Lect

①

Now we will cover Symmetric Matrix

$$A = A^T$$

Main point:

① The e'values are REAL

② The e'vectors are perpendicular (orthogonal)

why ① ⁈ for identity, every vector an e'vector.

Usual:

$$A = S\Lambda S^{-1}$$

But now matrix symmetric

Symmetric: orthogonal matrix inverse is the same as transpose.

Case $A = Q\Lambda Q^{-1} = Q\Lambda Q^T$

(spectrum theory)

orthonormal e'vectors, the right letter to use Q. ↳ column of Q.

Why are the e'values real?

2

$$Ax = \lambda x$$

from the moment λ or x or A can be complex.

conjugate

$$\bar{A} = \bar{x} = \bar{\lambda}\bar{x} \qquad \boxed{\overline{a+ib} = a - ib \text{ meaning}}$$

take the conjugate of everything.

$$\Rightarrow \bar{x}^T A^T = \bar{x}^T \bar{\lambda}$$

if matrix is symmetric then $A^T = A$.

$$\cancel{Ax = \lambda x}$$

$$\bar{x} A \bar{x} = \bar{\lambda}\bar{x}$$

$$\bar{x} A x = \bar{\lambda}\bar{x}^T x$$

$$Ax = \lambda x \qquad\qquad \bar{x}^T A = \bar{x}^T \bar{\lambda}$$

$$\bar{x}^T A x = \lambda \bar{x}^T x \qquad\qquad \bar{x}^T A x = \bar{x}^T \bar{\lambda} x$$

$$\lambda \bar{x}^T x \qquad = \qquad \bar{\lambda}\bar{x}^T x$$

3

$\therefore \lambda = \bar{\lambda}$ ← prove λ is real

is equal to its own complex conjugate, therefore it's no imaginary part.

$$\begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}, \text{ since } (\bar{x}^T x) = \begin{bmatrix} \bar{x}_1 & \bar{x}_2 & \dots & \bar{x}_n \end{bmatrix}$$

$\Rightarrow (\text{length})^2$

$$\bar{x}_1 x_1 + \bar{x}_2 x_2 + \dots$$

$$(a-ib)(a+ib) = a^2+b^2$$

the imaginary part is gone.

$A = A^T$

$$A = Q \Lambda Q^T$$

$$= \begin{bmatrix} q_1 & q_2 & \dots \end{bmatrix} \begin{bmatrix} \lambda_1 & & \\ & \lambda_2 & \\ & & \ddots \end{bmatrix} \begin{bmatrix} q_1^T \\ q_2^T \\ \vdots \end{bmatrix} = \lambda_1 q_1 q_1^T + \lambda_2 q_2 q_2^T \dots$$

↳ projection matrix

Every symmetric matrix is a comb. of

$\perp$ projection matrices.

For symmetric matrices

signs of pivots are same as

signs of λ's

\# of pos pivots = \# positive λ's.

mean: decent way to compute e'value.

Now we can narrow it down.

how many are positive / negative

Can shift matrix say 7 times to I,

shift all e'values by 7.

then take pivots of that matrix, and

then know how many e'values

of the original were above / below 7.

What's a positive definite matrix?

its symmetric.

⇒ with all e'values are positive

" " pivots are positive

eg

A $\begin{bmatrix} 5 & 2 \\ 2 & 3 \end{bmatrix}$ = pivot: 5, $\frac{11}{5}$ ✓ (all positive)

— symmetric, e'values are real, know signs of e'values, and know signs of pivots.

A Det = 11 ✓ product of pivots is its det

Now since all pivots positive, e' also positive ✓

$\det$

$$\begin{bmatrix} 5-\lambda & 2 \\ 2 & 3-\lambda \end{bmatrix} = \lambda^2 - 8\lambda + 11 = 0$$

$$\lambda = 4 \pm \sqrt{5}$$

know the signs, so we will know the stability.

(6)

What is the det?

- also positive

But $\begin{bmatrix} -1 & 0 \\ 0 & 3 \end{bmatrix}$ as det also positive, But pivots are negative, EV also negative.

Related fact now?

(3) All sub determinant are positive