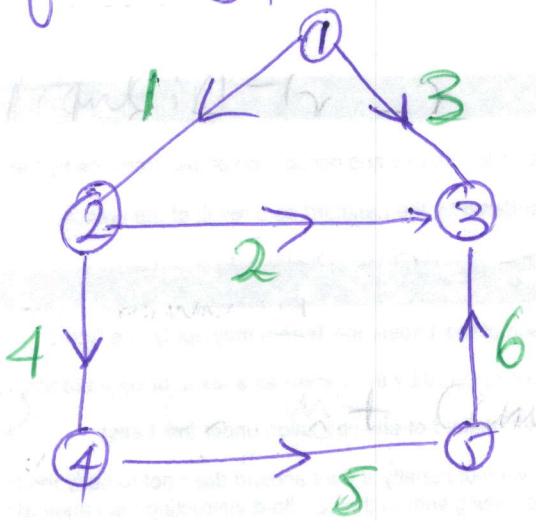
Rec

How we can use LA to describe graphs and networks, by looking at the following problem:



∴ 5 nodes
6 Edges

- Find incidence matrix A = (Calculate Kernel?)
- N(A) N(A^T) =
- Trace (A^T A) =

∴ What is an incidence matrix?
∴ it shows nodes connects to Edges
∴ it has as many rows as there is edges
" " cols as there is nodes

" "

- Wergang

- we going to full in row, and fill it in a format:
  - only use (-1, 1, 0)
    - -1 in i ← (j to i)
    - 1 in j (if edge connect i to j)

(A)

A =

Edge 1: node ① to ②
Edge 2: Node ② to ③
Edge 3: Node ① to ③
Edge 4: Node ② to ④
Edge 5: Node ④ to ⑤
Edge 6: Node ⑤ to ③

Now we will compute its null space,
⇒ what performing row operations.
In order to do it, its helpfull to look at graph as electric circuit.
and assign to each of nodes an Electric potential.

$Ax = 0$ → vectors: may entries as their are edges. and gives precisely the potential difference across edges of graph.

→ all potential difference be zero

~~Therefore~~ all nodes need to equal constant number.

⇒ here we can conclude: — constant 1,

$$N(A) = \left\{ \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{pmatrix} \right\}$$

5 (1's) correspond to 5 nodes.

What about Nullspace of A^T

$N(A^T) = ?$

Let's use the analogy of electric circuit

$\Rightarrow$ Currents flowing across edges of graph.

Let's use convention for currents:

$+V \Rightarrow$ flows in direction of Edge

$-V \Rightarrow$ flows opposite direction

$\therefore A^T y$ — where y is vector whose entry is a current on the edge

$A^T y \Rightarrow$ Total current flowing through each of nodes through graph.

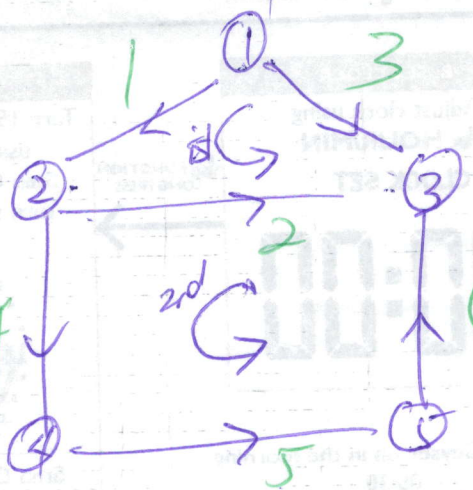
$\therefore A^T y = 0$ — [Balance] Ⓐ Current flowing in equal Current flowing node.

$\therefore$ Each to find such a configuration of currents.

— that satisfy the Ⓐ to Balanced Equation

— we do it by flowing around Loops of graph.

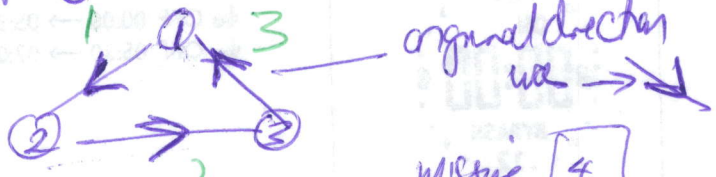
Eg. graph has 3 loop.



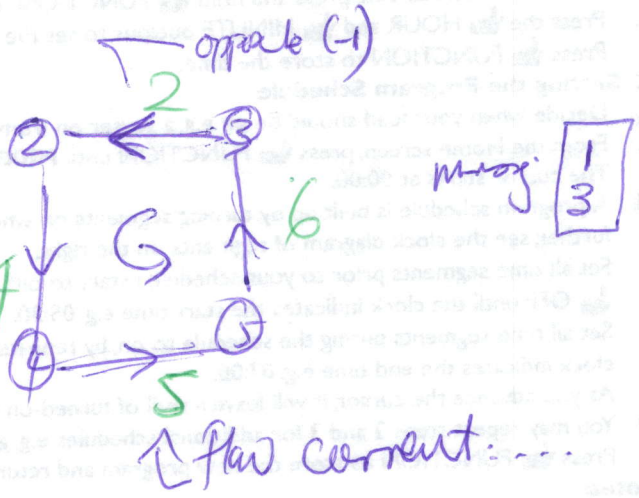
3rd ↺ around outside of graph.

↳ Can be thought of as super position of 1st & 2nd

Let's take the Configuration that Balance the 1st loop



$$y = \begin{pmatrix} 1 \\ 1 \\ -1 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$ ← direction opposite to scene ✓



$$y = \begin{pmatrix} 0 \\ -1 \\ 0 \\ 1 \\ 1 \\ 1 \end{pmatrix}$$

→ Null position ←

What is Trace (A^TA)?

∴ Trace of matrix is sum of its diagonal entries.

∴ diagonal entry of A^TA are precisely magnitude squared of columns of A.

Idea: Each column (entry) of the instance matrix is $(1, -1 \text{ or } 0)$
- so when we square the entry we get 1 or 0
- and when we add them up we get precisely non-trivial entries in column

∴ go Back to matrix A, and Count. [(A) on page (2)]

non-zero entries, ∴ equal to number of edges with node → called degree of node

∴ Trace (A^TA) ⇒ Sum of degrees of graph.

∴ 2 [2 edges connecting to (1)]
∴ + 3 [3 edges " " (2)]
+ 3 [3 " " " (3)]
+ 2 [2 " " " (4)]
+ 2 [2 " " " (5)]
= 12