Lineer Algebra (MIT)

What is the fundamental problem of lineer algebra?

→ to Solve a system of lineer Equations

What are linear Equations?

- it is an Equation of a Straight line in form:

$$y = mx + b$$ (square roots) [no exponents or irrational or fractions!]

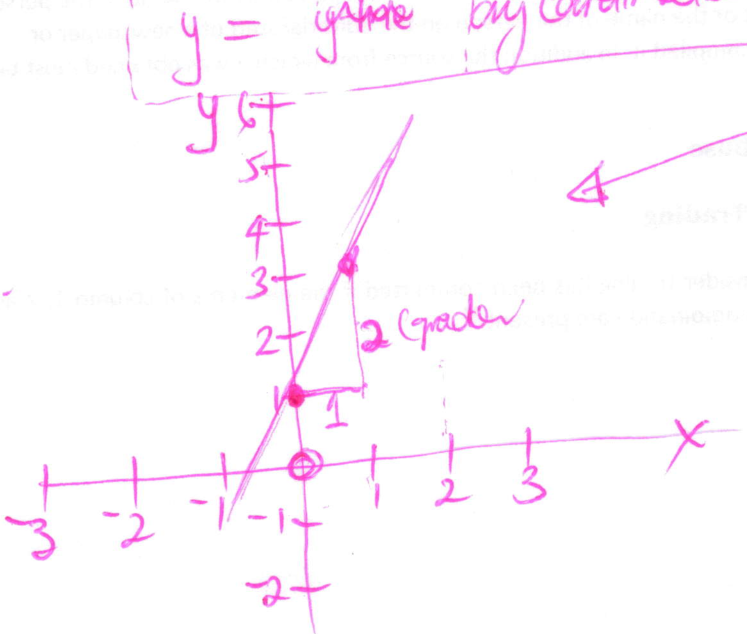
m = slope (gradient)

b = y-intercept [x=0, and lies on y-line]

x = x-line	any coordinate on the line
y = y-line	any coordinate on the line

[$y = mx + b$]

$y = 2x + 1$



$m = 2$ [slope]

$b = 1$

Workings

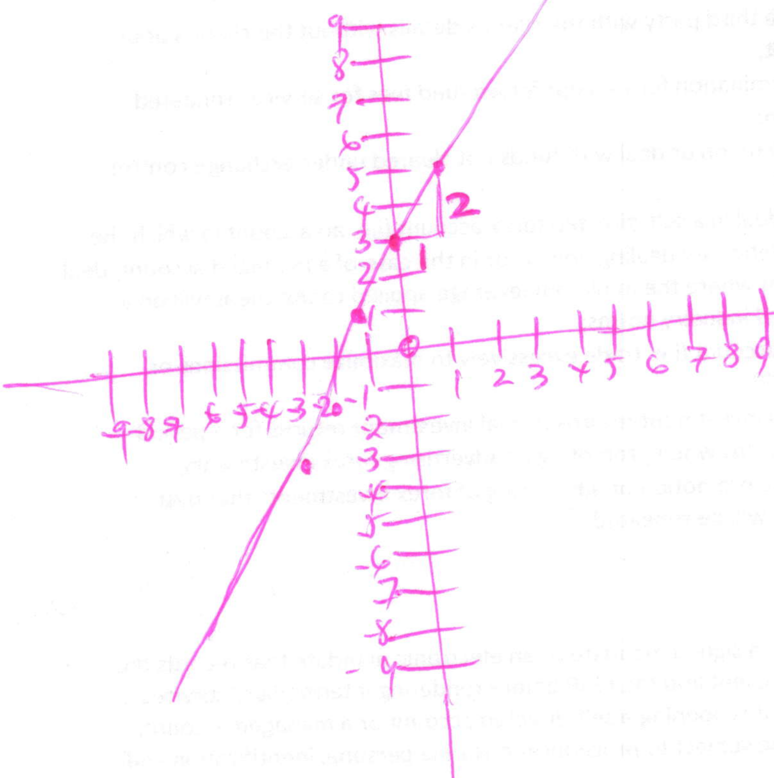
$y = mx + b$

$y = 2x + \boxed{3}$ — y intercept

(2-3) +3
-6+3
-3

x	-3	-2	-1	0	1	2	3
y	-3	-1	1	3	5	7	9

+2 +2 +2 ← (gradient) for every 1



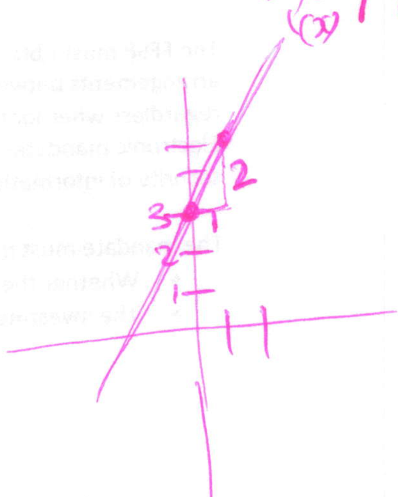
gradient = $\frac{\text{rise}}{\text{run}}$

$\frac{2 \text{ (rise)}}{1 \text{ (run)}}$

whole number

2

(y) $\frac{2}{1}$ rise / run (x)



⇒ also known as slope intercept formula!

3

∴ gradient was whole number

2

$$\therefore \frac{rise}{run} = \frac{2}{1}$$

∴ if gradient was fraction

eg $\frac{2}{3}$

$$\therefore \frac{rise}{run} = \frac{2}{3}$$ → up, → to right

if gradient was fraction

$-\frac{1}{2}$

$$\therefore \frac{rise}{run} = -\frac{1}{2} =$$ → go down, → going right.

we will be looking at the following:

1. Row picture
2. Column picture
3. Matrix form

Row picture with 2 equations: [n equations, n unknowns] *

$$2x - y = 0$$

$$-x + 2y = 3$$

What is (coefficient matrix)

→ what is matrix [2 rows, 2 column]

$$\begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$$

What is the unknown matrix, and right hand side

Vectors → $\begin{bmatrix} x \\ y \end{bmatrix}$ $\begin{bmatrix} 0 \\ 3 \end{bmatrix}$

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$$\begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 3 \end{bmatrix}$$

$$A \quad x = b$$

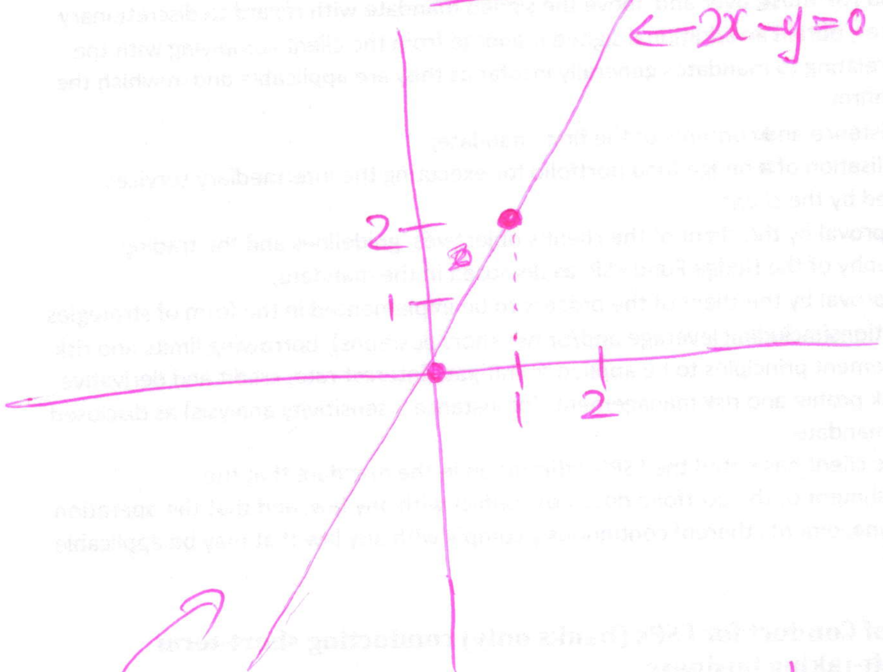
[idea is to Solve this, and Step Back to See Bigger picture]

A = Coeffient matrix

X = Vector of Unknown [mg huentupe unknown]

b = Vector

So what will the raw picture look like?



all the points will lie on this Straight here, as it is a linear equation

$2x - y = 0$

$\begin{bmatrix} x = 1 \\ y = 2 \end{bmatrix}$

$2(1) - 2 = 0$

$2 - 2 = 0$

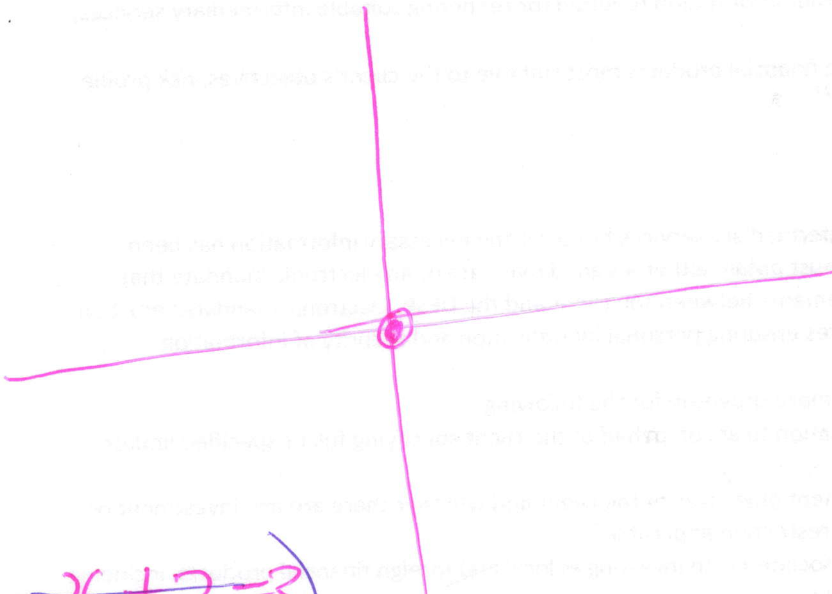
$0 = 0$ [

0 = y intercept

Fill in values for x & y to get 0.

But 2nd equation will not go through origin.

→ as $d = 3$.



~~$x + 2 = 3$~~

~~if $y = 0$~~

~~1+~~

(See workings Next page)

where the two point meet / interset

$$\boxed{\begin{aligned} x &= 1 \\ y &= 2. \end{aligned}}$$

2 Equations and 2 unknowns (x, y)

6.1

$2x - y = 0$

$-x + 2y = 3$

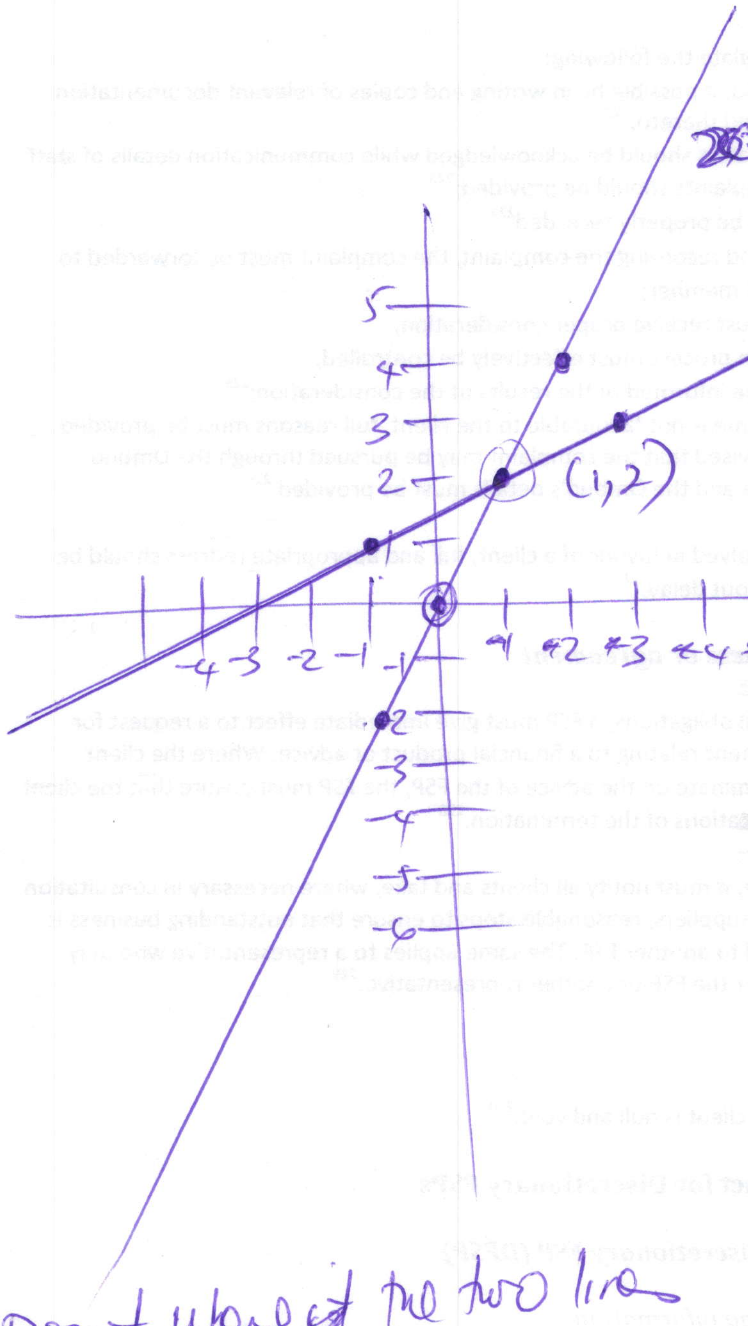


Table Coordinates

$\boxed{2x - y = 0}$

$x = 1, y = 2$

$2(1) - 2 = 0$

$2 - 2 = 0$

$0 = 0$

$x = 2, y = 4$

$2(2) - 4 = 0$

$4 - 4 = 0$

$0 = 0$

$x = -1, y = -2$

$2(-1) - (-2)$

$-2 + 2 = 0$

$0 = 0$

$-x + 2y = 3$

$x = 1 \quad y = 1 \quad =$

$-1(-1) + 2(1) = 3$

$1 + 2 = 3$

$3 = 3$

$x = 1 \quad y = 2$

$-1(1) + 2(2) = 3$

$-1 + 4 = 3$

$3 = 3$

$\cancel{x = 3, y = 2}$

$\cancel{-(3) + 2(3) = 3}$

$+6 =$

$x = 3, y = 3$

$-(3) + 2(3) = 3$

$-3 + 6 = 3$

$3 = 3$

$\therefore$ Point where the two lines intersect $x = 1$

$y = 2$

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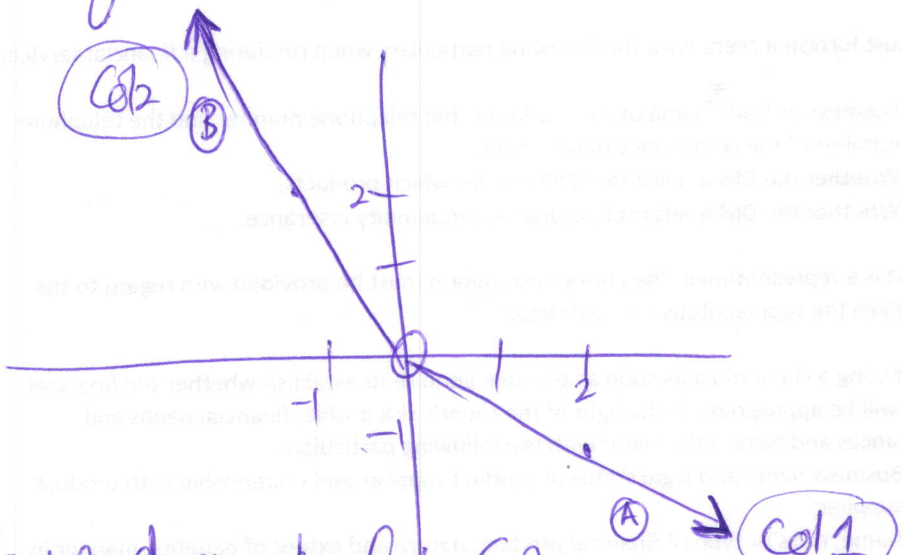
Lets look at the column picture:

$$x\begin{bmatrix}2\\-1\end{bmatrix} + y\begin{bmatrix}-1\\2\end{bmatrix} = \begin{bmatrix}0\\3\end{bmatrix}$$

what does the equation ask us to do:

∴ Combine x vector with y vector to get the result vector.

Now we need to draw the picture (geometry) of what it need to look like:



How do we draw the first column:

$$\begin{bmatrix}2\\-1\end{bmatrix} \quad \therefore \quad \left.\begin{matrix}x=2\\y=-1\end{matrix}\right\} A$$

Col 1

$$\begin{bmatrix}-1\\2\end{bmatrix} \quad \therefore \quad \left.\begin{matrix}x-1\\y=2\end{matrix}\right\} B$$

Col 2

Now we need to get the write combination to get the correct result $\begin{bmatrix} 0 \\ 3 \end{bmatrix}$

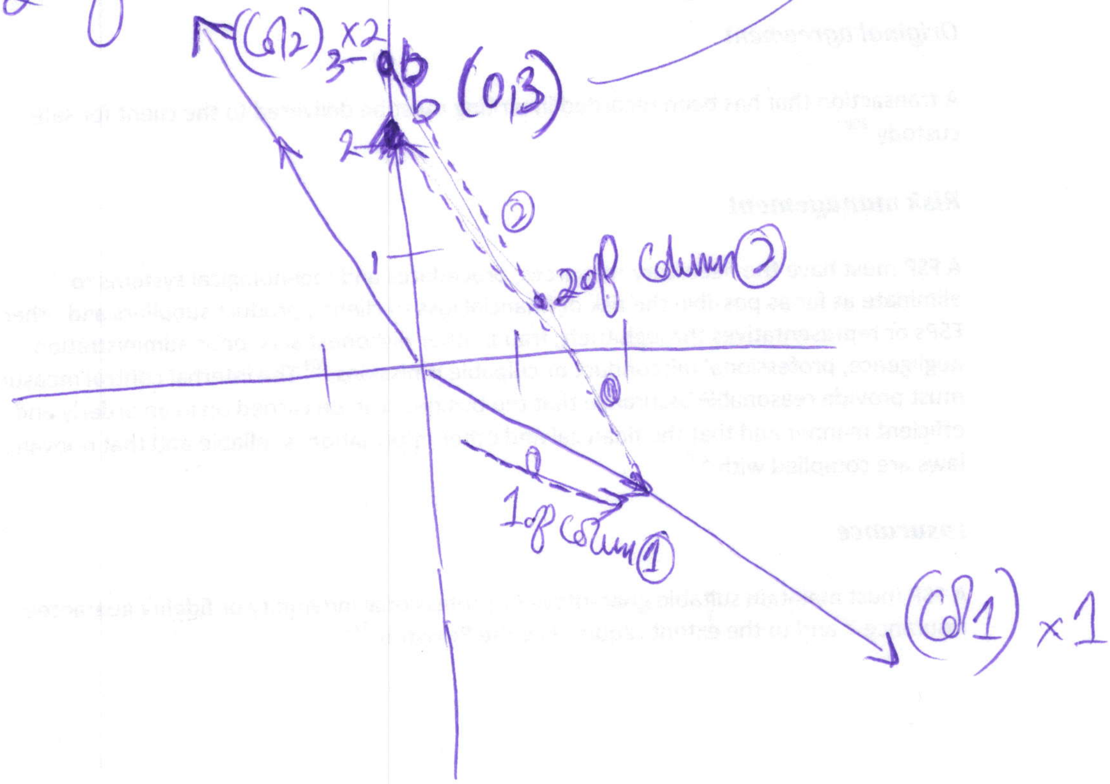
$x = 1,\ y = 2 \longrightarrow$ we know this correct combination per [illegible]

$$x \begin{bmatrix} 2 \\ -1 \end{bmatrix} + y \begin{bmatrix} -1 \\ 2 \end{bmatrix} = \begin{bmatrix} 0 \\ 3 \end{bmatrix}$$

$$1 \begin{bmatrix} 2 \\ -1 \end{bmatrix} + 2 \begin{bmatrix} -1 \\ 2 \end{bmatrix} = \begin{bmatrix} 0 \\ 3 \end{bmatrix}$$

1 of Column 1

and 2 of Col 2



But what are all Combinations?

for x and y, so for all Combinations it will fill the whole plane $\longrightarrow$ (Come Back)

Lets move to 3 equations, 3 unknowns! $\longrightarrow$

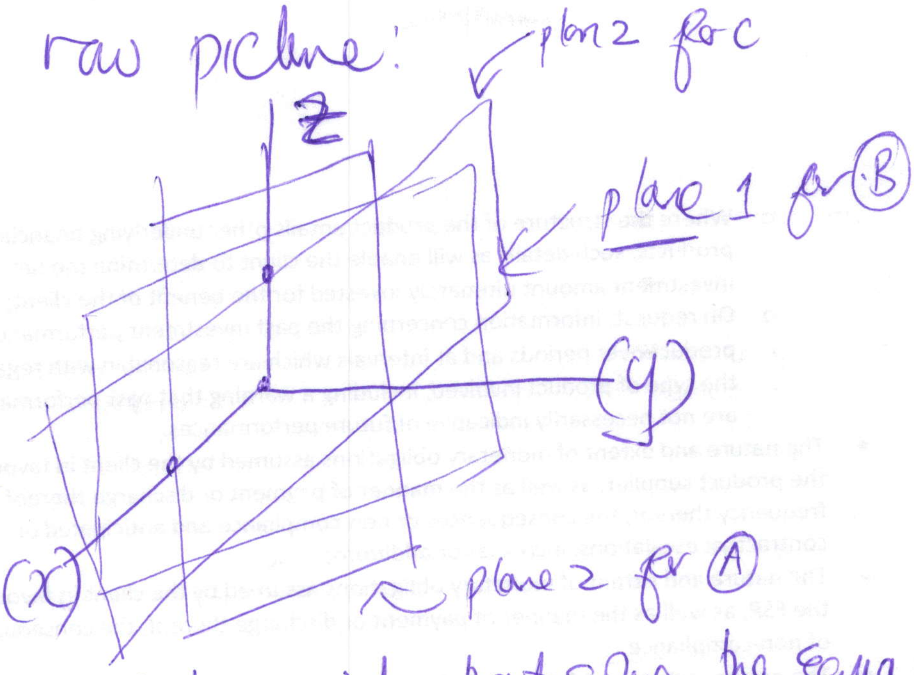
$$2x - y = 0$$
$$-x + 2y - z = -1$$
$$-3y + 4z = 4$$

Lets do matrix form:

$$\begin{array}{c} A \\ B \\ C \end{array} \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -3 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ -1 \\ 4 \end{bmatrix}$$

$$A \qquad x = b$$

Lets do raw picture:



(A) Need to get all the point that solve the equation.

∴ all the point that will satisfy the equation will be a PLANE

for ~~(equation)~~ each equation

∴ from the above (with all planes drawn) the raw picture becomes hard to see ⇒

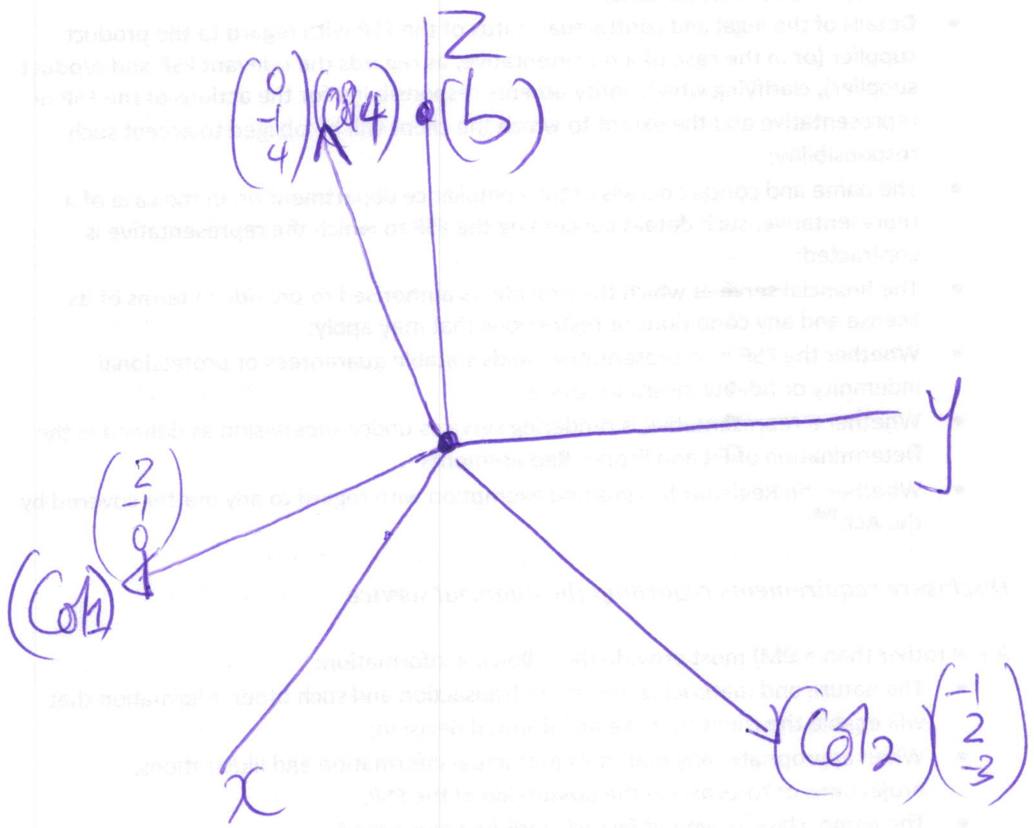
∴ Lets do Column Picture (Preferred)

Column Picture:

$$x\begin{bmatrix}2\\-1\\0\end{bmatrix}+y\begin{bmatrix}-1\\2\\-3\end{bmatrix}+z\begin{bmatrix}0\\-1\\4\end{bmatrix}=\begin{bmatrix}0\\-1\\4\end{bmatrix}$$

$$\begin{array}{ccc}\text{Col 1} & \text{Col 2} & \text{Col 3}\end{array}$$

∴ What combination of vectors on left will produce vector on right.



What combinations of coordinates do we use.

Cleverly selected → $\begin{bmatrix}0\\-1\\4\end{bmatrix}=\begin{bmatrix}0\\-1\\4\end{bmatrix}$

∴ other 2 vectors need to be $\emptyset$.

$$x\begin{bmatrix}2\\-1\\0\end{bmatrix}+y\begin{bmatrix}-1\\2\\-3\end{bmatrix}+z\begin{bmatrix}0\\-1\\4\end{bmatrix}=\begin{bmatrix}0\\-1\\4\end{bmatrix}$$

$x=0, \; y=0, \; z=1$

$\therefore \quad b=(z)$

But Next Lecture, we use elimination to solve this better!

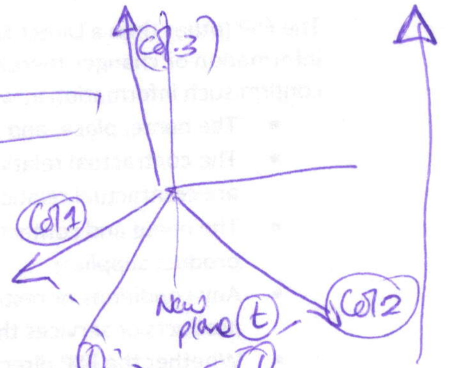
Lets look at bigger picture; Keep matrix on left same, and change the one on right!

$$x\underset{A}{\begin{bmatrix}2\\-1\\0\end{bmatrix}}\overset{\text{Same}}{+}y\underset{B}{\begin{bmatrix}-1\\2\\-3\end{bmatrix}}+z\underset{C}{\begin{bmatrix}0\\-1\\4\end{bmatrix}}=\begin{bmatrix}1\\1\\-3\end{bmatrix}\,(t)$$

how get this? adding A + B

$2-1=1$
$-1+2=1$
$0-3=-3$

$\therefore x=1 \quad y=1 \quad z=0$



How did I get $x=1, y=1$, since took 1 of x, and 1 of y, and 0 of x

Now can I solve $Ax = b$ for every b? (right hand side)

$\therefore$ Do the linear combinations of the columns fill 3-D space?

For this A, answer yes!

otherwise may not be true,

where plane coordinates lie in same plane!

eg eg Col(A) and Col(B) lie in same plane, so we get nothing more from it.

Matrix form of equation:

$$Ax = b$$

$A \to$ Matrix, $b \to$ Vector

How do we multiply a matrix by vector

$$\begin{array}{cc} \text{col 1} & \text{col 2} \\ \end{array}$$
$$\begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

Two ways to do it:

technique①

① take a column at a time

take: 1 (vector) of col 1
2 of col 2

then $(1\times2)+(2\times5)=12$
$(1\times1)+(2\times3)=7$

$$\begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} = 1\begin{bmatrix} 2 \\ 1 \end{bmatrix} + 2\begin{bmatrix} 5 \\ 3 \end{bmatrix} = \begin{bmatrix} 12 \\ 7 \end{bmatrix}$$

technique ②

- Take raw at time

$$
\begin{array}{c}
\text{raw1} \\
\text{raw2}
\end{array}
\begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix}
\begin{bmatrix} 1 \\ 2 \end{bmatrix}
=
\begin{bmatrix} 12 \\ 7 \end{bmatrix}
$$

raw1 $(2 \times 1) + (5 \times 2) = 12$

$(1 \times 1) + (3 \times 2) = 7$

Next System of Elimination