Working with ~~identifier~~ quantifiers $\exists$ $\forall$

"How does negation affect quantifiers"

Simple Quiz: (T or F) $\Rightarrow$ for real numbers!

$\Rightarrow \forall x(x+1 \geq x)$ (True) ✓

"for every x, $x+1 \geq x$"

Is it the case that.... (True) ✓

$\Rightarrow \exists x(x^2+1=3)$ — not true for rational, But true for real numbers

"Is it true "there is a real number x, $x^2+1=3$" (eg) $x=\sqrt{2}$

("we exist" written above "there is")

aside: real numbers include: rational, irrational and whole numbers

really any number you can think of, but not imaginary or infinity

$\sqrt{-1}$

$\Rightarrow \forall x(x^3+17x^2+6x+11 \geq 0)$

$\therefore$ $(-\text{something})^3$ is very large $\therefore$ False

$\forall x \exists y \, (x^3 + y^3 = 0)$

Is it true, that for all x, there is a y, $x^3 + y^3 = 0$

$\therefore y^3 = -x^3$ $\therefore$ (true) ✓

given any x, just let y be -negative x.

$\therefore$ when you cube positive number, its positive

" negative ", its negative

$\therefore$ you will get 0, by taking $-y$.

$\exists x \forall y^{(*)} \, (x + y = 0)$ — one x, cannot work for every y. ($\times$)

"Is there an x, such that for any y you add it it you get 0. (False)

$\Rightarrow$ quantifiers should be other way around.

Correct way (True)

$\forall y \exists x$ (✓)

For every y, there is an x, such that $x + y = 0$

$\hookrightarrow$ But for every y, there is an x

$\forall x \exists y (x \geq 0 \Rightarrow y^2 = x)$

Exceptions are for me.

for every x, there is a y, such that

<u>if</u> $x \geq 0$, then $y^2 = x$

truth table makes it (True)

eg $-x$, then false anticedent, (game over)

(True)

or non negative x, then $y^2 = x$

then any y will make it True

But $y^2 \neq x$

"Negating statement that have quantifiers"

By putting $\neg$ (negation) symbol in front

But also,

You will need to produce a <u>positive</u> assertion, not negative one

what is, rather than what is <u>not</u>

Let $A(x)$ be some property of x.
(A of x)

Eg. x is a real root of the equation $x^2+2x+1=0$

∴ root = solution to an equation

I'll show that $\neg[\forall x A(x)]$ is ← [not, For all x, Ax]

equivalent to $\exists x [\neg A(x)]$

(there exist x, such that not $A(x)$)

Eg. "It is not the case that all motorists run red lights" (true)

"is equivalent to" $\Longleftrightarrow$ "there is a motorist who does not run red lights."

very obvious that these two are equivalent.

Remember to prove equivalence,

(1) First prove implication from left to right $A \Rightarrow B$

(2) then from right to left.

1 left to right Case

$\Rightarrow$ Assume $\neg[\forall x\, A(x)]$
(not, for all x, $A(x)$)

$\therefore$ If it is not the case for all x, $A(x)$, then at least one x must fail to satisfy $A(x)$.

So for at least one x, $\neg A(x)$ is true

$\therefore \exists x[\neg(A(x)]$

There exist an x, such that not $A(x)$ is true (✓)

$\therefore$ That left to right implication $\square$

$\therefore$ I assume $\neg[\forall x\, A(x)]$, and I Concluded $\exists x[\neg A(x)]$

$\Leftarrow$

2 Assume right to left $\exists x[\neg A(x)]$

"There is an x for which $A(x)$ is false"

Then $A(x)$ cannot be true for all x

$\therefore \forall x\,(Ax)$ must be false

$\therefore \neg[\forall x\, A(x)]$

$\therefore$ Assume $\exists x[\neg A(x)]$, then

Conclude $\neg[\forall x\, A(x)]$

<u>Remember</u> the human brain is very good at doing logical reasoning about <u>familiar situation</u> (eg motorist/red light example)

When we turn situations into <u>abstract</u> version, the brain finds it difficult. [at least at first]

<u>what mathematicians do</u>:

, so taking familiar everyday reasoning, (that we don't even think about), and reproducing it in an abstract <u>situation</u>

<u>Another example</u>:

Show that $\neg[\exists x\, A(x)] \Leftrightarrow \forall x[\neg A(x)]$

First give everyday example.

(more) $\Rightarrow$ "It not case, that ~~every~~ there is a motorist run red lights

$\Leftrightarrow$ all motorist does not run red lights.

(same eg)

"~~All~~ domestic cars are badly made"

Let C be the set of all cars, $D(x)$ means x is domestic,

$M(x)$ means x is badly made.

$$(\forall x \in C)[D(x) \Rightarrow M(x)]$$

for all cars, if car is domestic, then its badly made

(what happens)

∴ when we negate a universal quantifier:

$$(\exists x \in C)[D(x) \not\Rightarrow M(x)]$$

there exist an x in C, ~~such~~ such that

$D(x)$ does not imply $M(x)$

this is abbreviated, correct:

$$\neg[D(x) \Rightarrow M(x)]$$

why not $(\exists x \notin C)$? ← don't negate this (wrong)!

We know $[D(x) \not\Rightarrow M(x)]$ is equivalent to

$D(x) \wedge \neg M(x)$ [when we did truth table]

So $\neg(\forall x \in C)[D(x) \Rightarrow M(x)]$ is equivalent to:

$(\exists x \in C)[D(x) \wedge \neg M(x)]$

mean: there is a car, which is domestic and is "not badly made"

"All Domestic ones are badly made"

from: $(\forall x \in C)[D(x) \Rightarrow M(x)]$

[negation] to: $[(\exists x \in C)[D(x) \wedge \neg(M(x))]]$