Rec

- Vec spaces are actually matrices or function.

Show that the set of 2×3 matrices ~~(the)~~ whose nullspace contains $\begin{bmatrix} 2 \\ 1 \end{bmatrix}$ is a Vector subspace, and find a basis for it

What about the set of those whose column space contains $\begin{bmatrix} 2 \\ 1 \end{bmatrix}$?

How do we show that something is a vector subspace?

- there are only 2 things we need to check?

1) 2 matrices are in that space, then there sum is in that space
2) and if you take vector, in this case matrix, multiply it by scalar, you still be in the space.

Suppose matrix a and b are in subspace (set)

$$\overset{2\times 3}{A} \overset{3\times 3}{\begin{bmatrix}2\\1\\1\end{bmatrix}} = \overset{2\times 1 \text{ (result)}}{\begin{bmatrix}0\\0\end{bmatrix}}$$

Suppose $\begin{bmatrix}2\\1\\1\end{bmatrix}$ is in the null space of A

and $B\begin{bmatrix}2\\1\\1\end{bmatrix}$ is also in the null space

if $B\begin{bmatrix}2\\1\\1\end{bmatrix} = \begin{bmatrix}0\\0\end{bmatrix}$

then what $(A+B)\begin{bmatrix}2\\1\\1\end{bmatrix} = A\begin{bmatrix}2\\1\\1\end{bmatrix} + B\begin{bmatrix}2\\1\\1\end{bmatrix} = \begin{bmatrix}0\\0\end{bmatrix}$

indeed $\begin{bmatrix}2\\1\end{bmatrix}$ is in the null space (A+B)

$A\begin{bmatrix}2\\1\\1\end{bmatrix}=\begin{bmatrix}0\\0\end{bmatrix}$ ← means its number

C as scalar

$(CA)\begin{bmatrix}2\\1\\1\end{bmatrix}$

→ then we want to check $(cA)\begin{bmatrix}2\\1\\1\end{bmatrix}$ is in null space (CA)

every entry A is multiplied by c

$(CA)\begin{bmatrix}2\\1\\1\end{bmatrix} = c\left(A\begin{bmatrix}2\\1\\1\end{bmatrix}\right)=\begin{bmatrix}0\\0\end{bmatrix}$

Matrix (cA) is also contained in set.

Set is indeed a vector subspace

2nd part of question
- Find Basis for Subspace?

- Condition for matrix to be in SubSpace, is that vector $\begin{bmatrix}2\\1\\1\end{bmatrix}$ is in null space.

$\therefore$ Each row of A must be $\begin{bmatrix}a & b & c\end{bmatrix}\begin{bmatrix}2\\1\\1\end{bmatrix}=0$

$\therefore$ $(2a+b+c=0)$

$\therefore$ must be $\begin{bmatrix}a & b & -2a-b\end{bmatrix} = \begin{bmatrix}a & 0 & -2a\end{bmatrix}+\begin{bmatrix}0 & b & -b\end{bmatrix}$

$\therefore$ must be lin. comb of $\begin{bmatrix}1 & 0 & -2\end{bmatrix}, \begin{bmatrix}0 & 1 & -1\end{bmatrix}$

Basis $\begin{bmatrix}1 & 0 & -2\\0 & 0 & 0\end{bmatrix}, \begin{bmatrix}0 & 1 & -1\\0 & 0 & 0\end{bmatrix}, \begin{bmatrix}0 & 0 & 0\\1 & 0 & -2\end{bmatrix} \begin{bmatrix}0 & 0 & 0\\0 & 1 & -1\end{bmatrix}$

Dim $=4$.

Last question!

What about the set of those whose column space contains (set) $\begin{bmatrix}2\\1\\1\end{bmatrix}$
- is that vector subspace?

Quick check: zero vector/matrix belongs to set.

Does $\begin{bmatrix}0 & 0 & 0\\0 & 0 & 0\end{bmatrix}$ belong to set $\begin{bmatrix}2\\1\\1\end{bmatrix}$, NO!