Ps : Subspaces / null spaces

which are subspaces of

$\mathbb{R}^3 = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$

1) $b_1 + b_2 - b_3 = 0$

any linear equation can be written as matrix.

$\begin{bmatrix} 1 & 1 & -1 \end{bmatrix} \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} = 0$

$b_1 b_2 b_3$ ⇒ Describes null space of $[1 \; 1 \; -1]$

2) $b_1 b_2 - b_3 = 0$ is this linear?

we can see $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ $\begin{bmatrix} 2 \\ 2 \\ 2 \end{bmatrix}$ ← also in subspace as any multiple of vector of subspace is inside subspace.

But is $\begin{matrix} 2 \\ 2 \\ 2 \end{matrix}$ described by equation?
No

Not subspace.

3) $$\begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} = \underset{A_1}{\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}} + c_1 \underset{A}{\begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}} + c_2 \underset{A}{\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}}$$

2

A ⇒ all linearly independent
so vector space is plane in R^3

⇒ A_1 is linear Combination of A and A

$$\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \underbrace{\frac{1}{2}\begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} + \frac{1}{2}\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}}$$

$$\Rightarrow \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} = c_1 \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} + c_2 \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} =$$

is vector space

4) → But $\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$ is not inside $c_1 \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} + c_2 \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$

Cannot find coefficient!

No