## 1.1   Individual Speed Test

**Morning, January 30, 2010**

*There are 20 problems, worth 3 points each, and 20 minutes to solve as many problems as possible.*

1. Evaluate $\dfrac{\sqrt{2} \cdot \sqrt{6}}{\sqrt{3}}$.

2. If 6% of a number is 1218, what is 18% of that number?

3. What is the median of $\{42, 9, 8, 4, 5, 1, 13666, 3\}$?

4. Define the operation $\heartsuit$ so that $i \heartsuit u = 5i - 2u$. What is $3 \heartsuit 4$?

5. How many 0.2-inch by 1-inch by 1-inch gold bars can fit in a 15-inch by 12-inch by 9-inch box?

6. A tetrahedron is a triangular pyramid. What is the sum of the number of edges, faces, and vertices of a tetrahedron?

7. Ron has three blue socks, four white socks, five green socks, and two black socks in a drawer. Ron takes socks out of his drawer blindly and at random. What is the least number of socks that Ron needs to take out to guarantee he will be able to make a pair of matching socks?

8. One segment with length 6 and some segments with lengths 10, 8, and 2 form the three letters in the diagram shown below. Compute the sum of the perimeters of the three figures.
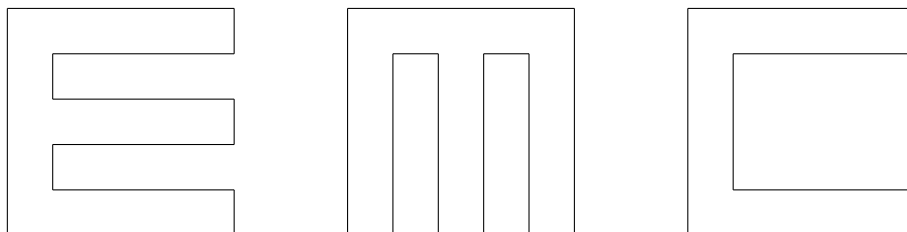


9. How many integer solutions are there to the inequality $|x - 6| \leq 4$?

10. In a land for bad children, the flavors of ice cream are grass, dirt, earwax, hair, and dust-bunny. The cones are made out of granite, marble, or pumice, and can be topped by hot lava, chalk, or ink. How many ice cream cones can the evil confectioners in this ice-cream land make? (Every ice cream cone consists of one scoop of ice cream, one cone, and one topping.)

11. Compute the sum of the prime divisors of $245 + 452 + 524$.

12. In quadrilateral $SEAT$, $SE = 2$, $EA = 3$, $AT = 4$, $\angle EAT = \angle SET = 90°$. What is the area of the quadrilateral?

13. What is the angle, in degrees, formed by the hour and minute hands on a clock at 10:30 AM?

14. Three numbers are randomly chosen without replacement from the set $\{101, 102, 103, \ldots, 200\}$. What is the probability that these three numbers are the side lengths of a triangle?

15. John takes a 30-mile bike ride over hilly terrain, where the road always either goes uphill or downhill, and is never flat. If he bikes a total of 20 miles uphill, and he bikes at 6 mph when he goes uphill, and 24 mph when he goes downhill, what is his average speed, in mph, for the ride?

16. How many distinct six-letter words (not necessarily in any language known to man) can be formed by rearranging the letters in EXETER? (You should include the word EXETER in your count.)

17. A pie has been cut into eight slices of different sizes. Snow White steals a slice. Then, the seven dwarfs (Sneezy, Sleepy, Dopey, Doc, Happy, Bashful, Grumpy) take slices one by one according to the alphabetical order of their names, but each dwarf can only take a slice next to one that has already been taken. In how many ways can this pie be eaten by these eight persons?

18. Assume that $n$ is a positive integer such that the remainder of $n$ is 1 when divided by 3, is 2 when divided by 4, is 3 when divided by 5, ..., and is 8 when divided by 10. What is the smallest possible value of $n$?

19. Find the sum of all positive four-digit numbers that are perfect squares and that have remainder 1 when divided by 100.

20. A coin of radius 1 cm is tossed onto a plane surface that has been tiled by equilateral triangles with side length $20\sqrt{3}$ cm. What is the probability that the coin lands within one of the triangles?