Assignment -3 Part -1

Roll Number used = 2019101006

Value of x = $1 - 0.17 = 0.83$
Value of y = 3

Table y tells

$P(\text{Observation} = \text{Red} \mid \text{state} = \text{Red}) = 0.85$

$P(\text{Observation} = \text{Green} \mid \text{state} = \text{Red}) = 0.15$

$P(\text{observation} = \text{Green} \mid \text{state} = \text{Green}) = 0.9$

$P(\text{observation} = \text{Red} \mid \text{State} = \text{Green}) = 0.1$

The formula used

$$b_i(s) = P(O \mid s = c)\left[\sum_{s'} T(s, a, s')\, b_{i-1}(s')\right]$$

$i \rightarrow$ iteration number.
$s \rightarrow$ state for which we are calculating
$T(s, a, s') \rightarrow$ probability to move s' from s via action a
$P(O \mid s = c) \rightarrow$ Probability the observed color is O given the state color as c.

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Iteration 1

$\begin{Bmatrix} \text{Action is right} \\ \text{color observed is green} \end{Bmatrix}$

$$b_0 = \left[\underset{s_1}{\tfrac{1}{3}},\ \underset{s_2}{0},\ \underset{s_3}{\tfrac{1}{3}},\ \underset{s_4}{0},\ \underset{s_5}{0},\ \underset{s_6}{\tfrac{1}{3}}\right]$$

Let the new belief array for this iteration be b_1

$$b_1(s_1) = 0.15\left[0.17 \times b_0(s_1) + 0.17 \times b_0(s_2) + 0 + 0 + 0 + 0\right]$$

$$= \frac{0.15 \times 0.17}{3} = \frac{17}{2000}$$

$$b_1(s_2) = 0.9\left[0.83 \times b_0(s_1) + 0 + 0.17 \times b_0(s_3) + 0 + 0 + 0\right]$$

$$= \frac{3}{10}$$

$$b_1(s_3) = 0.15\left[0 + 0.83 \times b_0(s_2) + 0 + 0.17 \times b_0(s_4) + 0 + 0\right]$$

$$= 0$$

$$b_1(s_4) = 0.9\left[0 + 0 + 0.83 \times b_0(s_3) + 0 + 0.17\, b_0(s_5) + 0\right]$$

$$= 249/1000$$

$$b_1(s_5) = 0.9\left[0 + 0 + 0 + 0.83\, b_0(s_4) + 0 + 0.17\, b_0(s_6)\right]$$

$$= 51/1000$$

$$b_1(s_6) = 0.15\left[0 + 0 + 0 + 0 + 0.83\, b_0(s_5) + 0.83\, b_0(s_6)\right]$$

$$= \frac{83}{2000}.$$

On normalizing i.e. $\sum_{S} b_1(s) = 1$

$$\delta = \sum_{S} b_1(s) = \frac{17}{2000} + \frac{3}{10} + 0 + \frac{249}{1000} + \frac{51}{1000} + \frac{83}{2000}$$

$$= \frac{13}{20}$$

So upon normalizing

$b_1(s_1) = 17/1300$

$b_1(s_2) = 6/13$

$b_1(s_3) = 0$

$b_1(s_4) = 249/650$

$b_1(s_5) = 51/650$

$b_1(s_6) = 83/1300$

Iteration 2 $\left\{ \begin{array}{l} \text{Action is left} \\ \text{color observed is red} \end{array} \right\}$

Let the belief array be b_2

$$b_1 = \left[\frac{17}{1300}, \frac{6}{13}, 0, \frac{249}{650}, \frac{51}{650}, \frac{83}{1300} \right]$$

$$b_2(s_1) = 0.85 \left[0.83\, b_1(s_1) + 0.83 \cdot b_1(s_2) + 0 + 0 + 0 + 0 \right]$$

$$= 870587/2600000$$

$$b_2(s_2) = 0.1 \left[0.17\, b_1(s_1) + 0 + 0.83\, b_1(s_3) + 0 + 0 + 0 \right]$$

$$= 289/1300000$$

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$$b_2(s_3) = 0.85\left[0 + 0.17b_1(s_2) + 0 + 0.83b_1(s_4) + 0 + 0\right]$$
$$= \frac{438039}{1300000}$$

$$b_2(s_4) = 0.1\left[0 + 0 + 0.17b_1(s_3) + 0 + 0.83b_1(s_5) + 0\right]$$
$$= 4233/650000$$

$$b_2(s_5) = 0.1\left[0 + 0 + 0 + 0.17b_1(s_4) + 0 + 0.83b_1(s_6)\right]$$
$$= 3071/260000$$

$$b_2(s_6) = 0.85\left[0 + 0 + 0 + 0 + 0.17b_1(s_5) + 0.17b_1(s_6)\right]$$
$$= 10693/520000$$

Now

$$S = \sum_s b_2(s) = \frac{36967}{52000}$$

So upon normalizing

$$b_2(s_1) = 870587/1848350$$
$$b_2(s_2) = 289/924175$$
$$b_2(s_3) = 62577/132025$$
$$b_2(s_4) = 8466/924175$$
$$b_2(s_5) = 3071/184835$$
$$b_2(s_6) = 10693/369670$$