Additional

LU Decomposition

Find the LU Factorization

of $A = \begin{bmatrix} 2 & 2 & 3 \\ 5 & 9 & 10 \\ 4 & 1 & 2 \end{bmatrix}$

goal write A as product

~~A =~~ with first factor

called L and second factor

will be called U

where L ; unit lower triangle

meaning 1's main diagonal, and

any real entries below diagonal,

But it must have zeros

above the diagonal

$$L = \begin{bmatrix} 1 & 0 & 0 \\ x & 1 & 0 \\ x & x & 1 \end{bmatrix}$$

[has 1's on the diagonal]

Called Lower, all non-zero sits <u>below</u> the diagonal

∴ where U, upper higher / triangular matrix

so any real entries <u>on</u> and above diagonal, but must have zeros Below the diagonal

$$U = \begin{bmatrix} x & x & x \\ 0 & x & x \\ 0 & 0 & x \end{bmatrix}$$

[also has pivots on diagonal]

∴ * = any real number

③

The way we going to find the factorization is by starting with matrix A and gradually turning it into matrix U

$A \rightsquigarrow U$

By doing only 1 type of row operation.

(PatrickMJT)

only allowed: Replace R_i by $R_i - kR_j$ or $-kR_j + R_i$

- not allowed to swap rows
- not allowed to multiply or divide

We get U First

$$\begin{bmatrix} 2 & 2 & 3 \\ 5 & 9 & 10 \\ 4 & 1 & 2 \end{bmatrix}$$

want 0's in here

How we going to do this?

$\therefore$ for row 2: $-\frac{5}{2}R_1 + R_2 \Rightarrow R_2$.

for row 3: $-2R_1 + R_3 \Rightarrow R_3$.

Remember, $k = \frac{5}{2}$ (R_2) } Later used

[Change signs] $\Rightarrow k = 2$ (R_3)

$$\begin{bmatrix} 2 & 2 & 3 \\ 5 & 9 & 10 \\ 4 & 1 & 2 \end{bmatrix} \rightarrow \begin{array}{c} \\ A \\ B \end{array} \begin{bmatrix} 2 & 2 & 3 \\ 0 & 4 & 5/2 \\ 0 & -3 & -4 \end{bmatrix}$$

$-\frac{5}{2}R_1 + R_2 \Rightarrow R_2$

A

$\boxed{R_{21}} = -\frac{5}{2}(2) + 5$
$= -5 + 5$
$= 0$

$\boxed{R_{22}} = -\frac{5}{2}(2) + 9$
$= -5 + 9$
$= 4$

$\boxed{R_{23}} = -\frac{5}{2}(3) + 10$
$= \frac{-15}{2} + \frac{10}{1} \begin{bmatrix} \times 2 \\ \times 2 \end{bmatrix}$
$= \frac{-15}{2} + \frac{20}{2}$
$= 5/2$

$-2R_1 + R_3 \Rightarrow R_3$

B

$\boxed{R_{31}} = -2(2) + 4$
$= -4 + 4$
$= 0$

$\boxed{R_{32}} = -2(2) + 1$
$= -4 + 1$
$= -3$

$\boxed{R_{33}} = -2(3) + 2$
$= -6 + 2$
$= -4$

5

$$\begin{bmatrix} 2 & 2 & 3 \\ 0 & 4 & 5/2 \\ 0 & -3 & -4 \end{bmatrix}$$

want 0

$\therefore \frac{3}{4} \cdot R_2 + R_3 \Rightarrow R_3$

Remember 1) $k = \frac{-3}{4}$ R_3 } Later used

$$\begin{bmatrix} 2 & 2 & 3 \\ 0 & 4 & 5/2 \\ 0 & -3 & -4 \end{bmatrix} \rightarrow \text{(A)} \begin{bmatrix} 2 & 2 & 3 \\ 0 & 4 & 5/2 \\ 0 & 0 & -17/8 \end{bmatrix}$$

(A) — $\frac{3}{4} R_2 + R_3 \Rightarrow R_3$

$\boxed{R_{31}} = 0$

$$\boxed{R_{32}} = \frac{3}{4}(4) + (-3) = 3 - 3 = 0$$

$$\boxed{R_{33}} = \frac{3}{4}\left(\frac{5}{2}\right) + (-4)$$

$$= \frac{15}{8} - 4 = \frac{4}{1}\,\begin{smallmatrix}\times 8\\ \times 8\end{smallmatrix}$$

$$= \frac{15}{8} - \frac{32}{8} = \frac{-17}{8}$$

How do we get L ?

But inverse

we use the K (multipliers)(Coefficients)

$$\begin{bmatrix} 1 & 0 & 0 \\ 5/2 & 1 & 0 \\ 2 & -3/4 & 1 \end{bmatrix}$$