Module 3 ①

multivariate Chain rule

In last module, we saw something called the Total derivative, which showed us that when we have a multivariable function such as $f(x,y,z)$, but variables themselves (x,y,z) each a function of some additional variable t and if we want to calculate the derivative of f, w.r.t. t, we can use the expression:

$$f(x,y,z) = \sin(x)e^{yz^2}$$

$$x = t-1;\ y = t^2;\ z = \frac{1}{t}$$

This expression:

$$\frac{df}{dt} = \frac{\partial f}{\partial x}\frac{dx}{dt} + \frac{\partial f}{\partial y}\frac{dy}{dt} + \frac{\partial f}{\partial z}\frac{dz}{dt}$$

which is simply the sum of the chains relating f to t, through each of its 3 variables

This allows us to calculate the result in a piecewise manner

$$\frac{df}{dt} = \cos(t-1)e$$

rather than substituting everything at start, and computers are really good at solving piecewise problems quickly.

What we're going to do now, is generalize this concept and also simplify the notation a little

Eg Have function of n variables $x_1, x_2, \ldots$ up to x_n, f can write just as $f(\mathbf{x})$

$$f(x_1, x_2, \ldots, x_n) = f(\mathbf{x})$$

But written the x in bold to help us remember that the x represents a sequence of variables, which we will now more conveniently think of as an n dimensional vector.

once again each of the component of our x vector, are themselves functions of some other variable t

and $x_1(t) = \ldots$

$x_2(t) = \ldots$

$x_3(t) = \ldots$

$x_n(t) = \ldots$

and what we would like to know the derivative of f w.r.t t

$$\frac{df}{dt} = ?$$

Lets start: $f(x) = f(x_1, x_2, \ldots, x_n)$

Remember, also, that each component of x is also a function of t

Here we again, want to build an expression linking f to t through the sum of changes of each of its variables

So we going to need all the partial derivatives A of f w.r.t x, as well as the derivatives B of each of the component of x w.r.t t

$$\frac{\partial f}{\partial x} = \underbrace{\begin{bmatrix} \partial f/\partial x_1 \\ \partial f/\partial x_2 \\ \vdots \\ \partial f/\partial x_n \end{bmatrix}}_{A} \qquad \frac{dx}{dt} \underbrace{\begin{bmatrix} dx_1/dt \\ dx_2/dt \\ \vdots \\ dx_n/dt \end{bmatrix}}_{B}$$

Once again, we going to store all of the object in a pair of n dimensional vectors

Finally we looking to build a multivariable chain rule expression

So we looking to find the sum of the product of each pair of terms in same position in each vector.

Thinking back to our LA, this is exactly what the dot product does

But there is no need for us to write out these vectors in full,

So we can simply write the dot of our two multivariable derivative expressions

$$\frac{df}{dt} = \frac{\partial f}{\partial \mathbf{x}} \cdot \frac{d\mathbf{x}}{dt}$$

and that's it

We now have our generalized form. for the multivariable chain rule, expressed nice and neatly

So we can now update our list of tools to reflect it

But all of the other 1st (3) multivariable time saving rules already work for multivariate problems as they are

— will put into practice soon