Rec

We now are becoming familiar with the geometric interpretation of determinants.

ABS $|\det|$ = Volume of parallelopiped "Box" span by the row vectors of matrix.

- we will apply this fact to solve the following problem:

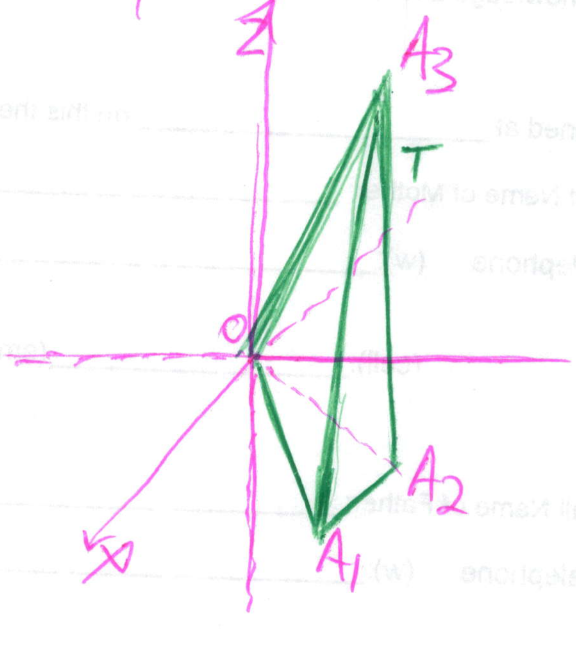
We have a tetrahedron T with vectors (in this 3d space)

(origin) $O\,(0,0,0)$

$A_1\,(2,2,-1)$

$A_2\,(1,3,0)$

$A_3\,(-1,1,4)$



Compute Vol(T) [using the determinant]

Q3 A_1, A_2 are fixed, but A_3 is moved to $A_3'(-201, -199, 104)$

∴ Compute Vol(T) again

Since we want to use the fact that the det is related to the volume, we have to figure out which volume we need to look at.

We know det is related to the volume of a parallelopiped. But here we have a tetrahedron?

∴ we need to find out <u>WHICH</u> parallelopiped we should be working with.

3

- Lets Complete the picture.

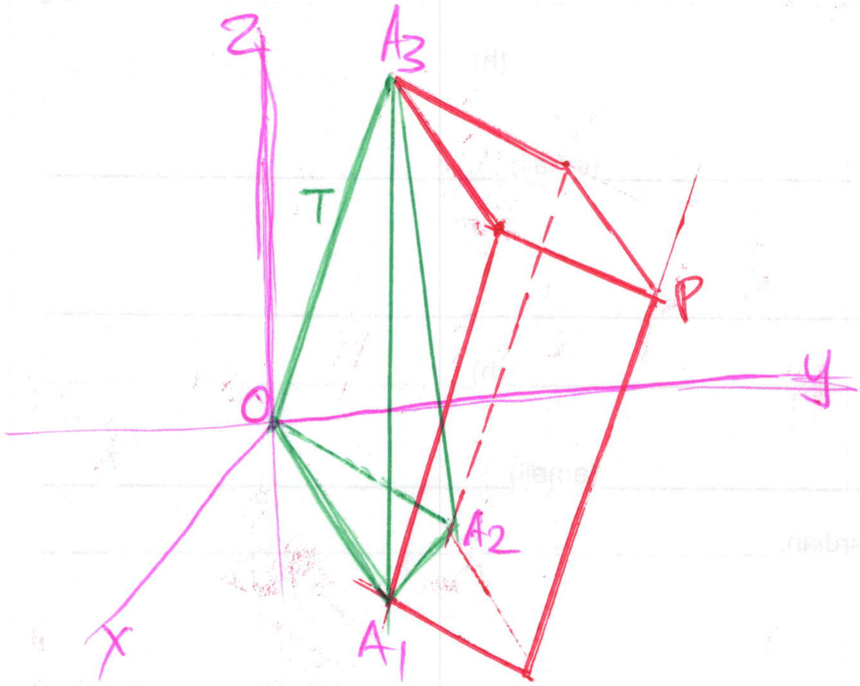
- we need a parallelopiped, so we can use the fact that the det is related to the volume.

- But here we have a tetrahedron.

Lets look at the following 3 edges

$O - A_1$
$O - A_2$
$O - A_3$
} all of them meet at the origin.

- why don't we just consider the parallelopiped span by the same 3 edges.



4

$\therefore$ Green is the original T

$\therefore$ Red is p. (parallelopiped) — will work with

- Now relate the volume of ~~PT~~ T to volume P

- What is volume of tetrahedron.

$$Vol(T) = \frac{1}{3} B \times h$$

$\therefore$ Can choose any side/face to be the base, for convenience we will choose triangle $O - A_1 - A_2$ to be the base.

$$Vol(T) = \frac{1}{3} A(\Delta OA_1A_2) \cdot h \rightarrow (A3)$$

to indicate Area.

$\Rightarrow$ Now lets see what volume of P is:

Choose parallelogram to be Base

$\downarrow$

two copies of $\Delta O A_1 A_2$

$$Vol(P) = \text{Area base} \times \text{height}$$
$$= 2A(\Delta OA_1A_2) \cdot h$$

~~$\therefore Vol(B)$~~

$$\therefore Vol(T) = \frac{1}{6} Vol(P)$$

only need to compute volume of P.

Vol (P) = related to det of Matrix

∴ row vectors of matrix is given by

these 3 edges (A_1, A_2 & A_3)

, and all of them start at zero

we only need coordinates of (A_1, A_2, A_3)

Absolute value

$$Vol(P) = \left| \det \begin{pmatrix} 2 & 2 & -1 \\ 1 & 3 & 0 \\ -1 & 1 & 4 \end{pmatrix} \right|$$

⇒ lets compute 3x3 matrix

$= 12.$

$Vol(T) = 12/6 = 2$

Now lets look at second Part:

$A_3 \xrightarrow{\text{New Point}} A_3'(-201, -199, 104)$

∴ point is far away from origin.

∴ follow same idea as above:

$$Vol(T') = \frac{1}{6} \left| \det \begin{pmatrix} 2 & 2 & -1 \\ 1 & 3 & 0 \\ -201 & 199 & 104 \end{pmatrix} \right|$$

→ difference

← $A' - A_3 = -100 A_1$

$= 2.$