

①

System of Equations we will be working with, and solve it:

$$x + 2y + z = 2$$

$$3x + 8y + z = 12$$

$$4y + z = 2$$

looking at:

1) Elimination [Not determinant because, this will cancel]

2) Back Substitution

3) Elimination Matrix

4) Matrix multiplication

$$\begin{bmatrix} 1 & 2 & 1 \\ 3 & 8 & 1 \\ 0 & 4 & 1 \end{bmatrix}$$

↑ matrix ↑

But Elimination can also Fail

Elimination was thought up by Gauss!

Here everything will be expressed as Matrix operations

$$Ax = b \text{ [to be solved]} \quad (2)$$

Step 1: Eliminate x [set ~~to~~ ^{as} ~~value~~ ^{value}] \rightarrow to value that will make $(2,1) = 0$

- 1) multiply the first row
- 2) Subtract the second row from first row (row 1, column 1)
- 3) Make $(1,1) = 1$

also called ~~(row)~~ First Pivot
and first row

4) Make all subsequent $x = 0$

$$\begin{array}{ccc} (1,1) & & \\ \boxed{1} & 2 & 1 \\ (2,1) \boxed{3} & 8 & 1 \\ 0 & 4 & 1 \end{array}$$

Need to multiply $(1,1)$ by 3

$$\begin{array}{ccc} (x3) & (x3) & (x3) \\ 1 & 2 & 1 \\ 0 & 2 & -2 \end{array} \quad \begin{array}{l} (x3) \\ \text{Subtract} \end{array} \quad \begin{array}{l} \text{row from (row 1)} \\ 1 \times 3 = 3 \\ \therefore 3 - 3 = 0 \text{ (2,1)} \\ 2 \times 3 = 6 \\ \therefore 8 - 6 = 2 \text{ (2,2)} \\ 1 \times 3 = 3 \\ \therefore 1 - 3 = -2 \text{ (2,3)} \end{array}$$

$(3-3)=0$

84 $(2 \times 3) = 6$ $8 - 6 = 2$
16 $(1 \times 3) = 3$ $1 - 3 = -2$

$$\boxed{0 \quad 4 \quad 1} \quad \begin{array}{l} \text{Stop} \\ \text{Now} \\ (3,1) = 0 \end{array}$$

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if 0.41 , if $(3,3) \neq 0$, then
had to be the same as we did
for the second row

Now x is eliminated
i.e we have it set = ~~to~~ one value

Step 2: Eliminate y [set $y = \text{one value}$]

- 1) Multiply First row
- 2) Subtract second row from F_1
- 3) Make $(3,2) = 0$

- 4) Make all subsequent $y = 0$
- 5) Here pivot = $(2,2)$

we always knock out (make zero)
the (row, col) item under pivot

\therefore here pivot is $(2,2)$

so we need to knock out (0^{row}) :
 $(3,2)$

$$\begin{array}{ccc}
 1 & 2 & 1 \\
 0 & \boxed{2} & -2 \\
 0 & \boxed{4} & 1
 \end{array}$$

next pivot

Remember
Pivots can't
be zero!

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what we
need to
know

need multiply (2,2) by
2 to knock out 4.

$$\begin{array}{ccc}
 1 & 2 & 1 \\
 0 & 2 & -2 \\
 0 & 4 & 1
 \end{array}
 \Rightarrow
 \begin{array}{ccc}
 \boxed{1} & 2 & 1 \\
 0 & \boxed{2} & -2 \\
 0 & 0 & \boxed{5}
 \end{array}$$

(3,3) \Rightarrow Next pivot.

$$2 \times 2 = 4$$

$$4 - 4 = 0$$

$$2 \times 1 = 2$$

$$2 \times -2 = -4$$

$$1 - (-4) =$$

$$1 + 4 = 5$$

But

we know have 3 pivots!

But how could this have Failed?

⑤

1 2 1

3 8 1

0 4 1

If (1,1) was zero, then maybe issue!

But how do we solve:

⇒ we switch rows / swap rows

OR 1 3 8 1

if that was 6,
then it would be zero, then it
could not use this row [as we
should never have pivot as zero]

⇒ again switch rows

non zero below, then use it.

there is temporary failure, where we
can do row exchange, or complete
failure, where zero below

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Next Back substitution

[Now we bring the right hand side in]

But all the operations we did for matrix we need to do for right hand side

$\therefore b$

$$\begin{array}{ccc|c} 1 & 2 & 1 & 2 \\ 3 & 8 & 1 & 12 \\ 0 & 4 & 1 & 2 \end{array} \Rightarrow \begin{array}{ccc|c} 1 & 2 & 1 & 2 \\ 0 & 2 & -2 & 6 \\ 0 & 4 & 1 & 2 \end{array} \Rightarrow \begin{array}{ccc|c} 1 & 2 & 1 & 2 \\ 0 & 2 & -2 & 6 \\ 0 & 0 & 5 & -10 \end{array} \begin{array}{l} \\ \\ (u) \end{array}$$

$$\begin{array}{r} 3 \times 2 = 6 \\ 12 - 6 = 6 \\ \hline 3 \times 2 \\ 6 \times 4 = 24 \\ 2 \\ 1 - 24 = -23 \end{array}$$

$$\begin{array}{rcl} x + 2y + z & = & 2 \\ 2y - 2z & = & 6 \\ 5z & = & -10 \end{array}$$

So we solve for z

← This is Back Substitution

Back Substitution
Doing equation into reverse order

$$\begin{array}{rcl} z & = & -2 \\ x + 2y + z & = & 2 \\ 2y - 2z & = & 6 \\ 5z & = & -10 \end{array} \begin{array}{l} z = -2 \\ x + 2(1) + (-2) = 2 = x + 0 = 2 \\ x = 2 \\ y = 1 \\ 2y - 2(-2) = 6 \\ 2y + 4 = 6 \\ 2y = 6 - 4 = 2 \\ y = 1 \end{array} \begin{array}{l} \\ \\ \\ \text{Back} \end{array}$$

$$\begin{array}{l} x = 2 \\ y = 1 \\ z = -2 \\ (C) \end{array}$$

⑨

⑩ Next piece is row matrices [Elimination]

- Now we need to express the elimination steps as matrices.
 ← want to do elimination on it.

$$\begin{bmatrix} 1 & 2 & 1 \\ 3 & 8 & 1 \\ 0 & 4 & 1 \end{bmatrix}$$

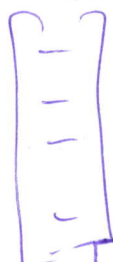
Previously how did we multiply (Matrix) with vector

$$\begin{bmatrix} \text{---} \\ \text{---} \\ \text{---} \\ \text{---} \end{bmatrix} \begin{bmatrix} 3 \\ 4 \\ 5 \end{bmatrix}$$

$$\text{matrix} \times \text{Column}^{(\text{vector})} = \text{Column}^{(\text{vector})}$$

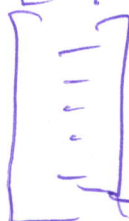
→ First column of matrix

3x



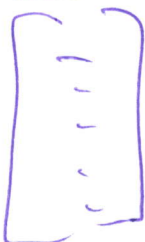
→ 2nd Column

4x



→ 3rd Column

5x



∴ Combining the Column.
 →

Same can be done with rows [in fact we will be dealing primarily with rows] ⑧

$$\begin{bmatrix} 1 & 2 & 7 \end{bmatrix}_{1 \times 3} \times \begin{bmatrix} - & - & - \\ - & - & - \\ - & - & - \end{bmatrix}_{3 \times 3} \text{ row } \times \text{ matrix} =$$

∴ Combining the rows.

∴ 1 x row 1

2 x row 2

7 x row 3

What's matrix step that subtracts 3 from Equation 2 [First step]
(and leaves other rows same)

Subtract 3 x row 1 (Equation 1) from row 2 (Equation 2)

$$\begin{bmatrix} 1 & 2 & 7 \\ 3 & 8 & 1 \\ 0 & 4 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 2 & -2 \\ 0 & 4 & 1 \end{bmatrix}$$

only row 2 will change

∴ what will be matrix

$$\begin{bmatrix} 1 & \textcircled{1} & 0 \\ -3 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & \textcircled{2} & 1 \\ 3 & 8 & 1 \\ 0 & 4 & 1 \end{bmatrix} = \begin{bmatrix} 1 & \textcircled{3} & 1 \\ 0 & 2 & \textcircled{2} \\ 0 & 4 & 1 \end{bmatrix} \textcircled{9} \textcircled{-2}$$

→ E = Elementary Matrix

row 1: $1 \quad 0 \quad 0$

row 3: $0 \quad 0 \quad 1$

row 2: $-3 \quad 1 \quad 0$

↑
to be subtracted from row 1.

$$\begin{bmatrix} -3 & 1 & 0 \\ -3 & 1 & 0 \\ -3 & 1 & 0 \end{bmatrix}$$

$$-3 + 3 = 0$$

A	B	B
row 1	row 1	row 2

$$1 + (-3) + 3 = 1$$

$$2 +$$

$$A - B - B$$

$$2 + 1 = 3$$

How to Check $\textcircled{2} [2, 3]$

look at row 2 of $\textcircled{1}$

and column 3 of $\textcircled{2}$

©

How to demonstrate the ~~how~~ whole
Elimination in matrix language.

Step 2: Subtract 2x row 2 from row 3

$$E_{(3,2)} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -2 & 1 \end{bmatrix} \quad \begin{bmatrix} 1 & 2 & 1 \\ 0 & 2 & -2 \\ 0 & 4 & -1 \end{bmatrix} \quad \begin{bmatrix} 1 & 2 & 1 \\ 0 & 2 & -2 \\ 0 & 0 & 5 \end{bmatrix}$$

Now puts all steps together into
matrix that does it all.

$$\therefore E_{\text{32}}(E_{21}A) = U$$

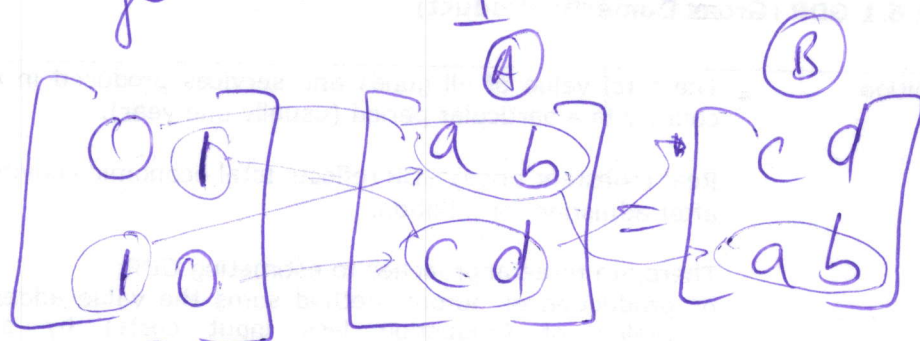
$$(E_{32}E_{21})A = U$$

P₂

What Permutation Matrix

(11)

Exchange rows 1 and 2



* for row operators
matrix goes on left!

P \rightarrow matrix to do row exchanges

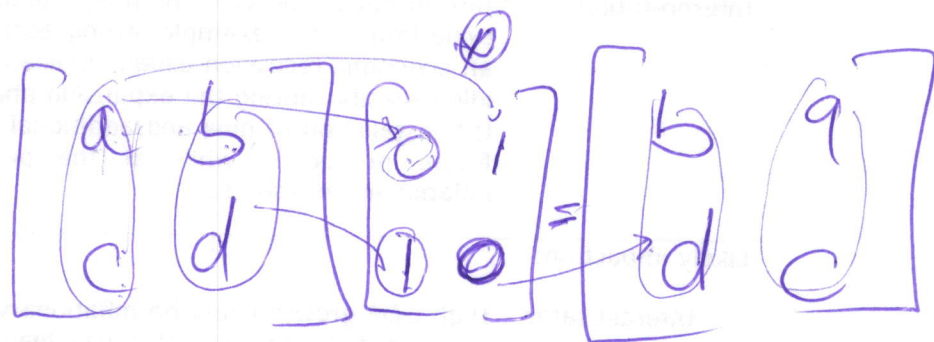
\therefore exchange rows in A to look like rows in B

What matrix would you use:

1 of c d $\begin{bmatrix} 0 & 1 \end{bmatrix}$

and 1 of a b $\begin{bmatrix} 1 & 0 \end{bmatrix}$

Suppose now I want to swap columns



to do columns operators

* we put matrix on right

(12)

What are inverse matrix

$$\begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ -3 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

What matrix does that undo this step:
(subtract 3 from row 1 from row 2)

matrix that undo the elimination

inverse add 3

} Do inverse