

1.4 Guts Round

Afternoon, January 30, 2010

1.4.1 Round 1

1. [5pts] Define the operation \clubsuit so that $a \clubsuit b = a^b + b^a$. Then, if $2 \clubsuit b = 32$, what is b ?
2. [5pts] A square is changed into a rectangle by increasing two of its sides by $p\%$ and decreasing the two other sides by $p\%$. The area is then reduced by 1%. What is the value of p ?
3. [5pts] What is the sum, in degrees, of the internal angles of a heptagon?
4. [5pts] How many integers in between $\sqrt{47}$ and $\sqrt{8283}$ are divisible by 7?



1.4.2 Round 2

5. [8pts] Some mutant green turkeys and pink elephants are grazing in a field. Mutant green turkeys have six legs and three heads. Pink elephants have 4 legs and 1 head. There are 100 legs and 37 heads in the field. How many animals are grazing?
6. [8pts] Let $A = (0, 0)$, $B = (6, 8)$, $C = (20, 8)$, $D = (14, 0)$, $E = (21, -10)$, and $F = (7, -10)$. Find the area of the hexagon $ABCDEF$.
7. [8pts] In Moscow, three men, Oleg, Igor, and Dima, are questioned on suspicion of stealing Vladimir Putin's blankie. It is known that each man either always tells the truth or always lies. They make the following statements:
 - (a) Oleg: I am innocent!
 - (b) Igor: Dima stole the blankie!
 - (c) Dima: I am innocent!
 - (d) Igor: I am guilty!
 - (e) Oleg: Yes, Igor is indeed guilty!

If exactly one of Oleg, Igor, and Dima is guilty of the theft, who is the thief??

8. [8pts] How many 11-letter sequences of E's and M's have at least as many E's as M's?

1.4.3 Round 3

9. [11pts] John is entering the following summation $31 + 32 + 33 + 34 + 35 + 36 + 37 + 38 + 39$ in his calculator. However, he accidentally leaves out a plus sign and the answer becomes 3582. What is the number that comes before the missing plus sign?
10. [11pts] Two circles of radius 6 intersect such that they share a common chord of length 6. The total area covered may be expressed as $a\pi + \sqrt{b}$, where a and b are integers. What is $a + b$?
11. [11pts] Alice has a rectangular room with 6 outlets lined up on one wall and 6 lamps lined up on the opposite wall. She has 6 distinct power cords (red, blue, green, purple, black, yellow). If the red and green power cords cannot cross, how many ways can she plug in all six lamps?
12. [11pts] Tracy wants to jump through a line of 12 tiles on the floor by either jumping onto the next block, or jumping onto the block two steps ahead. An example of a path through the 12 tiles may be: 1 step, 2 steps, 2 steps, 2 steps, 1 step, 2 steps, 2 steps. In how many ways can Tracy jump through these 12 tiles?



1.4.4 Round 4

13. [14pts] What is the units digit of the number $(2^1 + 1)(2^2 - 1)(2^3 + 1)(2^4 - 1) \dots (2^{2010} - 1)$?
14. [14pts] Mr. Fat noted that on January 2, 2010, the display of the day is 01/02/2010, and the sequence 01022010 is a palindrome (a number that reads the same forwards and backwards). How many days does Mr. Fat need to wait between this palindrome day and the last palindrome day of this decade?
15. [14pts] Farmer Tim has a 30-meter by 30-meter by $30\sqrt{2}$ -meter triangular barn. He ties his goat to the corner where the two shorter sides meet with a 60-meter rope. What is the area, in square meters, of the land where the goat can graze, given that it cannot get inside the barn?
16. [14pts] In triangle ABC , $AB = 3$, $BC = 4$, and $CA = 5$. Point P lies inside the triangle and the distances from P to two of the sides of the triangle are 1 and 2. What is the maximum distance from P to the third side of the triangle?

1.4.5 Round 5

17. [17pts] Let Z be the answer to the third question on this guts quadruplet. If $x^2 - 2x = Z - 1$, find the positive value of x .
18. [17pts] Let X be the answer to the first question on this guts quadruplet. To make a FATRON2012, a cubical steel body as large as possible is cut out from a solid sphere of diameter X . A TAFTRON2013 is created by cutting a FATRON2012 into 27 identical cubes, with no material wasted. What is the length of one edge of a TAFTRON2013?
19. [17pts] Let Y be the smallest integer greater than the answer to the second question on this guts quadruplet. Fred posts two distinguishable sheets on the wall. Then, Y people walk into the room. Each of the Y people signs up on 0, 1, or 2 of the sheets. Given that there are at least two people in the room other than Fred, how many possible pairs of lists can Fred have?
20. [17pts] Let A, B, C , be the respective answers to the first, second, and third questions on this guts quadruplet. At the Robot Design Convention and Showcase, a series of robots are programmed such that each robot shakes hands exactly once with every other robot of the same height. If the heights of the 16 robots are 4, 4, 4, 5, 5, 7, 17, 17, 17, 34, 34, 42, 100, A , B , and C feet, how many handshakes will take place?



1.4.6 Round 6

21. [20pts] Determine the number of ordered triples (p, q, r) of primes with $1 < p < q < r < 100$ such that $q - p = r - q$.
22. [20pts] For numbers a, b, c, d such that $0 \leq a, b, c, d \leq 10$, find the minimum value of $ab + bc + cd + da - 5a - 5b - 5c - 5d$.
23. [20pts] Daniel has a task to measure 1 gram, 2 grams, 3 grams, 4 grams, \dots , all the way up to n grams. He goes into a store and buys a scale and six weights of his choosing (so that he knows the value for each weight that he buys). If he can place the weights on either side of the scale, what is the maximum value of n ?
24. [20pts] Given a Rubik's cube, what is the probability that at least one face will remain unchanged after a random sequence of three moves? (A Rubik's cube is a 3 by 3 by 3 cube with each face starting as a different color. The faces (3 by 3) can be freely turned. A move is defined in this problem as a 90 degree rotation of one face either clockwise or counter-clockwise. The center square on each face—six in total—is fixed.)