## 1.3 Team Round

**Morning, January 30, 2010**

*There are 15 problems, worth 20 points each, and 30 minutes to solve as many problems as possible.*

1. A very large lucky number $N$ consists of eighty-eight 8s in a row. Find the remainder when this number $N$ is divided by 6.

2. If 3 chickens can lay 9 eggs in 4 days, how many chickens does it take to lay 180 eggs in 8 days?

3. Find the ordered pair $(x, y)$ of real numbers satisfying the conditions $x > y$, $x+y = 10$, and $xy = -119$.

4. There is pair of similar triangles. One triangle has side lengths 4, 6, and 9. The other triangle has side lengths 8, 12 and $x$. Find the sum of two possible values of $x$.

5. If $x^2 + \frac{1}{x^2} = 3$, there are two possible values of $x + \frac{1}{x}$. What is the smaller of the two values?

6. Three flavors (chocolate strawberry, vanilla) of ice cream are sold at Brian's ice cream shop. Brian's friend Zerg gets a coupon for 10 free scoops of ice cream. If the coupon requires Zerg to choose an even number of scoops of each flavor of ice cream, how many ways can he choose his ice cream scoops? (For example, he could have 6 scoops of vanilla and 4 scoops of chocolate. The order in which Zerg eats the scoops does not matter.)

7. David decides he wants to join the West African Drumming Ensemble, and thus he goes to the store and buys three large cylindrical drums. In order to ensure none of the drums drop on the way home, he ties a rope around all of the drums at their mid sections so that each drum is next to the other two. Suppose that each drum has a diameter of 3.5 feet. David needs $m$ feet of rope. Given that $m = a\pi + b$, where $a$ and $b$ are rational numbers, find sum $a + b$.

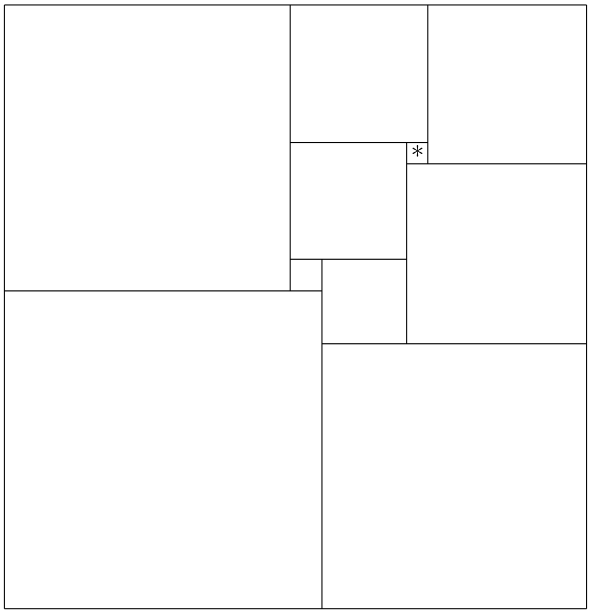
8. Segment $AB$ is the diameter of a semicircle of radius 24. A beam of light is shot from a point $12\sqrt{3}$ from the center of the semicircle, and perpendicular to $AB$. How many times does it reflect off the semicircle before hitting $AB$ again?

9. A cube is inscribed in a sphere of radius 8. A smaller sphere is inscribed in the same sphere such that it is externally tangent to one face of the cube and internally tangent to the larger sphere. The maximum value of the ratio of the volume of the smaller sphere to the volume of the larger sphere can be written in the form $\frac{a-\sqrt{b}}{36}$, where $a$ and $b$ are positive integers. Find the product $ab$.

10. How many ordered pairs $(x, y)$ of integers are there such that $2xy + x + y = 52$?

11. Three musketeers looted a caravan and walked off with a chest full of coins. During the night, the first musketeer divided the coins into three equal piles, with one coin left over. He threw it into the ocean and took one of the piles for himself, then went back to sleep. The second musketeer woke up an hour later. He divided the remaining coins into three equal piles, and threw out the one coin that was left over. He took one of the piles and went back to sleep. The third musketeer woke up and divided the remaining coins into three equal piles, threw out the extra coin, and took one pile for himself. The next morning, the three musketeers gathered around to divide the coins into three equal piles. Strangely enough, they had one coin left over this time as well. What is the minimum number of coins that were originally in the chest?

12. The diagram shows a rectangle that has been divided into ten squares of different sizes. The smallest square is $2 \times 2$ (marked with $*$). What is the area of the rectangle (which looks rather like a square itself)?




13. Let $A = (3, 2)$, $B = (0, 1)$, and $P$ be on the line $x + y = 0$. What is the minimum possible value of $AP + BP$?

14. Mr. Mustafa the number man got a $6 \times x$ rectangular chess board for his birthday. Because he was bored, he wrote the numbers 1 to $6x$ starting in the upper left corner and moving across row by row (so the number $x + 1$ is in the $2^{\text{nd}}$ row, $1^{\text{st}}$ column). Then, he wrote the same numbers starting in the upper left corner and moving down each column (so the number 7 appears in the $1^{\text{st}}$ row, $2^{\text{nd}}$ column). He then added up the two numbers in each of the cells and found that some of the sums were repeated. Given that $x$ is less than or equal to 100, how many possibilities are there for $x$?

15. Six congruent equilateral triangles are arranged in the plane so that every triangle shares at least one whole edge with some other triangle. Find the number of distinct arrangements. (Two arrangements are considered the same if one can be rotated and/or reflected onto another.)