

# Introduction to Matrices →

— Solving Simultaneous Equation Problems.

Matrices → objects that rotate and stretch vectors

, But also objects that let us solve  
the eg with apples and bananas personally

Eg lets visit again apple / Bananas problem.  
 $a \Rightarrow$  apples } walked into shop to buy  
 $b \Rightarrow$  Bananas }

$$\begin{array}{l} \text{(Day 1)} \\ \text{(Day 2)} \end{array} \quad \begin{array}{l} 2a + 3b = 8 \\ 10a + b = 13 \end{array}$$

Silly above, But in business above,

Price discovery happens all time  
(Complicated contracts, prices)

But these can be written in another way.



Using matrix:

(2)

$$\begin{bmatrix} 2 & 3 \\ 10 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 8 \\ 13 \end{bmatrix}$$

Matrix

∴ multiply row(element) with column(element)

$$\begin{pmatrix} 2a + 3b \\ 10a + 1b \end{pmatrix} = \begin{pmatrix} 8 \\ 13 \end{pmatrix}$$

(Back to original)

But they look like vectors

→ and we dealt with vectors before.

Matrix operates on vector to give result vector

⇒ so what vector (transform) / do we use to give the guy on right side (result)

Let's multiply matrix with, unit basis vector (x, y) axes

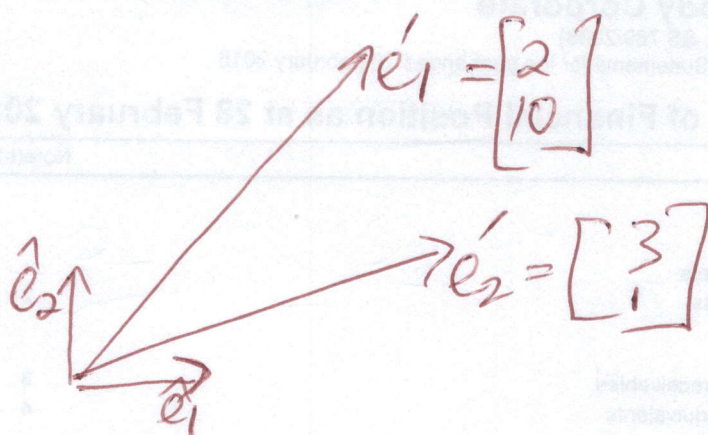
$$\begin{bmatrix} 2 & 3 \\ 10 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 2 \\ 10 \end{bmatrix}$$

$$\begin{matrix} 2 \times 1 + 3 \times 0 \\ 10 \times 1 + 1 \times 0 \end{matrix} =$$



Geometrically it does the following:

③



⇒ takes unit vector  $\hat{e}_1$ , and transforms it to  
another place  $\hat{e}'_1$

⇒ and do it with other basis vector  $(0,1)$

$$\begin{bmatrix} 2 & 3 \\ 10 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \end{bmatrix} \quad \text{also}$$

⇒ that means now, the other basis vector  $\hat{e}_2$ , gets  
transformed into  $\hat{e}'_2$

7/10 what the matrix does it ~~transform~~  
moves the basis vectors in some way, transforms  
them, changes the space.



So matrix  $\begin{bmatrix} 2 & 3 \\ 10 & 1 \end{bmatrix}$  is a function that  
operates on input vectors and gives  
as other output vectors

(4)

And set of simultaneous equations is  
saying "what vector do I need  
in order to get a transformed  
product at position  $\begin{bmatrix} 8 \\ 13 \end{bmatrix}$  in  
order to get output  $\begin{bmatrix} 8 \\ 13 \end{bmatrix}$ "

We can now see what we mean by  
linear algebra:

- takes input values ( $a$  and  $b$ )  
and multiply it with constants  
(so everything is linear)
- and its algebra that its notation  
describing mathematical objects  
in manipulation these notations



⑤  
LA, is a mathematical system of  
manipulating vectors ~~spaces~~ in space  
described by vectors

There is a deep Connection between

- ① Simultaneous equations (Called matrices)
- ② Vectors (Covered previously)

Key: to Solving Sim. equation problems  
is appreciating how vectors are transformed  
by matrices, which is the heart  
of LA.