

Qec Basis & Dimension

①

Find the dimension of the vector space spanned by the vectors

$$(1, 1, 2, 0, -1)$$

$$(1, 2, 0, 4, 1)$$

$$(0, 1, 3, -3, 2)$$

$$(2, 3, 0, -2, 0)$$

And find the Basis for that space.

∴ What's the Dimension:

— Number of vectors in Vector space.

So Problem is Backward

- 1) First Find Basis
- 2) Then find Dimension

(2)

So we need to find Basis that span
~~the~~ 4 vectors.

- So we may just want to say the basis for vector space is those 4 vectors, since they span the vector space.
- But there is another thing a basis has to satisfy?
- Elements of basis, has to be linearly independent.
- how do we check:
- put vectors as rows of matrix, then do elimination, then rows that have pivots, are independent ones.

$$\begin{bmatrix} 1 & 1 & -2 & 0 & -1 \\ 1 & 2 & 0 & -4 & 1 \\ 0 & 1 & 3 & -3 & 2 \\ 2 & 3 & 0 & -2 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 1 & -2 & 0 & -1 \\ 0 & 1 & 2 & -4 & 2 \\ 0 & 1 & 3 & -3 & 2 \\ 0 & 1 & 4 & -2 & 2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & -2 & 0 & -1 \\ 0 & 1 & 2 & -4 & 2 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 2 & 2 & 0 \end{bmatrix}$$

$$\begin{bmatrix} \textcircled{1} & 1 & -2 & 0 & -1 \\ 0 & \textcircled{1} & 2 & -4 & 2 \\ 0 & 0 & \textcircled{1} & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Next step
 has row disappear
 linearly independent ← 3 pivots

∴ Elements for Basis are rows with pivot. (3)

$$\text{Basis: } (1, 1, -2, 0, -1)$$

$$(0, 1, 2, -4, 2)$$

$$(0, 0, 1, 1, 0)$$

∴ What is the dimension?

$$\text{Dim} = 3$$

Another way to Solve? Couple of ways to find out which vectors are linearly indep.

— to write the initial vectors as columns of matrix, then perform elimination. That will work, as we will be working with transpose of matrix (row based)

∴ Same matrix, But written as Columns:

$$\begin{bmatrix} 1 & 1 & 0 & 2 \\ 1 & 2 & 1 & 3 \\ -2 & 0 & -3 & 0 \\ 0 & -4 & -3 & -2 \\ -1 & 1 & 2 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} \textcircled{1} & 1 & 0 & 2 \\ 0 & \textcircled{1} & 1 & 1 \\ 0 & 0 & \textcircled{1} & 2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

↑ ↑ ↑
Use these
3 columns as basis

x x x
1 1 1
— Not Use these
Columns as Basis