# Linear models in R

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## Thanks and preamble

Thanks to J. J. Valletta (now an Associate Lecturer in Statistics, University of St Andrews) and T. J. McKinley (Lecturer in Mathematical Biology, now at the Streatham Campus)

Extensive notes, handouts of these slides, and data files for the practicals are available at: https://exeter-data-analytics.github.io/StatModelling/

### The Team

- Dr Beth Clark
- Dr Dan Padfield
- Dr Matt Silk
- Dr Richard Inger



## What is a model and why do we need one?

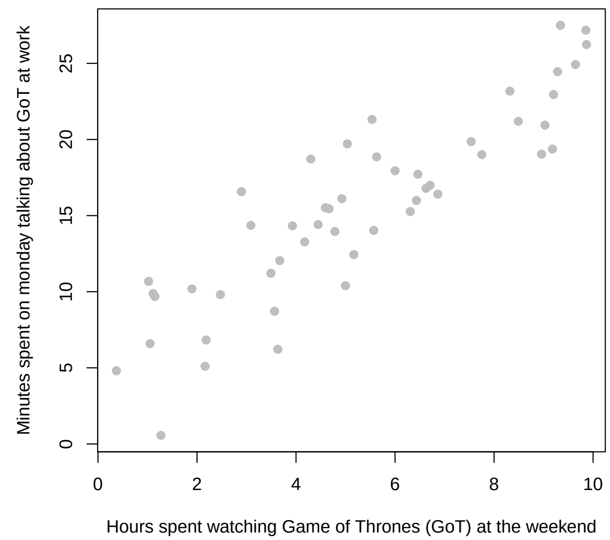
A **model** is a human construct/abstraction that tries to approximate the **data generating process** in some useful manner

### Models are built for

- enhancing our understanding of a complex phenomenon
- executing "what if" scenarios
- predicting/forecasting an outcome
- controlling a system



## Illustrative example



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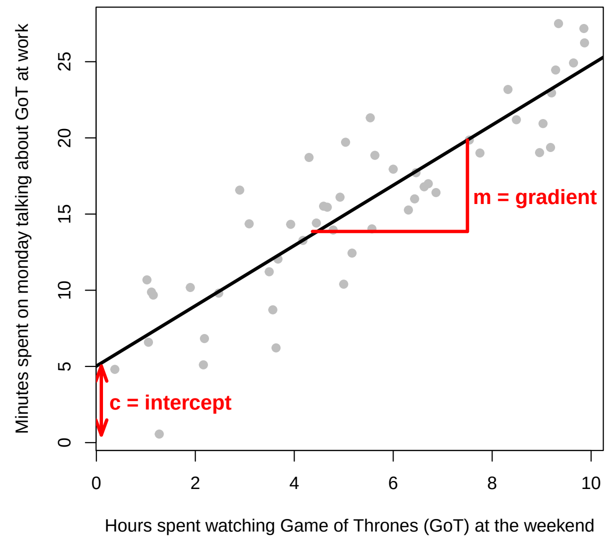




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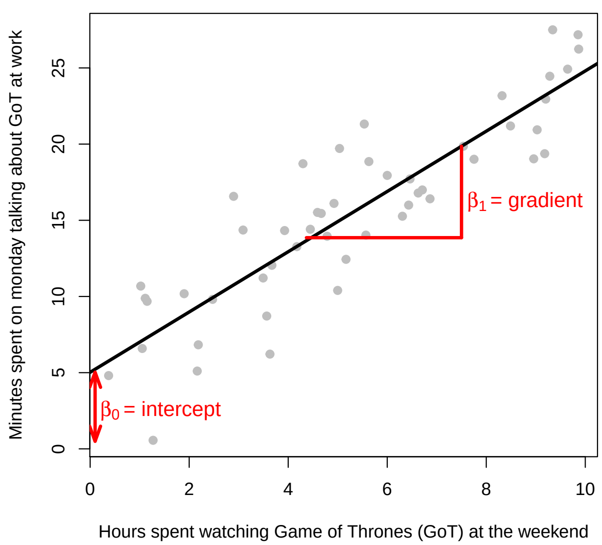





# Formal definition

$$y_i = \beta_0 + \beta_1 x_i + \epsilon_i$$
$$\epsilon_i \sim \mathcal{N}(0, \sigma^2)$$

## Observed data

- $y$ (outcome/response): minutes spent talking about GoT
- $x$ (explanatory): hours spent watching Game of Thrones (GoT)

## Parameters to infer

- $\beta_0$: intercept
- $\beta_1$: gradient wrt minutes talking about GoT

# Linear models in R

- Use the `lm()` function
- Requires a **formula** object
  `outcome ~ explanatory variable`

```r
# talk: minutes spent talking about GoT (outcome/response variable)
# watch: hours spent watching GoT (explanatory variable)

fit <- lm(talk ~ watch)

# If data is in a data frame called "df"
fit <- lm(talk ~ watch, df)
```



# Summary of fitted model

```r
summary(fit)
```

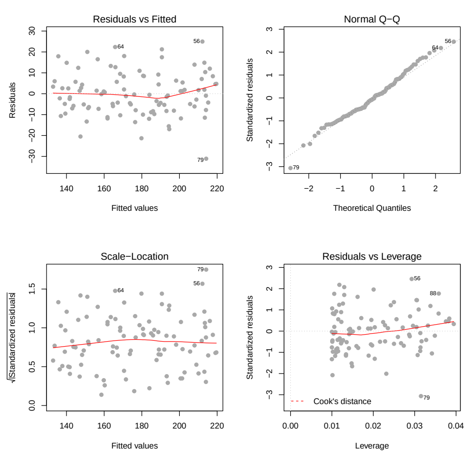
```
##
## Call:
## lm(formula = height ~ weight, data = df)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -31.089  -6.926  -0.689   6.057  24.967
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)  2.35229    7.11668   0.331    0.742
## weight       2.17446    0.08782  24.762   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 10.31 on 98 degrees of freedom
## Multiple R-squared:  0.8622, Adjusted R-squared:  0.8608
## F-statistic: 613.1 on 1 and 98 DF,  p-value: < 2.2e-16
```



# Model checking

In order to make **robust** inference, we must check the model fit

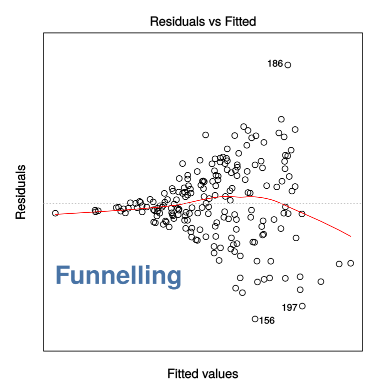
```r
plot(fit)
```

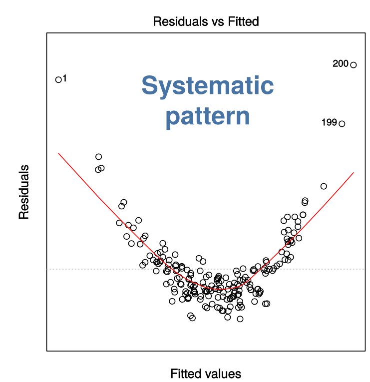





# Model checking

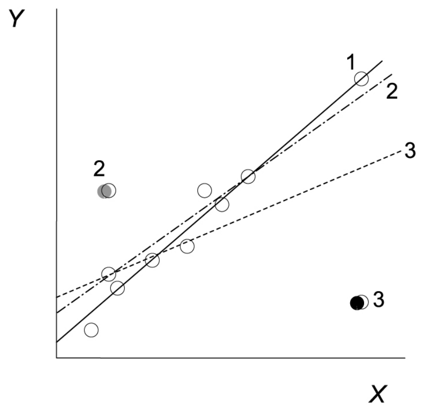
A couple of examples where the homogeneity of variance assumption is **violated**



## Model checking: Leverage and Influence

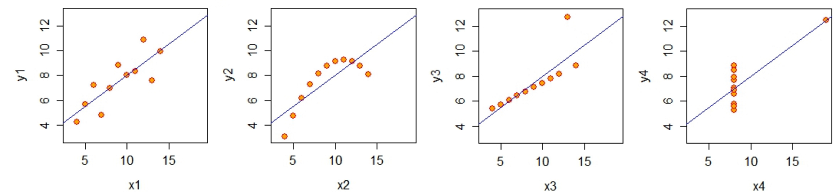
- Obs. 1 – **large leverage** (outlier in x and y), but not influential
- Obs. 2 – **large residual**, but not an outlier in x or y. Not influential
- Obs. 3 – not an outlier for y, but has large **leverage and large residual**: very influential (high Cook's distance).






## Model checking: Plot your data!

- **Always** visualise your data before fitting any model
- Assumption of a linear relationship...
- Scatterplots can indicate unequal variance, nonlinearity and outliers



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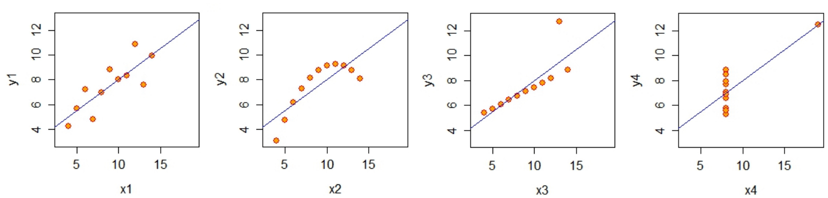
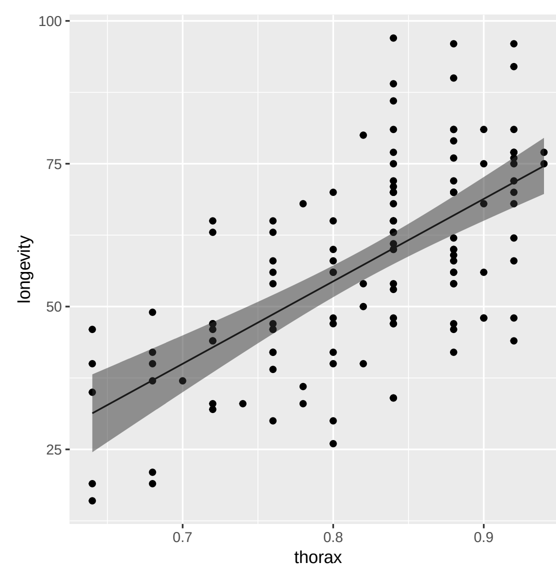



### Anscombe (1973) American Statistician 27: 17–21

- $r^2 = 0.68$ in all four cases
- Test that $H_0 = 0$ identical in all four cases ($t = 4.24$, $P = 0.002$)



## Confidence vs prediction intervals



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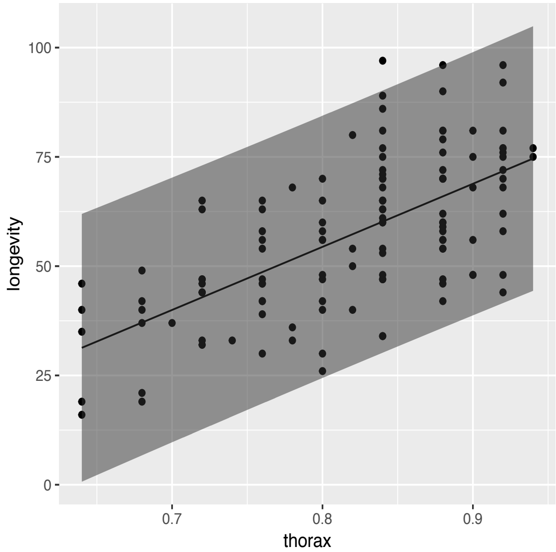
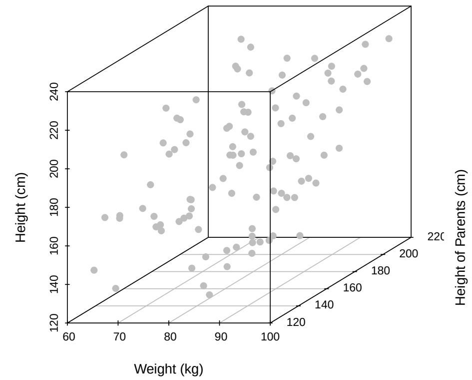





## Multiple linear regression

$$y_i = \beta_0 + \beta_1 x_{1i} + \ldots + \beta_p x_{pi} + \epsilon_i$$
$$\epsilon_i \sim \mathcal{N}(0, \sigma^2)$$



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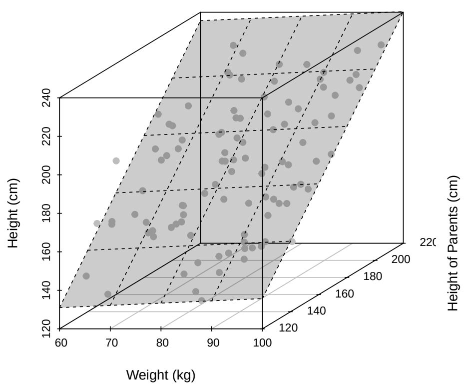
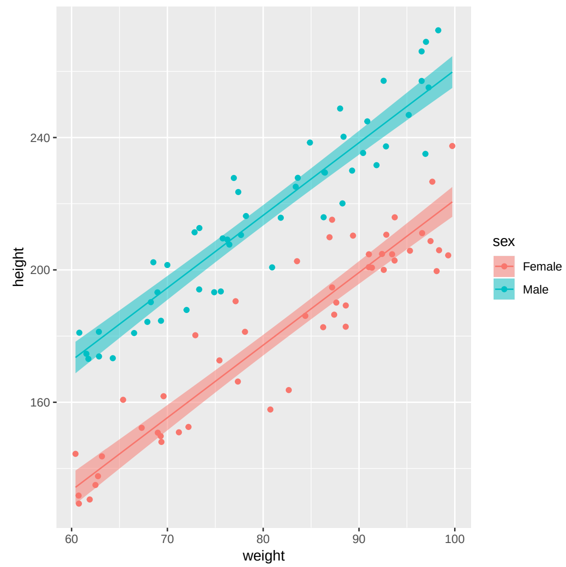





## Categorical variables



## Categorical variables

We need **dummy** variables

$$S_i = \begin{cases} 1 & \text{if } i \text{ is male,} \\ 0 & \text{otherwise} \end{cases}$$

Here, female is known as the **baseline/reference level**
The regression is:

$$y_i = \beta_0 + \beta_1 S_i + \beta_2 x_i + \epsilon_i$$

Or in English:

$$\text{height}_i = \beta_0 + \beta_1 \text{sex}_i + \beta_2 \text{weight}_i + \epsilon_i$$



## Categorical variables

The mean regression lines for male and female are:

- Female (`sex=0`)

$$\text{height}_i = \beta_0 + (\beta_1 \times 0) + \beta_2 \text{weight}_i$$
$$\text{height}_i = \beta_0 + \beta_2 \text{weight}_i$$

- Male (`sex=1`)

$$\text{height}_i = \beta_0 + (\beta_1 \times 1) + \beta_2 \text{weight}_i$$
$$\text{height}_i = (\beta_0 + \beta_1) + \beta_2 \text{weight}_i$$



## Summary

**Linear regression** is a powerful tool:

- It splits the data into **signal** (trend/mean) and **noise** (residual error)
- It can cope with **multiple variables**
- It can incorporate different **types** of variable
- It can be used to produce **point** and **interval** estimates for the parameters
- It can be used to assess the importance of variables

**But** always **check** that the model fit is sensible, that the assumptions are met and the results make sense!