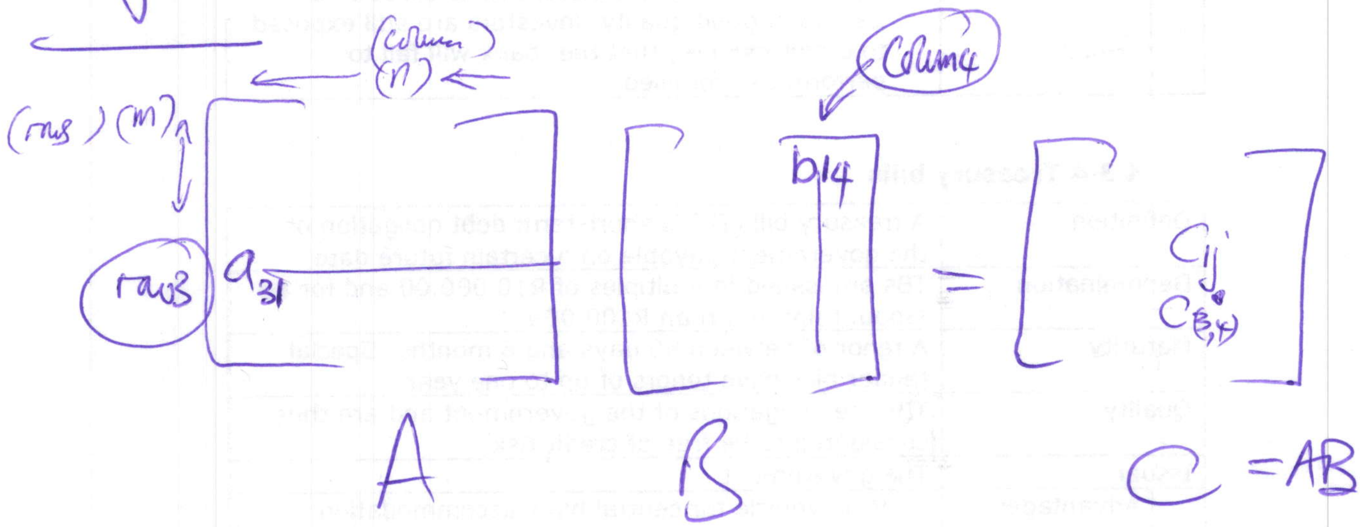
multiplication and inverse matrices *

So what are rules for matrix multiplication

⇒ 4 ways
– give same answer
– all important

Way 1



$C_{3,4}$ = Comes from row 3 and column 4

$$C_{34} = (\text{row 3 of } A) \cdot (\text{column 4 of } B)$$

$$= a_{31}b_{14} + a_{32}b_{24} + \dots = \sum_{k=1}^{n} a_{3k} b_{k4}$$

col row

eg k=1,2..

$$\sum_{k=1}^{n} a_{3k} b_{k4}$$

But when are we allowed to multiply
two matrices?

- they have rows have match other
  ones columns.
- Can be square or rectangle

$$A^{m \times n}$$
m — rows, n — column

$$B^{n \times p}$$
n (rows) → must match with A's (m) toward; p — columns

$$C^{m \times p} = AB$$
[result] m → rows, p — column

this is standard rule
for multiplying matrices

$$\begin{bmatrix} & & \\ & & \\ & & \end{bmatrix} \begin{bmatrix} & 1 & \\ & \vdots & \\ & & \end{bmatrix} = \begin{bmatrix} & 1 & \\ & \vdots & \\ & & \end{bmatrix}$$

A B C

$m \times n$ $n \times p$ $m \times p$

Lets look at whole column.

I know how to multiply matrix A by column 1 in B.

Column 1 is answer in C

Order

$$\begin{bmatrix} 2 & 3 & 1 \\ 2 & -7 & 4 \end{bmatrix} \cdot \begin{bmatrix} 3 & 4 & 5 \\ 1 & 1 & 4 \\ 2 & 1 & 4 \end{bmatrix}$$

row col. rows col

$m \times n$ $n \times p$

2×3 3×3

need to be same

To be able to multiply

give you size of new matrix

2×3

$$\begin{bmatrix} \boxed{2} & \boxed{3} & \boxed{1} \\ 2 & -7 & 4 \end{bmatrix} \cdot \begin{array}{ccc} ① & ② & ③ \\ \begin{bmatrix} 3 & 4 & 5 \\ 1 & 1 & 4 \\ 2 & 1 & 4 \end{bmatrix} \end{array} = \begin{bmatrix} ①+②+③ & ①②③ & ①②③ \\ & & \end{bmatrix}$$

∴ we multiply the First row [First Matrix] by First Colemn [2nd matrix]

$$\begin{bmatrix} 2 & 3 & 1 \\ 2 & -7 & 4 \end{bmatrix} \begin{bmatrix} 3 & 4 & 5 \\ 1 & 1 & 4 \\ 2 & 1 & 4 \end{bmatrix} = \begin{bmatrix} 6+3+2 & 8+3+1 & 10+12+4 \\ 6-7+8 & 8-7+4 & 10-28+16 \end{bmatrix}$$

- remember this is refering to matrix times vector
It works differently.

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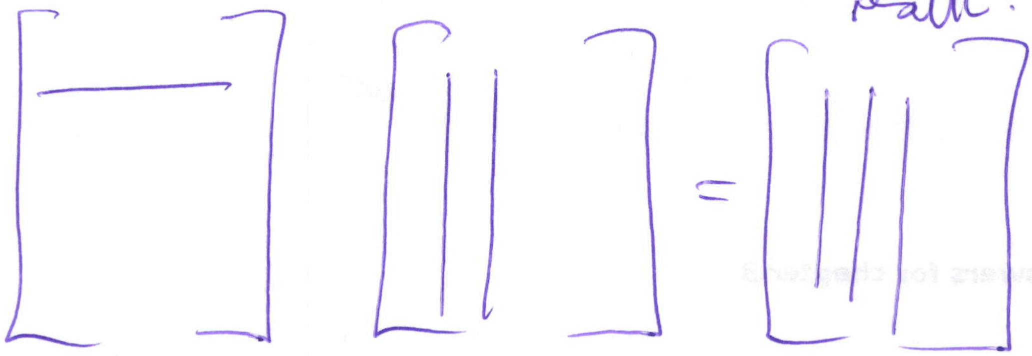
realtt.



→ different from vector × Matrix multiplication

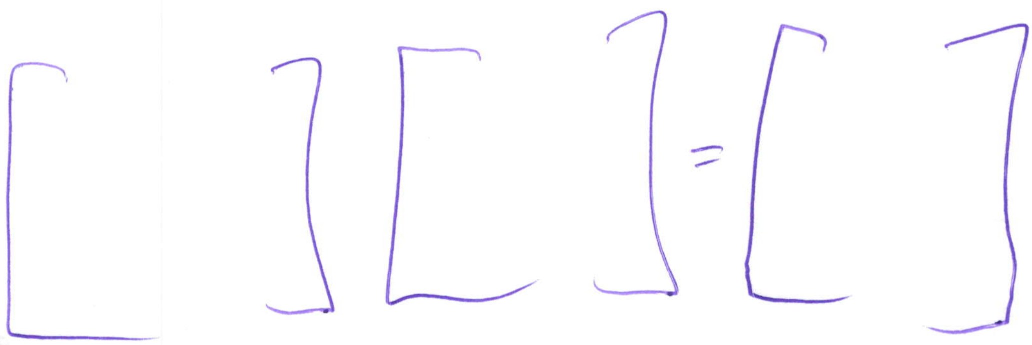
$A_{m\times n}$ $B_{n\times p}$ $C_{m\times p}$

answer will be in column form.

(columns of C)

Way 3 ___ (look at it by rows)

Way 4



Column of A $\times$ row of B

$$\begin{array}{ccc} m \times 1 & & 1 \times p \\ \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix} & \begin{bmatrix} 1, 6 \end{bmatrix} & = \begin{bmatrix} 2 & 12 \\ 3 & 18 \\ 4 & 24 \end{bmatrix} \\ A & B & C \end{array}$$

$\Rightarrow$ very special matrix!

- the Columns of result Matrix (C) are multiples of A.
- the rows of (C) are multiples of (B)

, But we getting a Full size matrix

- $\boxed{\text{4th way}}$

AB = Sum of (Cols of A) $\times$ (rows of B)

(1st col x 1st row) + (2nd col x 2nd row)

$$\begin{bmatrix} 2 & 7 \\ 3 & 8 \\ 4 & 9 \end{bmatrix} \begin{bmatrix} 1 & 6 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix} \begin{bmatrix} 1 & 6 \end{bmatrix} + \begin{bmatrix} 7 \\ 8 \\ 9 \end{bmatrix} \begin{bmatrix} 0 & 0 \end{bmatrix}$$

But you can also cut the matrix into blocks:

Block multiplication

$A_1B_1 + A_2B_3$

$$\underset{A}{\left[\begin{array}{c|c} A_1 & A_2 \\ \hline A_3 & A_4 \end{array}\right]} \underset{B}{\left[\begin{array}{c|c} B_1 & B_2 \\ \hline B_3 & B_4 \end{array}\right]} = \underset{C}{\left[\begin{array}{c|c} \quad & \quad \\ \hline \quad & \quad \end{array}\right]}$$

Chop into 1/4 and 1/4

Lets tackle inverse

⇒ First Square matrices ← not all matrices have inverse

$A^{-1} A = I$ ← produced identity

But only if it exists

$A A^{-1} = I$ (also true)

(invertible or (non-singular))

left inverse $\overset{\text{(same)}}{=}$ right inverse

only works for Square matrices

does not work for rectangular matrices

left inverse ≠ right inverse

$A^{-1}A \neq AA^{-1}$

Lets talk about case where there is no inverse

⇒ i.e Singular or no inverse.

$$A = \begin{bmatrix} 1 & 3 \\ 2 & 6 \end{bmatrix}$$

- Why does this matrix have no inverse?

$\Rightarrow$ I can find a vector x
with $Ax = 0$

$3 + (-3) = 0$
$(1 \times 3) + (-1 \times 3)$

e.g.

$$Ax = \begin{bmatrix} 1 & 3 \\ 2 & 6 \end{bmatrix} \begin{bmatrix} 3 \\ -1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \qquad x \neq 0$$

Now lets take matrix that does have inverse.

$$\underbrace{\begin{bmatrix} 1 & 3 \\ 2 & 7 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} a & \\ b & \end{bmatrix}}_{A^{-1}} = \underbrace{\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}}_{I}$$

$A \times$ Column j of A^{-1} = Column j of I

Guass-Jordan (solve 2 equations at once)

$$\begin{bmatrix} 1 & 3 \\ 2 & 7 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 3 \\ 2 & 7 \end{bmatrix} \begin{bmatrix} c \\ d \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$\left[\begin{array}{cc|cc} 1 & 3 & 1 & 0 \\ 2 & 7 & 0 & 1 \end{array}\right] \Rightarrow \left[\begin{array}{cc|cc} 1 & 3 & 1 & 0 \\ 0 & 1 & -2 & 1 \end{array}\right]$$

A I

$$\rightarrow \left[\begin{array}{cc|cc} 1 & 0 & 7 & -3 \\ 0 & 1 & -2 & 1 \end{array}\right]$$

I A^{-1}

1st

$$\begin{array}{cccc} 1 & 3 & 1 & 0 \\ 2 & 7 & 0 & 1 \end{array}$$

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