Rec:

Tackle Least Squares approximation.

Here problem:

Find the quadatic equation through the origin that is a best fit for the points $(1,1)$, $(2,5)$, $(-1,-2)$

Lets see first how the quadratic equation will look like.

$$ct + dt^2 = y$$

Note: if it was just any quadratic equation then we have constant term, but through origin, just mean constant term is 0.

Next step, lets set up a matrix equation.

Square of those point

second coordinate

$$A = \begin{pmatrix} 1 & 1 \\ 2 & 4 \\ -1 & 1 \end{pmatrix}, \quad \hat{x} = \begin{bmatrix} c \\ d \end{bmatrix}, \quad b = \begin{bmatrix} 1 \\ 5 \\ -2 \end{bmatrix}$$

first coordinate of 3 points, and put in first column of this matrix

why is the matrix setup like this?

$\Rightarrow$ multiply $A\hat{x}$, then

First coordinate, is 1 times c

and 1 time d

is same as plugging in the left hand side of equation

$$ct + dt^2$$

Similarly, if take 2nd coordinate,

- 2 times c

- 4 times d

b, comes y coordinate of 3 point $\begin{pmatrix} 1 \\ 5 \end{pmatrix}$

B

∴ Remember we can't solve $A\vec{x} = b$

∴ we need to find the best approximation to it. A

A ∴ $A\hat{x}$ = projection of b onto the column space of A.

∴ really same as solving: $A^TA\hat{x} = A^Tb$

∴ Computation:

$$A^TA = \begin{pmatrix} 1 & 2 & 1 \\ 1 & 4 & -1 \end{pmatrix} \begin{pmatrix} 1 & 1 \\ 2 & 4 \\ -1 & -1 \end{pmatrix} = \begin{pmatrix} 6 & 8 \\ 8 & 10 \end{pmatrix}$$

$$A^Tb = \begin{pmatrix} 1 & 2 & -1 \\ 1 & 4 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 5 \\ -2 \end{pmatrix} = \begin{pmatrix} 13 \\ 19 \end{pmatrix}$$

$$\begin{pmatrix} 6 & 8 \\ 8 & 10 \end{pmatrix} \hat{x} = \begin{pmatrix} 13 \\ 19 \end{pmatrix} \rightsquigarrow \text{using elimination.}$$

$$\begin{pmatrix} 6 & 8 \\ 0 & -2 \end{pmatrix} \hat{x} = \begin{pmatrix} 13 \\ 15 \end{pmatrix}$$

$d = -5/2, \quad C = 11/2$

∴ $y = \frac{11}{2}t - \frac{5}{2}t^2$ } Best quadratic equation through origin.

④

Review Key steps:

⇒ 1) First get general form of the equation:

$$ct + dt^2 = y$$

⇒ 2) Next write into matrices.

$$A = \ldots, \quad \hat{x} = \ldots, \quad b = \ldots$$

⇒ 3) Setup Projection Equation: $A^T A \hat{x} = A^T b$

⇒ 4) then just do Computation.