1.2 Individual Accuracy Test

Morning, January 28, 2012

There are 10 problems, worth 9 points each, to be solved in 30 minutes.

1. An 18oz glass of apple juice is 6% sugar and a 6oz glass of orange juice is 12% sugar. The two glasses are poured together to create a cocktail. What percent of the cocktail is sugar?

2. Find the number of positive numbers that can be expressed as the difference of two integers between -2 and 2012 inclusive.

3. An *annulus* is defined as the region between two concentric circles. Suppose that the inner circle of an annulus has radius 2 and the outer circle has radius 5. Find the probability that a randomly chosen point in the annulus is at most 3 units from the center.

4. Ben and Jerry are walking together inside a train tunnel when they hear a train approaching. They decide to run in opposite directions, with Ben heading towards the train and Jerry heading away from the train. As soon as Ben finishes his 1200 meter dash to the outside, the front of the train enters the tunnel. Coincidentally, Jerry also barely survives, with the front of the train exiting the tunnel as soon as he does. Given that Ben and Jerry both run at 1/9 of the train's speed, how long is the tunnel in meters?

5. Let ABC be an isosceles triangle with $AB = AC = 9$ and $\angle B = \angle C = 75^\circ$. Let DEF be another triangle congruent to ABC. The two triangles are placed together (without overlapping) to form a quadrilateral, which is cut along one of its diagonals into two triangles. Given that the two resulting triangles are incongruent, find the area of the larger one.

6. There is an infinitely long row of boxes, with a Ditto in one of them. Every minute, each existing Ditto clones itself, and the clone moves to the box to the right of the original box, while the original Ditto does not move. Eventually, one of the boxes contains over 100 Dittos. How many Dittos are in that box when this first happens?

7. Evaluate
$$26 + 36 + 998 + 26 \cdot 36 + 26 \cdot 998 + 36 \cdot 998 + 26 \cdot 36 \cdot 998.$$

8. There are 15 students in a school. Every two students are either friends or not friends. Among every group of three students, either all three are friends with each other, or exactly one pair of them are friends. Determine the minimum possible number of friendships at the school.

9. Let $f(x) = \sqrt{2x + 1 + 2\sqrt{x^2 + x}}$. Determine the value of
$$\frac{1}{f(1)} + \frac{1}{f(2)} + \frac{1}{f(3)} + \cdots + \frac{1}{f(24)}.$$

10. In square $ABCD$, points E and F lie on segments AD and CD, respectively. Given that $\angle EBF = 45^\circ$, $DE = 12$, and $DF = 35$, compute AB.