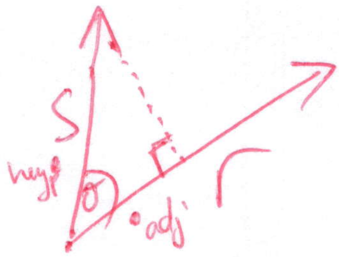
Vector projection:

$\Rightarrow$ for that we need triangle



$\exists$ have vector r, and another vector S

and drop right handed triangle (down here)
(90°)

, then I can do the following:

$\cos\theta$ (from "Sohcahtoa")

$$= \frac{\text{Adjacent length}}{\text{hypotenuse} \Rightarrow \text{size of } S}$$

$$= \frac{\text{adj}}{|S|}$$

and compare this to the definition of dot product.

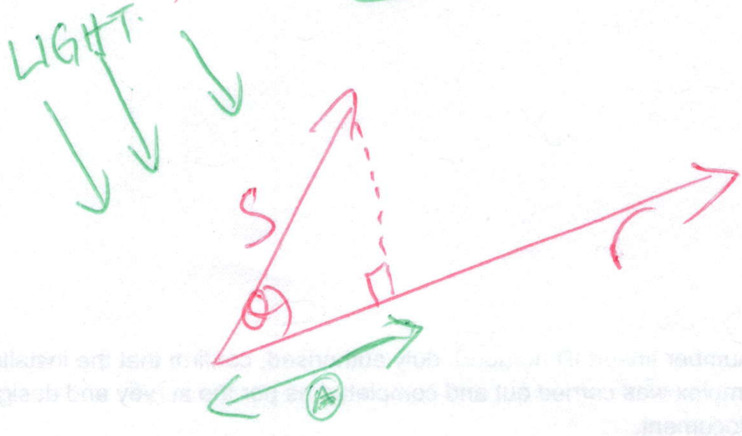
$\therefore \; r.S = |r||s| \cos\theta$ [def of dot Product]

$\therefore$ But $\cos\theta = \frac{adj}{|S|}$

factorize $\Rightarrow$ $Adj = |S|\cos\theta$

$\therefore \; r.S = |r| \underbrace{|s| \cos\theta}_{Adj}$

But what is adj side, say I have light coming from S, so shadow of S onto r ⊗



⊗ that is called the projection

So what the dot product gives us.

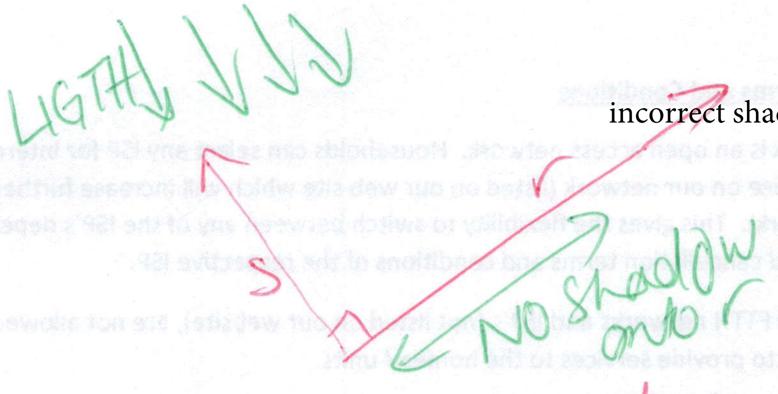
it gives us the projection of S onto r

$r \cdot S = |r||S| \cos\theta$

Adj

$|r| *$ projection A [How much S goes along r]

But if $\cos\theta$ was 90° or S was perpendicular there would be no shadow of



incorrect shadow, would be parallel to S

no projection

So ⇒ dot product gives us some projection, i.e. the shadow of S onto r

If we divide dotproduct r.s by length of r, we get |s|cos θ i.e Adjacent side

$$\frac{r.s}{|r|} = |s|\cos\theta$$

also called the Scalar projection

[How much s goes along r]

But that's why the dot product (r.s) is also called the projection product. (it takes the projection of one vector onto another, we just have to divide by length of r i.e |r|)

(And if r was unit vector (space of length 1) then r.s would just be the scalar projection of s onto r vector defining axis)

⑤

If we want to encode which way r was going into dot product, we can define:

vector projection : $\frac{r.s}{|r||r|} = \frac{r.s}{r.r}$

and gets multiplied by vector (r) itself

$r \cdot \frac{r.s}{|r||r|} = \frac{r.s}{r.r} \cdot r$

So what have we done above:

- we've taken:
  - the scalar projection $\frac{r.s}{|r|}$
  - (that means how much s goes along r

and multiplied it by r, divided by its length |r|

$r \; \frac{r.s}{|r||r|}$

- ~~that is vector~~ a that is multiplied by vector going the direction of r, but normalised to have a length of 1 (divided by length of itself)

- that vector projection is a number times a unit (that goes in direction of r)

$$\Rightarrow \frac{r.s}{r.r} r$$