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Four Fundamental Subspaces (for Matrix A)

— We know of Column Space } First 2
& Null Space } Sub Spaces

— Another 2 to follow

∴ The 4 Fundamental Sub Spaces:

Column Space $C(A)$
Null Space $N(A)$
Row Space
↳ all Combinations of rows.
↳ rows span the row space
(~~zrash~~)

, But all the vectors we dealt with has been Column Vectors.

, how do I get column vectors out of rows, we transpose to Column vectors

Rowspace = all Combinations of the Column of A^T

$$= C(A^T)$$

Nullspace of A^T $= N(A^T)$ (Called left nullspace of A)

<u>Given</u> A is $\overset{\text{colum}}{m} \times \overset{\text{rows}}{n}$

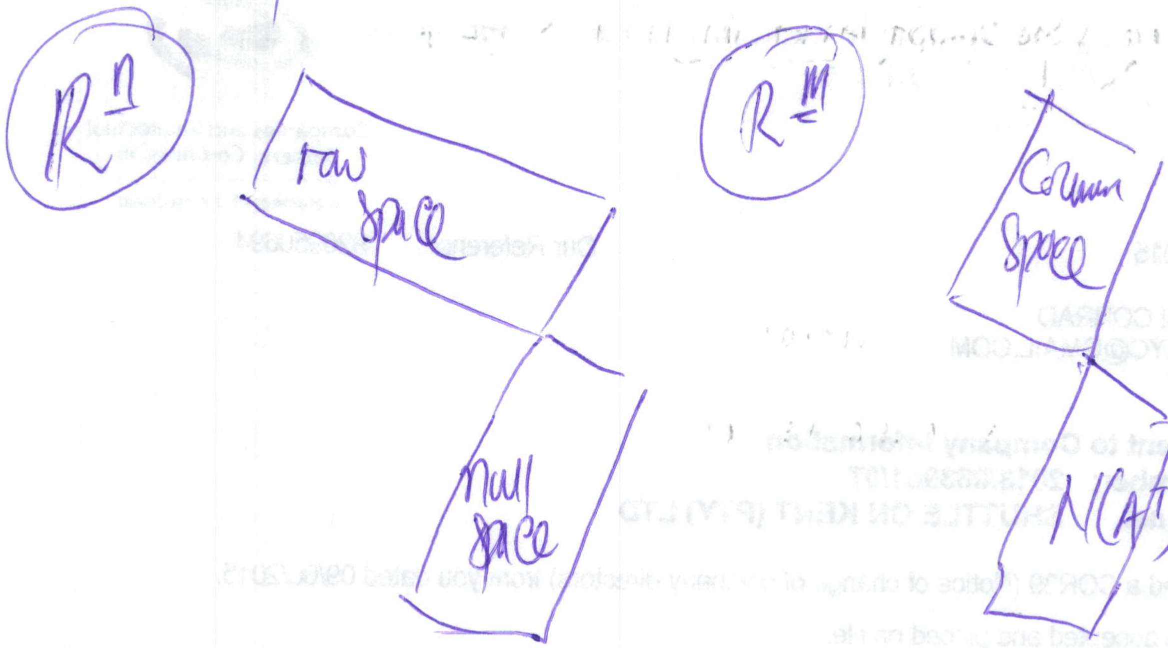
$N(A)$ in $\mathbb{R}^n$

$C(A)$ in $\mathbb{R}^m$

$C(A^T)$ in R^n

$N(A^T)$ in R^m

4 Subspaces



- What are the basis for those spaces (for each one spaces)?
- what systematic way (to find it):
- and what's their dimension?

∴ Dimension of Column space:

$$\dim C(A) = \text{rank } r$$

∴ Basis for Column space: pivot columns ⇒ C(A) - pivot colu what is the dim of row space? 4

⇒ also r

⇒ Columnspace and rowspace has same dim.

$r(A^T) = r$

What is dim of nullspace?

$\dim N(A) = $ ~~special solutions~~

~~$= n - f_i$ [no of free variables]~~

$= n - r$

Basis = special solutions [no free variables)

there is one for every free variable

$= n - r$

What is dim null space of A^T [left nullspace]

$\dim N(A^T) = m - r$

Basis?

Eg: Basis for row space:

$$A = \begin{bmatrix} 1 & 2 & 3 & 1 \\ 1 & 1 & 2 & 1 \\ 1 & 2 & 3 & 1 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 2 & 3 & 1 \\ 0 & -1 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\rightarrow \begin{bmatrix} 1 & 2 & 3 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \rightarrow \begin{array}{c} \begin{array}{cc} I & F \end{array} \\ \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{array} = R$$

can't go it!

$C(R) \neq C(A)$ ∴ different column space.

But they have same row space.

What's Basis for Original A?

What's Basis for R?

∴ Basis for A:

- Basis for rowspace for A or R is first r rows of R [not A]

∴ Vectors in rowspace $\leftrightarrow$ (1 0 1 1) and (0 1 1 0)

∴ the rowspace is sitting in $\mathbb{R}$.

Now want Basis for nullspace of A^T. [left nullspace]

∴ $A^T y = 0$

$$\begin{array}{ccc} \text{matrix} & \text{column} & \\ \begin{bmatrix} & \end{bmatrix} & \begin{bmatrix} \\ \\ \end{bmatrix} & = \begin{bmatrix} 0 \end{bmatrix} \end{array}$$

– Now want y to sit on left and want A instead A^T (transpose the equation)

$$y^T A^{TT} = 0^T$$

$$A =$$

$\rightarrow$ 0 lie down

row vector

$$\begin{bmatrix} y^T \end{bmatrix} \begin{bmatrix} A \end{bmatrix} = \begin{bmatrix} 0 \end{bmatrix}$$

left nullspace, had to make it into row vector

∴ How do we get a Basis?

Eg.

$$A = \begin{bmatrix} 1 & 2 & 3 & 1 \\ 1 & 1 & 2 & 1 \\ 1 & 2 & 3 & 1 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} R$$

What were steps taken above

∴ what whole matrix that took me from A to R.

⇒ how would we find it.

Remember Guass-Jordan (GJ) → where you tag on the identity matrix.

But still tag on identity matrix — record of what we did

$$\text{rref}\begin{bmatrix} A_{m\times n} & I_{m\times n} \end{bmatrix} \Rightarrow \begin{bmatrix} R_{m\times n} & E_{n\times m} \end{bmatrix}$$

(→ was square with GJ, But now matrix rectangular)

$$EA = R$$

in Chapter 2, R was $\underline{I}$
then E was A^{-1}

$$E = \begin{bmatrix} -1 & 2 & 0 \\ 1 & -1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$$

New Vector Space (M)

all 3x3 matrices
(my matrices are the vectors)

A+B, CA

not interested in AB for now

- What about subspaces

subspaces of $\underline{M}$ ||
1) upper triangular matrices
2) symmetric matrices
3) Diagonal matrices
(dim = 3)