Again using getting inverse of matrix using guass Jordan / (or called) Raw reduction

augment it

$$\left[\, A \;\middle|\; I \,\right] \rightarrow \left[\, I \;\middle|\; A^{-1} \,\right]$$

if we cant produce this, then dont have inverse

then do Bunch of Raw reduction

First two by two:

$$\begin{bmatrix} 1 & 3 \\ 2 & 5 \end{bmatrix}$$

$$\left[\, A \;\middle|\; I \,\right] \rightarrow \left[\, I \;\middle|\; A^{-1} \,\right]$$

②

$$\left[\begin{array}{cc|cc} 1 & 3 & 1 & 0 \\ 2 & 5 & 0 & 1 \end{array}\right] \rightarrow \left[\begin{array}{cc|cc} 1 & 3 & 1 & 0 \\ 0 & -1 & -2 & 1 \end{array}\right]$$

Ⓐ

$$\begin{array}{cccc} e & f & g & h \end{array}$$

∴ already have 1, top left, but need 0 below where 2 is.

∴ $-2 \cdot R_1 + R_2 \rightarrow R_2$

Ⓐ

$-2+2=0$ ✓ e

$\Rightarrow -2\times 3+5$

$-6+5=-1$ ✓ f

$\Rightarrow -2\times 1+0$

$-2+0$

$=-2$ ✓ g

$\Rightarrow -2\times 0+1$

$0+1$

$=1$ ✓ h

$\longrightarrow$

$$\left[\begin{array}{cc|cc} 1 & 3 & 1 & 0 \\ 0 & -1 & -2 & 1 \end{array}\right] = \text{(A)} \begin{array}{c} \left[\begin{array}{ccc|cc} 1 & & 3 & 1 & 0 \\ 0 & 1 & \cancel{2} & 2 & -1 \end{array}\right] \\ \begin{array}{ccccc} f & g & h & h & i \end{array} \end{array}$$

- Now we need -1 to be positive $\underline{1}$.

$\therefore \; -1 \cdot R_2 \rightarrow R_2$

(A)
$-1 \times 0 = 0$ ✓f

$-1 \times (1) = 1$ ✓g

$-1(-2) = 2h$ ✓

$-1(1) = -1\, i$ ✓

$$\left[\begin{array}{cc|cc} 1 & 3 & 1 & 0 \\ 0 & 1 & 2 & -1 \end{array}\right] \Rightarrow \text{(A)} \begin{array}{c} \begin{array}{cccc} f & g & h & i \end{array} \\ \left[\begin{array}{cc|cc} 1 & 0 & -5 & -3 \\ 0 & 1 & 2 & -1 \end{array}\right] \end{array}$$

- Now we need the 3 to be 1 to get identity matrix

$\therefore$ ~~$-3R_1$~~
$-3R_2 + R_1 \rightarrow R_1$

$\rightarrow$ (A) $\Rightarrow -3 \times 0 + 1$
$0 + 1 = 1\; f$ ✓

$\rightarrow -3 \times -1 + 0$
$-3 + 0$
$-3\; i$ ✓

$(-3 \times 1) + 3 = -3 + 3 = 0\; g$ ✓

$$\left[\begin{array}{cc|cc} 1 & 0 & -5 & -3 \\ 0 & 1 & 2 & -1 \end{array}\right]$$

$$A^{-1} = \begin{bmatrix} -5 & 3 \\ 2 & -1 \end{bmatrix}$$

→ Inverse ←

3x3 ⇒ (inverse) (5)

→

$$\begin{bmatrix} 2 & 3 & 0 \\ 1 & -2 & -1 \\ 2 & 0 & -1 \end{bmatrix}$$

$$[A \mid I] \rightarrow [I \mid A^{-1}]$$

same reduction to end up

$$\begin{array}{c} \\ Ⓐ \\ \\ \end{array}\left[\begin{array}{ccc|ccc} \boxed{2} & 3 & 0 & 1 & 0 & 0 \\ 1 & -2 & -1 & 0 & 1 & 0 \\ 3 & 0 & -1 & 0 & 0 & 1 \end{array}\right]$$

Need Ⓐ to begin

need 1 in top (row 1, col 1)

so we simply order change:

$$\left[\begin{array}{ccc|ccc} 1 & -2 & -1 & 0 & 1 & 0 \\ 2 & 3 & 0 & 1 & 0 & 0 \\ 2 & 0 & -1 & 0 & 0 & 1 \end{array}\right]$$

need zero here, but since they Both have same value:

$-2 \cdot R_1 + R_2 \to R_3$

$-2R_{\cancel{2}1} + R_3 \to R_3$

$$\begin{array}{cc} & \\ \text{raw2} & \left[\begin{array}{ccc|ccc} 1 & -2 & -1 & 0 & 1 & 0 \\ 0 & 7 & 2 & 1 & -2 & 0 \\ 0 & 4 & 1 & 0 & -2 & 1 \end{array}\right] \end{array}$$

raw2 : $-2(1)+2=$
$-2+2=0$

$-2(-2)+3 \to$
$4+3=7$

$-2(-1)+0$
$+2+0$

$-2(0)+1$
$0+1$

$-2(1)+0$
$-2+0$

raw3 : $-2(1)+2=$
$-2+2=0$

$-2(-2)+0$
$4+0=4$

$-2(-1)+(-1)=$
$2-1=1$

$-2(0)+0=0$

$-2(1)+0=$
$-2+0$

∴ need to get 7 to be 1.

∴ $\frac{1}{7} \cdot R_2 \rightarrow R_2$ ∴ divide everything in row 2 by 7

$$\left[\begin{array}{ccc|ccc} 1 & -2 & -1 & 0 & 1 & 0 \\ 0 & 1 & \frac{2}{7} & \frac{1}{7} & -\frac{2}{7} & 0 \\ 0 & 4 & 1 & 0 & -2 & 1 \end{array}\right]$$

← First row → 3rd row.

∴ we need 0 for -2 and 4

∴ $2 \cdot R_2 + R_1 \rightarrow R_1$ $\frac{4}{7} - 1$

$\frac{2}{7}$

$-4R_2 + R_3 \rightarrow R_3$.

$$\left[\begin{array}{ccc|ccc} 1 & 0 & -\frac{3}{7} & \frac{2}{7} & \frac{3}{7} & 0 \\ 0 & 1 & \frac{2}{7} & \frac{1}{7} & -\frac{2}{7} & 0 \\ 0 & 0 & -\frac{1}{7} & -\frac{4}{7} & -\frac{6}{7} & 1 \end{array}\right]$$

⇒ get 1 for $-\frac{1}{7}$

∴ $-7 \cdot R_3 \rightarrow R_3$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & -3/4 & 2/7 & 3/7 & 0 \\ 0 & 1 & 2/7 & 1/7 & -2/7 & 0 \\ 0 & 0 & 1 & 4 & 6 & -7 \end{array}\right]$$

Need $-3/4$ to be 0

$2/7$ to be 0

$3/7 \cdot R_3 + R_1 \to R_1$

$-2/7 \cdot R_3 + R_2 - R_2$

$12/7 + 2/7 = 14/7 = 2$

$3/7 \times 6 = 18/7 + 3/7 = 21/7 = 3$

$3/7 \times -7 = -3 + 0 = 3$

$-2/7 \times 4 = -8/7 + 1/7 = -7/7 = -1$

$\cancel{-2/7 \times 6 = 14/7 = 2 + 3/7}$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 2 & 3 & -3 \\ 0 & 1 & 0 & -1 & -2 & 2 \\ 0 & 0 & 1 & 4 & 6 & -7 \end{array}\right]$$

Inverse matrix.

$$A^{-1} = \begin{bmatrix} 2 & 3 & -3 \\ -1 & -2 & 2 \\ 4 & 6 & -7 \end{bmatrix}$$