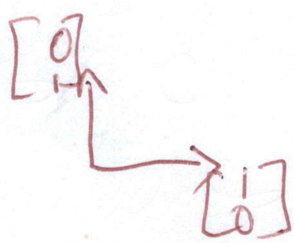
Types of matrix transformations

1. Special types of matrix that does simple things (Combine above)
2. Types of matrix that does complicated things

Lets use a matrix that does not change anything:

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix}$$

∴ Basis vector of space, then multiply it by (x, y) vector, it will not change vector (x, y)



∴ Composed of Basis vectors, and it does not change them.

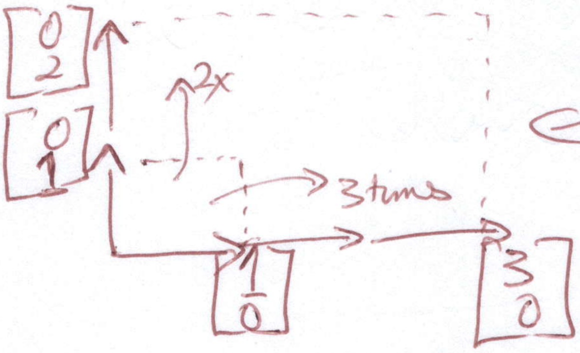
∴ its therefore called the Identity matrix

→ matrix that does nothing, and leaves everything preserved.

But what if I have different numbers along the leading diagonal eg (not 1 anymore, But keep 0's)

$$\begin{bmatrix} 3 & 0 \\ 0 & 2 \end{bmatrix}$$

it will scale the unit vectors by certain factor [by 3 or by 2]

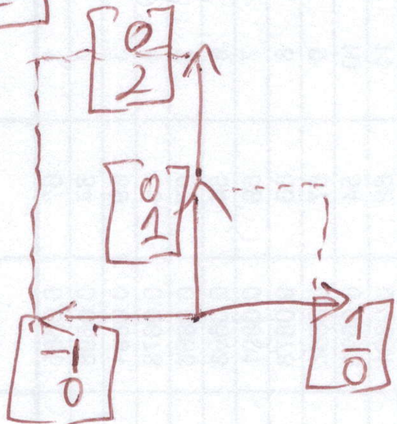


← now become rectangle

if it was fraction eg 1/3 then it would squish space

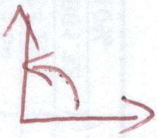
what if scale the unit vector by, -ve number.

eg $\begin{bmatrix}-1 & 0\\ 0 & 2\end{bmatrix}$



So original cube is flipped over, and become rectangle.
But what does it mean.

previously

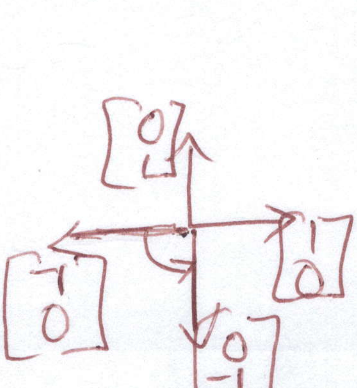


now

$\Rightarrow$ Change "sense" of the coordinate system.
when flipping it over.

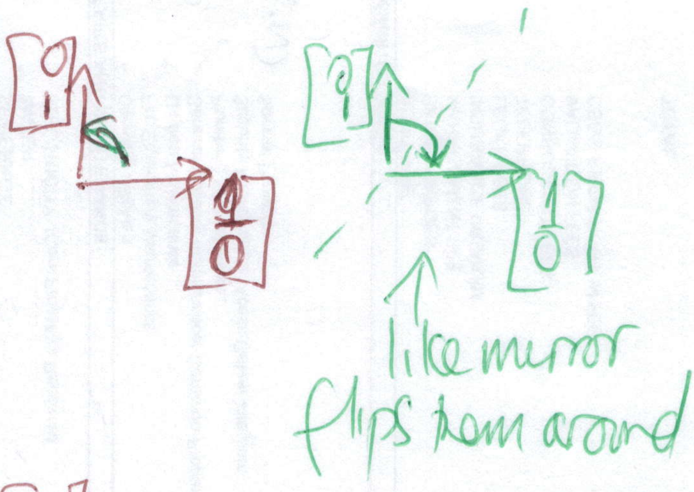
$\Rightarrow$ and what does following matrix produce.

eg $\begin{bmatrix}-1 & 0\\ 0 & -1\end{bmatrix}$

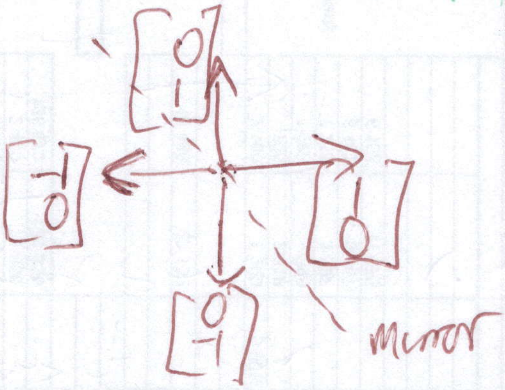


$\rightarrow$ Called Inversion
flips both axes

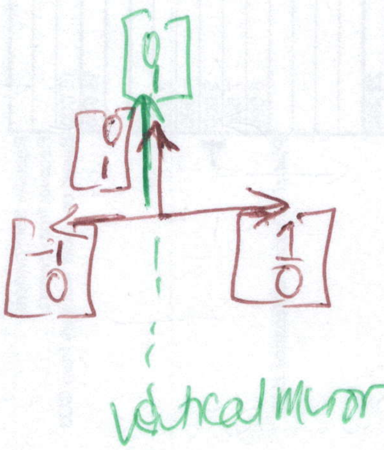
or $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$



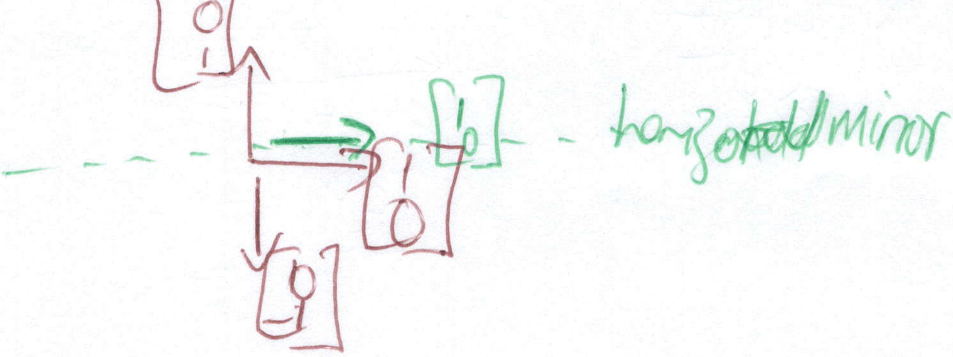
or $\begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$



$\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$

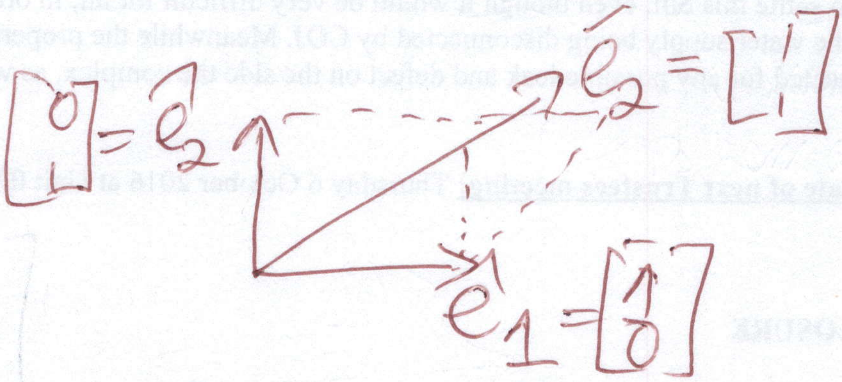


$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$



Lets look at Shears

$\Rightarrow$ keep $\hat{e}_1$ where it was, But move $\hat{e}_2$ over to $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$



What matrix will achieve this? $\hat{e}_2^*$.

$\hat{e}_1$ stay at $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$

$\hat{e}_2$ become $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$\therefore \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ $\Rightarrow$ this will be the transformation matrix

$\therefore$ <u>Shearing</u> the unit ~~squ~~ from been a square to been a parrallelogram.

$\Rightarrow$ this is basically how a shear will look.

Next one, a rotation (~~Leave done~~

$\longrightarrow$

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$\begin{bmatrix}0\\1\end{bmatrix}$ $\hat{e}_2$

$\hat{e}_1$ $\begin{bmatrix}1\\0\end{bmatrix}$

$\hat{e}_1'$ $\begin{bmatrix}0\\1\end{bmatrix}$

$\begin{bmatrix}-1\\0\end{bmatrix}$ $\hat{e}_2'$

90° rotation

$\begin{bmatrix}0 & -1\\1 & 0\end{bmatrix}$ "Transformation" matrix

in general, can write down a rotation....

θ

$\begin{bmatrix}\cos\theta & \sin\theta\\-\sin\theta & \cos\theta\end{bmatrix}$ ← general expression for rotation in 2D

application: stretch faces / transform faces for facial recognition.

⇒ mirrors, shears, rotations to get at all looking up, and not some funny angle.

Recap:

→ described all the possible sort of changes we can do with a matrix [in vectorspace]

→ Next: how do we combine eg rotation with stretch ⇒