Rec

E'val and E'vc: one of the many important applications of it is solving higher order linear differential equations with constant coefficient.

A typical example:

: y is a function of t

it includes y, y prime, and all the way to its 3rd ~~deriv~~ derivative

Solve this equation for its general solution: (A)

differential equation: $y''' + 2y'' - y' - 2y = 0$

Using the method of matrix.

: which matrix should we be working with.

After that we also need to say something about the exponential of matrix (At)

find out the first column of the Matrix Exponential ②

$\exp(TAt)$.

Lets put this problem into LA.

the big idea is to put y'', y' and y as vector:

$$\begin{bmatrix} y'' \\ y' \\ y \end{bmatrix}$$

$u(t)$

that's u, what is u'?

$$\begin{bmatrix} y''' \\ y'' \\ y' \end{bmatrix}$$

$u'(t)$

write u' as matrix.

③

$$u'(t) = A \cdot u(t)$$

what is A?: write matrix, by incorporating the differential equation ④

$$\begin{bmatrix} -2 & 1 & 2 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

A.

Furthermore:

$$\begin{bmatrix} y''' \\ y'' \\ y' \end{bmatrix} = \begin{bmatrix} -2 & 1 & 2 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} y'' \\ y' \\ y \end{bmatrix}$$

$$u'(t) = A \cdot u(t)$$

Lets solve the above Equation.

④

∴ we need the E'V and E'Vec for A (matrix)

∴ we need to complete the det of the following:

$$\det(A-\lambda I) = \det\begin{pmatrix} -2-\lambda & 1 & 2 \\ 1 & -\lambda & 0 \\ 0 & 1 & -\lambda \end{pmatrix} =$$

$$\Rightarrow (1-\lambda)(1+\lambda)(2+\lambda)$$

$*\ \lambda_1 = 1 \quad (A-I)\begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} -3 & 1 & 2 \\ 1 & -1 & 0 \\ 0 & 1 & -1 \end{pmatrix}\begin{pmatrix} a \\ b \\ c \end{pmatrix} = 0$

$x_1 = (1,1,1)^T$ ∴ last row ($b=c$) ∴ $a=b=c$

middle row ($a=b$)

$\lambda_2 = -1$

$x_2 = \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$

$\lambda_3 = -2$

$x_3 = \begin{pmatrix} 4 \\ -2 \\ 1 \end{pmatrix}$

Now we have everything to create the general solution for $u(t)$:

general solution for u(t)

$$u(t) = C_1 e^{t} x_1 + C_2 e^{-t} x_2 + C_3 e^{-2t} x_3$$

$$y(t) = C_1 e^{t} + C_2 e^{-t} + C_3 e^{-2t}$$

$\exp(At)$ = product of three matrices:

$$= S e^{\Lambda t} S^{-1}$$

$$S = \begin{bmatrix} x_1, & x_2, & x_3 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 4 \\ 1 & -1 & -2 \\ 1 & 1 & 1 \end{bmatrix} \quad e^{\Lambda t} = \begin{bmatrix} e^{t} & & \\ & e^{-t} & \\ & & e^{2t} \end{bmatrix}$$

$$\exp(At) = \begin{bmatrix} e^{t} x_1 & e^{-t} x_2 & e^{-2t} x_3 \end{bmatrix} \begin{bmatrix} \frac{1}{6} \cdots \\ -\frac{1}{2} \cdots \\ \frac{1}{3} \cdots \end{bmatrix}$$

$$S^{-1} = \frac{1}{\det S} C^{T} = \frac{1}{6} \begin{bmatrix} 1 \cdots \\ -3 \cdots \\ 2 \cdots \end{bmatrix}$$

$$\exp(At) = \begin{bmatrix} \frac{e^{t}}{6} x_1 \cancel{t} - \frac{e^{-t}}{2} x_2 + \frac{e^{2t}}{3} x_3 \\ \times \quad \times \quad \times \end{bmatrix}$$

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