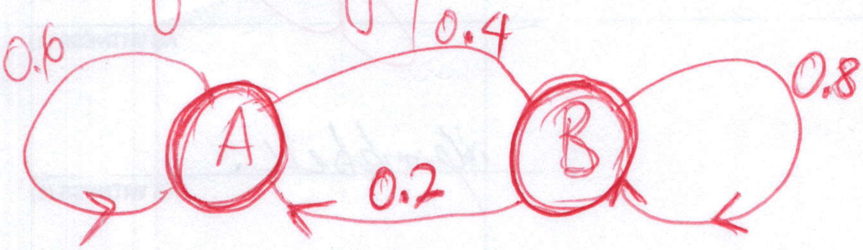
Markov Matrix:

Have particle that jumps between position A and B.
with following probabilities:



If it start at A, what is the probability it is at A and B after

i) 1 step ii) n steps iii) ∞ - steps

Convert to a matrix form: particle position A, B

$$A = \begin{array}{cc} \begin{array}{cc} A & B \\ \downarrow & \downarrow \end{array} & \\ \begin{pmatrix} 0.6 & 0.2 \\ 0.4 & 0.8 \end{pmatrix} & \begin{array}{l} \leftarrow A \\ \leftarrow B \end{array} \end{array}$$

Markov matrix:
- every element is +ve
- sum of each column is 1

These matrices come up all the time when we talk about probabilities

Introduce vector p

$p_0 \rightarrow$ time 0.

$$p_0 = \begin{array}{c} \begin{pmatrix}1\\0\end{pmatrix} \end{array} \begin{array}{l} \leftarrow A \\ \leftarrow B \end{array}$$

(i) 1 step (what's the probability going to be)

(after 1 step) $p_1 = Ap_0 = \begin{pmatrix}0.6\\0.4\end{pmatrix}$

(ii) n step

$p_1 = Ap_0,\ p_2 = Ap_1 = A^2 p_0$

∴ ~~general~~ general trend, after n steps.

$p_n = A^n p_0$ ∴ how to take n^{th} power matrix.

∴ this is where you use e'vc & e'vl

Recall $A = UDU^{-1}$ matrix whose column correspond to e'vectors of A.

diagonal Matrix

$\therefore$ Because Markove matrix: $\lambda = 1$.

$$\lambda = 1, \quad u_1 = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

$$\lambda = 0.4, \quad u_2 = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

$$U = \begin{pmatrix} 1 & 1 \\ 2 & -1 \end{pmatrix}$$

$$\Lambda = \begin{pmatrix} 1 & 0 \\ 0 & 0.4 \end{pmatrix}$$

$$U^{-1} = -\frac{1}{3}\begin{pmatrix} -1 & -1 \\ -2 & 1 \end{pmatrix} = \frac{1}{3}\begin{pmatrix} 1 & 1 \\ 2 & -1 \end{pmatrix}$$

$$P_n = A^n p_0 = U \Lambda^n U^{-1} p_0$$

$$= \begin{pmatrix} 1 & 1 \\ 2 & -1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & 0.4^n \end{pmatrix} \frac{1}{3}\begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

$$= \frac{1}{3}\begin{pmatrix} 2(0.4)^n + 1 \\ -2(0.4)^n + 2 \end{pmatrix}$$

iii) $$p_\infty = \frac{1}{3}\begin{pmatrix} 1 \\ 2 \end{pmatrix}$$