Re:

Looking at Similar Matrices

: Which of the following statements are true?

(a) If A and B are similar matrices, then $2A^3 + A - 3\underline{I}$ and $2B^3 + B - 3I$ are similar.

True (why)

⇒ What does it mean for A & B to be similar?

Know $\exists M$ s.t. $MAM^{-1} = B$.

$$M(2A^3 + A - 3\underline{I})M^{-1}$$

$$= 2(MAM^{-1}MAM^{-1}MAM^{-1}) + MAM^{-1} - 3M\underline{I}M^{-1}$$

$$= 2B^3 + B - 3\underline{I}$$

∴ general remark: if you have matrices A & B that are similar, then any polynomials in those matrices will be similar

(b) If A and B are 3x3 matrices with E'value 1, 0, -1 then A and B are Similar.

True

∵ matrix with distinct E'values are diagonalizable.

$A = S\Lambda S^{-1}$, $\Lambda = \begin{bmatrix} 1 & & \\ & 0 & \\ & & -1 \end{bmatrix}$

$B = T\Lambda T^{-1}$, Λ is the same, as they have same E'values

∴ two matrices are similar to same matrix. → transitive similar relation, then they similar to each other.

$(TS^{-1})A(TS^{-1})^{-1} = B$ ← then get B.

(c) The matrices $J_1 = \begin{pmatrix} -1 & 1 & 0 \\ 0 & -1 & 1 \\ 0 & 0 & -1 \end{pmatrix}$

and $J_2 = \begin{pmatrix} -1 & 1 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$

are similar

③

∴ They diff. matrices in Jordan Form
→ so they won't be similar

Why?
- One of the things that similarity preserves
are e'vectors and e'values.

[lets look at null space of matrix]

Ⓐ $J_1 + I = \begin{bmatrix} 0 & 1 & \\ & 0 & 1 \\ & & 0 \end{bmatrix}$ ← nullspace here's just 1-Dim

Ⓑ $J_2 + I = \begin{bmatrix} 0 & 1 & \\ & 0 & \\ & & 0 \end{bmatrix}$ ← nullspace is 2 Dim
2 indep. e'vectors

Ⓐ $\text{Dim } N(J_1 + I) = \underline{1}$

Ⓑ $\text{Dim } N(J_2 + I) = 2$

→ Can't be similar