Rec

Look at Pos def.

for which values of c is

$$B = \begin{bmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2+c \end{bmatrix}$$

→ positive definite?

→ positive semidefinite?

- There are several tests you can use of an matrix to find out they pos def and ps. semi Def.

⇒ demonstrate 3 here.

∴ one you can use, where you have very little time:

∴ determinant systems / test

∴ calculate det in upper left corner

$$\begin{bmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2+c \end{bmatrix}$$

- and will be positive iff > 0

- and pos semidef iff some zeros sneak in.

$$\overset{1st}{2}, \overset{2nd}{\det}\begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} = 4-1=3, \det B =$$

$$2 \cdot \begin{vmatrix} 2 & -1 \\ -1 & 2+c \end{vmatrix} - (-1) \cdot \begin{vmatrix} -1 & -1 \\ -1 & 2+c \end{vmatrix} +$$

$$+(-1) \cdot \begin{vmatrix} -1 & 2 \\ -1 & -1 \end{vmatrix} = 2 \cdot (4+2c) + (-2-c-1)$$

$$-(1+2) = 6+4c-3-c-3 = 3c$$

$\therefore$ 2 pos, 3 pos, 3 pos $\Big\}$ matrix is pos. def iff $c > 0$

and positive semi def. iff $c \geq 0$

$\Rightarrow$ use this if you in test

Lets do two more tests:

① Pivot test $\rightarrow$ Pivots $= 2, \frac{3}{2}, C$

$$\begin{bmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2+C \end{bmatrix} \sim \begin{bmatrix} 2 & -1 & -1 \\ 0 & \frac{3}{2} & -\frac{3}{2} \\ 0 & -\frac{3}{2} & \frac{3}{2}+C \end{bmatrix} \sim \begin{bmatrix} \boxed{2} & -1 & -1 \\ 0 & \boxed{\frac{3}{2}} & -\frac{3}{2} \\ 0 & 0 & \boxed{C} \end{bmatrix}$$

$\frac{1}{2}$, $+$

∴ Same as Before w.r.t C. pos def $C > 0$, pos sem. def $C \geq 0$

But, Before we had $2, 3$, and $3C$?

Last: Energy Test or / Completing the Square.

$$[x\,y\,z] \cdot B \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} \geq 0 \quad \left[\begin{matrix}\text{Here } x, y, z \text{ all} \\ \text{have to be } \neq 0\end{matrix}\right]$$

— above one of def. of pos. def.

$$[x\,y\,z] \cdot B \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 2x^2 + 2y^2 + (2+C)z^2 - 2xy^2 - 2xz - 2yz$$

Lets Complete using formula:

$$* \ (a+b+c)^2 = a^2+b^2+c^2+2ab+2ac+2bc$$

$$* \ (a+b)^2 = a^2+2ab+b^2$$

$$= 2x^2+2y^2+(2+c)z^2-2xy-2xz-2yz$$

(lets get something squared) $\Rightarrow$ that has all x's above

$$= 2\cdot\underbrace{\overbrace{\left(x-\tfrac{1}{2}y-\tfrac{1}{2}z\right)^2}^{\text{took up all x's}}}_{\text{Square of some real number pos.}}+\frac{3}{2}\cdot\underbrace{\overbrace{(y-z)^2}^{\text{took up all y's}}}_{\text{Square of some real number pos.}}+\underbrace{c\cdot z^2}_{\text{positive}} \geq 0$$

If $\boxed{c \geq 0}$

$$\begin{array}{c} 2 \\ \tfrac{3}{2} \\ c \end{array}\begin{bmatrix} 1 & -\tfrac{1}{2} & -\tfrac{1}{2} \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}$$

$\boxed{c=0}$

$$\begin{array}{c} 2 \\ \tfrac{3}{2} \\ \ \end{array}\begin{bmatrix} 1 & -\tfrac{1}{2} & -\tfrac{1}{2} \\ 0 & \boxed{1} & -1 \\ 0 & 0 & \underbrace{0}_{\text{free}} \end{bmatrix} \quad \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$