More on the application of e'values

∴ Markov matrices

then look at application of projections.

∴ Fourier series

What's a Markov matrix

$$A = \begin{bmatrix} .1 & .01 & .3 \\ .2 & .99 & .3 \\ .7 & 0 & .4 \end{bmatrix}$$

What makes it as such?

Two properties:

(1) All entries ≥ 0 (if square it, still ≥ 0)

(2) All columns add to 1 (")

②

∴ Powers of my matrix, are all Markov matrices

and, always interested in E'v and E'vc

and Question of Steady State will come up.

Back:

What was the E'value in the differential Equation state, that led to a Steady state, it was $\lambda = 0$

Here:

What in the powers case (here)

$\Rightarrow$ Steady state; $\lambda = 1$.

∴ this matrix has an E'value of 1

(related to step ② above)

columns add up to 1.

Keypoint:

1. $\lambda = 1$ is an E'value.
2. all other λ values smaller than 1 : $|\lambda_i| < 1$

Form:

$$u_k = A^k u_0 = c_1 \lambda_1^k x_1 + c_2 \lambda_2^k x_2 + \dots$$

3. E'vectors $x_1 \geq 0$

$$A - 1I = \begin{bmatrix} -.9 & .01 & .3 \\ .2 & -0.1 & .3 \\ .7 & & -.6 \end{bmatrix}$$

Proof it is singular. and why is it singular.

all columns of $A - I$ now add to zero $\longrightarrow$ $A - I$ is singular.

Columns are dependent.

$\Rightarrow$ they all to zero

$\Rightarrow$ Can also show the rows are dependent.

Singular because the rows are dependent, also $(1,1,1)$ is in nullspace $N(A)^T$, the E'vector is in nullspace of A.

what combination of columns give zero?

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eivalues of A } they ~~the~~ are the same
" " A^T }

$\Rightarrow$ get E'values of A: $\det(A-\lambda I)=0$

get E'values A^T: $\det(A^T-\lambda I)=0$

~~Markov~~ where does Markov matrices come from?

$u_{k+1} = A u_k$ A is Markov

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$$\begin{bmatrix} u_{cal} \\ u_{mass} \end{bmatrix} = \begin{bmatrix} 0.9 & .20 \\ .1 & .80 \end{bmatrix} \begin{bmatrix} u_{cal} \\ u_{mass} \end{bmatrix} = \begin{bmatrix} 0 \\ 1000 \end{bmatrix}$$

$$\begin{bmatrix} .9 & .2 \\ -1 & .8 \end{bmatrix} \quad \lambda = 1 \quad \lambda = .7$$

$$\text{Evectors} \begin{bmatrix} -.1 & .2 \\ 0.1 & .2 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

u after ~~k~~ steps:

$$u_k = C_1 1^k \begin{bmatrix} 2 \\ 1 \end{bmatrix} + C_2 (.7)^k \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

$$u(0) = \begin{bmatrix} 0 \\ 1000 \end{bmatrix} = C_1 \begin{bmatrix} 2 \\ 1 \end{bmatrix} + C_2 \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

$$= \frac{1000}{3} \begin{bmatrix} 2 \\ 1 \end{bmatrix} + \frac{2000}{3} \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

all for Markov Now

~~altof~~ New projections:

projections with orthonormal basis

$$q_1, \dots q_n$$

any:

(A) $$V = x_1 q_1 + x_2 q_2 + \quad + x_n q_n$$

∴ What's x_1 ___ what's formula for it?

∴ looking for expansion.

— expanding vector in basis

$$q_1^T V = x_1 \cancel{q_1^T} q_1 + 0 + \quad + 0$$

(A) in matrix language:

$$\begin{bmatrix} q_1 \dots q_n \end{bmatrix} \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix} = V$$

$$Q \quad X = V$$

$$\therefore X = Q^{-1} V = Q^T V$$

$$x_1 = q_1^T V$$

Fourier Series:

$f(x) = f(x+2\pi)$ periodic fn

← love function

$$f(x) = a_0 + a_1 \cos x + b_1 \sin x + a_2 \cos 2x + b_2 \sin 2x + \ldots$$

this problem is infinite.

he realized he could work in function space

- he could have $\perp$ functions

vectors are now functions.

- infinite dimensional space.

What does it now mean for $\perp$

for vectors:

$$v^T w = v_1 w_1 + \ldots + v_n w_n$$

Now functions: (what's inner product)

$$f^T g = \int_0^{2\pi} f(x) g(x)\, dx$$

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The first Fourier Coefficient?

How do I get a_1?

$$\int_0^{2\pi} f(x)\,(\cos x)^2\,dx$$