Rev 1 Geometry & Linear equations:

First Lecture:

Solve $\begin{cases} 2x+y=3 \\ x-2y=-1 \end{cases}$, and find out

its "raw picture" + Matrix form (as well)

" Column picture"

2 Equations with 2 unknown.

x & y must satisfy Both Equations

Simplify: $x-2y=-1$

(*) $\Rightarrow x = 2y-1$ [left to right]

Now Plot this into First Equations

$2x+y=3$

(*) $2(2y-1)+y=3$

$\Rightarrow 4y-2+y=3$

$5y-2=3 \Rightarrow 5y=5$

$\therefore$ go back to $x=2y-1$

$$x=(2)(1)-1$$
$$x=2-1$$
$$x=1$$

$\therefore$ $x=1,\ y=1$ solve Both Equations:

$\therefore$ Lets test 1): $2x+y=3$

$$2(1)+1=3$$
$$2+1=3$$
$$3=3 \quad \boxed{\checkmark}$$

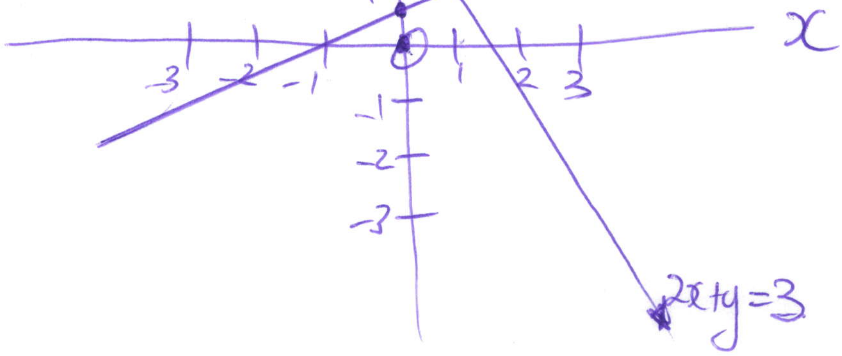
2) $x-2y=-1$

$$1-2(1)=-1$$
$$1-2=-1$$
$$-1=-1 \quad \boxed{\checkmark}$$

$\therefore$ Now lets plend at its raw picture and Column Picture.

Raw Picture

- look at each linear equations as raw
- which represent a straight line



(Eq 1) Coordinate
(0,3)
(1,1) } Enough to connect two points

(Eq 2) Coordinates
(0,1/2)
(1,1)

Now for second row:

$$x - 2y = -1$$

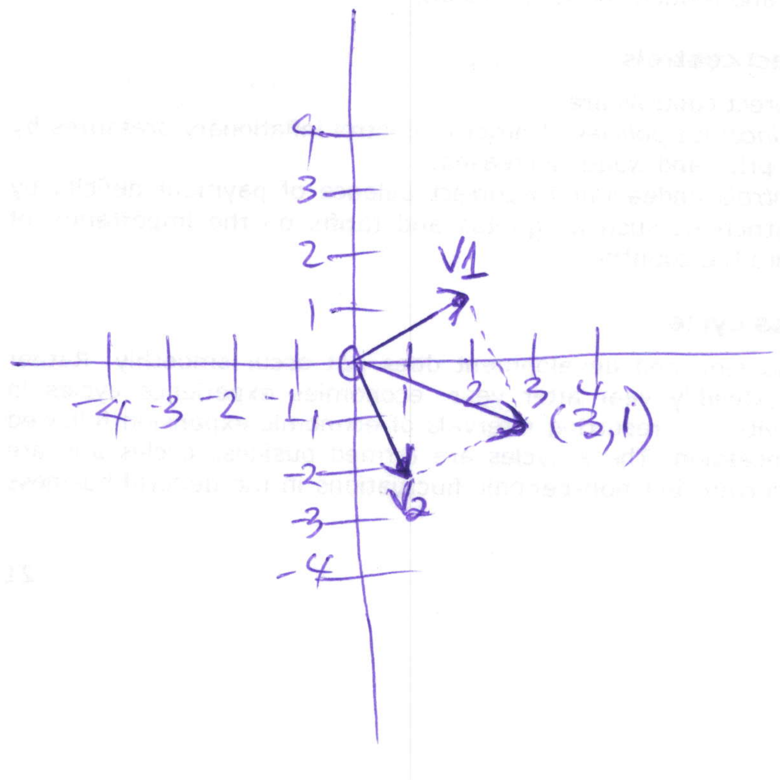
Convert the second Equation Straight line

∴ What does it mean to solve these two Equations?

⇒ By getting the <u>intersection</u> (point) where the two lines cross

∴ (1,1)

Now Lets move on to Column Picture: again we will need an $(x, y$ coordinate$)$



Where can I find my column?

we get the coefficients of (x, y)

$$\begin{array}{cc} V_1 & V_2 \\ \begin{bmatrix} 2 \\ 1 \end{bmatrix} x + & \begin{bmatrix} 1 \\ -2 \end{bmatrix} y = \begin{bmatrix} 3 \\ -1 \end{bmatrix} \end{array}$$

(See p3)

Now, let draw this picture on x, y coordinate system

But, $\begin{bmatrix} 2 \\ 1 \end{bmatrix}$ $2 \rightarrow$ refers to x, $1 \rightarrow$ refers to y

$\begin{bmatrix} 1 \\ -2 \end{bmatrix}$ $1 \rightarrow$ refers to x, $-2 \rightarrow$ refers to y

(has to be solved first)

$\therefore$ $V_1 + V_2$, we know $x = y = 1$ [this came from row picture]

$\therefore$ ~~as~~ sum one copy of V_1 + one copy of V_2

$\Rightarrow$ How do we indicate the sum of these two vectors

- You complete the "parallelogram" of the two vectors

$\therefore$ then vector by diagonal, is sum of $V_1 + V_2$.

⑤

∴ New vector should be $\begin{bmatrix} 3 \\ -1 \end{bmatrix} \begin{matrix} \to x \\ \to y \end{matrix}$

∴ $2 + 1 = 3$

$1 + {-2} = -1$

∴ $\underset{(2,1)}{V_1} + \underset{(1,-2)}{V_2} = \underset{(3,1)}{\begin{bmatrix} 3 \\ 1 \end{bmatrix}}$

what is the matrix form? →

I want to put the V_1 and V_2 as:

$[V_1 \; V_2] \Rightarrow$ Called A

∴ $\underset{A}{\begin{bmatrix} 2 & 1 \\ 1 & -2 \end{bmatrix}} \underset{x}{\begin{bmatrix} x \\ y \end{bmatrix}} = \underset{b}{\begin{bmatrix} 3 \\ -1 \end{bmatrix}}$ ← This is the Matrix Form of linear system.

∴ $Ax = b$

∴ if $A \neq 0$

$x = \frac{b}{a}$ — inverse

or

$x = a^{-1}b$ [can also be written as]

we want to apply a similar idea but to matrix.

⇒ we want to find matrix:

$A^{-1} \cdot A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ ← i.e. get an identity matrix [Come back]

∴ we want to find a matrix $A^{1}.A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ ← identity matrix

this idea will become more natural further into course

if such an inverse A^{1} matrix exist then what will be $\begin{bmatrix} x \\ y \end{bmatrix}$:

, then $\begin{bmatrix} x \\ y \end{bmatrix} = A^{-1} \begin{bmatrix} 3 \\ -1 \end{bmatrix}$ → this will give me the answer.

→ return to this latter in the course