Module 2

Length and distance Part 2.

Now that we know how to compute the length of vector, we can also compute the distance between any two vectors x and y

The distance between two vectors is defined as the length of the difference vector.

"we can unto the distance between x and y to be the norm of x minus y", and as we know this depends on the definition of an inner product

$$d(x,y) = \|x-y\| = \sqrt{\langle x-y, x-y \rangle}$$

(A)

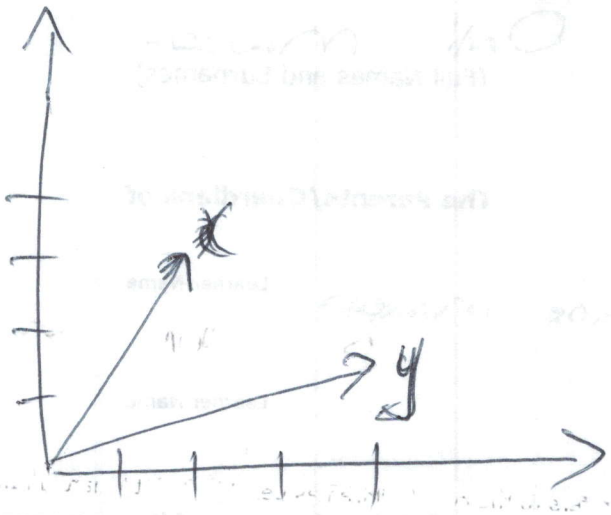
If we use the dot product, then the distance is called the Euclidean distance

Lets have a look at an example...

We look at two vectors x and y, we going to say $x = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$, and y is $\begin{bmatrix} 4 \\ 1 \end{bmatrix}$

$$\therefore \quad x = \begin{bmatrix} 2 \\ 3 \end{bmatrix} \quad y = \begin{bmatrix} 4 \\ 1 \end{bmatrix}$$

Let's draw it:



In order to compute the distance between these two vectors, the first thing we need to do is, is to have look at distance vector. (A page 1

$$x - y = \begin{bmatrix} 2-4 \\ 3-1 \end{bmatrix} \begin{matrix} \leftarrow \text{first component} \\ \leftarrow \text{second component} \end{matrix} = \begin{bmatrix} -2 \\ 2 \end{bmatrix} \text{ difference vector}$$

Now we can define inner products
Let's now use the dot product on our first example.

Dot Product.

If we use the dot product to compute the length of the difference vector $(x-y)$, we will get: square root of the first component squared plus the second component square,

$$\|x-y\| = \sqrt{-2^2 + 2^2} = \sqrt{4+4} = \sqrt{8}$$

and if we use a different inner product, let say we define:

"Inner product between x and y to be X transpose times $\begin{bmatrix} 1 & -\frac{1}{2} \\ -\frac{1}{2} & 1 \end{bmatrix}$ times y

$$\langle x, y \rangle = X^T \begin{bmatrix} 1 & -\frac{1}{2} \\ -\frac{1}{2} & 1 \end{bmatrix} y$$

Now, if we now use the difference vector here, we will be square root of 12

$$\therefore \|x-y\| = \sqrt{12}$$

So we can see that depending on the choice of our inner product, we'll get different answers of what the distance between x and y actually is.

In this section we computed distances between 2 vectors using inner products, and we saw that depending on the inner product, the distance between those two vectors can differ