

Additional: (extra)

0

Null Space:

Let A be an $m \times n$ matrix

The null space is the set of solutions to the homogeneous system $Ax=0$

Another way of saying this: $\xrightarrow{\text{set of vectors}}$

$$\underset{\substack{\uparrow \\ \text{null space of } A}}{N(A)} = \left\{ \underset{\substack{\uparrow \text{ so that}}}{x} \in \mathbb{R}^n : Ax=0 \right\}$$

Let's look at quick example:

$$x_1 + x_2 + x_3 + 0x_4 = 0$$

$$2x_1 + x_2 + 0x_3 + x_4 = 0$$

Let's find the null space.

for matrix A, we just take the coefficients: ②

$$A = \begin{bmatrix} 1 & 1 & 1 & 0 \\ 2 & 1 & 0 & 1 \end{bmatrix}$$

∴ we want to find the Null Space of that matrix A.

∴ Find $N(A)$

∴ we want to figure out the form of vectors, so that when we substitute them in [Equation] and do the arithmetic we get $= 0$.
(in Both of our Equations.)

So to do this, we first do some row reduction:

$$\begin{bmatrix} \boxed{1} & 1 & 1 & 0 & | & 0 \\ 2 & 1 & 0 & 1 & | & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 1 & 1 & 0 & | & 0 \\ 0 & \boxed{-1} & -2 & 1 & | & 0 \end{bmatrix}$$

Need one

$$-2R_1 + R_2 \Rightarrow R_2$$

$$\begin{bmatrix} 1 & 1 & 1 & 0 & | & 0 \\ 0 & -1 & -2 & 1 & | & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & -1 & 1 & | & 0 \\ 0 & -1 & -2 & 1 & | & 0 \end{bmatrix} \quad (3)$$

$$R_2 + R_1 \Rightarrow R_2$$

$$\begin{bmatrix} 1 & 0 & -1 & 1 & | & 0 \\ 0 & -1 & -2 & 1 & | & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & -1 & 1 & | & 0 \\ 0 & 1 & 2 & -1 & | & 0 \end{bmatrix}$$

↑ reduced echelon form.

$$\therefore -1R_2 \Rightarrow R_2$$

\therefore rewrite as system of equations.

$$\therefore 1x_1 + 0 - x_3 + 1x_4 = 0 \rightarrow x_1 = x_3 - x_4$$

$$0 + 2x_2 + 2x_3 - 1x_4 = 0$$

Here we have couple of free variables. $\hookrightarrow x_2 = -2x_3 + x_4$

\therefore so far we have done row reduction to find null space.

④

$$\text{let } x_3 = \alpha$$

$$x_4 = \beta$$

\therefore Now we have found the form vectors that comprised the null space.

$$X = \begin{bmatrix} \alpha - \beta \\ -2\alpha + \beta \\ \alpha \\ \beta \end{bmatrix} \begin{matrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{matrix}$$

$$\therefore x_1 = x_3 - x_4$$

we said $x_3 = \alpha$
 $x_4 = \beta$

$$\therefore x_2 = -2x_3 + x_4 \\ = -2\alpha + \beta$$

$$\therefore \alpha, \beta \in \mathbb{R}$$

we can Break it up:

eg factor α out:

$$x = \alpha \begin{bmatrix} 1 \\ -2 \\ 0 \end{bmatrix} + \beta \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix}$$

\therefore Vectors of this Form will be vectors that make up the Null Space.

\therefore Now you can pick a number for α or β

$$\therefore \alpha = 1, \beta = 1$$

$$\bar{V} = 1 \begin{bmatrix} 1 \\ -2 \\ 0 \end{bmatrix} + 1 \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix}$$

$$\left. \begin{array}{l} x_1 = 0 \\ x_2 = -1 \\ x_3 = 1 \\ x_4 = 1 \end{array} \right\}$$

plug back into original equation

\therefore must get 0

in First and 2nd Equation.

Note:

This is the idea of Null Space

⑥

∴ What \vec{v} can we substitute into
the original system of equations

so we get zero.

and to get it we just do \rightarrow row reduction.

Review:

①

1) Row Reduction

2) Row-Echelon Form

3) Reduced Row Echelon Form. (RREF) [Best]

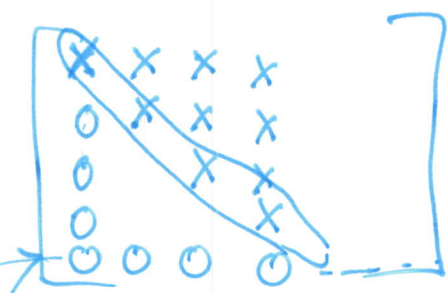
Rules of game:

1) Rescale a row

2) Add multiple of one row to another

3) Swap 2 rows.

Goal: Put matrix in REF or RREF

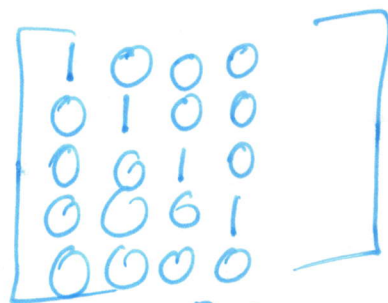


REF

$x \in \mathbb{R}$, where $x \neq 0$ or 1 .

only last row all zero

But diagonal you
can also skip, two pivots
apart.



RREF

Diagonal 1's and
zero below and top
of diagonal

\therefore rescale pivots
to make it 1.