

## Module 5: Eigen problems.

- we will draw on all of the ideas of the previous 4 modules.

- then applying this new information to re-create Google PageRank  
i.e. taking internet search results and ordering them by relevance.

What are Eigen values and eigen vectors -

Eigen  $\Rightarrow$  German  $\Rightarrow$  Characteristic

$\therefore$  Eigen problem?  $\Rightarrow$  finding the characteristic

properties of something.

But Characteristic of what?

Will explain the eigenness through geometric interpretation

$\therefore$  we'll look at geometry / images, then to get tangled up by maths / calculation



Idea here: as soon as you know  
how to sketch "problem" the  
rest is just algebra.

So we have learned that concept of  
linear algebra transformation can be  
expressed through matrices

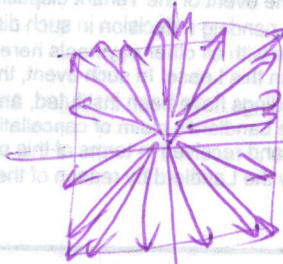
operation which can include:

- Scaling
- Rotation
- Shears.

When applying these transformations, we  
determine what they map to a SPECIFIC  
vector.

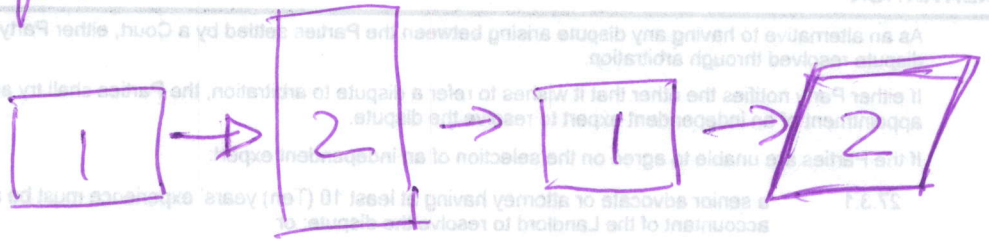
, But also how it may look like when  
it's applied to EVERY vector.





and this can be easily visualized by drawing a square  
- and see how your shape is distorted, when  
applying the transformation.

eg applying a scaling of 2 in vertical direction  
∴ square will become rectangle



and the applying a horizontal shear

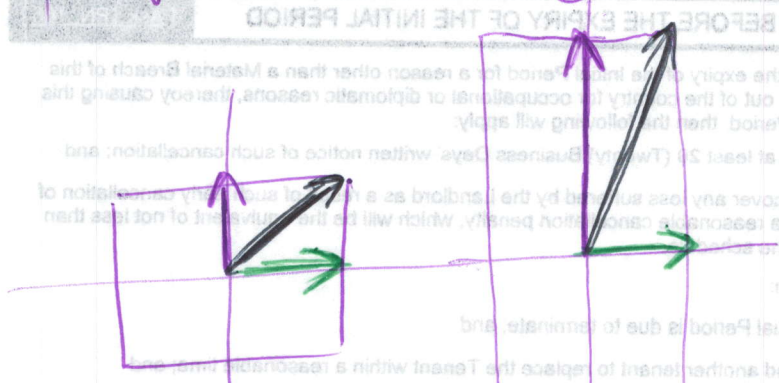
Here's Key Concept:

① we use the square to help us visualize  
what is happening to MANY vectors

But notice that some vectors end up lying  
on the same line that they started on.



Where others do not.  
To highlight this, we going to draw 3 (iii)  
Specific vector on to our initial square



Let's consider our vector scaling again and think what will happen to the 3 vectors.

① Horizontal green vector is unchanged  
⇒ pointing same direction  
⇒ and having the same length.

② Vertical purple vector:  
⇒ points in same direction  
⇒ But length has doubled

③ Diagonal brown vector  
⇒ angle (baseline  $45^\circ$ ) increased  
⇒ length also increased



Beside the Horizontal and Vertical vectors,  
any other vector direction will

Change by the vertical scaling

Horizontal & Vertical vectors are  
Special, i.e. Characteristic of the  
particular transform.

and they are referred to as EIGEN VECTORS

And since the horizontal vector length  
was unchanged, it has a  
Corresponding Eigenvalue of 1

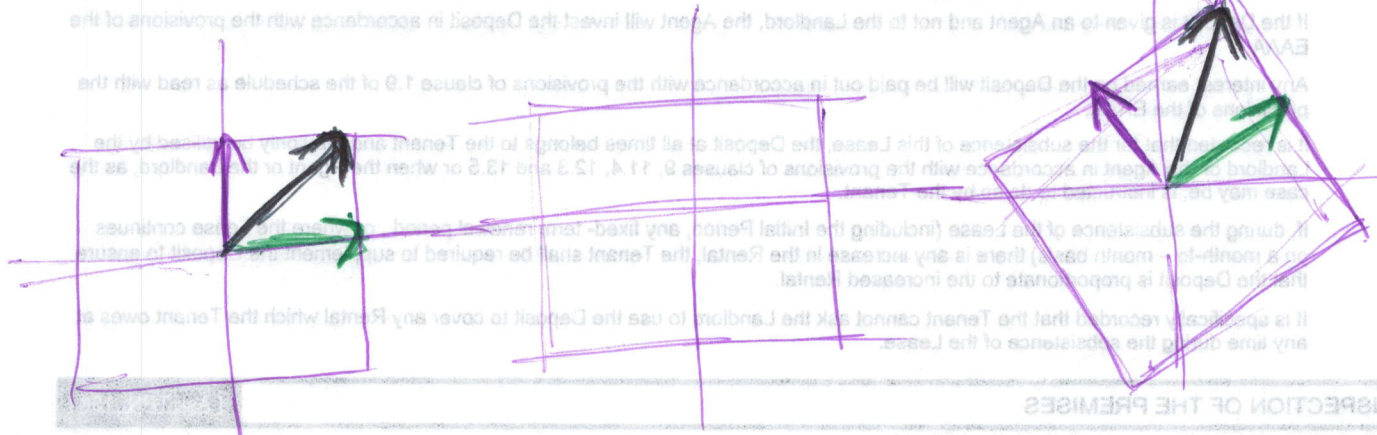
Vertical vector length double in length  
and thus has Eigenvalue of 2

From a Conceptual perspective, that's it,  
for 2D Eigen Problems.  
i.e. we take transform, and we look  
for vectors



all the other vectors are shifted.

lets look at rotation



Here there are NO Eigen vectors, as all vectors have been rotated; off their original span.

With 3D space more space, the Concept is the same

→ next we'll look at special case  
→ and what we observe in mathematical terms