factorization into LU →

So we got inverse of A = A^{-1}

But if I multiply two matrices together

AB what is its inverse →

$$A \cdot A^{-1} = I = A^{-1}A$$

formula $A = LU$ (No row exchanges)

AB in reverse order.

$$(AB)\,B^{-1}A^{-1} = I$$ → will give the identity.

or $$B^{-1}A^{-1}(AB) = I$$

Ok lets do 2x2 (first)

$A = LU$ the most basic factorization of a matrix

Addition

(2.1) (3)

$$\begin{array}{c} A \\ \begin{bmatrix} 2 & 1 \\ 8 & \boxed{4} \end{bmatrix} \end{array} \qquad \text{(A)} \begin{array}{c} U \\ \begin{bmatrix} 2 & 1 \\ 0 & \boxed{0} \end{bmatrix} \end{array}$$

Do not want this 4 here

Singular, But pivot cannot be zero

to get 8 as zero and have 2 as pivot.

$-4R_1 + R_2 \Rightarrow R_2$

multiplier

U $\Rightarrow$ Leave top Row

(A) $-4(2)+8 \Rightarrow$

$-8+8=0$

$-4(1)+4 \Rightarrow$

$-4+4=0$

So we make it:

$$\begin{bmatrix} 2 & 1 \\ 8 & 7 \end{bmatrix} \qquad \begin{array}{c} U \\ \begin{bmatrix} 2 & 1 \\ 0 & 3. \end{bmatrix} \end{array}$$

$-4R_1+R_2 \Rightarrow R_2$

$-4(2)+8 \Rightarrow$

$-8+8 \Rightarrow 0$

$-4(1)+7$

$-4+7=3$

2,2

$$= \cancel{\begin{bmatrix}1 & 0\\ & \end{bmatrix}\begin{bmatrix} & \\ & \end{bmatrix}}$$

Inverse

E_{21}

$$\begin{bmatrix}1 & 0\\ -4 & 1\end{bmatrix} = \begin{bmatrix}2 & 1\\ 8 & 7\end{bmatrix}\begin{bmatrix}2 & 1\\ 0 & 3\end{bmatrix}$$

2x2 matrix:

A

$$\begin{bmatrix} 2 & 1 \\ 8 & 7 \end{bmatrix}$$

First pivot.

$$\begin{bmatrix} \boxed{2} & 1 \\ 8 & 4 \end{bmatrix} \qquad A \begin{bmatrix} 2 & 1 \\ 0 & 0 \end{bmatrix}$$

matrix will be singular $\Rightarrow$ or pun

$-4R_1 + R_2 \Rightarrow R_2$

multiplier

$A \Rightarrow (-4 \times 2) + 8 \Rightarrow$

$-8 + 8 = 0$

$\Rightarrow (-4 \times 1) + 4 \Rightarrow$

$-4 + 4 =$

(2x2) Matrix

②

$$\underset{E_{21}}{\begin{bmatrix} 1 & 0 \\ -4 & 1 \end{bmatrix}} \underset{A}{\begin{bmatrix} 2 & 1 \\ 8 & 7 \end{bmatrix}} = \underset{U}{\begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix}}$$

Should not put 4 there as 2nd pivot would not exist

$$\underset{A}{\begin{bmatrix} 2 & 1 \\ 8 & 7 \end{bmatrix}} = \underset{L\,(?)}{\begin{bmatrix} 1 & 0 \\ 4 & 1 \end{bmatrix}} \underset{U}{\begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix}}$$

Inverse of $E = \begin{bmatrix} 1 & 0 \\ 4 & 1 \end{bmatrix}$ (this is L)

U = Upper triangular

L = Lower triangular

How do we do Elimination? where we place zero's

3rd 2nd 1st

$$E_{32}E_{31}E_{21}A = U \quad \text{(No row exchange)}$$

what's been done

$$A = E_{21}^{-1}E_{31}^{-1}E_{32}^{-1}\,U$$

$$= LU$$

$$\boxed{A = LU}$$

If no row exchanges, multipliers go directly into L

Question: on an $n \times n$ matrix how many operations can we do.

eg 100×100

Count

$$n^2 + n^3 \dots + 1^2$$

$\approx$ Cost of n

$$= \tfrac{1}{3} n^3 \; n$$

Permutation (3X3) 6P's

$$\begin{bmatrix} 1 & & \\ & 1 & \\ & & 1 \end{bmatrix}, \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

If we multiply two of them together, what will happen?

$\Rightarrow$ will get answer in group.

$P^{-1} = P^T \Rightarrow$ inverse is its transpose.

for 4×4 = ? P's

= 24 P's