Reservoir Computing Stuff

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Hallo

1 todo

1.1 things on the todo list.

1.1.1 programming work

1.1.2 research work

- check different weights
- check symmetry breaking nodes
- check "standard" topologies
- check breaking of standard topologies (extra links)

1.1.3 graphics/plots

- stuart landau oscillator: complex plain, driven by input, with delayed feedback
- linear memory recall curve
- feed in plus system response for different ratios of theta and lambda
- rN from 1 to 32, rN from 1 to 32 with plot.

here comes the intro and all the important references... small world [WAT98].

1.1.4 theory

- reservoir computing. definitions RC with delay, cc, masks, virtual nodes (more later)
- virtual networks through multiplexing. breakdown of analogy (random masks)
- tasks, training, covariance
- linear memory, legendre polynomials, narma
- linear regression. result: weights (plot weights for linear memories).
- networks

1.1.5 results

- increasing network size
- increasing virtualization
- increasing input strength

2 one

2.1 Introduction

Reservoir Computing encompasses the field of machine learning in which the capacity to store and compute transform data is investigated in a wide variety of dynamical systems (reservoirs). Reservoir Computing (RC) is particularly interesting as computations can be performed by the physical systems directly. Today "classical" computers of the van-Neumann-type (or more generally of the Turing-machine-type) need rely on the existence of a digital space in which everything can be represented by a combination of discrete values (0 and 1). In order to establish such a computing space in our analogue world

in the sense of the Turing-machine as a model is more like a virtual space created in our physical space. Within this virtual space everything is defined only in discrete units of "0" and "1". In order to create such a virtual space the usual unpredictability that inhabits the scales in which modern computer circuits exist in has to be tamed in order to create this virtual computing space. As the scales of modern transistors shrink they rapidly approach the scales in which quantum effects become problematic. The maintaining of this virtual space of 0 and 1 in which all our computations are performed is increasingly difficult. Modern CPU manufacturing has to take into account many error compensation algorithms in. Simultaneously manufacturing is becoming more challenging as well since the scales of modern transistors are so small the light sources needed for lithography are becoming rare.

Reservoir computing is the circumventing of this virtual space of discrete values in order to perform computations directly in physical systems. A wide variety of systems can be used e.g. a literal bucket of water can act as a reservoir that performs computations [FER03]. Albeit the most interesting applications lie in potential optical computers. RC offers a way of utilizing the highly complex dynamics of optical systems in order to perform computation on them. Optical Computers in the form of lasers appear to be an ideal application of RC, because of the timescales that laser dynamics. Optical reservoir computers already perform classification tasks on very timescales unmatched by modern silicon electronics [BRU13a] Another expected benefit would be the vastly lower power consumption.

In recent years reservoir computing has received a lot of attention as bridge between machine learning and physics. As the end of "Moore's Law" is slowly encroaching we are in the waning years of an age of staggering performance leaps in silicon electronic circuits. Reservoir Computing as a way of utilizing the much smaller timescales at which optical systems operate offer a way of building drastically faster computers. There

[SAN17a] overview-paper

[LAR12] i fischer.

[ROE18a] paper von André. (has normalization?)

[JAE01] let's not train the networks.

[APP11] - original paper introducint the delay-based reservoir approach.

[ANT19] ? lesen!

[STE20] ? lesen

[ATI00] - NARMA10 task introduction.

2.1.1 blabla

Analyzing timeseries data is an important part of machine learning tasks. Many datasets can be regarded as a timeseries. Temperature, pressure and wind evolve over time and are fed into complicated climate prediction models in order to continue this timeseries into the future - predicting tomorrows weather patterns. The applications of extracting information from timeseries are vast. Predicting the stock market, driving an autonomous car based on a video stream and other vehicle metrics, recognizing types of heart arrhythmia or more general heart diseases and infections through analysis of electrocardiography by machine learning algorhithms (https://ieeexplore.ieee.org/abstract/document/7164783). Last, but not least: the most apparent machine learning application to date: voice recognition algorithms are predicting the meaning of audio information everytime we utter the magical words "Hi, Siri", "Ok, Google" or "Alexa...". It can also be argued that classical prediction tasks that do not involve time-conscious datasets like images fed into one directional feed forward networks

2.1.2 An example of reservoir computing

hier kommt das beispiel mit dem stein und dem teich.

To understand the idea of reservoir computing one can imaging a pond of water. Now a stone is thrown into, creating ripples that propagate over the surface, reflect along edges and interfere with one another. The pond has some kind of short memory as ripples take time in order to disappear. It is easy to imagine that from the pattern of ripples a spectator could estimate the position of a stone that was thrown in shortly before.

2.2 Theory

2.2.1 Stuart-Landau-Oscillator

The Stuart-Landau oscillator is a dynamical system often used to model basic class 1 lasers. It can be written either as a single complex differential equation (2.1) or a set of two equations written in polar coordinates (2.2). From the equation in polar coordinates easy to see that the equation has rotational symmetry as the radial differential equation does not change with the dynamical variable ϕ.

$$\dot{z} = (\lambda + i\omega + \gamma|z|^2)\ z \tag{2.1}$$

$$\begin{aligned} \dot{r} &= \lambda r + \mathrm{Re}(\gamma)\ r^3 \\ \dot{\phi} &= i\omega + \mathrm{Im}(\gamma)\ r^2 \end{aligned} \tag{2.2}$$

For the radial dynamical variable the Stuart-Landau oscillator has two fixed points where the derivative $\dot{r}$ vanishes $r = 0$ and $r = \sqrt{-\lambda / \mathrm{Re}(\gamma)}$ whose stability depends on λ and $\mathrm{Re}(\gamma)$. For $\mathrm{Re}(\gamma) < 0$ (supercritical case).

The limit cycle (LC) which is shown in fig 2.1 is depending on the ratio or λ and $Re\,[\gamma]$.

As can be seen in (eq. 2.1), the equation has a linear and a nonlinear term regarding the absolute value of z.

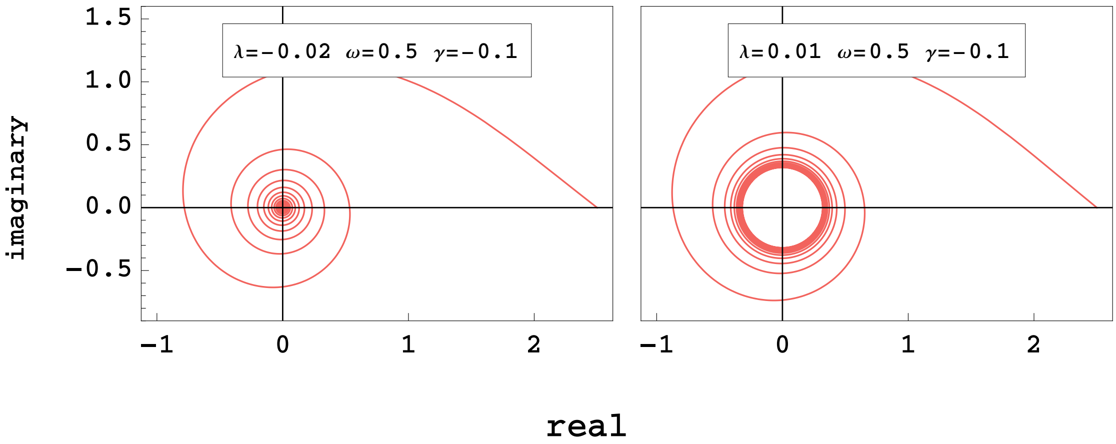

Figure 2.1: 2 very basic scenarios of the Stuart-Landau oscillator: Decay towards a single fixed point (left) or towards a limit cycle (right).

2.2.2 Networks

Vertices blabla Edges blabla.

circulant Matrix

A circulant matrix has the same entries its row vectors, but with its entries rotated one element to the right relative to the previous row.

2.2.3 virtual Nodes and multiplexing

here: papers for explanation! [KUR18]

[STE20] // off-resonance = better! -> reason for choosing 17 * 12.

By multiplexing the input signal one can create virtual nodes in a network. The analogy to a real network can be best understood if the input signal is masked with a binary mask containing only values of either 0 or 1.

Different Mask types: discrete values with constant interpolation: binary, uniform - easy to implement) continuous: any function or repeating noise-patterns. (more difficult, but)

here add dependency of total linear memory on number of nodes and virtal nodes.

2.2.4 Dynamics of rings of stuart landau oscillators

pony-states (von André)

2.2.5 Reservoir computing

Measuring computation performance

We can measure how well a dynamical system perform computations by testing it in a variety of benchmarks. Dynamical systems are continuously evolving in time, thus reservoir computation mostly is mostly tested on timeseries data. There exist also approaches of using RC on datasets that do not involve datasets without a temporal dimension e.g. image classification of single images []

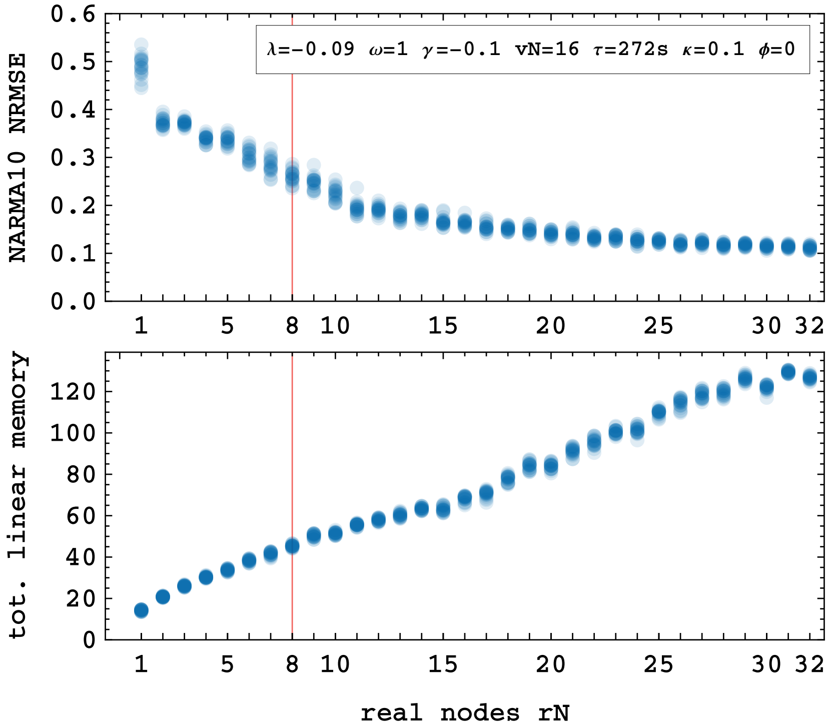

Figure 2.2: changing rc performance for increasing number or real nodes rN in unidirectianally coupled ring networks. (see some plot).

2.2.6 Reservoir computing tasks

The reservoir computing performance of a given reservoir can be quantified by testing its predictions for certain tasks. The word "prediction" not necessarily means to predict the future value of something e.g. extend a timeseries into the future. Instead it often means the estimation of a value. A weather model which has been fed past temperature data can be tasked to "predict" the past humidity values. In machine learning the task is usually to predict a certain value or set of values from a set of inputs. Ideally the prediction can then be compared to the base truth and the difference between prediction and ground truth quantifies the error. The closer the prediction to the ground truth, the better the system performs a given task. It is important to note that usually these predictions are not of singular values, but give a vector of probabilities. A neural network used for image classification will output a vector with values e.g. it is quantifying the "dog-ness", the "tree-ness" or the "car-ness" of an input image. The actual decision is made by choosing the entry with the highest probability. For predictions based on timeseries data the same applies. They are represented by continuous values that the system puts out. Even if the desired output is of discrete nature e.g. "yes" or "no" the system will usually output a real number.

In general a sequence of inputs u is drawn from a distribution. In this work only uniform distributions have been used. This sequence is used as input of a transformation which maps the sequence u onto its target values o.

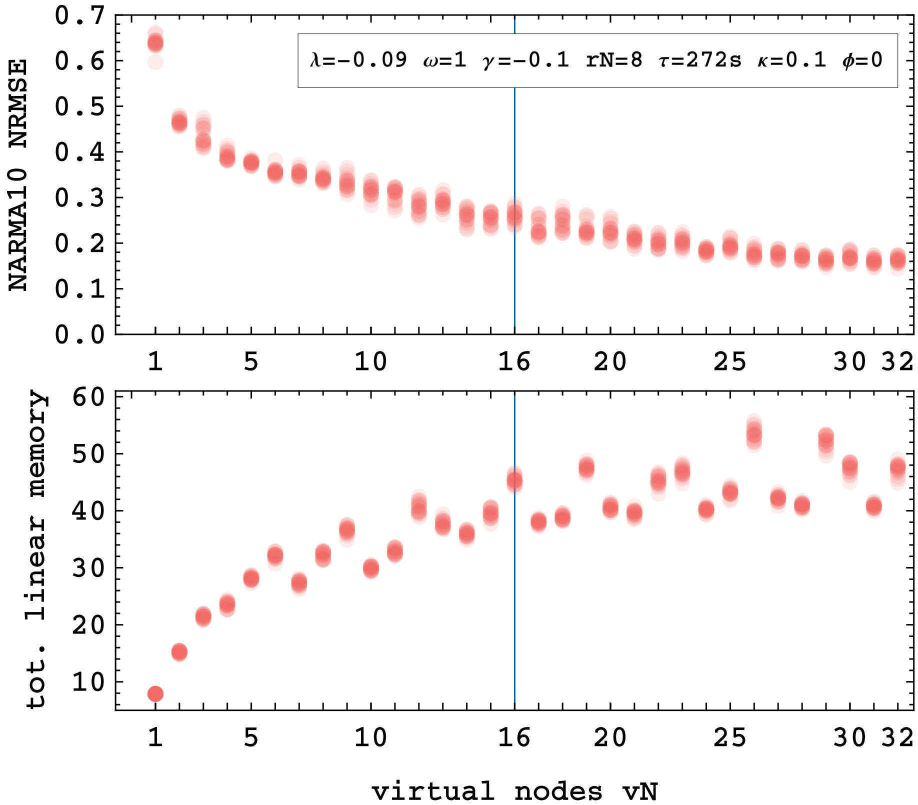

Figure 2.3: changing rc performance for increasing number or virtual nodes rN in unidirectionally coupled ring networks. (see some plot).

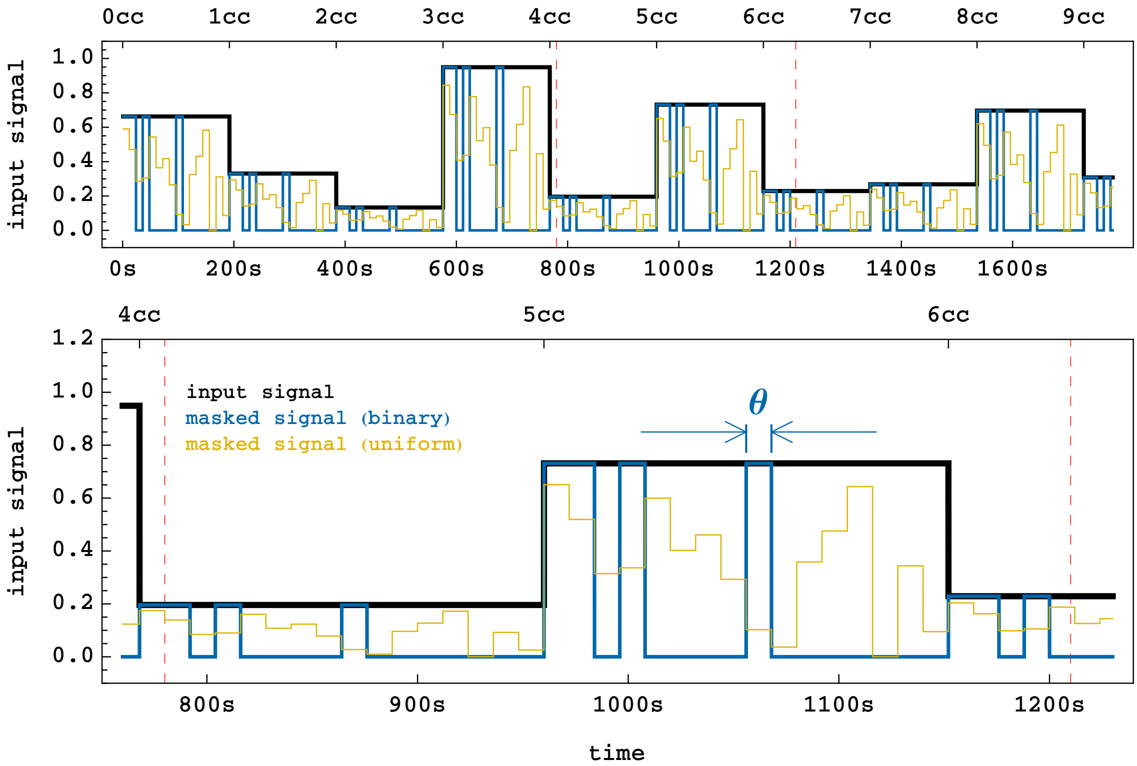

Figure 2.4: A timeseries (black) with constant interpolation ("sample & hold") between samples and the corresponding masked signal (blue). The mask length is counted in clockcycles (cc) and the time per virtual node is counted in θ. Here $\theta = 12s$ and $1cc = 16\theta = 272s$

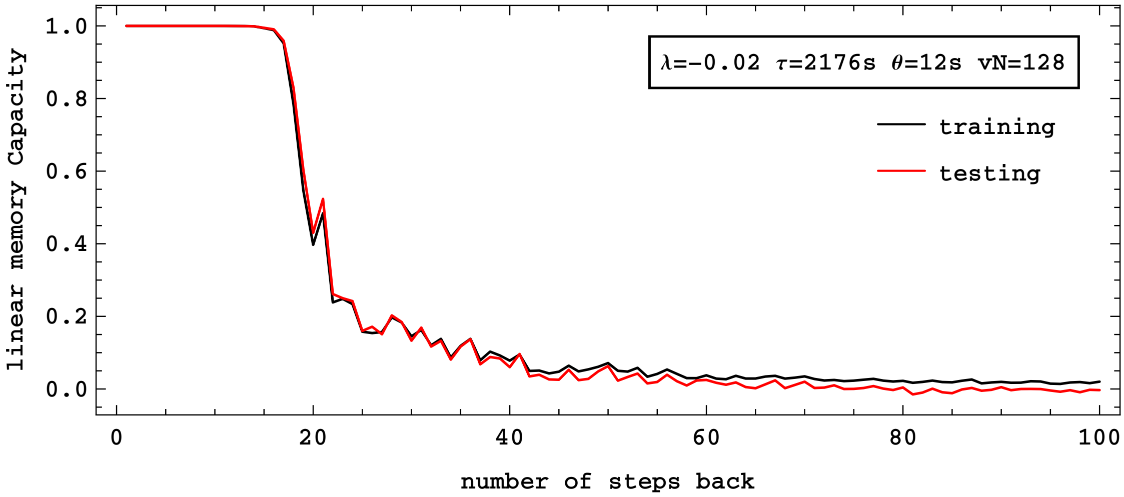

Figure 2.5: The linear memory capacities for differently many steps into the past. The system is able to perfectly reproduce inputs up until 12 steps into the past. $N = 1, vN = 128, \lambda = -0.02, \omega = 1, \gamma = -0.1, \theta = 12, \tau = 2176$.

Linear Memory Recall

The simplest task a reservoir can perform is to repeat the the information that was fed into it at a certain point in time.

2.2.7 Legendre polynomials as Nonlinear Transformations

In order to investigate the nonlinear transformation capabilities one can use Legendre Polynomials **??** in order to transform a given input into a system. Legendre Polynomials have the useful property of being orthonormal to every other polynomial within an interval $[-1, 1]$. This makes them ideal in order to measure linearily independent nonlinear (but also linear) transformation capacities. Legendre polynomials $L_d(x)$ for degrees $d \in \{1-5\}$ are shown in fig.2.7). Depending on the definition they are scaled so that $L_d(1) = 1$. The Legendre polynomial $L_d(x)$ is of course simply the identity. The idea of measuring nonlinear (as well as linear) transformation capacities through the means of Legendre polynomials is largely inspired from the publication [DAM12]. Hence the terminology used in the latter shall be used here as well. As all nonlinear transformations of an input sequence u^h can be expressed as a linear combination of Legendre polynomials with inputs in u^h it is necessary to test all combinations. To elaborate: For the product of 2 Legendre polynomials $L_{d_1}(u1)$ and $L_{d_2}(u2)$ with degrees d_1 and d_2 all combinations of $u1$ and $u2$ have to be calculated.

Argh! kompliziert.

In order to investigate the different nonlinear transformation capabilities of a system it is necessary to iterate over all possible combinations of Products of Legendre polynomials

They are used in Neural networks as well blabla citecite findfind. [**?**]

NARMA10 task

Lastly, the performance was investigated by measuring its capacity to compute the NARMA10 task. The Nonlinear Autoregressive Moving Average Task [HER12] is used in many publications as a benchmark. The sequence is calculated using an average of its last 10 steps while also being fed a product of a random sequence taken at two different positions.

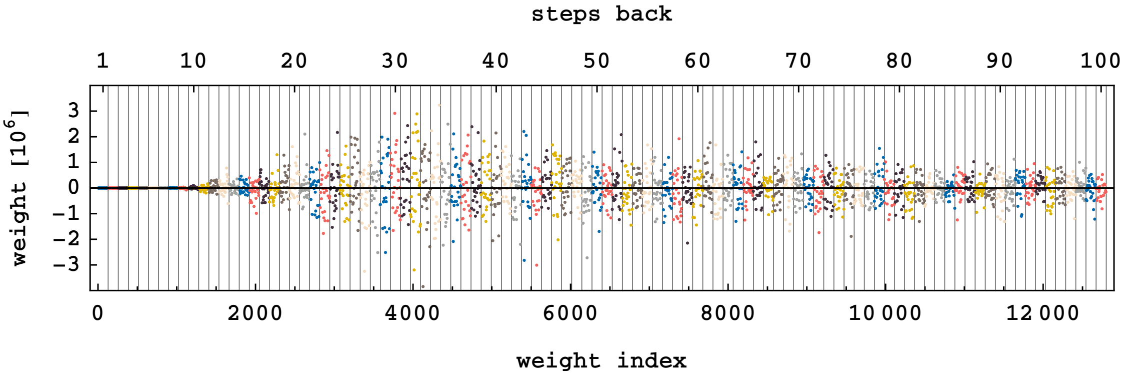

Figure 2.6: All weights $W_{i,s}$ attained through linear regression of the linear memory recalls $s \in [1, 100]$ steps back. The system was a unidirectional ring with of $N_{real} = 8$ and $N_{virtual} = 16$ nodes. The total read-out dimension is 128. For reconstruction of more recent inputs the inputs are small, but become enormous for inputs further in the past. This is the equivalent of "grasping at straws" as the system tries to extract information by multiplying microscopic fluctuations in the system state to linearily combine them to values between $[-1, 1]$.

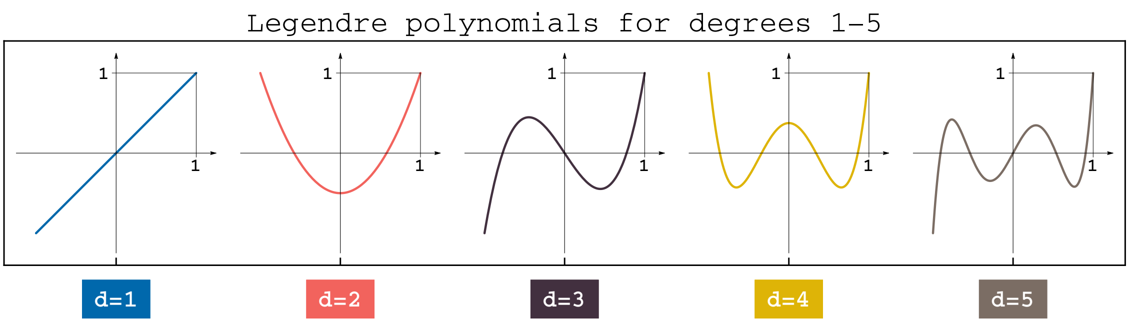

Figure 2.7: Legendre polynomials $L_d(x)$ for degrees $d \in \{1 - 5\}$ each shown for $x \in [-1, 1]$
.

In order to perform well, systems need memory up to 10 steps (hence the "10") into the past as well as nonlinear transformation capacities. Recently it has been shown that the task is not ideal as its difficulty depends non-trivially on the shape of the distribution used [?].
The NARMA10 sequence is created by the iterative formula given by 2.3. It is fed 2 inputs from a sequence u which is drawn from a uniform distribution $U(0, 0.5)$ on interval $[0, 0.5]$.

$$A_{n+1} = 0.3A_n + 0.05A_k \left(\sum_{i=0}^{9} A_{k-9} \right) + 1.5u_{k-9}u_k + 0.1 \tag{2.3}$$

2.3 Results

2.3.1 Highly symmetrical network topologies

Ring networks

Rings with from N=1(edge case) to N=16 Bidirectional Rings Bidirectional Rings with self-feedback and diffuse coupling

2.3.2 All to all coupled networks

N=3 - N=16 row normalization

2.3.3 Less symmetrical network topologies

unidirectional rings with jumps

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