Module 4 ①

Power Series

In this session, we getting a taster of what the Taylor series is doing, before we try and write down anything like a formal definition.

This will allow us to have a go at a graphical question first, which is much like how we approached differentiation at the start of sessions

Taylor Series are also referred to as Power Series, this is because they composed of coefficients in front of increasing powers of x

So we can write a simple, generalized expression for a power series

as g of x, equals a plus bx plus cx2 plus dx3 etc
Potentially going off for infinitely many terms
depending on what function we considering.

$$g(x) = a + bx + cx^2 + dx^3 + \dots$$

When we calculate the Taylor series in
Next session, we will build up Coefficient
by Coefficient, where each term that
we add improve the approximation
In many cases, we will then be able to see
a pattern emerges in the coefficient, which
thankfully saves a the trouble of calculating
infinitely many term.
However many of the applications of the Taylor Series
involves just making use of the First
few terms of the series, in the hope
that this will be a good enough approximation
for certain application.

Starting from just a single term, we call these expressions, zeroth, 1st, 2nd, 3rd order of approximations, etc

Collectively these short sections of the series are called truncated series

$$g(x) = a + bx + cx^2 + dx^3 + \dots$$

$$g_0(x) = a$$

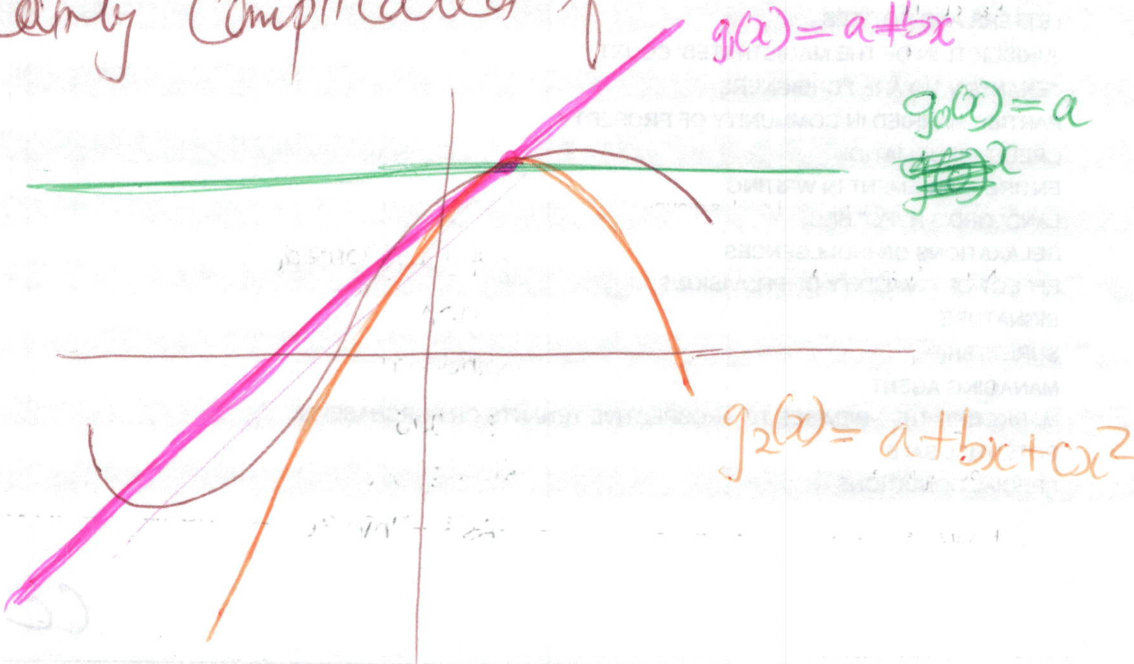
$$g_1(x) = a + bx$$

$$g_2(x) = a + bx + cx^2$$

$$g_3(x) = a + bx + cx^2 + dx^2$$

$$g_n(x) =$$

So lets begin by looking at some arbitary but fairly complicated function



May have this shape...

all we going to do, is focus on one particular point on this curve (•)

Then we going to start building our function, by trying to make it, more and more, like the point we have chosen

So, as the first term of our generalized power series, is just a number a, and we ignoring all the other terms for now. A (page 5)

We know, that our opening approximation must just be a number a that goes through the same point

So we can just add our zeroth order approximation function to our plot

Clearly, this did not do a great job to approximating our curve (Brown)

So, then lets go to our 1st order approximation, (page 5) B which can also have a gradient

And if we would like to match our function, at this point, it should have the same gradient

A $g_0(x) = a$

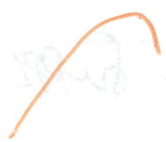
B $g_1(x) = a + bx$

Clearly our approximation has improved a little in the region around our point, although there is still plenty of room for improvement

We can also move on to the second order function which we can see is a parabola

$$g_2(x) = a + bx + cx^2$$

But at this point things get a little tough to draw, and matching second derivatives by eye is not easy, But may look like



Hopefully, without going into any of the details about the maths, we will now be able to match up some wacky mystery functions to their corresponding truncated Taylor series approximations, in next exercise

In next session, we going to walk through the detail derivation of the terms.

But the above was an attempt to allow us not to loose sight what we trying to achieve at end.