1.2 Individual Accuracy Test

Morning, January 26, 2013

There are 10 problems, worth 9 points each, to be solved in 30 minutes.

1. Find the largest possible number of consecutive 9's in which an integer between 10,000,000 and 13,371,337 can end. For example, 199 ends in two 9's, while 92,999 ends in three 9's.

2. Let $ABCD$ be a square of side length 2. Equilateral triangles ABP, BCQ, CDR, and DAS are constructed inside the square. Compute the area of quadrilateral $PQRS$.

3. Evaluate the expression $7 \cdot 11 \cdot 13 \cdot 1003 - 3 \cdot 17 \cdot 59 \cdot 331$.

4. Compute the number of positive integers c such that there is a non-degenerate obtuse triangle with side lengths $21, 29$, and c.

5. Consider a 5×5 board, colored like a chessboard, such that the four corners are black. Determine the number of ways to place 5 rooks on black squares such that no two of the rooks attack one another, given that the rooks are indistinguishable and the board cannot be rotated. (Two rooks attack each other if they are in the same row or column.)

6. Let $ABCD$ be a trapezoid of height 6 with bases AB and CD. Suppose that $AB = 2$ and $CD = 3$, and let F and G be the midpoints of segments AD and BC, respectively. If diagonals AC and BD intersect at point E, compute the area of triangle FGE.

7. A *regular octahedron* is a solid with eight faces that are congruent equilateral triangles. Suppose that an ant is at the center of one face of a regular octahedron of edge length 10. The ant wants to walk along the surface of the octahedron to reach the center of the opposite face. (Two faces of an octahedron are said to be opposite if they do not share a vertex.) Determine the minimum possible distance that the ant must walk.

8. Let $A_1A_2A_3$, $B_1B_2B_3$, $C_1C_2C_3$, and $D_1D_2D_3$ be triangles in the plane. All the sides of the four triangles are extended into lines. Determine the maximum number of pairs of these lines that can meet at 60° angles.

9. For an integer n, let $f_n(x)$ denote the function $f_n(x) = \sqrt{x^2 - 2012x + n} + 1006$. Determine all positive integers a such that $f_a(f_{2012}(x)) = x$ for all $x \geq 2012$.

10. Determine the number of ordered triples of integers (a, b, c) such that $(a + b)(b + c)(c + a) = 1800$.