Recap: $Ax = b$. → for non-homogeneous system

Finding solutions, depending on b_1, b_2, b_3:

$$x - 2y - 2z = b_1$$
$$2x - 5y - 4z = b_2$$
$$4x - 9y - 8z = b_3$$

System not only has numbers & unknowns; it also has parameters, $b_{1,2,3}$

⇒ the solution will depend on these parameters.

∴ find a solution, and when it exist — depending on values b_1, b_2, b_3

⇒ lets solve the system, as if b_1, b_2, b_3 were #'s

$$\left[\begin{array}{ccc|c} 1 & -2 & -2 & b_1 \\ 2 & -5 & -4 & b_2 \\ 4 & -9 & -8 & b_3 \end{array}\right] \to \left[\begin{array}{ccc|c} 1 & -2 & -2 & b_1 \\ 0 & -1 & 0 & -2b_1+b_2 \\ 0 & -1 & 0 & -4b_1+b_3 \end{array}\right]$$

⇒ using elimination ⇒

$$\left[\begin{array}{ccc|c} 1 & -2 & -2 & b_1 \\ 0 & -1 & 0 & -2b_1+b_2 \\ 0 & 0 & 0 & -b_2-2b_1+b_3 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & -2 & -2 & b_1 \\ 0 & 1 & 0 & -2b_1+b_2 \\ 0 & 0 & 0 & -2b_1-b_2+b_3 \end{array}\right]$$

$\therefore$ If $-2b_1-b_2+b_3 \neq 0$

$\Rightarrow$ No Solutions

However: $-2b_1-b_2+b_3=0$

$\therefore$ let make 1, 1

also changes

$$\left[\begin{array}{ccc|c} 1 & 0 & -2 & 5b_1-2b_2 \\ 0 & 1 & 0 & 2b_1-b_2 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

Became zero

$(x, y) \rightarrow$ Pivots free variable (z)

reason why we did this, is to get Identity matrix here

$\therefore$ Now we going to calculate the solution(s), by picking particular values for z, then calculating the actual values for x and y.

Two kinds of solution:

*1) Particular solution

(solves) $Ax = b$

$\therefore$ by setting $z = 0$

$$\Rightarrow x_p = \begin{bmatrix} 5b_1 - 2b_2 \\ 2b_1 - b_2 \\ 0 \end{bmatrix}$$

2) Special solution

solves $Ax = 0$

$\therefore$ by setting $z = 1$.

$$x_s = \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}$$

$\therefore$ <u>All solution:</u>

$$\vec{x} = \vec{x}_p + \underbrace{c \cdot \vec{x}_s}_{\text{any multiple}}$$