Index Mutual Funds and Exchange-Traded Funds

A comparison of two methods of passive investment.

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Exchange-traded funds (ETFs), once a phenomenon, have emerged as a viable alternative for investors seeking to tie their holdings to a major market index. By the end of 2000, the market for ETFs totaled over $75 billion, up 82% from the previous year, this in a climate of less than stellar stock returns. Just one ETF, the S&P Depository Receipts 500, has assets of over $28 billion. While ETFs still represent only a small slice of the $1.5 trillion index fund pie, their growth in popularity among retail and large-scale investors prompts more research on their advantages and disadvantages.[1]

One subject not adequately understood is the explicit and implicit costs incurred by ETFs and how these compare to the costs of index mutual funds. I develop a simple one-period model that is useful in examining the major differences between ETFs and index funds, depending on investor trading preferences, tax implications, and other characteristics. Then I expand the one-period model to multiple periods, and also review some qualitative differences between ETFs and index funds that cannot be incorporated into this model.

PRIOR RESEARCH

Mutual fund performance has certainly not been ignored in the economic literature. Ever since mutual funds emerged in the early 1960s, the question of their performance and fund manager selectivity skills has interested economists. Sharpe [1966], Treynor [1966], and

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Jensen [1968] conclude that mutual fund performance, net of expenses and after risk adjustment, is poorer than what investors could achieve using a naive buy-and-hold strategy. While authors like Chen and Stockum [1986] and Lee and Rahman [1990] find that a limited number of fund managers have the selectivity and market-timing skills required to beat the market, analysis by Malkiel [1995] and Bogle [1998] has shown that without prior knowledge of these few superior fund managers, investors would do best to stay in index funds. As Bogle writes [1998, p. 38], an investor would be "a bit of a fool" not to seriously consider limiting fund selection to low-expense funds. The most recent study by Frino and Gallagher [2001] once again concludes that in the past five years, S&P 500 index mutual funds earned a better risk-adjusted, expense-adjusted return than actively managed funds.

Of course, it would be wrong to say that views on index fund superiority are unanimous. Minor [2001] notes that, depending on the time horizon of data, it is possible to find periods when active funds outperformed their index fund cousins.

Whichever view one favors, the keys to comparing active funds and index funds are the costs of activity: turnover costs, expense ratios, and transaction costs. This 1% to 2% per year can often make the difference between beating the market or falling behind it. As a result, research on transaction costs has been substantial.

Ferris and Chance [1987] find that 12b-1 charges (fees charged by mutual funds to pay for sales and advertising) are a deadweight loss borne by the shareholders. Grinblatt and Titman [1994] conclude that there is no correlation between loads and performance; i.e., there is no additional return premium for buying funds with higher costs. Finally, Dellva and Olson find that "in general, investors should not select funds with front-end loads, 12b-1 fees, deferred sales charges, and redemption fees, but they should not expect that the avoidance of these fees will coincide with superior risk-adjusted return" [1998, p. 101]. On balance, the research suggests that avoidance of extra fees removes deadweight losses, and thus improves returns.

Another area of research deals specifically with the costs of index funds and exchange-traded funds. While all the research cited suggests that active fund managers generally do not have superior selectivity skills, but instead incur extra costs that penalize fund shareholders, analysts have not examined the problems inherent in indexed investments. As Frino and Gallagher point out, "Despite the significant attention to active funds in the performance evaluation literature, empirical research evaluating index funds is surprisingly scarce" [2001, p. 45].

Frino and Gallagher discuss the main problem of tracking error. The main factors driving index fund tracking error are transaction costs, fund cash flows, dividends, benchmark volatility, corporate activity, and index composition changes.[2] These factors prevent index funds from perfectly replicating the performance of the underlying index.

One of the most surprising findings in Frino and Gallagher [2001] is that the extent of tracking errors is seasonal in nature. This suggests that some seasonal effects like December tax-loss selling and quarterly dividend distributions have particularly strong effects on index fund tracking error.

Since the appearance of ETFs in early 1999, much has been written about them in the popular business journals.[3] *Barron's*, *BusinessWeek, Money,* and *Forbes* have all praised ETFs for their efficiency and versatility. Gastineau [2002], one of the developers of ETFs at the American Stock Exchange, outlines their history and mechanics.

The only academic article I am aware of is Dellva [2001], who compares ETFs with index funds, and concludes that ETFs are not attractive for small investors because of brokerage commission costs. Because Dellva [2001] does not attempt to quantitatively model the differences in costs, I focus my attention on that issue.

WORKINGS OF INDEX FUNDS AND ETFs

The goal of index funds and ETFs is essentially the same: to provide investors with a way to own a well-diversified indexed portfolio by using economies of scale to buy large quantities of stock at low cost. They accomplish this goal in two very different ways.

Index funds work exactly like other active mutual funds. They accept cash deposits from outside investors, and in return issue shares of the net asset value (NAV) of the fund. Then, they use these deposits to purchase shares of stock in firms in the index or to pay back investors who are redeeming shares. Clearly, for most investors, this is far superior to paying huge transaction costs for buying 30 or 500 or even 5,000 different stocks in the index that they want to track.

As the Vanguard 500 Fund Prospectus points out, however:

> An index fund does not always perform exactly like its target index. Like all mutual funds, index funds

> have operating expenses and transactions costs. Market indexes do not, and therefore will usually have a slight performance advantage over funds that track them.[4]

It is important to enumerate these operating expenses and transaction costs that make index funds imperfect instruments for tracking indexes.

Index Funds

Bid-ask spreads and other liquidity costs are the primary source of tracking error for index fund managers. For example, when there is a large inflow of funds, managers must invest these funds, paying fees (in the form of bid-ask spreads) to market makers. Likewise, when there are redemptions that cannot be met with the cash available on hand, fund managers have to sell stocks and once again incur costs. Very often, some constituent stocks of an index are illiquid, forcing managers to suffer high costs to trade in them.

The movement of cash in and out of index funds is a secondary cause of tracking error. An effect known as *cash drag* arises because index fund managers have to keep a certain percentage of assets uninvested to meet redemption needs. Furthermore, because it's impossible to immediately invest all incoming funds, there is a short period when inflows remain in cash. Futures are often used to alleviate cash drag, but if futures aren't used or are unavailable, cash drag could become a significant source of tracking error.

Critics may argue that this effect is insignificant compared to the large price movements that occur in the stock market every day. Yet competition in the index-tracking industry is so intense that every basis point in deviation from the target index can be significant.

A third factor causing tracking error lies in dividend policies. Some paper indexes assume an immediate reinvestment on the ex-dividend date, but because index funds must wait a certain time to receive these cash dividends, there is often a short lag that contributes to tracking error. This effect has diminished in previous years, as dividend yields have fallen to their lowest levels in many decades. Yet, for some indexes full of high-dividend stocks, the effect is not negligible, and must be included as a component of tracking error.

Research has suggested that in-and-out trading can be a sizable cost drag for long-term mutual fund shareholders. Since most mutual funds allow trading until 4:00 PM and calculate NAVs as of that time, it is often possible for arbitrageurs to time their trades to take advantage of momentum and stale prices.

Zitzewitz [2002] estimates it is possible for these arbitrageurs to earn excess returns of 40% to 70% in international funds at the expense of other shareholders. Edelen [1999] relates in-and-out trading to liquidity, showing that the indirect costs of providing liquidity to investors in an asymmetrically informed market can have a significant negative impact on mutual fund returns. Although this problem isn't as important for domestic index funds, and is not relevant at all for the Vanguard index funds, it can still be a meaningful influence on index fund tracking error.[5]

Finally, the last important factor contributing to tracking error is rebalancing costs due to index changes or corporate activity. If a company leaves an index because it merges with a different firm, for example, timing mismatches can occur between the time the company leaves the index and when the index fund is able to sell all its shares and buy the shares of the company replacing it. If corporate activity such as a spin-off drastically changes the market value of a firm, the index fund must suffer transaction costs in rebalancing its portfolio.

Exchange-Traded Funds

An exchange-traded fund works in a completely different way. Unlike an index fund, an ETF does not need to pay to obtain shares of constituent stocks, operating instead through a process known as creation/redemption in-kind. This means that large investors can purchase a sizable number of shares of ETFs only by supplying a stock portfolio that matches the target index in weights and that has the same value as those shares. For example, the SPDR ETF that matches the S&P 500 can be created only in 50,000 share chunks (and redemptions work in the same way).

The advantage for the ETF is that it gets constituent shares without liquidity costs. The advantage for the large investor is that one can obtain a large number of ETF shares without moving its price in the secondary market.

Creations/redemptions in-kind are also important because they provide arbitrage opportunities that prevent the ETF price from diverging too much from the net asset value of the constituent shares. If there is a substantial deviation, arbitrageurs will step in and create or redeem shares, bringing the market back to equilibrium. Most small investors, however, are unable to meet the size requirements for creations and redemptions in kind, and must conduct all transactions in the secondary market.

Fund transaction costs are nearly non-existent because of creation/redemption in kind, although there is some cash drag, far smaller than the 2% estimated in index funds. Because the prices of ETFs and constituent stocks change nearly every second, any difference between the value of the round number of shares of the ETF (e.g., 50,000) and the value of the supplied portfolio must be equalized with a cash component. The cash-balancing amount can be positive or negative, and it is this uninvested component that can contribute to the tracking error of ETFs.

The problems arising from ETF dividend policy are similar to those for index funds. They face the same costs and timing mismatches as index funds when a constituent firm is replaced in an index or when corporate activity such as a secondary public offering changes the market cap of a stock and increases its weight in the index. These three sources of tracking error, although minor in comparison to market movements, are impossible to avoid in whatever form of index tracking an investor chooses.

Non-Tracking Error Differences

Now, let's assume that ETFs and index funds are able to perfectly replicate the performance of the market. An investor would still have an important choice to make because of three non-tracking error differences between ETFs and index funds: management fees, shareholder transaction costs, and taxation costs. While tracking error sources are nearly impossible to quantify, it is fairly simple to model the effect of these three non-tracking error sources on investor returns.

Management fees are an inescapable cost of indirect investment in the stock market. For active mutual funds, the expense ratio, which measures management fees as a percentage of total managed assets, can be as high as 2%, but for index funds, expense ratios are usually below 0.5% per year. Exchange-traded funds have been able to offer even lower expense ratios than the cheapest of index funds.

For example, the SPDR ETF has an expense ratio of 0.12% while the Vanguard 500 Fund has an expense ratio of 0.18%. The Barclays iShares500 ETF, which tracks the S&P 500, has an even lower expense ratio of only 0.09%.

The main reason ETFs are able to offer lower expense ratios is that they are not in charge of shareholder accounting. The task of keeping track of shareholder transactions and other such paperwork is a large percentage of the expense ratio; for ETFs, these tasks are performed by the brokerage house of the shareholder. Gastineau [2001] suggests that the elimination of shareholder accounting can save ETFs anywhere from 5 to 35 basis points in expense costs.

Shareholder transaction cost is another factor that is different for ETFs and index funds. No-load index funds do not charge commissions on transactions, and since the vast majority of index funds are no-load, an investor can easily find an index fund that does not charge a load.

ETFs, on the other hand, have to be purchased on the secondary market (except for large investors who can perform creations/redemptions in-kind) where the investor has to pay a commission to the brokerage house and a fee to the market makers through the mechanism of the bid-ask spread. Brokerage house commissions can be as high as 2% for full-service brokerages like Merrill Lynch, although competition among discount brokers and on-line brokers has cut commissions dramatically. It is possible to make transactions now for extremely low flat rate commissions.

Bid-ask spreads on ETFs are the other component of transaction costs for shareholders. As of now, the largest ETFs (SPDRs and QQQs) are so liquid that bid-ask spreads are estimated to be below 2 cents per share. Smaller ETFs are much less liquid, and experts believe that in the future they may suffer even worse liquidity (and higher bid-ask spreads) as volume shifts to the more popular ETFs.

The last factor that distinguishes ETFs and index funds is their tax efficiency. When redemptions exceed additions, the index fund manager is forced to sell stocks and distribute capital gains to shareholders. These capital gains are immediately taxed and create substantial costs for the shareholders. An ETF, on the other hand, rarely if ever distributes capital gains.[6]

Because of creation/redemption in-kind, ETFs always give away the stock with the lowest basis (the one that appreciated the most and has the most capital gains taxes to be paid), and keep the stock with the highest basis. When they need to sell stocks in order to rebalance, they can sell that stock and not incur capital gains because it has a high basis. Of course, Congress may some time change the law to close this loophole, but until then tax efficiency favors the ETF, and taxable investors shouldn't ignore this advantage.

EXHIBIT 1
Summary of Cost Comparisons

Types of Costs	Exchange-Traded Funds	Traditional Index Funds
	Fund Costs	
Fund Transaction Costs on Purchases and Sales by the Fund	*None*. All creations and redemptions are in-kind	Bid-Ask spreads (as fees to market makers, etc.)
Cash Inflows and Outflows	Deviations in value of creations and redemption in-kind are paid in cash	Cash drag. Small percentage (~2%) of assets is uninvested.
Dividend Policy	Lag between ex-dividend date and receipt of dividends	Lag between ex-dividend date and receipt of dividends
In-and-Out Arbitrage Trading	*None*. Arbitrage eliminated by creation/redemption in-kind	Can be important for some domestic index funds. None at Vanguard.
Index Fund Changes	ETFs must incur costs to rebalance	Similar costs to rebalance
Corporate Activity	ETFs must incur costs to rebalance	Similar costs to rebalance
Management Fees	ETFs have very low expense ratios because all accounting is done at the shareholder level	Index funds have slightly higher expense ratios because shareholder accounting is done at the fund level
	Shareholder Costs	
Shareholder Transaction Costs	Brokerage transaction fees + bid-ask spreads on ETFs	*None*, except for index funds with loads, which is rare
Taxation Costs	Capital gains are distributed very rarely (almost never)	Significant share of capital gains gets distributed especially in bull markets

Cost Comparisons

Exhibit 1 provides a summary of ETF and index fund costs. There are important differences on many levels.

ONE-PERIOD MODEL

Although the differences in the costs of ETFs and index funds are small, they are still very important to analyze. For actively managed funds, investors can always make the argument that they are paying a higher cost for a better manager; for the passively managed funds that we are looking at, though, costs are the only factor in deciding which instrument to pick. Thus, we see funds competing by reducing their expense ratios only a few basis points and substantially increasing their market share.

Take an investor who wants to invest an amount I in an index tracking asset for a period of length t. Either because he wants to use dollar-cost averaging or because he receives this sum in installments over the entire period (e.g., he is investing a part of his monthly salary over an entire year), he makes N purchases at prices $P_0, \ldots, P_{n-1}$. (P_i is the price of the fund at each transaction.) He also pays a flat rate C in commissions for each purchase.

At the end of period t, a share α of his capital gains is distributed. We assume he earned k in capital gains, of which αk is distributed and a percentage $(\alpha k)\tau_k$ is paid in taxes to the government before reinvesting. Also, at the end of period t, a part of his initial investment is distributed in dividends d and he must pay the percentage $d\tau_d$ in taxes to the government before reinvesting. Finally, what's left over is charged an expense ratio e. Note that d and k are not dollar values but ratios of the total post-commission investment $(I - CN)$, while C is a constant that is independent of the initial investment I.

Using this information, we can develop a formula for the final value of the investment. The capital gains earned are directly related to the price distribution by:

$$k = \frac{\left(\frac{1}{P_0} + \frac{1}{P_1} + \ldots + \frac{1}{P_{n-1}}\right)P_n}{N} - 1 \tag{1}$$

The value of the investment at time t before dividend and capital gains taxes are paid is:

$$(I - CN)(1 + k + d) \tag{2}$$

The value of the capital gains taxes that have to be paid is:

$$\alpha\{(I - CN)k\}\tau_k \tag{3}$$

The value of the dividend taxes that have to be paid is:

$$\{(I - CN)d\}\tau_d \tag{4}$$

The value before expenses is simply Expression (2) – Expression (3) – Expression (4):

$$(I - CN)(1 + k + d) - \alpha\{(I - CN)k\}\tau_k - \{(I - CN)d\}\tau_d \tag{5}$$

We take out the $I - CN$ term and multiply Expression (5) by $(1 - e)$ to get the final value:

$$\text{Final Value} = (1 - e)(I - CN) \times \{1 + k(1 - \alpha\tau_k) + d(1 - \tau_d)\} \tag{6}$$

This formula works for both index funds and ETFs. For example, for index funds, one would usually set $C = 0$ because index funds almost never charge commissions on transactions. For ETFs, one would usually set $\alpha = 0$ because ETFs almost never distribute capital gains.

I make four significant assumptions that must be explained (see Appendix A for a summary). First, I am assuming that the investor reinvests all after-tax dividends and distributed capital gains. If the investor sold all his shares in the asset, he would have to immediately pay capital gains taxes on the capital gains that weren't distributed, and this would eliminate the advantage that the tax-efficient (low α) funds have over tax-inefficient (high α) funds.

Second, I assume that dividends are paid as a percentage of the initial investment and not of the final investment. This is irrelevant for my analysis and is chosen purely for purposes of simplification.

Third, I am assuming that C is constant and is thus independent of the value of the transaction. While this is actually untrue for ETFs, which have bid-ask spreads, I assume that we are discussing only liquid ETFs like QQQs or SPDRs, which trade at negligible bid-ask spreads compared to commissions charged by brokers. Also, I assume that these charges are flat-rate commissions like most discount and on-line brokerage fees.

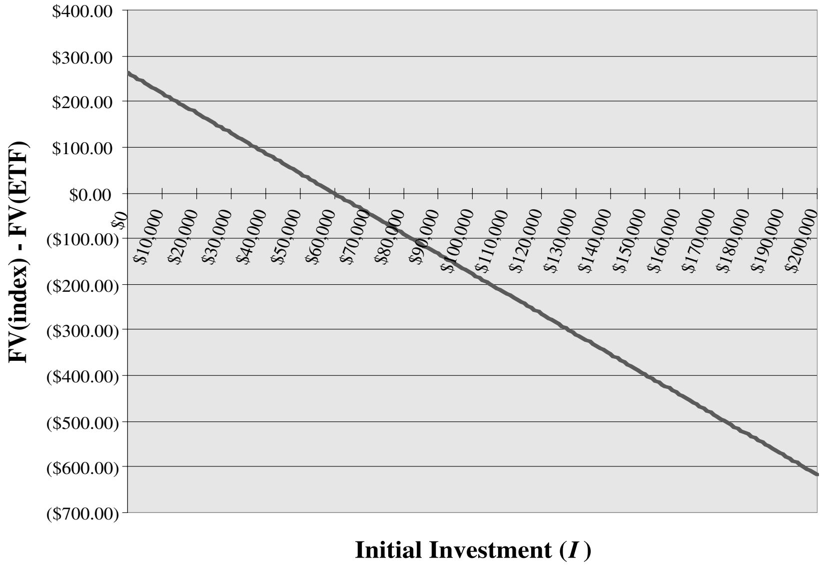
Finally, I assume N is not correlated with k, so increasing N is a deadweight loss (since it increases only the total brokerage fees paid). In reality, Equation (1) shows that this is not true, and increasing N can have unpredictable effects on k, depending on the price distribution. Because my purpose is not to model the effectiveness of dollar-cost averaging, and because changes in N have the same effect on k for both index funds and ETFs, it is not incorrect to make this assumption for comparisons.

The nine independent variables in the model are summarized in Exhibit 2. I call a variable that is the same across all investors and across all funds tracking the same target index *global*. Because tracking error is very difficult to model, I will assume that ETFs and index funds can track the market identically, and thus the returns k and d (the return on capital gains and the dividend yield) are equal for all funds tracking the same index. If one doesn't wish to assume perfect tracking, it's still possible to consider k and d as *global* by including the difference in tracking error as part of the difference in the expense ratio e.

EXHIBIT 2
Analysis of Variables

Formula Variable	Variable Type	Variable Description
k = return from capital gains	Global	Equal to the capital gains return on the target index
d = return from dividends	Global	Equal to the dividend return on the target index
C = flat brokerage commission	Semiglobal	$0 for index funds. ETFs: Ranges from $10 to 2%
e = expense ratio	Semiglobal	Can include tracking error
α = capital gains distribution ratio	Semiglobal	0 for most ETFs. Up to 0.3 for some index funds
I = initial investment	Local	Assume less than $500,000
N = # of purchases	Local	Assume N is uncorrelated with k; see below
τ_k = tax rate on capital gains	Local	Usually 20% for most investors
τ_d = tax rate on dividends	Local	Can range from 0% (for pensions) to nearly 40%
Distribution of prices P_i	Dependent	Dependent on k
t = time	Other	Subscript: Indicates prices at different times.

EXHIBIT 3
Changes in Initial Investment



I call a variable that is the same for all investors but differs for assets semiglobal. The flat-rate brokerage fee C, expense ratio e, and capital gains distribution ratio α are all variables that are asset-dependent. Finally, I call a variable that differs among investors *local*. The initial investment I, the number of purchases N, and the tax rates τ_k and τ_d vary for different investors, and are thus *local* variables.

The one set of variables not included in this list is the distribution of prices P_0, ..., P_n. These variables are included in k.

An examination of Exhibit 1 indicates that all the major differences between index funds and ETFs are represented in the model's *semiglobal* variables. The difference in management fees is in e; the difference in shareholder transaction fees is in C; and the difference in taxation efficiency is in α. If we assume that the difference in tracking error is a part of the expense ratio e, this model fully accounts for all the key quantitative differences between index funds and ETFs.

An investor will choose an index fund over an ETF if the Final Value (index fund) – Final Value (ETF) > 0. The only variables different for the FV (index fund) and FV (ETF) are the *semiglobal* variables. We will indicate all *semiglobal* variables for index funds by the subscript (i) and all *semiglobal* variables for ETFs by the subscript (e). For example, the expense ratio of an index fund would be e_i, and the expense ratio of an ETF would be e_e.

The investor choice equation is:

$$FV_i - FV_e = \left[(1-e_i)(I-C_iN)\left\{1+k(1-\alpha_i\tau_k)+d(1-\tau_d)\right\}\right] - \left[(1-e_e)(I-C_eN)\left\{1+k(1-\alpha_e\tau_k)+d(1-\tau_d)\right\}\right] \tag{7}$$

Although this equation may look complicated, it's simply the difference in value between buying an index fund and buying an ETF. By adding subscripts to Equation (6), we can quantify the profit of holding index funds instead of ETFs (a negative number would indicate the profit of holding an ETF instead of an index fund).

What happens if we graph the value of Equation (7) with changing I, while keeping all the other variables constant? The results are shown in Exhibit 3. (See Appendix B for the actual values of the constants and why the values were chosen.).

First, we can clearly assert that those investing more than $59,635 will choose to invest in ETFs, while those investing less will choose index funds. This value is actually not very interesting because the threshold level where Equation (7) = 0 is dependent on the values chosen for the other variables (see Appendix B).

More important, the first derivative of Equation (7) with respect to I is negative and constant, so this graph will slope down linearly for all reasonable choices of con-

EXHIBIT 4
Boundary Condition Analysis

(1)	$\alpha_i = 0$	$\alpha_i = 0.1$	$\alpha_i = 0.2$	$\alpha_i = 0.3$	$\alpha_i = 0.4$	$\alpha_i = 0.5$
Minimum ***I*** (1st Deriv.)	\$217,876 (-1.203)	\$93,641 (-2.799)	**\$59,635** (-4.395)	\$43,748 (-5.991)	\$34,546 (-7.587)	\$28,542 (-9.183)

(2)	C_e = \$0	C_e = \$10	C_e = \$20	C_e = \$35	C_e = \$50	C_e = \$200
Minimum ***I*** (1st Deriv.)	\$0 (-4.395)	\$29,817 (-4.395)	**\$59,635** (-4.395)	\$104,362 (-4.395)	\$149,089 (-4.395)	\$596,357 (-4.395)

(3)	$e_i = 0.05\%$	$e_i = 0.15\%$	$e_i = 0.25\%$	$e_i = 0.4\%$	$e_i = 0.6\%$	$e_i = 1\%$
Minimum ***I*** (1st Deriv.)	\$118,372 (-2.214)	\$79,313 (-3.305)	**\$59,635** (-4.395)	\$43,461 (-6.031)	\$31,919 (-8.211)	\$20,846 (-12.573)

(4)	$N = 1$	$N = 2$	$N = 4$	$N = 6$	$N = 12$	$N = 24$
Minimum ***I*** (1st Deriv.)	\$4,969 (-4.395)	\$9,939 (-4.395)	\$19,878 (-4.395)	\$29,816 (-4.395)	**\$59,635** (-4.395)	\$119,271 (-4.395)

(5)	$\tau_k = 0\%$	$\tau_k = 10\%$	$\tau_k = 20\%$	$\tau_k = 28\%$		
Minimum ***I*** (1st Deriv.)	\$217,876 (-1.203)	\$93,641 (-2.799)	**\$59,635** (-4.395)	\$46,210 (-5.672)		

stants. The value of the first derivative for the choice of constants in Appendix B is 0.00439 so for every extra \$10,000 to be invested, ETFs will provide an extra \$43.90 more in value than index funds.

Finally, Exhibit 3 shows that as initial investment size grows, ETFs become far superior to index funds. For example, for a person investing \$500,000, ETFs will provide \$1,935 more than an index fund, suggesting that ETFs should be an important and useful tool for large investors' portfolios.

The second element of the analysis is to adjust several *semiglobal* and *local* variables and see the effect of this on the threshold level. Exhibit 4 provides a summary of the minimum investment (I) needed to make ETFs preferable to index funds.

The underlined and boldfaced figures are identical to the minimum I in Exhibit 3 because they are simply derived from the constants in Appendix B. The numbers in parentheses show the first derivative with respect to I multiplied by 1,000. In other words, this is the change in dollars in Equation (7) upon increasing the initial investment by \$1,000.

Most of the results seen in Exhibit 4 are as would be expected. Increasing index fund alpha, index fund expense ratios, and capital gains tax rates has both absolute and marginal benefits for ETFs. In all cases, the derivative is negative, because higher initial investments should always benefit ETFs.

Exhibit 4 also highlights some interesting implications that are not as obvious. First, note that increasing N or increasing C increases the absolute value of the minimum I needed to switch to ETFs, but has no marginal effects (since the derivative stays constant). This indicates that whatever benefits dollar-cost averaging provides (which I don't look into here) must be weighed against an initial fixed cost if one decides to invest in ETFs.

Another interesting implication is that the tax rate on capital gains and the capital gains distribution ratio have an identical effect on the minimum value of I. Thus, in this model, lowering α from 0.2 to 0.1 is the same as if the government had reduced the capital gains tax rate by half, from 20% to 10%. This is an important implication that shows just how critical tax efficiency can be. In multiperiod models, this one-to-one correspondence disappears because taxes on the undistributed capital gains will eventually have to be paid when the investor sells the investment.

The last result that I find surprising is the superiority of index funds over ETFs for small investors under almost any conditions. Assuming rather unusual conditions such as an index fund expense ratio of 1% or transaction fees as low as \$10 still does not change the preference of small investors (< \$20,000) for index funds over ETFs. This suggests that there is no reason for small investors who want to invest for a short period of time to choose ETFs.

MULTIPERIOD MODEL

Does a multiperiod model change the conclusions, or should small investors always prefer index funds to ETFs? Take an investor who wants to invest an amount I in an index-tracking asset for a period of n years. Again, she makes N purchases at prices $P_0, \ldots, P_{n-1}$ (P_i is the price of the fund at each transaction) all during the first year. Then, in all the following years, she simply reinvests all her distributed dividends and capital gains and does not add any new money. After n years, she sells her shares and pays capital gains taxes on the difference between her final value and the cost basis.

If we assume all the variables are the same as in the one-period model, the value of the investment at the end of the first year is given by Equation (6). In all the remaining years, she receives capital gains k_t and dividends d_t, which vary from year to year. The final value after time t (where t can be any number from 1 to n) is thus:

$$\text{Value}_t = (1-e)^t (I - CN) \times \prod_{t=1}^{t}\left[1 + k_t(1-\alpha\tau_k) + d_t(1-\tau_d)\right] \tag{8}$$

We get this by compounding the returns for each year (using Π), and multiplying each time by the expense ratio deduction $(1 - e)$.

The cost basis is the part of the final value that is not taxable because it was either already taxed or because it is part of the initial investment:

$$\text{Cost Basis}_t = \left\{(1-e)\text{Value}_0\right\} + \sum_{t=1}^{t}\left[\text{Value}_{t-1}\left\{\alpha k_t(1-\tau_k) + d_t(1-\tau_d)\right\}\right] \tag{9}$$

The final value after redemption (i.e., where $t = n$) is the value from Equation (8) minus the liquidation taxes that have to be paid on the undistributed capital gains minus the transaction fee for conducting that last transaction of selling one's shares:

$$\text{Final Value}_n = \text{Value}_n - \tau_k\left(\text{Value}_n - \text{Cost Basis}_n\right) - C \tag{10}$$

What are some interesting improvements of the multiperiod model over the one-period model? First, we can look at how different time horizons affect investor choices by seeing if a higher value of n favors ETFs or index funds. Second, we can look at how different statistical distributions of capital gains ($k_1, \ldots, k_n$) affect investor choices. For example, if the capital gains returns of a target index have a high variance (e.g., the Nasdaq), we can examine whether this favors ETFs or index funds.

Finally, we can eliminate the assumption of non-redemption included in the one-period model (Assumption 1, Appendix A). This allows our model to better approximate non-theoretical real-world conditions. Assumptions 2-6 in Appendix A are still necessary.

We start the analysis by exploring investor choice over different values of n. We can do this by simply graphing the difference between index funds and ETFs ($FV_i - FV_e$) using the final values from Equation (10) against the time horizon.

Exhibit 5 shows these relationships for different choices of the initial investment I. Once again, all other variables have the values shown in Appendix B. All k_t are equal to 8%, and all d_t are equal to 2%.

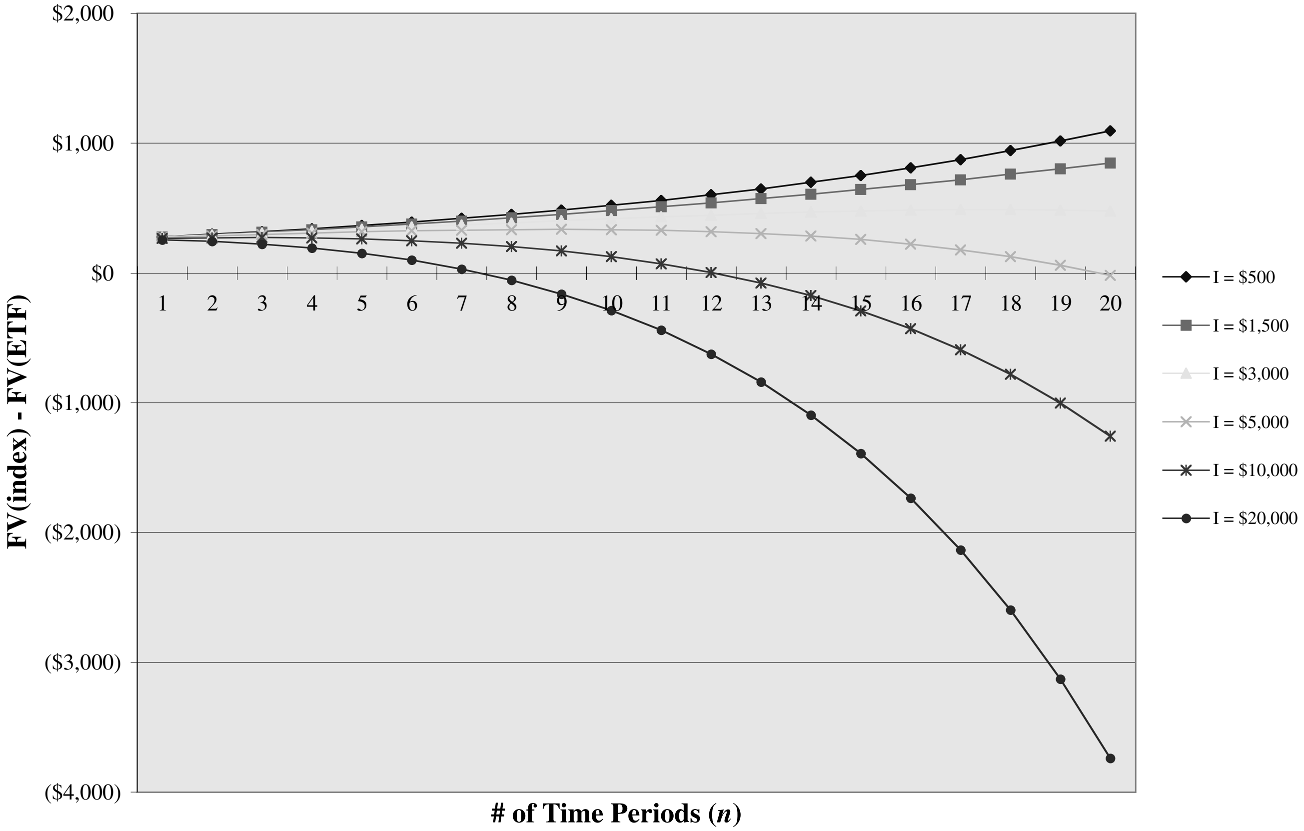
The graph in Exhibit 5 highlights some intriguing properties of investor choice in the multiperiod model. We find that changing the time horizon has a significant effect on whether index funds or ETFs are preferred, but the effect is not linear like the effect of I on investor choice (see Exhibit 3), but rather quadratic. As n rises initially, index funds become better off since the initial fixed-cost advantage is multiplied, but after an extended period of time, the superior tax-efficiency and lower expense ratios of ETFs cause a dramatic drop in the graph.

Although it looks as if the graph for $I = \$500$ keeps rising, one can calculate that ETFs will eventually become better than index funds after 171 periods. Only if $I < C_e N$ will index funds be superior for all time horizons, because in that case, ETF investors will have to pay out their entire initial investment in brokerage fees.

Another interesting conclusion is just how much the multiperiod model favors ETFs more than the one-period model. Keeping all other variables constant, one needs at least \$59,635 in investments to prefer ETFs to index funds for a holding period of one year. If an investor wants to stay in the tracking instrument for ten years, however, one needs only \$13,019. As most small investors fall in between these two numbers, this has an important ramification for our comparisons.

Even for investors who want to stay in the tracking

EXHIBIT 5
Changes in Time Horizon



instrument for only five years, $33,787 is the minimum investment, which is still well below the one-year threshold level. As with the one-period model, large investors strongly favor ETFs over index funds.

The last interesting point to note about Exhibit 5 is derived from the property that the relationship between the $FV_i - FV_e$ and n is quadratic. As a result, the slope increases quickly in magnitude (becoming more negative), making marginal decisions over the long term very important.

For example, if we look at the I = $20,000 part of Exhibit 5, lengthening the holding period by one year from 19 years to 20 years reduces $FV_i - FV_e$ by more than $600. If we look at the more extreme case of I = $1,000, lengthening the holding period by one year from 76 years to 77 years (unrealistic numbers, I do agree) would change an investor from preferring index funds by more than $1,400 to preferring ETFs by more than $600. This again reminds us how important marginal decisions can be over the long term.

The second part of the analysis compares the effect on investor choice of changing the variance of capital gains returns. We assign n the value of 10, and all other variables have the values given in Appendix B. Then, instead of k_t = 8% for all t, k_t alternates between a high value and a low value (with the average remaining at 8%). For example, to obtain a standard deviation of 2%, we have $k_{even\ years}$ = 9.897% and $k_{odd\ years}$ = 6.103% for n years (ten in this case).

Exhibit 6 displays the effect of differing standard deviation of k on the minimum initial investment I needed to make ETFs preferable to index funds. From this graph, we can see that the variance (or standard deviation) of capital gains can be very important for investors deciding whether index funds or ETFs should be preferred. For more stable indexes like the S&P 500, there is little effect of additional instability, but for volatile indexes like those in developing markets or the Nasdaq Composite Index, index funds have a dramatic advantage.

It is difficult to say whether the results in Exhibit 6 are simply the consequence of choosing certain values for variables, or if they hold for any reasonable variable values for ETFs and index funds. I can try to suggest some basis behind the index fund advantage. Because an ETF distributes less of its capital gains, the tax burden is impounded in the value of the shares until redemption. More volatile returns thus make the tax burden of ETFs more volatile, which diminishes their value compared to the more stable index fund tax burden. Of course, it's impossible to guess the validity of this hypothesis without more analysis of taxation's effect on Exhibit 6, a subject beyond my objectives.

In the one-period model, because non-distributed capital gains were never taxed, changing the capital gains distribution ratio of index funds α_i had the same effect as changing the capital gains tax rate τ_k. Clearly, this should not occur in the multiperiod model, as undistributed capital gains are later taxed at redemption.

Exhibit 7 (similar to Exhibit 4) compares the minimum value of *I* needed to make ETFs preferable to index funds for the multiperiod model with $n = 10$. Derivatives are not given because they are no longer constant.

Not surprisingly, the one-to-one correspondence between α_i and τ_k disappears in the multiperiod model. Instead, we see that raising the capital gains tax rate has less of an effect on investor choice than increasing the capital gains distribution of index funds. This is because raising the capital gains tax rate depresses the value of both ETFs and index funds (unlike the one-period case where capital gains taxes were charged only on the index fund), while increasing α_i increases the tax burden of only the index fund.

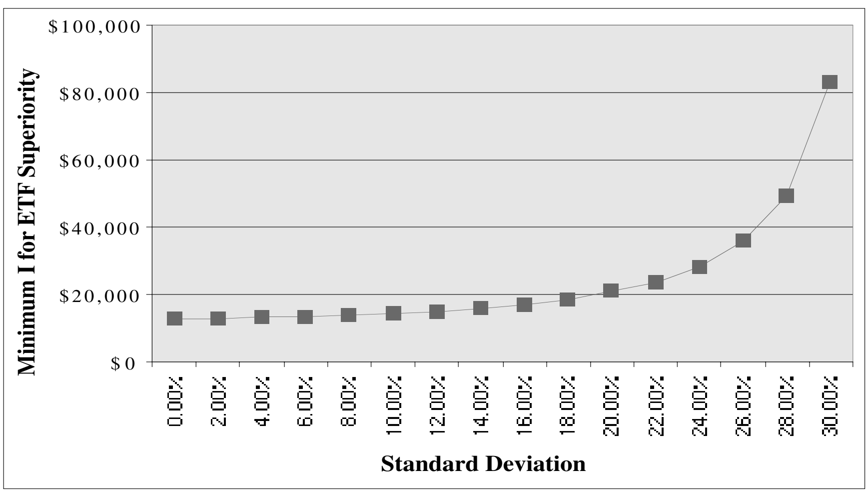
QUALITATIVE COMPARISON

As I indicated earlier, the models I propose are unable to perfectly replicate real-world conditions. First, there are simplifying assumptions that must be made in order to be able to conduct mathematical comparisons, and second, there are factors investors consider that just cannot be expressed in terms of numbers. Often, these factors have as much of a part to play in decision-making as the explicit cost comparison.

One important qualitative advantage of ETFs is convenience. One can sell or buy at any time of day instead of waiting until the end of the day for an index fund. Carty [2001] suggests an example. Imagine an investor who put in an order to sell his S&P index mutual fund on the morning of October 19, 1987, expecting to get a price around that of the previous day. What did he think when, the next day, he discovered that his proceeds were 22.4% less than he expected? Certainly, the 1987 crash isn't a daily occurrence, but the ease of getting out of an ETF can be a very important advantage, especially for more active larger investors.

A second advantage of ETFs is the ability to buy on margin and sell short. For many investors, this ability can be vital, and because ETFs are exempt from the sell-short uptick rule, they can also be used to great advantage in hedging strategies.

EXHIBIT 6
Changes in Standard Deviation



S.D.	Min I
0.00%	$13,019
2.00%	$13,047
4.00%	$13,179
6.00%	$13,422
8.00%	$13,788
10.00%	$14,297
12.00%	$14,979
14.00%	$15,878
16.00%	$17,064
18.00%	$18,644
20.00%	$20,795
22.00%	$23,830
24.00%	$28,345
26.00%	$35,651
28.00%	$49,272
30.00%	$83,018

EXHIBIT 7
Multivariable Boundary Condition Analysis

(1)	$\alpha_i = 0$	$\alpha_i = 0.1$	$\alpha_i = 0.2$	$\alpha_I = 0.3$	$\alpha_i = 0.4$	$\alpha_i = 0.5$
Minimum ***I***	\$22,738	\$16,539	\$13,019	\$10,750	\$9,167	\$7,998

(2)	$\tau_k = 0\%$	$\tau_k = 10\%$	$\tau_k = 20\%$	$\tau_k = 30\%$	$\tau_k = 40\%$	$\tau_k = 50\%$
Minimum ***I***	\$22,653	\$16,309	\$13,019	\$11,063	\$9,829	\$9,050

Finally, the fact that ETFs are traded allows investors to place stop, stop-loss, and limit orders on them. These are frequently used tools, and using them on ETFs is something many investors can take advantage of to prevent market makers from providing them bad prices.

Of course, index funds also have useful features that are attractive to investors. The most important feature is simplicity. For ETFs, one needs to open a brokerage account, deposit cash in the account, set a market or limit order, and make sure it's executed. For average investors, it is far simpler to send a check to an index fund, and wait for reports in the mail to see how one's investment is doing.

It is often surprising to Wall Street experts how much small investors value simplicity and are willing to sacrifice a lower expense ratio or superior tax-efficiency to achieve it.

CONCLUSION

As Gastineau [2002] points out, ETFs are still evolving. In the near future, fixed-income ETFs and actively managed ETFs may once again change the world of finance as equity index ETFs have in the last decade.

My research shows that the key areas of difference between the two instruments are management fees, shareholder transaction fees, taxation efficiency, and other qualitative differences. Tracking error is difficult to model because there isn't a true benchmark for comparison. Comparison with the paper indexes is fallacious because they assume efficient paper transactions.

Instead, my one-period model specifically looks at the other three quantitative factors and examines their effects on investor choices. A multiperiod model can deal with various problems in the one-period model. Finally, I consider that qualitative differences between ETFs and index funds can be extremely significant for decision-making.

Some additional analysis would be interesting, such as the effects of changing tax rates on investor choices. Other questions are why increasing variances helps index funds over ETFs in the multiperiod model, or different values of the dividend rate and a different mean capital gains rate affect investor choices, or whether it is possible to relax some of the simplifying assumptions.

APPENDIX A

Simplifying Assumptions for the One-Period Model

1. The final value is left invested in the asset and not sold to get cash. This is important because it creates an advantage for funds with a lower α.
2. The dividend d is paid on the total after-brokerage fee investment $I - CN$.
3. C is a flat dollar rate and unrelated to the transaction value.
4. k and d are the same for all ETFs and index funds that track the same index. Thus, tracking error is not included, or it is included as part of the expense ratio e.
5. k is unrelated to the value of N.
6. All distribution reinvestments are made without transaction costs.

APPENDIX B

Choice of Constants

I = \$5,000 (this can be amended for different investors).
N = 12 (assumes monthly investing for one year).
e_i = 0.25% (this incorporates expense ratio and tracking error).
e_e = 0.14% (this incorporates expense ratio and tracking error).
α_i = 0.2 (from historical data of index fund distributions).
α_e = 0 (ETFs almost never distribute capital gains).
τ_k = 20% (usual capital gains tax rate).
τ_d = 32% (usual income tax rate).
C_i = \$0 (index funds almost never charge brokerage fees).
C_e = \$20 (usual discount brokerage fee for buying stocks).
k_t = 8% (from historical data of index capital gains).
d_t = 2% (from recent historical data of index dividends).

Some of these values are assigned by looking at current (and historical) data on averages, and others are assigned at random. Thus, values are liable to change for different indexes, funds, and investors, but my analysis tries to focus on general characteristics rather than on actual values.

ENDNOTES

The author thanks Gary Gastineau, Alex Kostovetsky, and Marciano Siniscalchi for their excellent advice, contributions, and criticism.

[1]Statistics come, variously, from Williams [2001]; www.indexfunds.com; and Frino and Gallagher [2001].

[2]These factors are enumerated in Chiang [1998].

[3]The SPDR S&P 500 ETF has been around since 1993 although there was no real demand for it until the mid- or late 1990s.

[4]Vanguard 500 Fund Prospectus, at www.vanguard.com.

[5]Vanguard tries to protect shareholders by not allowing transactions in the last hours of the trading day.

[6]The SPDR has not distributed capital gains in the last four years, and the QQQ has not had a capital gains distribution since its inception two years ago. Vanguard, however, has passed along 4% of its value to its shareholders in the past three years.

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