Module 3 1:22.

Projection onto general subspaces

In last session we derived the formula for orthogonal projection of vectors onto n dimensional subspace.

In this session we will run through a simple example

"we define x to be a 3D vector, given by (2,1,1), and we define two basis vectors for our two dim subspace b_1 to be (1,2,0) and b_2 to be (1,1,0)

$$\therefore x = \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix}$$

$$b_1 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} \quad b_2 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$$

(see pic)

Ratmean U which is spanned by b_1 and b_2, is going to be (~~hyperplane~~) the plain (see pic) (This call is U)

$$U = [b_1, b_2]$$

The (~~orthogonal~~) orthogonal projection is given as "Π_U of x is B times λ, and we define B now to be b_1 and b_2 concatenated, which is (1,2,0), (1,1,0), and λ was given as B transpose B inverse times B transpose x"

$$\Pi_U(x) = B\lambda$$

$$B = [b_1 | b_2]$$

$$= \begin{bmatrix} 1 & 1 \\ 2 & 1 \\ 0 & 0 \end{bmatrix}$$

$$\lambda = (B^T B)^{-1} B^T x$$

"B transpose X is given as (4,3) vector"

$$B^T X = \begin{bmatrix} 4 \\ 3 \end{bmatrix}$$

"B transpose B is two by two matrix which is (5,3), (3,2)"

$$B^T B = \begin{bmatrix} 5 & 3 \\ 3 & 2 \end{bmatrix}$$

"Now we solve for λ as B transpose B inverse time B transpose X", which mean we find λ

$$B^T B \lambda = B^T X$$

Using gaussian elimination, we arrive at

λ equals $\begin{bmatrix} -1 \\ 3 \end{bmatrix}$

$$\Rightarrow \lambda = \begin{bmatrix} -1 \\ 3 \end{bmatrix}$$

"this implies our projection of x onto space spanned by two b vectors"

$$\Rightarrow \Pi_U(x) = -1b_1 + 3b_2 = \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}$$

Another diagram we expect will correspond to: (see pic), this vector here

This result make sense because our projected point has a third component, the zero

And our subspace requires ~~the~~ that the ~~our~~ 3rd component is always zero

Our projected vector $\begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}$ is still a 3D vector, but we can represent it using 2 coordinates if we use the basis defined by b_1 and b_2

Therefore that is the compact representation of the projection of X onto the two lower dimensional subspace.

In this session, we looked at a concrete example of the orthogonal projection of the 3D coin vector onto a 2D coin subspace

In next session we going to exploit orthogonal projection and derive a dimensionality reduction algorithm called Principal Component Analysis (PCA)