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Vector Space

Why do we need it?

it has properties:

- Commutative
- Associative
- Identity
- Every Element has additive inverse
- distributive

Addition and Scalar multiplication:

Addition Elements $u+v \in V$

multiplication

~~Element~~ Scalar $\lambda v \in V$

$\lambda \in F,\ v \in V$

Formal Definition:

Commutative

$u+v=v+u$

Associative

$(u+v)+w=u+(v+w)$

Additive Identity:
$v+0=v$

Additive inverse:
$v+w=0$

multiplicative inverse

~~$\lambda v+0=$~~

$1v+0=v$

Distributive:
$a(u+v)=au+av$

$\therefore$ all properties of complex numbers applies to element of vector space. (no need to repeat) ②

$\therefore$ when we think geometrically element in vector space are called vectors or points

$\therefore$ Scalar multiplication in vector space depends on F

remember correctly:

V is vector space "over" in F

eg $\mathbb{R}^n$ is vector space over $\mathbb{R}$

$\mathbb{C}^n$ is vector space over $\mathbb{C}$

Definition:

Vector space over $\mathbb{R}$ is called Real vector space

Vector space over $\mathbb{C}$ is called a Complex vector space.

Example:

F^∞ is list or sequence of all Element of F

$$F^\infty = \{(x_1, x_2 \ldots) : x_j \in F \text{ for } j = 1, 2 \ldots\}$$

Addition and Scalar multiplication of F^∞

: (adding the corresponding coordinate)

$$(x_1, x_2 \ldots) + (y_1, y_2 \ldots) = (x_1+y_1, x_2+y_2 \ldots)$$

& Scalar follow distributative property:

$$\lambda(x_1, x_2 \ldots) = \lambda x_1, \lambda x_2 \ldots$$

Notation FS.

Elementary properties of vector spaces:

Vector space has a unique additive identity:

Suppose 0 and $0'$ are both additive identities in vector space V

Then $0' = 0' + \underline{0} = 0 + \underline{0'} = 0$ $\quad \underline{\therefore 0' = 0}$

$\therefore$ has only $\underline{1}$ additive identity

Also, each element in V in vector space has an additive inverse only $\underline{1}$ additive inverse.

Proof

Suppose V is vector space. Let $v \in V$, and suppose w and w' are additive inverses of v

$$w = w + 0 = \underbrace{w + (v + w')}_{} = \underbrace{(w + v)}_{\text{Associative Rule}} + w' = 0 + w' = w'$$

$\therefore w = w'$

The following notation now makes sense:

$v, w \in V$ $\qquad v + (-v) = 0$

1 $-v$ denotes additive inverse of v

2 $w - v$ is defined as $w + (-v)$ $\Rightarrow$ (?)

Number 0 times a vector: $\qquad \boxed{0} \times i = i$

$$0v = 0$$
for every $v \in V$

$a \times i = a.$

A number times a vector 0

$$a0 = 0$$
for every $a \in \cancel{V} F$

The number -1 times a vector

$(-1)v = -v$ for every $v \in V$

Subspace:

A subset U of V is called a subspace of V if U is also a vector space (using same addition and multiplication as on V)

Example:

$\{(x_1, x_2, 0) : x_1, x_2, 0 \in F\}$ is a subspace of F^3

Conditions for subspace:

subset U of V is subspace of V, if and only if U satisfies the following 3 conditions:

additive identity: must be an element of U

$0 \in U$

closed under addition:

$u, w \in U$, implies $u + w \in U$

closed under scalar multiplication:

$a \in F$ and $u \in U$ implies $au \in U$

take any scalar, the product also in subspace

Other Conditions we also need to check its Commutativity and associativity

$\Rightarrow$ But if true V, then so also U.

$\therefore$ only the 3 above conditions need to be satisfied.

Example:

$b \in F$, then

$$\{(x_1, x_2, x_3, x_4) : x_3 = 5x_4 + b\}$$

, then U is subspace of F^4, if and only if $b = 0$

Property ①

$\therefore$ we must have the additive identity in set U

Direct Sum:

Sum $U_1 + \dots + U_m$, where each element can only be written in only one way as sum $u_1 + \dots u_m$, where u_j is in U_j

$\oplus$ direct sum sign

Example:

$U = \{ \overbrace{(x, y, 0)}^{\text{vectors}} \in F^3, x, y \in F)$ and

$W = \{ \overbrace{(0, 0, z)}^{\text{vectors}} \in F^3 : z \in F \}$

Both U and W are subspaces of F^3

U and W are a direct sum,

Can only be written in one way.

How can we check if sum is a direct sum.

$\Rightarrow$ whats the condition:

Suppose $U_1, \ldots U_m$ are subspaces of V,

then $U_1 + U_2 \ldots + U_m$, is a direct sum

if and only if the only way to write 0

as a sum $u_1 + \ldots u_m$, by ~~teac~~ taking

each u_j ~~eeo~~ equal to 0.

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Remember U is Subset of V.

Sums of Subspaces:

$U_1, \ldots U_m$, Subset of V

$U_1 + U_2 \ldots + U_m = \{u_1 + \ldots + u_m : u_1 \in U_1 \ldots u_m \in U_m\}$