Re

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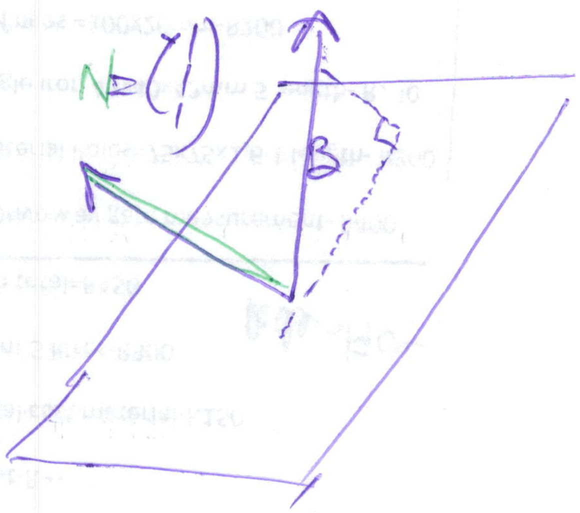
we going to compute projection matrix onto the plane given:

$$x + y - z = 0$$



∴ Recall what a projection matrix is:

→ in free space (3Dim)

takes any vector and project it down onto plane [3Dim subspace]

∴ Formula for the Projection Matrix:

$$P = A(A^TA)^{-1}A^T$$

where A is a matrix, that somehow encodes the subspace we projecting on.

$\therefore A = \begin{pmatrix} | & | \\ a_1 & a_2 \\ | & | \end{pmatrix}$ has column a_1, a_2, a basis for the plane we projecting on.

$\therefore$ So we need to find two such vectors that span the plane, (and compute matrix)

$\therefore a_1 = \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix} \quad a_2 = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$

$A = \begin{pmatrix} 1 & 1 \\ -1 & 0 \\ 0 & 1 \end{pmatrix}$ 3x2 matrix.

$\therefore A^T A = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$

Lets invert it:

1) switch diagonal entries
2) ~~sw~~ flip signs

$(A^T A)^{-1} = \frac{1}{3} \begin{pmatrix} 2 & -1 \\ -1 & 2 \end{pmatrix}$

$P = \begin{pmatrix} 1 & 1 \\ -1 & 0 \\ 0 & 1 \end{pmatrix} \frac{1}{3} \begin{pmatrix} 2 & -1 \\ -1 & 2 \end{pmatrix} \begin{pmatrix} 1 & -1 & 0 \\ 1 & 0 & 1 \end{pmatrix}$

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$$P = \frac{1}{3}\begin{pmatrix} 2 & -1 & 1 \\ -1 & 2 & 1 \\ 1 & 1 & 2 \end{pmatrix}$$

How to check if answer makes sense

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Lets look for an easier approach:

- Lets look at graph again.

any vector is sum of 2 components

- 1st comp is projection onto plane
- 2nd comp is othological component of plane i.e onto normal vector

in language of linear algebra:

$b \triangleq$ [illegible]

Projection onto plane

othonlognal component of plane.

$$I b = P b + P_n b$$

$$I = P + P_n$$

$$\Rightarrow P = I - P_n$$ ⟸ much easer to compute

$$P_N = N(N^T N)^{-1} N^T$$

$$= \begin{pmatrix}1\\1\\-1\end{pmatrix} \frac{1}{3} \begin{pmatrix}1 & 1 & -1\end{pmatrix}$$

$$P_n = \frac{1}{3}\begin{pmatrix}1 & 1 & -1\\1 & 1 & -1\\-1 & -1 & 1\end{pmatrix}$$

$$P = I - P_n$$