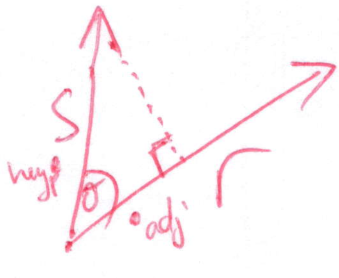
Vecter projection:

$\Rightarrow$ for that we need triangle



$\exists$ have vecter r, and another vecter S

and drop right handed triangle (down here)
(90°)

, then I can do the following:

$\cos\theta$ (from "Sohcahtoa")

$$= \frac{\text{Adjacent length}}{\text{hypotenuse} \Rightarrow \text{size of } S}$$

$$= \frac{\text{adj}}{|S|}$$

and Compare this to the definition of dot product.

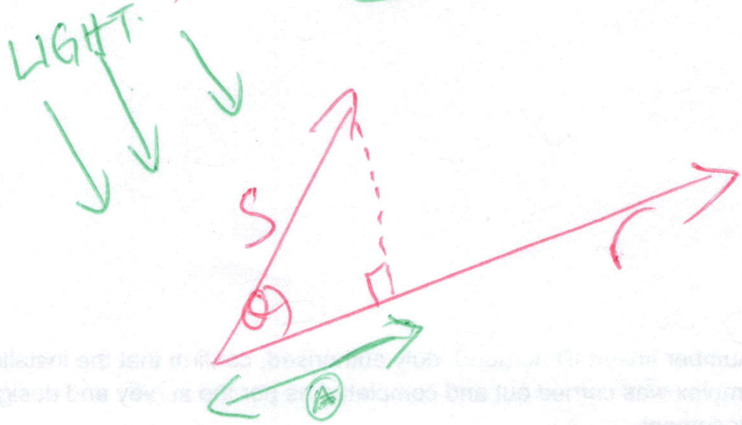
$\therefore \; r \cdot s = |r||s|\cos\theta$ [def of dot Product]

$\therefore$ But $\cos\theta = \frac{adj}{|s|}$

factorize $\Rightarrow$ $Adj = |s|\cos\theta$

$$\therefore \; r \cdot s = |r|\underbrace{|s|\cos\theta}_{Adj}$$

But what is adj side, say I have light coming from S, so shadow of S onto r ⊗



⊗ that is called the projection

So what the dot product gives us.

It gives us the projection of S onto r

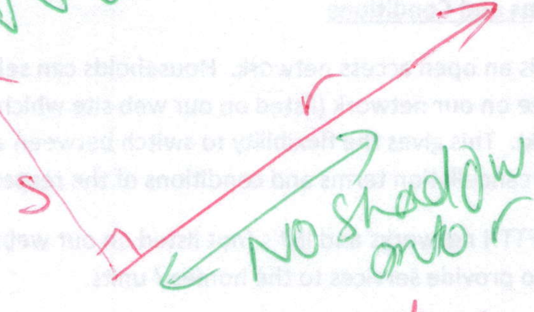
$$r \cdot S = |r||S| \cos\theta$$

Adj

|r| * projection (A) [How much S goes along r]

But if cos θ was 90° or S was perpendicular there would be no shadow of

LIGTH ↓↓↓



no projection

So ⇒ dot product gives us some projection, i.e. the shadow of S onto r

If we divide dotproduct $r.s$ by length of r, (4)
we get $|s|\cos\theta$ i.e Adjacent side

$$\frac{r.s}{|r|} = |s|\cos\theta$$

also called the scalar projection

[How much s goes along r]

But that's why the dot product ($r.s$) is also called the projection product.
(it takes the projection of one vector onto another, we just have to divide by length of r i.e $|r|$)

(and if r was unit vector, (vector of length 1) then $r.s$ would just be the scalar projection of s onto vector defining axis)

If we want to encode which way r was going into dot product, we can define:

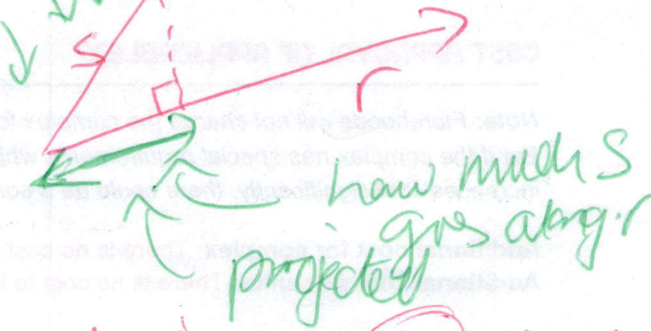
vector projection: $\frac{r.s}{|r||r|} = \frac{r.s}{r.r}$

and gets multiplied by vector (r) itself

$r.\frac{r.s}{|r||r|} = \frac{r.s}{r.r}.r$

So what have we done above:

- we've taken:
  - the scalar projection $\frac{r.s}{|r|}$
  - (that means how much s goes along r



and multiplied it by r, divided by its length |r|

$r \; \frac{r.s}{|r||r|}$

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~~that is vector~~ that is multiplied by vector
going the direction of r, but normalised
to have a length of 1 (divided by length
of itself)

that vector projection is a number
times a unit (that goes in direction of r)

$$\Rightarrow \frac{r.s}{r.r} \, r$$