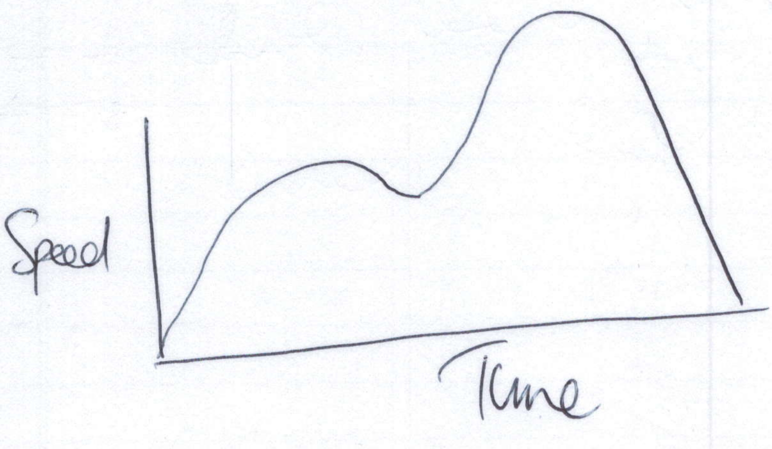
Module 1

Rise over Run

Remember Calculus is just a set of tools for describing the relationship between function and change in its variables

∴ Here we going to explore what this means and how it may be useful

Eg. Speed v Time graph for CAR:



∴ Graph tells us Car speed is not Constant

∴ Constant speed will be a flat horizontal line

from graph, Calculus will allow us to extract more than just speed of graph...

∴ from graph:

— car accellerates at start

— car also decellerate at end...

we said, horizontal line, is a constant speed.

and, more line is sloping up, the greater the acceleration

acceleration can be defined as local gradient of a speed/time graph

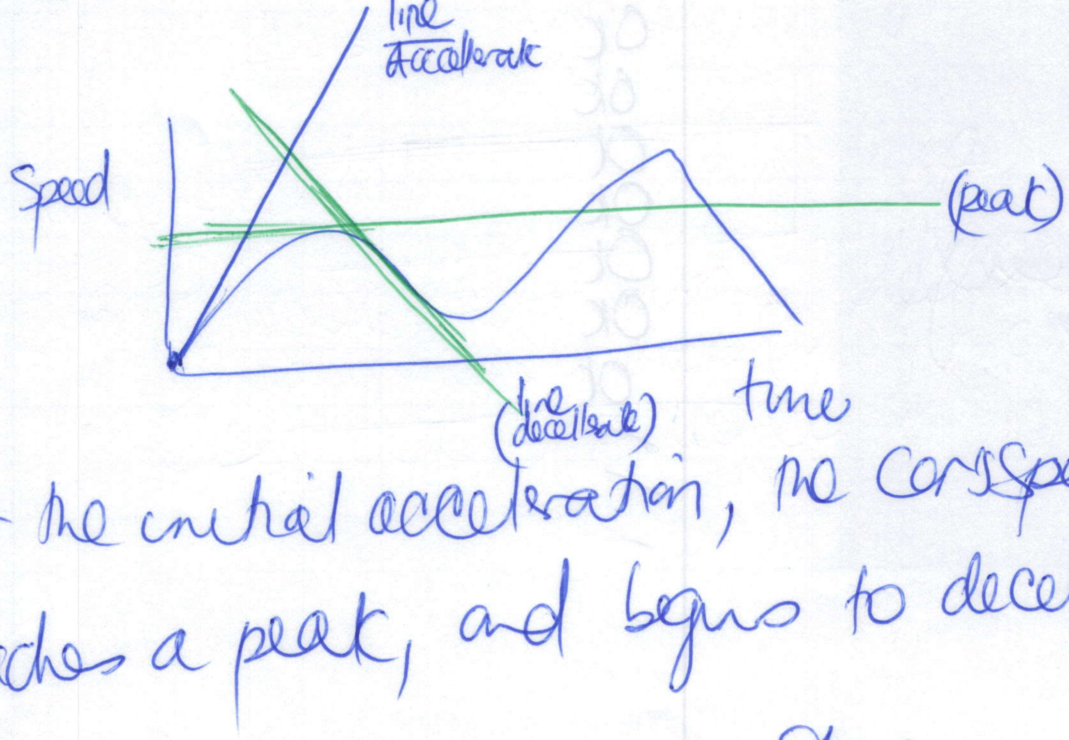
* Clearly acceleration itself is also a function of time (in our graph)

we refer to gradient at a single point as the localgradient

- and we can illustrate this concept by drawing a tangent line

① its a straight line that touches the curve at point

② and it also the same gradient of curve at that point



after the initial acceleration, the carspeed reaches a peak, and begins to decellerate

∴ Decelleration has a negative slope, and we record the slope of these tangent lines at every point, we plot an entirely new graph, which will show us accelleration against time, rather than speed against time.

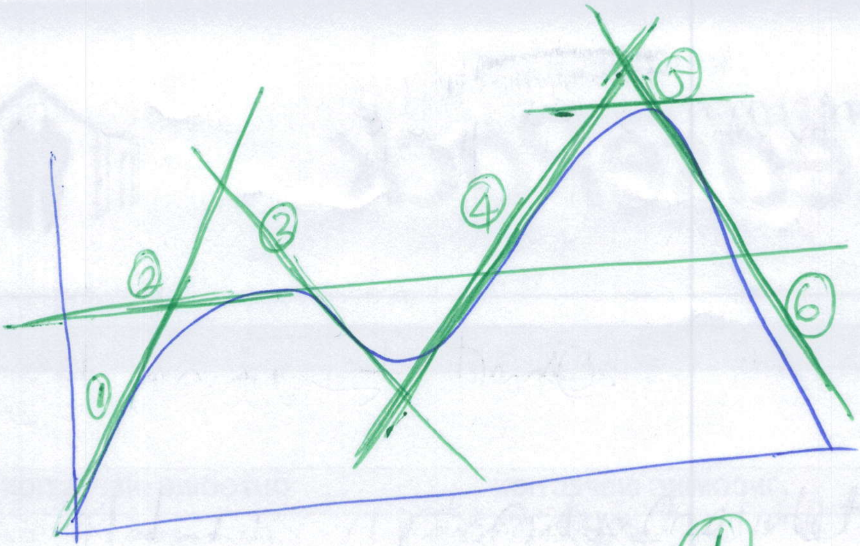
Before we ~~plot~~ plot this for complex case, lets look at simple case, and how the accelleration time graph will look like for ~~car~~ car travelling at CONSTANT SPEED.

∴ Constant speed, we'll have flat horizontal on speed / time graph.

∴ gradient will be 0

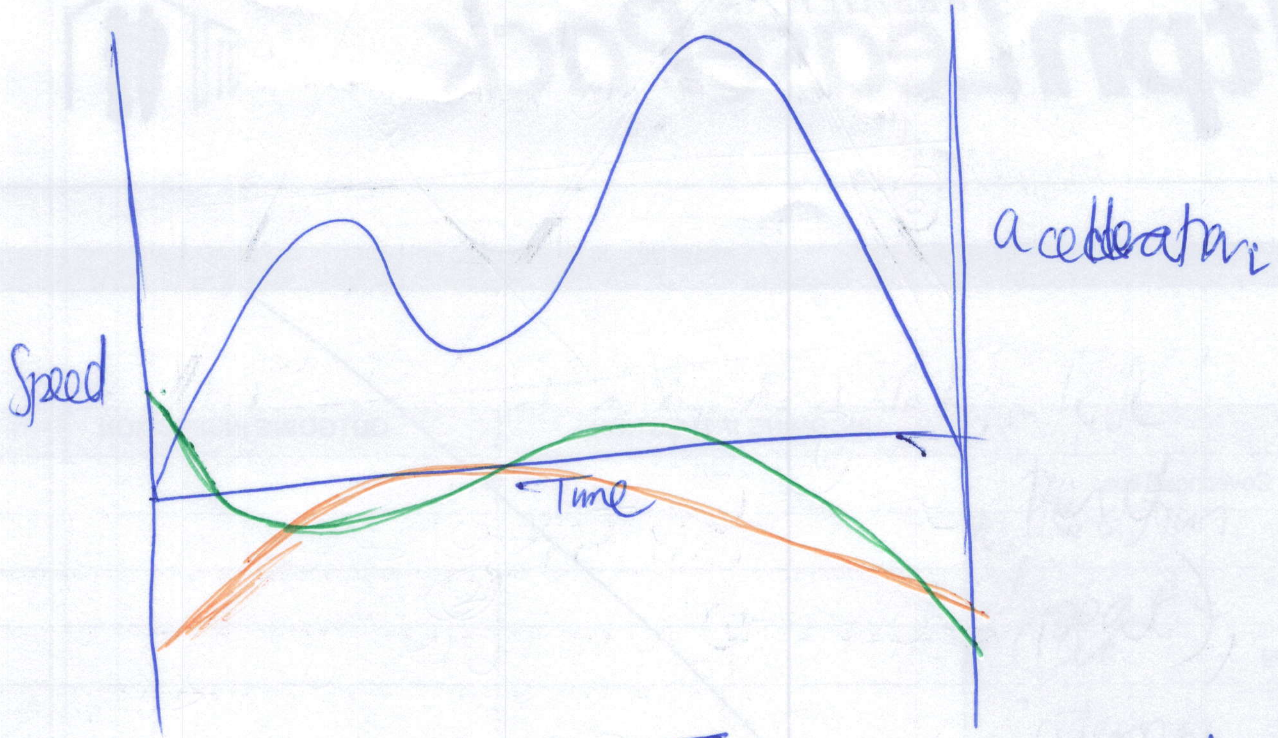
∴ So accelleration time graph will also just be a horizontal line — But with accellation = 0

Revisiting the complex case....



- initially gradient is constant and positive (1), then it "drops" to zero (2) (when car reaches peak), then it becomes negative (3), before returning to zero, etc.....

Now let have look at graph for accelleration vs time, overlayed onto speed/time graph



- the vertical axes for ~~blue~~ blue line is speed, and will have unit of distance per time.
- whereas the vertical axes for green is accelleration and will have units of distance per time squared.

As they have diffferent units we can Scale each of these two lines vertically and meaning will still be identical
However, these have been scaled just to make most used of plot area (limited) available.

we can see the points where the accelleration function point is zero (0)

i.e where it crosses ~~the~~ ~~crosses~~ the horizontal axis, coincides with where the speed / time graph is flat and has zero gradient. (as we would expect)

⇒ this is the essence of calculus

∴ where we look at continuous function and describe its slope at every point by constructing a new function which is its derivative.

we can in principle ~~of~~ plot the derivative of the accelleration function of ~~from~~ same procedure:

– where we take slope of the acceleration at every point

ie this is the rate of change of the accelleration
– which is the second derivative of speed, and also referred to as the "jerk" of car.

∴ jerky motion of car as it stops/starts
⇒ cranging line

So this is the derivative of the accelleration curve
⇒ to approximate or sketch the jerk

also we can take the 'baseline' speed function and try and imagine what function this would be the gradient of

∴ applying the inverse procedure of the one we just discussed.

Can thought of as the 'anti derivative'

This is closely related to the "integral"

For our example, it will represent the distance of the car from its starting position

This will make more sense when one consider the change in distance w.r.t time

<u>i.e</u> slope of distance / time graph

<u>i.e</u> how much distance do I cover per unit time, <u>which</u> is SPEED

This <u>analysis</u> of SLOPES is <u>ALL</u> we going to discuss here

- Even though we have not touched the formal def. of calculus, we should be able to discuss lots of gradient type questions in following exercises

⟶ B.