Reservoir Computing Stuff

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Hallo

1 todo

1.1 things on the todo list.

1.1.1 programming work

1.1.2 research work

- check different weights
- check symmetry breaking nodes
- check heterogenous edges
- check heterogenous nodes
- check

1.1.3 graphics/plots

- feed in plus system response for different ratios of theta and lambda

here comes the intro and all the important references... small world[WAT98]

2 one

2.1 Introduction

2.1.1 blabla

here comes the intro and all the important references...
[TER17b] [COU17] [PER16] [LAM17] [GIE17] [LIC17] [KAZ17] [YU16] [JOS18]

2.2 Theory

2.2.1 Stuart-Landau-Oscillator

The Stuart-Landau oscillator is a dynamical system often used to model basic class 1 lasers. It can be written either as a single complex differential equation (2.1) or a set of two equations written in polar coordinates (2.2). From the equation in polar coordinates easy to see that the equation has rotational symmetry as the radial differential equation does not change with the dynamical variable ϕ.

$$\dot{z} = \left(\lambda + i\omega + \gamma|z|^2\right) z \tag{2.1}$$

$$\begin{aligned} \dot{r} &= \lambda r + \operatorname{Re}(\gamma)\, r^3 \\ \dot{\phi} &= i\omega + \operatorname{Im}(\gamma)\, r^2 \end{aligned} \tag{2.2}$$

For the radial dynamical variable the Stuart-Landau oscillator has two fixed points where the derivative $\dot{r}$ vanishes $r = 0$ and $r = \sqrt{-\lambda / \operatorname{Re}(\gamma)}$ whose stability depends on λ and $\operatorname{Re}(\gamma)$. In the supercritical case ($\operatorname{Re}(\gamma) < 0$) the oscillator has a focus at $r = 0$ for $lambda < 0$ and a stable limit cycle in the case of $\lambda > 0$.

The limit cycle (LC) which is shown in (2.1, left) is depending on the ratio or λ and $Re\,[\gamma]$.

As can be seen in (eq. 2.1), the equation has a linear and a nonlinear term regarding the absolute value of z.

2.2.2 Networks

In mathematics networks are collections of vertices and edges which connect the former. Vertices can have a name or some descriptor that makes them uniquely identifiable. Usually they are indexed $1..N$ for a network of size N. An edge $i \to j$ (for $i, j \in$

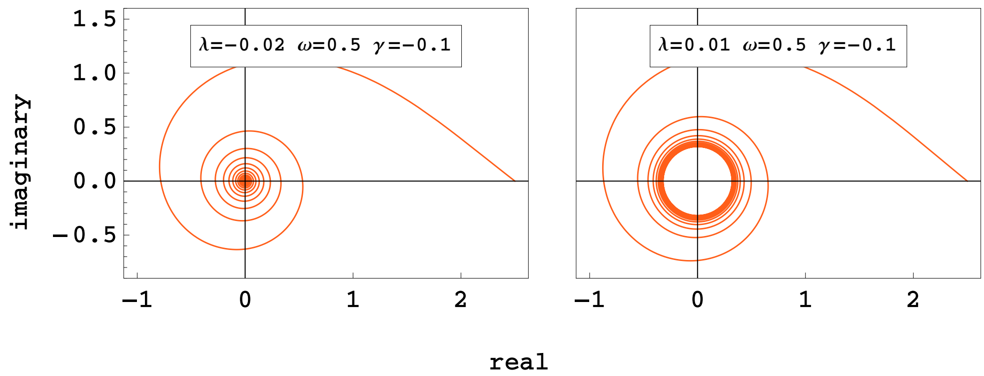

Figure 2.1: 2 very basic scenarios of the Stuart-Landau oscillator: Decay towards a single fixed point (left) or towards a limit cycle (right).

$[1..N]$)is connecting vertex i with vertex j. In a directed network there is a difference between $i \to j$ and $j \to i$.

Vertices blabla Edges blabla.

circulant Matrix

A circulant matrix has the same entries its row vectors, but with its entries rotated one element to the right relative to the previous row.

2.2.3 virtual Nodes and multiplexing

here: papers for explanation! By multiplexing the input signal one can create virtual nodes in a network. The analogy to a real network can be best understood if the input signal is masked with a binary mask containing only values of either 0 or 1.

here add dependency of total linear memory on number of nodes and virtal nodes.

2.2.4 Small world networks

The term small world network was introduced in [WAT98a] and describes a predominately locally coupled network with few non-local edges.

2.2.5 Dynamics of rings of stuart landau oscillators

pony-states (von André)

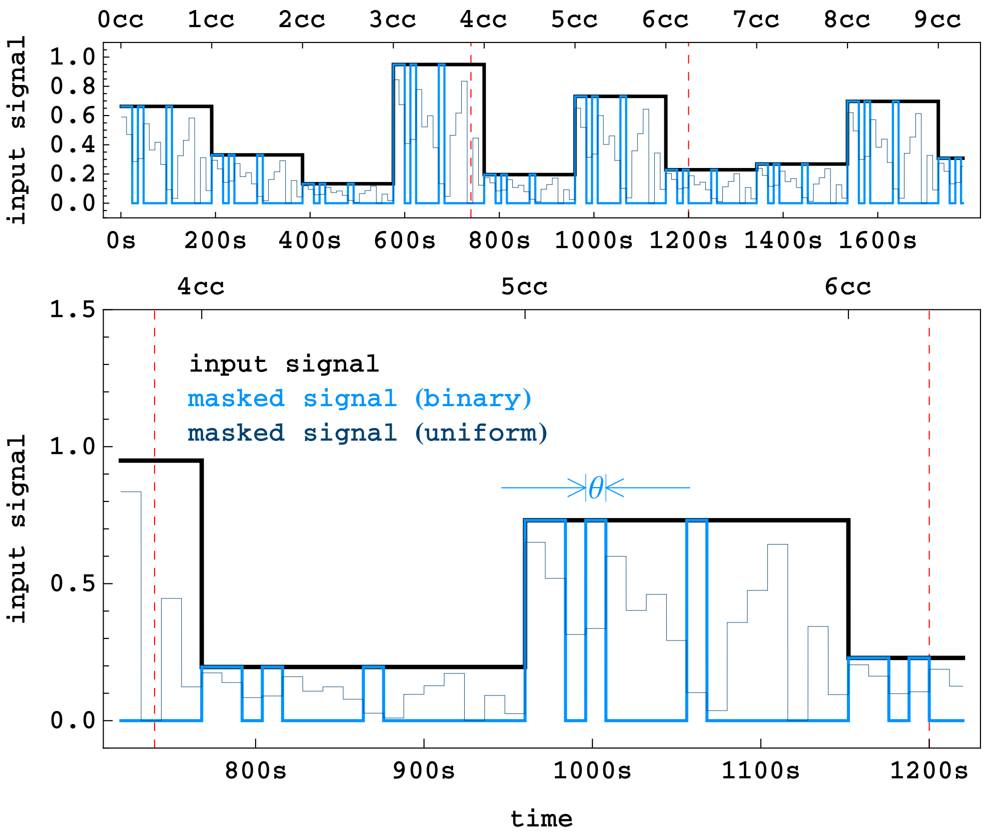

Figure 2.2: A timeseries (black) with constant interpolation ("sample & hold") between samples and the corresponding masked signal (blue). The mask length is counted in clockcycles (cc) and the time per virtual node is counted in θ. Here $\theta = 12s$ and $1cc = 16\ \theta = 272s$

2.3 Results

2.3.1 Highly symmetrical network topologies

Ring networks

Rings with from N=1(edge case) to N=16 Bidirectional Rings Bidirectional Rings with self-feedback and diffuse coupling

2.3.2 All to all coupled networks

N=3 - N=16 row normalization

2.3.3 Less symmetrical network topologies

unidirectional rings with jumps

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