Lec1

1

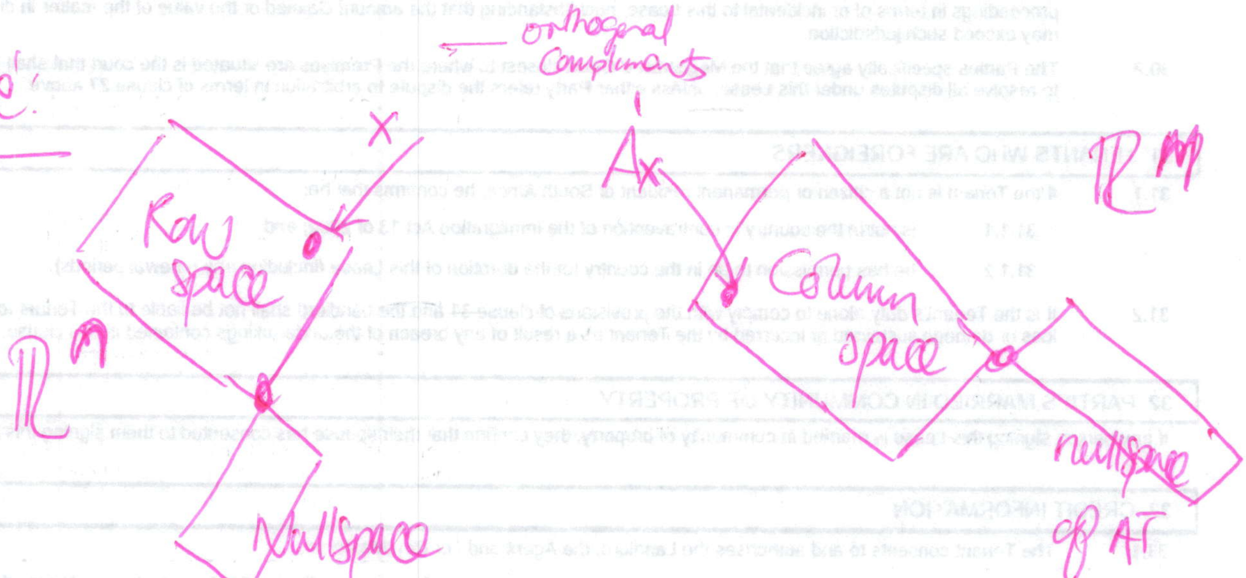
Cover 4 Subspaces

Left-inverse

Right-inverse

"Pseudoinverse"

Recall pic:



Let's speak about inverses, and identify the different possibilities.

⇒ when does a matrix have a perfect inverse, 2 sided

2 sided inverse (just call inverse)

$$AA^{-1} = I = A^{-1}A$$

- matrix that produces an I, whether we write it on both when we have an invertible matrix, how are the following related?

- rank (r)
- # columns (m)
- # rows (n)

They are all same ⟹

what are their relationship? (for the nice case)

[Square, full rank]

$r = m = n$

Then we started to deal with matrices that were not full rank, then we focused on full column rank.

to get inverse:

full column rank → column independent But not rows.

now $r = n$ (columns are independent) (But we probably have more rows)

nullspace = $\{\vec{0}\}$ ←

NO combination of columns give 0, except (they independent)

0 or 1 solutions to $Ax=b$

$(A^TA)^{-1}A^T\,A = I$ left inverse

$A^{-1}A = I_{n\times n}$

all good if you m

(But AA^T is Bad here)

Now Right Inverse

Full row rank

$r = m < n$ (rows are independent But not cols)

$\text{nullspace}(A^T) = \{0\}$ → independent rows

(will always have a solution)

Dim $N(A)$

– How many free variables: $n-m$

(∞ many solutions to $Ax=b$)

$A\,\underbrace{A^T(AA^T)^{-1}} = I$

$AA^{-1} = I$

right inverse

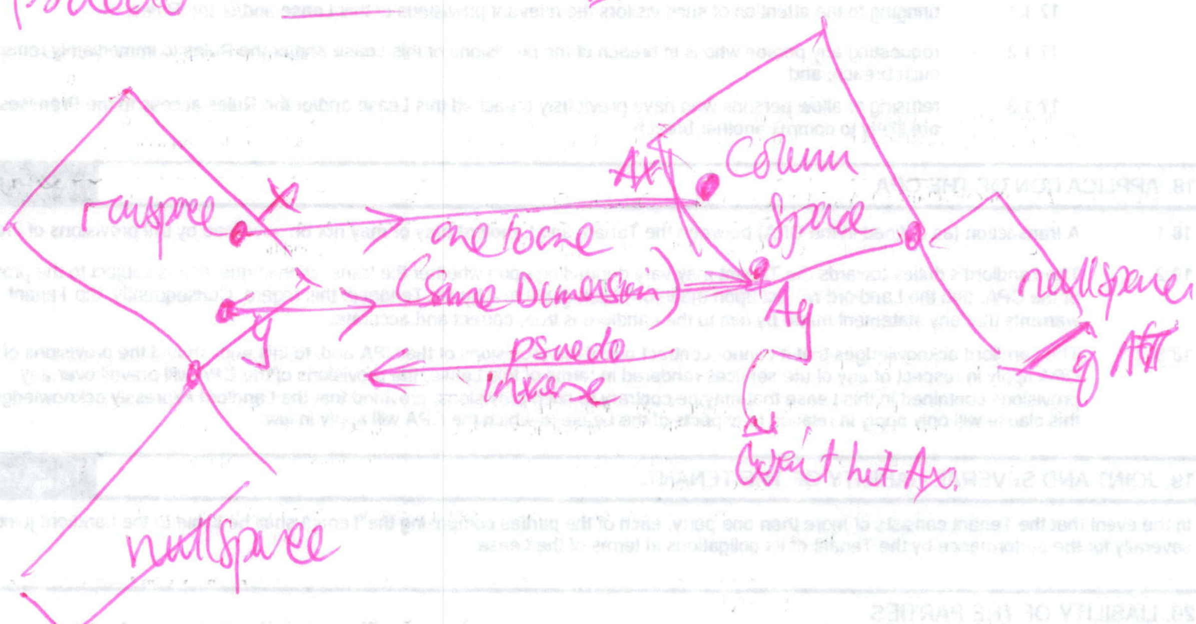
from lefthand side

$\therefore A(A^TA)^{-1}A^T$ = Projection ~~matrix~~ onto column space.

from righthand side

$\therefore A^T(AA^T)^{-1}A$ = Projection onto row space

Now Psuedo Inverse



$\therefore$ if null space that are screwing up inverses! of matrix, take vector to zero, there is no way an inverse can bring it back to life.

What's best inverse we can have?
eg vector x in rowspace, multiply by A, where does output land up ... Ax in columnspace

If x, y are in different rowspace, then

$Ax \neq Ay$ [Both in column space, but diff]

from rowspace to columnspace

A is perfect, it's an invertible matrix.

if we limit it to these space, and its inverse, is what we can call pseudo inverse

Pseudo Inverse, ~~inverse~~

from $x \xrightarrow{\text{row}} \text{column}$

$\text{row} \xleftarrow{\text{column}}$ (going back, inverse, pseudo)

→ other direction ← reverse

A^+ (denote inverse)

Suppose they $Ax = Ay$

$A(x-y) = 0$

sits in nullspace, But then sits also in rowspace

∴ its zero vector

6

How do I find this pseudo inverse A^+

① Start from SVD: $A = U\Sigma V^T$ $\quad \Sigma = \begin{bmatrix} \sigma_1 & 0 & \\ 0 & \sigma_2 & \\ & & \sigma_2 \end{bmatrix}_{m\times n}$

n = cols

rank r

m rows

$$\Sigma^+ = \begin{bmatrix} 1/\sigma_1 & 0 \\ 0 & 1/\sigma_r \\ & 0 \end{bmatrix}_{n\times m}$$

→ Pseudo inverse, the best I can come to inverse

$$\Sigma\Sigma^+ = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}_{m\times m}$$

project matrix onto column space

Q2 (different order)

$$\underset{n\times m}{\Sigma^+}\ \underset{m\times n}{\Sigma} = \begin{bmatrix} 1 & & \\ & 1 & \\ & & 0 \end{bmatrix}_{n\times n}$$

← Projection matrix onto row space

result

Pseudoinverse of A itself:

$$A = U\Sigma V^T$$

$$A^+ = V\Sigma^+ U^T$$

Recap: →