Module 2

Inner Products (Part 1)

In order to measure angles, lengths and distance we need to equip the vector space with inner product, which will allow us to talk about geometric properties in vector space

An example of inner product that we may know already is the dot product, between 2 vectors x and y.

If x and y is two vectors in $\mathbb{R}^n$ then dot product is defined $x^T y$ = sum $i=1$ to N of $x_i y_i$

where ~~x and~~ $x, y \in \mathbb{R}^n$

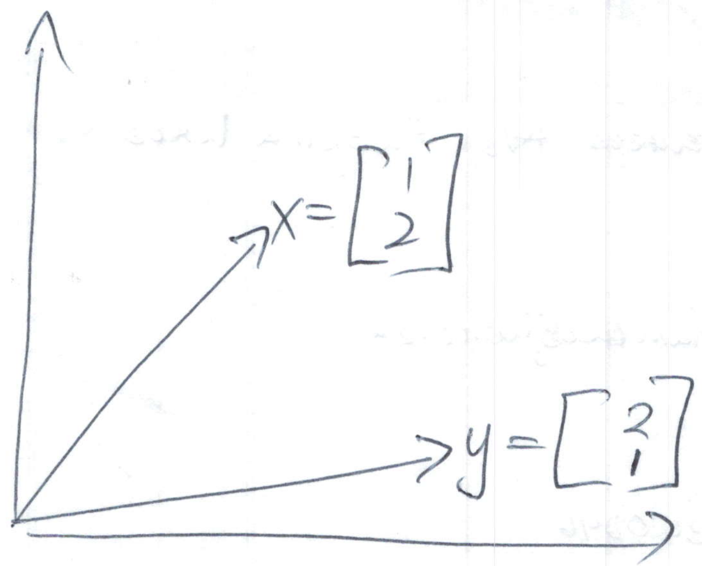
$$x^T y = \sum_{i=1}^{N} x_i y_i \ , \ x, y \in \mathbb{R}^N$$

The length of x is then defined as the square root of dot product of x with itself.

∴ "Length of x is square root of $x^T x$, which we can also write as square root of ~~a~~ sum of $i=1$ to N, of x_i squared"

$$\|x\| = \sqrt{x^T x} = \sqrt{\sum_{i=1}^{N} x_i^2}$$

Lets have a look at an example: we take vector x, as (1,2) and vector y as (2,1) in 2 Dim plane.



Then we can compute the length of vector x,

$$\|x\| = \sqrt{1^2+2^2} = \sqrt{5}$$

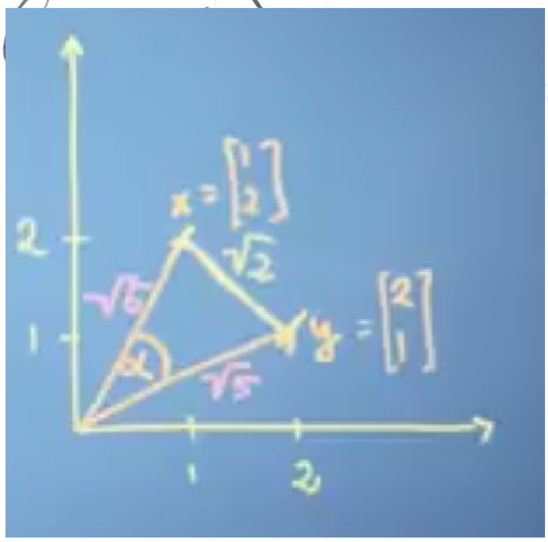
$$\|y\| = \sqrt{2^2+1^2} = \sqrt{5}$$

If we interested in the distance between vector x, y we simply compute the length of the different distance vector.

we generally define the distance between x, y to be length or norm of x−y, which is square root of (x−y) transpose times (x−y)

$$d(x,y) = \|x-y\| = \sqrt{(x-y)^T(x-y)}$$

Lets compute the distance between our two vectors over here:



The distance between these two vectors are effectively the length of difference.

$$d(x,y) = \left\|\begin{bmatrix}1\\2\end{bmatrix} - \begin{bmatrix}2\\1\end{bmatrix}\right\| = \left\|\begin{bmatrix}-1\\1\end{bmatrix}\right\| = \sqrt{1+1} = \sqrt{2}$$

The last thing we still interested in is the (length) angle between the two vectors,

And we can compute the angle, also using the dot product

Cos of angle (α), given by X transpose Y, divided by length of X times length of Y.

$$\cos\alpha = \frac{x^T y}{\|x\| \, \|y\|}$$

$$\cos\alpha = \frac{4}{5} \approx 0.64 \text{ radians}$$

In this lesson, we looked at dot product, a special case of inner product, to compute length of vectors, compute distances between vectors, and angles between two vectors

Part 2 we will look at general inner product, to compute exactly the same quantities