Pb : Inverse Matrices

Find the condition on a and b that makes the matrix A invertible, and find A^{-1} when it exists

$$A = \begin{bmatrix} a & b & b \\ a & a & b \\ a & a & a \end{bmatrix}$$

If it is invertible what is A^{-1}

Easy ~~test~~ to spot, when Matrix is not invertible:

1) Column of 0's
2) row of 0's

OR

1) two columns
2) two rows

} that are same

Lets Check our matrix

$$A = \begin{bmatrix} a & b & b \\ a & a & b \\ a & a & a \end{bmatrix}$$

$\therefore$ A is not invertible if $a=0$ [row 3]

or $a=b$

How to find inverse of matrix:

write giant matrix $\rightarrow \left[\, A \;\middle|\; I \,\right] \dashrightarrow \left[\, I \;\middle|\; A^{-1} \,\right]$

then perform Elimination steps,

and stop once you find I

Lets do Computations:

A 3

$$\begin{array}{l} \text{raw1} \\ \text{raw2} \\ \end{array}\left[\begin{array}{ccc|ccc} a & b & b & 1 & 0 & 0 \\ a & a & b & 0 & 1 & 0 \\ a & a & a & 0 & 0 & 1 \end{array}\right] \rightarrow \begin{array}{l} \\ B- \\ C \end{array}\left[\begin{array}{ccc|ccc} a & b & b & 1 & 0 & 0 \\ 0 & a-b & 0 & -1 & 1 & 0 \\ 0 & a-b & a-b & -1 & 0 & 1 \end{array}\right]$$

want to make 0

A want to make $(2,1)=0$

so we subtract raw 1 from raw 2

$\therefore$ raw2 − raw1

→ First write out raw 1 as is

→ $\therefore a-a=0$

above B

$a-b=a-b$

$\therefore b-b=0$

$0-1=-1$

$1-0=1$

$0-0=0$

→ then do the same for 3rd raw

$\therefore$ raw 3 − raw 1

above C $\therefore \rightarrow$

$\therefore a-a=0$

$a-b=a-b$

$0-1=-1$

$0-0=0$

$1-0=1$

$$
\begin{array}{c}
\\ \text{row2} \\ \text{row3}
\end{array}
\left[\begin{array}{ccc|ccc}
a & b & b & 1 & 0 & 0 \\
0 & a-b & 0 & -1 & 1 & 0 \\
0 & \underset{\text{(make zero)}}{a-b} & a-b & -1 & 0 & 1
\end{array}\right]
=
\begin{array}{c}
\\ \\ \text{A}
\end{array}
\left[\begin{array}{ccc|ccc}
a & b & b & 1 & 0 & 0 \\
0 & a-b & 0 & -1 & 1 & 0 \\
0 & 0 & a-b & 0 & -1 & 1
\end{array}\right]
$$

Now need to Eliminate $(a-b)$ in row 3

$\therefore$ First row stays same
2nd row " "

A Subtract row3 from row2

~~row3 - row2 - row3~~
row3 - row2

$\therefore$

$0-0=0$

$(a-b)-(a-b)=0$

$(a-b)-0=a-b$

$-1-(1)=$

$-1+1=0$

$0-1=-1$

$\cancel{(a-b)-0=(a-b)}$

oops

$1-0=1$

$$\left[\begin{array}{ccc|ccc} a & b & b & 1 & 0 & 0 \\ 0 & a-b & 0 & -1 & 1 & 0 \\ 0 & 0 & a-b & 0 & -1 & 1 \end{array}\right] \Rightarrow \left[\begin{array}{ccc|ccc} 1 & b/a & b/a & 1/a & 0 & 0 \\ 0 & 1 & 0 & -\frac{1}{a-b} & \frac{1}{a-b} & 0 \\ 0 & 0 & 1 & 0 & -\frac{1}{a-b} & \frac{1}{a-b} \end{array}\right] \quad 5$$

Now we need to turn all diagonal entries into 1. indicated by A

$\Rightarrow$ row 1 to turn $a = 1$ we divide by a

$\therefore$ we need to ensure that $a \neq 0$

(row 2) to turn $a-b$ $= 1$ we divide by $a-b$

$\therefore$ we need to ensure $a-b \neq 0$

row 3 to turn $a-b$ $= 1$, we divide in $a-b$

Now How do we eliminate b/a

$\downarrow$ Bit Complex.

$$\Rightarrow \text{raw } 1 - \frac{b}{a}(\text{raw}2 + \text{row}3)$$

$$\begin{array}{c|c}
 & \begin{array}{cccccc} & A & B & C & & \end{array} \\
\begin{array}{c} \\ \rightarrow \text{raw}2 \\ \text{row}3 \end{array} & \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & \frac{1}{a-b} & 0 & \frac{-b}{a(a-b)} \\ 0 & \boxed{1} & \boxed{0} & -\frac{1}{a-b} & \frac{1}{a-b} & 0 \\ 0 & \boxed{0} & \boxed{1} & 0 & \frac{-1}{a-b} & \frac{1}{a-b} \end{array}\right]
\end{array}$$

$$\Rightarrow A = \left[\frac{\cancel{b}}{a} - \frac{b}{a}(1+0)\right]$$

$$= \frac{b}{a} - \frac{b}{a}(1)$$

$$= 0$$

$$\Rightarrow B = \left[\frac{b}{a} - \frac{b}{a}(0+1)\right]$$

$$= \left[\frac{b}{a} - \frac{b}{a}(1)\right]$$

$$= 0$$

$$\Rightarrow \frac{1}{a} - \frac{b}{a}\left(\frac{-1}{a-b} + \overset{+0}{\cancel{\left(\frac{1}{a-b}\right)}}\right)$$

$$\frac{1}{a} - \frac{b}{a}\left(\frac{-1}{a-b}\right)$$

Lets clean it up:

$$A^{-1} = \frac{1}{a-b}\begin{bmatrix} 1 & 0 & -b/a \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix}$$