

## Quiz 4

● Graded

Student

Rey Stone

Total Points

7 / 20 pts

Question 1

Question 1

4 / 4 pts

1 (4 pts)

$$P(J|K) = \frac{P(J,K)}{P(K)}$$

✓ - 0 pts Correct. Answer is  $P(J|K) = \frac{P(J,K)}{P(K)}$

Question 2

Question 2a

3 / 8 pts

2a (8 pts)

You draw one coin at random from the box and flip it.

Define the following events:

$Coin_1$ : You draw one of the coins where Heads comes up twice as often as tails. (Thus  $P(A) = 5/12$ )

$Coin_2$ : You draw one of the coins where Heads comes up three times as often as tails. (Thus  $P(B) = 3/12$ )

$Coin_3$ : You draw an Unbiased coin. (Thus  $P(U) = 4/12 \approx 0.368$ )

We also know:

$$P(T|Coin_1) = 1/3$$

$$P(T|Coin_2) = 1/4 \text{ and}$$

$$P(T|Coin_3) = 1/2$$

By the Law of Total Probability:

$$P(T) = P(T, Coin_1) + P(T, Coin_2) + P(T, Coin_3)$$

$$= P(T|Coin_1)P(Coin_1) + P(T|Coin_2)P(Coin_2) + P(T|Coin_3)P(Coin_3)$$

$$= \left(\frac{1}{3}\right)\left(\frac{5}{12}\right) + \left(\frac{1}{4}\right)\left(\frac{3}{12}\right) + \left(\frac{1}{2}\right)\left(\frac{4}{12}\right) = \frac{53}{144} \approx 0.368$$

✓ - 1 pt Incorrect  $P(Tails|Coin_1) = 1/3$

✓ - 1 pt Incorrect  $P(Tails|Coin_2) = 1/4$

✓ - 1 pt Incorrect  $P(Tails|Coin_3) = 1/2$

✓ - 2 pts Need to use Law of Total Probability.

Question 3

Question 2b

0 / 8 pts

2b (8 pts)

You draw one coin at random from the box and flip it.

Define the following events:

A: You draw one of the coins where Heads comes up twice as often as tails. (Thus  $P(A) = 5/12$ )

B: You draw one of the coins where Heads comes up three times as often as tails. (Thus  $P(B) = 3/12$ )

U: You draw an Unbiased coin. (Thus  $P(U) = 4/12$ )

We also know:

$$P(T|A) = 1/3$$

$$P(T|B) = 1/4$$

$$P(T|U) = 1/2$$

Thus:

$$P(U|T) = \frac{P(U,T)}{P(T)} = \frac{P(T|U)P(U)}{P(T)} \text{ (Bayes' Thm)} = \frac{\left(\frac{1}{2}\right)\left(\frac{4}{12}\right)}{\left(\frac{1}{3}\right)\left(\frac{5}{12}\right) + \left(\frac{1}{4}\right)\left(\frac{3}{12}\right) + \left(\frac{1}{2}\right)\left(\frac{4}{12}\right)} = \frac{24}{53}$$

✓ - 4 pts Incorrect numerator. Should be  $P(Tails|Unbiased)P(Unbiased) = \left(\frac{1}{2}\right)\left(\frac{4}{12}\right) = \frac{4}{24}$

✓ - 4 pts Incorrect or missing  $P(T) = P(Tails|A)P(A) + P(Tails|B)P(B) + P(Tails|Unbiased)P(Unbiased)$  in denominator

---

Write clearly and in the box:

Name: <i>Ray Stone</i>	Student ID: <i>1110 44637 (?)</i>
------------------------	-----------------------------------

**Quiz Rules:**

**DO NOT TURN THIS PAGE OVER UNTIL THE QUIZ BEGINS.**

- All cell phones must be stored in your backpack. If you have a cell phone anywhere on your body or at your desk during this quiz you will receive a 0 on this quiz.
- You are allowed a two-sided 8.5" x 11" crib sheet with hand-written (not typed) notes
- You are allowed a calculator
- You are allowed to use the Data Wrangling with Pandas Cheatsheet from the Canvas Modules
- **No tablets, smartphones, smartwatches or any other electronic devices allowed.**
- No collaboration with other students is allowed during this quiz.
- **Show all work and simplify your answers!**
- You have **15 minutes** for this quiz.

---

Once the quiz begins you can use this extra space for your work if you need more space.

Answer:

1. (4 pts) Let  $J$  and  $K$  be events with  $P(J) > 0$  and  $P(K) > 0$ . What is the **mathematical definition** of the conditional probability of  $J$  given  $K$ ?  
Give your answer in the box provided using correct mathematical notation.

$$P(J|K) = \frac{P(J \cap K)}{P(K)}$$

2. You have a box with 12 coins in it:

- $\frac{5}{12}$  A • 5 of the coins are biased such that **heads comes up twice as often** as tails.  
 $\frac{3}{12}$  B • 3 of the coins are biased such that **heads comes up three times as often** as tails.  
 $\frac{4}{12}$  C • The remaining coins are unbiased, so heads and tails come up equally as often.

ANSWER:

$$\left(\frac{5}{24}\right) \left(\frac{1}{12}\right) \left(\frac{4}{12}\right)$$

T = tails

- (a) (8 pts)

You choose a coin at random from this box and flip it.

What is the probability that it lands on **tails**?

Show work in the space below, justifying all steps. Answers without any work/justification will receive 0 points. You can leave your answer unsimplified.

Prob A:  $\frac{5}{12}$   $(A|T) = \left(\frac{5}{12} \cdot \frac{1}{2}\right) = \frac{5}{24}$

Prob B:  $\frac{3}{12}$   $(B|T) = \left(\frac{3}{12} \cdot \frac{1}{3}\right) = \frac{1}{12}$

Prob C:  $\frac{4}{12}$   $(C|T) = \left(\frac{4}{12} \cdot 1\right) = \frac{1}{3}$

- (b) (8 pts) You choose a coin at random from this box and flip it.

**It comes up tails.** Given this information, what is the probability that the coin that you chose was one of the **unbiased** coins?

Show work in the space below, justifying all steps. Answers without any work/justification will receive 0 points. You can leave your answer unsimplified.

ANSWER:

$$\left(\frac{4}{24}\right) \left(\frac{5}{24}\right) \left(\frac{1}{12}\right)$$

Event A: unbiased coins

Event B = tails

$$P(B|A) = \frac{P(A|B) P(B)}{P(A)}$$

Event A:  $\frac{4}{12}$

Event B:  $\left(\frac{5}{24}\right) \left(\frac{1}{12}\right) \left(\frac{4}{12}\right)$

Event A|B:  $\frac{4}{12} \cdot \frac{1}{2} = \frac{1}{6}$

$$\frac{\left(\frac{4}{24}\right) \left(\frac{5}{24}\right) \left(\frac{1}{12}\right) \left(\frac{4}{12}\right)}{\left(\frac{4}{12}\right)}$$