Changing of Basis

①

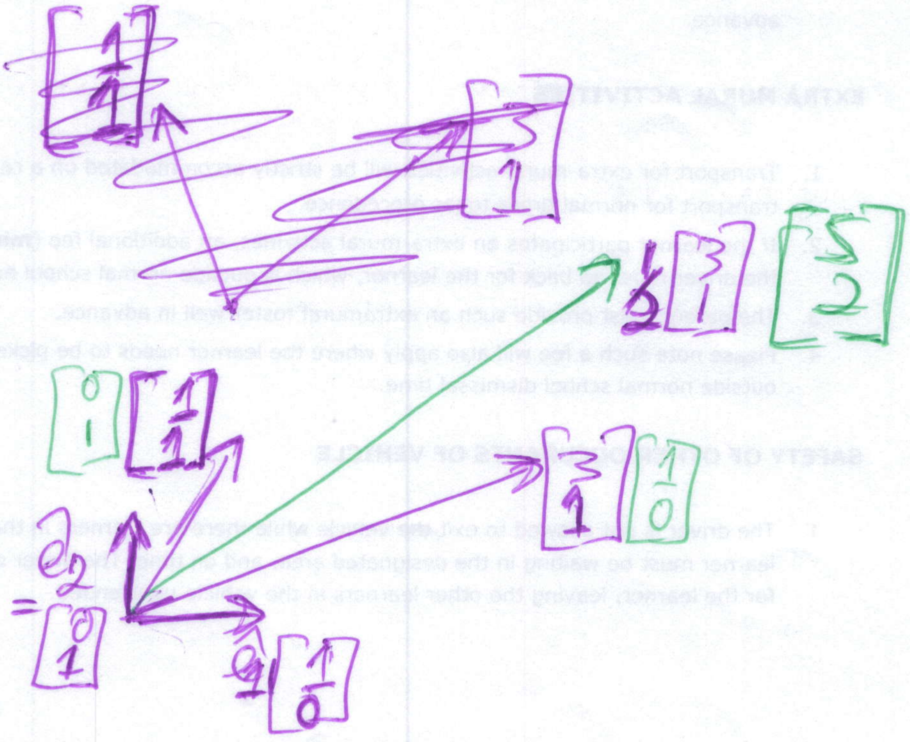
we said columns of (transformation) matrix are the axis of the new basis vectors of mapping in coordinate system.

⇒ Now we going to look at how to transform a vector from one set of Basis vectors to another.

Eg Two new Basis vectors, that describe ~~tran from~~ the world of Panda Bear, and Panda's world is orange. [two following Basis vectors]

$\begin{bmatrix}3\\1\end{bmatrix}$ $\begin{bmatrix}1\\1\end{bmatrix}$

Panda's Basis vectors



- P. Bears Basis vectors

$\begin{bmatrix}3\\1\end{bmatrix}, \begin{bmatrix}1\\1\end{bmatrix}$ in my frame

- Bear's Transformation matrix:

$$\begin{bmatrix}3&1\\1&1\end{bmatrix}$$

Lets take some vector, I want to transform, which in Bears world is $\frac{1}{2}\begin{bmatrix}3\\1\end{bmatrix}$

$$\begin{bmatrix}3&1\\1&1\end{bmatrix}\begin{bmatrix}\frac{3}{2}\\\frac{1}{2}\end{bmatrix}=\begin{bmatrix}5\\2\end{bmatrix} \leftarrow \text{in my world.}$$

my vector.

Bears vector

Bear's Basis in my Coordinate system → So it transforms Bears vector into my world

this is a problem ..... Usually I want to translate my world into Bears world.

So we need to figure out how to go the other way.

How do we perform this reverse process.

⇒ it will involve the INVERSE

$B \Rightarrow B^{-1}$

⇒ Need to get inverse of B

∴ So we have $\begin{bmatrix} 3 & 1 \\ 1 & 1 \end{bmatrix}$

∴ Inverse $\frac{1}{2}\begin{bmatrix} 1 & -1 \\ -1 & 3 \end{bmatrix}$

this is my Basis vectors in Bears coordinate or Bear's ~~world~~ world

∴ $\hat{e}_1 \begin{bmatrix} 1 \\ 0 \end{bmatrix} \Rightarrow$ this will be $\frac{1}{2}\begin{bmatrix} 1 \\ -1 \end{bmatrix}$ in Bears world

$\hat{e}_2 \begin{bmatrix} 0 \\ 1 \end{bmatrix} \Rightarrow$ will be $\frac{1}{2}\begin{bmatrix} -1 \\ 3 \end{bmatrix}$ in Bears world

∴ take my vecter $\begin{bmatrix}5\\2\end{bmatrix}$

$$\frac{1}{2}\begin{bmatrix}1 & -1\\ -1 & 3\end{bmatrix}\begin{bmatrix}5\\2\end{bmatrix} = 5\begin{bmatrix}\frac{1}{2}\\ -\frac{1}{2}\end{bmatrix} + 2\begin{bmatrix}-1\\3\end{bmatrix} = \underbrace{\frac{1}{2}\begin{bmatrix}3\\2\end{bmatrix}}_{\text{Bears vecter}}$$

Recap:

① If we ~~need~~ wat to take Bears vecter in my world.
— we need Bears basis in my Coordinate system

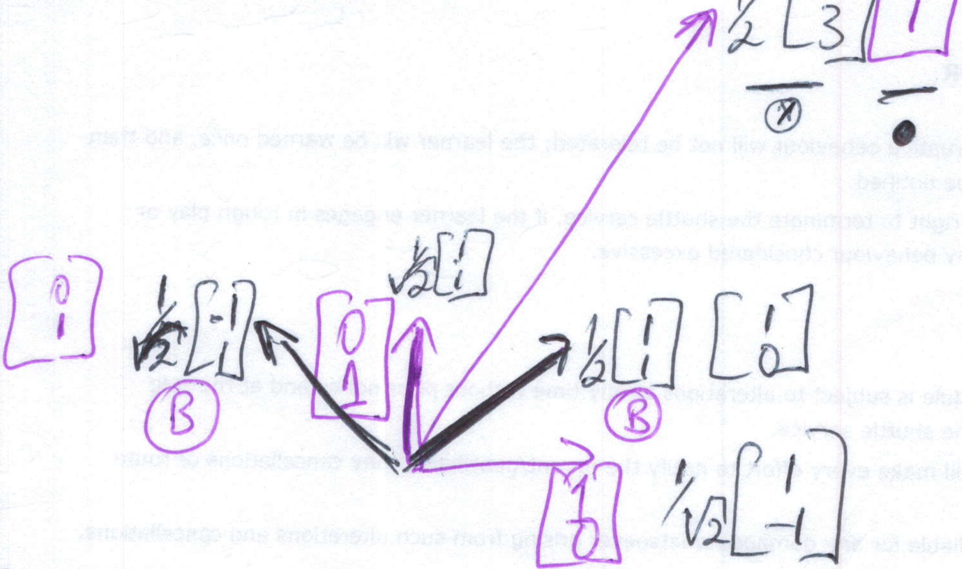
② If I want my Basis in Bears coordinate system
— for reversal.

⟶ Bit Counterintuitive.

Lets do another Example

eg

where Beans world will be an orthonormal basis vector set



Transformation Matrix

B

$\frac{1}{\sqrt{2}}\begin{bmatrix}1 & -1\\1 & 1\end{bmatrix}$ this transforms a vector of Bean — $\begin{bmatrix}2\\1\end{bmatrix}$ in Bean world.

$$\frac{1}{\sqrt{2}}\begin{bmatrix}1 & -1\\1 & 1\end{bmatrix}\begin{bmatrix}2\\1\end{bmatrix} = \frac{1}{\sqrt{2}}\underset{\otimes}{\begin{bmatrix}1\\3\end{bmatrix}}$$ ← vector in my world.

To get Reverse: B^{-1}

and B is an orthonormal Basis: that means determinant = 1

B^{-1}

$$\frac{1}{\sqrt{2}}\begin{bmatrix}1 & 1\\-1 & 1\end{bmatrix}\frac{1}{\sqrt{2}}\begin{bmatrix}1\\3\end{bmatrix} = \frac{1}{2}\begin{bmatrix}4\\2\end{bmatrix} = \begin{bmatrix}2\\1\end{bmatrix}$$

$\frac{1}{\sqrt{2}}\begin{bmatrix}1\\3\end{bmatrix}$ ← vector in my world

$\begin{bmatrix}2\\1\end{bmatrix}$ ← vector in Bean world

(and change signs)

This was all prep for Fun part, which is said before, if new Basis vectors are orthogonal, we can then use projections.

Take my version of vector and dot it with Bears axis then will get answer of vector in Bears world

$$\frac{1}{\sqrt{2}}\begin{bmatrix}1\\3\end{bmatrix} \cdot \frac{1}{\sqrt{2}}\begin{bmatrix}1\\1\end{bmatrix} = \frac{1}{2}4 = \begin{bmatrix}2\\ \end{bmatrix}$$

→ first component of Bears vector as its first of Bears axis.

$$\frac{1}{\sqrt{2}}\begin{bmatrix}1\\3\end{bmatrix} \cdot \frac{1}{\sqrt{2}}\begin{bmatrix}-1\\1\end{bmatrix} = \frac{1}{2}2 = \begin{bmatrix}\\1\end{bmatrix}$$

used projection to translate my vector to Bears vector just using the dot product.

Remember: with vector product, have to remember to normalize by their lengths (But lengths are 1)

⇒ demonstrated don't have to do complicated matrix maths just use dot product (But only if Bears vectors are orthogonal)