Rec

We covered determinants, there were 3 main ones, then 7 that followed.

Main 3 three:

1. Det of I matrix always = 1
2. If you switch two rows in matrix, the det switches sign
3. Det is a linear function of each row separately

Todays problem:

Finding the determinant of matrices by using these properties.

∴ Here we have 4 matrices

$$A = \begin{bmatrix} 101 & 201 & 301 \\ 102 & 202 & 302 \\ 103 & 203 & 303 \end{bmatrix}$$

(Vandermonde matrix)

$$B = \begin{bmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \begin{bmatrix} 1 & -4 & 5 \end{bmatrix}$$

$$D = \begin{bmatrix} 0 & 1 & 3 \\ -1 & 0 & 4 \\ -3 & 4 & 0 \end{bmatrix}$$

$$A = \begin{bmatrix} 101 & 201 & 301 \\ 102 & 202 & 302 \\ 103 & 203 & 303 \end{bmatrix}$$

⇒ lets do a bit of elimination, also doing this does not change the det's of matrix.

$$\therefore |A| = \begin{bmatrix} 101 & 201 & 301 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} = 0$$

Subtract row2 from row1.
Subtract row 3 from row1.

∴ if you have 2 equal rows in matrix then det = 0

$$|B| = \begin{bmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{bmatrix} \Rightarrow$$ lets do elimination again.

$$|B| = \begin{bmatrix} 1 & a & a^2 \\ a & (b-a) & (b-a)(b+a) \\ 0 & (c-a) & (c-a)(c+a) \end{bmatrix}$$ — subtract from row1 — subtract from row1.

lets use the 3rd property ....

③

∴ det is linear on each row separately.

= See factor $b-a$ it shows up in every entry in row 2.
∴ Pull out this factor of $b-a$

∴ Likewise with $c-a$ in third row.

$$= (b-a)(c-a)\begin{vmatrix} 1 & a & a^2 \\ 0 & 1 & b+a \\ 0 & 1 & c+a \end{vmatrix}$$

We need to get rid of $c+a$, to make it upper triangular to get rule $\boxed{}$ Come Back

∴ do another Elementary Step

$$= (b-a)(c-a)\begin{vmatrix} 1 & a & a^2 \\ 0 & 1 & b+a \\ 0 & 0 & c-b \end{vmatrix} = (b-a)(c-a)(c-b)$$

$$C = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \begin{bmatrix} 1 & -4 & 5 \end{bmatrix}$$

$\therefore$ rank 1 matrix

$\therefore$ column vector $\times$ row vector.

$\therefore$ result will be 3×3 matrix

ie $1 \times \quad 1, -4, 5$

$2 \times \quad 1, - -$

$3 \times \quad 1, - -$

$\therefore$ all the rows will be linearly independent.

$\therefore$ matrix is singular.

$\therefore$ if singular $\quad \det = 0$.

$|C| = 0$

$$\Delta = \begin{bmatrix} 0 & 1 & 3 \\ -1 & 0 & 4 \\ -3 & -4 & 0 \end{bmatrix}$$

$\therefore$ this matrix is skew symmetric.

(not Δ)

$\therefore$ if do $\Delta^T = -\Delta$

But $\Delta^T = -\Delta$ (are same matrix)

$|\Delta| = |\Delta^T| = |-\Delta| = -|\Delta|$

not true

But why not three

①

- the determinant is linear on each row separately

→ So you can't just pull out factor, you have to pull out for each row of matrix. (3 rows)

$$\therefore |-D| = (-1)^3 |D| = -|D|$$

$$|D| = 0$$

Is true that det of all skew matrices $= 0$?

→ is true in every case