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Module 2

The Sandpit

We've seen that the gradient describe the gradient of a multivariable system, and if you calculate it for a scalar value in a ~~multivariate system~~^{single}, you get a raw vector pointing up the direction of greater slope, with length proportional to local steepness.

Here we going to introduce a gradient playground.

To further develop our intuition of the gradient.

In everyday language we use the word optimization to describe the process of trying to make something as good as it can be.

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In maths optimization basically means the same thing as much of the research is dedicated to finding the input values that corresponds to either a maximum or minimum of system.

Examples of mathematical optimization in action in real world includes:

- planning of routes through busy street
- scheduling of production in factories
- strategy for selecting stock when trading

If we go back to the simplest function we
 saw in last section, and we said that
 we wanted to find the location of the maximum,
 we can simply solve the system analytically
 by first building the function and then
 finding the values of x, y , which make
 it equal to 0.

$$f(x,y) = e^{-(x^2+y^2)}$$

$$\nabla = \left[-2xe^{-(x^2+y^2)}, -2ye^{-(x^2+y^2)} \right] = 0$$

$$\Rightarrow x=0, y=0$$

However, when the function gets a bit more
 complicated, finding the maximum or minimum
 can be tricky.

$$z(x,y) = 3(1-x^2)e^{-x^2-(y+1)^2} - 10\left(\frac{x}{5}-x^3-y^5\right)e^{-x^2-y^2}$$

$$- \frac{1}{3}e^{-(x+1)^2-y^2}$$

(4)

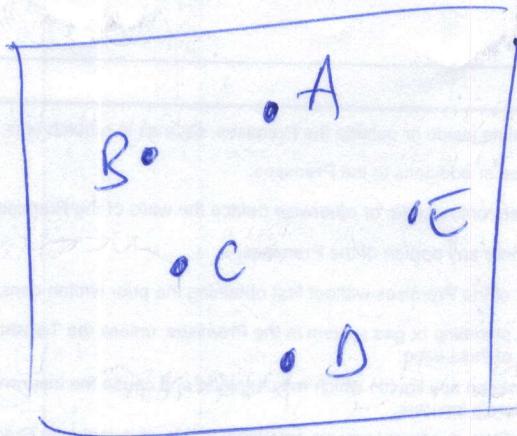
If, as in this case, we still have an analytical expression, then we can at least still find a general expression for the Jacobian

But now, just simply setting it to 0, is not much more complicated, but it also is not enough, as the function has multiple locations with 0 gradient

We assume that all of the maxima and minima can be seen in the region we are plotting (graph)....

Then just look at the surface plot of our function, makes it very clear, where the tallest peak and deepest trough are
we refer to all peaks as maxima,
But in this case, if have a single tallest peak A, which we will call the global maximum, as well as several local maxima, at C, and E.

(5)



Similarly we refer to all though as maxima, and we also have a single deepest point D, which we call global minimum, as well as local minimum at point B.

All fairly straightforward, however, there's one very important point here, that is to do with, we may have missed it...

Imagine standing on glass, with its hills and valleys, and we trying to climb to top of highest peak that no problem, we look around, spot tallest mountain, and walk towards it

(6)

But what if we walking at night,
rates similar, where we did not have
an analytical expression for a function?
So we simply ~~and~~ ^{can't} also to plot the
whole function and look around.

Perhaps each data point is a result of
a week long simulation on a super computer,
or maybe be at some of actual real
world experiment.

These night time scenarios very commonly
arise in optimization, and can be very
challenging to solve.

However, if we lucky we might find that
using a torch, we can see the Jacobian
vectors painted on road signs all around us.

But note, although the Jacobian's all point
uphill, they don't necessarily point to top
of tallest hill.

and you can find yourself walking up to
one of local maxima, or even
worse, when you get there you can find that
all of road signs are pointing directly at you.

The night time hell walking analogy is
often used when discussing a problem
of optimization.

However, it does have some misleading features,
such as the effect that when you are really
evaluating a function, it's no problem to
effectively transport all over the map
by teleportation, as you try the function at
many different places, but there is
no need to evaluate everywhere in between.
And Calculating Cost the same, no matter
how far the points are

So, we're not really walking, instead we'll switch to the analogy of Sandpit, with an uneven bed.

In next exercise, ~~(if we do)~~ we will try to find the deepest point of a sandpit, by measuring depth at various points, using a long stick.

This is very deep sandpit so if you push the stick down to bottom, there's no way to move it around sideways, just have to pull it out, and by some wise else

Also, crucially, like walking scenario, you have no idea what peaks and troughs look like, at bottom of pit, because you can't see, the sand is in the way.

As we work through exercises, we will hopefully
pickup some of the subtleties of approximation

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