Roc

Find the singular value decomposition (SVD) of the matrix

$$C = \begin{bmatrix} 5 & 5 \\ -1 & 7 \end{bmatrix}$$

Want: $C = U \Sigma V^T$

U, V will be orthogonal matrices; these columns are orthonormal sets.

$\therefore$ we need 2 equations

- $C^T C = V \Sigma^T \Sigma V^T$
- $CV = U\Sigma$

Two equations to use

Σ is diagonal matrix with non negative entries

$$C^T C = \begin{bmatrix} 5 & -1 \\ 5 & 7 \end{bmatrix} \begin{bmatrix} 5 & 5 \\ -1 & 7 \end{bmatrix} = \begin{bmatrix} 26 & 18 \\ 18 & 74 \end{bmatrix}$$

$$\det(C^T C - \lambda I) = \det \begin{bmatrix} 26-\lambda & 18 \\ 18 & 74-\lambda \end{bmatrix}$$

$$= \lambda^2 - 100\lambda + 1600 = (\lambda - 20)(\lambda - 80)$$

$\therefore$ Eigenvalues are 20, 80

(2)

What are Eigenvectors: — need now find vector in this nullspace

$$C^TC - 20I = \begin{bmatrix} 6 & 18 \\ 18 & 54 \end{bmatrix},$$

$$v_1 = \begin{bmatrix} -3/\sqrt{10} \\ 1/\sqrt{10} \end{bmatrix}$$

$$C^TC - 80I = \begin{bmatrix} -54 & 18 \\ 18 & -6 \end{bmatrix}, \quad v_2 = \begin{bmatrix} 1/\sqrt{10} \\ 3/\sqrt{10} \end{bmatrix}$$

But we have sigma is roll which is sqrts of $\lambda = 20, 80$

$$V = \begin{bmatrix} -3/\sqrt{10} & 1/\sqrt{10} \\ 1/\sqrt{10} & 3/\sqrt{10} \end{bmatrix}, \quad \Sigma = \begin{bmatrix} 2\sqrt{5} & 0 \\ 0 & 4\sqrt{5} \end{bmatrix}$$

Now need to find U

Use second equation: $CV = U\Sigma$

← $U\Sigma$, But need U

$CV = U\Sigma$

$$\begin{bmatrix} 5 & 5 \\ -1 & 7 \end{bmatrix} \begin{bmatrix} -3/\sqrt{10} & 1/\sqrt{10} \\ 1/\sqrt{10} & 3/\sqrt{10} \end{bmatrix} = \begin{bmatrix} -\sqrt{10} & 2\sqrt{10} \\ 10/\sqrt{10} & 20/\sqrt{10} \end{bmatrix}$$

(3)

$$= \begin{bmatrix} -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix} \begin{bmatrix} 2\sqrt{5} & \\ & 4\sqrt{5} \end{bmatrix}$$

$$U = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}$$

Recap

Remember we were looking for:

$$C = U \Sigma V^T$$

, where U and V are orthogonal matrices

, and Σ is diagonal, with non negative entries.

and find it using the following equations:

$$C^T C = V \Sigma^T \Sigma V^T$$

$$CV = U\Sigma$$