

Rec

①

Calculate Concentration Powers of Matrix

Find a formula for the k th

Power of matrix C

2×2 matrix, that depends
on variable a and b

2nd Part, Calculate C^{100} , where a and $b = -1$

What's first step in finding power of matrix?

— we need ~~the~~ to find ~~power of~~
 $E'V$ and $E'V$ of this matrix

How do we do that?

$$\det(C - \lambda I) = \det \begin{pmatrix} (2b-a) - \lambda & a-b \\ 2b-2a & (2a-b) - \lambda \end{pmatrix}$$

$$= \lambda^2 - (a+b)\lambda + ab = (\lambda - a)(\lambda - b)$$

factor b:2

$\therefore \lambda$ values a, b

Now find E' vectors:

→ now find null space for matrix.

$$C - aI = \begin{pmatrix} 2b-2a & a-b \\ 2b-2a & a-b \end{pmatrix} \quad (A)$$

∴ has same columns / same rows.

$$V_1 = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

$$C - bI = \begin{pmatrix} b-a & a-b \\ 2b-2a & 2a-2b \end{pmatrix}$$

$$V_2 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

Now we can take powers of it.

$$C = S \Lambda S^{-1} = \begin{pmatrix} 1 & 1 \\ 2 & 1 \end{pmatrix} \begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix} \begin{pmatrix} -1 & 1 \\ +2 & -1 \end{pmatrix}$$

$$C^k = S \Lambda^k S^{-1} = \begin{pmatrix} 1 & 1 \\ 2 & 1 \end{pmatrix} \begin{pmatrix} a^k & 0 \\ 0 & b^k \end{pmatrix} \begin{pmatrix} -1 & 1 \\ 2 & -1 \end{pmatrix}$$

$$= \begin{pmatrix} a^k & b^k \\ 2a^k & b^k \end{pmatrix} \begin{pmatrix} -1 & 1 \\ 2 & -1 \end{pmatrix} = \begin{pmatrix} 2b^k - a^k & a^k - b^k \\ 2b^k - 2a^k & 2a^k - b^k \end{pmatrix} \quad (3)$$

Let's check, plug in $k=1$

→ agrees with what we started with

(A)

$$a=b=-1, k=100$$

$$C^{100} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = I$$