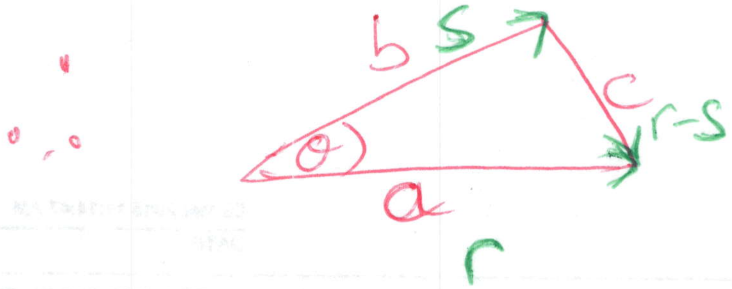
①

- Cosine rule
- Dot product.

Lets take cosine rule of algebra: (we have triangle with sides a, b, c



Cosine says: – angle between sides

$$c^2 = a^2 + b^2 - 2ab\cos\theta$$

this can be translated into vector notation.

$a = r$ vector
$b = s$ vector
$c = r-s$ vector

c^2 = modulus of $r-s$ – modulus of r or r^2

$$\Rightarrow |r-s|^2 = |r|^2 + |s|^2 - 2|r||s|\cos\theta$$

$\boxed{\text{Now we can multiply this out using dot product}}$

because we know;

$|r-s|^2$ is same as

$(r-s)\cdot(r-s)$

$\therefore$ size square is same as $(r-s)$ dotted with itself $\Rightarrow$.

$$\therefore (r-s)\cdot(r-s) = r\cdot r - s\cdot r - s\cdot r - s\cdot -s$$
$$= \cancel{|r|^2} - 2s\cdot r - \cancel{|s|^2}$$

Compare this to right hand side.

right hand side: $\cancel{|r|^2} + \cancel{|s|^2} - 2|r||s|\cos\theta$

$$\therefore \cancel{-}\cancel{2}S\cdot R = \cancel{+}\cancel{2}|r||s|\cos\theta$$

$$\boxed{S\cdot R = |r||s|\cos\theta}$$

tells us about something of direction of vectors in same direction

~~if $\cos\theta = 0$,~~

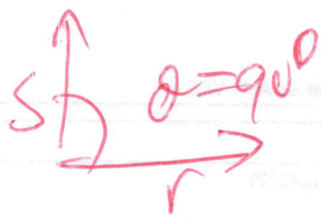
~~if $\cos\theta$~~

if $\theta = 0$, then $\cos\theta = 1$

$\therefore S\cdot r = |r||s|$

size of two vector multiplied together.

If 2 vectors were 90% from each other (3)



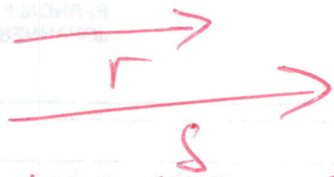
$\cos 90 = 0$

$\boxed{r \cdot s = |r||s| \cdot 0}$ 90%

they are orthogonal to eachother.

then dot product will be zero.

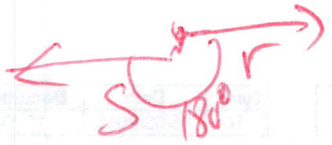
⇒ if they pointed in same direction.



and angle between the two is 0

$\boxed{\therefore \cos \emptyset = 1}$ same direction

$r \cdot s = |r||s|$



– going in opposite direction and angle between two are 180°

∴ $\boxed{\cos 180^\circ = -1}$ opposite direction.

∴ $r \cdot s = -|r||s|$

∴ When dot product $= 0$, they 90° orthogonal.

" " going same way, ∴ +ve answer

" " opposite direction, −ve answer.