Rec

Leened so far at Determinant and its property, and also derived the formula to compute it.

Lets put what we learned into practice.

Find the determinant of the 2 (5x5) matrices

$$A = \begin{bmatrix} x & y & 0 & 0 & 0 \\ 0 & x & y & 0 & 0 \\ 0 & 0 & x & y & 0 \\ 0 & 0 & 0 & x & y \\ y & 0 & 0 & 0 & x \end{bmatrix} \qquad B = \begin{bmatrix} x & y & y & y & y \\ y & x & y & y & y \\ y & y & x & y & y \\ y & y & y & x & y \\ y & y & y & y & x \end{bmatrix}$$

~~Matr~~ Matrix A is x along its diagonal and y next to x, except last raw, and y in last raw, and zero entries everywhere else.

B also has x along its diagonal, and y everywhere else.

method:

1) You can use Elimination to transform your matrix into upper triangular matrix

2) OR use By Σ (summation) formula

$$\sum \pm a_{1\alpha} \cdot a_{2\beta} \cdots a_{5\varphi}$$

3) Or by Co-factors

OR Combine method

Lets look at matrix A:

- there are alot of zero entries in Matrix A.

  so you dont need elimination to introduce more zero's

- we also notice that if we cover the last row $(y\ 0\ 0\ 0\ x)$ and first column $\begin{pmatrix} x \\ 0 \\ 0 \\ 0 \\ y \end{pmatrix}$, whats left over is simply a 4x4 lower triangular matrix.

Similarly, if Cover the First row and First Column, what's left over is a 4x4 upper triangular matrix.

This is telling us we should Calculate $|A|$ using the third method

Using Co-factors
(Expand along the first Column of A)

(1,1) of A times the Co-factor of that spot, which is the det of the left over 4x4 matrix — upper triangular...

$$\det A = x \cdot x^4 + y \cdot (-1)^{5+1} y^4$$

times co-factor of that spot, det. of lower triangular.

only nonzero entry in Column
@ very bottom

$$A = x \cdot x^4 + y \cdot (-1)^{5+1} \cdot y^4$$
$$= x^5 + y^5$$

— not too bad.

(N)

$$B = \begin{array}{c}\begin{bmatrix} x & y & y & y & y \\ y & x & y & y & y \\ y & y & x & y & y \\ y & y & y & x & y \\ y & y & y & y & x \end{bmatrix}\end{array}\begin{array}{l} (d) \\ (c) \\ (B) \\ (A) \\ \end{array} \quad \begin{array}{c}\begin{bmatrix} x & y & y & y & y \\ y-x & x-y & 0 & 0 & 0 \\ 0 & y-x & x-y & 0 & 0 \\ 0 & 0 & y-x & x-y & 0 \\ 0 & 0 & 0 & y-x & x-y \end{bmatrix}\end{array}\begin{array}{l} \\ \\ (C) \\ (B) \\ (A) \end{array}$$

- there are no zero's, so we need elimination,

How do we do elimination:

- Find pivot
- Eliminate rows (make zero's)

- Trick / short cut:

∴ Compare eg. row 4 and 5, they have a lot of entries in common, the only difference

→ last 2 columns (xy or yx)

(A) ∴ Subtract 4th row from 5th row

∴ simple operation, introduced 3 zero's at once!

(B) ∴ do the same for 3rd and 4th row.

(C) do the same for 2nd and 3rd row

(d) subtract 1st row from 2nd row.

(N) Lets observe the pattern of this new matrix

adding 3 column (D)

∴ Test Calc

(E)

x x x x x

$$\begin{bmatrix} x & y & y & y & y \\ y-x & x-y & 0 & 0 & 0 \\ 0 & y-x & x-y & 0 & 0 \\ 0 & 0 & y-x & x-y & 0 \\ 0 & 0 & 0 & y-x & x-y \end{bmatrix}$$

Add (B) (C) +1 (add) (A)

∴ we have 2 non zero entries, except the First row, and only difference is sign.

∴ find way to sum them up, then get more non zero entries

(C) Add these two

$$\rightarrow \begin{bmatrix} 2y & 2y & y \\ 0 & 0 & 0 \\ x-y & 0 & 0 \\ 0 & x-y & 0 \\ y-x & 0 & x-y \end{bmatrix} \rightarrow \begin{bmatrix} x+4y & 4y & 3y & 2y & y \\ 0 & x-y & 0 & 0 & 0 \\ 0 & 0 & x-y & 0 & 0 \\ 0 & 0 & 0 & x-y & 0 \\ 0 & 0 & 0 & 0 & x-y \end{bmatrix}$$

(B) (A) called this spot

(E) (D) (C)

∴ upper triangular matrix

$\therefore \det B = $ Det of upper triangular matrix.

$$= (x+4y)(x-y)^4$$