CSC311: Math Notes

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Euclidean Distance/L2 Norm

For a given vector v, the Euclidean distance (or L2 norm) is defined as the square root of the sum of the squares of its components. It is a measure of the straight-line distance between two points in Euclidean space.

$$\begin{aligned} ||v||_2 &= \sqrt{v_1^2 + v_2^2 + \ldots + v_D^2} \\ &= \sqrt{\sum_{i=1}^{D} v_i^2} \end{aligned}$$

If we have two vectors x and y, written as $x = (x_1, x_2, \ldots, x_n)$ and $y = (y_1, y_2, \ldots, y_n)$, the Euclidean distance between them is given by:

Euclidean Distance

$$\begin{aligned} d(x, y) &= \sqrt{\sum_{i=1}^{n} (x_i - y_i)^2} \\ &= \sqrt{(x_1 - y_1)^2 + (x_2 - y_2)^2 + \ldots + (x_n - y_n)^2} \end{aligned}$$

Vector Transpose

The transpose of a vector v is denoted as v^T. For a column vector $v = \begin{pmatrix} v_1 \\ v_2 \\ \vdots \\ v_n \end{pmatrix}$, the transpose is a row vector:

$$v^T = \begin{pmatrix} v_1 & v_2 & \ldots & v_n \end{pmatrix}$$

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Baye's Theorem

Baye's theorem describes the probability of an event based on prior knowledge of conditions that might be related to the event. It is expressed as:

Baye's Theorem

$$
\begin{aligned}
P(A|B) &= \frac{P(B|A) \cdot P(A)}{P(B)} \\
P(A|B) &= \text{Posterior Probability} \\
P(B|A) &= \text{Likelihood} \\
P(A) &= \text{Prior Probability} \\
P(B) &= \text{Marginal Likelihood}
\end{aligned}
$$

Where:

- $P(A|B)$ is the probability of event A given that B is true.
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- $P(A)$ is the probability of event A.
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To remember Baye's formula, the intuition says that the posterior probability is proportional to the likelihood times the prior probability. The denominator $P(B)$ is a normalizing constant that ensures the probabilities sum to 1.