Solomon Spires

Professor Clark and Professor Tueben

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Lab 1 Report: The Apollo Missions

Introduction

In order to successfully run the Apollo program, we must understand the gravitational dynamics between Earth and Moon. This would then help us accurately predict rocket performance carrying the capsule Saturn V. Gravitational fields affect spacecraft trajectories and calculations on the rocket's performance. Understanding the ascent performance of Saturn V's first stage involves an equation suited for this matter. This report provides detailed evaluations of the gravitational potential and force fields within the Earth-Moon system, as well as the numerical analysis of Saturn V's initial stage ascent dynamics.

To understand this, there are some basic physics concepts to know. We have Newton's law of universal gravitation, which states that gravitational force is proportional to the product of two masses and inversely proportional to the square of the distance between them. We also have the rocket ascent calculations based on the Tsiolkovsky rocket equation, accounting for thrust, mass change due to fuel consumption, and Earth's gravity. In this report, the numerical calculations rely on Python libraries: NumPy and Matplotlib. NumPy efficiently handles numerical arrays and mathematical operations essential for simulating physical systems. Matplotlib generates clear, informative visualizations, enabling intuitive interpretation of complex results. NumPy and Matplotlib tools make advanced computations accessible.

The Gravitational Potential of the Earth-Moon System

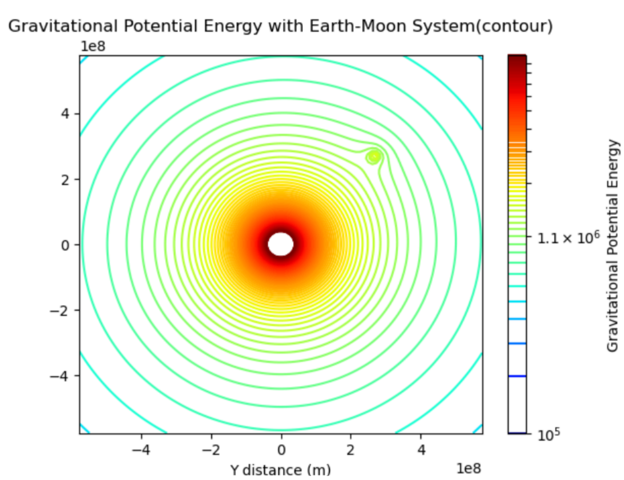

Figure 1

For this data, I used the Gravitational potential formula: $\Phi(r) = - GM/r$. I have the Earth with a mass of 5.9×10^{24} kg and the Moon with a mass of 7.3×10^{22} kg modeled as point masses. The point masses are located at (0,0) and (d_EM/ 2, d_EM/ 2) with d_EM = 3.8×10^8 m. d_EM, for context, is the distance from Earth to Moon. Figure 1 conveys the gravitational potential of the Earth-Moon system as a contour plot visualized on a logarithmic scale. The Earth is represented by the central deep-red region and it dominates the gravitational field due being significantly larger than the Moon. The Moon is represented as a smaller secondary light green region and is visible as an indentation in the potential contours in the upper right quadrant of the plot. This also highlights the Moon's gravitational attraction which forms a gravitational dip within the Earth's gravitational field.

The concentric circles emanating from the Earth display regions of uniform gravitational potential which convey how the gravitational strength decreases progressively with distance from the Earth's center. The color gradient from deep red to orange, yellow, green, cyan, and blue represents the gravitational potential energy. High gravitational potential energy is near deep red transitioning to lower gravitational energy which is near blue and/or cyan in the far outer region. This effectively portrays where the spacecraft would experience stronger gravitational pull. This figure is vital for mission planning due to clearly demonstrating the spatial variation of gravitational potential. Therefore this would aid in strategizing trajectory designs and fuel efficiency for navigating between the Earth and the Moon.

The Gravitational Force of the Earth-Moon System

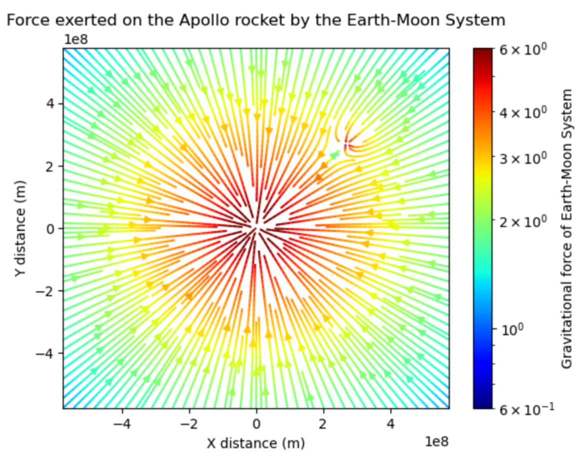

Figure 2

For this data, I used the Gravitational force formula: $F(r) = GMmr/\mathbf{r}^2$ with mass M exerting on a mass m and with **r** as the displacement vector from M to m. This gravitational force is acting on a 5500 kg Apollo command module and is computed as the vector sum of the forces from Earth and Moon. Figure 2 conveys the gravitational force vectors acting on the Apollo rocket within the Earth-Moon system. This figure is displayed as a vector field and the arrows designate the direction and magnitude of gravitational force exerted on the rocket at various positions in space.

The Earth, located at the center, generates the strongest gravitational force, directing all vectors radially inward toward itself. Near the Earth, the forces are densely packed and that indicates a strong gravitational pull. The Moon, on the other hand, is weaker due to its smaller mass and it appears as a slight deviation in vector alignment in the upper right quadrant. This means that the Moon will alter trajectories and it brings us back to reflect on the complexity of navigating spacecraft in the Earth-Moon system. The color gradient, progressing similarly as Figure 1, shows that the gravitational force is stronger near deep red and decreases as it approaches cyan/blue. This figure is necessary for precise modeling of gravitational fields for mission success.

Projected Performance of the Saturn V Stage 1

The Saturn V Stage 1(S-1C) performance was estimated with the Tsiolkovsky equation: $\Delta v(t) = v_e \ln(m_0/m(t)) - gt$. In this equation, we have:

- m_0 is the initial “wet” mass(fuel + rocket parts + payload).
- $m(t)$ is $m_0 +$ **m**t which is mass at time t. ($m_0 = 2.8 \times 10^6$ kg)
    - **m**t is the fuel burn rate(assumed to be constant, **m** = 1.3×10^4 kg)
- v_e is the fuel exhaust velocity. ($v_e = 2.4 \times 10^3$ m/s)
- g is the gravitational acceleration

Utilizing the Total Burn Time formula: $T = (m_0 - m_f)/$**m** with $m_f = 7.5 \times 10^5$ kg, I was able to find that the total burn time = 157.7 seconds. To find the end of burn altitude h, you can integrate $\Delta v(t)$ from 0 to T. Doing this numerical integration, I found that the S-1C achieves a burnout altitude of 74,218.3 meters. In Figure 3, you'll see the results I’ve gained from utilizing Python libraries. The results are essential for fine-tuning ascent trajectories, optimizing fuel usage, and devising mission plan objectives for the Apollo program.

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The height the rocket reaches at burnout is 74218.3 meters
The time before burnout is 157.7 seconds
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Figure 3

Discussion and Future Work

The calculations given in this report include several clarifying approximations: gravitational model simplification, neglecting relativistic effects, and atmospheric drag omission. For gravitational model simplification, we modeled the Earth and the Moon as point masses(neglecting their true spherical shape and internal density variations). For neglecting relativistic effects, the gravitational calculations were processed using Newtonian mechanics(neglecting the relativistic correction). For atmospheric drag omission, the rocket ascent calculations presume an idealized scenario of vertical ascent(neglecting atmospheric drag and aerodynamic resistance).

Comparing these idealized theoretical results to recent empirical test data from NASA reveals important insights. The prototype test of Saturn V indicated a burn duration of about 160 seconds, reaching an altitude of approximately 70 kilometers. Our analytical calculations predicted a slightly shorter burn time (157.7 seconds) and a higher altitude (74.2 kilometers). The main reason for the difference in altitude is primarily due to neglecting atmospheric drag. Drag forces significantly reduce rocket altitude by dissipating kinetic energy and slowing the ascent. Additionally, the assumption of constant gravitational acceleration leads to a minor overestimation of the rocket's performance.

To improve the accuracy of future analyses, we can incorporate realistic atmospheric conditions (altitude-dependent air density and drag coefficients). These improvements would significantly enhance predictive accuracy and reliability, guaranteeing safer and more efficient mission planning for future Apollo programs.