



Rainbow Unicode Characters  
Team Reference Document  
Lund University

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## 1. ACHIEVING AC ON A SOLVED PROBLEM

### 1.1. WA.

- Check that minimal input passes.
- Can an int overflow?
- Reread the problem statement.
- Start creating small test cases with python.
- Does cout print with high enough precision?
- Abstract the implementation.

### 1.2. TLE.

- Is the solution sanity checked?
- Use pypy instead of python.
- Rewrite in C++ or Java.
- Can we apply DP anywhere?
- To minimize penalty time you should create a worst case input (if easy) to test on.
- Binary Search over the answer?

### 1.3. RTE.

- Recursion limit in python?
- Arrayindex out of bounds?
- Division by 0?
- Modifying iterator while iterating over it?
- Not using a well defined operator for Collections.sort?
- If nothing makes sense and the end of the contest is approaching you can binary search over where the error is with try-except.

### 1.4. MLE.

- Create objects outside recursive function.
- Rewrite recursive solution to iterative with an own stack.

## 2. IDEAS

### 2.1. A TLE solution is obvious.

- If doing dp, drop parameter and recover from others.
- Use a sorted data structure.
- Is there a hint in the statement saying that something more is bounded?

### 2.2. Try this on clueless problems.

- Try to interpret problem as a graph (D - NCPC2017).
- Can we apply maxflow, with mincost?
- How does it look for small examples, can we find a pattern?
- Binary search over solution.
- If problem is small, just brute force instead of solving it cleverly. Some times its enough to iterate over the entire domains instead of using xgcd.

## 3. CODE TEMPLATES

### 3.1. .bashrc. Aliases.

```
alias p2=python2
alias p3=python3
alias nv=vim
alias o="xdg-open ."
setxkbmap -option 'nocaps:ctrl'
```

### 3.2. .vimrc. Tabs, line numbers, wrapping

```
set nowrap
syntax on
set tabstop=8 softtabstop=0 shiftwidth=4
set expandtab smarttab
set autoindent smartindent
set rnu number
set scrolloff=8
filetype plugin indent on
```

### 3.3. .run.sh. Bash script to run all tests in a folder.

```
#!/bin/bash
# make executable: chmod +x run.sh
# run: ./run.sh A pypy A.py
folder=$1;shift
for f in $folder/*.in; do
    echo $f
    pre=${f%.in}
    out=$pre.out
    ans=$pre.ans
    $* < $f > $out
    diff $out $ans
done
```

**3.4. Java Template.** A Java template.

```
import java.util.*;
import java.io.*;
public class A {
    void solve(BufferedReader in) throws Exception {
    }
    int toInt(String s) {return Integer.parseInt(s);}
    int[] toInts(String s) {
        String[] a = s.split(" ");
        int[] o = new int[a.length];
        for(int i = 0; i < a.length; i++)
            o[i] = toInt(a[i]);
        return o;
    }
    public static void main(String[] args)
    throws Exception {
        BufferedReader in = new BufferedReader
            (new InputStreamReader(System.in));
        (new A()).solve(in);
    }
}
```

**3.5. Python Template.** A Python template

```
from collections import *
from itertools import permutations #No repeated elements
import sys, bisect
sys.setrecursionlimit(10**5)
inp = raw_input

def err(s):
    sys.stderr.write('{}\n'.format(s))

def ni():
    return int(inp())

def nl():
    return [int(_) for _ in inp().split()]

# q = deque([0])
# a = q.popleft()
```

```
# q.append(0)
```

```
# a = [1, 2, 3, 3, 4]
# bisect.bisect(a, 3) == 4
# bisect.bisect_left(a, 3) == 2
```

**3.6. C++ Template.** A C++ template

```
#include <bits/stdc++.h>
using namespace std;

#define rep(i, a, b) for(int i = a; i < (b); ++i)
#define trav(a, x) for(auto& a : x)
#define sz(x) (int)(x).size()
typedef long long ll;
typedef pair<int, int> pii;
typedef vector<int> vi;
typedef long long ll;
ll smod(ll a, ll b){
    return (a % b + b) % b;
}
int main() {
    cout.precision(9);
    cin.sync_with_stdio(0); cin.tie(0);
    cin.exceptions(cin.failbit);
    int N;
    cin >> N;
    cout << 0 << endl;
}
```

**4. DATA STRUCTURES**

**4.1. Binary Indexed Tree.** Also called a Fenwick tree. Builds in  $\mathcal{O}(n \log n)$  from an array. Query sum from 0 to  $i$  in  $\mathcal{O}(\log n)$  and updates an element in  $\mathcal{O}(\log n)$ .

```
private static class BIT {
    long[] data;
    public BIT(int size) {
        data = new long[size+1];
    }
    public void update(int i, int delta) {
        while(i < data.length) {
            data[i] += delta;
```

```

    i += i&-i; // Integer.lowestOneBit(i);
}
}
public long sum(int i) {
    long sum = 0;
    while(i>0) {
        sum += data[i];
        i -= i&-i;
    }
    return sum;
}
}

```

4.2. **Segment Tree.** More general than a Fenwick tree. Can adapt other operations than sum, e.g. min and max.

```

private static class ST {
    int li, ri;
    int sum; //change to max/min
    ST lN;
    ST rN;
}
static ST makeSgmTree(int[] A, int l, int r) {
    if(l == r) {
        ST node = new ST();
        node.li = l;
        node.ri = r;
        node.sum = A[l]; //max/min
        return node;
    }
    int mid = (l+r)/2;
    ST lN = makeSgmTree(A, l, mid);
    ST rN = makeSgmTree(A, mid+1, r);
    ST root = new ST();
    root.li = lN.li;
    root.ri = rN.ri;
    root.sum = lN.sum + rN.sum; //max/min
    root.lN = lN;
    root.rN = rN;
    return root;
}

```

```

static int getSum(ST root, int l, int r) { //max/min
    if(root.li>=l && root.ri<=r)
        return root.sum; //max/min
    if(root.ri<l || root.li > r)
        return 0; //minInt/maxInt
    else //max/min
        return getSum(root.lN, l, r) + getSum(root.rN, l, r);
}
static int update(ST root, int i, int val) {
    int diff = 0;
    if(root.li==root.ri && i == root.li) {
        diff = val-root.sum; //max/min
        root.sum=val; //max/min
        return diff; //root.max
    }
    int mid = (root.li + root.ri) / 2;
    if (i <= mid) diff = update(root.lN, i, val);
    else diff = update(root.rN, i, val);
    root.sum+=diff; //ask other child
    return diff; //and compute max/min
}

```

4.3. **Lazy Segment Tree.** More general implementation of a segment tree where its possible to increase whole segments by some diff, with lazy propagation. Implemented with arrays instead of nodes, which probably has less overhead to write during a competition.

```

class LazySegmentTree {
    private int n;
    private int[] lo, hi, sum, delta;
    public LazySegmentTree(int n) {
        this.n = n;
        lo = new int[4*n + 1];
        hi = new int[4*n + 1];
        sum = new int[4*n + 1];
        delta = new int[4*n + 1];
        init();
    }
    public int sum(int a, int b) {
        return sum(1, a, b);
    }
}

```

```

private int sum(int i, int a, int b) {
    if(b < lo[i] || a > hi[i]) return 0;
    if(a <= lo[i] && hi[i] <= b) return sum(i);
    prop(i);
    int l = sum(2*i, a, b);
    int r = sum(2*i+1, a, b);
    update(i);
    return l + r;
}

public void inc(int a, int b, int v) {
    inc(1, a, b, v);
}

private void inc(int i, int a, int b, int v) {
    if(b < lo[i] || a > hi[i]) return;
    if(a <= lo[i] && hi[i] <= b) {
        delta[i] += v;
        return;
    }
    prop(i);
    inc(2*i, a, b, v);
    inc(2*i+1, a, b, v);
    update(i);
}

private void init() {
    init(1, 0, n-1, new int[n]);
}

private void init(int i, int a, int b, int[] v) {
    lo[i] = a;
    hi[i] = b;
    if(a == b) {
        sum[i] = v[a];
        return;
    }
    int m = (a+b)/2;
    init(2*i, a, m, v);
    init(2*i+1, m+1, b, v);
    update(i);
}

```

```

private void update(int i) {
    sum[i] = sum(2*i) + sum(2*i+1);
}

private int range(int i) {
    return hi[i] - lo[i] + 1;
}

private int sum(int i) {
    return sum[i] + range(i)*delta[i];
}

private void prop(int i) {
    delta[2*i] += delta[i];
    delta[2*i+1] += delta[i];
    delta[i] = 0;
}
}

```

4.4. **Union Find.** This data structure is used in various algorithms, for example Kruskal's algorithm for finding a Minimal Spanning Tree in a weighted graph. Also it can be used for backward simulation of dividing a set.

```

private class Node {
    Node parent;
    int h;
    public Node() {
        parent = this;
        h = 0;
    }
    public Node find() {
        if(parent != this) parent = parent.find();
        return parent;
    }
}

static void union(Node x, Node y) {
    Node xR = x.find(), yR = y.find();
    if(xR == yR) return;
    if(xR.h > yR.h)
        yR.parent = xR;
    else {
        if(yR.h == xR.h) yR.h++;
        xR.parent = yR;
    }
}

```

```

    }
}

```

4.5. **Monotone Queue.** Used in sliding window algorithms where one would like to find the minimum in each interval of a given length. Amortized  $\mathcal{O}(n)$  to find min in each of these intervals in an array of length  $n$ . Can easily be used to find the maximum as well.

```

private static class MinMonQue {
    LinkedList<Integer> que = new LinkedList<>();
    public void add(int i) {
        while(!que.isEmpty() && que.getFirst() > i)
            que.removeFirst();
        que.addFirst(i);
    }
    public int last() {
        return que.getLast();
    }
    public void remove(int i) {
        if(que.getLast() == i) que.removeLast();
    }
}

```

4.6. **Treap.** Treap is a binary search tree that uses randomization to balance itself. It's easy to implement, and gives you access to the internal structures of a binary tree, which can be used to find the  $k$ 'th element for example. Because of the randomness, the average height is about a factor 4 of a perfectly balanced tree.

```

class Treap{
    int sz;
    int v;
    double y;
    Treap L, R;

    static int sz(Treap t) {
        if(t == null) return 0;
        return t.sz;
    }
    static void update(Treap t) {
        if(t == null) return;
        t.sz = sz(t.L) + sz(t.R) + 1;
    }
}

```

```

static Treap merge(Treap a, Treap b) {
    if (a == null) return b;
    if(b == null) return a;
    if (a.y < b.y) {
        a.R = merge(a.R, b);
        update(a);
        return a;
    } else {
        b.L = merge(a, b.L);
        update(b);
        return b;
    }
}
//inserts middle in left half
static Treap[] split(Treap t, int x) {
    if (t == null) return new Treap[2];
    if (t.v <= x) {
        Treap[] p = split(t.R, x);
        t.R = p[0];
        p[0] = t;
        return p;
    } else {
        Treap[] p = split(t.L, x);
        t.L = p[1];
        p[1] = t;
        return p;
    }
}
//use only with split
static Treap insert(Treap t, int x) {
    Treap m = new Treap();
    m.v = x;
    m.y = Math.random();
    m.sz = 1;
    Treap[] p = splitK(t, x-1);
    return merge(merge(p[0],m), p[1]);
}

//inserts middle in left half

```

```

static Treap[] splitK(Treap t, int x) {
    if (t == null) return new Treap[2];
    if (t.sz < x) return new Treap[]{t, null};
    if (sz(t.L) >= x) {
        Treap[] p = splitK(t.L, x);
        t.L = p[1];
        p[1] = t;
        update(p[0]);
        update(p[1]);
        return p;
    } else if (sz(t.L) + 1 == x){
        Treap r = t.R;
        t.R = null;
        Treap[] p = new Treap[]{t, r};
        update(p[0]);
        update(p[1]);
        return p;
    } else {
        Treap[] p = splitK(t.R, x - sz(t.L)-1);
        t.R = p[0];
        p[0] = t;
        update(p[0]);
        update(p[1]);
        return p;
    }
}
//use only with splitK
static Treap insertK(Treap t, int w, int x) {
    Treap m = new Treap();
    m.v = x;
    m.y = Math.random();
    m.sz = 1;
    Treap[] p = splitK(t, w);
    t = merge(p[0], m);
    return merge(t, p[1]);
}
//use only with splitK
static Treap deleteK(Treap t, int w, int x) {
    Treap[] p = splitK(t, w);
    Treap[] q = splitK(p[0], w-1);

```

```

        return merge(q[0], p[1]);
    }

    static Treap Left(Treap t) {
        if (t == null) return null;
        if (t.L == null) return t;
        return Left(t.L);
    }

    static Treap Right(Treap t) {
        if (t == null) return null;
        if (t.R == null) return t;
        return Right(t.R);
    }
}

```

4.7. **RMQ.**  $\mathcal{O}(1)$  queries of interval min, max, gcd or lcm.  $\mathcal{O}(n \log n)$  building time.

```

import math
class RMQ:
    def __init__(self, arr, func=min):
        self.sz = len(arr)
        self.func = func
        MAXN = self.sz
        LOGMAXN = int(math.ceil(math.log(MAXN + 1, 2)))
        self.data = [[0]*LOGMAXN for _ in range(MAXN)]
        for i in range(MAXN):
            self.data[i][0] = arr[i]
            for j in range(1, LOGMAXN):
                for i in range(MAXN - (1<<j)+1):
                    self.data[i][j] = func(self.data[i][j-1],
                                            self.data[i + (1<<(j-1))][j-1])

    def query(self, a, b):
        if a > b:
            # some default value when query is empty
            return 1
        d = b - a + 1
        k = int(math.log(d, 2))
        return self.func(self.data[a][k], self.data[b - (1<<k)+1][k])

```

## 5. GRAPH ALGORITHMS

**5.1. Dijkstras algorithm.** Finds the shortest distance between two Nodes in a weighted graph in  $\mathcal{O}(|E|\log|V|)$  time.

```
from heapq import heappop as pop, heappush as push
# adj: adj-list where edges are tuples (node_id, weight):
# (1) --2-- (0) --3-- (2) has the adj-list:
# adj = [[(1, 2), (2, 3)], [(0, 2)], [0, 3]]
def dijk(adj, S, T):
    N = len(adj)
    INF = 10**10
    dist = [INF]*N
    pq = []
    dist[S] = 0
    push(pq, (0, S))

    while pq:
        D, i = pop(pq)
        if D != dist[i]: continue
        for j, w in adj[i]:
            alt = D + w
            if dist[j] > alt:
                dist[j] = alt
                push(pq, (alt, j))

    return dist[T]
```

**5.2. Bipartite Graphs.** The Hopcroft-Karp algorithm finds the maximal matching in a bipartite graph. Also, this matching can together with König's theorem be used to construct a minimal vertex-cover, which as we all know is the complement of a maximum independent set. Runs in  $\mathcal{O}(|E|\sqrt{|V|})$ .

```
import java.util.*;
class Node {
    int id;
    LinkedList<Node> ch = new LinkedList<>();
    public Node(int id) {
        this.id = id;
    }
}
public class BiGraph {
```

```
    private static int INF = Integer.MAX_VALUE;
    LinkedList<Node> L, R;
    int N, M;
    Node[] U;
    int[] Pair, Dist;
    int nild;
    public BiGraph(LinkedList<Node> L, LinkedList<Node> R){
        N = L.size(); M = R.size();
        this.L = L; this.R = R;
        U = new Node[N+M];
        for(Node n: L) U[n.id] = n;
        for(Node n: R) U[n.id] = n;
    }
    private boolean bfs() {
        LinkedList<Node> Q = new LinkedList<>();
        for(Node n: L)
            if(Pair[n.id] == -1) {
                Dist[n.id] = 0;
                Q.add(n);
            } else
                Dist[n.id] = INF;

        nild = INF;
        while(!Q.isEmpty()) {
            Node u = Q.removeFirst();
            if(Dist[u.id] < nild)
                for(Node v: u.ch) if(distp(v) == INF){
                    if(Pair[v.id] == -1)
                        nild = Dist[u.id] + 1;
                    else {
                        Dist[Pair[v.id]] = Dist[u.id] + 1;
                        Q.addLast(U[Pair[v.id]]);
                    }
                }
        }
        return nild != INF;
    }
    private int distp(Node v) {
        if(Pair[v.id] == -1) return nild;
        return Dist[Pair[v.id]];
    }
```



```

}
private boolean dfs(Node u) {
    for(Node v: u.ch) if(distp(v) == Dist[u.id] + 1) {
        if(Pair[v.id] == -1 || dfs(U[Pair[v.id]])) {
            Pair[v.id] = u.id;
            Pair[u.id] = v.id;
            return true;
        }
    }
    Dist[u.id] = INF;
    return false;
}
public HashMap<Integer, Integer> maxMatch() {
    Pair = new int[M+N];
    Dist = new int[M+N];
    for(int i = 0; i<M+N; i++) {
        Pair[i] = -1;
        Dist[i] = INF;
    }
    HashMap<Integer, Integer> out = new HashMap<>();
    while(bfs()) {
        for(Node n: L) if(Pair[n.id] == -1)
            dfs(n);
    }
    for(Node n: L) if(Pair[n.id] != -1)
        out.put(n.id, Pair[n.id]);
    return out;
}
public HashSet<Integer> minVTC() {
    HashMap<Integer, Integer> Lm = maxMatch();
    HashMap<Integer, Integer> Rm = new HashMap<>();
    for(int x: Lm.keySet()) Rm.put(Lm.get(x), x);
    boolean[] Z = new boolean[M+N];
    LinkedList<Node> bfs = new LinkedList<>();
    for(Node n: L) {
        if(!Lm.containsKey(n.id)) {
            Z[n.id] = true;
            bfs.add(n);
        }
    }
}

```

```

while(!bfs.isEmpty()) {
    Node x = bfs.removeFirst();
    int nono = -1;
    if(Lm.containsKey(x.id))
        nono = Lm.get(x.id);
    for(Node y: x.ch) {
        if(y.id == nono || Z[y.id]) continue;
        Z[y.id] = true;
        if(Rm.containsKey(y.id)){
            int xx = Rm.get(y.id);
            if(!Z[xx]) {
                Z[xx] = true;
                bfs.addLast(U[xx]);
            }
        }
    }
}
HashSet<Integer> K = new HashSet<>();
for(Node n: L) if(!Z[n.id]) K.add(n.id);
for(Node n: R) if(Z[n.id]) K.add(n.id);
return K;
}
}

```

**5.3. Network Flow.** Ford-Fulkerson algorithm for determining the maximum flow through a graph can be used for a lot of unexpected problems. Given a problem that can be formulated as a graph, where no ideas are found trying, it might help trying to apply network flow. The running time is  $\mathcal{O}(C \cdot m)$  where  $C$  is the maximum flow and  $m$  is the amount of edges in the graph. If  $C$  is very large we can change the running time to  $\mathcal{O}(\log C m^2)$  by only studying edges with a large enough capacity in the beginning.

```

from collections import defaultdict
class Flow:
    def __init__(self, sz):
        self.G = [defaultdict(int) for _ in range(sz)]

    def add_edge(self, i, j, w):
        self.G[i][j] += w

    def dfs(self, s, t, FLOW):

```

```

    if s in self.V: return 0
    if s == t: return FLOW
    self.V.add(s)
    for u, w in self.G[s].items():
        if w and u not in self.dead:
            F = self.dfs(u, t, min(FLOW, w))
            if F:
                self.G[s][u] -= F
                self.G[u][s] += F
                return F
    self.dead.add(s)
    return 0

def max_flow(self, s, t):
    flow = 0
    self.dead = set()
    while True:
        pushed = self.bfs(s, t)
        if not pushed: break
        flow += pushed
    return flow

```

5.4. Dinitz Algorithm. Faster flow algorithm.

```

from collections import defaultdict
class Dinitz:
    def __init__(self, sz, INF=10**10):
        self.G = [defaultdict(int) for _ in range(sz)]
        self.sz = sz
        self.INF = INF

    def add_edge(self, i, j, w):
        self.G[i][j] += w

    def bfs(self, s, t):
        level = [0]*self.sz
        q = [s]
        level[s] = 1
        while q:
            q2 = []
            for u in q:

```

```

                for v, w in self.G[u].items():
                    if w and level[v] == 0:
                        level[v] = level[u] + 1
                        q2.append(v)
            q = q2
        self.level = level
        return level[t] != 0

    def dfs(self, s, t, FLOW):
        if s in self.V: return 0
        if s == t: return FLOW
        self.V.add(s)
        L = self.level[s]
        for u, w in self.G[s].items():
            if u in self.dead: continue
            if w and L+1==self.level[u]:
                F = self.dfs(u, t, min(FLOW, w))
                if F:
                    self.G[s][u] -= F
                    self.G[u][s] += F
                    return F
        self.dead.add(s)
        return 0

```

```

    def max_flow(self, s, t):
        flow = 0
        while self.bfs(s, t):
            self.dead = set()
            while True:
                self.V = set()
                pushed = self.dfs(s, t, self.INF)
                if not pushed: break
                flow += pushed
            return flow

// C++ implementation of Dinic's Algorithm
// O(V*V*E) for generall flow-graphs. (But with a good constant)
// O(E*sqrt(V)) for bipartite matching graphs.
// O(E*min(V**(2/3),E**(1/3))) For unit-capacity graphs
#include<bits/stdc++.h>

```

```

using namespace std;
typedef long long ll;
struct Edge{
    ll v ;//to vertex
    ll flow ;
    ll C;//capacity
    ll rev;//reverse edge index
};
// Residual Graph
class Graph
{
public:
    ll V; // number of vertex
    vector<ll> level; // stores level of a node
    vector<vector<Edge>> adj; //can also be array of vector with global size
    Graph(ll V){
        adj.assign(V,vector<Edge>());
        this->V = V;
        level.assign(V,0);
    }

    void addEdge(ll u, ll v, ll C){
        Edge a{v, 0, C, (int)adj[v].size()};// Forward edge
        Edge b{u, 0, 0, (int)adj[u].size()};// Back edge
        adj[u].push_back(a);
        adj[v].push_back(b); // reverse edge
    }

    bool BFS(ll s, ll t){
        for (ll i = 0 ; i < V ; i++)
            level[i] = -1;
        level[s] = 0; // Level of source vertex
        list< ll > q;
        q.push_back(s);
        vector<Edge>::iterator i ;
        while (!q.empty()){
            ll u = q.front();
            q.pop_front();
            for (i = adj[u].begin(); i != adj[u].end(); i++){
                Edge &e = *i;
                if (level[e.v] < 0 && e.flow < e.C){
                    level[e.v] = level[u] + 1;
                    q.push_back(e.v);
                }
            }
        }
        return level[t] < 0 ? false : true; //can/cannot reach target
    }

    ll sendFlow(ll u, ll flow, ll t, vector<ll> &start){
        // Sink reached
        if (u == t)
            return flow;
        // Traverse all adjacent edges one -by - one.
        for ( ; start[u] < (int)adj[u].size(); start[u]++){
            Edge &e = adj[u][start[u]];
            if (level[e.v] == level[u]+1 && e.flow < e.C){
                // find minimum flow from u to t
                ll curr_flow = min(flow, e.C - e.flow);
                ll temp_flow = sendFlow(e.v, curr_flow, t, start);
                // flow is greater than zero
                if (temp_flow > 0){
                    e.flow += temp_flow;//add flow
                    adj[e.v][e.rev].flow -= temp_flow;//sub from reverse edge
                    return temp_flow;
                }
            }
        }
        return 0;
    }

    ll DinicMaxflow(ll s, ll t){
        // Corner case
        if (s == t) return -1;
        ll total = 0; // Initialize result
        while (BFS(s, t) == true){//while path from s to t
            // store how many edges are visited
            // from V { 0 to V }
            vector <ll> start;
            start.assign(V,0);
            // while flow is not zero in graph from S to D

```

```

    while (ll flow = sendFlow(s, 999999999, t, start))
        total += flow; // Add path flow to overall flow
    }
    return total;
}
};

```

5.5. **Min Cost Max Flow.** Finds the minimal cost of a maximum flow through a graph. Can be used for some optimization problems where the optimal assignment needs to be a maximum flow.

```

class MinCostMaxFlow {
    boolean found[];
    int N, dad[];
    long cap[][], flow[][], cost[][], dist[], pi[];

    static final long INF = Long.MAX_VALUE / 2 - 1;

    boolean search(int s, int t) {
        Arrays.fill(found, false);
        Arrays.fill(dist, INF);
        dist[s] = 0;

        while (s != N) {
            int best = N;
            found[s] = true;
            for (int k = 0; k < N; k++) {
                if (found[k]) continue;
                if (flow[k][s] != 0) {
                    long val = dist[s] + pi[s] - pi[k] - cost[k][s];
                    if (dist[k] > val) {
                        dist[k] = val;
                        dad[k] = s;
                    }
                }
            }
            if (flow[s][k] < cap[s][k]) {
                long val = dist[s] + pi[s] - pi[k] + cost[s][k];
                if (dist[k] > val) {
                    dist[k] = val;
                    dad[k] = s;
                }
            }
        }
    }
}

```

```

    }
    if (dist[k] < dist[best]) best = k;
    }
    s = best;
}
for (int k = 0; k < N; k++)
    pi[k] = Math.min(pi[k] + dist[k], INF);
return found[t];
}

long[] mcmf(long c[][], long d[][], int s, int t) {
    cap = c;
    cost = d;

    N = cap.length;
    found = new boolean[N];
    flow = new long[N][N];
    dist = new long[N+1];
    dad = new int[N];
    pi = new long[N];

    long totflow = 0, totcost = 0;
    while (search(s, t)) {
        long amt = INF;
        for (int x = t; x != s; x = dad[x])
            amt = Math.min(amt, flow[x][dad[x]] != 0 ?
                flow[x][dad[x]] : cap[dad[x]][x] - flow[dad[x]][x]);
        for (int x = t; x != s; x = dad[x]) {
            if (flow[x][dad[x]] != 0) {
                flow[x][dad[x]] -= amt;
                totcost -= amt * cost[x][dad[x]];
            } else {
                flow[dad[x]][x] += amt;
                totcost += amt * cost[dad[x]][x];
            }
        }
        totflow += amt;
    }
}

```

```
return new long[]{ totflow, totcost };
}
}
```

5.6. **2-Sat.** Solves 2sat by splitting up vertices in strongly connected components.

# used in sevenkingdoms, illumination

```
import sys
sys.setrecursionlimit(10**5)
class Sat:
    def __init__(self, no_vars):
        self.size = no_vars*2
        self.no_vars = no_vars
        self.adj = [[] for _ in range(self.size)]
        self.back = [[] for _ in range(self.size)]
    def add_imply(self, i, j):
        self.adj[i].append(j)
        self.back[j].append(i)
    def add_or(self, i, j):
        self.add_imply(i^1, j)
        self.add_imply(j^1, i)
    def add_xor(self, i, j):
        self.add_or(i, j)
        self.add_or(i^1, j^1)
    def add_eq(self, i, j):
        self.add_xor(i, j^1)

    def dfs1(self, i):
        if i in self.marked: return
        self.marked.add(i)
        for j in self.adj[i]:
            self.dfs1(j)
        self.stack.append(i)

    def dfs2(self, i):
        if i in self.marked: return
        self.marked.add(i)
        for j in self.back[i]:
            self.dfs2(j)
        self.comp[i] = self.no_c
```

```
def is_sat(self):
    self.marked = set()
    self.stack = []
    for i in range(self.size):
        self.dfs1(i)
    self.marked = set()
    self.no_c = 0
    self.comp = [0]*self.size
    while self.stack:
        i = self.stack.pop()
        if i not in self.marked:
            self.no_c += 1
            self.dfs2(i)
    for i in range(self.no_vars):
        if self.comp[i*2] == self.comp[i*2+1]:
            return False
    return True
```

# assumes is\_sat.

# If xi is after xi in topological sort,

# xi should be FALSE. It should be TRUE otherwise.

# <https://codeforces.com/blog/entry/16205>

```
def solution(self):
    V = []
    for i in range(self.no_vars):
        V.append(self.comp[i*2] > self.comp[i*2+1])
    return V
```

```
if __name__ == '__main__':
    S = Sat(1)
    S.add_or(0, 0)
    print(S.is_sat())
    print(S.solution())
```

5.7. **Min Cost Max Bipartite Matching.** The Hungarian algorithm runs in  $\mathcal{O}(n^3)$  with a low constant, giving us the minimum cost matching. If the maximum cost is wanted you can just negate the weights.

#include <bits/stdc++.h>

using namespace std;

```

#define rep(i, a, b) for(int i = a; i < (b); ++i)
#define trav(a, x) for(auto& a : x)
#define sz(x) (int)(x).size()
#define all(x) x.begin(), x.end()
typedef long long ll;
typedef pair<int, int> pii;
typedef vector<int> vi;

typedef vector<ll> vd;
bool zero(ll x) { return x == 0; }
// vector<vd> cost(sz, vd(sz, 0));
// Max Cost found by negating weights.
double MinCostMatching(const vector<vd>& cost) {
    int n = sz(cost), mated = 0;
    vd dist(n), u(n), v(n);
    vi dad(n), seen(n);

    /// construct dual feasible solution
    rep(i,0,n) {
        u[i] = cost[i][0];
        rep(j,1,n) u[i] = min(u[i], cost[i][j]);
    }
    rep(j,0,n) {
        v[j] = cost[0][j] - u[0];
        rep(i,1,n) v[j] = min(v[j], cost[i][j] - u[i]);
    }

    /// find primal solution satisfying complementary slackness
    vi L(n, -1);
    vi R(n, -1);
    rep(i,0,n) rep(j,0,n) {
        if (R[j] != -1) continue;
        if (zero(cost[i][j] - u[i] - v[j])) {
            L[i] = j;
            R[j] = i;
            mated++;
            break;
        }
    }
}

```

```

for (; mated < n; mated++) { // until solution is feasible
    int s = 0;
    while (L[s] != -1) s++;
    fill(all(dad), -1);
    fill(all(seen), 0);
    rep(k,0,n)
        dist[k] = cost[s][k] - u[s] - v[k];

    int j = 0;
    for (;;) { /// find closest
        j = -1;
        rep(k,0,n){
            if (seen[k]) continue;
            if (j == -1 || dist[k] < dist[j]) j = k;
        }
        seen[j] = 1;
        int i = R[j];
        if (i == -1) break;
        rep(k,0,n) { /// relax neighbors
            if (seen[k]) continue;
            auto new_dist = dist[j] + cost[i][k] - u[i] - v[k];
            if (dist[k] > new_dist) {
                dist[k] = new_dist;
                dad[k] = j;
            }
        }
    }

    /// update dual variables
    rep(k,0,n) {
        if (k == j || !seen[k]) continue;
        auto w = dist[k] - dist[j];
        v[k] += w, u[R[k]] -= w;
    }
    u[s] += dist[j];

    /// augment along path
    while (dad[j] >= 0) {
        int d = dad[j];
        R[j] = R[d];
    }
}

```

```

    L[R[j]] = j;
    j = d;
}
R[j] = s;
L[s] = j;
}
auto value = vd(1)[0];
rep(i,0,n) value += cost[i][L[i]];
return value;
}

```

## 6. DYNAMIC PROGRAMMING

**6.1. Longest Increasing Subsequence.** Finds the longest increasing subsequence in an array in  $\mathcal{O}(n \log n)$  time. Can easily be transformed to longest decreasing/non decreasing/non increasing subsequence.

```

def lis(X):
    N = len(X)
    P = [0]*N
    M = [0]*(N+1)
    L = 0
    for i in range(N):
        lo, hi = 1, L + 1
        while lo < hi:
            mid = (lo + hi) >> 1
            if X[M[mid]] < X[i]:
                lo = mid + 1
            else:
                hi = mid
        newL = lo
        P[i] = M[newL - 1]
        M[newL] = i
        L = max(L, newL)
    S = [0]*L
    k = M[L]
    for i in range(L-1, -1, -1):
        S[i] = X[k]
        k = P[k]
    return S

```

**6.2. String functions.** The z-function computes the longest common prefix of  $t$  and  $t[i:]$  for each  $i$  in  $\mathcal{O}(|t|)$ . The border function computes the longest common proper (smaller than whole string) prefix and suffix of string  $t[:i]$ .

```

def zfun(t):
    z = [0]*len(t)
    n = len(t)
    l, r = (0,0)
    for i in range(1,n):
        if i < r:
            z[i] = min(z[i-l], r-i+1)
        while z[i] + i < n and t[i+z[i]] == t[z[i]]:
            z[i]+=1
        if i + z[i] - 1 > r:
            l = i
            r = i + z[i] - 1
    return z

```

```

def matches(t, p):
    s = p + '#' + t
    return filter(lambda x: x[1] == len(p),
                  enumerate(zfun(s)))

```

```

def borders(s):
    b = [0]*len(s)
    for i in range(1, len(s)):
        k = b[i-1]
        while k>0 and s[k] != s[i]:
            k = b[k-1]
        if s[k] == s[i]:
            b[i] = k+1
    return b

```

**6.3. Josephus problem.** Who is the last one to get removed from a circle if the  $k$ 'th element is continuously removed?

```

# Rewritten from  $J(n, k) = (J(n-1, k) + k) \% n$ 
def J(n, k):
    r = 0
    for i in range(2, n+1):
        r = (r + k)%i
    return r

```

6.4. **Floyd Warshall.** Constructs a matrix with the distance between all pairs of nodes in  $\mathcal{O}(n^3)$  time. Works for negative edge weights, but not if there exists negative cycles. The `nxt` matrix is used to reconstruct a path. Can be skipped if we don't care about the path.

*# Computes distance matrix and next matrix given an edgelist*

```
def FloydWarshall(n, edges):
    INF = 10000000000
    dist = [[INF]*n for _ in range(n)]
    nxt = [[None]*n for _ in range(n)]
    for e in edges:
        dist[e[0]][e[1]] = e[2]
        nxt[e[0]][e[1]] = e[1]
    for k in range(n):
        for i in range(n):
            for j in range(n):
                if dist[i][j] > dist[i][k] + dist[k][j]:
                    dist[i][j] = dist[i][k] + dist[k][j]
                    nxt[i][j] = nxt[i][k]
    return dist, nxt
```

*# Computes the path from i to j given a nextmatrix*

```
def path(i, j, nxt):
    if nxt[i][j] == None: return []
    path = [i]
    while i != j:
        i = nxt[i][j]
        path.append(i)
    return path
```

## 7. ETC

7.1. **System of Equations.** Solves the system of equations  $Ax = b$  by Gaussian elimination. This can for example be used to determine the expected value of each node in a markov chain. Runs in  $\mathcal{O}(N^3)$ .

*# monoid needs to implement*

*# \_\_add\_\_, \_\_mul\_\_, \_\_sub\_\_, \_\_div\_\_ and isZ*

```
def gauss(A, b, monoid=None):
    def Z(v): return abs(v) < 1e-6 if not monoid else v.isZ()

    N = len(A[0])
```

```
for i in range(N):
    try:
        m = next(j for j in range(i, N) if Z(A[j][i]) == False)
    except:
        return None #A is not independent!
    if i != m:
        A[i], A[m] = A[m], A[i]
        b[i], b[m] = b[m], b[i]
    for j in range(i+1, N):
        sub = A[j][i]/A[i][i]
        b[j] -= sub*b[i]
        for k in range(N):
            A[j][k] -= sub*A[i][k]

for i in range(N-1, -1, -1):
    for j in range(N-1, i, -1):
        sub = A[i][j]/A[j][j]
        b[i] -= sub*b[j]
    b[i], A[i][i] = b[i]/A[i][i], A[i][i]/A[i][i]
return b
```

7.2. **Convex Hull.** From a collection of points in the plane the convex hull is often used to compute the largest distance or the area covered, or the length of a rope that encloses the points. It can be found in  $\mathcal{O}(N \log N)$  time by sorting the points on angle and the sweeping over all of them.

```
def convex_hull(pts):
    pts = sorted(set(pts))

    if len(pts) <= 2:
        return pts

    def cross(o, a, b):
        return (a[0] - o[0]) * (b[1] - o[1]) - (a[1] - o[1]) * (b[0] - o[0])

    lo = []
    for p in pts:
        while len(lo) >= 2 and cross(lo[-2], lo[-1], p) <= 0:
            lo.pop()
        lo.append(p)
```



```

hi = []
for p in reversed(pts):
    while len(hi) >= 2 and cross(hi[-2], hi[-1], p) <= 0:
        hi.pop()
    hi.append(p)

return lo[:-1] + hi[:-1]

```

### 7.3. Number Theory.

```

def gcd(a, b):
    return b if a%b == 0 else gcd(b, a%b)

```

*# Assumes MOD is a prime from Fermat's small theorem*

```

def inv(a, MOD):
    return pow(a, MOD - 2, MOD)

```

*# returns g = gcd(a, b), x0, y0,  
# where g = x0\*a + y0\*b*

```

def xgcd(a, b):
    x0, x1, y0, y1 = 1, 0, 0, 1
    while b != 0:
        q, a, b = (a // b, b, a % b)
        x0, x1 = (x1, x0 - q * x1)
        y0, y1 = (y1, y0 - q * y1)
    return (a, x0, y0)

```

```

def crt(la, ln):
    assert len(la) == len(ln)
    for i in range(len(la)):
        assert 0 <= la[i] < ln[i]
    prod = 1
    for n in ln:
        assert gcd(prod, n) == 1
        prod *= n
    lN = []
    for n in ln:
        lN.append(prod//n)
    x = 0
    for i, a in enumerate(la):
        print(lN[i], ln[i])

```

```

_, Mi, mi = xgcd(lN[i], ln[i])
x += a*Mi*ln[i]
return x % prod

```

*# finds x^e mod m*

```

def modpow(x, m, e):
    res = 1
    while e:
        if e%2 == 1:
            res = (res*x) % m
        x = (x*x) % m
        e = e//2
    return res

```

*# Divides a list of digits with an int.*

*# A lot faster than using bigint-division.*

```

def div(L, d):
    r = [0]*(len(L) + 1)
    q = [0]*len(L)
    for i in range(len(L)):
        x = int(L[i]) + r[i]*10
        q[i] = x//d
        r[i+1] = x-q[i]*d
    s = []
    for i in range(len(L) - 1, 0, -1):
        s.append(q[i]%10)
        q[i-1] += q[i]//10

```

```

while q[0]:
    s.append(q[0]%10)
    q[0] = q[0]//10
s = s[::-1]
i = 0
while s[i] == 0:
    i += 1
return s[i:]

```

*# Multiplies a list of digits with an int.*

*# A lot faster than using bigint-multiplication.*

```
def mul(L, d):
    r = [d*x for x in L]
    s = []
    for i in range(len(r) - 1, 0, -1):
        s.append(r[i]%10)
        r[i-1] += r[i]//10
    while r[0]:
        s.append(r[0]%10)
        r[0] = r[0]//10
    return s[::-1]
```

```
large_primes = [
5915587277,
1500450271,
3267000013,
5754853343,
4093082899,
9576890767,
3628273133,
2860486313,
5463458053,
3367900313,
10000000000000061,
10**16 + 61,
10**17 + 3
]
```

7.4. **FFT.** FFT can be used to calculate the product of two polynomials of length  $N$  in  $\mathcal{O}(N \log N)$  time. The FFT function requires a power of 2 sized array of size at least  $2N$  to store the results as complex numbers.

```
import cmath
# A has to be of length a power of 2.

def FFT(A, inverse=False):
    N = len(A)
    if N <= 1:
        return A
    if inverse:
        D = FFT(A) # d_0/N, d_{N-1}/N, d_{N-2}/N, ...
        return map(lambda x: x/N, [D[0]] + D[:0:-1])
```

```
    evn = FFT(A[0::2])
    odd = FFT(A[1::2])
    Nh = N//2
    return [evn[k*Nh]+cmath.exp(2j*cmath.pi*k/N)*odd[k*Nh]
            for k in range(N)]
```

*# A has to be of length a power of 2.*

```
def FFT2(a, inverse=False):
    N = len(a)
    j = 0
    for i in range(1, N):
        bit = N>>1
        while j&bit:
            j ^= bit
            bit >>= 1
        j ^= bit
        if i < j:
            a[i], a[j] = a[j], a[i]

    L = 2
    MUL = -1 if inverse else 1
    while L <= N:
        ang = 2j*cmath.pi/L * MUL
        wlen = cmath.exp(ang)
        for i in range(0, N, L):
            w = 1
            for j in range(L//2):
                u = a[i+j]
                v = a[i+j+L//2] * w
                a[i+j] = u + v
                a[i+j+L//2] = u - v
                w *= wlen
            L *= 2
    if inverse:
        for i in range(N):
            a[i] /= N
    return a

def uP(n):
    while n != (n&-n):
```

```

    n += n&-n
    return n

# C[x] = sum_{i=0..N}(A[x-i]*B[i])
def polymul(A, B):
    sz = max(uP(len(A), len(B)))
    A = A + [0]*(sz - len(A))
    B = B + [0]*(sz - len(B))
    fA = FFT(A)
    fB = FFT(B)
    fAB = [a*b for a, b in zip(fA, fB)]
    C = [x.real for x in FFT(fAB, True)]
    return C

```

## 8. NP TRICKS

**8.1. MaxClique.** The max clique problem is one of Karp's 21 NP-complete problems. The problem is to find the largest subset of an undirected graph that forms a clique - a complete graph. There is an obvious algorithm that just inspects every subset of the graph and determines if this subset is a clique. This algorithm runs in  $\mathcal{O}(n^2 2^n)$ . However one can use the meet in the middle trick (one step divide and conquer) and reduce the complexity to  $\mathcal{O}(n^2 2^{\frac{n}{2}})$ .

```

static int max_clique(int n, int[][] adj) {
    int fst = n/2;
    int snd = n - fst;
    int[] maxc = new int[1<<fst];
    int max = 1;
    for(int i = 0; i<(1<<fst); i++) {
        for(int a = 0; a<fst; a++) {
            if((i&1<<a) != 0)
                maxc[i] = Math.max(maxc[i], maxc[i^(1<<a)]);
        }
        boolean ok = true;
        for(int a = 0; a<fst; a++) if((i&1<<a) != 0) {
            for(int b = a+1; b<fst; b++) {
                if((i&1<<b) != 0 && adj[a][b] == 0)
                    ok = false;
            }
        }
        if(ok) {

```

```

            maxc[i] = Integer.bitCount(i);
            max = Math.max(max, maxc[i]);
        }
    }
    for(int i = 0; i<(1<<snd); i++) {
        boolean ok = true;
        for(int a = 0; a<snd; a++) if((i&1<<a) != 0) {
            for(int b = a+1; b<snd; b++) {
                if((i&1<<b) != 0)
                    if(adj[a+fst][b+fst] == 0)
                        ok = false;
            }
        }
        if(!ok) continue;
        int mask = 0;
        for(int a = 0; a<fst; a++) {
            ok = true;
            for(int b = 0; b<snd; b++) {
                if((i&1<<b) != 0) {
                    if(adj[a][b+fst] == 0) ok = false;
                }
            }
            if(ok) mask |= (1<<a);
        }
        max = Math.max(Integer.bitCount(i) + maxc[mask],
            max);
    }
    return max;
}

```

## 9. COORDINATE GEOMETRY

**9.1. Area of a nonintersecting polygon.** The signed area of a polygon with  $n$  vertices is given by

$$A = \frac{1}{2} \sum_{i=0}^{n-1} (x_i y_{i+1} - x_{i+1} y_i)$$

**9.2. Intersection of two lines.** Two lines defined by

$$a_1 x + b_1 y + c_1 = 0$$

$$a_2 x + b_2 y + c_2 = 0$$

Intersects in the point

$$P = \left( \frac{b_1 c_2 - b_2 c_1}{w}, \frac{a_2 c_1 - a_1 c_2}{w} \right),$$

where  $w = a_1 b_2 - a_2 b_1$ . If  $w = 0$  the lines are parallel.

**9.3. Distance between line segment and point.** Given a line segment between point  $P, Q$ , the distance  $D$  to point  $R$  is given by:

$$\begin{aligned} a &= Q_y - P_y \\ b &= Q_x - P_x \\ c &= P_x Q_y - P_y Q_x \\ R_P &= \left( \frac{b(bR_x - aR_y) - ac}{a^2 + b^2}, \frac{a(aR_y - bR_x) - bc}{a^2 + b^2} \right) \\ D &= \begin{cases} \frac{|aR_x + bR_y + c|}{\sqrt{a^2 + b^2}} & \text{if } (R_{P_x} - P_x)(R_{P_x} - Q_x) < 0, \\ \min |P - R|, |Q - R| & \text{otherwise} \end{cases} \end{aligned}$$

**9.4. Picks theorem.** Find the amount of internal integer coordinates  $i$  inside a polygon with picks theorem  $A = \frac{b}{2} + i - 1$ , where  $A$  is the area of the polygon and  $b$  is the amount of coordinates on the boundary.

**9.5. Trigonometry.** Sine-rule

$$\frac{\sin(\alpha)}{a} = \frac{\sin(\beta)}{b} = \frac{\sin(\gamma)}{c}$$

Cosine-rule

$$a^2 = b^2 + c^2 - 2bc \cdot \cos(\alpha)$$

Area-rule

$$A = \frac{a \cdot b \cdot \sin(\gamma)}{2}$$

Rotation Matrix, rotate a 2D-vector  $\theta$  radians by multiplying with the following matrix.

$$\begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix}$$

**9.6. Implementations.**

```
import math
```

```
# Distance between two points
```

```
def dist(p, q):
    return math.hypot(p[0]-q[0], p[1] - q[1])
```

```
# Square distance between two points
```

```
def d2(p, q):
    return (p[0] - q[0])**2 + (p[1] - q[1])**2
```

```
# Converts two points to a line (a, b, c),
```

```
# ax + by + c = 0
```

```
# if p == q, a = b = c = 0
```

```
def pts2line(p, q):
    return (-q[1] + p[1],
            q[0] - p[0],
            p[0]*q[1] - p[1]*q[0])
```

```
# Distance from a point to a line,
```

```
# given that a != 0 or b != 0
```

```
def distl(l, p):
    return (abs(l[0]*p[0] + l[1]*p[1] + l[2])
            /math.hypot(l[0], l[1]))
```

```
# intersects two lines.
```

```
# if parallel, returns False.
```

```
def inters(l1, l2):
    a1,b1,c1 = l1
    a2,b2,c2 = l2
    cp = a1*b2 - a2*b1
    if cp != 0:
        return float(b1*c2 - b2*c1)/cp, float(a2*c1 - a1*c2)/cp
    else:
        return False
```

```
# projects a point on a line
```

```
def project(l, p):
    a, b, c = l
    return ((b*(b*p[0] - a*p[1]) - a*c)/(a*a + b*b),
            (a*(a*p[1] - b*p[0]) - b*c)/(a*a + b*b))
```

```
# Intersections between circles
```

```
def intersections(c1, c2):
    if c1[2] > c2[2]:
        c1, c2 = c2, c1
    x1, y1, r1 = c1
    x2, y2, r2 = c2
```

```

if x1 == x2 and y1 == y2 and r1 == r2:
    return False

dist2 = (x1 - x2)*(x1-x2) + (y1 - y2)*(y1 - y2)
rsq = (r1 + r2)*(r1 + r2)
if dist2 > rsq or dist2 < (r1-r2)*(r1-r2):
    return []
elif dist2 == rsq:
    cx = x1 + (x2-x1)*r1/(r1+r2)
    cy = y1 + (y2-y1)*r1/(r1+r2)
    return [(cx, cy)]
elif dist2 == (r1-r2)*(r1-r2):
    cx = x1 - (x2-x1)*r1/(r2-r1)
    cy = y1 - (y2-y1)*r1/(r2-r1)
    return [(cx, cy)]

d = math.sqrt(dist2)
f = (r1*r1 - r2*r2 + dist2)/(2*dist2)
xf = x1 + f*(x2-x1)
yf = y1 + f*(y2-y1)
dx = xf-x1
dy = yf-y1
h = math.sqrt(r1*r1 - dx*dx - dy*dy)
norm = abs(math.hypot(dx, dy))
p1 = (xf + h*(-dy)/norm, yf + h*(dx)/norm)
p2 = (xf + h*(dy)/norm, yf + h*(-dx)/norm)
return sorted([p1, p2])

# Finds the bisector through origo
# between two points by normalizing.
def bisector(p1, p2):
    d1 = math.hypot(p1[0], p2[1])
    d2 = math.hypot(p2[0], p2[1])
    return ((p1[0]/d1 + p2[0]/d2),
            (p1[1]/d1 + p2[1]/d2))

# Distance from P to origo
def norm(P):
    return (P[0]**2 + P[1]**2 + P[2]**2)**(0.5)

# Finds distance between point p
# and line A + t*u in 3D
def dist3D(A, u, p):
    AP = tuple(A[i] - p[i] for i in range(3))
    cross = tuple(AP[i]*u[(i+1)%3] - AP[(i+1)%3]*u[i]
                  for i in range(3))
    return norm(cross)/norm(u)

def vec(p1, p2):
    return p2[0]-p1[0], p2[1] - p1[1]

def sign(x):
    if x < 0: return -1
    return 1 if x > 0 else 0

def cross(u, v):
    return u[0] * v[1] - u[1] * v[0]

# s1: (Point, Point)
# s2: (Point, Point)
# Point : (x, y)
# returns true if intersecting s1 & s2 shares at least 1 point.
def segment_intersect(s1, s2):
    u = vec(*s1)
    v = vec(*s2)
    p1, p2 = s1
    q1, q2 = s2
    d1 = cross(u, vec(p1, q1))
    d2 = cross(u, vec(p1, q2))
    d3 = cross(v, vec(q1, p1))
    d4 = cross(v, vec(q1, p2))
    if d1 * d2 * d3 * d4 == 0:
        return True
    return sign(d1) != sign(d2) and sign(d3) != sign(d4)

```

---

10. PRACTICE CONTEST CHECKLIST

- Operations per second in py2
  - Operations per second in py3
  - Operations per second in java
  - Operations per second in c++
  - Operations per second on local machine
  - Is MLE called MLE or RTE?
  - What happens if extra output is added? What about one extra new line or space?
  - Look at documentation on judge.
  - Submit a clarification.
  - Print a file.
  - Directory with test cases.
-