

what does "implies" mean.

Φ implies Ψ

remember
 $\Phi = \text{"phi"}$
 $\Psi = \text{"psi"}$

①

\Rightarrow the truth of Ψ "follows" from truth of Φ

④ Φ : $\sqrt{2}$ is irrational
" Φ is the statement "

⑤ Ψ : $0 < 1$
"let Ψ be statement"

so "Is statement Φ implies Ψ true

so we know Φ is true { to be proved later course }
and we know Ψ is true

\Rightarrow But does that mean Φ implies Ψ ?
it does not

there is no relationship.

that is, the truth of ⑤ Does not
follow from the truth of ④

②
The rule:

"implication involves Causality"

↳ this is complex

= with conjunction and disjunction, it did not matter
⇒ 1:2 relationship between 2 conjuncts or

2 disjuncts!
eg (conjunction) $(\overset{(T)}{\text{Julius Caesar is dead}}) \wedge (\overset{(F)}{1+1=3}) = F \checkmark$
(Disjunction) $(\overset{(T)}{\text{Julius Caesar is dead}}) \vee (\overset{(F)}{1+1=3}) = T \checkmark$

— one is statement of dead individual,
the other is mathematical statement

⇒ clearly no relationship

But we do not require it for \wedge or \vee to work.

⇒ play no role in what truth value was!

↳ this is not the same with
implication

⇒ ① it has Truth Part

② and it has a Causation Part

"Conditional is always defined"

④

(Quiz?)

Truth table

| | ϕ | ψ | $\phi \Rightarrow \psi$ |
|---|--------|--------|-------------------------|
| ① | T | T | T ✓ |
| ② | T | F | F |
| ③ | F | T | T |
| ④ | F | F | T |

"The truth of ~~ϕ~~
follows from the
truth of ψ "

① $\phi: N > 7$

$\psi: N^2 > 40$

readings: assume
 ϕ is that statement
 $N > 7$

assume ψ is the
statement $N^2 > 40$

\Rightarrow if ϕ is true, that is $N > 7$
then N^2 is also larger than 40
(since) $7^2 = 49 > 40$

$\therefore \phi \Rightarrow \psi$

\Rightarrow here the two follow from each other!

\Rightarrow this we will encounter in maths!

(4.1)

Q : Julius Caesar is dead

let Q be shorthand
Julius Caesar is dead

ψ : $\pi > 3$

let ψ be shorthand
 $\pi > 3$

if Q is true, and ψ is true

$\therefore Q \implies \psi$

$\therefore (\text{Julius Caesar is dead}) \implies (\pi > 3)$

But we are only interested in truth part (non-emptiness)

\Rightarrow there is no connection!

\Rightarrow No meaning r.t.o. implication, the truth
value is well defined.

\hookrightarrow But then we want have in math

②

| | | |
|--------|--------|-------------------------|
| Φ | Ψ | $\Phi \Rightarrow \Psi$ |
| T | F | F |

(exception)

⑤

"remember: ~~the truth of Φ follows from~~
~~truth of Ψ~~ the truth of Ψ follows
 from truth of Φ

$$\Phi = T$$

$$\Psi = F$$

$\Phi \Rightarrow \Psi$ should thus be T, not F

$$\therefore \Phi \Rightarrow \Psi = (F)$$

③

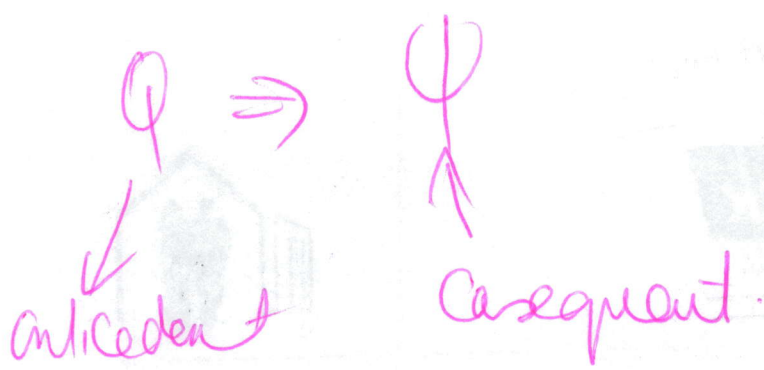
| | | |
|--------|--------|-------------------------|
| Φ | Ψ | $\Phi \Rightarrow \Psi$ |
| F | T | T |
| F | F | T |

Rule: for all circumstances $\Phi \Rightarrow$ (false)

$\Phi \Rightarrow \Psi$ will be T

$\Phi \Rightarrow \Psi$ is true, whenever:

- ✓ (1) Φ and Ψ are both true (T)
- ✓ (2) Φ is false and Ψ is true T
- ✓ (3) Φ and Ψ are both false T
- ✓ (4) Φ is true and Ψ false (F)



means: \emptyset
Cannot get false
consequence, from
True assumption / case

\therefore When Ψ (Consequent is False)
(provided the Q (Antecedent is not false)
the $Q \Rightarrow \Psi$ will be false
 \Rightarrow that is only occurrence where
it will be false

All other Circumstances it will be true.

⑦

$$\begin{matrix} \text{True} \\ (11 > 2) \Rightarrow (11 > 1.2) \Rightarrow \text{True} \\ \text{antecedent} \quad \text{consequent} \end{matrix}$$

$$\begin{matrix} \text{False} \\ (11 < 0) \Rightarrow (11 = 3) = \text{False tree} \\ \text{antecedent} \quad \text{consequent} \end{matrix}$$

$$\begin{matrix} \text{True} \\ (11 > 0) \Rightarrow (1 + 2 = 4) \quad \text{False} \\ \text{antecedent} \quad \text{consequent} \end{matrix}$$

antecedent

consequent \Rightarrow "can't get False consequence from true assumption or answer"