→ and I would like to know the determinant.

of upper triangular matrix.

∴ product of d's.

This is how mathLab will compute the determinant.

Also product of pivots, if there was no row exchange.

If there was, then we need to watch signs.

→ go to (row exchanges)

Property 8 $\det A = 0$

Exactly when A is singular

→ go to $U \to d_1, d_2 \ldots d_n$

∴ $\det A \neq 0$

when A is invertible

∴ matrix is invertible when elimination produces a full set of pivots.

→ now we know the determinant is product of those non-zero #s

Lets check:

What's pivot of 2x2 matrix

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \Rightarrow \begin{bmatrix} a & b \\ 0 & d-\frac{c}{a}b \end{bmatrix}$$

∴ what does elimination do, to the 2x2 matrix.

∴ @ is 1st pivot

∴ $ad-bc \Rightarrow$ determinant

So far properties listed, are connected with our elimination process.

Now Properties 9 and 10 product by multiplying two matrices.

Property 9 $\det AB = (\det A)(\det B)$

~~But we so far have not multiplying matrices.~~

∴ what $\det A^{-1}$ [using new property 9]

→ we know $A^{-1}A = I$

∴ So take the det of Both side:

∴ $\det I = 1$

and

and what the determinant of

$A^{-1}A$?

∴ its product of two matrices (A&B)

its the product of the determinant...

$$(\det A^{-1})(\det A) = 1$$

$$\det A^{-1} = 1/\det A$$

How do we check?

⇒ we know $A = \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}$ A

∴ $A^{-1} = \begin{bmatrix} 1/2 & 0 \\ 0 & 1/3 \end{bmatrix}$ A ✓

property 9 ⇒ for diagonal matrices (works)

What is the determinant of A^2. → $\det A^2 = (\det A)^2$

det gets squared

and if I double the matrix:

$$\det 2A = 2^{?} \det A$$

try why 2^n and not just 2.

— I am factoring ~~xx~~ out 2 from every row.

∴ factor out 2 from 1st row

then " factor out 2 from 2nd row

" " 2 from 3rd row ....

∴ all those 2's coming out

Property (10) $\det A^T = \det A$.

Check $\begin{vmatrix} a & b \\ c & d \end{vmatrix} = \begin{vmatrix} a & c \\ b & d \end{vmatrix}$ 'transpose'

∴ transposing did not change the determinant

∴ remember if rows are all zero's, $\det = 0$.

But now with rule (10), what if column is all zero's, what det. $= 0$

∴ all those properties about rows.

i.e. changing two rows, reverse signs

, now exchanging two column reverses the signs

∴ nothing special about row 1,
now here ⇒ nothing special about rows, that's equally true for columns, qed

Lets do quick proof for No 10: by using 1 to 9 rules/properties

$|A^T| = |A|$ ∴ $\det A^T = \det A$.

$|U^T L^T| = |LU|$ prove these two are the same.

see product here, so use rule 9

$|U^T||L^T| = |L||U|$ ✓

whats det of L = 1

∴ triangular = 1.