$

Kumaresh Selvakumar
kumaresh@exaslate.com

ExaSlate
Develop · Guide · Collab

Numerical Stability

$$\frac{d\rho}{dx} = -c\rho$$



Stability Condition

$$\left|\frac{\rho_{i+1}}{\rho_i}\right| < 1$$

$$\rho_{i+1} = \rho_i(1 - c\Delta x)$$

$$\left|\frac{\rho_{i+1}}{\rho_i}\right| = |(1 - c\Delta x)| < 1$$

$$-1 < (1 - c\Delta x) < 1$$

$$0 < \Delta x < 2/c$$

$$\boxed{\Delta x < 2/c}$$

$$\Delta x < \frac{2}{c}$$

$$\frac{2}{c} = \frac{2}{4} = 0.5$$

Kumaresh Selvakumar
kumaresh@exaslate.com

Numerical Stability

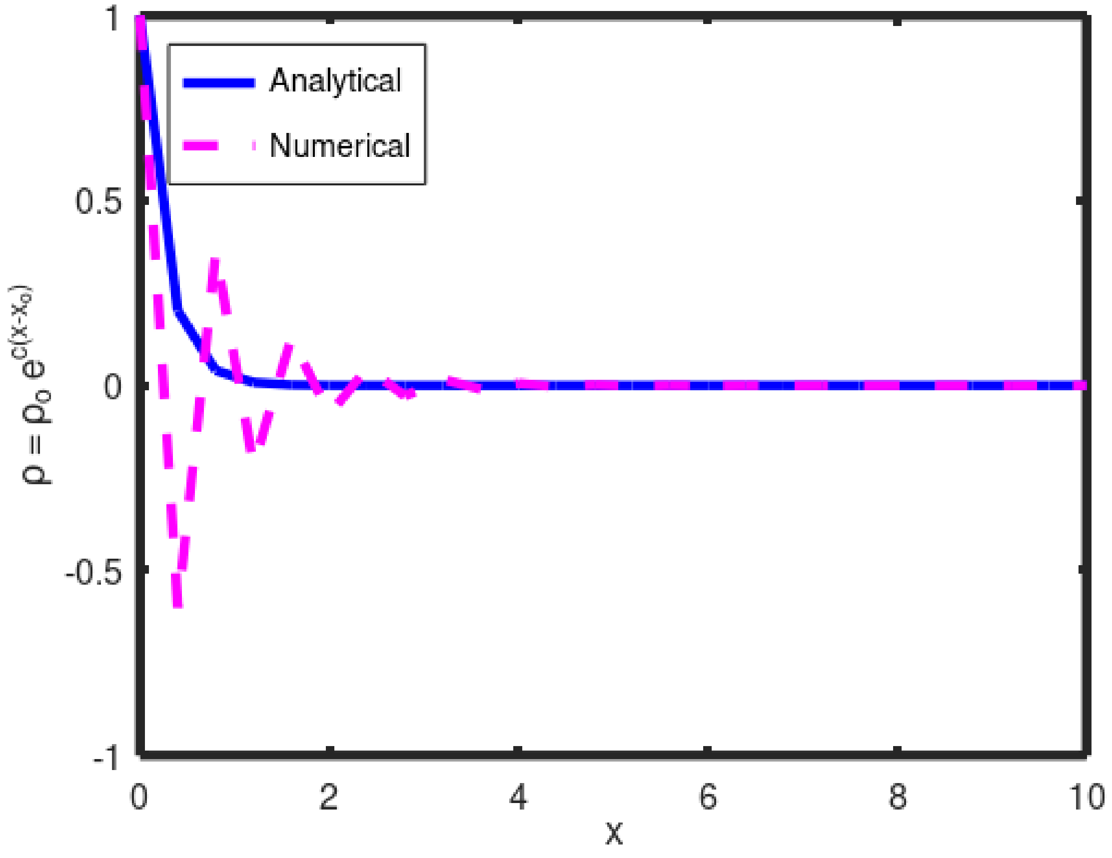
$$\frac{d\rho}{dx} = -c\rho$$

$$\rho_{i+1} = \rho_i(1 - c\Delta x)$$

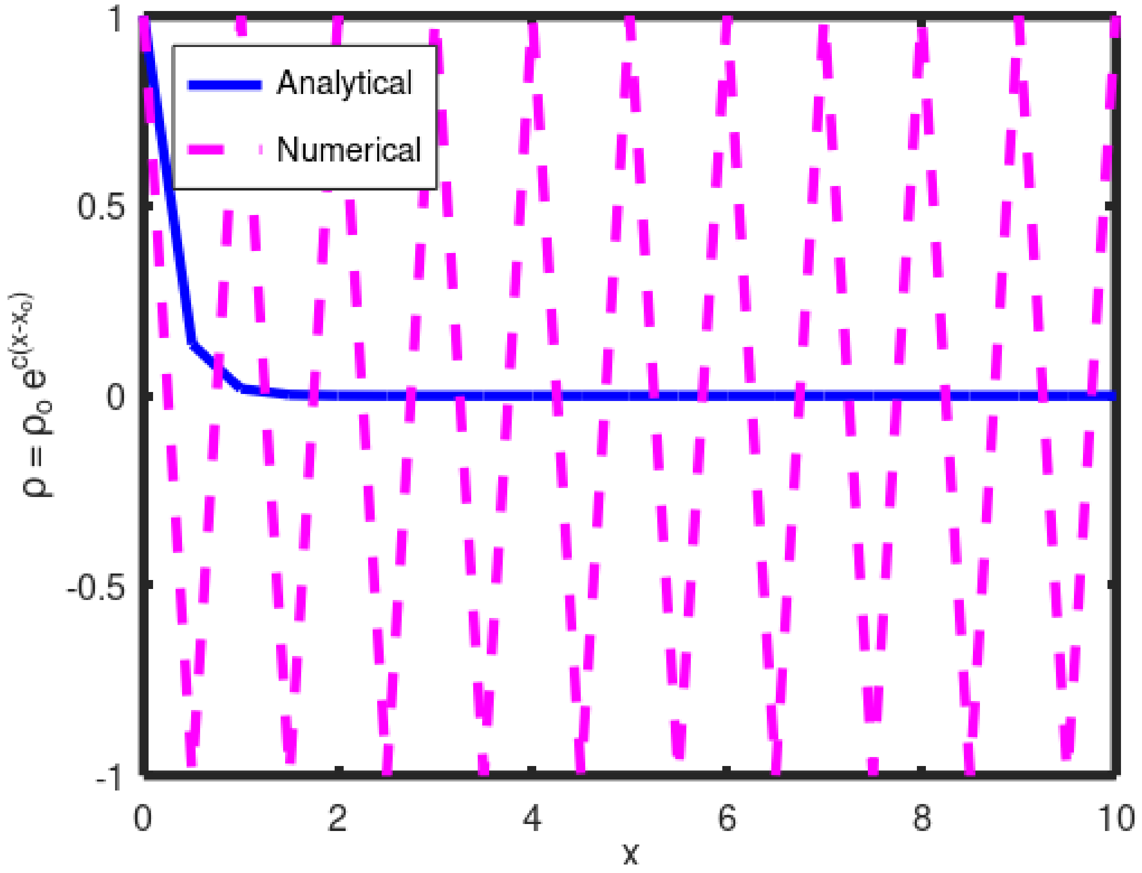
x_{i-1} x_i x_{i+1} x_{i+2}

$$\Delta x < \frac{2}{c}$$

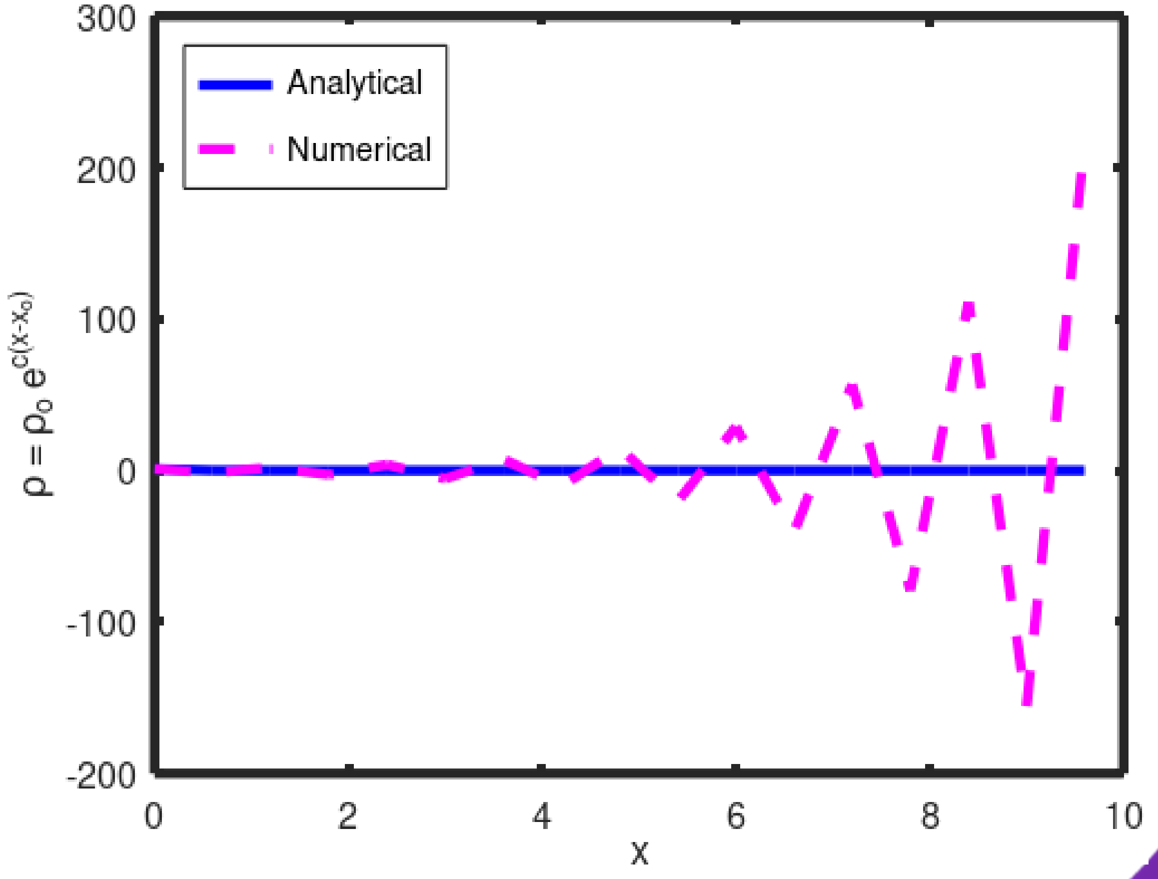
$$\frac{2}{c} = \frac{2}{4} = 0.5$$



$c = 4, x_o = 0, \rho_o = 1, x \in [0,10]$,
$\Delta x = 0.4$



$c = 4, x_o = 0, \rho_o = 1, x \in [0,10]$,
$\Delta x = 0.5$



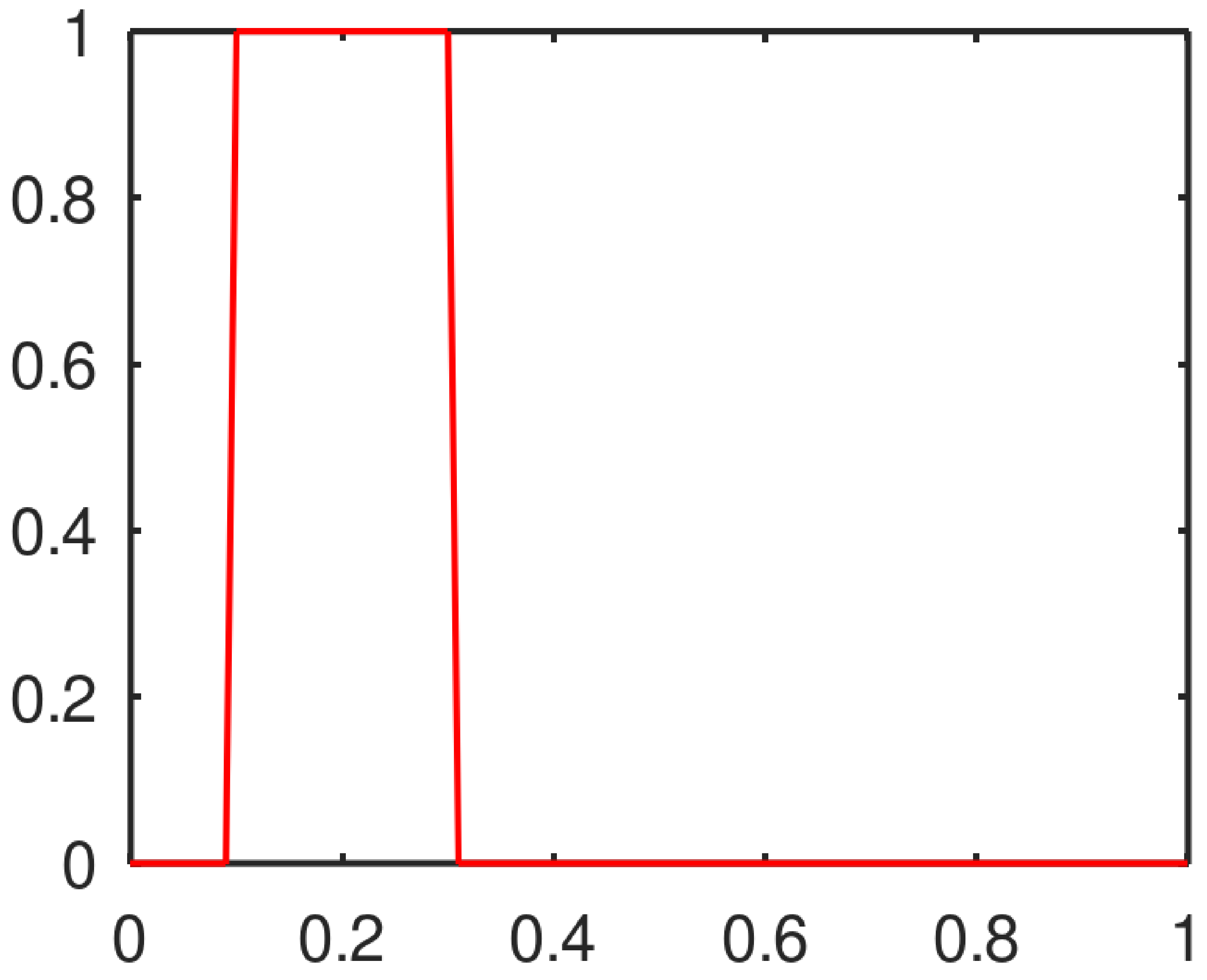
$c = 4, x_o = 0, \rho_o = 1, x \in [0,10]$,
$\Delta x = 0.6$

Kumaresh Selvakumar
kumaresh@exaslate.com

Kumaresh Selvakumar
kumaresh@exaslate.com

ExaSlate
Develop = Guide = Collab

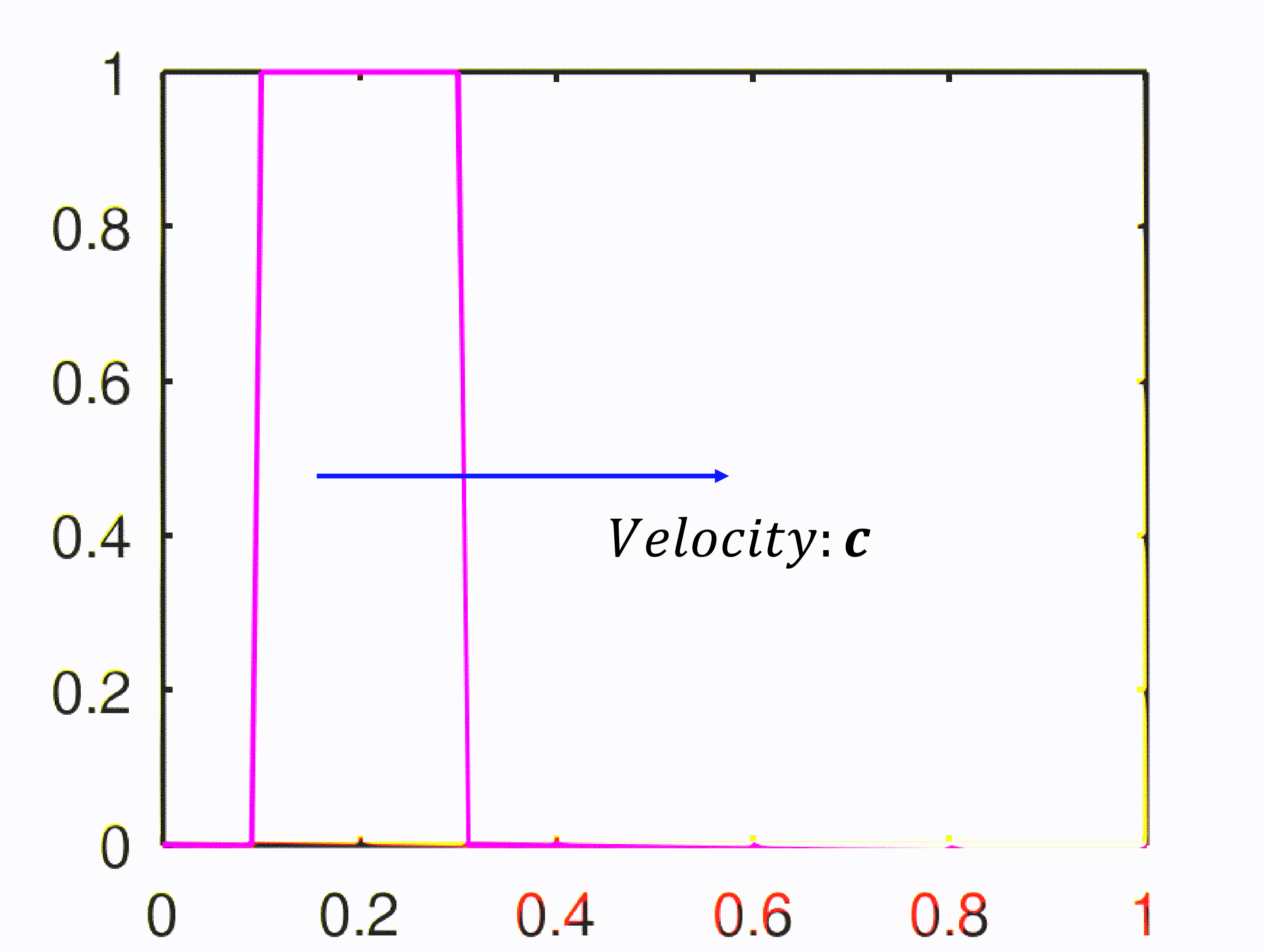
Numerical Stability - Advection equation




```
for i = 1 : length(x)
  if (x(i, 1) >= 0.1) && (x(i, 1) <= 0.3)
    u(i, 1) = 1;
  endif
end
```


Kumaresh Selvakumar
kumaresh@exaslate.com

Numerical Stability - Advection equation



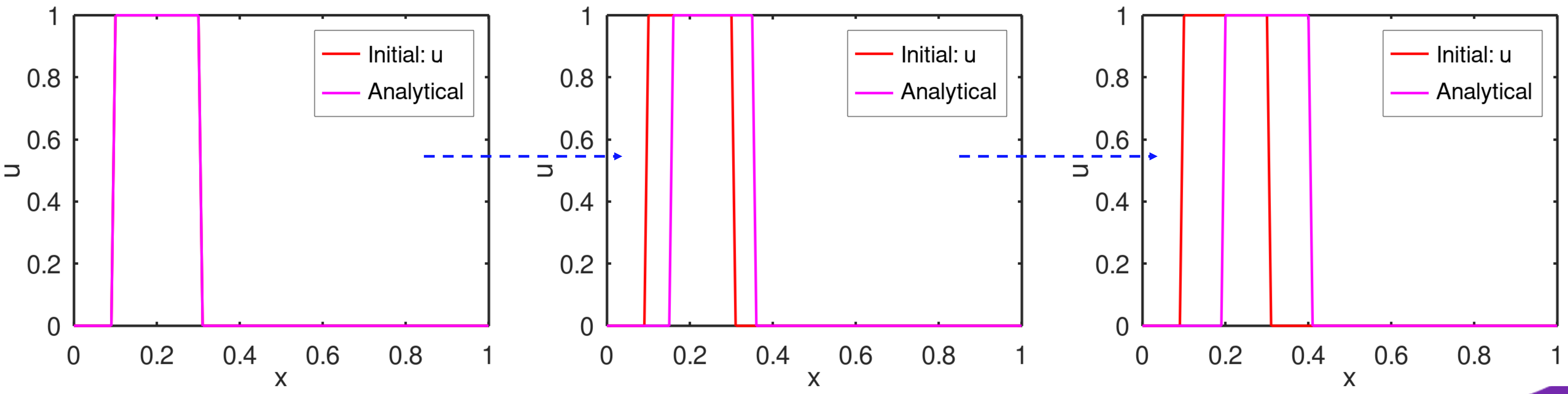
Kumaresh Selvakumar
kumaresh@exaslate.com

ExaSlate
Develop Guide Collab

Numerical Stability - Advection equation

$$\frac{\partial u}{\partial t} + c\frac{\partial u}{\partial x} = 0$$ Advection equation

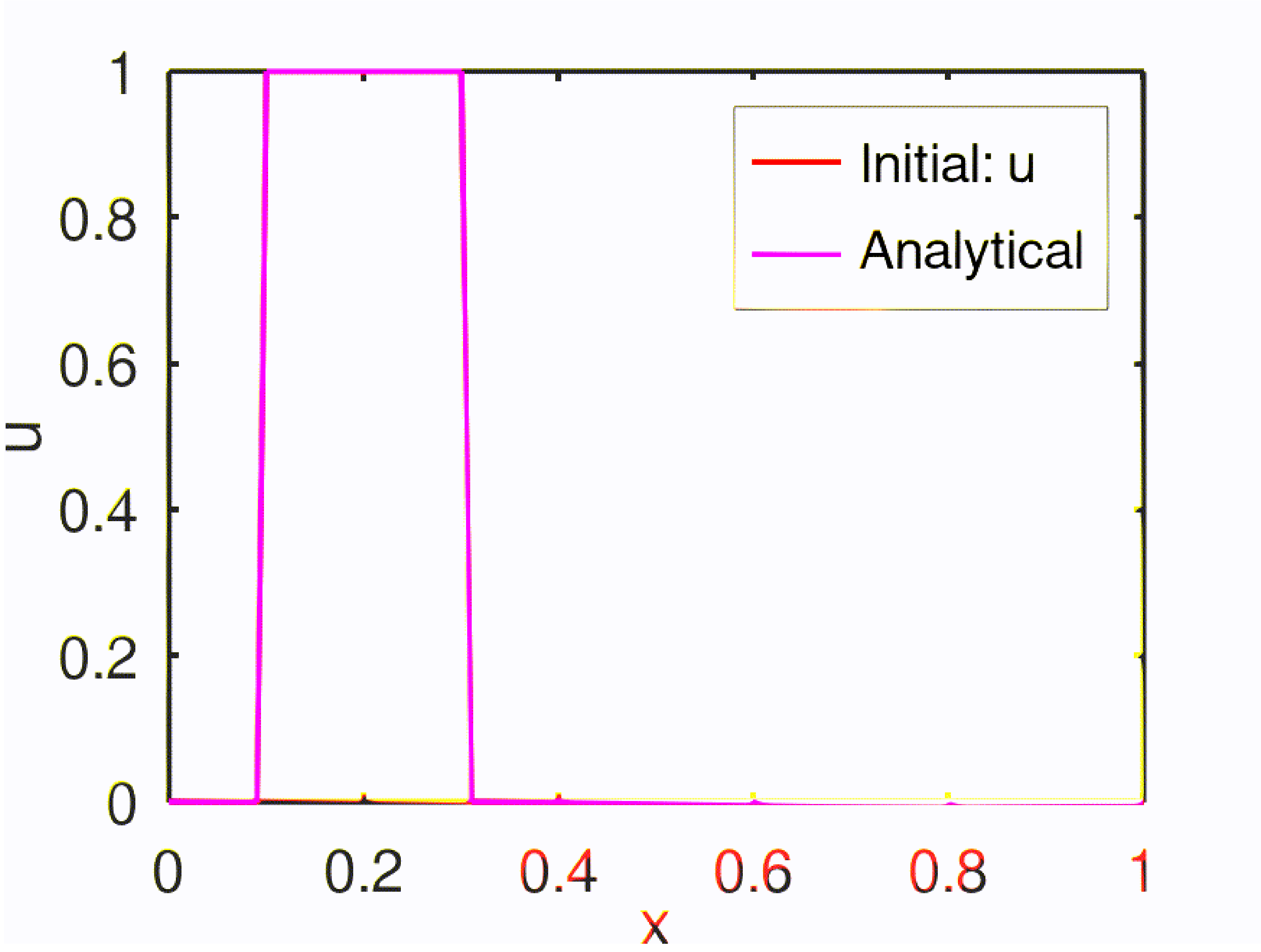
```
for i = 1 : length(x)
  if (x(i, 1) >= 0.1+c*t) && (x(i, 1) <= 0.3+c*t)
    u_analytical(i, 1) = 1;
  endif
end
```



Kumaresh Selvakumar
kumaresh@exaslate.com

ExaSlate
Develop = Guide = Collab

Numerical Stability - Advection equation




```
for i = 1 : length(x)
  if (x(i, 1) >= 0.1+c*t) && (x(i, 1) <= 0.3+c*t)
    u_analytical(i, 1) = 1;
  endif
end
```


Kumaresh Selvakumar
kumaresh@exaslate.com

Exercise – 5 (i)

1. Solve the following advection equation analytically in octave

$$\frac{\partial u}{\partial t} + c\frac{\partial u}{\partial x} = 0$$

2. Upload in GitHub

Kumaresh Selvakumar
kumaresh@exaslate.com

ExaSlate
Develop = Guide = Collab

Numerical Stability - Advection equation

$$\boxed{\frac{\partial u}{\partial t} + c\frac{\partial u}{\partial x} = 0}$$

$$\boxed{\frac{u_i^{n+1} - u_i^n}{\Delta t} + c\left(\frac{\partial u}{\partial x}\right)_i^n = 0}$$

x_{i-1} x_i x_{i+1} x_{i+2}

x_{i-1} x_i x_{i+1} x_{i+2}

$$\left(\frac{d\rho}{dx}\right)_i \approx \frac{\rho(x_{i+1}) - \rho(x_i)}{\Delta x_i} \qquad \left(\frac{d\rho}{dx}\right)_i \approx \frac{\rho(x_{i+1}) - \rho(x_{i-1})}{2\Delta x_i}$$

$$u_i^{n+1} = u_i^n - c\Delta t\left(\frac{\partial u}{\partial x}\right)_i^n$$

$$\left(\frac{\partial u}{\partial x}\right)_i^n \approx \frac{u_{i+1}^n - u_i^n}{\Delta x_i}$$

Simple forward difference scheme

$$\left(\frac{\partial u}{\partial x}\right)_i^n \approx \frac{u_{i+1}^n - u_{i-1}^n}{2\Delta x_i}$$

Central difference

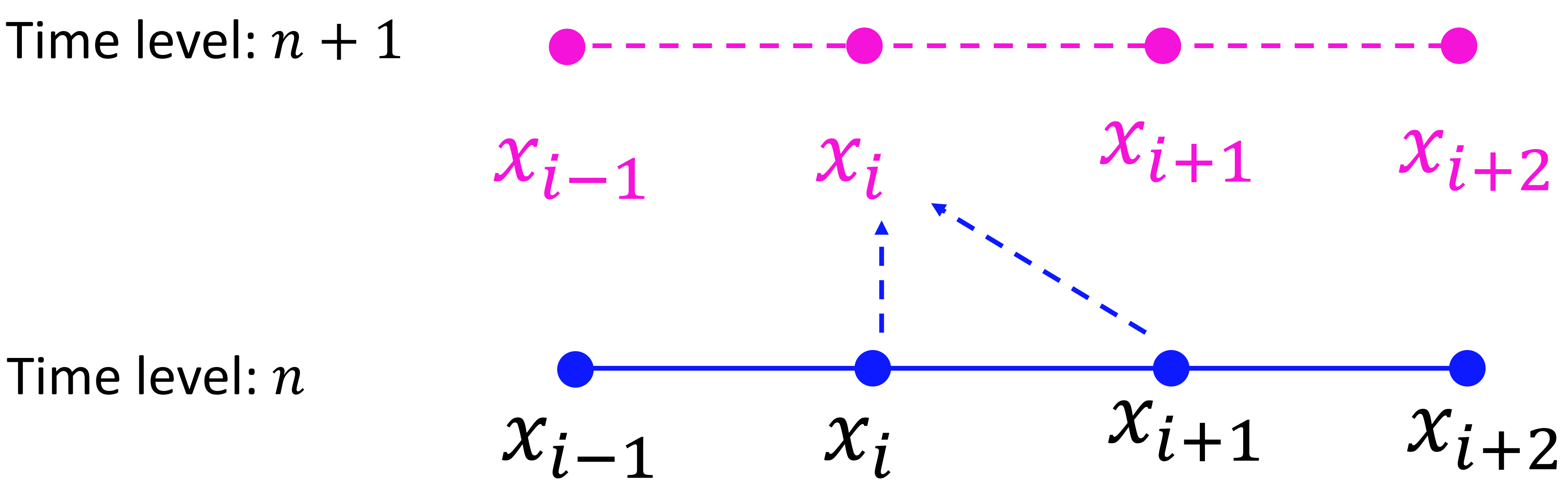
(Explicit) First order - Forward Euler

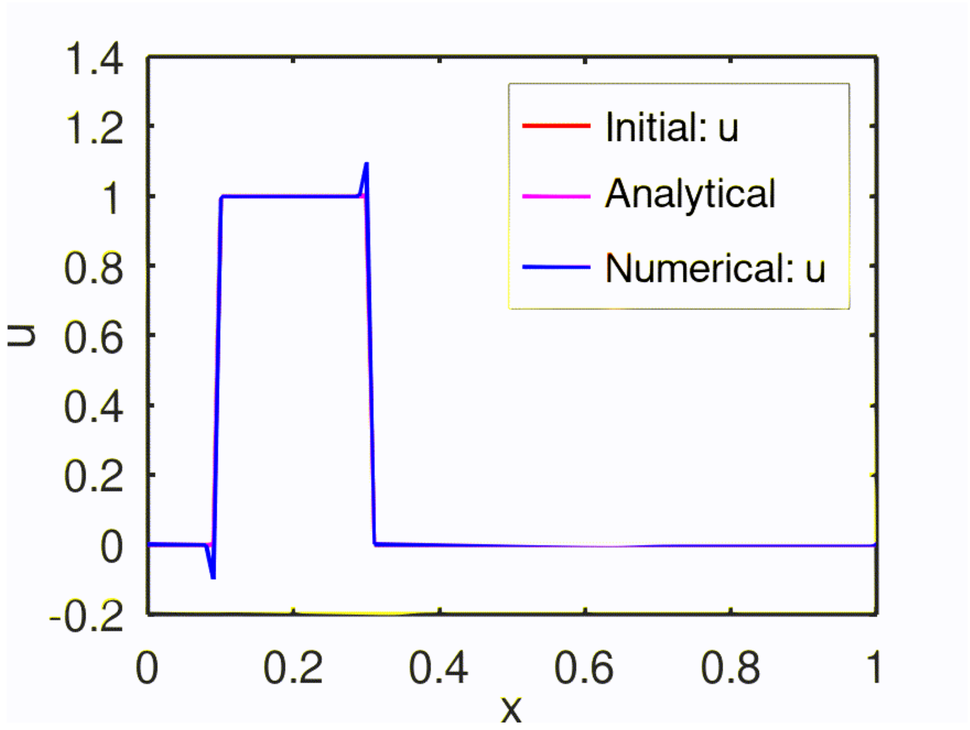
(only one unknown (n+1) with other knowns at n^{th} node)
→ conditionally stable

Kumaresh Selvakumar
kumaresh@exaslate.com

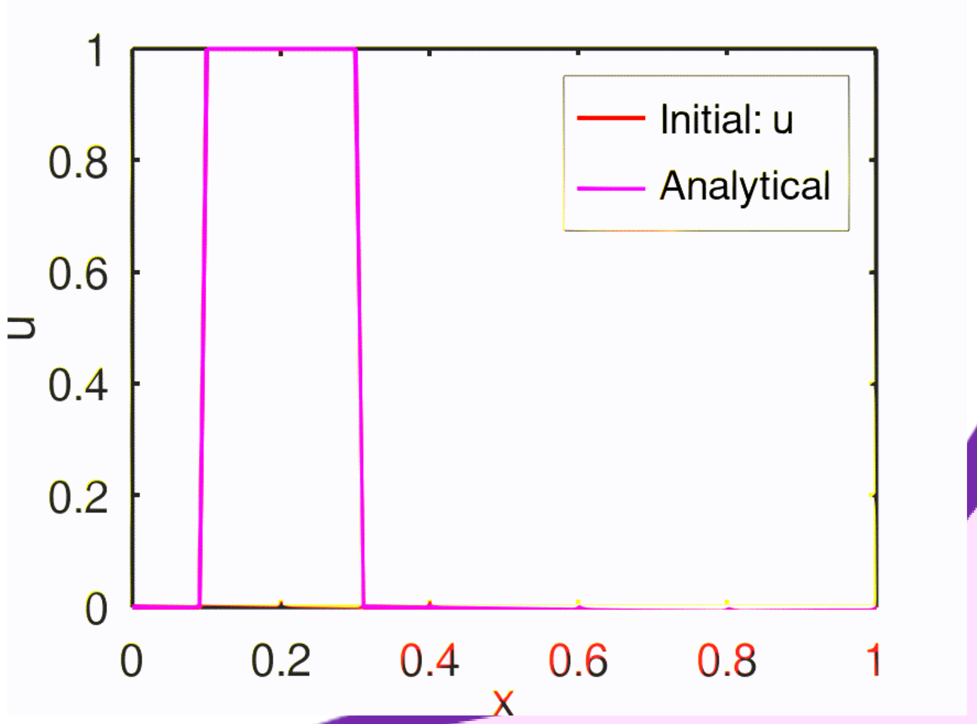
ExaSlate
Develop · Guide · Collab

Numerical Stability - Advection equation





Information from right to left end



$$u_i^{n+1} = u_i^n - c\Delta t \boxed{\left(\frac{\partial u}{\partial x}\right)_i^n}$$

$$\left(\frac{\partial u}{\partial x}\right)_i^n \approx \frac{u_{i+1}^n - u_i^n}{\Delta x_i}$$

Simple forward difference scheme

$$\left(\frac{\partial u}{\partial x}\right)_i^n \approx \frac{u_{i+1}^n - u_{i-1}^n}{2\Delta x_i}$$

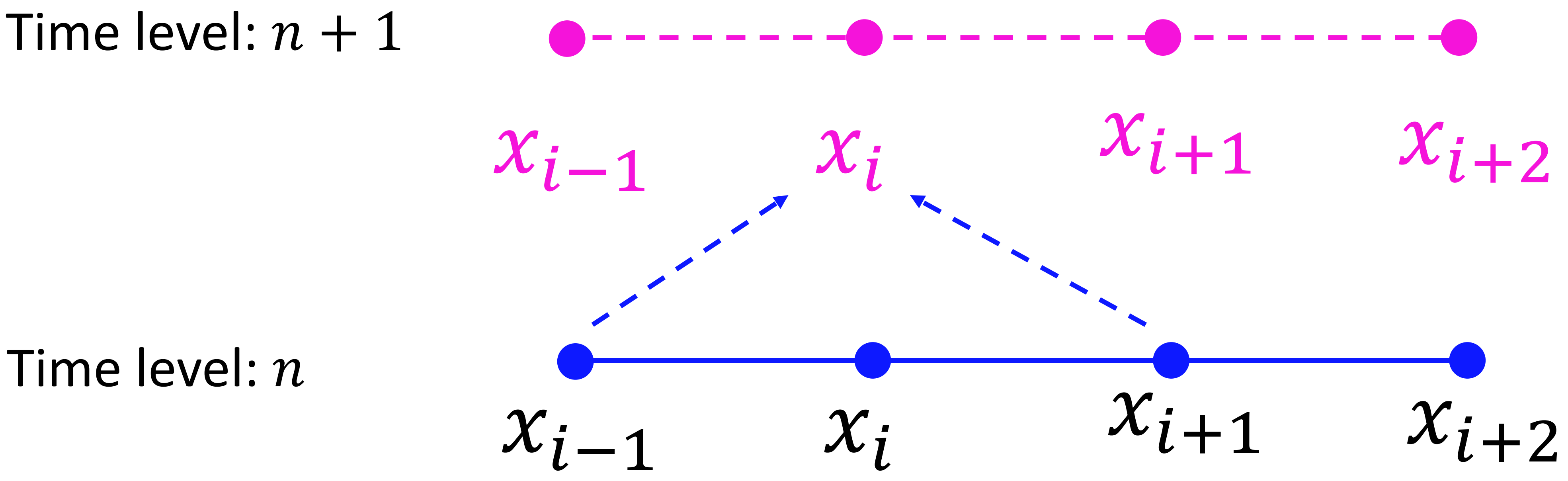
Central difference

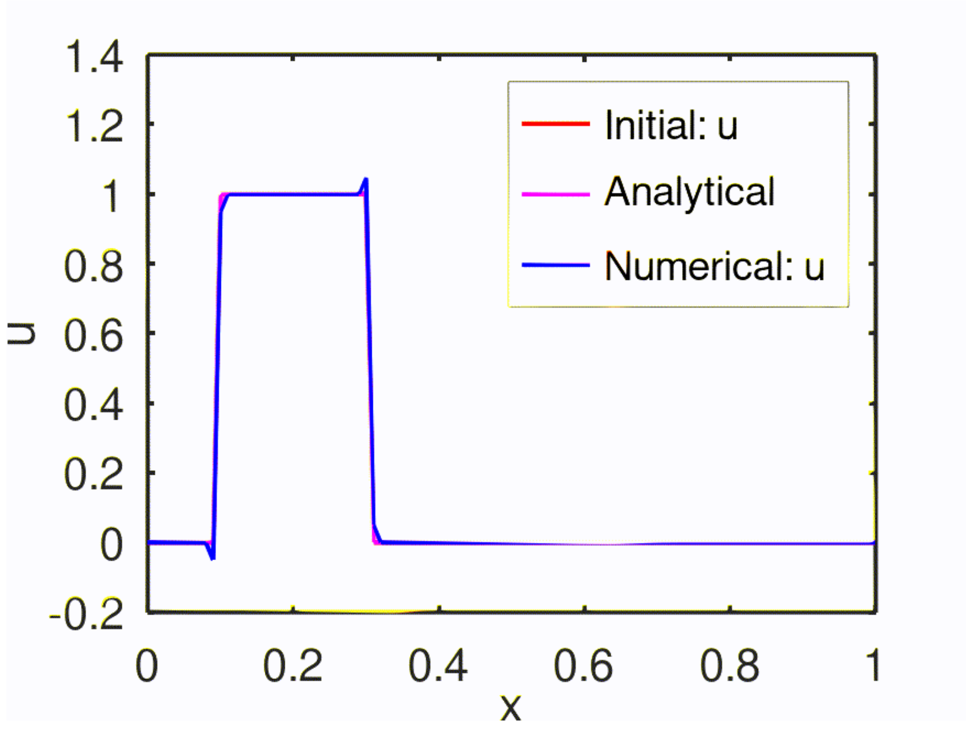
Kumaresh Selvakumar
kumaresh@exaslate.com

ExaSlate
Develop = Guide = Collab

Application: Differential equation used in physics, weather forecasting, stoke markets behaviors (based on probability, past and present information and predicting future information)

Numerical Stability - Advection equation



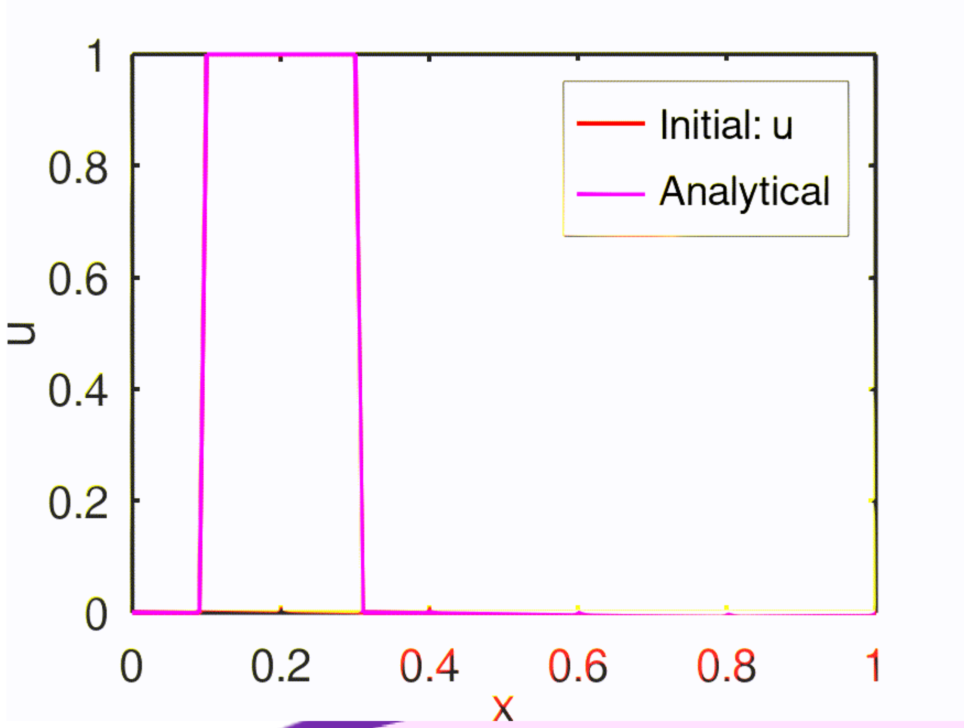


Information from left and right ends

$$u_i^{n+1} = u_i^n - c\Delta t \boxed{\left(\frac{\partial u}{\partial x}\right)_i^n}$$

$$\left(\frac{\partial u}{\partial x}\right)_i^n \approx \frac{u_{i+1}^n - u_i^n}{\Delta x_i}$$ Simple forward difference scheme

$$\left(\frac{\partial u}{\partial x}\right)_i^n \approx \frac{u_{i+1}^n - u_{i-1}^n}{2\Delta x_i}$$ Central difference

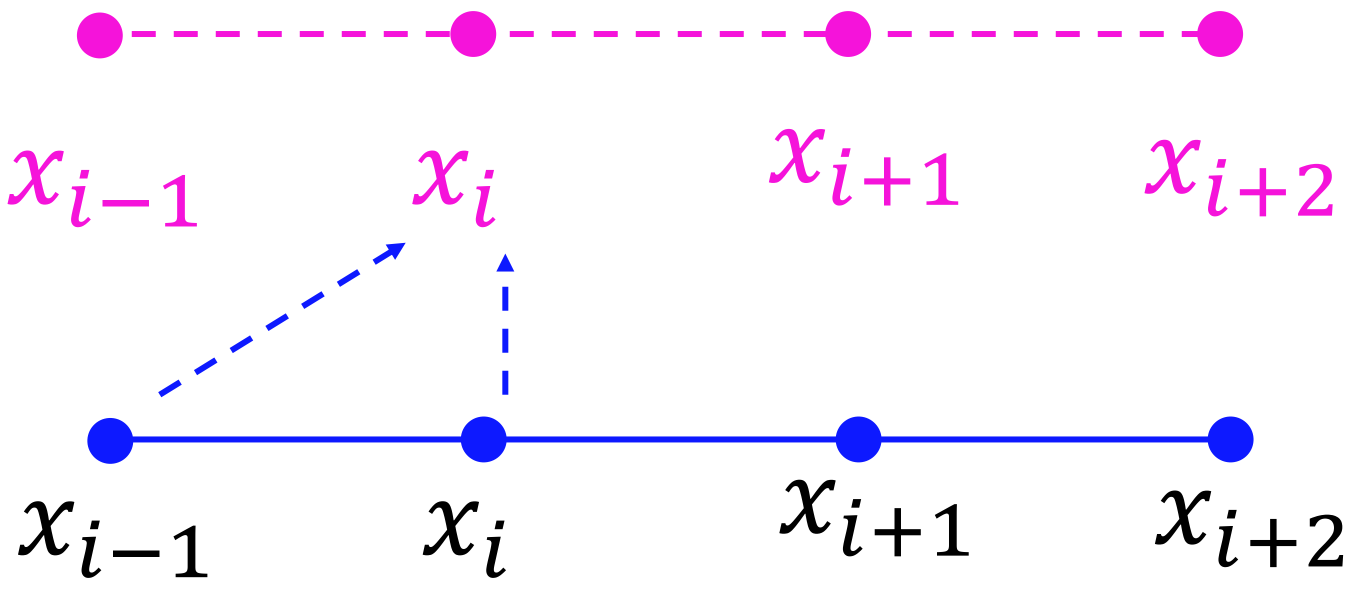


Kumaresh Selvakumar
kumaresh@exaslate.com

Numerical Stability - Advection equation

Time level: $n+1$

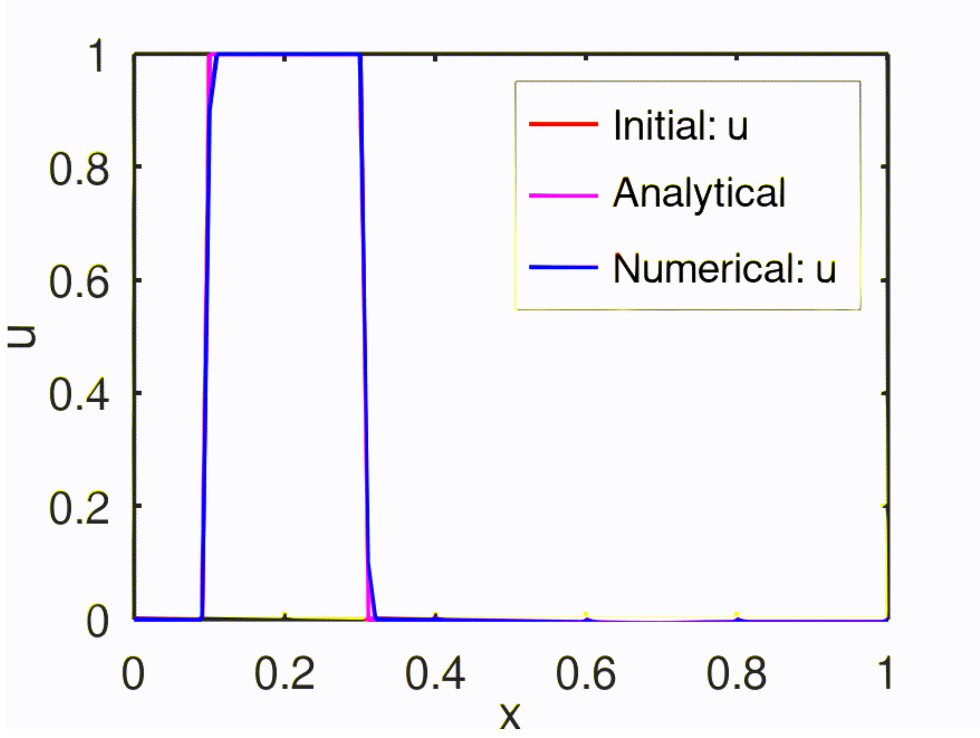
Time level: n

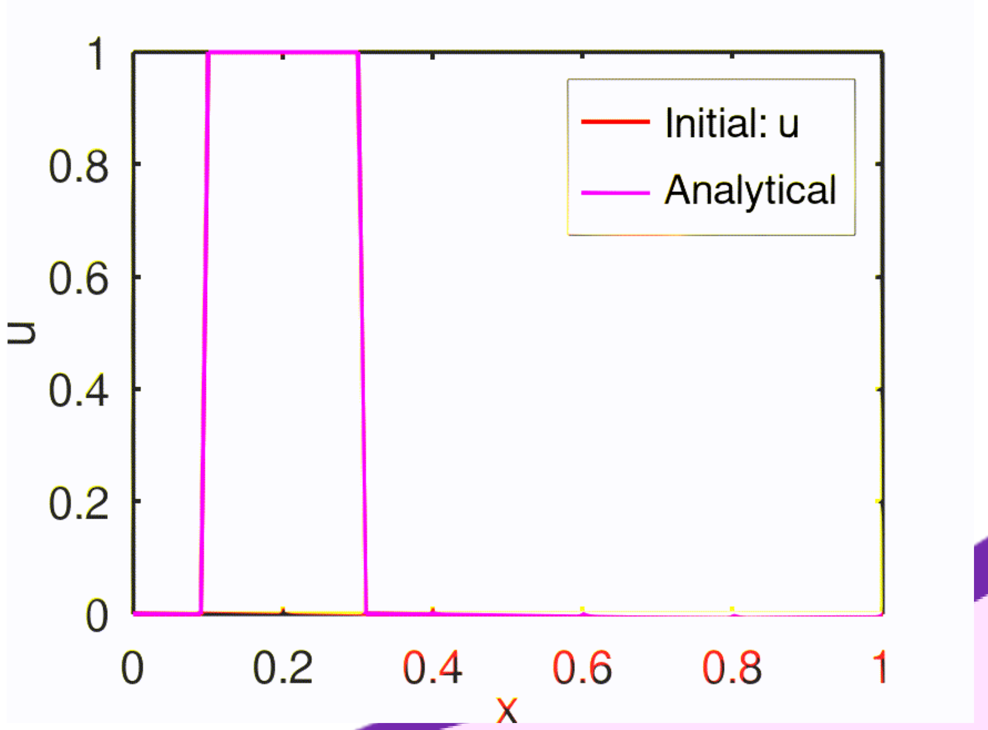


Information from left to right end
Wind is flowing from left end (bird moves from left to right)

$$u_i^{n+1} = u_i^n - c\Delta t \boxed{\left(\frac{\partial u}{\partial x}\right)_i^n} \longrightarrow \left(\frac{\partial u}{\partial x}\right)_i^n \approx \frac{u_i^n - u_{i-1}^n}{\Delta x_i}$$

Simple backward difference scheme





Kumaresh Selvakumar
kumaresh@exaslate.com

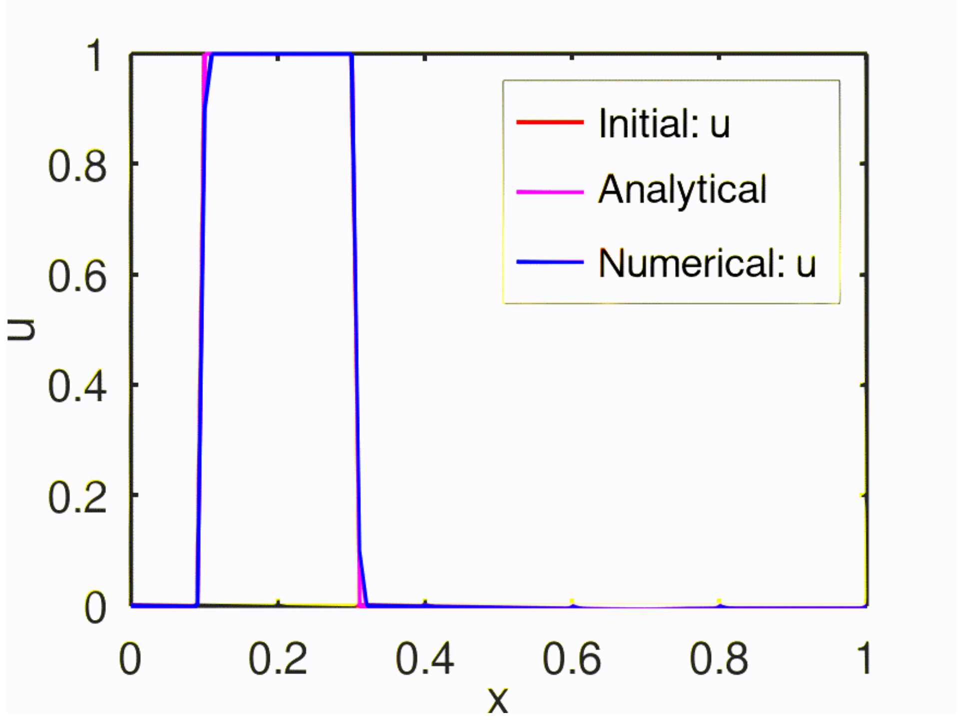
Numerical Stability - Advection equation

$$\boxed{\frac{\partial u}{\partial t} + c\frac{\partial u}{\partial x} = 0}$$

$$u_i^{n+1} = u_i^n - c\Delta t\left(\frac{\partial u}{\partial x}\right)_i^n \longrightarrow \boxed{\left(\frac{\partial u}{\partial x}\right)_i^n \approx \frac{u_i^n - u_{i-1}^n}{\Delta x_i}}$$

Simple backward difference scheme

Upwind scheme



Information from left to right end

Wind is flowing from left end (Flowing towards the source of the wind)

$$u_i^{n+1} = u_i^n - c\Delta t\left(\frac{\partial u}{\partial x}\right)_i^n \longrightarrow \left(\frac{\partial u}{\partial x}\right)_i^n \approx \frac{u_{i+1}^n - u_i^n}{\Delta x_i}$$

Simple forward difference scheme

Downwind scheme

Wind is flowing from right end (Flowing away from the source of wind)

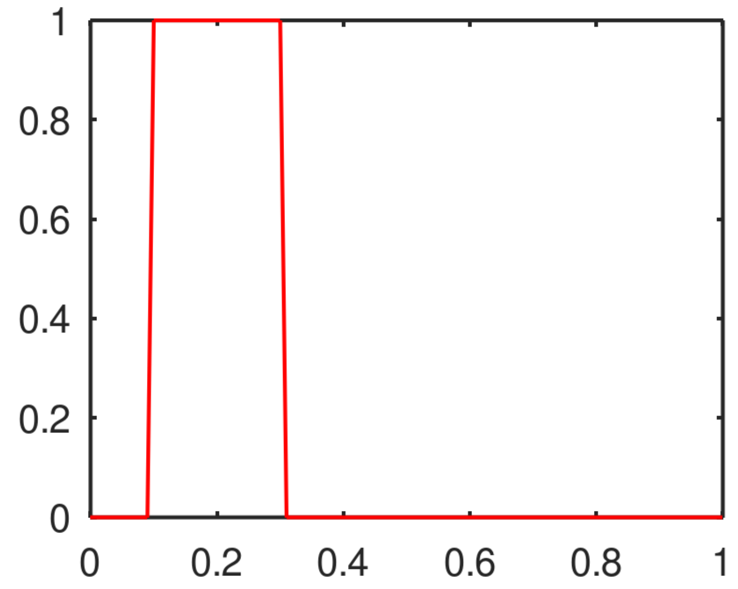
CFL = 0.1 $\quad CFL: \dfrac{c\Delta t}{\Delta x}$

Courant - Friedrichs - Lewy Number

Kumaresh Selvakumar
kumaresh@exaslate.com

ExaSlate
Develop = Guide = Collab

Numerical Stability - Advection equation

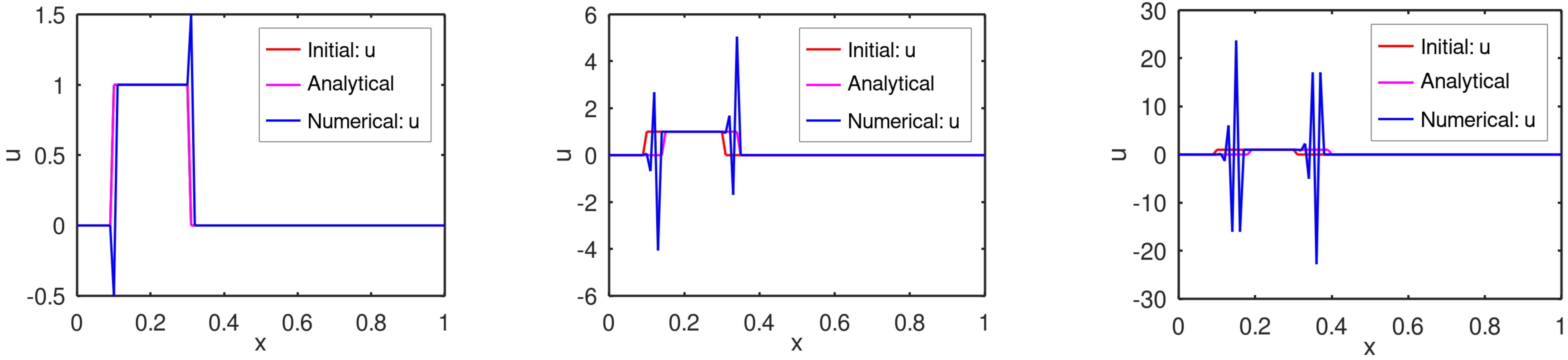


$$u_i^{n+1} = u_i^n - c\Delta t \left(\frac{\partial u}{\partial x}\right)_i^n \longrightarrow \left(\frac{\partial u}{\partial x}\right)_i^n \approx \frac{u_i^n - u_{i-1}^n}{\Delta x_i}$$

Simple backward difference scheme

Upwind scheme

$$CFL: \frac{c\Delta t}{\Delta x}$$

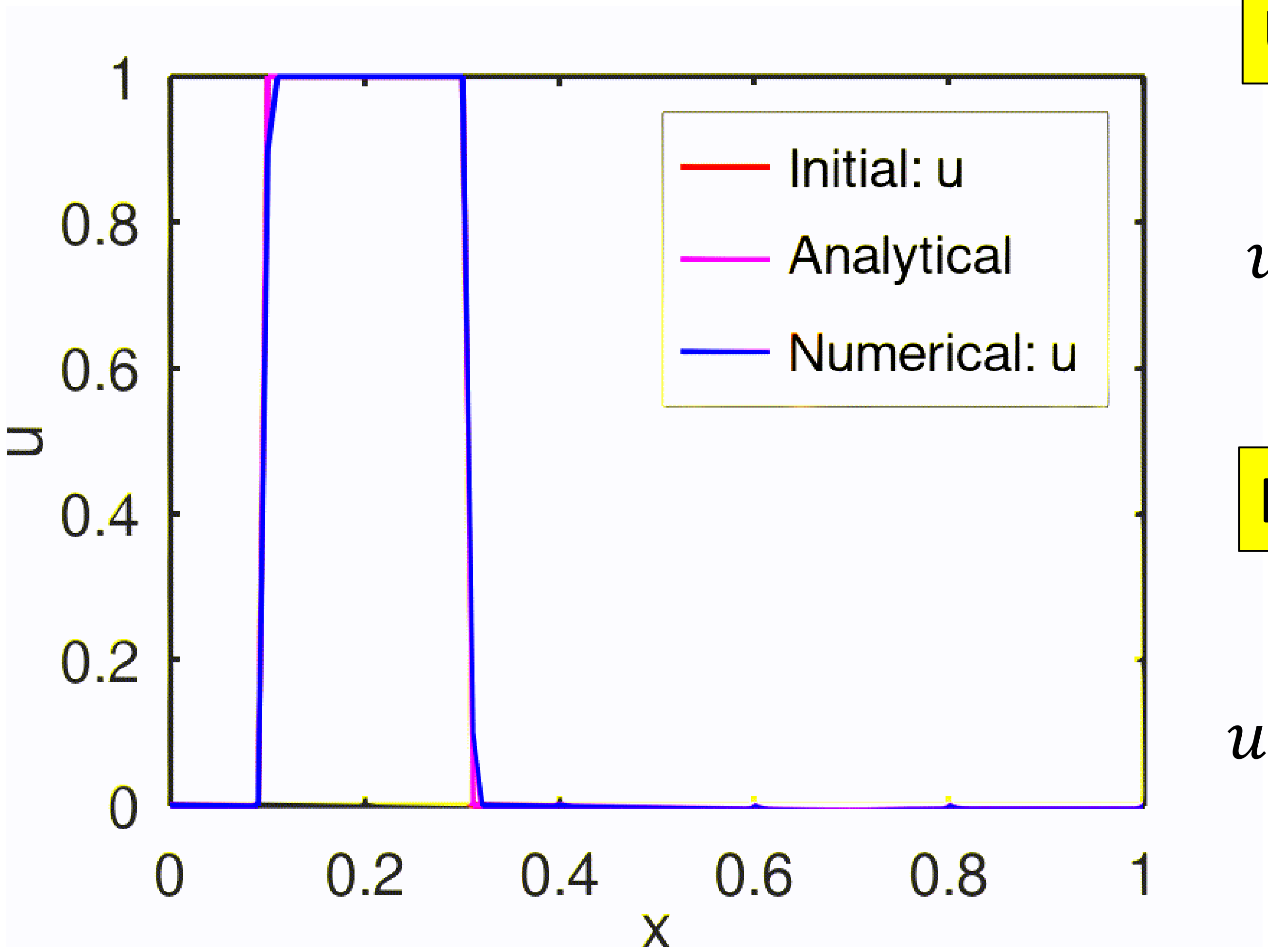


CFL = 1.5 Δt **increase**

Kumaresh Selvakumar
kumaresh@exaslate.com

What did we discuss ?

- Proper discrete approximations need to be chosen based on the velocity field.
- CFL number is critical to ensure numerical stability.



Upwind scheme

$$u_i^{n+1} = u_i^n - c\Delta t \boxed{\left(\frac{\partial u}{\partial x}\right)_i^n} \longrightarrow \left(\frac{\partial u}{\partial x}\right)_i^n \approx \frac{u_i^n - u_{i-1}^n}{\Delta x_i}$$

Simple backward difference scheme

Downwind scheme

$$u_i^{n+1} = u_i^n - c\Delta t \boxed{\left(\frac{\partial u}{\partial x}\right)_i^n} \longrightarrow \left(\frac{\partial u}{\partial x}\right)_i^n \approx \frac{u_{i+1}^n - u_i^n}{\Delta x_i}$$

Simple forward difference scheme

Kumaresh Selvakumar
kumaresh@exaslate.com

ExaSlate
Develop - Guide - Collab

Exercise – 5 (ii)

1. Solve the following advection equation numerically in octave

$$\frac{\partial u}{\partial t} + c\frac{\partial u}{\partial x} = 0$$

   a) Central difference with CFL = 0.1 (dx = 0.01, c = 0.01, dt = 0.1, t_final = 5)

2. Upwind scheme (backward difference) with CFL = 0.1 (dx = 0.01, dt = 0.1, t_final = 5). Change the "c" value between 0.01 and -0.01 and analyze the stability. Hint: Upwind scheme with c = -0.01 becomes unstable and act as downwind.

3. Downwind scheme (forward difference) with CFL = 0.1 (dx = 0.01, dt = 0.1, t_final = 5). Change the "c" value between 0.01 and -0.01 and analyze the stability. Hint: Downwind scheme with c = -0.01 becomes stable and act as upwind.

4. Examine CFL numbers. Analyse the upwind scheme with CFL = 0.1, 1.0, and 10. Analyze the stability. (dx = 0.01, c = 0.01, dt = 0.1, t_final = 5)

5. Upload in GitHub.

Kumaresh Selvakumar
kumaresh@exaslate.com

THANK YOU