Rec

Play around with the basis of

~~Evalue~~

Eigenvalues and Eigenvectors

We give the following invertible matrix A, but we need to find the EVL and EVe not of A,

But of A^2, $A^{-1} - I$

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 0 & 1 & -2 \\ 0 & 1 & 4 \end{pmatrix}$$

But we can observe the following:

$$Av = \lambda v$$

$$\therefore A^2 v = A(Av) = A(\lambda v) = \lambda(Av) = \lambda^2 v$$

$\therefore$ If v is an Evector for A, then its also Evector for A^2

$$\xrightarrow{\lambda^2 v}$$

$$A^{-1}v = A^{-1}\frac{Av}{\lambda} = \underbrace{A^{-1}A}_{I}\frac{v}{\lambda} = \frac{1}{\lambda}v \quad (\lambda \neq 0)$$

$$\Rightarrow (A^{-1} - I)v = (\lambda^{-1} - 1)v$$

∴ Now we just need to ~~figure~~ find Evalues and Evectors of A, then we can find it for A^2, A^{-1} ... etc.

∴ How do we find the Evalue?

∴ what does it mean for λ to be an Evalue of A?

mean that matrix $A - \lambda I$ is singular. Which is precisely the case when its determinant is zero.

$$0 = \det(A - \lambda I) = \det\begin{pmatrix} 1-\lambda & 2 & 3 \\ 0 & 1-\lambda & -2 \\ 0 & 1 & 4-\lambda \end{pmatrix}$$

∴ we should use Column 1, to expand the det, because we only have 1 non zero entry.

$$= (1-\lambda)\det\begin{pmatrix} 1-\lambda & -2 \\ 1 & 4-\lambda \end{pmatrix} \Rightarrow$$

$$= (1-\lambda)(\lambda^2-5\lambda+6)$$

$$= (1-\lambda)(\lambda-2)(\lambda-3)$$

$$\Rightarrow \lambda = 1, 2, 3. \leftarrow \text{Eigenvalues}$$

"Now associate e'vectors"

$\lambda = 1$

have to be in nullspace)

$$0 = (A-I)v = \begin{pmatrix} 0 & 2 & 3 \\ 0 & 0 & -2 \\ 0 & 1 & 3 \end{pmatrix} v$$

$$\Rightarrow v = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

Same can be done for $\lambda = 2$ and 3

	A	A^2	$A^{-1}-I$
e-values	λ	λ^2	$\lambda^{-1}-1$
e vectors	v	v	v

$\leftarrow$ will be same