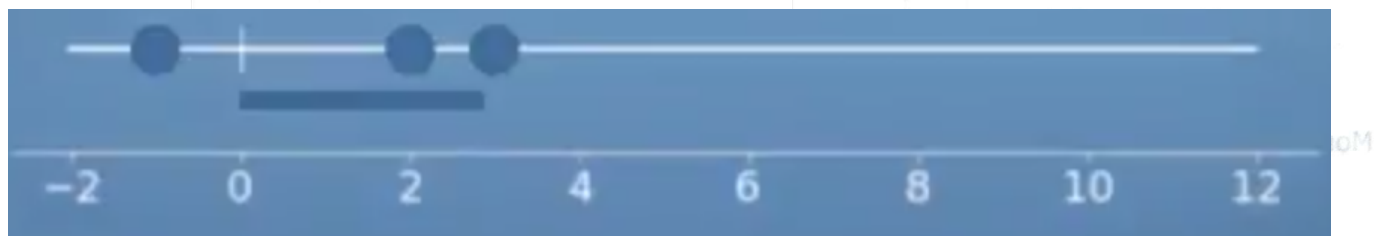


Modulus

Linear Transformation of variance (Part 2)

We've seen what shifting and scaling does to the mean,  
now let's take a look at the effect to the variance.

Remember the variance is spread of the data.  
What do we expect when a dataset is shifted?  
Let's have a look at dataset (see pc)



We have 3 datapoints :  $(-1, 2, 3)$ , and now  
we shifting the dataset toward the right.

The variance of dataset is indicated by  
Blue bar at bottom (see pc)



The shift of data sets given by red dots (see pic)



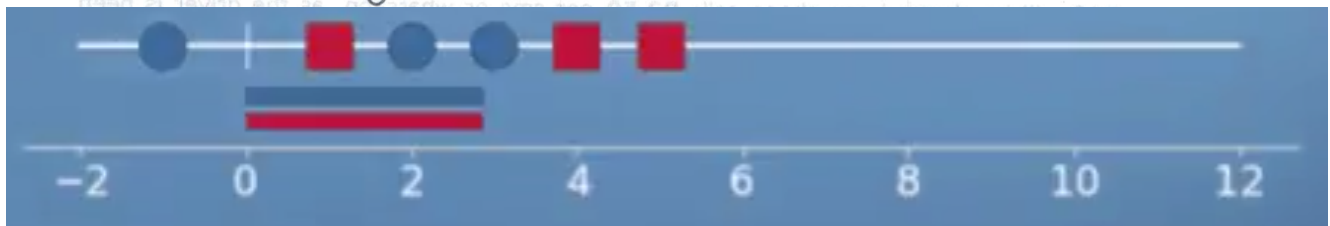
and we shift every individual data point by 2.

Question now: what happens to variance?

Remember, the shift of data, does not really affect the relation of the data points amongst themselves, therefore the variance does not change.

$\therefore$  variance is identical (see pic)

The variance of blue data set, is identical to variance of red data set. (see pic)



Prove a general result that, if we have variance of  $D$  is exactly the same as variance of our dataset  $D+a$ , where  $a$  is an offset applied to every individual element of  $D$ .



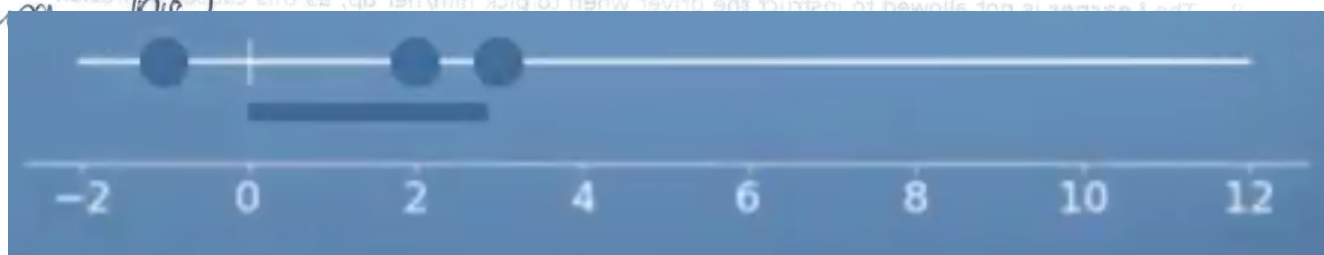
$$\text{Var}[\Delta] = \text{Var}[\Delta + a]$$

lets now scale the dataset, and see what effect it has on variance of data

we going to take exactly the same dataset as before, the variance is indicated by blue bar, and we going to scale every individual datapoint by 2

Question: what's variance of 2 times  $\Delta$ ?

Prun (0.0)



New (1.0)



New datasets indicated by those 2 dots,

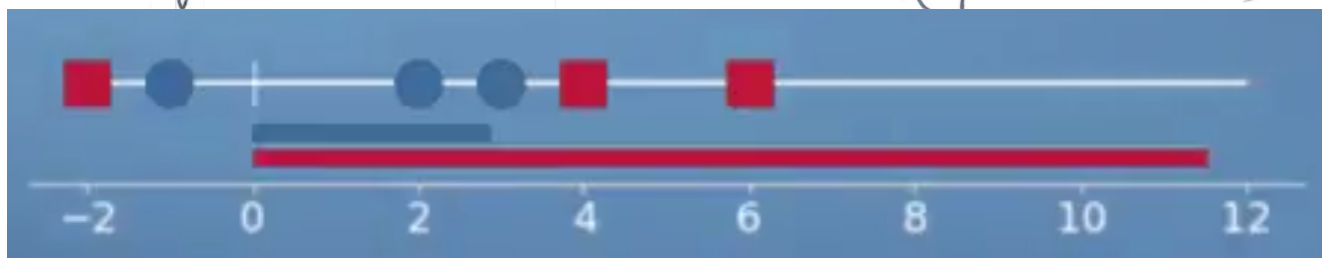
Remember the variance is average square

distance of datapoint from mean.

If we scale the dataset by factor of 2, the

distance of every data point to mean is scaled by 2.

But the square distance, is scaled by 4. (see pc) 4.



and variance is therefore, 4 times as big,  
as it used to be

Our next result: Variance of  $\alpha$  times  $\Delta$ , is  
 $\alpha$  squared times the variance of  $\Delta$ , where  $\alpha$   
is a real number that scales every individual

Data set  $\Delta$ .

$$\text{Var}[\alpha\Delta] = \alpha^2 \text{Var}[\Delta]$$

Now let's have a look at high dimensions and problems:

As usual we have dataset  $\Delta$ , which is a  
collection of datapoints  $x_1$  to  $x_n$ , and  $x_i$   
live in  $\mathbb{R}^p$

$$\Delta = \{x_1, \dots, x_n\}, x_i \in \mathbb{R}^p$$



Remember the variance of the data set is given by  
Co-variance matrix.

If you perform a linear transformation of  
every datapoint, say  $Ax_i + b$ , for given  
matrix  $A$  and offset vector  $b$ , the question is  
what happens to our data set:...

$$Ax_i + b$$

If we do it to every single data point  
well, we get the Covariance matrix of the  
transformed dataset, as follows:  
"variance of  $A$  times  $D$  plus  $b$  is  $A$   
times variance of  $D$  times  $A$  transpose"

$$\text{Var}[AD+b] = A \text{Var}[D] A^T.$$

In this session we say what effects the linear transformation of dataset has on the mean and variance.

In pen & paper, we saw that shifting data has only an effect on mean, not variance. Here scaling the data effects both the mean and variance.