Mixture of LA that comes with Change of Basis, from one Basis to another

$\Rightarrow$ this is what you really do (Change Basis) in applications

eg Such applications:

Compression of Image

" " Signal

$\therefore$ that is exactly what is, a Change of Basis

Main theme here: the connection between a Lin Trans which does not have to have coordinates, and Matrix that tells us that transformation w.r.t coordinates.

$\therefore$ matrix is Coordinate Base description of the Lin Trans.

Aside:

Image Compression ⇒ everyone needs compression

eg videos - compression, motion is "jerky"

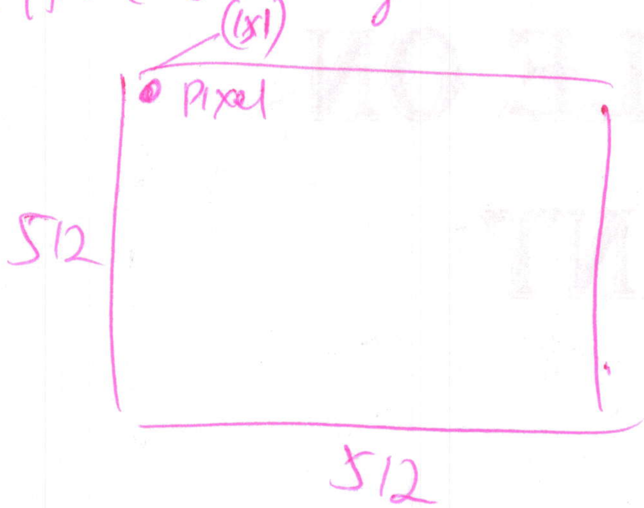
Eg Lets look at still image ⇒

Some compression can be done with no loss (lossless compression)

But here we talking about <u>lossy</u> compression

Eg Here is image: Consist of lots pixels 512×512

$2^9 \times 2^9$



x_i = real number on scale 0–255.

2^8 possibilities.

$0 \le x_i \le 255$

∴ 8 bits (A)

∴ Pixel will tell us grayscale (0–255) of <u>BW</u>

(A) But we have that for every pixel.

∴ $X \in R^n$

$n = (512)^2$

③

Pixels in vector that gives us picture of image.

or image is vector of that length. $x \in \mathbb{R}^n$

If Colour image, we will have it 3 times the length

∴ we need 3 coordinates to get Colour.

∴ $n = (512)^3$.

Big amount of information.

∴ we need Compression.

Standard Compression called JPEG.

what is JPEG. ⇒ its a change of basis

the Standard Basis: every pixel gives a value

eg $x = \begin{bmatrix} 1 \\ 121 \\ 134 \\ \vdots \end{bmatrix}$ 512 long

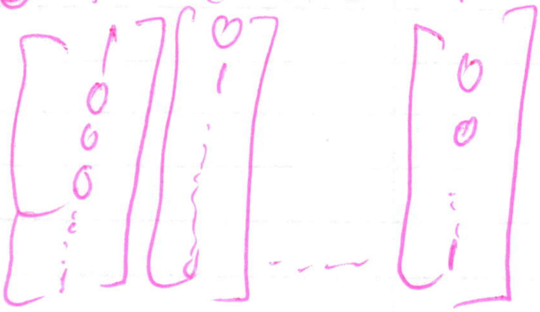
eg Something with same Colour, there is no need to show

Such image with same color, pixel by pixel.

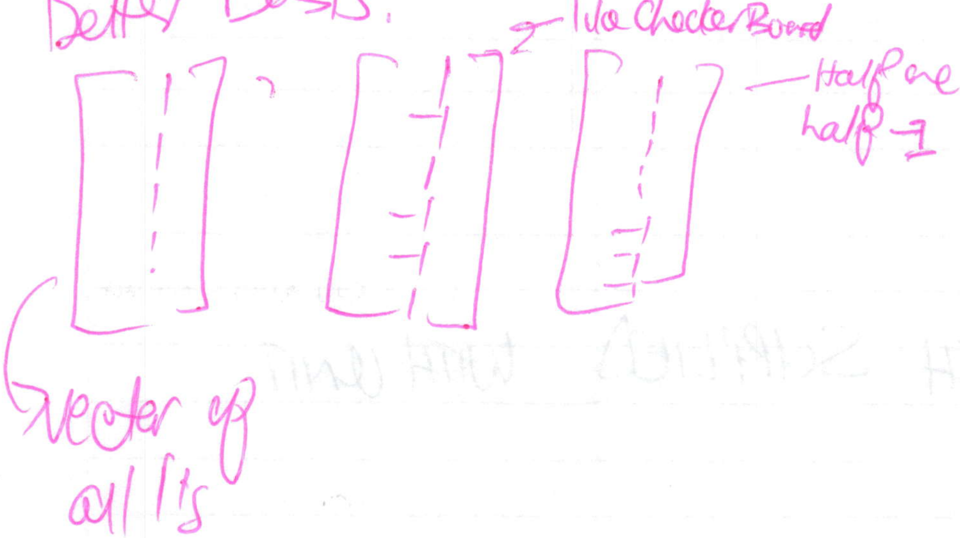
i.e.: all gray levels are same.

∴ So in that case the Standard Basis is not good.

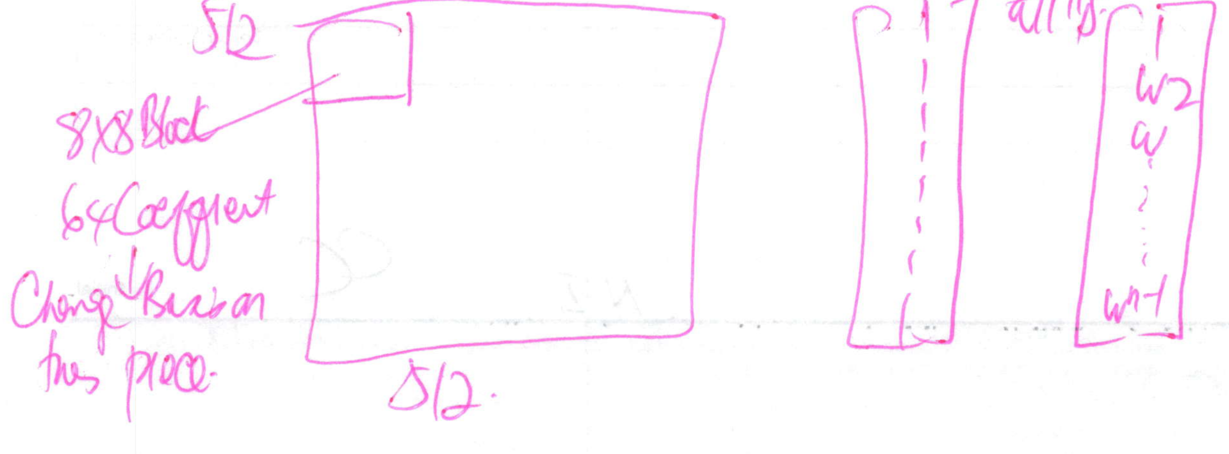
So Standard Basis



Better Basis:



What does JPEG use: Fourier Basis (8x8)



∴ What happens to image for video?

Broken up into 8x8 Blocks

– Within each Block, we have 64 coeff.

64 Basis, 64 pixels.

and we change Basis, in 64 Dim Space

∴ That was lossless step

Eg In comes signal x

↓ we change Basis

Coeffs C ← 64 pixels come in

↓ 64 coeff. come out

Lossless through change Basis

Now comes compression

↓ Here we through away coeffs. thresholding ↳ eye can't pic up.

Lossy now we loose

to produce

new compressed set coeffs.

$\hat{C}$ (many zero)

Left with fewer coeffs

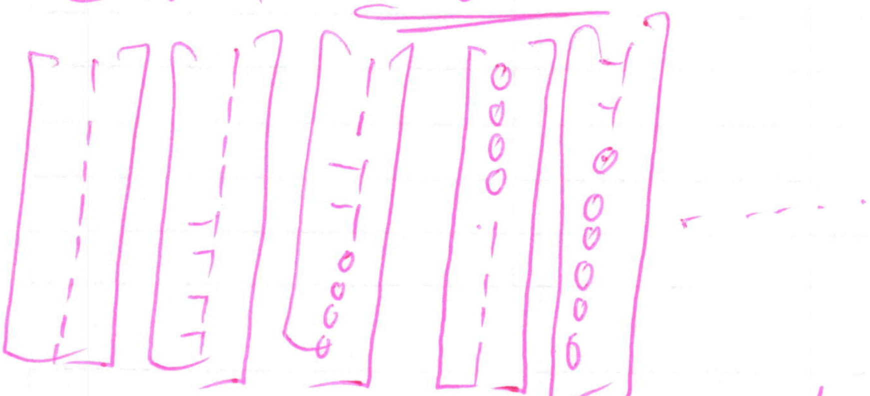
↓

reconstruct a signal using those coeffs.

$$\hat{x} = \sum \hat{c}_i v_i$$

But this sum does not have 64 term anymore, probably has 2 or 3 term instead of 3, from 64 to 3 that's compression of 21 : 1 [21 to 1]
- that's the compression you looking for.

New Basis ⇒ competition for Fourier ⇒
Called Wavelets: 8x8 Core R^8



∴ 8 vectors in 8 dimensional space
Called wavelets
eg How would I write Fourier as Combination of Wavelets

Eg Transform space

$$P = c_1 w_1 + c_8 \ldots w_8$$

Change from Standard Basis

$$\begin{bmatrix} p_1 \\ p_2 \\ \vdots \\ p_8 \end{bmatrix} = c_1 \begin{bmatrix} \\ \\ \\ \end{bmatrix} + c_2 \begin{bmatrix} \\ \\ \\ \end{bmatrix}$$

wavelets Basis

coeffs.

$$P \quad \begin{bmatrix} & & & \\ & & & \\ & & & \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ \vdots \\ c_8 \end{bmatrix}$$

So transform: Solve: $P = Wc$

and Coeffs: $C = W^{-1}P$

Good Basis has a nice fast <u>inverse</u>

Good Basis mean what: FFT. [Fast Fourier Transform]

① Fast [must be able to multiply By W and inverse fast]

FFT. and New FWT
Fast wavelet transform.

[Basis vectors are orthogonal, But not orthonormal]

What is W^{-1} : Inverse is Same as transpose

For orthonormal Columns

2nd Requirement

② Few is enough ∫ i.e. good Compression. ⑧

— few Basis vectors

— to reproduce the image/video

Review Change of Basis idea ⟹

Columns of W = be the new basis vectors

vector $\begin{bmatrix} x \end{bmatrix}$ old Basis $\xrightarrow{\text{converts}}$ $\begin{bmatrix} c \end{bmatrix}$ new Basis

$x = Wc$

[matrix W that gives a change of Basis]

Suppose (Lin. Trans) ($n \times n$ matrix) (8×8)

w.r.t $v_1, \ldots v_8$

it has matrix A.

Lets Say its a ROTATION

w.r.t $w_1 \ldots w_8$

it has matrix B

What is the Connection between A & B

- Remember How to create matrix A. (9)
- Same method to create B, now what is the relation.

∴ have same T, and ~~computed it~~ computed its matrix in one basis, then another basis

∴ those two matrices are similar

What does similar matrix mean?

$$B = M^{-1} A M$$

I take same matrix, B and I can compute another matrix A, using M^{-1} on one side, and M on other side } of A.

M will be change of basis matrix

∴ What does it mean for A to be the matrix of this transformation T?

if I change to different Basis

two things happen:

- every vector has new coordinate
- every matrix changes

Recap: Come Back ----

What is A? using a basis $V_1 \ldots V_8$

if I know what the transformation does to those 8 Basis vectors, I know it completely.

I know T completely from knowing what T does to $V_1\ V_2$

$T(V_1), T(V_2) \ldots T(V_8)$

Why? Because is linear Transformation and how does linearity work?

every X is some ~~kind of~~ combination of those basis vectors

$X = C_1V_1 + C_2V_2 + \ldots C_8V_8$

[every vector is combination of Basis vectors] what is T of X: — [we claimed we knew T(x) completely] (11)

$$T(x) = C_1 T(V_1) + C_2 T(V_2) + \dots$$

$T(V_1)$: output of V_1

$T(V_2)$: output of V_2

∴ we know everything when we know what T does to every Basis vector.

∴ write

1st column → $T(V_1) = a_{11}V_1 + a_{21}V_2 + \dots a_{81}V_8$

[write first output as combination of Basis vectors]

2nd Column → $T(V_2) = a_{12}V_1 + a_{22}V_2 + \dots$

∴ we writing the matrix A.

$$[A] = \begin{bmatrix} a_{11} & a_{21} & \\ \vdots & \vdots & \text{—} \\ a_{18} & a_{28} & \end{bmatrix}$$

So we get:

1. Basis
2. Transformation

then we compute:

1. Σ of each Basis
2. Expand result in Basis

gives 64 numbers

Suppose $V_1 \ldots V_8$ are the E'vectors.

∴ E'vector Basis

∴ $T(V_i) = \lambda_i V_i$

⇒ is in the same direction

∴ what is A?

Lets carry through steps: → will take more time (But we know)

- they may look for E'vectors
- so Best Basis is E'vector Basis
- Whats matrix?
- what 1st column of matrix
- How do I get 1st column

$$A = \begin{bmatrix} \lambda_1 & 0 & & \\ 0 & \lambda_2 & & \\ 0 & 0 & \ddots & \\ 0 & 0 & & \\ 0 & 0 & & \\ 0 & 0 & & \end{bmatrix}$$

∴ matrix is diagonal.

- take first basis vector V_1
- & look to see what does the transformation do to it
- the output is Lambda λ_1, V_1
- & express the output as combination:

- So first input is v_1
- its output is $\lambda_1 v_1$
- now write $\lambda_1 v_1$ as a combination of basis vectors, But it's already done
- ∴ its just $\lambda_1 v_1$ ... and zero's
- So first Column will have λ_1 and zero
- 2nd input is v_2, output is $\lambda_2 v_2$
- write that output as a combination of v's, already done.
- ∴ just $\lambda_2 v_2$
- 2nd Column we have λ_2 ...
- See what's coming, the matrix is diagonal.
- perfect Basis for imageprocessing