Row Elimination of Matrices

Solve following, using method of Elimination.

4 equations, and 4 unknown.

$$x - y - z + 4 = 0$$
$$2x + 2z = 8$$
$$-y - 2z = -8$$
$$3x - 3y - 2z + 4 = 7$$

∴ replace the above equation, with Equivalent solution by series of row operations.

∴ row operation does not change the solution to system

"$\Rightarrow$", change order of rows

) multiply equation with non-zero number.

) add non-zero multiple of one equation to other.

So when we do operations:

- lets hide name of variables (x, y, z, w)
  or " equality signs (=)

But we keep the important information:

- Coefficients in matrix

right side, augmented matrix

$$\left[\begin{array}{cccc|c} \boxed{1} & -1 & -1 & 1 & 0 \\ 2 & 0 & 2 & 0 & 8 \\ 0 & -1 & -2 & 0 & -8 \\ 3 & -3 & -2 & 4 & 7 \end{array}\right]$$

- Each row is equation
- Each column is unknown

∴ lets reduce matrix [to "upper triangular one"]

∴ lets start with (1,1) ⇒ called pivot, to get rid of number under it.

i.e get 0's.

1) X2) the 1st row
2) and add 1st + 2nd row

(X-2 multiplication) →

1	−1	−1	1	0
2	0	2	0	8
0	−1	−2	0	−8
3	−3	−2	4	7

(Addition) →

(X-2)
(X-3)

0	2	4	−2	8
0	−1	−2	0	−8
0	0	1	1	7

(Has zero) Copied →

Copied →

1	−1	−1	1	0
0	2	4	−2	8
0	−1	−2	0	−8
0	0	1	1	7

Next pivot ←

$$\left[\begin{array}{cccc:c} 1 & -1 & -1 & 1 & 0 \\ 0 & 2 & 4 & -2 & 8 \\ \hline 0 & 0 & 0 & -1 & -4 \\ 0 & 0 & 1 & 1 & 7 \end{array}\right]$$

$\times \frac{1}{2}$)

Copy a (4,2) already)

$\therefore \times \frac{1}{2}$)

→ Now we modify the row below pivot number, which 3rd row

→ Next pivot, But zero can never be pivot.

∴ Swap row 3 with row 4

$$\left[\begin{array}{cccc:c} 1 & -1 & -1 & 1 & 0 \\ 0 & 2 & 4 & -2 & 8 \\ 0 & 0 & 0 & -1 & -4 \\ 0 & 0 & & & \end{array}\right]$$

$$\left[\begin{array}{cccc|c} 1 & -1 & -1 & 1 & 0 \\ 0 & 2 & 4 & -2 & 8 \\ 0 & 0 & \boxed{1} & 1 & 7 \\ 0 & 0 & 0 & \boxed{-1} & -4 \end{array}\right]$$

- Now we modify the row below pivot, row 4, and get rid of number below pivot i.e. make zero
But already zero

∴ Now starting from last equation, lets solve all unknowns.

1) $-u = -4 \Rightarrow \boxed{u=4}$ [row 4]

2) $z+u = 7$

$2+4 = 7$

$\boxed{z=3}$ [row 3]

3) $2y + 4z - 2u = 8$

$2y + 4(3) - 2(4) = 8$

$2y + 12 - 8 = 8 \quad 2y + 4 = 8$

$2y = +4$

$\boxed{y=2}$ [row 2]

$x - 2 - 3 + 4 = 0 \Rightarrow \boxed{x = 1}$ (6)

$\therefore$ $x = 1$
$y = 2$
$z = 3$
$u = 4$