Res. $Ax=0$ ①

Today's problem is about solving Homeogenous system $Ax=0$:

The set S of points $P(x,y,z)$ such that (st) $x-5y+2z=9$ is a ________ in $\mathbb{R}^3$.

It is ________ to be (in certain relation) ________. So of $P(x,y,z)$ that satisfy the following Equation $x-5y+2z=0$

after we solved this we have second part of problem:

all points of S have a specific form:

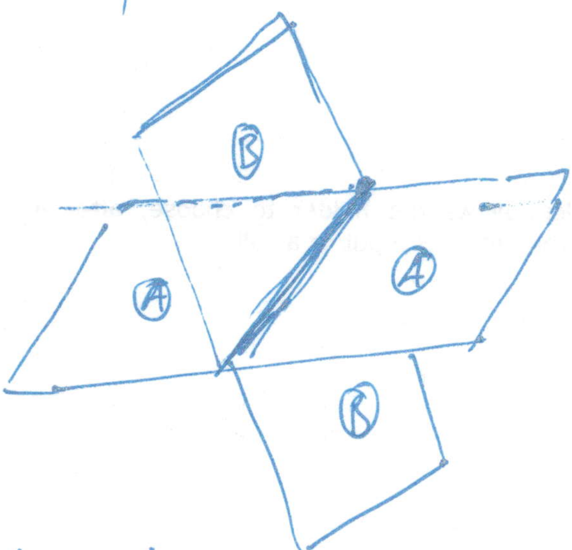
$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \\ 0 \\ 0 \end{bmatrix} + c_1 \begin{bmatrix} \\ 1 \\ 0 \end{bmatrix} + c_2 \begin{bmatrix} \\ 0 \\ 1 \end{bmatrix}$$

If you have a 3 dimensional space with 3 degrees of freedom and put in 1 equation, then you get something that has 2 degrees of freedom (i.e 2dimensional)

∴ this ε is called a Plane in $\mathbb{R}^3$

∴ Two planes in $\mathbb{R}^3$: Lets look at the general positions that planes in $\mathbb{R}^3$ can be:

(1)



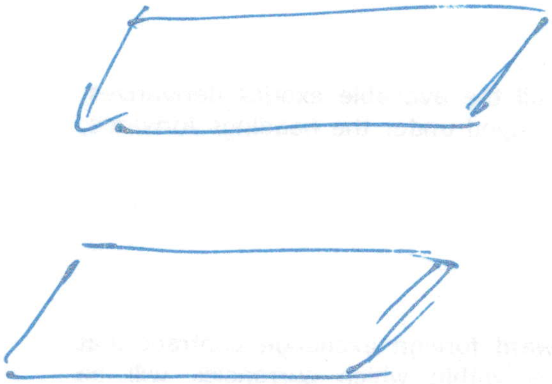
∴ all points in Plane A are points whose coordinates satisfy the equation of that plane

∴ all points in Plane B are points whose coordinates will satisfy the equation of that plane

and points on line where system satisfy A & B

Other position planes can be is that they not intersecting at all

(ii)



and they parallel.

Lets try to find the intersection (double line) in (i)

- the equation of one Plane is:

$$x - 5y + 2z = 9$$

- the equation of other Plane is

$$x - 5y + 2z = 0$$

$$\therefore \begin{cases} x - 5y + 2z = 9 \\ x - 5y + 2z = 0 \end{cases}$$

You can look at it and see how many solutions it ~~may~~ have OR

use elimination and after one step of elimination, get $0 = 9$ which never happen.

$\therefore$ There cannot exist number x, y, z such that this combination (2 equation) produces 0 AND same combination produces 9 at same time

$\therefore$ the double intersection line in (i) does not exist

$\therefore$ relationship between the two plane has to be parallel!

Complete sentence $\Rightarrow$ $\therefore$ It is parallel to the plane

∴ Lets move to the other half of the problem:

It says all points of S has this specific form:

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \underbrace{\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}}_{P_0} + c_1 \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} + c_2 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

We can plug in $c_1 = c_2 = 0$

$\Rightarrow$ we get that x, y, z or P_0 is point on plane S.

∴ $\Rightarrow P_0$ is in S!

∴ what do we know of the point P_0

- the fact that it is in S that its coordinates $x - 5y + 2z = 9$ [that equation of S]

- we also know that y and $z = 0$ $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \end{bmatrix}$ P_0

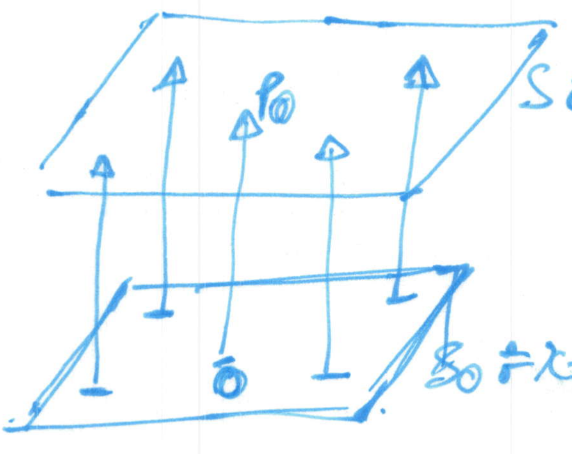
$$\begin{cases} y = 0 \\ z = 0 \\ x - 5y + 2z = 9 \end{cases}$$

$\Rightarrow x = 9$

$$\therefore \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \underbrace{\begin{bmatrix} 9 \\ 0 \\ 0 \end{bmatrix}}_{P_0}$$

So now we have 2 blanks to be filled:

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} + c_1 \begin{bmatrix} 1 \\ 0 \end{bmatrix} + c_2 \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$



has P_0 (just figured this out)

$S = x - 5y + 2z = 9$

this vector connect the two planes (to zero bottom)

$S_0 = x - 5y + 2z = 0$

has zero

$\therefore$ to get to any point in S, can go to point in S_0, then up by that vector.

$\therefore$ any point in S is of the form:

$= P_0 + \left(\begin{array}{c}\text{any point in} \\ S_0\end{array}\right)$

$$\therefore \underbrace{c_1 \begin{bmatrix} 1 \\ 0 \end{bmatrix} + c_2 \begin{bmatrix} 0 \\ 1 \end{bmatrix}}_{\text{hostable point in } S_0}$$

How can we view $x - 5y + 2z = 0$ differently:

$$\begin{bmatrix} 1 & -5 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = [0]$$

$\therefore$ Lets think of this as matrix of system

$\therefore$ matrix $\cdot$ Vector $= 0$

and try to find all solutions to system

$\therefore$ Row Reduction:

$$\begin{bmatrix} \overset{\text{pivot}}{\boxed{1}} & 5 & 2 \end{bmatrix}$$

free variables

$y \quad z$

∴ For each free variable we get one particular (case) solution.

∴ $y = 1, z = 0$ (A) or $y = 0, z = 1$ (B)

Plug in:

$$\begin{bmatrix} 1 & -5 & 2 \\ & y & z \end{bmatrix}$$

(A)

$x + 1 + -5 \times 1 + 2 \times 0$

$= x - 5 + 0 = 0$

$x = 5$

(B)

$x - 5(0) + 2(1) = 0$

$x - 0 + 2 = 0$

$x = -2.$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} \therefore \; + c_1 \begin{bmatrix} 5 \\ 1 \\ 0 \end{bmatrix} + c_2 \begin{bmatrix} -2 \\ 0 \\ 1 \end{bmatrix}$$