Le

①

Now we start using Complex numbers.

Since, even a real matrix, can have Complex e'values

∴ What do you do, when numbers become Complex numbers

∴ What happens when vectors are Complex, or matrices are Complex

∴ inner product or dot product of two Complex vectors

⇒ ∴ What will be the change.

∴ Most Important Example of Complex Matrices

∴ Comes in Fourier matrix.

The most Imp Complex matrix

: Fast Fourier Transform (FFT)
— Used in thousand places.
— transformed whole industries

Example:

$Z = \begin{bmatrix} z_1 \\ z_2 \\ \vdots \\ z_n \end{bmatrix}$ in $\mathbb{C}^n$ ⇒ in complex space

Length : Now $Z^T Z$ is no good.

(No longer in $\mathbb{R}^n$)

of transpose : $[z_1, z_2 \dots z_n]$ But not correct.

⇒ Remember Lenght squared should be positive

But what if $z_1 + i$ ---??

But I really need $\bar{Z}$ ~ Conjugate.

$\bar{Z}^T Z =$ is good.

⇒ and will give a positive length.

Eg $\underbrace{[1 \; -i]}_{\text{Conjugate}} \begin{bmatrix} 1 \\ i \end{bmatrix} = 1+1 = 2.$

∴ one symbol to do both

$$z^H z \qquad = |z_1|^2 + \dots + |z_n|^2$$

Hermitian (Hermite)

∴ above is length squared

∴ What is inner product? They should match.

for real vectors used to be: $y^T x$

Complex vectors now: $y^H x$

Symmetric matrix $A^T = A$ [real vectors] Eigen

But Complex: ; $\bar{A}^T = A = \begin{bmatrix} 2 & 3+i \\ 3-i & 5 \end{bmatrix}$

∴ Hermitian Matrix

∴ $A^H = A$ Has: ① Real E'values ② $\perp$ E'vectors

Perpendicular vectors (Real): $q_1 q_2 \dots q_n$

Now $\bar{q}_i^T q_j = \begin{cases} 0 & i \neq j \\ 1 & i = j \end{cases}$

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Create matrix out of these guys:

$$Q = \begin{bmatrix} | & | & & | \\ q_1 & q_2 & \cdots & q_n \\ | & | & & | \end{bmatrix}$$

$Q^T Q = I$ orthogonal matrix. | Unitary Matrix.

But what has changed now?

$$Q^H Q = I$$

Matrix that's all around us? Fourier...

Vector 1's First Column

$n \times n$

$$F_n = \begin{bmatrix} 1 & 1 & 1 & & 1 \\ 1 & w & w^2 & & w^{n-1} \\ 1 & w^2 & w^4 & & w^{2(n-1)} \\ 1 & & & & \\ 1 & w^{n-1} & w^{2(n-1)} & & w^{(n-1)^2} \end{bmatrix}$$

$(F_n)_{ij} = w^{ij}$

all entries are powers $\underline{w}$

none is zero

w very special number.

$$w^n = 1$$

$$w = e^{i 2\pi/n}$$

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$$F_4 = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & i & i^2 & i^3 \\ 1 & i^2 & i^4 & i^6 \\ 1 & i^3 & i^6 & i^9 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & i & -1 & \cdot i \\ 1 & -1 & 1 & -1 \\ 1 & -1 & i & i \end{bmatrix}$$

∴ Column are orthogonal $\longrightarrow$

to make it orthonormal $\quad F_4 \tfrac{1}{2}\begin{bmatrix} \quad \end{bmatrix} = \tfrac{1}{2}\begin{bmatrix} \quad \end{bmatrix}$

∴ Column now are orthonormal

$$F_4^H F_4 = \underline{I}$$

$\longrightarrow$

Eg

$$\begin{bmatrix} F_{64} \end{bmatrix} \begin{bmatrix} I & D \\ I & -D \end{bmatrix} \begin{bmatrix} F_{32} & 0 \\ 0 & F_{32} \end{bmatrix} \begin{bmatrix} \quad \end{bmatrix}^{P} \qquad D = \begin{bmatrix} 1 & & & \\ & w & & \\ & & w^2 & \\ & & & \ddots \; w^{31} \end{bmatrix}$$

$$\left| \tfrac{1}{2} n \log_2 n \right|$$

1000 times n

(6)

$n = 1024 = 2^{10}$

$n^2 > 1\,000\,000 \rightarrow$ time $\boxed{1024 \times 1024}$

5×n.

$\frac{1}{2} n \log_2 n = (1024)\frac{10}{2} = \boxed{5 \times 1024}$

reduced the Calculation by factor of 200