New vector space

Previously we talked about vector space

, But the things inside of it are not really vectors?

- You can add it and multiply it
  - they really matrices.
  - ∴ we had matrix space.
  - ∴ 3x3 matrices.
- Look at n dimensional space.
- Same should work as look as you can add & multiply by scalars

New vectors space:

M = all 3 by 3 matrices

∴ - Add it
  - multiply by scalars.

- Can even multiply the two of them together

But I do not do it
, that is not part of the vector space picture

— Vector Space part

— adding the matrices, and

— multiplying by numbers

— we stay within 3x3 matrices.

i.e. Subspace is symmetric.

or Subspace is upper triangular (3x3)

(~~lower~~)

What is the basis for that subspace

" Dimension "

" " of that whole space

There is a natural basis for all 3x3 matrices

Basis for M = all 3x3's

Dim = 3x3 = 9.

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \cdots \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$\therefore$ Space same as 9 dimensional space.

∴ ~~12~~ 9 numbers are written in square &

∴ Dim of M = 9

for symmetric Vector Space

Dim S = 6

Upper triangular ~~S~~ Vector space

Dim U = 6

~~Set~~:

$S \cap U$ = symmetric and upper triangular

— what's the dimension of this space.

its diagonal (3X3)

intersect.

$\mathrm{Dim}(S \cap U) = 3$

— what is bad about $S \cup U$?

its not a subspace

— they headed in different directions, we can't just combine them.

④

$S+U$ = not their union, but their sum

= any element of S + any element of U.

= this is a subspace

= all 3×3's

what's $\dim(S+U) = 9$

$\therefore$ - $\dim(S)=6 + \dim(U)=6 =$

$\dim(S\cap U)=3 + \dim(S+U)=9$

$6+6 \Rightarrow$

$\Rightarrow 3+9=$

Key number associated with matrices

is the rank.

- We know the rank is not bigger than m

- and not bigger than n

rank of matrix.

eg. $A = \begin{bmatrix} 1 & 4 & 5 \\ 2 & 8 & 10 \end{bmatrix}$

Suppose 2x3

possible rows, for this matrix to have rank 1?
(for 2nd row)
→ its a multiple of the first row.
↳ it is not independent.

- What is a basis for the row space?
  - the first row: (1 4 5)
- What's the basis for column space?
  - What Dim of Column space = 1. = r

Can write matrix:

$$\underset{(2\times1)}{= \begin{bmatrix} 1 \\ 2 \end{bmatrix}} \underset{(1\times3)}{\begin{bmatrix} 1 & 4 & 5 \end{bmatrix}} = \underset{\text{result } (2\times3)}{\begin{bmatrix} 1 & 4 & 5 \\ 2 & 8 & 10 \end{bmatrix}}$$

1 Column × 1 row

∴ Rank 1 matrices has form:

$$A = U V^T$$

Some column × Some row

column vector, but make into row, by putting in transpose

∴ rank 1 matrices are the building blocks of all matrices.

If I took any matrix, 5×17 of rank 4, then it's pretty likely that matrix can be broken ~~down~~ up as a combination of rank 1 matrices.

∴ and many of those would I need?

∴ need 4. rank 1 matrices.

Would rank 4 matrix ~~form~~ form a subspace? (7)

Lets take all 5×17 matrices, and

Think of subset of rank 4 matrices...

Matrix = all 5×17 matrices

Subset of rank 4 matrices $\Rightarrow$ is that a subspace?

eg add 2 rank 4 matrices, is sum rank 4
(not usually)
– the rank could be 5

$\Rightarrow$ If subset of rank 1 matrices $\Rightarrow$ is that a subspace?

$\therefore$ rank 1 to rank 1 (No, mostly likely it would have rank 2)

$\therefore$ Not a subspace.

Suppose I am in R4 — (4 Component)

$$V = \begin{bmatrix} V1 \\ V2 \\ V3 \\ V4 \end{bmatrix} \qquad S = \text{all } V \text{ in } R4 \text{ with } V_1+V_2+V_3+V_4 = 0$$

(*) Suppose I take the subspace, whose component add to 0

Is it a subspace? Yes.

How do we see it? How do we check?

eg one component $V \times 0$

6×0 = still zero.

— What is dim and basis subspace?

of subspace S,

$Av = 0$

Dim = 3.

(A) This is nullspace of $A = \begin{bmatrix} 1 & 1 & 1 & 1 \end{bmatrix}$

If we speak about S, we speak about the nullspace of (A) above.

rank = 1 = r

general formula for Dim of nullspace:

$$\text{Dim } N(A) = n - r = 4 - 1$$

Lets take the 4 fundamental subspaces of this eg.

$$A = \begin{array}{c} \text{pivot} \\ \begin{bmatrix} \boxed{1} & 1 & 1 & 1 \end{bmatrix} \end{array}$$

- rowspace is 1 dimensional, all multiples of that row.

- nullspace is 3 dimensional $n-r=4-1=3$

what is basis for null space

- special solution, get free variables.
- pivot 1
- free variable col_2, col_3, col_4.

Basis for S: [we list 3 vectors, naturals]

special three

add to zero

$$\begin{bmatrix} \boxed{-1} \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

3 special solutions

Column space ?

- Column space is subspace of R^1

$m = 1$.

$C(A)$ is somewhere in space R^1

$= R^1$

What nullspace (left hand) side

$N(A^T) = [0]$

Lets look at Dimensions:

Nullspace Dim $= 3$

rowspace Dim $= 3 + 1 = 4 = n$ (colum)

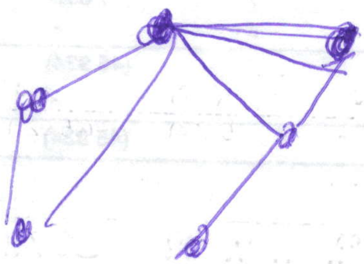
Column space Dim $= 1 + 0 = 1 = m$ (rows)

Nullspace (left hand) Dim $= 0$

What is a graph?

— Not talking here about Calculus (sine curve)

Bunch of nodes & Edges connecting nodes.



— Suppose the graph has lots nodes.
— Suppose there is ~~ordin~~ an edge (line / point) between nodes if Nodes are Friends.

∴ How many steps does it take from one node to next.
— what two nodes are furtherest appart?