"Logical Equivalence"
"similar to implication"

- This is a central notion in maths

⇒ Equivalence is to logic

as Equations are to arithmetic & algebra

- two statements ϕ, ψ are said to be logically Equivalent if each implies the other!

⇒ formal version of Equivalence is called the bi conditional.

$$\phi \Longleftrightarrow \psi$$

denoted by double arrow

also means

same as $(\phi \Rightarrow \psi) \wedge (\psi \Rightarrow \phi)$

3:40 – 4:35
78 kcy

So Biconditional can also be defined i.t.o. truth value (2)

$\Phi \iff \Psi$ is true

- iff Φ, Ψ are both true or Both False ← denote (capital) $\bar{\Phi}$, $\bar{\Psi}$

must show $\bar{\Phi}, \bar{\Psi}$ have same truth table

E.g.

$$\overbrace{(\Phi \wedge \Psi) \vee (\neg \Phi)}^{\bar{\Phi}\ (A)} \text{ is equivalent } \overbrace{\Phi \Rightarrow \Psi}^{\bar{\Psi}\ (B)}$$

'Φ conjunct with Ψ or not Φ' is equivalent $\Phi \Rightarrow \Psi$

(A) capital $\bar{\Phi}$

(B) capital $\bar{\Psi}$

we need to look at truth table for (A) and (B) and show they are same

Φ	Ψ	$\Phi \wedge \Psi$	$\neg\Phi$	$(\Phi \wedge \Psi) \vee (\neg\Phi)$	③ $\Phi \Rightarrow \Psi$
T	T	T	F	T	T
T	F	F	F	F	F
F	T	F	T	T	T
F	F	F	T	T	T

From memory

Now compare these two

$\Rightarrow$ Colums are same

now we can conclude

$\Phi \wedge \Psi \vee (\neg\Phi)$ is equivalent $\Phi \Rightarrow \Psi$ ✓

But proving Equivalence via a truth table is very unusual!

What is difficult with Equivalence is mastering the different non-mechanisms!

("genuine implication")

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we start out with:

$$\phi \Rightarrow \psi$$

"different notations"

the following all mean "ϕ implies ψ"

1. if ϕ, then ψ

check word "sufficient"

2. ϕ is sufficient for ψ

3. ϕ only if ψ [Not same as if ψ then ϕ]

("e.g. you can ride TDF, only if you ~~are a professional rider~~ have a bicycle"

Not same as if have bicycle, you can ride TDF.

$\Rightarrow$ must be careful.

4. ψ if ϕ [flipped order]

[ϕ — original antecedent
ψ — consequent]
(~~flipped~~) stay same

5. ψ whenever ϕ [again order flipped]

6. ψ is necessary for ϕ [again order flipped]

But they still same

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Impontant to master the languages or temenology (used in science, legal document, etc) not just maths)

[same can be said for equivalence]

a) Φ is equivalent to φ is itself equivalent to

b) a) Φ is <u>necessary and suffecent</u> for φ

b) Φ <u>if and only</u> if φ iff = if and only iff

Quiz:

Which of the following conditions is necessary for natural numbers n to be a multiple of 10?

" Does n being multiple of 10 imply "the statement" ..... (below)

✓ 1. n is a multiple of 5 (yes)

✗ 2. n is a multiple of 20 (NO $\Rightarrow$ 10 is not a multiple of 20)

✓ 3. n is even and multiple of 5

✗ 4. $n = 100$

✓ 5. n^2 is multiple of 100

Que 2:

which of the following conditions is sufficient for the natural number n to be a multiple of 10?

"Does the statement imply n is a multiple of 10"

does this statement imply that n is a multiple of 10

(No) ✗ 1. n is a ~~not~~ multiple of 5

5 is not multiple of 10

yes ✓ 2. n is a multiple of 20

yes ✓ 3. n is even and a multiple of 5

✓ 4. n = 100

✓ 5. n^2 is multiple of 100

→ Que: which of following conditions is necessary and sufficient for the natural number n to be a multiple of 100

→ Combined two before!

N	(and)	S	
~~✓~~		✓	1
~~✓~~		✓	2
✓		✓	3
~~✓~~		✓	4
✓		✓	5

(where Both is true) ③ ⇒ ⑤ ⇒

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Identify the antecedent in each of the conditionals

i.e what does the "implying"

∴ antecedent (first part)

consequent (second part)

["truth of ϕ" is determined by ψ]

1. If the alarm rings, everyone leaves

∴ If ϕ, then ψ

antecedent — consequent

The alarm rings — Everyone leaves

Does the implying

2. Everyone leaves, if the alarm rings

ψ if ϕ

Everyone leaves — The alarm rings

Does the implying

3. Chez cycles only if sun shines

ϕ only if ψ [Flipped]

Chez cycles — The sun shines

Does the implying

4. Joe leaves whenever Amy arrives

ψ whenever ϕ [flipped]

Joe leaves. Amy arrives

does the implying