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look at Diagonalizing a Complex matrix

given $A \begin{pmatrix} 2 & 1-i \\ 1+i & 3 \end{pmatrix} = \bar{A}^T = A^H$

find its E'~~vector~~ value Matrix Λ and

E'vector matrix S.

if take $\bar{A}^T$ Conjugate transpose it equals to itself A^H

$\longrightarrow$

$\det(A - \lambda I) = 0$

Note Entries of A is complex

$\det\begin{pmatrix} 2-\lambda & 1-i \\ 1+i & 3-\lambda \end{pmatrix} = 0$

$(2-\lambda)(3-\lambda) - (1+i)(1-i) = 0$

$6 - 5\lambda + \lambda^2 - 2 = 0$

$\lambda^2 - 5\lambda + 4 = 0 \implies (\lambda-1)(\lambda-4) = 0$

$\lambda = 1, 4$

E'values are real

But Started out matrix are Complex

general Property of H matrix

Hermitian matrices always have real e'values

Find e'vectors:

$\lambda = 1$:

$(A - \lambda I)v = 0$

$$\begin{pmatrix} 1 & 1-i \\ 1+i & 2 \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = 0$$

$v_1 = (1-i)$

$v_2 = -1$

$$v = \begin{pmatrix} 1-i \\ -1 \end{pmatrix}$$

$\lambda = 4$

$(A - \lambda I v) = 0$

$$\begin{pmatrix} -2 & 1-i \\ 1+i & -1 \end{pmatrix} \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} = 0$$

$$u = \begin{pmatrix} 1 \\ 1+i \end{pmatrix}$$

$$\Lambda = \begin{pmatrix} 1 & 0 \\ 0 & 4 \end{pmatrix}$$

$$S = \frac{1}{\sqrt{3}} \begin{pmatrix} 1-i & 1 \\ -1 & 1+i \end{pmatrix}$$

$$S^{-1} = \bar{S}^{T}$$

③

$$A = S\Lambda S^{-1} = S\Lambda \bar{S}^{T}$$

$$A = \frac{1}{\sqrt{3}}\begin{pmatrix} 1-i & 1 \\ -1 & 1+i \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & 4 \end{pmatrix}\frac{1}{\sqrt{3}}\begin{pmatrix} 1+i & -1 \\ 1 & \cancel{i+}1-i \end{pmatrix}$$

$$\longrightarrow A.$$