How to read mathematical formulas (Binding preference)

1. Quantifiers bind whatever comes next

$(\forall L)(\ldots\ldots)$

next

- so does negation

2. $\neg(\ldots\ldots)$

Red predicate that applies to Balls

$\forall B\ Red(B)$

~~For~~ all Balls that are ~~Read~~

" For all B Red(B)

∴ $\forall B$ only applies to Red(B)

if there was something after Red(B)

forall will not apply to it

∴ $\forall B\ Red(B)(\ldots\ldots)$

does not apply to it

if want to apply, need to add parenthesis!

for any doubts, add parenthesis
and mix it with [ ] ← Square Bracket.

3 Next priority is Conjuction

$$(\dots) \wedge (\dots)$$

4 then comes disjuction and implication (conditional)

$$(\dots) \vee (\dots)$$

$$\underset{\text{antecedent}}{(\dots)} \Rightarrow \underset{\text{consequent}}{(\dots)} \qquad (\dots) \Leftrightarrow (\dots)$$

Equivalence: its just a (and) Conjuction
of two implications!

∴ "biconditional"

Lets take an Example

$$(\forall L)\left[(\exists S_1)\,\mathrm{Valid}(L,S_1) \Rightarrow (\forall S_2)\,\mathrm{Valid}(L,S_2)\right]$$

$\forall \Rightarrow$ for all applies to everything that is adjacent to it

[ ] what is in parenthesis or Square Brackets] applies to

∴ any licence - - - - -

∴ that L is bound by the "for all" quantifier.

⇒ "for all licence L, ~~if there is a state~~, if L is valid in some (atleastone) state,

implication ⇒ then L is valid in every (all) state" ←

⇒ again, $\exists S_1$, exist only Binds Valid(L,S_1)

⇒ likewise, $\forall S_2$ only Binds Valid(L,S_2)

∴ ⇒ a licence, that is valid in one state, is valid in all state another example

$$(\forall L)[(\exists s_1)\,valid(L,s_1) \wedge (\forall s_2)\,valid(L,s_2)]$$

for any licence L, there is a state in which L is valid, and L is valid in every state (False)

⇒ what about invalid licence
⇒ not every licence is valid!

Difference Between First two

1. if Licence is valid [anticedent part]

2. there is a state in which L is valid [and part]

* important difference; they the same sentence but we read it differently.

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for "implication" ($\Rightarrow$)

$\Rightarrow$ for all licence L, if L is valid in some (or at least one) state, then L is valid in every (all) state

For conjunction ($\wedge$)

$\Rightarrow$ for all licence L, there is state in L licence is valid, and L is valid in every state [False]

3 another example: [no parenthesis] orphaned

$$(\forall L)(\exists S_1)\ Valid(L, S_1) \Rightarrow (\forall S_2)\ Valid(L, S_2)$$

(If) all licences are valid in (at least one) state, then licence L are valid in all states

But L is orphaned, or unbound variable as it does not have a quantifier another example

$$(\forall L)(\forall S_1)(\forall S_2)[\text{Valid}(L, S_1) \wedge \text{Valid}(L, S_2)]$$ (false)

"all licences are valid in all State

(also false, as we have invalid licence)

, but redundancy $\Rightarrow S_2$ we do not need it

$\Rightarrow$ redundant clauses

AGAIN

$$(\forall x)[P(x) \Rightarrow Q(x)]$$

(occurs alot eg in maths)

"for all (every) x, if (+ satisfies) $P(x)$, then $Q(x)$"

$\therefore$ x satisfies P, then it will (definitely) satisfy (Q)

truth of Q follows from truth of P

~~$(\forall x)$~~

$$(\forall x)[P(x) \wedge Q(x)]$$ $\Rightarrow$ same as: $\forall x P(x) \wedge \forall x Q(x)$

"for all (every) x, $P(x)$ and $Q(x)$"

don't see this very often.

$(\exists x)[P(x) \wedge Q(x)]$ → very common in maths

"There is an x, for which $P(x)$ and $Q(x)$"

⇒ you can find single x, that satisfies P and Q.

or has property P or property Q.

$(\exists x)[P(x) \Rightarrow Q(x)]$ ⇒ (not very common)

(exist with implication ⊗ does not make sense)

"There is an x, if $P(x)$, then $Q(x)$" [weak]

! confused !

(x) satisfies

also why is it weak?

If you can find an x that does not satisfy P, (False)

then $P(x) P(x)$ becomes true!

Conditional will be true.

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