1.2 Individual Accuracy Test

Morning, January 30, 2010

There are 10 problems, worth 9 points each, and 30 minutes to solve as many problems as possible.

1. Calculate $\left(\frac{1}{2}+\frac{1}{3}+\frac{1}{4}\right)^2$.

2. Find the 2010^{th} digit after the decimal point in the expansion of $\frac{1}{7}$.

3. If you add 1 liter of water to a solution consisting of acid and water, the new solutions will contain of 30% water. If you add another 5 liters of water to the new solution, it will contain $36\frac{4}{11}\%$ water. Find the number of liters of acid in the original solution.

4. John places 5 indistinguishable blue marbles and 5 indistinguishable red marbles into two distinguishable buckets such that each bucket has at least one blue marble and one red marble. How many distinguishable marble distributions are possible after the process is completed?

5. In quadrilateral $PEAR$, $PE = 21$, $EA = 20$, $AR = 15$, $RE = 25$, and $AP = 29$. Find the area of the quadrilateral.

6. Four congruent semicircles are drawn within the boundary of a square with side length 1. The center of each semicircle is the midpoint of a side of the square. Each semicircle is tangent to two other semicircles. Region $\mathcal{R}$ consists of points lying inside the square but outside of the semicircles. The area of $\mathcal{R}$ can be written in the form $a - b\pi$, where a and b are positive rational numbers. Compute $a + b$.

7. Let x and y be two numbers satisfying the relations $x \geq 0$, $y \geq 0$, and $3x + 5y = 7$. What is the maximum possible value of $9x^2 + 25y^2$?

8. In the Senate office in Exie-land, there are 6 distinguishable senators and 6 distinguishable interns. Some senators and an equal number of interns will attend a convention. If at least one senator must attend, how many combinations of senators and interns can attend the convention?

9. Evaluate $(1^2 - 3^2 + 5^2 - 7^2 + 9^2 - \cdots + 2009^2) - (2^2 - 4^2 + 6^2 - 8^2 + 10^2 - \cdots + 2010^2)$.

10. Segment EA has length 1. Region $\mathcal{R}$ consists of points P in the plane such that $\angle PEA \geq 120°$ and $PE < \sqrt{3}$. If point X is picked randomly from the region $\mathcal{R}$, the probability that $AX < \sqrt{3}$ can be written in the form $a - \frac{\sqrt{b}}{c\pi}$, where a is a rational number, b and c are positive integers, and b is not divisible by the square of a prime. Find the ordered triple (a, b, c).