Module 1:

Defenition of derivative

So what we just learnt about gradients, we Going to construct a mathematical notation...

Here goes.... the uncomfortable bit.... ☹

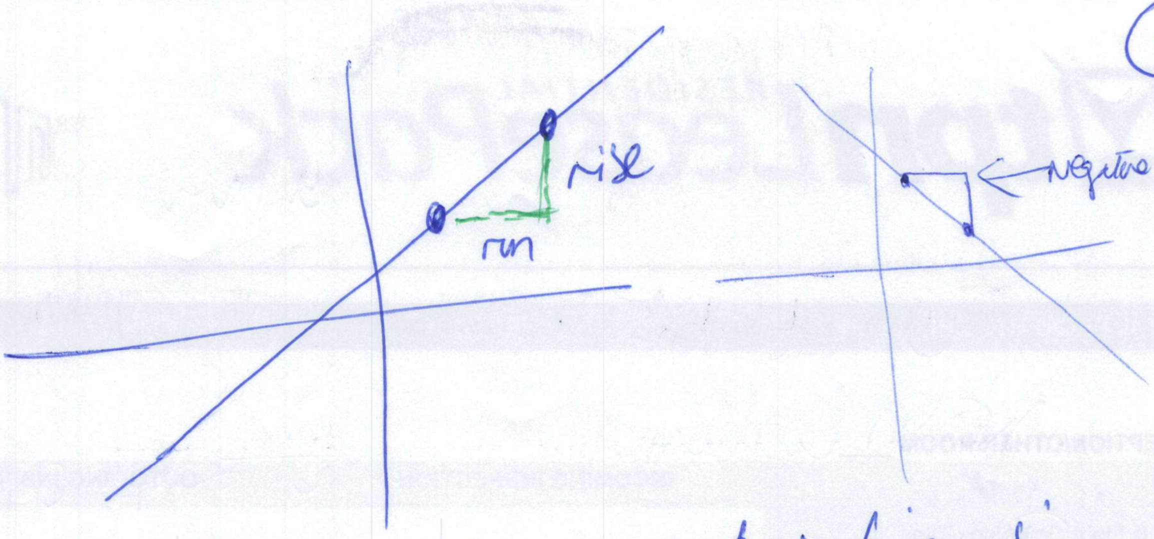
We know:

⇒ Horizontal lines have gradient = 0

⇒ Upward/Downwards ~~two~~ Sloping lines have +ve ~~or~~ -ve gradients

⇒ Lets write down a defenition of this Concept by taking the example of a linear function which has the same gradient everywhere

Lets start by picking any two (2) points



∴ we can say that gradient of this line, is equal to the amount of that function that increases in this interval, divided by length of interval we considering

∴ this description is condensed into "rise over run"

~~[illegible]~~

① where rise is the increase in vertical direction
② and run is the distance on the horizontal axis

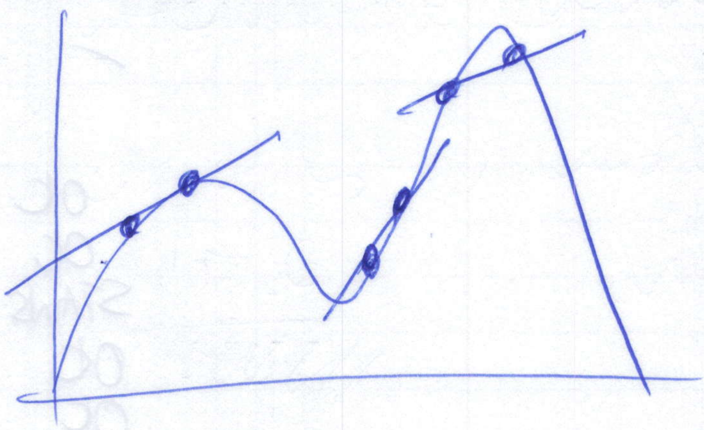
if our function was sloping down, and we pick points the same horizontal locations then run would be the same, but rise would be negative

∴ gradient = $\frac{rise}{run}$

But how does it relate to more complicated function (covered previously)

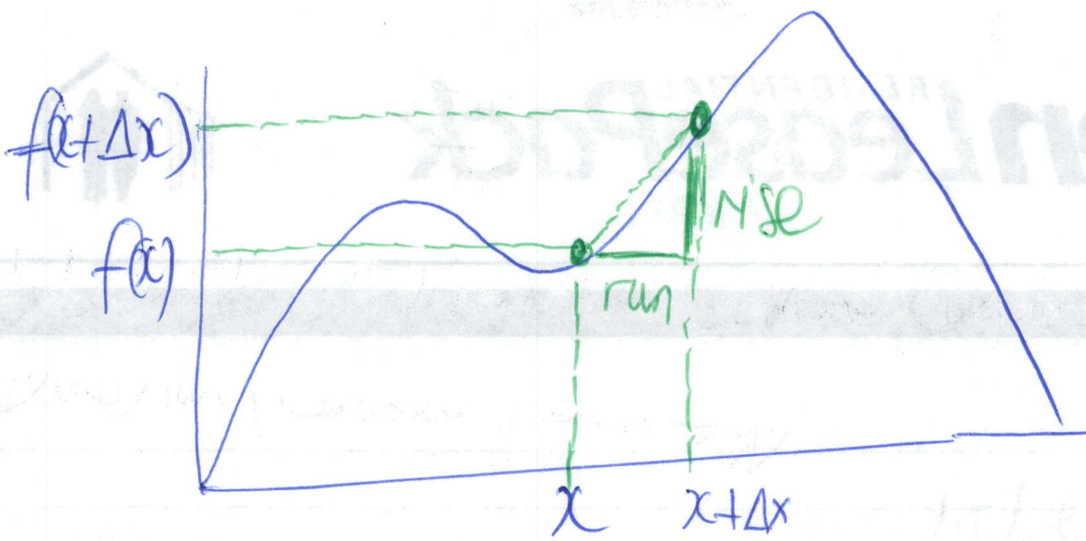
∴ where gradient is different at every point.

∴ rise over run gradient varies depending on where we choose our point



Lets take a single (pick) point where we want to know the gradient, which we say is at point x on the horizontal axis

∴ the value of our function at this point is therefore clearly just $f(x)$



Using the same logic as before, we need to pick a second point, to get our rise over run triangle (second point in green)

- we can call the horizontal distance between two points delta (Δ) x
  - delta (Δ) is used to express a small change in something
- So our second point must be at position $x + \Delta x$
- we can also write down the vertical position as a function, evaluated at new location,

$$f(x+\Delta x)$$

So now, we can build an expression for the APPROXIMATE gradient at our point x, Based on rise over run gradient, between point x and any second point

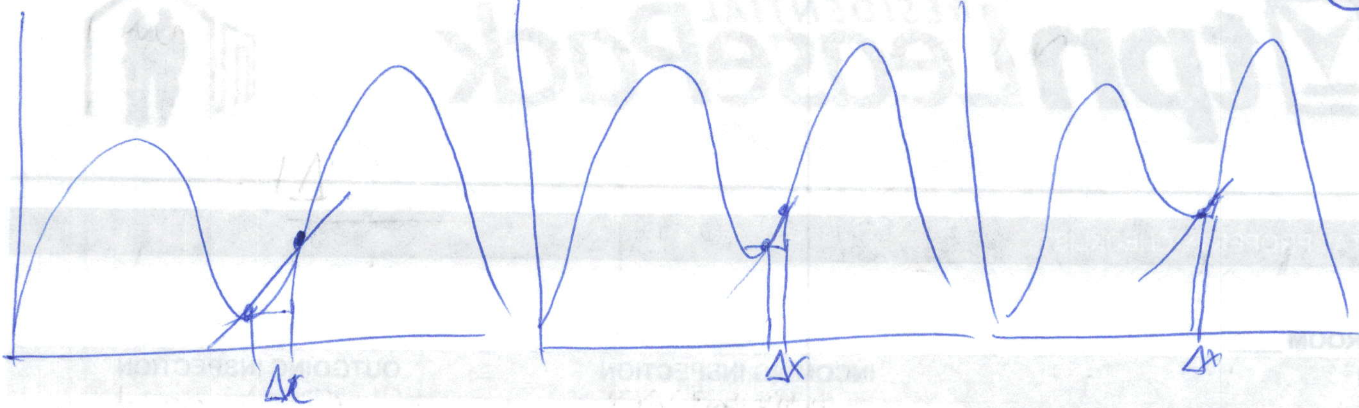
$$\therefore \text{gradient at } x \approx \frac{\text{Rise}}{\text{Run}}$$

$\therefore$ run = "distance Δx"
rise = "difference in height between two points"

$$\therefore \text{gradient at } x = \frac{\text{Rise}}{\text{Run}} = \frac{f(x+\Delta x) - f(x)}{(x+\Delta x) - x} = \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

But we not dealing with straight line, But continuous smooth line, so as distance between two points get smaller, or as Δx get smaller, the line connecting two points becomes a better and better approximation of the actual gradient at our point x.

(6)



$\therefore$ Δx becomes smaller.

$$\therefore \text{gradient at } x = \lim_{\Delta x \to 0} \left(\frac{f(x+\Delta x) - f(x)}{\Delta x} \right)$$

we can express this concept formula, by using the limit notation scheme

– which says that as Δx goes to 0, our expression will give us a function for our gradient at any point we choose

which we write as

$$\left.\begin{aligned} f'(x) &= \ldots\ldots \\ \text{or} \quad \frac{df}{dx} &= \ldots\ldots \end{aligned}\right\} \text{refer to same thing}$$

Strange Concept :- we not saying $\Delta x = 0$

(~~~~ since dividing by zero is undefined)

$\Delta x \to 0$, x extremely close to 0

This process of "extreme" rise of run is what differentiation is

So we (if we are) out to differentiate a function, this means substitute function into this expression

$$\frac{df}{dx} = f'(x) = \lim_{\Delta x \to 0}\left(\frac{f(x+\Delta x) - f(x)}{\Delta x}\right)$$

Lets put the derivative function into practice

- Lets practice on linear function.
- we know the answer should just be constant.

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whats the gradient of:

$$f(x) = 3x+2$$

$$f'(x) = \lim_{\Delta x \to 0}\left(\frac{f(x+\Delta x) - f(x)}{\Delta x}\right)$$

$$f'(x) = \lim_{\Delta x \to 0}\left(\frac{3(x+\Delta x)+2 - (3x+2)}{\Delta x}\right)$$

$$= \lim_{\Delta x \to 0}\left(\frac{\cancel{3x}+3\Delta x\cancel{+2} \cancel{-3x}\cancel{-2}}{\Delta x}\right) = \lim\left(\frac{3\cancel{\Delta x}}{\cancel{\Delta x}}\right)$$

$$= \lim_{\Delta x \to 0}(3)$$

theres no Δx

$$= 3.$$

since Δx has disappeared, the limit has not effect, so we ignore it

the gradient is constant, $=3$

From this simple example, we differentiated 2 things at once:

— $3x$ bit

— $+2$ bit

Could have differentiated it ~~separate~~ separately and then added together, as still got same result.

This interchangeability approach is

Call the <u>Sum Rule</u>

$$\frac{d}{dx}\left(f(x)+g(x)\right)=\frac{df(x)}{dx}+\frac{dg(x)}{dx}$$

Lets look at:

$$f(x) = 5x^2$$

$$f'(x) = \lim_{\Delta x \to 0} \left(\frac{5(x+\Delta x)^2 - 5x^2}{\Delta x} \right)$$

$$= \lim_{\Delta x \to 0} \left(\frac{\cancel{5x^2} + 10\Delta x + 5\Delta x^2 \cancel{- 5x^2}}{\cancel{\Delta x}} \right)$$

$$= \lim_{\Delta x \to 0} \left(10x + 5\Delta x \right)$$

only second term has Δx in it...

so as Δx goes to 0, we going to forget about this term, and become extremely small

$$= 10x \quad \leftarrow \left[\text{derivative of } 5x^2 \right]$$

So we can generalize this, apply a rule for handling functions with powers of x

Power Rule

if $\quad f(x) = ax^b$

then $\quad f'(x) = abx^{b-1}$

∴ the original power gets multiplied to front $^{(3)}$
and new power is just 1 less than before.
∴ known as power rule

We've seen two example of which we have applied the limit Reserver Ren ~~rule~~ method
to differentiate two simple functions
But if we want to differentiate long, complicated expression, this process will become quite tedious
Later lessons we'll see more rules (in addition to sum and power rules)
to help us speed up process

~~First~~, lets look at magical special case function which differentiate in interesting way.