Linear models in R

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Researcher Development

Thanks and preamble

Thanks to J. J. Valletta (now an Associate Lecturer in Statistics, University of St Andrews) and T. J. McKinley (Lecturer in Mathematical Biology, now at the Streatham Campus)

Extensive notes, handouts of these slides, and data files for the practicals are available at: https://exeter-data-analytics.github.io/StatModelling/

The Team

- Dr Beth Clark
- Dr Dan Padfield
- Dr Matt Silk
- Dr Richard Inger

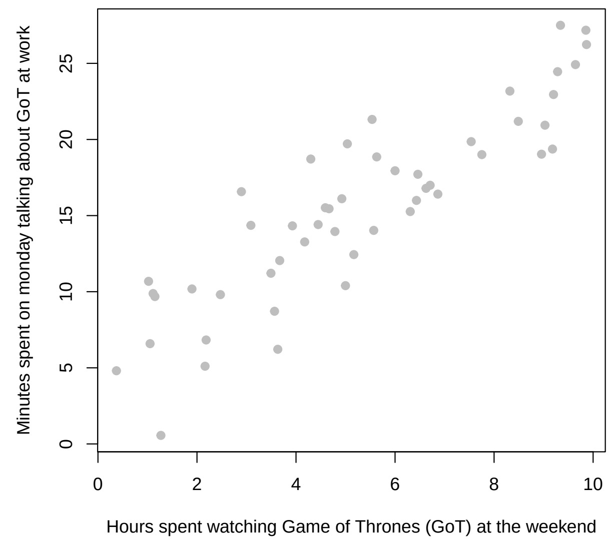
What is a model and why do we need one?

A **model** is a human construct/abstraction that tries to approximate the **data generating process** in some useful manner

Models are built for

- enhancing our understanding of a complex phenomenon
- executing "what if" scenarios
- predicting/forecasting an outcome
- controlling a system

Illustrative example



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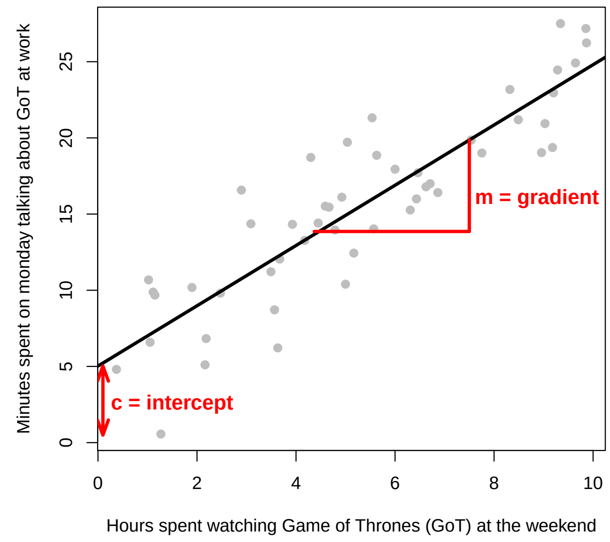

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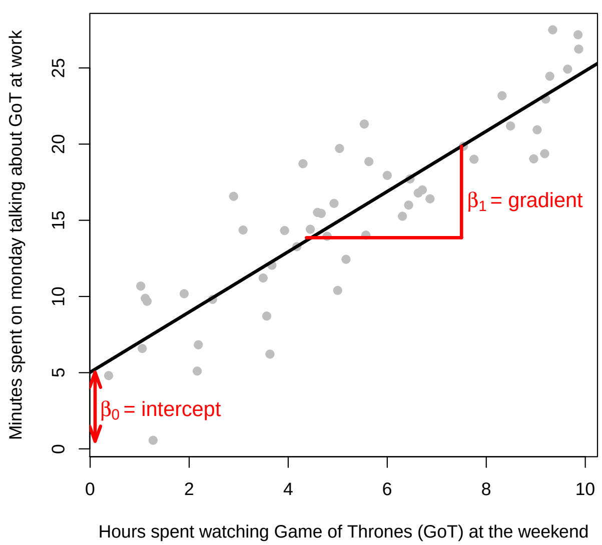


Formal definition

$$y_i = \beta_0 + \beta_1 x_i + \epsilon_i$$
$$\epsilon_i \sim \mathcal{N}(0, \sigma^2)$$

Observed data

- y (outcome/response): minutes spent talking about GoT
- x (explanatory): hours spent watching Game of Thrones (GoT)

Parameters to infer

- β_0: intercept
- β_1: gradient wrt minutes talking about GoT

Linear models in R

- Use the `lm()` function
- Requires a **formula** object
  `outcome` ∼ `explanatory variable`

```r
# talk: minutes spent talking about GoT (outcome/response variable)
# watch: hours spent watching GoT (explanatory variable)

fit <- lm(talk ~ watch)

# If data is in a data frame called "df"
fit <- lm(talk ~ watch, df)
```


Summary of fitted model

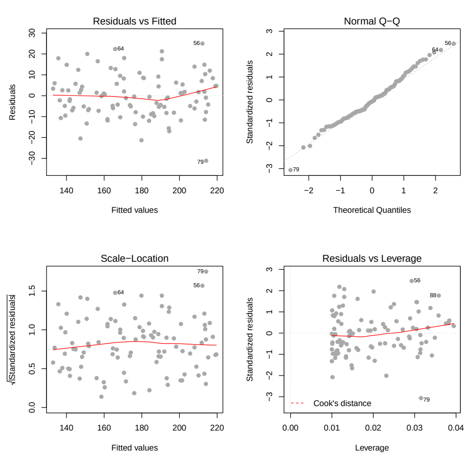
```r
summary(fit)
```

```
##
## Call:
## lm(formula = height ~ weight, data = df)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -31.089  -6.926  -0.689   6.057  24.967
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)  2.35229    7.11668   0.331    0.742
## weight       2.17446    0.08782  24.762   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 10.31 on 98 degrees of freedom
## Multiple R-squared:  0.8622, Adjusted R-squared:  0.8608
## F-statistic: 613.1 on 1 and 98 DF,  p-value: < 2.2e-16
```

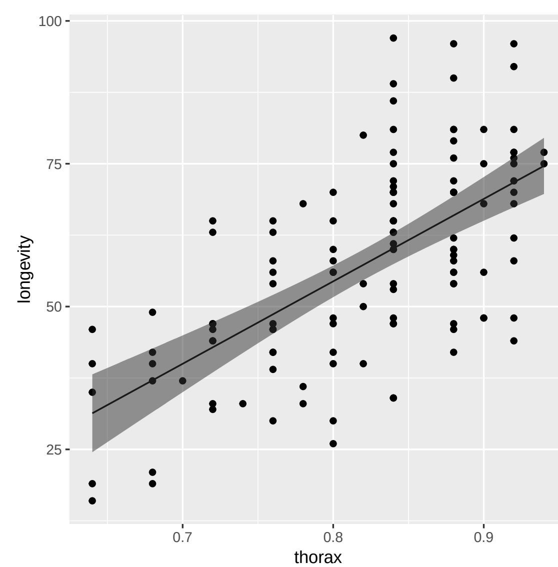

Model checking

In order to make **robust** inference, we must check the model fit

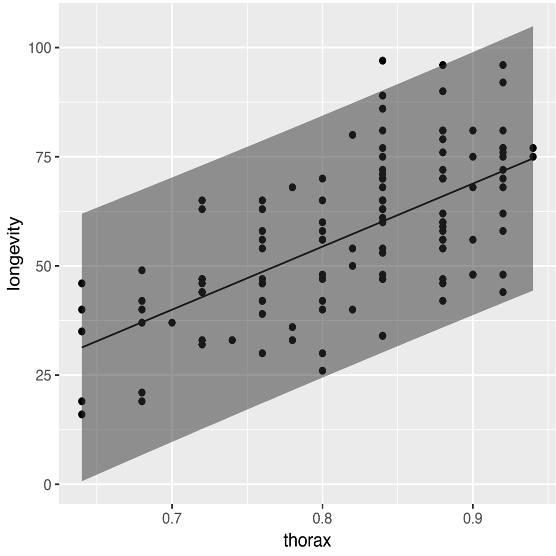
```r
plot(fit)
```



Confidence vs prediction intervals

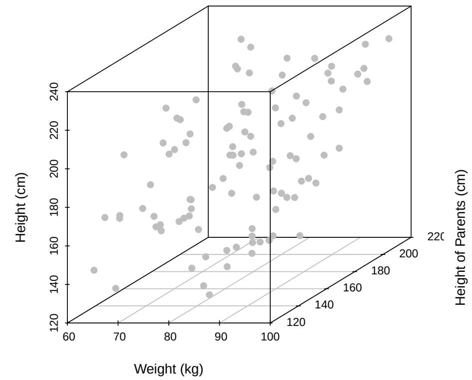


Confidence vs prediction intervals



Multiple linear regression

$$y_i = \beta_0 + \beta_1 x_{1i} + \ldots + \beta_p x_{pi} + \epsilon_i$$
$$\epsilon_i \sim \mathcal{N}(0, \sigma^2)$$



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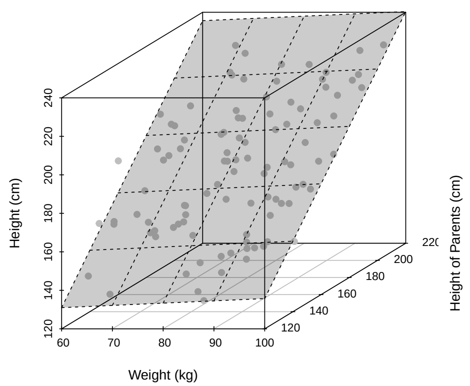
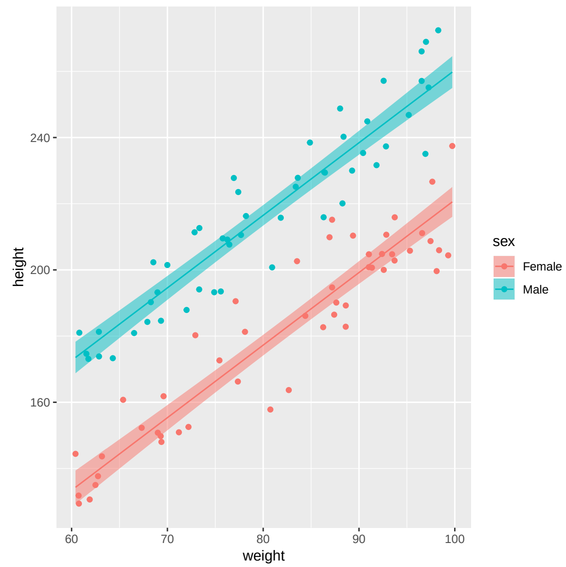


Categorical variables



Categorical variables

We need **dummy** variables

$$S_i = \begin{cases} 1 & \text{if } i \text{ is male,} \\ 0 & \text{otherwise} \end{cases}$$

Here, female is known as the **baseline/reference level**
The regression is:

$$y_i = \beta_0 + \beta_1 S_i + \beta_2 x_i + \epsilon_i$$

Or in English:

$$\text{height}_i = \beta_0 + \beta_1 \text{sex}_i + \beta_2 \text{weight}_i + \epsilon_i$$

Categorical variables

The mean regression lines for male and female are:

- Female (`sex=0`)

$$\begin{aligned} \text{height}_i &= \beta_0 + (\beta_1 \times 0) + \beta_2 \text{weight}_i \\ \text{height}_i &= \beta_0 + \beta_2 \text{weight}_i \end{aligned}$$

- Male (`sex=1`)

$$\begin{aligned} \text{height}_i &= \beta_0 + (\beta_1 \times 1) + \beta_2 \text{weight}_i \\ \text{height}_i &= (\beta_0 + \beta_1) + \beta_2 \text{weight}_i \end{aligned}$$