Strong scaling for Poisson Equation in 3D

1 Aims:

2 Experiment:

Consider Poisson's equation in a 3D domain $\Omega = [0,1]^3 \subset \mathbb{R}^3$ with homogeneous Dirichlet boundary conditions:

$$\begin{cases} -\nabla^2 u = f & \text{on } \Omega \\ u = 0 & \text{on } \partial\Omega \end{cases} \tag{1}$$

With a manufactured solution

$$u(x,y,z) = \sin(\pi x)\tan\left(\frac{\pi x}{4}\right)\sin(a\pi y)\sin(b\pi z) \tag{2}$$

corresponding to a right-hand side

$$f(x,y,z) = \frac{\pi^2}{2}\left(2\cos(\pi x) - \cos\left(\frac{\pi x}{2}\right) - 2(a^2+b^2)\sin(\pi x)\tan\left(\frac{\pi x}{4}\right)\right)\sin(a\pi y)\sin(b\pi z). \tag{3}$$

For these experiments we fix $a = 1$ and $b = 2$.

3 Setup:

```
mpiexec -N 32 /tmp/firedrake/firedrake/bin/python poisson_gmg.py --resultsdir
    results/poisson_telescope_new --baseN 12 --nref 3 --solver_params "MG F-cycle PatchPC
    telescope" --telescope_factor 1 -log_view
    :results/poisson_telescope_new/32_logview.py:ascii_info_detail
```

Listing 1: Run configuration (on one node)

Solver options for 1 node shown.
Total DOFs: 24137569

```
{'ksp_type': 'preonly',
 'mg_coarse_assembled': {'mat_type': 'aij',
                         'pc_telescope_reduction_factor': 1,
                         'pc_telescope_subcomm_type': 'contiguous',
                         'pc_type': 'telescope',
                         'telescope_pc_factor_mat_solver_type': 'mumps',
                         'telescope_pc_type': 'lu'},
 'mg_coarse_pc_python_type': 'firedrake.AssembledPC',
 'mg_coarse_pc_type': 'python',
 'mg_levels': {'ksp_convergence_test': 'skip',
               'ksp_max_it': 2,
               'ksp_norm_type': 'unpreconditioned',
               'ksp_type': 'chebyshev',
               'patch_pc_patch_construct_dim': 0,
               'patch_pc_patch_construct_type': 'star',
               'patch_pc_patch_dense_inverse': True,
               'patch_pc_patch_partition_of_unity': False,
               'patch_pc_patch_precompute_element_tensors': True,
               'patch_pc_patch_save_operators': True,
               'pc_python_type': 'firedrake.PatchPC',
               'pc_type': 'python'},
 'pc_mg_log': None,
 'pc_mg_type': 'full',
 'pc_type': 'mg',
 'snes_view': None}
```

Listing 2: Solver options

4 Raw results:

call nprocs	KSPSolve	MatMult	PCApply	PCSetUp	ParLoopExecute
32	93.07	0.8136	16.49	89.19	24.99
64	49.53	0.4347	9.503	46.93	14.56
128	33.71	0.2135	10.53	27.58	7.508
256	32.54	0.3413	6.738	28.38	4.164

Table 1: Multiple Node Scaling for mmsldc3d, MG F-cycle PatchPC telescope

call nprocs	KSPSolve	MatMult	MatSolve	PCApply	ParLoopExecute
32	3.178	0.9129	0.05638	2.599	0.462
64	2.123	0.5572	0.04886	1.825	0.2724
128	5.802	2.222	0.05461	5.617	0.1423
256	3.896	2.157	0.07033	3.666	0.1189

Table 2: Multiple Node Scaling for mmsldc3d, MG Apply Calls

call nprocs	MGSetup Level 0	MGSetup Level 1	MGSetup Level 2	MGSetup Level 3
32	0.05613	0.9761	6.393	34.39
64	0.04358	0.5761	3.259	17.58
128	1.527	1.495	1.824	9.217
256	0.1122	2.376	13.53	4.843

Table 3: Multiple Node Scaling for mmsldc3d, MGSetup

call nprocs	MGSmooth Level 0	MGSmooth Level 1	MGSmooth Level 2	MGSmooth Level 3
32	1.526	0.06913	0.3301	1.256
64	1.293	0.03912	0.1764	0.6224
128	3.464	0.03291	0.1224	2.187
256	1.554	0.2029	0.2023	1.959

Table 4: Multiple Node Scaling for mmsldc3d, MGSmooth

call nprocs	MGInterp Level 1	MGInterp Level 2	MGInterp Level 3
32	0.1942	0.07971	0.3801
64	0.2083	0.06865	0.219
128	0.1974	0.0262	0.1044
256	0.2284	0.042	0.07605

Table 5: Multiple Node Scaling for mmsldc3d, MGInterp

call nprocs	MGResid Level 1	MGResid Level 2	MGResid Level 3
32	0.003335	0.01794	0.07211
64	0.00205	0.008525	0.037
128	0.001918	0.004606	0.01939
256	0.02017	0.01235	0.01253

Table 6: Multiple Node Scaling for mmsldc3d, MGResid

5 Plots:

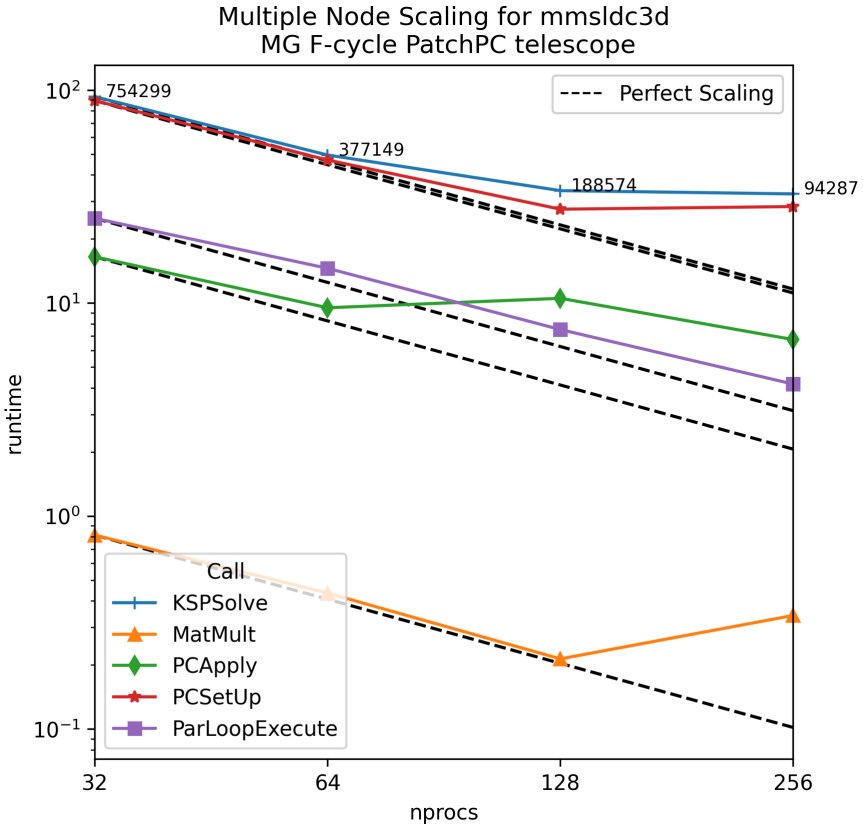

Figure 1:

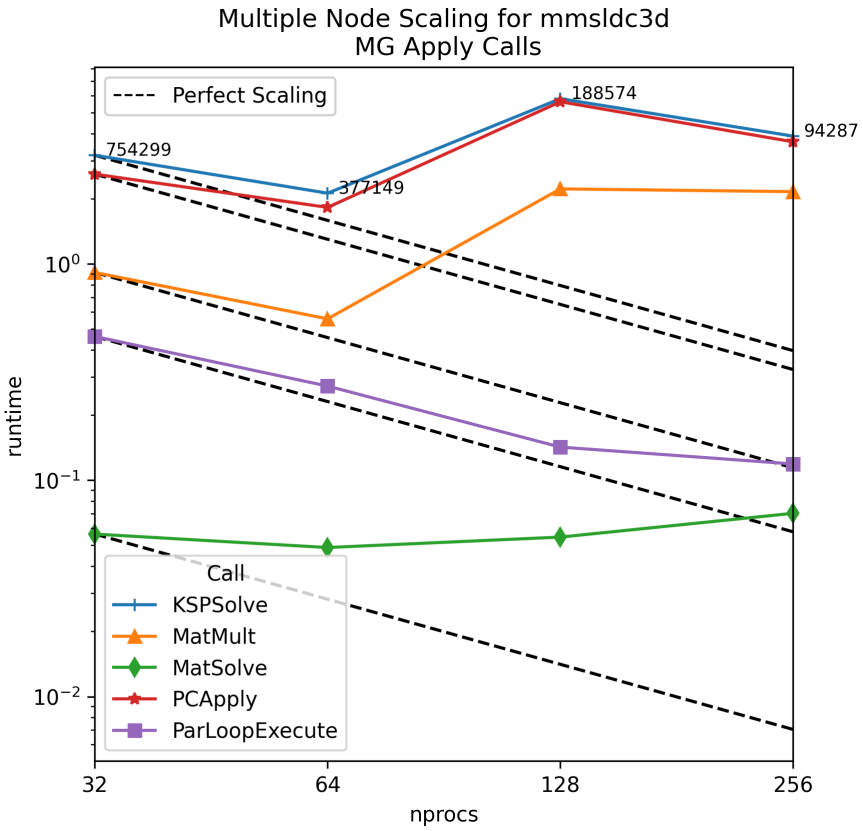

Figure 2:

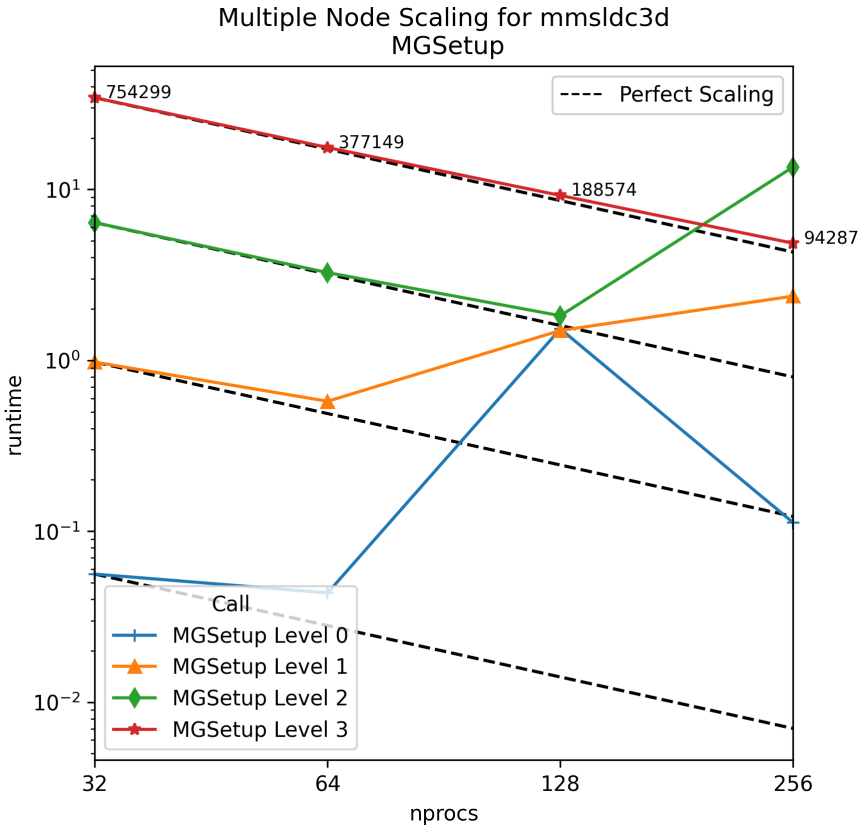

Figure 3:

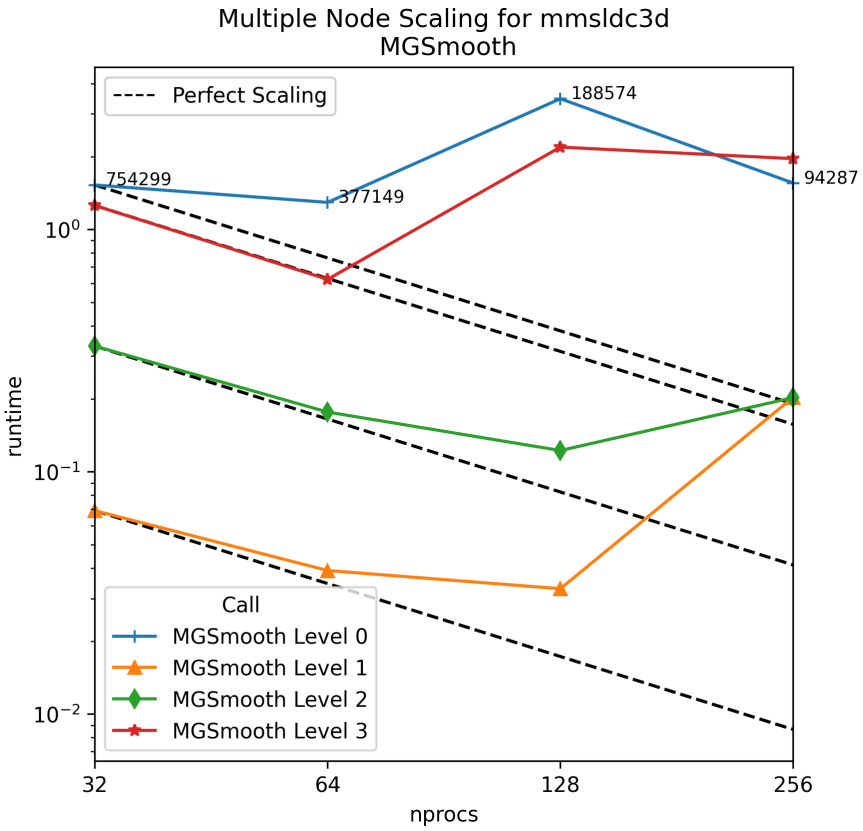

Figure 4:

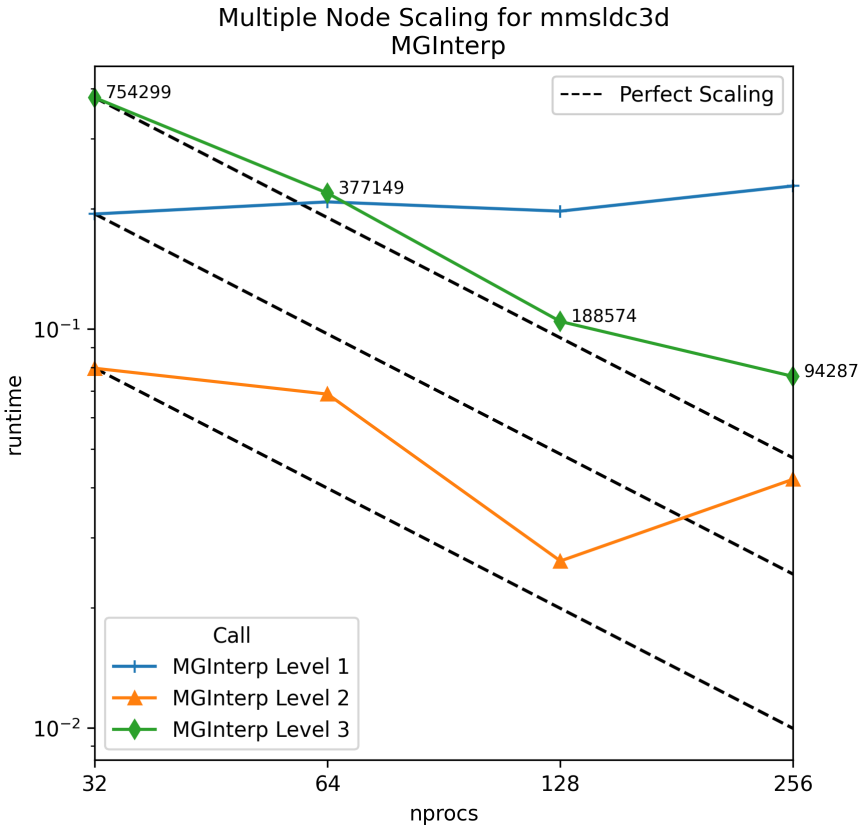

Figure 5:

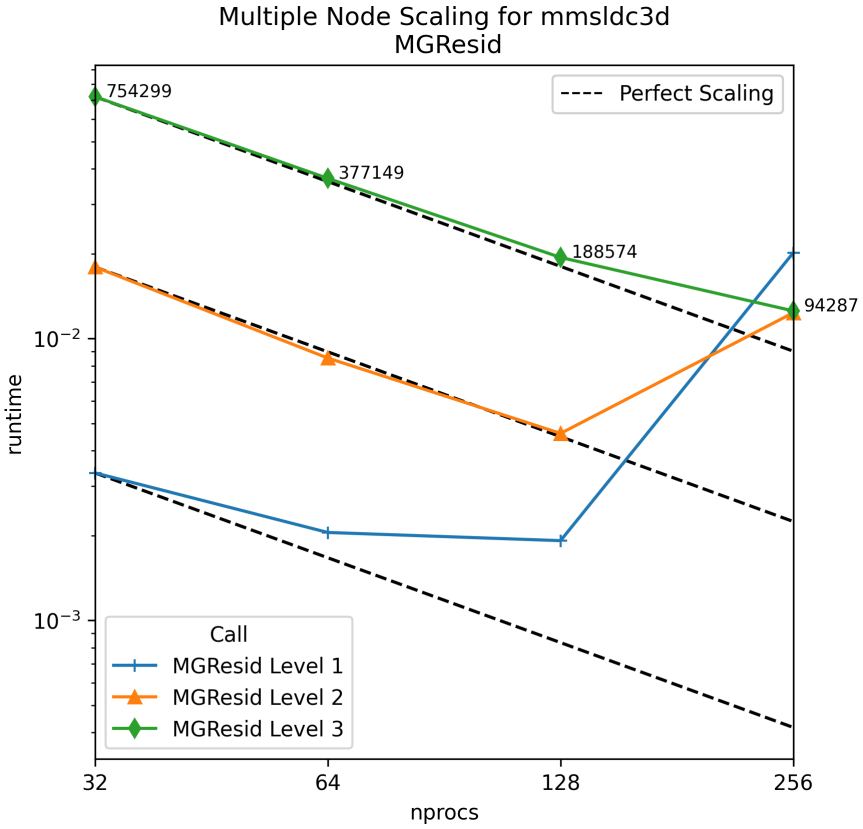
Multiple Node Scaling for mmsldc3d
MGResid
Perfect Scaling
754299
377149
188574
94287
10^{-2}
10^{-3}
runtime
Call
MGResid Level 1
MGResid Level 2
MGResid Level 3
32
64
128
256
nprocs

Figure 6:

6 Conclusions and Next steps:

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