Module 3 1:22. 1

Projection onto general subspace

In last session we derived the (general) orthogonal projection of vectors onto n dimensional subspace.

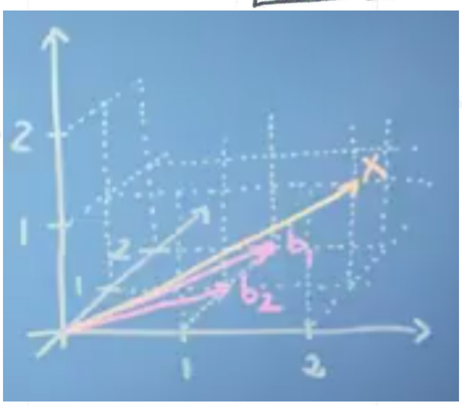
In this session we will run through a simple example

"we define x to be a 3 Dim vector given by (2,1,1) and we define two basis vectors for our two dim subspace b_1 to be (1,2,0) and b_2 to be (1,1,0)

$$\therefore \quad x = \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix}$$

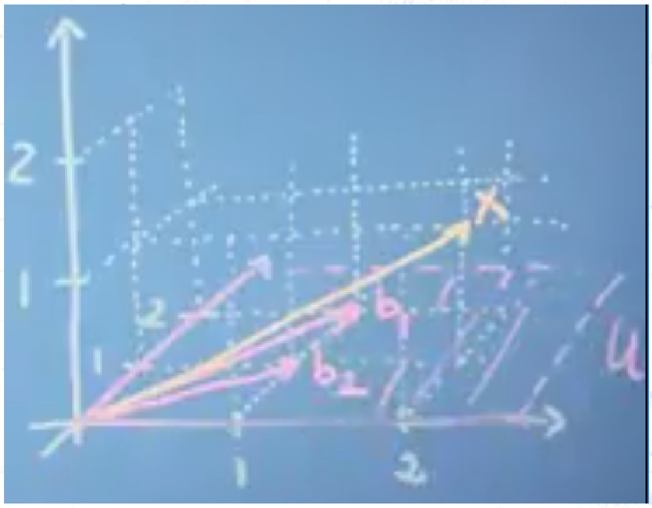
$$b_1 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} \quad b_2 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$$

(See pic)



That means U which is spanned by b_1 and b_2, is going to be (~~the plane~~) the plain (see pic) (This call is U)

$$U = [b_1, b_2]$$



The (~~orthogonal~~) orthogonal projection is given as "π_U of x is B times λ, and we define B now to be b_1 and b_2 concatenated, which is $(1,2,0)$, $(1,1,0)$, and λ was given as B transpose B inverse times B transpose X"

$$\pi_U(x) = B\lambda$$

$$B = [b_1 | b_2] = \begin{bmatrix} 1 & 1 \\ 2 & 1 \\ 0 & 0 \end{bmatrix}$$

$$\lambda = (B^T B)^{-1} B^T x$$

"B transpose X is given as (4,3) vector"

$$B^T X = \begin{bmatrix} 4 \\ 3 \end{bmatrix}$$

"B transpose B is two by two matrix which is (5,3), (3,2)"

$$B^T B = \begin{bmatrix} 5 & 3 \\ 3 & 2 \end{bmatrix}$$

"Now we solve for λ as B transpose B inverse times B transpose X" which means we find λ

$$B^T B \lambda = B^T x$$

Using Gaussian elimination, we arrive at

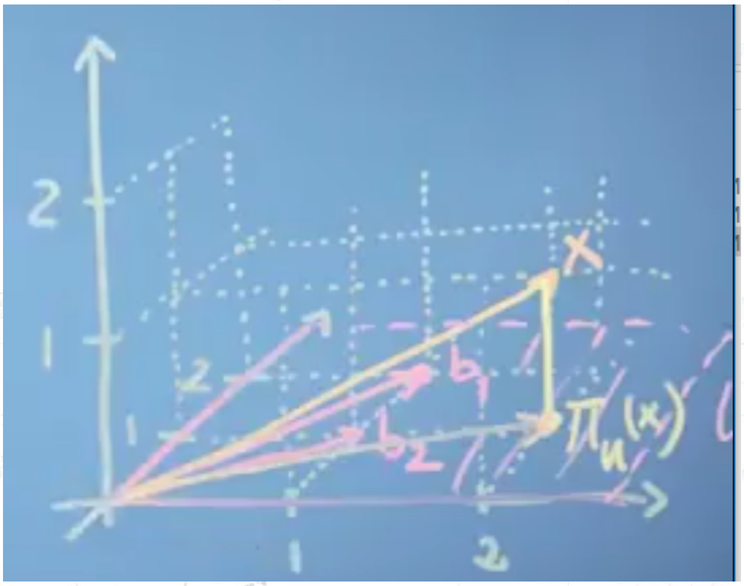
λ equals $\begin{bmatrix} -1 \\ 3 \end{bmatrix}$

$$\Rightarrow \lambda = \begin{bmatrix} -1 \\ 3 \end{bmatrix}$$

"This implies our projection of x onto space spanned by two b vectors"

$$\Rightarrow \Pi_u(x) = -1b_1 + 3b_2 = \begin{bmatrix}2\\1\\0\end{bmatrix}$$

In our diagram over here it will correspond to: (see pic), this vector here



This result make sense because our projected point has a third component, the zero (0)

And our subspace requires ~~the~~ that the ~~all~~ 3rd component is always zero

our projected vector $\begin{bmatrix}2\\1\\0\end{bmatrix}$ is still a 3 Dim vector, but we can represent it using 2 coordinates if we use the basis defined by b_1 and b_2

Therefore that's the compact representation of the projection of X onto ~~the~~ this lower dimensional subspace.

In this session, we looked at a concrete example of the orthogonal projection of the 3 Dim vector onto a 2 Dim subspace

In next session we going to exploit orthogonal projections and derive a dimensionality reduction algorithm called Principal Component Analysis (PCA)