lec

Linear Transformation.

Here we have a matrix A and produce its transpose.

Let $T(A) = A^T$, A is 2×2

1) why is T linear? What is T^{-1}?

- What are the abstract properties that a linear operator satisfies?

What happens when T acts on sum of two matrices

$$T(A+B) = (A+B)^T = A^T + B^T = T(A) + T(B)$$

multiple of transformation

$$T(cA) = (cA)^T = cA^T = cT(A)$$

What does it is a linear operator.

What does transpose do? [what is its inverse]

— takes column $\begin{bmatrix} \cdot \\ \cdot \end{bmatrix}$, and flips it into row $[\cdot\ \cdot]$

and if we apply the operation once again? ②

it flip it from row [$\sim\sim$] Back to Column [$\wr$]

$\therefore$ applying transformation twice

$\Rightarrow$ we Come Back to the original position

$\therefore T^2 = I \Rightarrow T^{-1} = T$ [what is T^{-1}]

2) write down the matrix of T in [Following two Basis Standard Basis]

$$v_1 = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, v_2 = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, v_3 = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}, v_4 = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$$

The way we compute the matrix, we first compute: what T does to each of the basis element.

$$\left.\begin{aligned} Tv_1 &= v_1 \\ Tv_2 &= v_3 \\ Tv_3 &= v_2 \\ Tv_4 &= v_4 \end{aligned}\right\}$$

Encode this into a matrix

$$M_T = \begin{array}{c} \begin{array}{cccc} Tv_1 & Tv_2 & Tv_3 & Tv_4 \end{array} \\ \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \end{array}$$

(How Tv_1 is expresed as a ~~basis~~ Linear Combination of the Basis Element)

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \rightarrow \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}$$

$$\downarrow T$$

$$\begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix} \rightarrow \begin{bmatrix} 1 \\ 3 \\ 2 \\ 4 \end{bmatrix}$$

M_T multiply Top, we get Bottom

Now Lets compute T in basis, vectors $w_1, w_2\ w_3, w_4$

$$w_1 = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \quad w_2 = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \quad w_3 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \quad w_4 = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$

4

$$\left.\begin{aligned} Tw_1 &= w_1 \\ Tw_2 &= w_2 \\ Tw_3 &= w_3 \\ Tw_4 &= -w_4 \end{aligned}\right\} M_T = \begin{bmatrix} 1 & & & \\ & 1 & & \\ & & 1 & \\ & & & -1 \end{bmatrix}$$

3) Eigen value | vectors of T [lin. Trans]

we looking for vectors

$$Tv = \lambda v$$

Look Back at w_1, w_2, w_3, w_4

w_1, w_2, w_3 are E'vectors for T.

with E'value $= 1$.

and w_4, is E'vector for T

with E'value $= -1$.