Rec

tackle problem in orthogonal subspace.

Given a subspace S, and suppose S is spanned by $(1\ 2\ 2\ 3)$ and $(1\ 3\ 3\ 2)$ (vectors)

i) Find a basis for $S^{\perp}$ "perp" ← subspace orthogonal to S

ii) Can every $\vec{v}$ (vector) in $\mathbb{R}^4$ be uniquely written l.c. of S and $S^{\perp}$?

What does it mean for vector to be in $S^{\perp}$

iff (vector) x in $S^{\perp}$

∴ x is going to be orthogonal to every vector in S.

Specifically S is spanned by 'those two vectors'

so it is sufficient that x is

perpendicular to these two vectors in S. (2)

Specifically can take

$\begin{pmatrix} 1 & 2 & 2 & 3 \end{pmatrix} \overset{\text{dot}}{x} = 0$ treating x as column vector.

In addition:

$\begin{pmatrix} 1 & 3 & 3 & 2 \end{pmatrix} x = 0$

x is orthogonal for $\begin{pmatrix} 1 & 3 & 3 & 2 \end{pmatrix}$

∴ we can write this as a matrix notation.

$$\begin{pmatrix} 1 & 2 & 2 & 3 \\ 1 & 3 & 3 & 2 \end{pmatrix} x = 0$$

∴ what we saying is find all x that are in null space this matrix.

∴ we can row reduce matrix and try find Basis for Nullspace

, also by row reduction we don't change the change null space of matrix

A

$$\Rightarrow \begin{pmatrix} 1 & 2 & 2 & 3 \\ 0 & 1 & 1 & -1 \end{pmatrix} \text{(Sublaction)(top-Bottom)} \underline{x} = 0$$

→ to parametized the nullspace, write x out as component.

$$\underline{x} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix}$$

let x_3, x_4 be constant →

$x_4 = b$

$x_3 = a$

→ we see A has rank of 2

we looking ~~for~~ at vector that live in $\mathbb{R}^4$

So we know the nullspace have

Dim = 4 − 2

∴ must be 2 vectors in nullspace of matrix.

$\therefore\ x_2 = -x_3 + x_4 = a + b$

$x_1 = -2x_2 - 2x_3 - 3x_4$

$$\begin{pmatrix} 0 & 1 & 1 & -1 \\ 1 & 2 & 2 & 3 \end{pmatrix} \quad 4$$

Now lets substitute:

$= 2\cdot(-a+b) - 2a - 3b$

$= -5b$

$$\underline{X} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} -5b \\ -a+b \\ a \\ b \end{pmatrix} = \overbrace{a\begin{pmatrix} 0 \\ -1 \\ 1 \\ 0 \end{pmatrix} + b\begin{pmatrix} -5 \\ 1 \\ 0 \\ 1 \end{pmatrix}}^{S^{\perp}}$$

∴ Successfully achieved parameterization of ~~matrix~~ nullspace of matrix as some constant a( )+b( ) thus is the entire space $S^{\perp}$

Notice: if I take these two vectors in S, and dot/multiply with any vector in nullspace by construction, it automatically vanishes

ii) Yes.

Have vector V, can write it as ~~constant~~ C1

$$V = C_1 \underbrace{\begin{pmatrix}1\\2\\2\\3\end{pmatrix} + C_2 \begin{pmatrix}1\\3\\3\\2\end{pmatrix}}_{S} + C_3 \underbrace{\begin{pmatrix}0\\-1\\1\\0\end{pmatrix} + C_4 \begin{pmatrix}-5\\1\\0\\1\end{pmatrix}}_{S^{\perp}}$$

∴ Given any V, can ~~I~~ find C_1, C_2, C_3, C_4, such that this equation holds.

Yes

∴ rewrite in matrix notation.

$$\begin{pmatrix}1 & 1 & 0 & -5\\2 & 3 & -1 & 1\\2 & 3 & 1 & 0\\3 & 2 & 0 & 1\end{pmatrix}\begin{pmatrix}C_1\\C_2\\C_3\\C_4\end{pmatrix} = V$$

these vectors are linearly independent

↓

∴ matrix is invertible

↓

∴ for any V on right hand side, we can invert this matrix and obtain unique coefficients $C_1 \ldots C_4$.