HEILBRONN UNIVERSITY
OF APPLIED SCIENCES

Autonomous Systems – Path Planning and Control

Lab Project Documentation (2.5 ECTS)

Students: Semyon Kondratev (207612)
Daniel Vetter (190238)

Professor: Prof. Dr.-Ing. Frank Tränkle

Term: Winter term 2021/22

Inhalt

1 Exercise 5 - Vehicle Dynamics

1.1 Exercise 5.1 Longitudinal dynamic model

1.1.1 Exercise 5.1 b) Calculation of the parameters

The following formula was given as a solution to exercise 5) a):

$$\frac{m_{tot}}{k_v}\frac{dv_r(t)}{dt} + v_r(t) = \frac{u_g c_m u_{max}}{rRk_v}(t - T_t) - \frac{1}{k_v}F_{wr}(t)\,; t > 0$$

with the starting condition:

$$v_r(0) = 0\frac{m}{s}$$

calculation of T, k_u, T_t :

$$T = \frac{m}{\frac{c_m^2 u_g^2}{Rr^2} p c_w A\, \bar{v}_r} = 0.361\ s$$

$$k_u = \frac{u_g c_m u_{max}}{rR\left(\frac{c_m^2 u_g^2}{Rr^2} + p c_w A\, \bar{v}_r\right)} = 2.51\ \frac{m}{s}$$

$$T_t = 0.100\ s$$

1.1.2 Exercise 5.1 c) Calculation of the transfer function

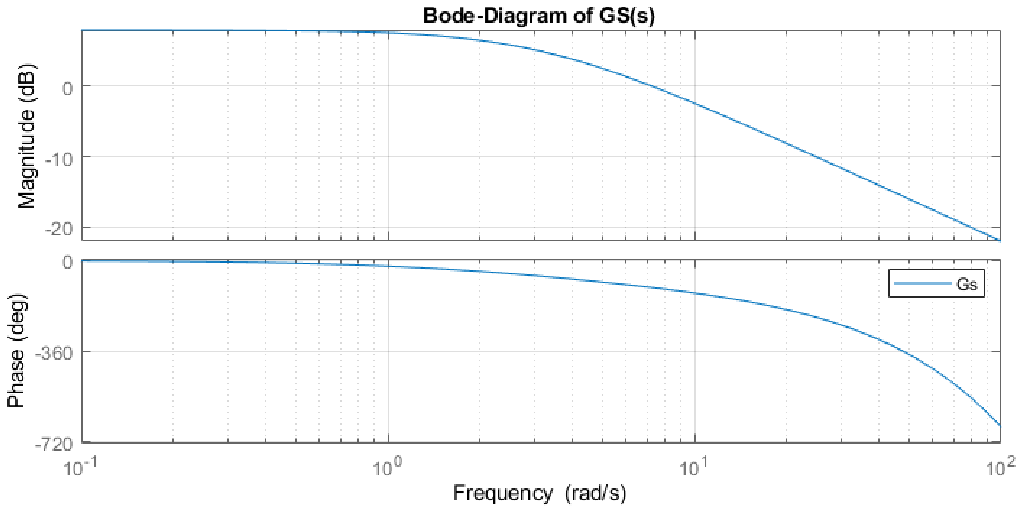
With the parameters calculated in 5) b) the transfer function can be calculated as follows.

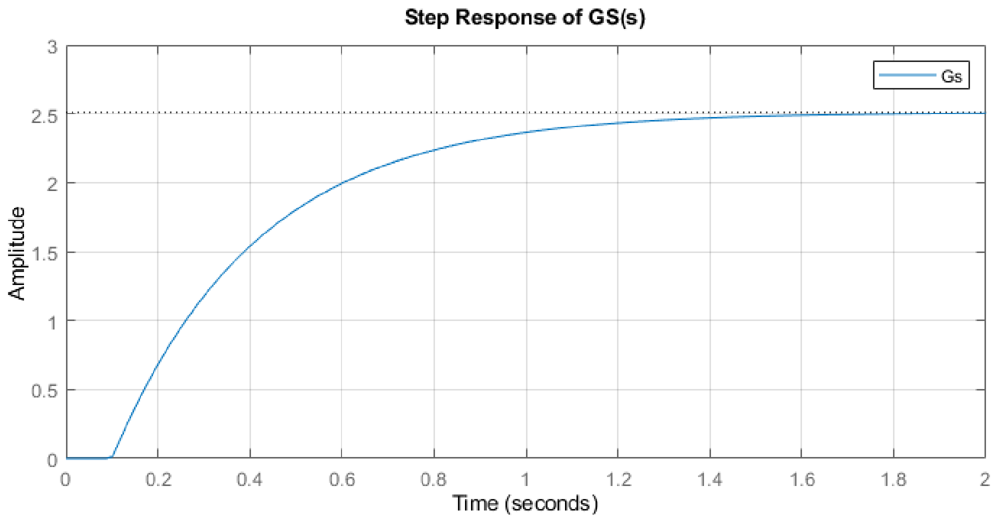
$$G_s(s) = \frac{Y(s)}{U(s)} = \frac{V_r(s)}{U_n(s)} = \frac{k_u e^{-sT_t}}{Ts + 1}$$

1.1.3 Exercise 5.1 d) Bode plot and step response

See the following files:

- ex5_1.m





1.2 Exercise 5.2 vehicle simulation

1.2.1 Exercise 5.2 b) Simulink model of the vehicle

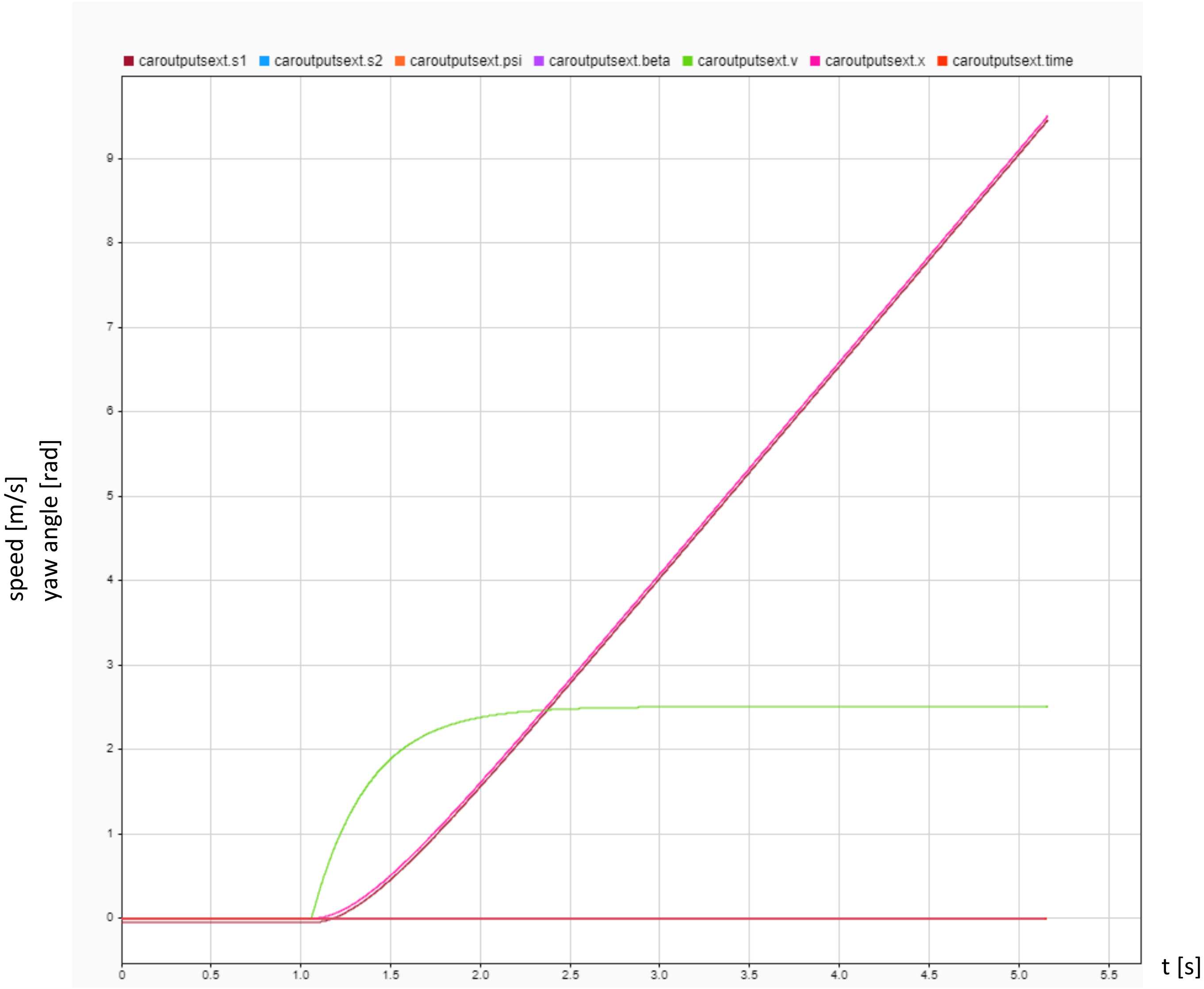
See the following files:

- s6_template.slx (created with version R2020b)
- s6_template_2019a.slx (converted to version R2019a)

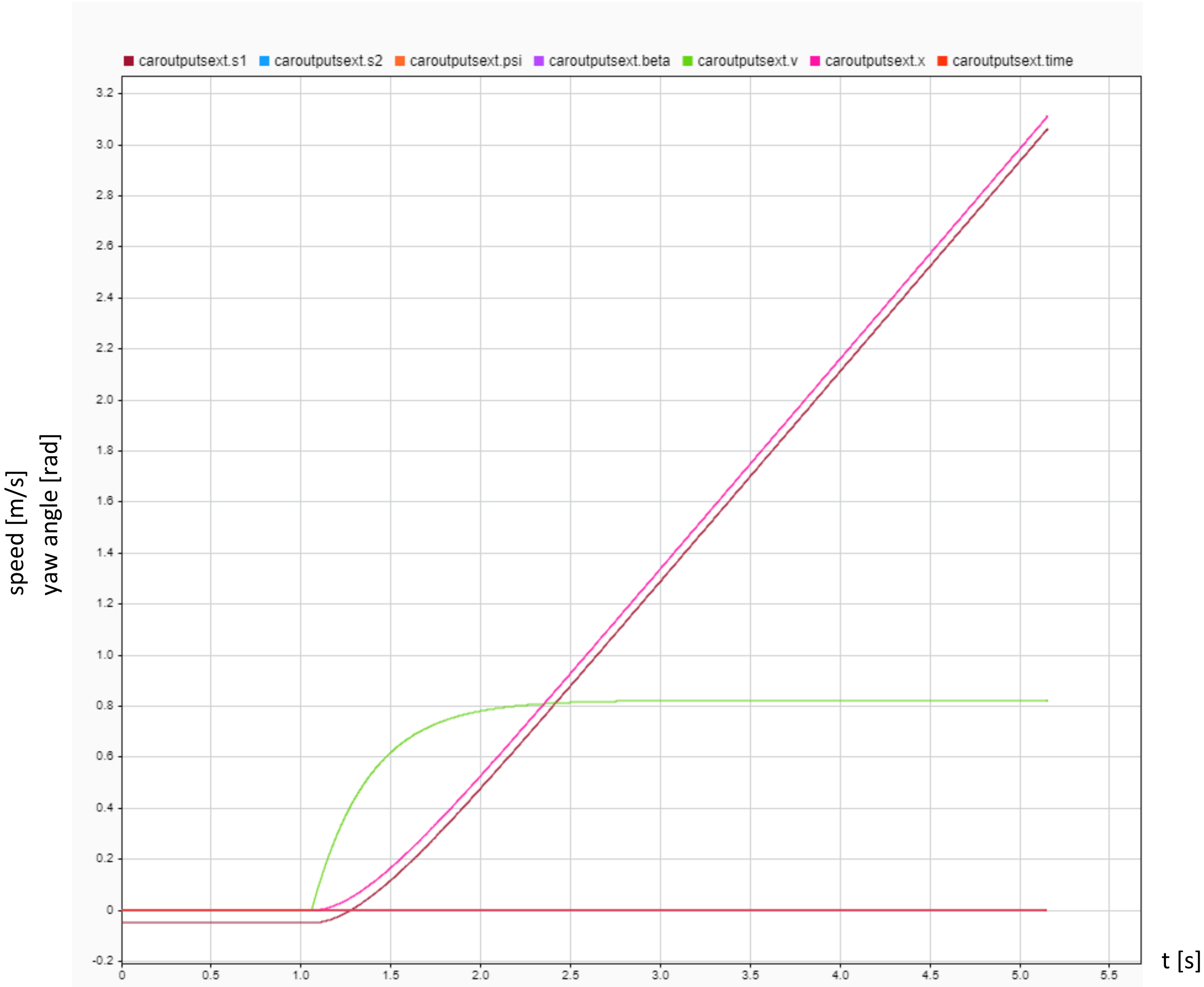
1.2.2 Exercise 5.2 c) Test

Below we inserted a few diagrams with different parameters for the pedals, steering and command

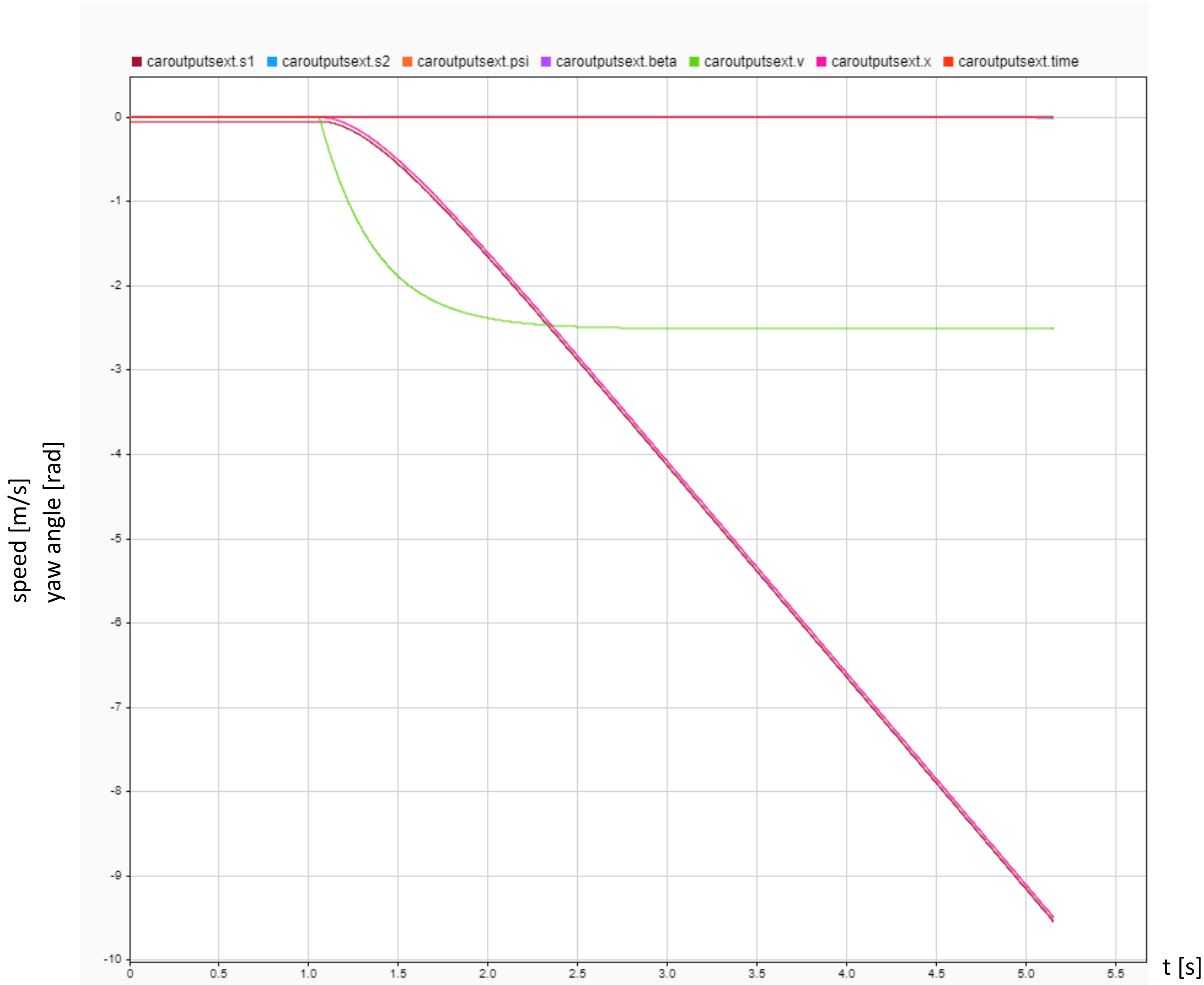
1. CarInputsCommandForward, pedals = 1, δ = 0



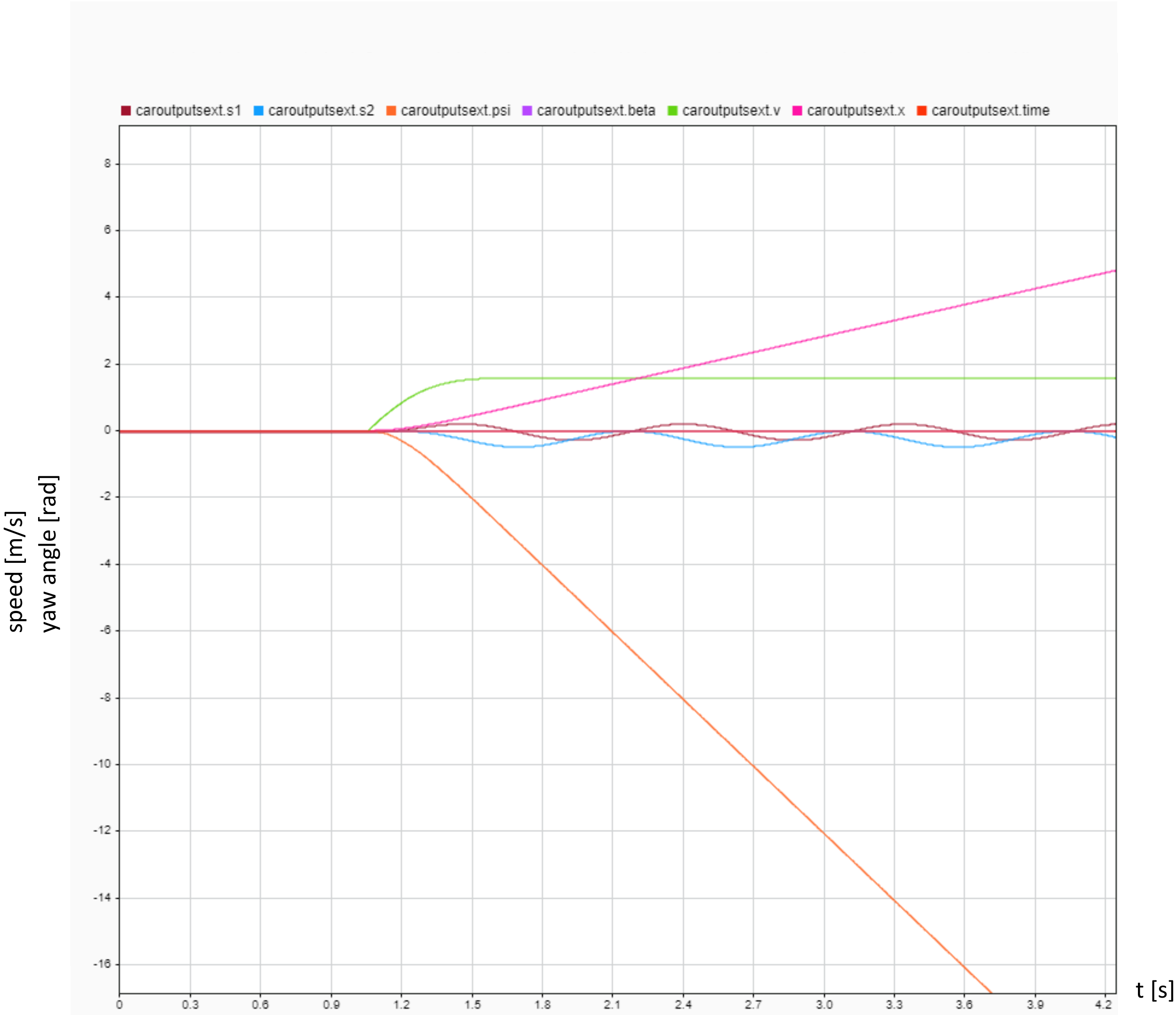
2. CarInputsCommandSlow, pedals = 1, δ = 0



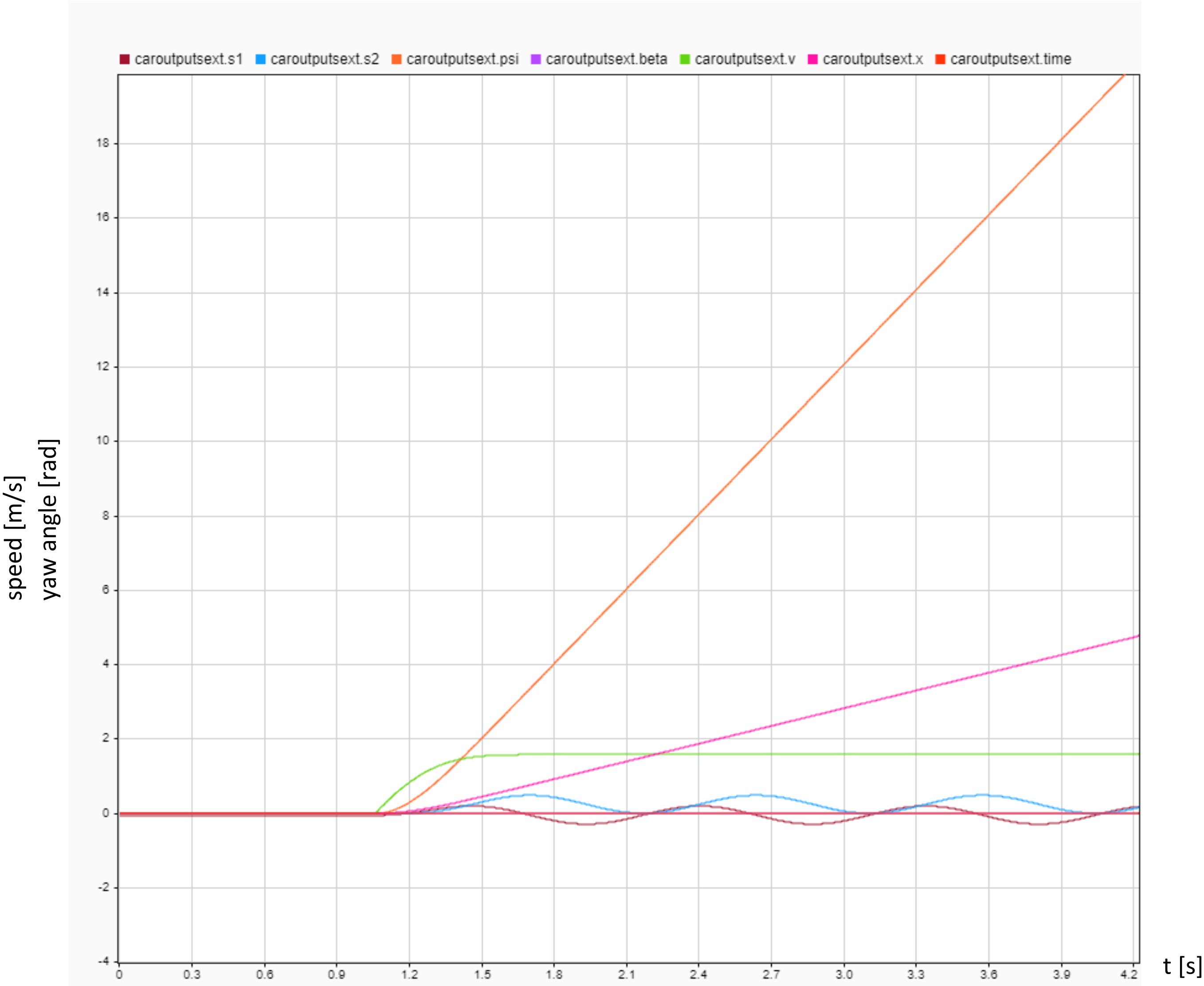
3. CarInputsCommandReverse, pedals = -1, δ = 0



4. CarInputsCommandForward, pedals = 1, δ = -0.5



5. CarInputsCommandForward, pedals = 1, δ = 0.5



1.3 Exercise 6.1 vehicle simulation

1.3.1 Exercise 6.1 a) Parameter calculation

plant transfer function:

$$G_s(s) = \frac{k_u}{Ts+1} e^{-T_t s} = \frac{k_u}{j\omega T + 1}(\cos(T_{t^\omega}) - j\sin(T_{t^\omega}))$$

Controller transfer function:

$$G_R(s) = k_R \frac{(T_i s + 1)}{T_i s} \Rightarrow G_R(\omega) = \frac{j\omega k_R T_i + k_R}{j\omega T_i}$$

Open-loop transfer function:

$$G_0(s) = k_u k_R \frac{(T_i s + 1)}{(Ts+1)T_i s} e^{-T_t s}$$

Phase of plant:

$$\varphi_s(\omega) = arctan\left(\frac{-k_u\, cos(T_t\omega)}{k_u\, sin(T_t\omega)}\right) - arctan(T\omega)$$

$$= arctan(-\,tan(T_t\omega))\, arctan(T\omega)$$

$$= -T_t\omega - arctan(T\omega)$$

Phase of controller:

$$\varphi_R(\omega) = arctan\left(\frac{k_{R^\omega} T_i}{k_R}\right) - \left(-\frac{\pi}{2}\right)$$

Open-loop phase:

$$\varphi_0(\omega) = \varphi_s(\omega) + \varphi_R(\omega) = -\omega T_t - arctan(\omega T) + arctan(\omega T_i) - \frac{\pi}{2}$$

Phase margin:

$$\varphi_{Re\,s} = \pi + \varphi_0(\omega_D) = \pi - \frac{\pi}{2} - T_t\omega_D - arctan(\omega_D T) + arctan(\omega_D T_i)$$

$$= \frac{\pi}{2} - \omega_D T_t - arctan(\omega_D T) + arctan(\omega_D T_i)$$

$$T_i = \frac{tan(\varphi_{Re\,s} - \frac{\pi}{2} + \omega_D T_t + arctan(\omega_D T))}{\omega_D} = 0{,}2468s$$

Open-loop frequency response:

$$G_0(\omega) = k_u k_R \frac{j\omega T_i + 1}{(j\omega T + 1) j\omega T_i} e^{-j\omega T_t} = k_u k_R \frac{j\omega T_i + 1}{-\omega^2 T T_i + j\omega T_i} e^{-j\omega T_t}$$

Open-loop magnitude response:

$$A_0(\omega) = k_u k_R \frac{\sqrt{1 + (\omega T_i)^2}}{\sqrt{(\omega^2 T T_i)^2 + (\omega T_i)^2}}$$

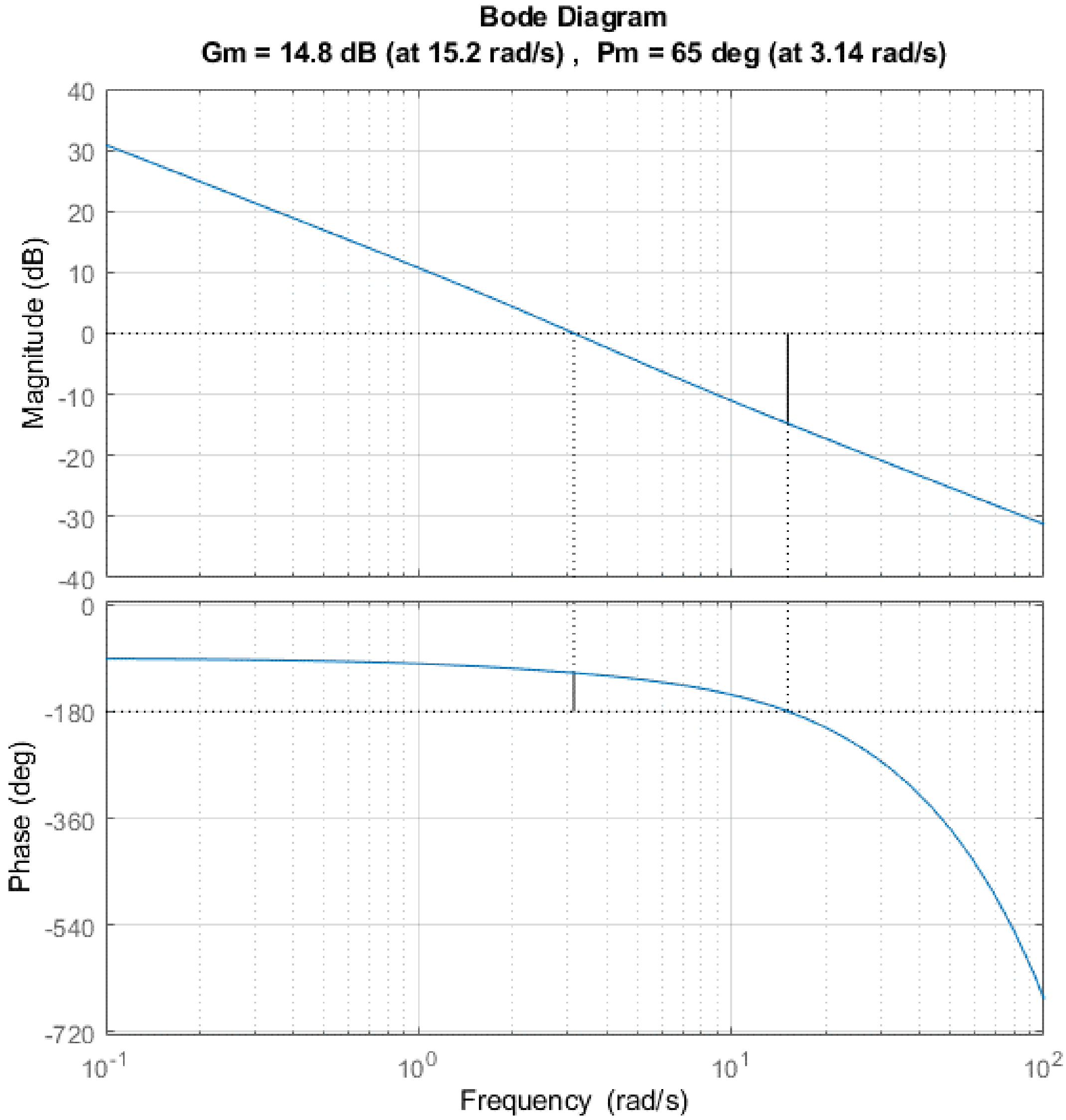
$$A_0(\omega_D) = 1$$

$$k_R = \frac{\sqrt{(\omega_D^2 T T_i)^2 + (\omega_D T_i)^2}}{\sqrt{1 + (\omega_D T_i)^2} k_u} = 0{,}3440 \frac{s}{m}$$

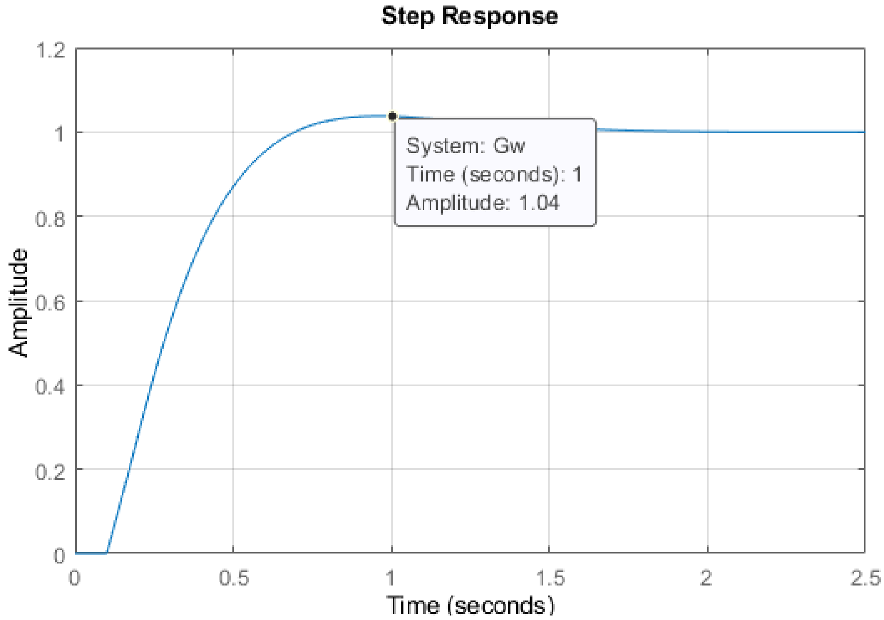
1.3.2 Exercise 6.1 b) Bode plot of $G_0(s)$

See the following files:

- ex6_1.m



1.3.3 Exercise 6.1 c) Step Response of $G_w(s)$




```
   RiseTime: 0.3941
SettlingTime: 1.3211
 SettlingMin: 0.9046
 SettlingMax: 1.0376
   Overshoot: 3.7564
  Undershoot: 0
        Peak: 1.0376
    PeakTime: 0.9594
```

1.3.4 Exercise 6.1 d) discretization

$$G_R(s) = k_R \cdot \left(1 + \frac{1}{T_i s}\right), \text{ with } s = \frac{1-z^{-1}}{T_A}$$

$$G_R(z) = \frac{U(z)}{E(z)} = k_R \left(1 + \frac{1}{T_i \frac{1-z^{-1}}{T_A}}\right)$$

$$G_R(z) = k_r \cdot \left(1 + \frac{T_A}{T_i \cdot (1 - z^{-1})}\right)$$

$$u_{p,k} = e_k \cdot k_r$$

$$u_{i,k} = u_{i,k-1} + k_r \frac{T_A}{T_i} \cdot e_k$$

$$u_k = u_{p,k} + u_{i,k} = k_r \cdot e_k + k_r \frac{T_A}{T_i} \cdot e_k + u_{i,k-1}$$

1.4 Exercise 6.2 Speed Control simulation

Step responses of different vehicle speeds

See also the following files:

- s6_data.m
- s7_template.slx (created with version R2020b)
- s7_template.slx (converted to version R2019a)

1.5 Exercise 8.1 Path definition of straight lines

1.5.1 Exercise 8.1 a) Derivation of parameterized curve definition

$$s(x) = \begin{pmatrix} s_1(x) \\ s_2(x) \end{pmatrix} = \begin{pmatrix} s_{0,1} + cos(\psi_0) \cdot x \\ s_{0,2} + sin(\psi_0) \cdot x \end{pmatrix}$$

1.5.2 Exercise 8.1 b) Calculation of curvature, tangent vector and normal vector

Tangent vector:

$$t(x) = \frac{ds}{dx} = \begin{pmatrix} s'_1(x) \\ s'_2(x) \end{pmatrix} = \begin{pmatrix} cos(\psi_0) \\ sin(\psi_0) \end{pmatrix}$$

Normal vector:

$$n(x) \times t(x) = 0$$

$$\cos(\psi_0) \cdot t_1(x) + \sin(\psi_0) \ \cdot t_2(x) = 0$$

$$n(x) = \begin{pmatrix} -sin(\psi_0) \\ cos(\psi_0) \end{pmatrix} \ and \ \begin{pmatrix} sin(\psi_0) \\ -cos(\psi_0) \end{pmatrix}$$

Curvature:

$$\kappa(x) = \left| \frac{d^2 s}{dx^2} \right| = \sqrt{s''_1 + s''_2} = \sqrt{0 + 0} = 0$$

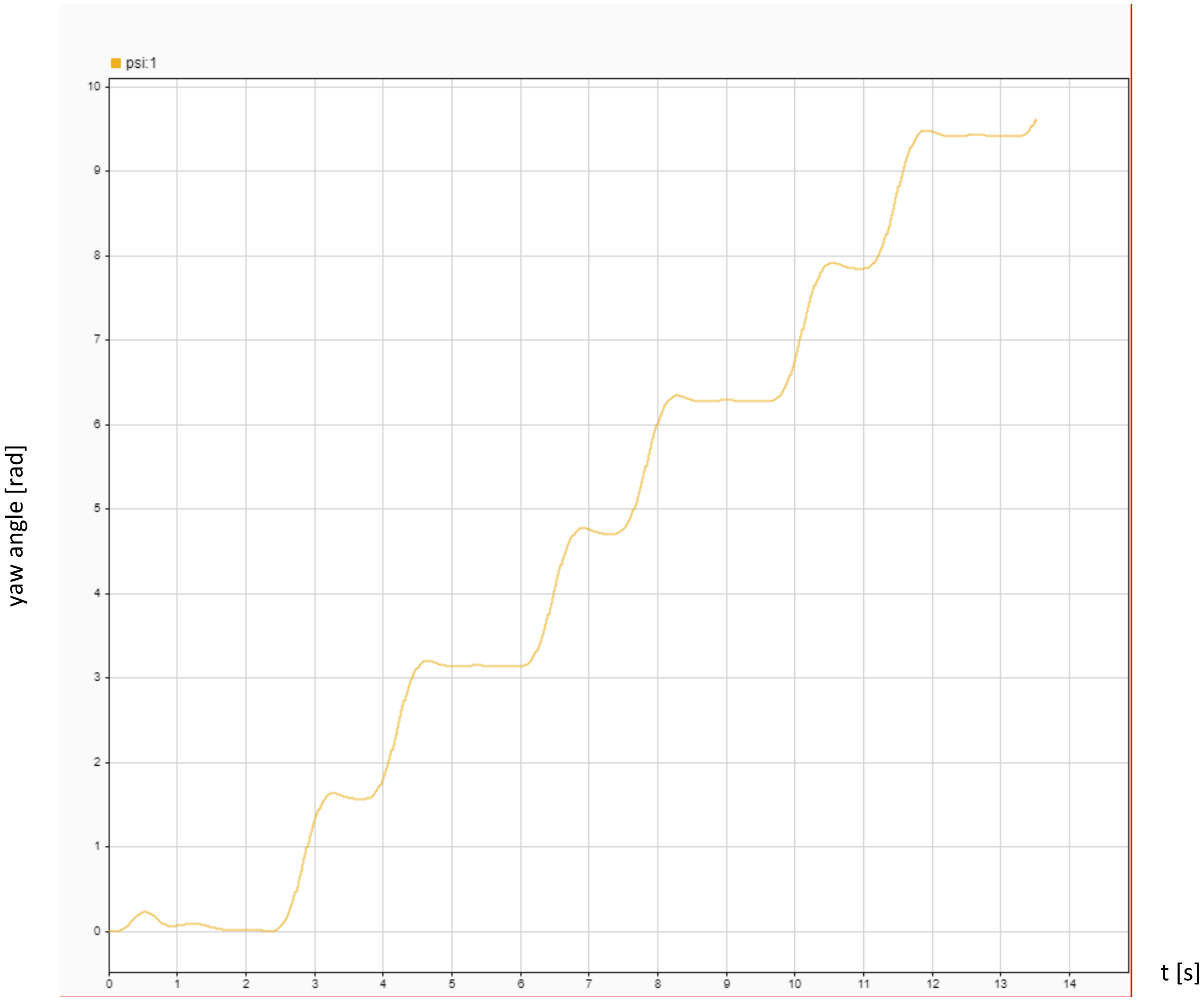
1.6 Exercise 8.2 MODBAS CAR Function for Clothoids (not required for 2.5 ECTS)

1.6.1 See the following matlab-files:

- mbc_clothoid_create.m
- mbc_clothoid_get_points.m
- s6_data.m

1.7 Exercise 9.1 Path Following Controller

1.7.1 Yaw-angle diagram



See the following files:

- s9_template.slx (created with version R2020b)
- s9_template.slx (converted to version R2019a)
- s6_data.m