①

Composition or Combination of Matrix Transformation

If I want to do any kind of shape alteration, eg of all pixels in image in a face, then you can make that shape / face change through a $\boxed{\text{COMBINATION}}$ of rotation, shears, stretches or inverses.

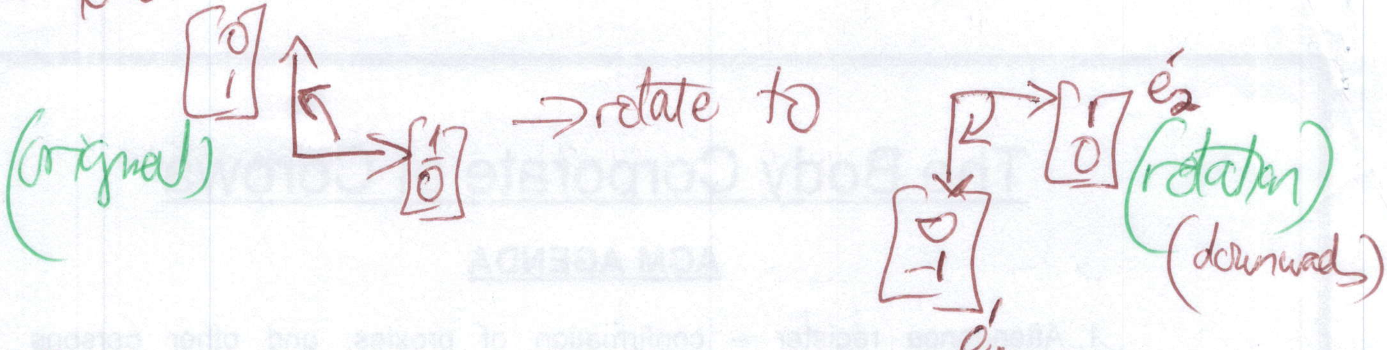
eg apply a transformation to vector $\vec{r}$, we can use A_1, which will make the first shape change, then apply some other transformation A_2, to that (resultant) vector.

eg

$$A_1\vec{r}$$
$$\downarrow$$
$$A_2(A_1\vec{r})$$

Done first A_1, then A_2

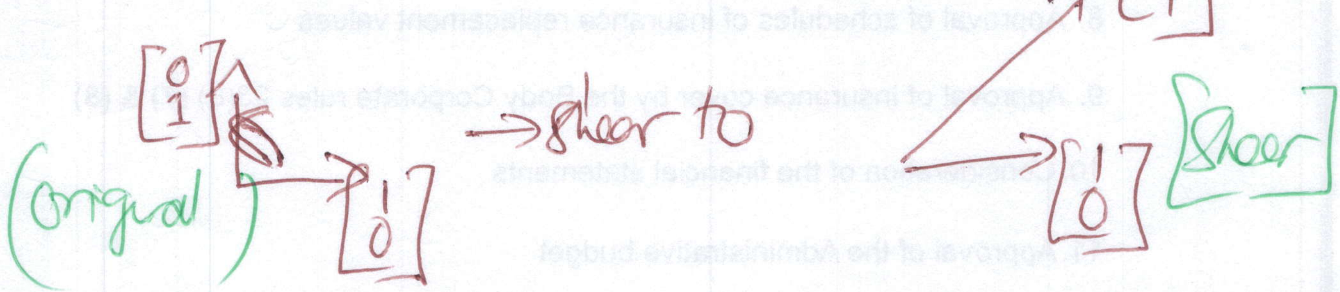
Rotate 90°



So first transformation matrix:

$$A_1 = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$

⇒ Lets do shear now



leave first axis where it is $\begin{bmatrix}1\\0\end{bmatrix}$, and shear to $\begin{bmatrix}1\\1\end{bmatrix}$

So second transformation matrix:

$$A_2 = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$$

③

Now lets do formula

$$A_2(A_1,)$$

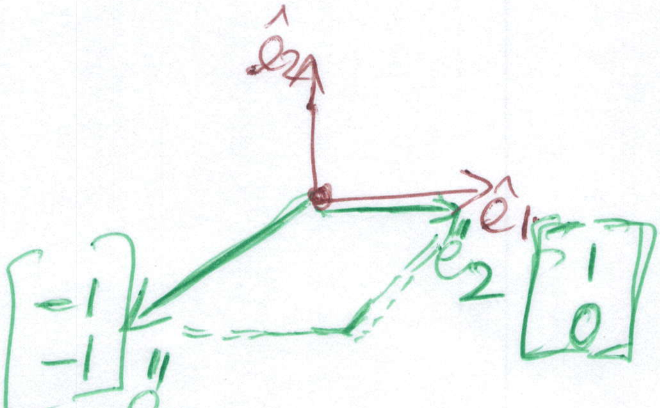
First do $\hat{e}_1$ times A_2.

$$\begin{array}{cc} A_2 & \acute{e}_1 \\ \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} & \begin{bmatrix} 0 \\ -1 \end{bmatrix} \end{array} = \begin{bmatrix} -1 \\ -1 \end{bmatrix} \Rightarrow e_1''$$

Then do $\hat{e}_2$ times A_2

$$\begin{array}{cc} A_2 & e_2' \\ \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} & \begin{bmatrix} 1 \\ 0 \end{bmatrix} \end{array} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \Rightarrow e_2''$$

New Basis/unit vectors



This shows the new axis (basis/unit vector) looks like, a parallelogram

⇒ and Combination of these two vectors (4)

Overall:

$$\begin{bmatrix} -1 & 1 \\ -1 & 0 \end{bmatrix}$$

$$= A_2 A_1$$

∴ above steps is same as:

$$\begin{array}{cc} A_2 & A_1 \\ \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} & \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \end{array} = \begin{bmatrix} -1 & 1 \\ -1 & 0 \end{bmatrix}$$

(rows x columns)

So individual steps we combined now

⇒ First Transformation: Rotation

⇒ Second Transformation: Shear

or n transformation to get final

But is $A_2(A_1 r)$ same as $A_1(A_2 r)$

Can we reverse the order

NO

There is an order dependence in doing matrix operations

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- rotate then shear $\neq$ shear then rotate

- need to be very careful with matrix multiplication. $\Rightarrow$

Recap:

- deep connection between simultaneous equations (called matrices) and vectors (of last session)

- turn out that solving sim. eq. probs ↳ appreciated how vectors are transformed by matrices, which heart of LA.