1 Tensor Basics

1.1 Definitions

TODO

1.2 Tensor Manipulations and Operations

TODO

2 Tensor Derivatives

let T be a contravariant tensor, then:

$\bar{T}^i = T^r \frac{\partial \bar{x}^i}{\partial x^r}$, where $T = T^i(x(t))$

Take derivative wrt. t:

$\frac{\partial \bar{T}^i}{\partial t} = \frac{\partial T^r}{\partial t} \frac{\partial \bar{x}^i}{\partial x^r} + T^r \frac{\partial^2 \bar{x}^i}{\partial x^s \partial x^r} \frac{\partial x^s}{\partial t}$ (not a general tensor)

$\bar{x}^i$ being a linear functions of $x^r \implies \frac{\partial \bar{T}^i}{\partial t} = \frac{\partial T^r}{\partial t} \frac{\partial \bar{x}^i}{\partial x^r}$ (it's a general tensor)

Goal: Would like the curvature of a curve to be an intrinsic concept, independent of coordinate systems. Use Christoffel symbols to solve it.

2.1 Christoffel Symbol of the 1st Kind

$\Gamma_{ijk} = \frac{1}{2} \left(-\frac{\partial(g_{ij})}{\partial x^k} + \frac{\partial(g_{jk})}{\partial x^i} + \frac{\partial(g_{ki})}{\partial x^j} \right)$

Let $g_{ijk} = \frac{\partial(g_{ij})}{\partial(x^k)}$

$\Gamma_{ijk} = \frac{1}{2}(-g_{ijk} + g_{jki} + g_{kij})$

Cyclic permute and add:

$\Gamma_{ijk} + \Gamma_{jki} = g_{kij}$

Driving transformation law for Γ_{ijk}:
By using g_{ij} derivative and its cyclic permutatations,
$\bar{\Gamma}_{ijk} = \frac{1}{2}(-\bar{g}_{ijk} + \bar{g}_{jki} + \bar{g}_{kij})$

where:

$\bar{g}_{ijk} = \frac{\partial}{\partial \bar{x}^k} \left(g_{rs} \frac{\partial x^r}{\partial \bar{x}^i} \frac{\partial x^s}{\partial \bar{x}^j} \right)$ (expand this)

Rearrange and simplify to:

$\bar{\Gamma}_{ijk} = \Gamma_{rst} \frac{\partial x^r}{\partial \bar{x}^i} \frac{\partial x^s}{\partial \bar{x}^j} \frac{\partial x^t}{\partial \bar{x}^k} + g_{rs} \frac{\partial^2 x^r}{\partial \bar{x}^i \partial \bar{x}^j} \frac{\partial x^s}{\partial \bar{x}^k}$ (not a general tensor)

2.2 Christoffel Symbol of the 2nd Kind

TODO

2.3 Covariant Derivative of Covariant Vector

2.4 Covariant Derivative of Contravariant Vector

TODO

2.5 Absolute Differentiation Along Curve

TODO