HW1 Manually Graded: Upload PDF here

● Graded

Student

Rey Stone

Total Points

12 / 15 pts

Question 1

Question 9

3 / 3 pts

Question 9 (3 pts)

```python
def summation(n):
    """Compute the summation i^3 + 3 * i^2 for 1 <= i <= n."""
    # BEGIN SOLUTION
    return sum((np.arange(1, n + 1) ** 3) + (3 * np.arange(1, n + 1) ** 2))
    # END SOLUTION

#Do not change the following cells:
print("summation(5) =  ", summation(5))
print("summation(200) = ", summation(200))

```

```
summation(5) =   390
summation(200) =  412070100
```

✔ **– 0 pts** Completely correct

Question 2

Question 10

2 / 3 pts

Question 10:

Solution Option 1:

We can see that this is the area of a large triangle from $4 \le x \le 7$ minus the smaller triangle from $4 \le x \le 6$:

Area of triangle = $\frac{1}{2}(base)(height)$

Large triangle minus small triangle = $\frac{1}{2}(3)\frac{1}{4} - \frac{1}{2}(2)(\text{height at } 6)$

Thus we need to find the height of the line when $x = 6$.

To do this we find the equation of the line that passes through the points $(4, 0.25)$ and $(7, 0)$.

This line is given by: $y - 0 = \frac{0.25-0}{4-7}(x-7) \implies y = -\frac{1}{12}(x-7)$.

Thus when \$x=6\$, the height of the line is: $y = -\frac{1}{12}(6-7) = \frac{1}{12}$

Thus: Area of trapezoid = Large triangle minus small triangle = $\frac{1}{2}(3)\frac{1}{4} - \frac{1}{2}(1)(\frac{1}{12}) = \frac{1}{3}$

Thus the total area = $\frac{3}{16} + \frac{1}{3} = \boxed{\frac{25}{48}} \approx 0.520833$

Solution Option 2:

The area of a trapezoid is $\frac{1}{2}(b)(h_1 + h_2)$.

We can see visually that $b = 2$ and $h_1 = 0.25$, however we need to find h_2.

Thus we need to find the height of the line when $x = 6$. (See work shown in Option 1 above):

Thus when $x = 6$, the height of the line is: $y = -\frac{1}{12}(6-7) = \frac{1}{12}$

Thus the area of the trapezoid = $\frac{1}{2}(2)(\frac{1}{4} + \frac{1}{12}) = \frac{1}{3}$

Thus the total area = $\frac{3}{16} + \frac{1}{3} = \frac{25}{48} \approx 0.520833$

Option 3: Using integration:

Area = $\int_{3.25}^{4} \frac{1}{4}\,dx + \int_4^6 -\frac{1}{12}(x-7)\,dx$

$$= \frac{3}{16} - \left(\frac{1}{24}(x-7)^2 \Big|_4^6 \right) = \frac{3}{16} - \left(\frac{1}{24} - \frac{9}{24}\right) = \frac{3}{16} + \frac{1}{3} = \frac{25}{48} \approx 0.520833$$

✔ **– 1 pt** Incorrect or incomplete work justifying area of trapezoid portion of region

1 Please include steps as to how you got this formula.

Question 3

Question 11b

1 / 3 pts

11b (3 pts)

Solution

To find the probability of at most adding to 9 we will calculate:

$1 - P(\text{sum of } 10) - P(\text{sum of } 11) - P(\text{sum of } 12)$

$P(\text{sum of } 12) = \frac{1}{36}$ (since there are (6)(6)=36 possible outcomes and only the outcome with 2 sixes adds to 12)

$P(\text{sum of } 11) = \frac{2}{36}$ (i.e. either 5 and 6 or 6 and 5)

$P(\text{sum of } 10) = \frac{3}{36}$ (either (6, 4), (4, 6), (5,5))

Thus $P(\text{sum of at most } 9) = 1 - \frac{3}{36} - \frac{2}{36} - \frac{1}{36} = \frac{30}{36} = \boxed{\frac{5}{6}}$

✔ **– 2 pts** Insufficient justification (need to show much more detailed steps explaining how you got to the final answer).

Question 4

Question 11c

3 / 3 pts

11c (3 pts)

Solution

On any given question, you have a $1/5$ probability of guessing the correct answer and a $4/5$ probability of guessing incorrectly.

There are $C(10, 3) = \frac{10!}{7!3!} = \frac{10*9*8}{3*2} = 120$ different ways to select the exact 3 questions out of 10 you get correct.

The probability of any one of these is $(\frac{1}{5})^3(\frac{4}{5})^7$.

Thus the total probability is $C(10, 3)(\frac{1}{5})^3(\frac{4}{5})^7 = \boxed{120\left(\frac{4^7}{5^{10}}\right)}$

$\approx 20.13\%$

✔ **– 0 pts** Correct

Question 5

Question 11d

3 / 3 pts

11d (3 pts)

SOLUTION We'll use the addition rule for non-disjoint events:

Number of strings with 01 in the beginning + number of strings with 100 at the end minus the number with both (because we've double counted the number with both).

$2^7 + 2^6 - 2^4 = 176$

✔ **– 0 pts** Completely correct

Question assigned to the following page: 1

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0.1 Question 9 (3 pts)

Write a function `summation(n)` that uses vectorization in Numpy to evaluate the following summation for $n \geq 1$:

$$\sum_{i=1}^{n} (i^3 + 3i^2)$$

Note: You should **NOT** use ANY `for` loops in your solution. You may find `np.arange` helpful for this question!

```
In [330]: def summation(n):
              """Compute the summation i^3 + 3 * i^2 for 1 <= i <= n."""
              return (np.sum(np.arange(1, n+1)**3)) + 3*(np.sum(np.arange(1, n+1)**2))

          #Do not change the following cells:
          print("summation(5) =  ", summation(5))
          print("summation(200) = ", summation(200))
```

```
summation(5) =   390
summation(200) =  412070100
```

```
In [331]: grader.check("q9")
```

```
Out[331]: q9 results: All test cases passed!
```

Question assigned to the following page: 2

0.2 Question 10 (3 pts):

Areas under curves will be used in this class when we calculate some probabilities.

Consider the function $f(x)$ shown below:

Calculate $\int_{3.25}^{6} f(x)\,dx$. You can use geometry and/or calculus. Show all steps/explain your reasoning.

Question 10 Solution) **Type your work answering to Question 10** in this cell (use LaTeX for any math equations). Show all of your steps and fully justify your answer. Do not add any additional cells to this part.

From 3.25 to 4, the slope of the line, is just $y = 0.25$, so the integral would look like:

$\int_{3.25}^{4} 0.25\,dx$

$\implies 0.25x\Big|_{3.25}^{4}$

$\implies 0.25(4) - 0.25(3.25) = 1 - 0.8125 = 0.1875 = \frac{3}{16}$

From 4 to 6, the slope of the line is $y = -\frac{1}{12}x + \frac{7}{12}$, so the integral would look like:

$\int_{4}^{6} -\frac{1}{12}x + \frac{7}{12}\,dx$

$\implies -\frac{x^2}{24} + \frac{7}{12}x\Big|_{4}^{6}$

$\implies [-\frac{6^2}{24} + \frac{7}{12}(6)] - [-\frac{4^2}{24} + \frac{7}{12}(4)]$

$\implies -\frac{36}{24} + \frac{42}{12} - [-\frac{16}{24} + \frac{28}{12}] = 2 - \frac{5}{3} = \frac{1}{3}$

Now that we have both parts integrated, all we have to do is add them together:

$\frac{3}{16} + \frac{1}{3} = \frac{25}{48}$

Question assigned to the following page: 3

0.3 Question 11b (3 pts)

What is the probability that if I roll two 6-sided dice they add up to **at most** 9? Use LaTeX (not code) in the cell directly below to show all of your steps and fully justify your answer.

Let's consider all the possibilities of which if I roll two 6-sided dice, they are 9 or greater.

Since there are 36 combinations of all outcomes, we can model these outcomes as such:

–	**1**	**2**	**3**	**4**	**5**	**6**
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

There are only 6 combinations that are greater than 9, so, the probability that two 6-sided dice add up to at most 9 are:

$\frac{30}{36}$

Question assigned to the following page: 4

0.4 Question 11c (3 pts)

Suppose you uncharacteristically show up to a quiz completely unprepared. The quiz has 10 questions, each with 5 multiple choice options. You decide to guess each answer in a completely random way. What is the probability that you get exactly 3 questions correct? Use Markdown and LaTeX (not code) in the cell directly below to show all of your steps and fully justify your answer.

We can use the Binomial Distribution equation to answer this question.

The Binomial equation is as follows:

$\binom{n}{x} p^x q^{n-x}$

Where n = number of questions

x = successful question answered

p = probability of successful question

q = probability of failing

We can just plug in all our values, knowing that the probability of getting one correct answer is $\frac{1}{5}$

$\binom{10}{3} \frac{1}{5}^3 \frac{4}{5}^{10-3} = \frac{393216}{1953125}$

Question assigned to the following page: 5

0.5 Question 11d (3 pts):

A 9-bit string is sent over a network. The receiver only accepts strings that either start with 01 and/or end with 100. How many unique 9-bit strings will the receiver accept? Explain/justify using math.

Enter your answer for part 11d) in this cell (double click on this cell and write all steps using Markdown and LaTex):

First we should consider strings that all start with 01

0 1 _ _ _ _ _ _ _ $= 2^7 = 128$ possible combinations

Now we can consider strings that all end with 100:

_ _ _ _ _ _ 1 0 0 $= 2^6 = 64$ possible combinations

Now, we have to use the Inclusion-Exclusion principle, to not double count strings that start with 01 AND end with 100

Since the receiver accepts strings that start with both 01 and end with 100, there are:

0 1 _ _ _ _ 1 0 0 $= 2^4 = 16$ possibilities

Thus, we have $128 + 64 - 16 = 176$