

Re

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We covered determinants, there were 3 main ones, then 7 that followed.

Main 3 three:

- ① Det of I matrix always = 1
- ② If you switch two rows in matrix, the det switches sign
- ③ Det is a linear function of each row & column

Today's problem:

Finding the determinant of matrices by using these properties.

∴ Here we have 4 matrices

$$A = \begin{bmatrix} 1 & 0 & 1 & 2 & 0 & 1 & 3 & 0 & 1 \\ 1 & 0 & 2 & 2 & 0 & 2 & 3 & 0 & 2 \\ 1 & 0 & 3 & 2 & 0 & 3 & 3 & 0 & 3 \end{bmatrix}$$

(Vander Monde matrix)

$$B = \begin{bmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \begin{bmatrix} 1 & -4 & 5 \end{bmatrix}$$

$$D = \begin{bmatrix} 0 & 1 & 3 \\ -1 & 0 & 4 \\ -3 & 4 & 0 \end{bmatrix}$$

$$A = \begin{bmatrix} 101 & 201 & 301 \\ 102 & 202 & 302 \\ 103 & 203 & 303 \end{bmatrix}$$

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⇒ let's do a bit of elimination, also
 as this does not have the det's of matrix.

$$\therefore |A| = \begin{bmatrix} 101 & 201 & 301 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} = 0$$

Subtract row 2 from row 1.

Subtract row 3 from row 1.

∴ if you have 2 equal rows in matrix
 then $\det = 0$

$$B = \begin{bmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{bmatrix} \Rightarrow \text{let's do elimination again.}$$

$$|B| = \begin{bmatrix} 1 & a & a^2 \\ 0 & (b-a)(b+a) & -Subtract \text{ from row 1} \\ 0 & (c-a)(c+a) & -Subtract \text{ from row 1} \end{bmatrix}$$

let's use the 3rd property - - -

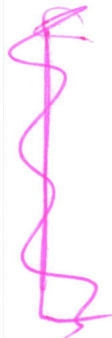
\therefore det is linear in each row separately. (3)


= See factor $b-a$ it shows up in every

entry in row 2.

\therefore Pull out this factor of $b-a$

Like wise with $c-a$ in third row.


$$= (b-a)(c-a) \begin{vmatrix} 1 & a & a^2 \\ 0 & 1 & b+a \\ 0 & 1 & c+a \end{vmatrix}$$

We need to get rid of $c+a$, to make it
upper triangular to get rule  Come Back
 \therefore do another Elementary Step

$$= (b-a)(c-a) \begin{vmatrix} 1 & a & a^2 \\ 0 & 1 & b+a \\ 0 & 0 & c-b \end{vmatrix} = (b-a)(c-a)(c-b)$$

$$C = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \begin{bmatrix} 1 & -4 & 5 \end{bmatrix}$$

(4)

\therefore rank 1 matrix

\therefore column vector \times row vector.

\therefore result will be 3×3 matrix.

$$1 \times 1 \times 1, -4, 5$$

$$2 \times 1, \dots$$

$$3 \times 1, \dots$$

\therefore all the rows will be linearly independent.

\therefore matrix is singular.

\therefore if singular $\det = 0$,
 $|C| = 0$

$$A = \begin{bmatrix} 0 & 1 & 3 \\ -1 & 0 & 4 \\ -3 & -4 & 0 \end{bmatrix}$$

\therefore this matrix is skew symmetric.

\therefore if do $A^T = -A$ (not A)

But $A^T = -A$ (are same matrix)

$$|A| = |A^T| = |-A| = \uparrow |A|$$

But why not true

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— the determinant is linear on each row separately

→ So you can't just pull out factor, you have to pull out δ for each row of matrix.

$$\therefore | -D | = (-1)^{\text{(rows)}} | D | = -| D |$$

$$| D | = 0$$

Is true that det of all ~~row~~ matrices = 0?
⇒ is true in every case