Reservoir Computing Stuff

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Hallo

1 todo

1.1 things on the todo list.

1.1.1 programming work

1.1.2 research work

- check different weights
- check symmetry breaking nodes
- check heterogenous edges
- check heterogenous nodes
- check

1.1.3 graphics/plots

- feed in plus system response for different ratios of theta and lambda

here comes the intro and all the important references... small world[WAT98]

2 one

2.1 Introduction

Reservoir Computing encompasses the field of machine learning in which a wide variety of systems (reservoirs) are used to contain information and perform calculations on it. Reservoir Computing is particularly interesting as computations can be performed by the physical system directly. The "classical" Turing-machine as a model is more like a virtual space created in our physical space. Within this virtual space everything is defined only in discrete units of "0" and "1". In order to create such a virtual space the usual unpredictability that inhabits the scales in which modern computer circuits exist in has to be tamed in order to create this virtual computing space. As the scales of modern transistors shrink they rapidly approach the scales in which quantum effects become problematic. The maintaining of this virtual space of 0 and 1 in which all our computations are performed is increasingly difficult. Modern CPU manufacturing has to take into account many error compensation algorithms in. Simultaneously manufacturing is becoming more challenging as well since the scales of modern transistors are so small the light sources needed for lithography are becoming rare.

Reservoir computing is the circumventing of this virtual space of discrete values in order to perform computations directly in physical systems. A wide variety of systems can be used e.g. a literal bucket of water can be used as reservoir [FER03].

2.1.1 blabla

here comes the intro and all the important references...

2.2 Theory

2.2.1 Stuart-Landau-Oscillator

The Stuart-Landau oscillator is a dynamical system often used to model basic class 1 lasers. It can be written either as a single complex differential equation (2.1) or a set of two equations written in polar coordinates (2.2). From the equation in polar coordinates easy to see that the equation has rotational symmetry as the radial differential equation does not change with the dynamical variable ϕ.

$$\dot{z} = \left(\lambda + i\omega + \gamma|z|^2\right) z \tag{2.1}$$

$$\begin{aligned} \dot{r} &= \lambda r + \mathrm{Re}(\gamma)\, r^3 \\ \dot{\phi} &= i\omega + \mathrm{Im}(\gamma)\, r^2 \end{aligned} \tag{2.2}$$

For the radial dynamical variable the Stuart-Landau oscillator has two fixed points where the derivative $\dot{r}$ vanishes $r = 0$ and $r = \sqrt{-\lambda / \mathrm{Re}(\gamma)}$ whose stability depends on λ and $\mathrm{Re}(\gamma)$. For $\mathrm{Re}(\gamma) < 0$ (supercritical case).

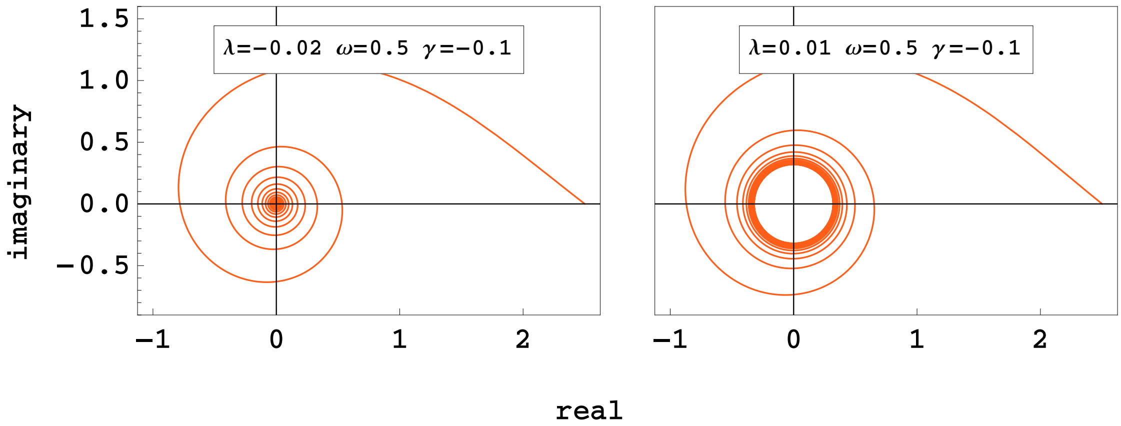

Figure 2.1: 2 very basic scenarios of the Stuart-Landau oscillator: Decay towards a single fixed point (left) or towards a limit cycle (right).

The limit cycle (LC) which is shown in fig 2.1 is depending on the ratio or λ and $Re\,[\gamma]$.

As can be seen in (eq. 2.1), the equation has a linear and a nonlinear term regarding the absolute value of z.

2.2.2 Networks

Vertices blabla Edges blabla.

circulant Matrix

A circulant matrix has the same entries its row vectors, but with its entries rotated one element to the right relative to the previous row.

2.2.3 virtual Nodes and multiplexing

here: papers for explanation! By multiplexing the input signal one can create virtual nodes in a network. The analogy to a real network can be best understood if the input signal is masked with a binary mask containing only values of either 0 or 1.

here add dependency of total linear memory on number of nodes and virtal nodes.

2.2.4 Dynamics of rings of stuart landau oscillators

pony-states (von André)

2.2.5 Reservoir computing

It is therefor distinct from the "classical" van-Neumann computer architecture where The term "reservoir" is abstract as the actual physical systems reservoir can be a literal bucket of water [FER03]

2.2.6 Reservoir computing tasks

The reservoir computing performance of a given dynamic system can be quantified by testing its predictions for certain tasks. In machine learning the task is usually to predict a certain value from an initial set of inputs. Ideally the prediction can then be compared

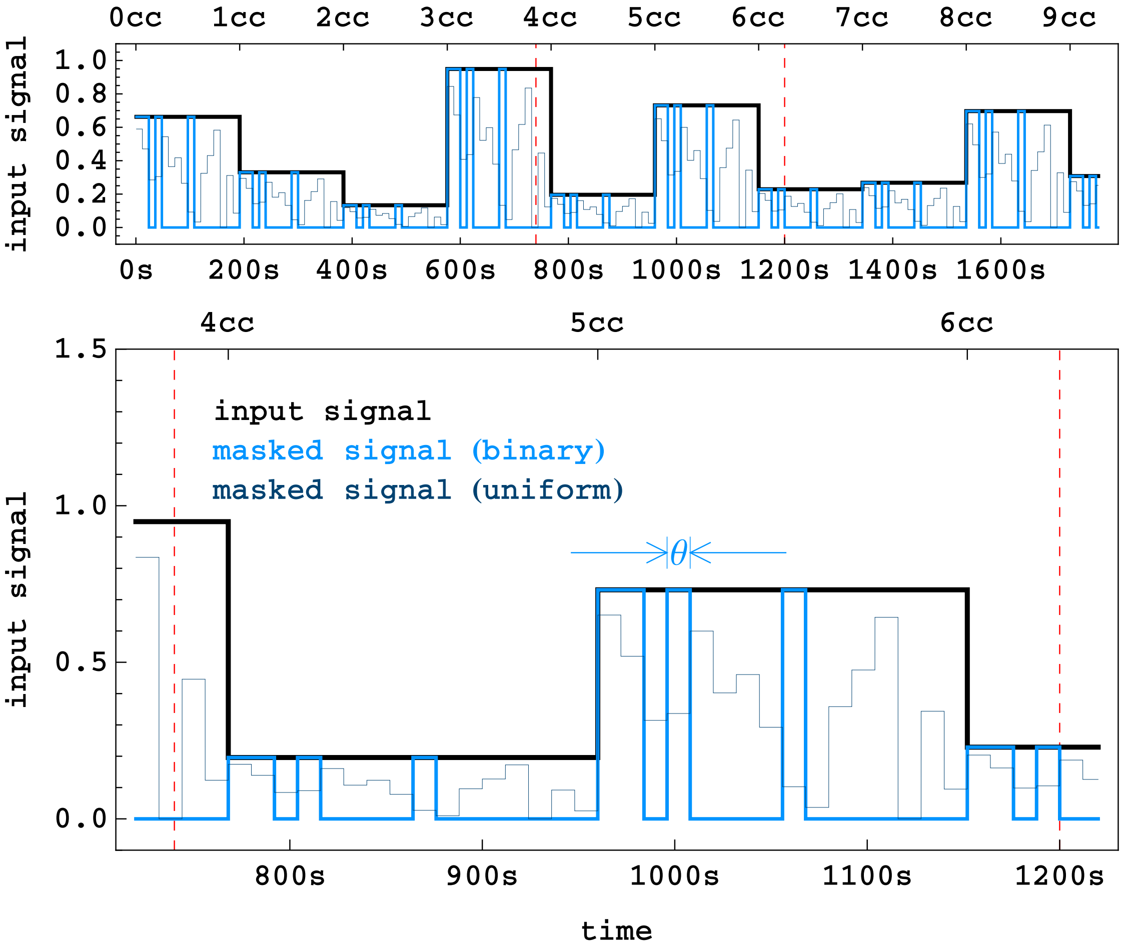

Figure 2.2: A timeseries (black) with constant interpolation ("sample & hold") between samples and the corresponding masked signal (blue). The mask length is counted in clockcycles (cc) and the time per virtual node is counted in θ. Here $\theta = 12s$ and $1cc = 16\theta = 272s$

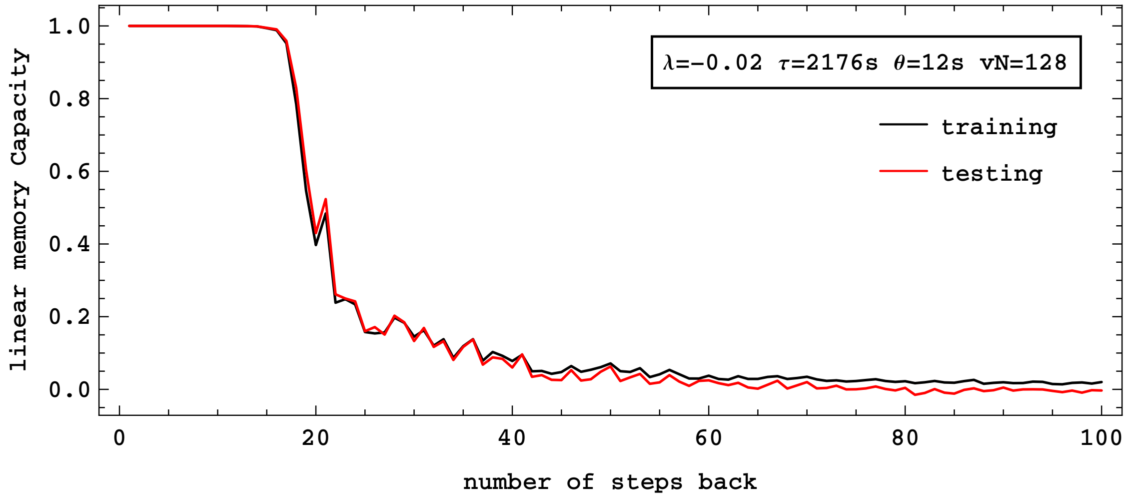

Figure 2.3: The linear memory capacities for differently many steps into the past. The system is able to perfectly reproduce inputs up until 12 steps into the past. $N = 1, vN = 128, \lambda = -0.02, \omega = 1, \gamma = -0.1, \theta = 12, \tau = 2176$.

to the base truth and the difference between prediction and base truth is quantified as the error. The closer the prediction is to the ground truth, the better the system performs a given task.

Linear Memory Recall

The simplest task a reservoir can perform is to repeat the the information that was fed into it at a certain point in time.

NARMA10 NRMSE

2.3 Results

2.3.1 Highly symmetrical network topologies

Ring networks

Rings with from N=1(edge case) to N=16 Bidirectional Rings Bidirectional Rings with self-feedback and diffuse coupling

2.3.2 All to all coupled networks

N=3 - N=16 row normalization

2.3.3 Less symmetrical network topologies

unidirectional rings with jumps

Bibliography

[FER03] C. Fernando and S. Sojakka. Pattern Recognition in a Bucket. In *Advances in Artificial Life*, Seiten 588–597, 2003.

[WAT98] D. J. Watts and S. H. Strogatz. Collective dynamics of 'small-world' networks. Nature **3**93, 440–442 (1998).

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[FER03] C. Fernando and S. Sojakka. Pattern Recognition in a Bucket. In *Advances in Artificial Life*, Seiten 588–597, 2003.

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