Rec

①

Analyse some several properties of Positive definite matrices.

Why true:

a) Every positive definite matrix is invertible

b) The only positive definite projection matrix is $P = I$

c) D is diagonal with positive entries is positive definite

d) S symmetric with $\det S > 0$ might not be positive definite

a) Every positive definite matrix is invertible

simpler

A invertible $\Leftrightarrow$ $\det A$ $\lambda_1 \lambda_2 \cdots \lambda_n \neq 0$

$\lambda_1, \ldots, \lambda_n \to$ Evalues of A.

A is pos. definite [what does it say about Evalues A]

$\Leftrightarrow \lambda_1, \lambda_2 \ldots \lambda_n > 0$

$\therefore \det A = \lambda_1 \cdot \lambda_2 \cdot \lambda_3 \ldots \lambda_n > 0 \neq 0$

Hence A must be invertible

b) The only positive definite projection matrix is $P = I$

$\Rightarrow$ How do we tackle this $\Rightarrow$ we look at E'value.

① P is projection $\Rightarrow$ Evalues of $P = 0$ or 1.

② P is pos def $\Rightarrow$ Evalues > 0

$\therefore$ E'values of $P = 1$

$\therefore$ Only matrix ~~that~~ where $P = 1$, is I matrix
$P = I$

If P is diagonalizable
$\therefore P = U I U^{-1}$
$P = U U^{-1} = I$ } check.

c) D is diagonal with positive entries is positive definite.

$\therefore D = diag(d_1, d_2 \ldots d_n)$ $\therefore$ D is diagonal matrix.

to be positive def.?
$\Rightarrow$ for any vector $x \neq 0$

$$x^T D x > 0$$

$$\therefore x = (x_1, x_2 \ldots x_n)$$

$$x^T D x = d_1 x_1^2 + d_2 x_2^2 + \ldots d_n x_n^2$$
$$> 0$$

d) S symmetric with det $S > 0$ might not be positive definite.

$$S = \begin{pmatrix} \boxed{-3} & 1 \\ 1 & -2 \end{pmatrix}$$ $\Rightarrow$ pick negative numbers on the diagonal.

$$\therefore \det S = 6 - 1 = 5 > 0$$

$$x^T S x \rightarrow x = (1 \ 0)^T$$

$$x^T S x = -3.$$