from Gaussian Elimination
to finding the inverse Matrix

How can we use the idea of Elimination,
to finding the inverse matrix.
⇒ which will allow us to solving
a more general problem
⇒ irrespective of what we write
for the right hand side answer.

Remember: $\boxed{A^{-1} A = I}$

Have 3x3 matrix A, ~~[illegible]~~ its
inverse $B = I$

$$A \quad B = I$$

$$\begin{bmatrix} 1 & 1 & 3 \\ 1 & 2 & 4 \\ 1 & 1 & 2 \end{bmatrix} \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{bmatrix} = I$$

②

$\therefore b_{12} \Rightarrow$ where 1st digit represents the row
$\hookrightarrow$ " 2nd digit " " col.

$\therefore B = A^{-1}$

$\therefore AA^{-1} = \underline{I}$

inverse is special, can apply it on left or right and will still work.

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$$\underbrace{\begin{bmatrix} 1 & 1 & 3 \\ 1 & 2 & 4 \\ 1 & 1 & 2 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{23} \end{bmatrix}}_{B} = I$$

$= I =$

where $B = A^{-1}$

and $I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$$\begin{bmatrix} A \end{bmatrix} \begin{bmatrix} b_{11} \\ b_{21} \\ b_{31} \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$ ← will get first column of I

⇒ Can solve this now using Elimination / Back substitution and plugging $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ different values here

then I can do it again for second $\begin{bmatrix} b_{12} \\ b_{22} \\ b_{32} \end{bmatrix}$

for second $\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$

then do it for 3rd one

or I can do it all at once.

(3)

$$\begin{bmatrix}1&1&3\\1&2&4\\1&1&2\end{bmatrix}\begin{bmatrix}b_{11}&b_{12}&b_{13}\\b_{21}&b_{22}&b_{23}\\b_{31}&b_{32}&b_{33}\end{bmatrix}=\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}$$

(A) take first row off second row, then (B) take first row off third row

$$\begin{array}{c}\\ (A)\\ (B)\end{array}\begin{bmatrix}1&1&3\\0&1&1\\0&0&-1\end{bmatrix}\qquad\begin{bmatrix}1&0&0\\-1&1&0\\-1&0&1\end{bmatrix}$$

Now multiply 3rd row by (-1)

$$\begin{bmatrix}1&1&3\\0&1&1\\0&0&1\end{bmatrix}\qquad\begin{bmatrix}1&0&0\\-1&1&0\\1&0&-1\end{bmatrix}$$

(4) Substitute the 3rd row back into 2nd and (B) 1st row

makes 3?

$$\begin{array}{r}\text{"row1} - 3\,\text{row3"}\ (B)\\ \text{row2} - \text{row3}\ (A)\\ \\ \end{array}\begin{bmatrix}1&1&0\\0&1&0\\0&0&1\end{bmatrix}\qquad\begin{array}{c}\\ \text{row2} - \text{row3}\\ \\ \end{array}\begin{bmatrix}-2&0&3\\-2&1&1\\1&0&-1\end{bmatrix}$$

$$\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}\qquad\begin{bmatrix}0&-1&2\\-2&1&1\\1&0&-1\end{bmatrix}$$

∴ transformed A into I matrix

and I (righthand side) have changed.

But we know I times itself (b/s) is just itself.

$$\begin{bmatrix} 0 & -1 & 2 \\ -2 & 1 & 1 \\ 1 & 0 & -1 \end{bmatrix} \Leftarrow \text{is answer } A^{-1} \text{ or } B.$$

find answer using row elimination / Back substitution.

∴ test row $AA^{-1} = I$

$$\begin{bmatrix} 1 & 1 & 3 \\ 1 & 2 & 4 \\ 1 & 1 & 2 \end{bmatrix} \begin{bmatrix} 0 & -1 & 2 \\ -2 & 1 & 1 \\ 1 & 0 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

For 100 rows/columns, computationally better to do it this way. ⇒ Computationally efficient.

or Can use another decomposition method (may be faster)

eg pass command inv(A) → program will then choose best method.

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Recap

- figured out how to solve a linear equation (~~to get~~) for a specific answer
- figured out a general method, ~~usin~~ by finding an inverse, for all cases. (for whatever is on the right hand side)