another Example:

"All prime numbers are odd"

$\Rightarrow$ we all know this is false e.g. 2 is not odd

Lets tease apart the logic, or in situations where we do not know the answer.

Let $P(x)$: "x is prime"
(mean that x is prime)

$Q(x)$: x is odd
(mean that x is odd)

So the sentence can be re written symbolically

$$\forall x\,[P(x) \Rightarrow Q(x)]$$

"For all of x, if x is prime, then x is odd"

If I negate this:

$$\neg\forall x\,[P(x) \Rightarrow Qx] \Leftrightarrow \exists x\,[P(x) \not\Rightarrow Qx]$$

or

$$\Leftrightarrow \exists x\,[P(x) \wedge \neg Qx]$$

In words:

i.e There is a prime that is not odd ✱

So in order to prove this is false: (original statement)

"All prime numbers are odd" or prove ✱ is ~~false~~.

that is to prove the negation is true

$$(\neg\forall x[P(x)\Rightarrow Ox]$$

The logic says:

"Find a prime that is not odd" ← this is the negation we need prove this true!

~~But we can't, 2 is prime that is not odd~~

~~we cannot~~!

easy to prove, we just pulled 2 out of the hat.

therefore the negation is true

therefore the original statement is false lets modefy it:

"all prime numbers bigger than 2 are odd"

$$(\forall x > 2)[P(x) \Rightarrow O(x)]$$

"For all $x > 2$, if x is a prime, then x is odd"

What is the negation of this statement?

$$(\exists x > 2)(P(x) \wedge \neg O(x))$$

$\therefore$ "there exist number > 2, which is prime and not odd"

Let x denote a person

$P(x)$: "x plays for sport team T"
(denote)

$H(x)$: "x is healthy"

$\exists x [P(x) \wedge \neg (Hx)]$

meaning in everyday sentence

"There is an player playing for sport team T that is unhealthy"

(there is an unhealthy player on team T)

Lets negate it:

- when we negate a $\exists$, it turns into $\forall$

$\therefore \forall x \neg [Px \wedge \neg H(x)]$ and negation moves inside

not the way to do it

$\therefore \forall x [\neg P(x) \vee H(x)]$

$\neg$ moves inside

$\wedge$, become $\vee$

$\neg$ falls away, Baadtoonga

But that looks familiar:

$p \Rightarrow q$ p conditional q

Means same as: ← may have the same truth table

$\neg p \vee q$

$\therefore \forall x[\neg P(x) \vee H(x)]$

Can be rewritten as:

$\forall x[P(x) \Rightarrow H(x)]$

I saw this!

$\therefore$ in English

"all players on team T are healthy"

another:

$$\forall x \, [x > 0 \Rightarrow \exists y \, (xy = 1)]$$

"for all x, if $x > 0$, then there is a y, such that $xy = 1$"

$\therefore$ the $\forall$ (quantifier) only tells you something, "If you know what the variable denote"

- associated with any quantifier we a domain of quantification (which tells us what the x denote?

if the domain of qualification is obvious (?)
- if we have set it ~~at~~ in advance or context makes it clear what it is,
- then we can make statements (make conclusion)

- Otherwise we need to state the (explicit) domain of qualification

(rational numbers)

$$\text{"}(\forall x \in R)\, [x > 0 \Rightarrow \exists y \, (xy = 1)]$$

$\therefore$ I now have true statement that is unambiguous.

But yo not explicit, (we need to deal with that) ⑦

$$(\forall x \in \mathbb{R})[x>0 \Rightarrow (\exists y \in \mathbb{Q})(xy=1)]$$

But Sometimes Mathematicians OMIT the quantifier:

(Avoid doing this)

$x \geq 0 \Rightarrow \sqrt{x} \geq 0$ ⟸ implicit quantification!

what that mean is following:

$$(\forall x \in \mathbb{R})[x \geq 0 \Rightarrow \sqrt{x} \geq 0]$$

[to be avoided]

Now we will combine quantifiers with conjunction and disjunction

Let $\mathbb{N}$ "natural numbers" be the domain of quantification

Let $E(x)$: x is even
$O(x)$: x is odd

$\forall x\,[E(x) \vee O(x)]$ [True] [For every natural number it is either even or odd]

~~$\forall x\,[E(x) \wedge O(x)]$~~

$\forall x E(x) \vee \forall x (O(x))$ — Every natural number is even every natural number is odd. [False]

$\cancel{\exists E(x)}$

$\therefore$ $\exists_x [E(x) \wedge O(x)]$ → there is x, which is both even AND ODD (8) [False]

$\exists_x E(x) \wedge \exists_{(x)} O(x)$ → there is an x that even and there is an x that odd [True]

→ How things go badly wrong for both $\exists$ and $\forall$ when mixing it with quantifiers (T becomes F)

Quiz

"we just said"

$\forall x [A(x) \vee B(x)]$ (True) (Disjunction) is not equivalent

to $\forall x A(x) \vee \forall x B(x)$ (False)

$\therefore$ But is $\forall x [A(x) \wedge B(x)]$ (Conjunction) equivalent to

$\forall x A(x) \wedge \forall x B(x)$?

Yes! they are equivalent

Lets look at an example to demonstrate:

"All athletes are big and strong"

Let $A(x)$: athletes are big

$B(x)$: athletes are strong ← (Not so disjunction)

with conjunction, then Both True

⟶

We just saw $\exists x[A(x) \wedge B(x)]$ is not equivalent to $\exists x A(x) \wedge \exists x B(x)$

But is:

$\exists x[A(x) \vee B(x)]$ equivalent to $\exists x A(x) \vee \exists x B(x)$ Yes (True)

There is players who is a good attacker or a good defender

is the same 1) there is good attacker ($\vee$)

2) There is good defender

$\Rightarrow$ $\Leftarrow$] They equivalent

$\forall$ is "like" $\wedge$ (conjunction) — (for) all true

$\exists$ is "like" $\vee$ (disjunction) — at least one to true

When we have $\exists$ with $\wedge$ [name same as above], then things fall apart.

$\longleftrightarrow$ domain of quantification

Real numbers : eg x, y, z

rational numbers x $\quad \exists x \in \mathbb{Q}$ or $\forall y \in \mathbb{Q}$

Suppose the domain of quantification is the set of animals

"Every leopard has spots"

$(\forall x \in L)\, S(x)$

"For all x in set Leopards, x has spots"

But later I may have lions, tigers, so we will have a lot of domain(s) of quantifiers floating around

$\Rightarrow$ remember it should not be about leopard, tigers etc, it should be about <u>ANIMALS</u>

domains of Quantification shall be about animals

instead of $(\forall x \in L)(Sx)$ x

it shall be:

$(\forall x \in A)$

$\forall x\, [L(x) \Rightarrow S(x)]$

"For all animals, if that animal is leopard, then it has spots"

which will allow me to say

$\exists x\, [H(x) \wedge S(x)]$

"There is animal, which is horse, and has spots"

$\forall x\, [T(x) \Rightarrow \neg S(x)]$

"For all animals, if x is tiger, then x does not have spots"

– we is $T(x)$, $H(x)$ $\Rightarrow$ this refers to properties of particular kinds of animals