Res. LU Decomposition

Find LU - Decomposition of the matrix

$$A = \begin{bmatrix} 1 & 0 & 1 \\ a & a & a \\ b & b & a \end{bmatrix}$$

For which real numbers a and b does it exist?

has variables as well as # (numbers)

Remember: Previously we found U

in Form: $\begin{bmatrix} x & x & x \\ 0 & x & x \\ 0 & 0 & x \end{bmatrix}$

where x ~~was~~ in diagonal was pivot values and below diagonal there were zero value.

But now we need to find L as well.

$$\therefore \begin{bmatrix} 1 & 0 & 1 \\ a & a & a \\ b & b & a \end{bmatrix} \longrightarrow A \begin{bmatrix} 1 & 0 & 1 \\ 0 & a & 0 \\ b & b & a \end{bmatrix}$$

$\therefore$ R_{21} we want zero

How do we get it?

~~$-aR_2+$~~

$-aR_1 + R_2 \Rightarrow R_2$ $\therefore$ Rember $R_2 \Rightarrow k = a$ [inverse]

A

$\boxed{R_{21}} = -a(1) + a$
$= -a + a$
$= 0$

$\boxed{R_{22}} = -a(0) + a$
$= 0 + a$
$= a$

$\boxed{R_{23}} = -a(1) + a$
$= -a + a$
$= 0$

$\therefore$ Keep track of Elementary

$$E_{21} = \begin{bmatrix} 1 & & \\ -a & 1 & \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & a & 0 \\ b & b & a \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & 1 \\ 0 & a & 0 \\ 0 & b & (a-b) \end{bmatrix} \quad A$$

want zero

Remember $x = b$ [inverse]

$\therefore -bR_1 + R_3 \Rightarrow R_3$

(Elementary) $R_{31} = -b$

$\boxed{R_{31}} = -b(1)+b$
$= -b+b$
$= 0$

$$E_{31} \begin{bmatrix} 1 & & \\ -a & 1 & \\ -b & 0 & 1 \end{bmatrix}$$

$\boxed{R_{32}} = -b(0)+b$
$0 + b$
$= b$

$\boxed{R_{33}} = -b(1)+a$
$-b+a$
or $(a-b)$

$$\longrightarrow$$

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & a & 0 \\ 0 & b & a-b \end{bmatrix}$$

(b/a) swap to get rid.

zero

Assume $a \neq 0$, we need it to be a pivot.

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & a & 0 \\ 0 & b & a-b \end{bmatrix} \rightarrow \text{A} \begin{bmatrix} 1 & 0 & 1 \\ 0 & a & 0 \\ 0 & 0 & a-b \end{bmatrix}$$

$\therefore -\frac{b}{a}R_2 + R_3 \Rightarrow R_3$

Remember $K = \frac{b}{a}$

$\text{Element}_{32} = -\frac{b}{a}$

$$E_{32} \begin{bmatrix} 1 & & \\ -a & 1 & \\ -b & -\frac{b}{a} & 1 \end{bmatrix}$$

A

$\boxed{R_{31}} = 0$

$\boxed{R_{32}} = -\frac{b}{a}(a) + b = -b + b = 0$

$\boxed{R_{33}} = -\frac{b}{a}(0) + (a-b)$
$(a-b)$

$$\therefore U = \begin{bmatrix} 1 & 0 & 1 \\ 0 & a & 0 \\ 0 & 0 & a-b \end{bmatrix}$$

$$L = \begin{bmatrix} 1 & 0 & \\ a & 1 & 0 \\ b & \frac{b}{a} & 1 \end{bmatrix}$$

OR we been doing all the way:

$$E_{32} E_{31} E_{21} A = U$$

$$\Rightarrow A = E_{21}^{-1} E_{31}^{-1} E_{32}^{-1} U$$