LoC

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Beginning of Second half of Course.

Previously, we paid special attention to rectangular matrices

Now we will be concentrating on Square matrices.

Two Big topics here:

- Determinant of Square matrix
- Eigen values

Reason we need determinants is for getting the Eigen values

Determinant is a number associated with every square matrix.

∴ written as det A or |A|

Note: this number packs in alot of information about matrix

The other fact, we mentioned earlier that was important:

- Matrix is invertible, when the determinant is not zero.
- matrix is singular, when the determinant IS zero

∴ determinant is test for invertibility. (But it has a lot more to it)

What are the three properties of the determinant?

(1) det I = 1

(2) Exchange rows: reverse the sign of det

∴ what happens when you exchange two rows of a matrix

what happens to the determinant?

⇒ reverse sign of det. (± sign)

∴ applies to every permutation matrix.

stands for permutation

$\det P = 1$ or -1

(depending on # of exchanges we even (1)

" " odd (−1))

∴ Property ① said, in 2×2 case:

① $\begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = 1$ this matrix, has det of 1.

② Property ②, says that this matrix:

$\begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix} = -1$ has det of −1

General 2×2:

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$$

Property 3

3 (a) If multiple one of rows (say first row) by a number t, and leave the other $n-1$ rows alone, what happens to the determinant?

$$\begin{vmatrix} ta & tb \\ c & d \end{vmatrix} = t \begin{vmatrix} a & b \\ c & d \end{vmatrix}$$

leave alone...

Factor t comes out

∴ t x (times) this determinant $| \; |$

(b) and if I am always keeping the last row the same, then this breaks up into sum of this determinant

$$\begin{vmatrix} a+a' & b+b' \\ c & d \end{vmatrix} = \begin{vmatrix} a & b \\ c & d \end{vmatrix} + \begin{vmatrix} a' & b' \\ c & d \end{vmatrix}$$

∴ the determinant is a linear function of the first row if all the other rows stay the same

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I am not saying: det(A+B)
= det A + det B.

⇒ not saying this!

∴ I am only linearity of each row

∴ From these 3 properties, we going to learn much more about determinant

∴ like to Learn:
if 2 rows are equal, the determinant is zero

Property ④ 2 equal rows ⇒ det = 0

Suppose: I have a matrix, and two rows are equal, how do I see that the determinant has to be zero.

∴ I do an exchange.

∴ Property ② is okay for this

∴ use property ②

Exchange rows ⟹ get same matrix

∴ determinant did not change

On other hand, property ② says that sign did change. → i.e. does change / does not change.

∴ $\underline{\underline{det = 0}}$

Property ⑤ The elimination step that we always doing: Subtract a multiple of $l \times$ row i from another row k

∴ So as to produce 0 (in the position underpart).

Pants: Determinant does not change.

Eg 2x2 case

$$\begin{vmatrix} a & b \\ c - la & d - lb \end{vmatrix} = \begin{vmatrix} a & b \\ c & d \end{vmatrix} + \begin{vmatrix} a & b \\ -la & -lb \end{vmatrix}$$

determinant

Property 3(b):

$\Rightarrow$ kept one row the same, and had a combination in the second, in the other row and just separated it out

How do we use 3(a): factor out a minus l

$$\begin{vmatrix} a & b \\ c-la & d-lb \end{vmatrix} = \begin{vmatrix} a & b \\ c & d \end{vmatrix} - l \begin{vmatrix} a & b \\ a & b \end{vmatrix}$$

then we use property (4)

this determinant $= 0$

Property (6) Row of zeros $\xRightarrow[\text{means}]{\text{lead to}}$ $\det A = 0$

Property (7) Suppose my matrix is triangular: [what is determinant?]

$$U = \begin{bmatrix} d_1 & x & x & x \\ 0 & d_2 & x & x \\ 0 & 0 & \ddots & x \\ 0 & 0 & 0 & d_n \end{bmatrix} = (d_1)(d_2),\ldots,(d_n)$$

Know this is all zero.

$\therefore$ only look at the diagonal.