

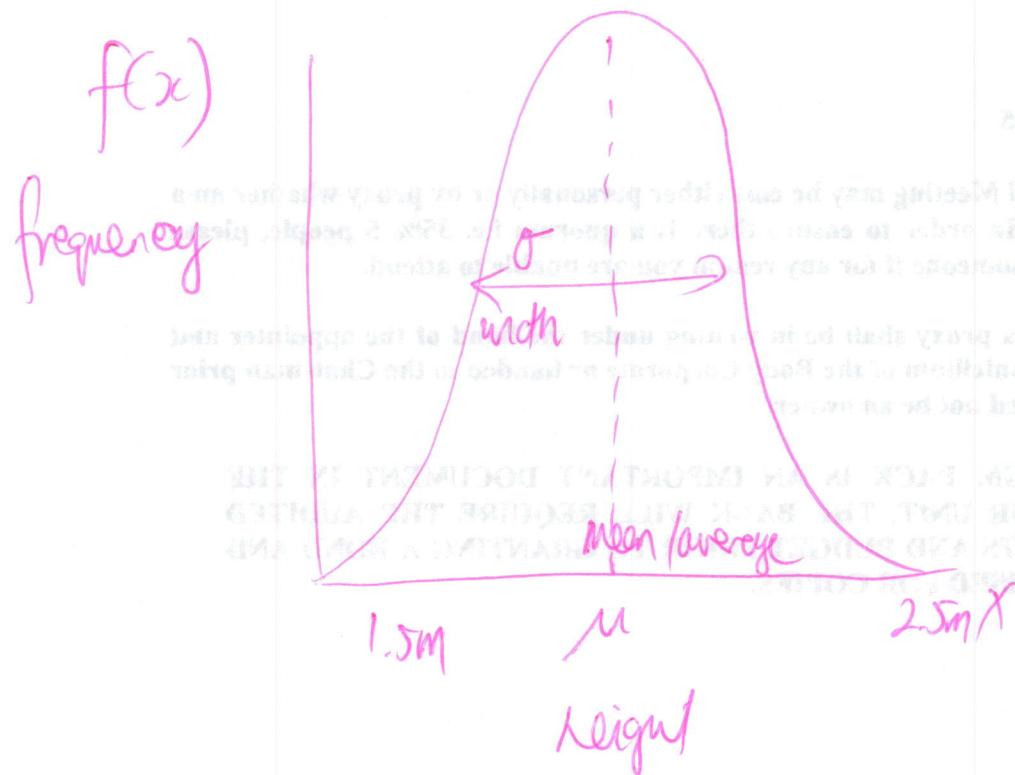
Module 5

Intro

David Dye says!: ~~we will start to do the Calc~~
Yes, in this module, we'll start to do the Calc
We have done, and put together with
lectures, mind to solve equations

In this first session, we will look at example
and, where we need to find no gradient,
at the derivative, in order to solve equation.

Using, what's called, the Newton-Raphson Method



Say we have, we know the distribution of Length again with mean, average $\rightarrow \mu(\mu)$ and width (σ) and we want to fit an equation to fit that distribution.

So we don't have to bother about carrying round all the data points we just have a model with 2 (data points) parameters, a mean and width. And we can just do everything using just the model and that will be loads faster and simpler, and would let us make predictions and so on and would be much, much nice, but, how do we find the right parameters for the model?

How do we find the best μ and σ we can.

What we going to do is find some expression for how well the model fits the data

And then look at how that goodness of fit

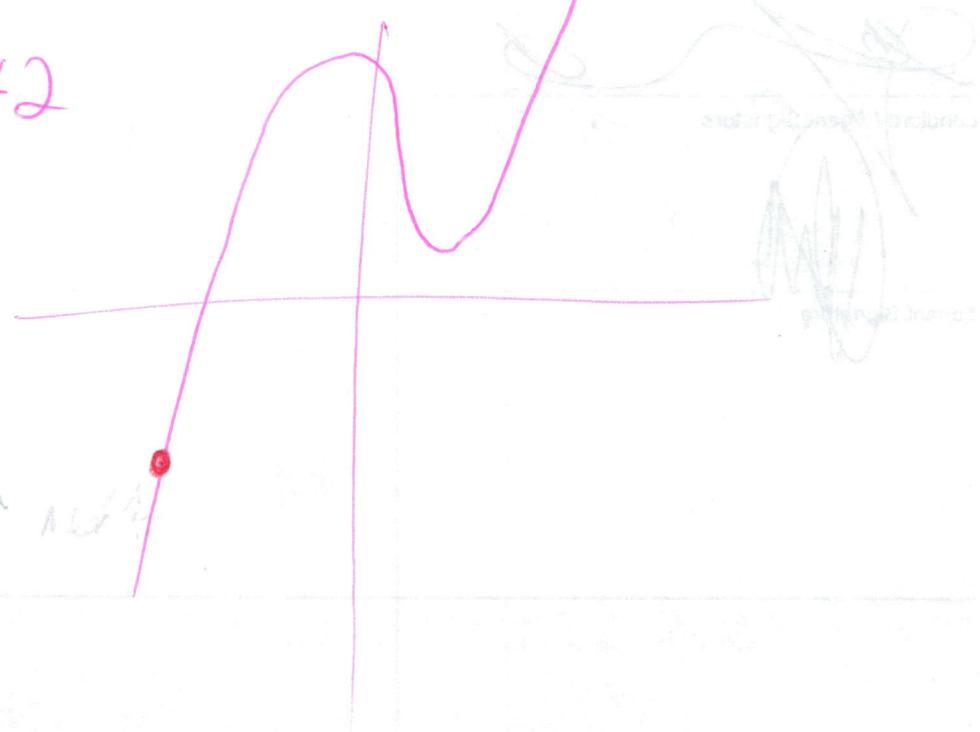
varies as the fitting parameters μ and σ change

try to solve an equation where the fitting parameters would be in the equation

But in order to get there, in next module, we first need to do a bit more calculus

So let's look at the equation of line:

$$y = k^3x - 2x + 2$$



If we differentiate this equation, we get quadratic equation: $3x^2 - 2$

$$\frac{dy}{dx} = 3x^2 - 2$$

And that that quadratic will have 2 Spurts

∴ two turning points will exist

one maximum, one minimum

Say I do not know what the equation will look like
and I do not have enough computer resources
to graph the values ^{out} at every point

... and exist in so many dim, I cannot visualise it at all.

But say I only need to find the solution to

equation where $y = 0$

$$\text{Where } x^3 - 2x^2 = 0$$

$$[x^3 - 2x^2 = 0]$$

We can see that there actually only 1 solution here

on the graph, but there could be more.

Depending on the exact equation we want to solve.

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Now, say that I have an idea that I want to hunt for a solution somewhere near some initial guess (red dot •)

For instance,
The gradient is very pretty small and positive, so my guess that I may need a slightly negative value of x , say -2 , maybe somewhere near a solution

Now, if I can evaluate the value of the function at my guess of $x = -2$, I find the function has a value of -2

And if I look what the gradient is at that value $x = -2$, I find that the gradient is positive and it's 10

Now, I can extrapolate the gradient, to the intercept with y -axis, which is will be my first guess of the solution of equation, ... there it find intercept with y -axis.

So, I can use that value of x at the intercept as a new estimate for what the solution to the equation is.

Effectively I am guessing that the function is a straight line

and I am using the gradient to extrapolate to find the solution.

It isn't really a straight line, of course, but 1st order approximation would be that a straight line, and we are not to update our guess, then go again and evaluate

So I can write down an equation for my guess:
 $x_i + 1$, Based on my previous guess, x_i ;
As been my original guess, minus the value of function, divided by the gradient

$$f(x_i + 1) = f(x_i) - \frac{f(x_i)}{f'(x_i)}$$

So let's see how it plays out..

We can make a table starting with our initial guess $x=0$, then we can find the gradient, intercept and intercept.

i	x_i	$y(x_i)$	$\frac{dy(x_i)}{dx}$
0	-2	-2	10
1	1.8	-0.23	7.7
2	-1.77	-0.005	7.4
3	-1.769	-2.3×10^{-6}	

and as not to generate a new guess

lets say,

In this case $-2 - (-2)$ divided by 10, gives us 1.8
then we can evaluate the result for next guess,
and offend that's just a little less than 0, -0.23
and gradient is 7.7

so we've gone from been out by 2 on y, by
just been out by 0.23. So we got 10 times better

in an estimate, just in a first go.

If we carry on, then we get no next guess for x_2
and we're -1.77 and that's just 0.005 away

from axis, which is really close

and if we have another go, after the

3 iterations we get an answer for x

-1.769 , which is just 2.3×10^{-6} from axis

so in just 3 iterations we pretty much solve problem

The method is called Newton-Raphson method

$$f(x_{i+1}) = f(x_i) - \frac{f(x_i)}{f'(x_i)}$$

and it's really pretty neat.

To solve an equation, all we need to be able to do, is evaluate and differentiate. We don't need to graph and visualize it everywhere, calculate it lots and lots of times and don't need to solve it algebraically either. Much if we have lots of data to dataset and big multibillion function we want to fit to that data, it will be much too expensive to solve it analytically, and even plot it out for all possible values for variables. This sort of method, where we try to solve a function and evaluate it, and try it with new guess, and evaluate that, and again, again, again, called iteration.

This is a very fundamental Computational approach

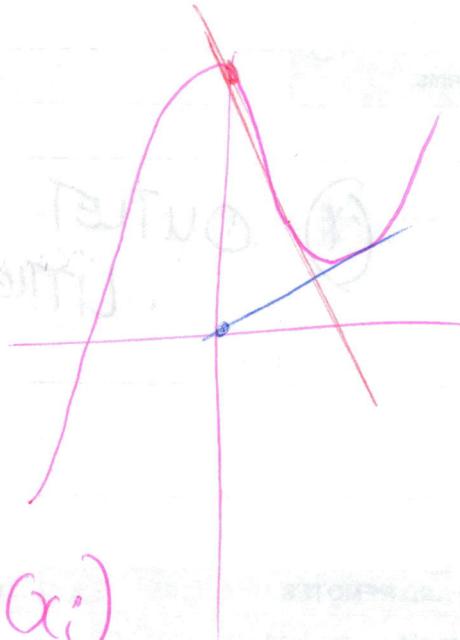
There are some things that can go wrong with this method, so let's briefly look at those.

$$\therefore y = x^3 - 2x + 2$$

$$f(x_{i+1}) = f(x_i) + \frac{f(x_i)}{f'(x_i)}$$

$$\frac{dy}{dx} = 3x^2 \rightarrow$$

$$\begin{array}{cccc} i & x_i & y(x_i) & \frac{dy(x_i)}{dx} \\ \hline 0 & 0 & 2 & -2 \\ 1 & 1 & 1 & 1 \\ 2 & 0 & 2 & \end{array}$$



Say I start off with guess of $x=0$, which evaluates to $y=2$

When find gradient for that, and extrapolated.

It takes me away from solution to the other side of turning point, and gives me a new guess at $x=1$

When we evaluate that one, then I get value for y at $x=1$, of 1, went find he gradient and extrapolate that, then new estimate lands me back at $x=0$

So I have a problem, so I have magically landed in a closed loop, where my estimates just cycle back

between $x=0$ and $x=1$

and I never get close, and I never even go anywhere near the solution of $x = -1.769$.

Then comes another problem, which is that I am

close to a turning point, \curvearrowleft to a minimum and maxima, then because my gradient will be very small, when I divide by gradient in NR equation my next estimate will take me zapping off to some crazy value, then if it won't converge and dive off somewhere

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To math the Newton Raphson method, we iterate to a solution to the equation, by each making an a new estimate from the solution, using the gradient to extrapolate towards the solution, and going again, again, again etc. Most of time, it works well, as a means to step towards the solution.

Recap:

- look at method for using just the gradient, to step our way towards solving a problem
- Called Newton Raphson method
- Really powerful way to solve an equation
- Just by evaluating & and gradient a few times
- ~~It's like~~ If you standing on hill, in fog, you knew your height and knew to callly make going on around you, but you don't see the whole landscape around you, i.e. ~~can't~~ don't know how foggy down mountain

To what you do, you guess based on how steep the hill is, locally around you, which way to go, and you take steps blindfolded, then when you get here, you ask again what height you are, and keep making more steps down hill.

P.S. You don't need to know what the pants you don't need to know what the landscape looks like, (the gradient) \Rightarrow just need to altimeter, ^(altimeter) the value of pressure, to altimeter, the value of pressure, to feel with your feet, what the gradients really

Next session, look at how to apply this, where we have variables, not just x , and that will involve finding the gradient vector

i.e. how to go down a hill on a Contour plot: