System of equations we will be working with, and solve it:

$$x + 2y + z = 2$$
$$3x + 8y + z = 12$$
$$4y + z = 2$$

Looking at:

1) Elimination [Not determinant for now, this will come late]
2) Back Substitution
3) Elimination Matrix
4) Matrix multiplication

$$A = \begin{bmatrix} 1 & 2 & 1 \\ 3 & 8 & 1 \\ 0 & 4 & 1 \end{bmatrix}$$

matrix

But Elimination can also Fail

Elimination was thought up by Guass!

Here everything will be expressed as Matrix operations

$Ax = b$ [to be solved]

Step 1: Eliminate x [set = ~~take~~ value] → to value that will make $(2,1) = 0$

1) multiply the First row
2) Subtract the second row from First row
3) Make $(~~2~~1,1) = 1$ (row 1, Column 1)

also called ~~(Pivote)~~ First Pivote and Pivot row

4) Make all subsequent $x = 0$

$$\begin{array}{cc} & (1,1) \\ & \begin{matrix} \boxed{1} & 2 & 1 \end{matrix} \\ (2,1) & \begin{matrix} \boxed{3} & 8 & 1 \end{matrix} \\ & \begin{matrix} 0 & 4 & 1 \end{matrix} \end{array}$$

Need to multiply $(1,1)$ by 3

$$\begin{array}{ccc} (\times 3) & (\times 3) & (\times 3) \\ 1 & 2 & 1 \\ 0 & 2 & -2 \\ 0 & 4 & 1 \end{array}$$

(X3) Subtract this row from (row1)

$(3-3)=0$

~~$8-(2\times3)=6-6=2$~~

~~$1-(1\times3)=-2$~~

$\boxed{0 \quad 4 \quad 1}$ stay this was as $(3,1)=0$

$1\times3=3$
$\therefore 3-3=0$ (2,1)

$\therefore 2\times3=6$
$\therefore 8-6=2$ (2,2)

$1\times3=3$
$\therefore 1-3=-2$ (2,3)

if 0 . 4 1 , if $(3,3) \neq 0$, then
had to do the same as we did
for the second raw
Now x is eliminated
i.e we have it set = one value

Step 2: 1) Eliminate y [set y = one value]
1) Multiply first row
2) Subtract second raw from this
3) Make $(3,2) = 0$
4) Make all subsequent $y = 0$
5) Here pivot $= (2,2)$

we always knock out (make zero) the (raw, col) item under pivot
here pivot is $(2,2)$

so we need to knock out $\left(\overset{\text{make}}{0}\right)$
$(3,2)$

Remember pivots can't be zero!

$$\begin{matrix} 1 & 2 & 1 \\ 0 & \boxed{2} & -2 \\ 0 & \boxed{4} & 1 \end{matrix}$$

Next pivot

what we need to knock out

Need multiply (2,2) by 2 to knock out 4.

$$\begin{matrix} 1 & 2 & 1 \\ 0 & 2 & -2 \\ 0 & 4 & 1 \end{matrix} \Rightarrow \begin{matrix} \boxed{1} & 2 & 1 \\ 0 & \boxed{2} & -2 \\ 0 & 0 & \boxed{5} \end{matrix}$$

(3,3) ⇒ Next pivot.

$2 \times 2 = 4$

$4 - 4 = 0$

$\cancel{2 \times 1 = 2}$

$2 \times -2 = -4$

$1 - (-4) =$

$1 + 4 = 5$

we know have 3 pivots!

Bud

But how could this have failed?

$$\begin{matrix} 1 & 2 & 1 \\ 3 & 8 & 1 \\ 0 & 4 & 1 \end{matrix}$$

If (1,1) was zero, then maybe issue!

But how do we solve:

⇒ we switch rows / swap rows

OR if 3 $\boxed{8}$ 1

if that was 6, then it would be zero, then it could not use this row [as we should never have pivot as zero]

⇒ again switch rows

non zero below, then use it.

There is temporary failure, where we can do row exchange, or complete failure, where zero below

Next Back Substitution

[Now we bring the right hand side in]

, But all the operations we did for matrix we need to do for right hand side

b

$$\left[\begin{array}{ccc|c} 1 & 2 & 1 & 2 \\ 3 & 8 & 1 & 12 \\ 0 & 4 & 1 & 2 \end{array}\right] \Rightarrow \left[\begin{array}{ccc|c} 1 & 2 & 1 & 2 \\ 0 & 2 & -2 & 6 \\ 0 & 4 & 1 & 2 \end{array}\right] \Rightarrow \left[\begin{array}{ccc|c} 1 & 2 & 1 & 2 \\ 0 & 2 & -2 & 6 \\ 0 & 0 & 5 & -10 \end{array}\right]$$

(U)

$3 \times 2 = 6$

$12 - 6 = 6$

3×2

$6 \times 4 = 24$

$1 - 24 = -23$

$$x + 2y + z = 2$$
$$2y - 2z = 6$$
$$5z = -10$$

This is Back Substitution

So we solve for z

~~z = -2~~

Back Substitution | Solving equation into reverse order

$$x + 2y + z = 2 \qquad x + 2(1) + (-2) = 2 = x + 0 = 2, \; x = 2$$
$$y = 1$$
$$2y - 2z = 6 \qquad 2y - (2 \cdot -2) = 6 \qquad \cancel{2y - 4 = 6 = 2y = 6 + 4 = 10 = y = 5}$$

Back

$$5z = -10 = z = -2$$

$x = 2$

$y = 1$

$z = -2$

(C)

* Next piece is now Matrices [Elimination]

- Now we need to express the Elimination steps as matrices.

want to do elimination on it.

$$\begin{bmatrix} 1 & 2 & 1 \\ 3 & 8 & 1 \\ 0 & 4 & 1 \end{bmatrix}$$

Previously how did we multiply (Matrix) with vector)

$$\begin{bmatrix} - & - & - \\ - & - & - \\ - & - & - \end{bmatrix}\begin{bmatrix} 3 \\ 4 \\ 5 \end{bmatrix}$$

∴ matrix x column (vector) = column (vector)

→ first column of matrix

$3 \times \begin{bmatrix} - \\ - \\ - \\ - \end{bmatrix}$ → 2nd column

$4 \times \begin{bmatrix} - \\ - \\ - \\ - \end{bmatrix}$ → 3rd column

$5 \times \begin{bmatrix} - \\ - \\ - \\ - \end{bmatrix}$

∴ combining the column.

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Same can be done with raws [in fact we will be dealing primarily with raws]

$$\begin{bmatrix} 1 & 2 & 7 \end{bmatrix} \times \begin{bmatrix} - & - & - \\ - & - & - \\ - & - & - \end{bmatrix}$$

1x3   3x3

raw x matrix =

-- Combining the raws.

1 x raw 1

2 x raw 2

7 x raw 3

What is matrix step that subtract 3 from Equation 2 [First step] (and leaves other raws same)

Subtract 3x raw 1 (Equation 1) from raw 2 (Equation 2).

$$\begin{bmatrix} ? \\ ? \\ \end{bmatrix} \begin{bmatrix} 1 & 2 & 1 \\ 3 & 8 & 1 \\ 0 & 4 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 2 & -2 \\ 0 & 4 & 1 \end{bmatrix}$$

only raw 2 will change

∴ what will be matrix

$$\begin{array}{cccc} ① & ② & & ③ \\ \begin{bmatrix}1&0&0\\-3&1&0\\0&0&1\end{bmatrix} & \begin{bmatrix}1&2&1\\3&8&1\\0&4&1\end{bmatrix} & = & \begin{bmatrix}1&2&1\\0&2&-2\\0&4&1\end{bmatrix}\end{array} \quad \begin{array}{c} 9 \\ -2 \\ \end{array}$$

→ E = Elementary Matrix

row 1 : 1 0 0

row 3 : 0 0 1

row 2 : -3 1 0

to be subtracted from row 1.

How to Check 2 [2,3]

look at row 2 of 1

and column 3 of 2

1

~~-3+1+3~~

~~-3+1+3~~

~~-3+1+3~~

~~-3+1+3~~ =

A	B	B
row1	row2	row2

1 ~~+(-3)+3~~ = 1

2 +

A ~~B B~~

~~2+1=8~~

How to demonstrate the ~~how~~ whole elimination in matrix language.

Step2: Subtract 2xraw2 from raw3

$$E_{(3,2)} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 1 \\ 0 & 2 & -2 \\ 0 & 4 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 1 \\ 0 & 2 & -2 \\ 0 & 0 & 5 \end{bmatrix}$$

Now puts all steps together into matrix that does it all.

$$E_{32}(E_{21}A) = U$$

$$(E_{32}E_{21})A = U$$

Pe

What Permutation Matrix

exchange rows 1 and 2

$$\underset{P}{\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix}}\overset{A}{\begin{bmatrix}a & b\\ c & d\end{bmatrix}} = \overset{B}{\begin{bmatrix}c & d\\ a & b\end{bmatrix}}$$

P → matrix to do row exchanges

* for row operations matrix goes on left!

exchange rows in A to look like rows in B

What matrix would you use?

1 of c d   0 1

and 1 of a b   1 0

Suppose now I want to swap columns

$$\begin{bmatrix}a & b\\ c & d\end{bmatrix}\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix} = \begin{bmatrix}b & a\\ d & c\end{bmatrix}$$

to do column operations

* we put matrix on right

What are inverse matrix

$$\begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}_{E^{-1}} \begin{bmatrix} 1 & 0 & 0 \\ -3 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

What matrix does that undoes this step:
(Subtract 3 from row 1 from row 2)

Matrix that undo the elimination

inverse
add 3

} Do inverse