

Reservoir Computing Stuff

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Hallo

1 todo

1.1 things on the todo list.

1.1.1 programming work

1.1.2 research work

- check different weights
- check symmetry breaking nodes
- check "standard" topologies
- check breaking of standard topologies (extra links)

1.1.3 graphics/plots

- stuart landau oscillator: complex plain, driven by input, with delayed feedback
- linear memory recall curve
- feed in plus system response for different ratios of theta and lambda
- rN from 1 to 32, rN from 1 to 32 with plot.

here comes the intro and all the important references... small world [WAT98].

2 one

2.1 Introduction

Reservoir Computing encompasses the field of machine learning in which a wide variety of systems (reservoirs) are used to contain information and perform calculations on it. Reservoir Computing is particularly interesting as computations can be performed by the physical system directly. The "classical" Turing-machine as a model is more like a virtual space created in our physical space. Within this virtual space everything is defined only in discrete units of "0" and "1". In order to create such a virtual space the usual unpredictability that inhabits the scales in which modern computer circuits exist in has to be tamed in order to create this virtual computing space. As the scales of modern transistors shrink they rapidly approach the scales in which quantum effects become problematic. The maintaining of this virtual space of 0 and 1 in which all our computations are performed is increasingly difficult. Modern CPU manufacturing has to take into account many error compensation algorithms in. Simultaneously manufacturing is becoming more challenging as well since the scales of modern transistors are so small the light sources needed for lithography are becoming rare.

Reservoir computing is the circumventing of this virtual space of discrete values in order to perform computations directly in physical systems. A wide variety of systems can be used e.g. a literal bucket of water can act as a reservoir that performs computations [FER03]. Albeit the most interesting applications lie in potential optical computers. RC offers a way of utilizing the highly complex dynamics of optical systems in order to perform computation on them. Optical Computers in the form of lasers appear to be an ideal application of RC, because of the timescales that laser dynamics. Optical reservoir computers already perform classification tasks on very timescales unmatched by modern silicon electronics. Another expected benefit would be the vastly lower power consumption.

In recent years reservoir computing has received a lot of attention as bridge between machine learning and physics. As the end of "Moore's Law" is slowly encroaching we are in the waning years of an age of staggering performance leaps in silicon electronic circuits. Reservoir Computing as a way of utilizing

[SAN17a]

[LAR12] i fischer.

2.1.1 blabla

here comes the intro and all the important references...

2.2 Theory

2.2.1 Stuart-Landau-Oscillator

The Stuart-Landau oscillator is a dynamical system often used to model basic class 1 lasers. It can be written either as a single complex differential equation (2.1) or a set of

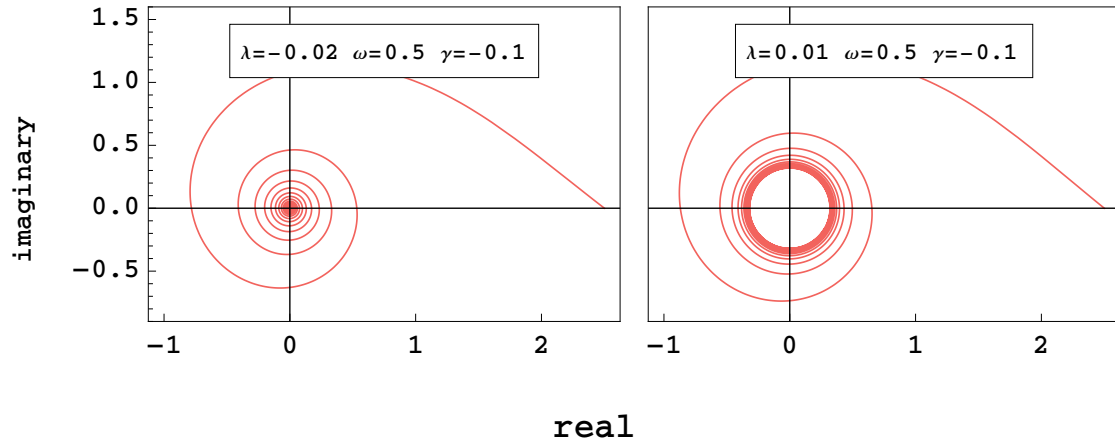


Figure 2.1: 2 very basic scenarios of the Stuart-Landau oscillator: Decay towards a single fixed point (left) or towards a limit cycle (right).

two equations written in polar coordinates (2.2). From the equation in polar coordinates easy to see that the equation has rotational symmetry as the radial differential equation does not change with the dynamical variable ϕ .

$$\dot{z} = (\lambda + i\omega + \gamma|z|^2) z \quad (2.1)$$

$$\begin{aligned} \dot{r} &= \lambda r + \text{Re}(\gamma) r^3 \\ \dot{\phi} &= i\omega + \text{Im}(\gamma) r^2 \end{aligned} \quad (2.2)$$

For the radial dynamical variable the Stuart-Landau oscillator has two fixed points where the derivative \dot{r} vanishes $r = 0$ and $r = \sqrt{-\lambda/\text{Re}(\gamma)}$ whose stability depends on λ and $\text{Re}(\gamma)$. For $\text{Re}(\gamma) < 0$ (supercritical case).

The limit cycle (LC) which is shown in fig 2.1 is depending on the ratio of λ and $\text{Re}[\gamma]$.

As can be seen in (eq. 2.1), the equation has a linear and a nonlinear term regarding the absolute value of z .

2.2.2 Networks

Vertices blabla Edges blabla.

circulant Matrix

A circulant matrix has the same entries its row vectors, but with its entries rotated one element to the right relative to the previous row.

2.2.3 virtual Nodes and multiplexing

here: papers for explanation! By multiplexing the input signal one can create virtual nodes in a network. The analogy to a real network can be best understood if the input signal is masked with a binary mask containing only values of either 0 or 1.

here add dependency of total linear memory on number of nodes and virtual nodes.

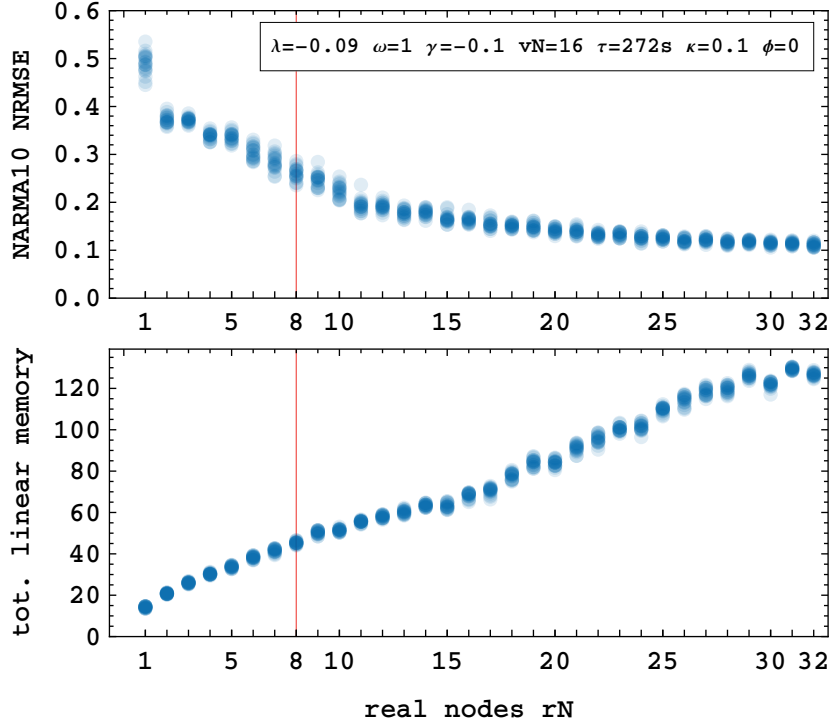


Figure 2.2: changing rc performance for increasing number of real nodes rN in unidirectionally coupled ring networks. (see some plot).

2.2.4 Dynamics of rings of stuart landau oscillators

pony-states (von André)

2.2.5 Reservoir computing

Measuring computation performance

We can measure how well a dynamical system perform computations by testing it in a variety of benchmarks. Dynamical systems are continuously evolving in time, thus reservoir computation mostly is mostly tested on timeseries data. There exist also approaches of using RC on datasets that do not involve datasets without a temporal dimension e.g. image classification of single images []

2.2.6 Reservoir computing tasks

The reservoir computing performance of a given dynamic system can be quantified by testing its predictions for certain tasks. In machine learning the task is usually to predict a certain value from an initial set of inputs. Ideally the prediction can then be compared to the base truth and the difference between prediction and base truth is quantified as the error. The closer the prediction is to the ground truth, the better the system performs a given task.

Linear Memory Recall

The simplest task a reservoir can perform is to repeat the the information that was fed into it at a certain point in time.

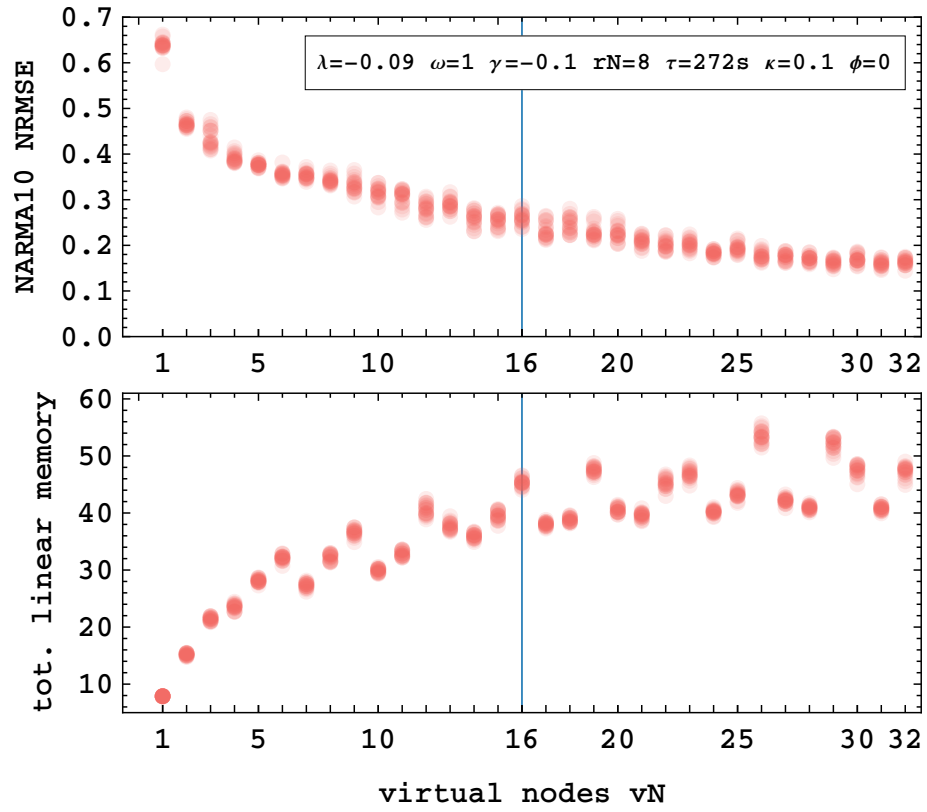


Figure 2.3: changing rc performance for increasing number or virtual nodes rN in unidirectionally coupled ring networks. (see some plot).

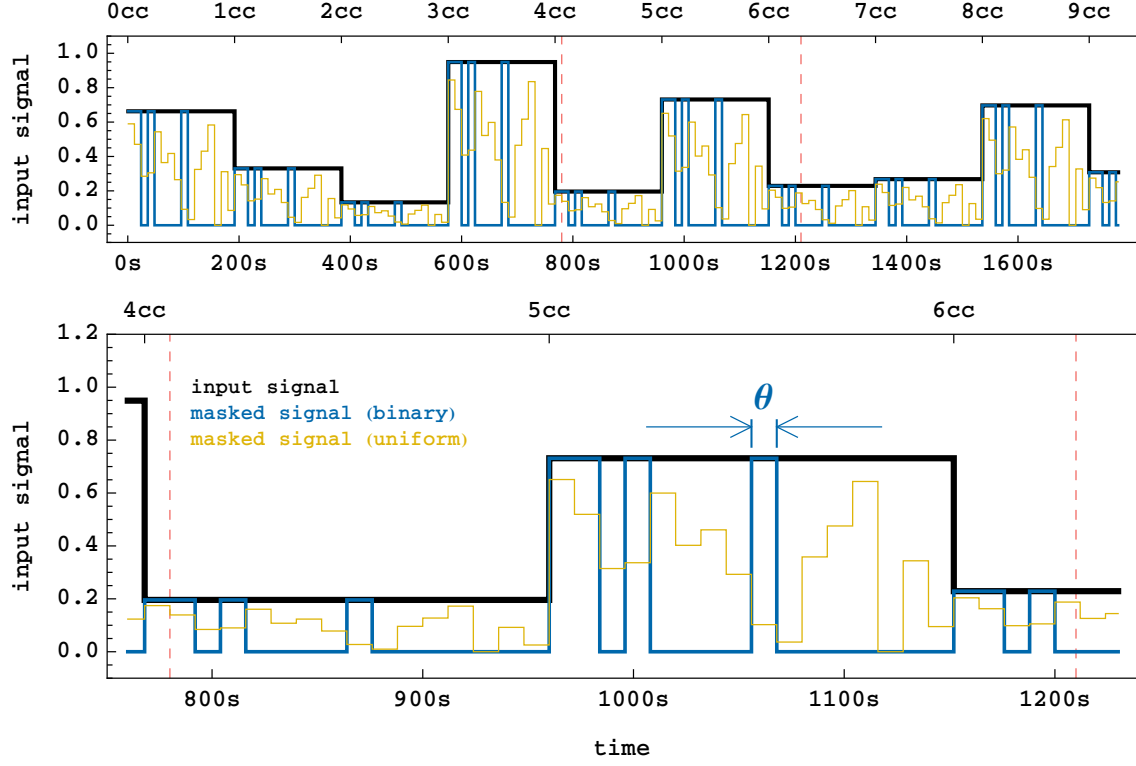


Figure 2.4: A timeseries (black) with constant interpolation ("sample & hold") between samples and the corresponding masked signal (blue). The mask length is counted in clockcycles (cc) and the time per virtual node is counted in θ . Here $\theta = 12s$ and $1cc = 16\theta = 272s$

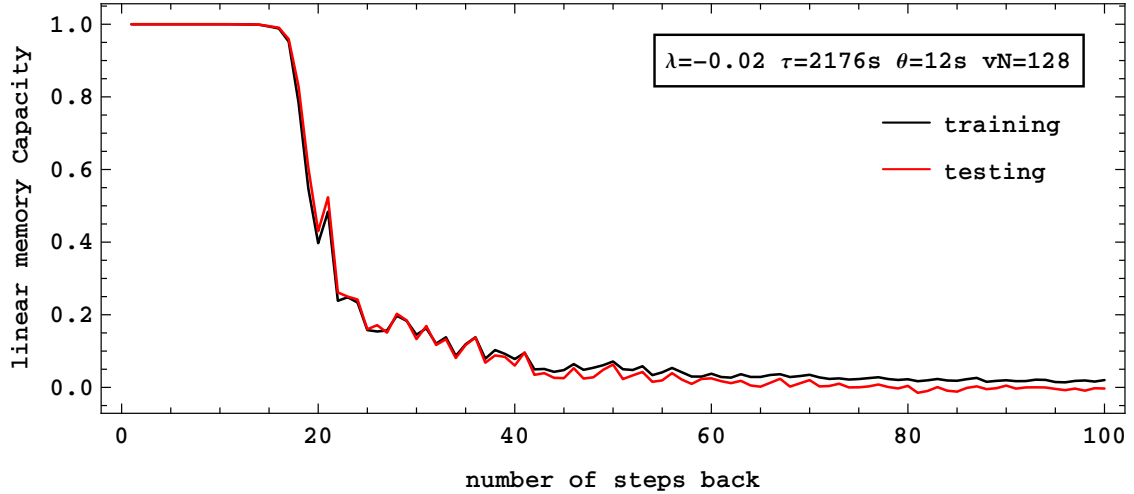


Figure 2.5: The linear memory capacities for differently many steps into the past. The system is able to perfectly reproduce inputs up until 12 steps into the past. $N = 1, vN = 128, \lambda = -0.02, \omega = 1, \gamma = -0.1, \theta = 12, \tau = 2176$.

NARMA10 NRMSE

2.3 Results

2.3.1 Highly symmetrical network topologies

Ring networks

Rings with from $N=1$ (edge case) to $N=16$ Bidirectional Rings Bidirectional Rings with self-feedback and diffuse coupling

2.3.2 All to all coupled networks

$N=3$ - $N=16$ row normalization

2.3.3 Less symmetrical network topologies

unidirectional rings with jumps

Bibliography

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