Additional: ([illegible])

Null Space:

Let A be an $m \times n$ matrix

The null space is the set of solutions to the homogenous system $Ax = 0$

another way of saying this:

set vectors

$$N(A) = \{ \bar{x} \in \mathbb{R}^n : Ax = 0 \}$$

so that

null space of A

Lets look at quick example:

$$x_1 + x_2 + x_3 + 0x_4 = 0$$

$$2x_1 + x_2 + 0x_3 + x_4 = 0$$

Lets find the null space.

for matrix A, we just take the coefficients.

$$A = \begin{bmatrix} 1 & 1 & 1 & 0 \\ 2 & 1 & 0 & 1 \end{bmatrix}$$

∴ we want to find the null space of that matrix A.

∴ Find $N(A)$

∴ we want to figure out the form of vector, so that when we substitute them in [equation] and do the arithmetic we get $=0$. (in both of our equations.

So to do this, we first do some row reduction:

$$\left[\begin{array}{cccc|c} \overset{\text{(pivot)}}{\boxed{1}} & 1 & 1 & 0 & 0 \\ 2 & 1 & 0 & 1 & 0 \end{array}\right] \sim \left[\begin{array}{cccc|c} 1 & 1 & 1 & 0 & 0 \\ 0 & \underset{\text{Need one}}{\boxed{-1}} & -2 & 1 & 0 \end{array}\right]$$

$-2R_1 + R_2 \Rightarrow R_2$

∴

3

$$\left[\begin{array}{cccc|c} 1 & 1 & 1 & 0 & 0 \\ 0 & -1 & -2 & 1 & 0 \end{array}\right] = \left[\begin{array}{cccc|c} 1 & 0 & -1 & 1 & 0 \\ 0 & -1 & -2 & 1 & 0 \end{array}\right]$$

$R_2 + R_1 \Rightarrow R_1$

$$\left[\begin{array}{cccc|c} 1 & 0 & -1 & 1 & 0 \\ 0 & -1 & -2 & 1 & 0 \end{array}\right] = \left[\begin{array}{cccc|c} 1 & 0 & -1 & 1 & 0 \\ 0 & 1 & 2 & -1 & 0 \end{array}\right]$$

↑ reduced echelon form.

$\therefore -1R_2 \Rightarrow R_2$

$\therefore$ rewrite as system of equations.

$\therefore 1x_1 + 0 - x_3 + 1x_4 = 0 \rightarrow x_1 = +x_3 - x_4$

$0 + 2x_2 + 2x_3 - 1x_4 = 0$

$\hookrightarrow x_2 = -2x_3 + x_4$

Here we have couple of free variables.

$\therefore$ So far we have done row reduction to find null space

Let $x_3 = \alpha$

$x_4 = \beta$

$\therefore$ now we have found the form vectors that comprised the null space.

$\therefore$

$\therefore x_1 = x_3 - x_4$

we said $x_3 = \alpha$

$x_4 = \beta$

$\therefore x_2 = -2x_3 + x_4$

$= -2\alpha + \beta$

$$X = \begin{array}{cc} \begin{bmatrix} \alpha - \beta \\ -2\alpha + \beta \\ \alpha \\ \beta \end{bmatrix} & \begin{array}{c} x_1 \\ x_2 \\ x_3 \\ x_4 \end{array} \end{array}$$

$\therefore \alpha, \beta \in \mathbb{R}$

we can Break it up:
eg factor the α out:

$$x = \underbrace{\alpha \begin{bmatrix} 1 \\ -2 \\ 1 \\ 0 \end{bmatrix} + \beta \begin{bmatrix} -1 \\ 1 \\ 0 \\ 1 \end{bmatrix}}$$

∴ Vectors of this Form will be vectors that make up the null space.

∴ Now you can pick a number for α or β

∴ $\alpha = 1$, $\beta = 1$

$$\bar{V} = 1 \begin{bmatrix} 1 \\ -2 \\ 1 \\ 0 \end{bmatrix} + 1 \begin{bmatrix} -1 \\ 1 \\ 0 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ -1 \\ 1 \\ 1 \end{bmatrix}$$

$\left.\begin{matrix} x_1 = 0 \\ x_2 = -1 \\ x_3 = 1 \\ x_4 = 1 \end{matrix}\right\}$ plug Back into original equation
∴ must get $\underline{0}$ in First and 2nd Equation.

Note:

This is the idea of Null Space

∴ What $\bar{V}$ can we substitute into the original system of Equation so we get zero.

and to get it we just do $\rightarrow$ row reduction.

Revise:

1) Row Reduction
2) Row-Echelon Form
3) Reduced Row Echelon Form. (RREF) [Best]

Rules of game:

1) Rescale a row
2) Add multiple of one row to another
3) Swap 2 rows.

Goal: Put matrix in REF or RREF

$$\begin{bmatrix} x & x & x & x & \\ 0 & x & x & x & \\ 0 & & x & x & \\ 0 & & & x & \\ 0 & 0 & 0 & 0 & \dots \end{bmatrix}$$

REF

$x \in \mathbb{R}$, where $x \neq 0$ or 1.

only last row all zero

But diagonal you can also skip, two pivots apart.

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

RREF

Diagonal 1's and zero below and top of diagonal

$\therefore$ rescale pivots to make it 1.