Quiz 5

Student

Rey Stone

Total Points

18 / 20 pts

Question 1

Question 1   **8** / 8 pts

Question 2 (8 pts)

Start by finding $P(S, W)$. Using the inclusion-exclusion principal we have:

$$P(S \cup W) = P(S) + P(W) - P(S, W)$$

$$\implies 0.60 = 0.50 + 0.40 - P(S, W)$$

$$\implies P(S, W) = .5 + .4 - .6 = 0.30$$

In this case, the events S and W are DEPENDENT. Need to mathematically show any one of the 3 inequalities shown below.

$$\left(P(S, W) = \tfrac{3}{10}\right) \neq \left(P(S)P(W) = \tfrac{1}{2} \cdot \tfrac{2}{5}\right)$$

$$\left(P(S|W) = \tfrac{3}{4}\right) \neq \left(P(S) = \tfrac{1}{2}\right)$$

$$\left(P(W|S) = \tfrac{3}{5}\right) \neq \left(P(W) = \tfrac{2}{5}\right)$$

✔ **− 0 pts** Correct!

Question 2

2.1 **Question 2a** **5** / 7 pts

Question 2a (7 pts)

k	$P(X = k)$
$-\$1$	$\dfrac{11}{18}$
$\$1.50$	$\dfrac{11}{36}$
$\$2$	$\dfrac{1}{12}$

✔ **− 2 pts** Incorrect $P(X = 1.50) = \frac{11}{36}$ or missing justification

2.2 **Question 2b** **5** / 5 pts

Question 2b (5 pts)

$E[X] = 2P(X = 2) + 1.50P(X = 1.50) + (-1)P(X = -1)$

$= \left(2 \cdot \frac{1}{12}\right) + \left(1.50 \cdot \frac{11}{36}\right) + \left(-1 \cdot \frac{11}{18}\right) = \frac{1}{72} \approx 0.014$

Units: Dollars

✔ **− 0 pts** Correct or follows from above

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Quiz Rules:

DO NOT TURN THIS PAGE OVER UNTIL THE QUIZ BEGINS.

- All cell phones must be stored in your backpack. If you have a cell phone anywhere on your body or at your desk during this quiz you will receive a 0 on this quiz.

- You are allowed a two-sided 8.5" x 11" crib sheet with hand-written (not typed) notes

- You are allowed a calculator

- **No tablets, smartphones, smartwatches or any other electronic devices allowed.**

- No collaboration with other students is allowed during this quiz.

- **Show all work and simplify your answers!**

- You have **15 minutes** for this quiz.

Once the quiz begins you can use this extra space for your work if you need more space.

1. (8 pts) Suppose you are given the following information about the weather probabilities for tomorrow:

   - The probability of snow tomorrow is 50%, $P(S) = 50\%$

   - The probability of strong winds tomorrow is 40%, W

   - The probability of snow **or** strong winds tomorrow is 60%. $P(S \text{ or } W) = 60\%$

   Are the events "snow tomorrow" (S), and "strong winds tomorrow" (W)
   independent or dependent? Justify your answer using the **mathematical definition of**
   independence. *Show work in the space below, justifying all steps. Correct answers without
   any work/justification will receive 0 points.*

   ANSWER:

   not independent

   DEPENDENT

   if $P(S,W) = P(S) \cdot P(W)$ then independent

   $P(S) = 0.5$

   $P(W) = 0.4$

   $P(S) \cdot P(W) = 0.2$

   $P(S,W) = 0.3$ ✗

   $P(S) P(W) = 0.2$ ✗

   $P(S \text{ or } W) = P(S) + P(W) - P(S \text{ and } W)$

   $0.6 = 0.5 + 0.4 - P(S \cap W)$

   $0.3 = P(S \cap W)$

   2

2. Suppose a casino introduces a new game. One round of the game consists of rolling a **pair** of fair 6-sided dice.
   Here are the payouts for the game based on a $1 bet:

   A
   - If you roll **at least one** 5, your net winnings is $1.50 (i.e. you get your original dollar back, plus $1.50)
   - If the sum of the outcomes on the two dice add up to 4, your net winnings is $2 (i.e. you get your original dollar back, plus $2)
   - Otherwise you lose your dollar (your net winnings is −$1).

   Let X be the random variable that represents your net winnings from betting $1 and playing one round of this game.

   (a) (7 pts) What is the **probability mass function of** X? Give your answer as a table.
   Show work in the space below, justifying all steps. Answers without any work/justification will receive 0 points:

k	$P(X=k)$
1.5	2/36
2	3/36
−1	31/36

   $P(D_1) = 5 = \frac{1}{6}$ ⟩ $\frac{1}{36}$
   $P(D_2) = 5 = \frac{1}{6}$

   $D_1 + D_2 = 4$
   $(1,3),(3,1)(2,2) = 3/36$

k	$P(X=k)$
1.5	3/36
2	3/36
−1	30/36

   Answer:

   $1.5\left(\frac{3}{36}\right) + 2\left(\frac{3}{36}\right) + (-1)\left(\frac{30}{36}\right)$

   (b) (5 pts) What is the **expected value** of the net winnings you would make from betting $1 and playing one round of this game?
   Include units. You can leave your final answer **unsimplified**.
   Show work in the space below, justifying all steps. Answers without any work/justification will receive 0 points:

   Units:

   $ (dollars)

   $E[X] = 1.5\left(\frac{3}{36}\right) + 2\left(\frac{3}{36}\right) + (-1)\left(\frac{30}{36}\right)$