Linear models in R

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UNIVERSITY OF EXETER | DOCTORAL COLLEGE

Researcher Development

What is a model and why do we need one?

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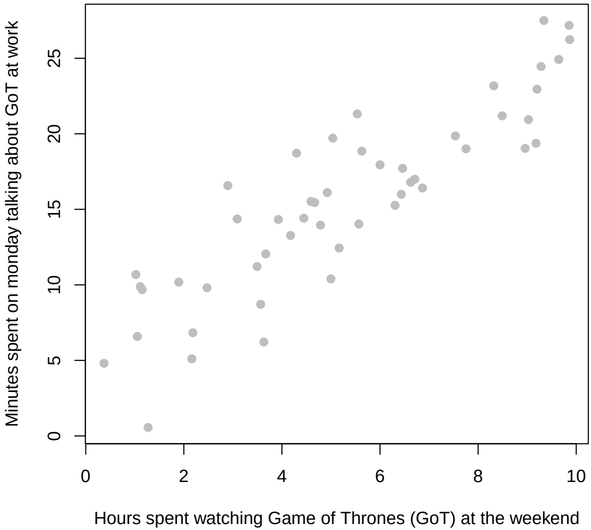
Illustrative example



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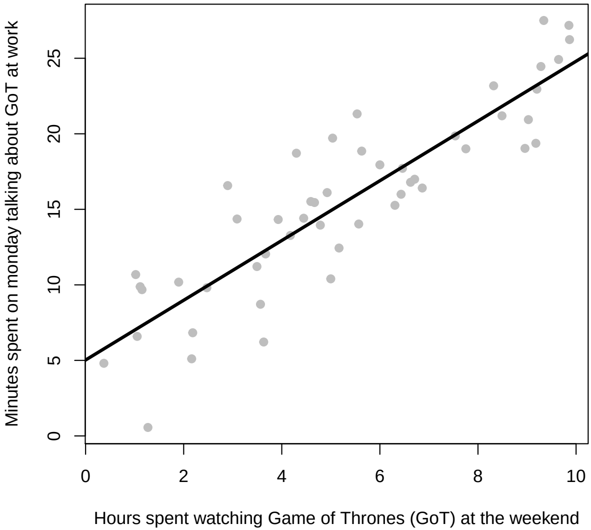

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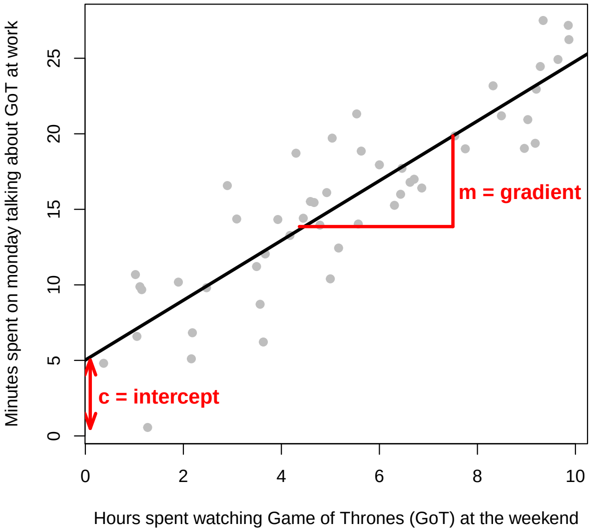

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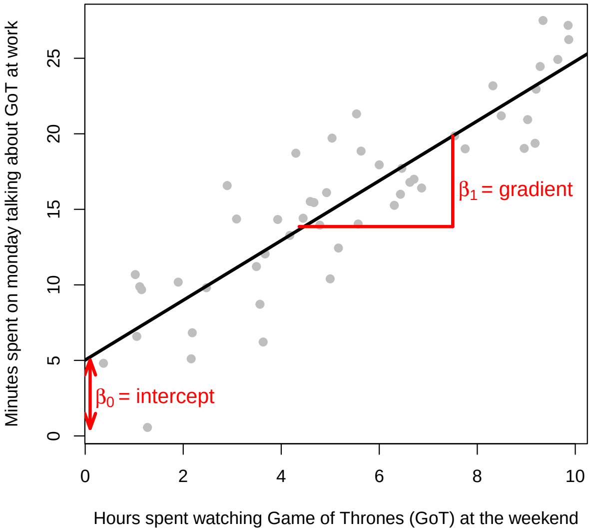

Formal definition

$$
\begin{aligned}
y_i &= \beta_0 + \beta_1 x_i + \epsilon_i \\
\epsilon_i &\sim \mathcal{N}(0, \sigma^2)
\end{aligned}
$$

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Observed data

- y (outcome/response): minutes spent talking about GoT
- x (explanatory): hours spent watching Game of Thrones (GoT)

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Parameters to infer

- β_0: intercept
- β_1: gradient wrt minutes talking about GoT

Linear models in R

- Use the `lm()` function
- Requires a **formula** object
  `outcome ~ explanatory variable`

```r
# talk: minutes spent talking about GoT (outcome/response variable)
# watch: hours spent watching GoT (explanatory variable)

fit <- lm(talk ~ watch)

# If data is in a data frame called "df"
fit <- lm(talk ~ watch, df)
```

Summary of fitted model

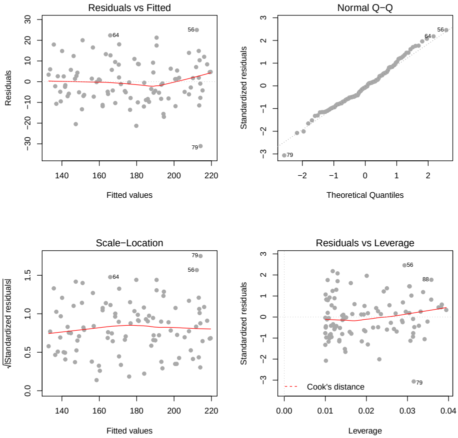
```r
summary(fit)
```

```
##
## Call:
## lm(formula = height ~ weight, data = df)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -31.089  -6.926  -0.689   6.057  24.967
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)  2.35229    7.11668   0.331    0.742
## weight       2.17446    0.08782  24.762   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 10.31 on 98 degrees of freedom
## Multiple R-squared:  0.8622, Adjusted R-squared:  0.8608
## F-statistic: 613.1 on 1 and 98 DF,  p-value: < 2.2e-16
```

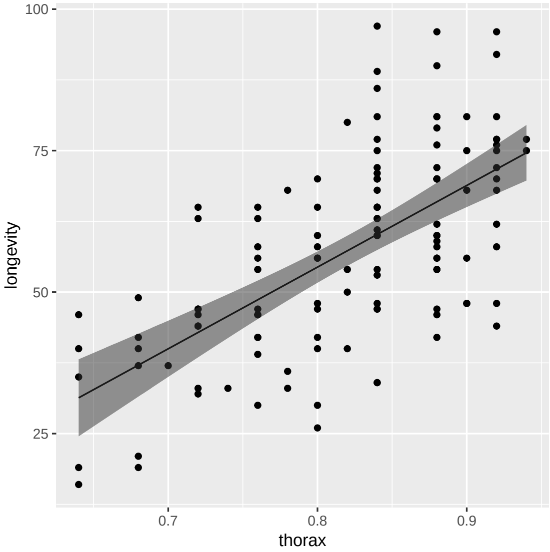
Model checking

In order to make **robust** inference, we must check the model fit

```r
plot(fit)
```



Confidence vs prediction intervals



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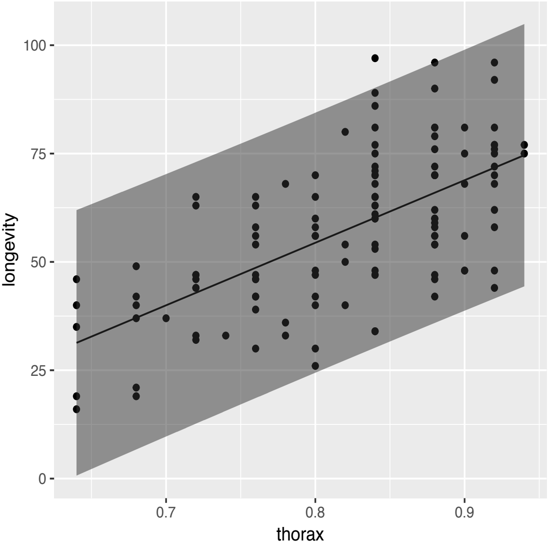
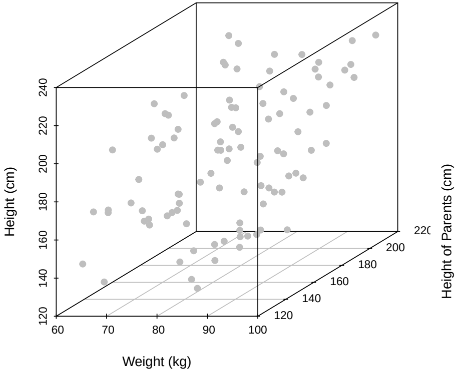


Multiple linear regression

$$y_i = \beta_0 + \beta_1 x_{1i} + \ldots + \beta_p x_{pi} + \epsilon_i$$
$$\epsilon_i \sim \mathcal{N}(0, \sigma^2)$$



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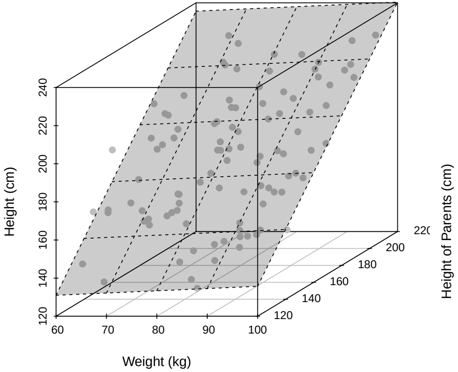

Categorical variables

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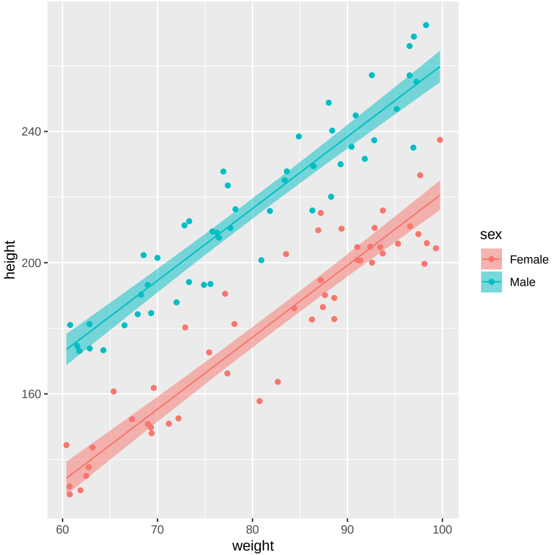

Categorical variables

We need **dummy** variables

$$S_i = \begin{cases} 1 & \text{if } i \text{ is male,} \\ 0 & \text{otherwise} \end{cases}$$

Here, female is known as the **baseline/reference level**
The regression is:

$$y_i = \beta_0 + \beta_1 S_i + \beta_2 x_i + \epsilon_i$$

Or in English:

$$\text{height}_i = \beta_0 + \beta_1 \text{sex}_i + \beta_2 \text{weight}_i + \epsilon_i$$

Categorical variables

The mean regression lines for male and female are:

- Female (`sex=0`)

$$\text{height}_i = \beta_0 + (\beta_1 \times 0) + \beta_2 \text{weight}_i$$
$$\text{height}_i = \beta_0 + \beta_2 \text{weight}_i$$

- Male (`sex=1`)

$$\text{height}_i = \beta_0 + (\beta_1 \times 1) + \beta_2 \text{weight}_i$$
$$\text{height}_i = (\beta_0 + \beta_1) + \beta_2 \text{weight}_i$$