Module 2

Variables, Constants & Context

In first Module, we developed a strong, visual intuition, relating derivatives to gradient of function at each point

We then followed this up by describing 4 Handy Rules to speed up process of finding derivatives.

However, all of the examples we looked at was for systems involving a single variable

Now apply the same idea for system with many variables, known as multivariate systems

But what is a variable?

Previously (we showed system) one of variable was function of other.

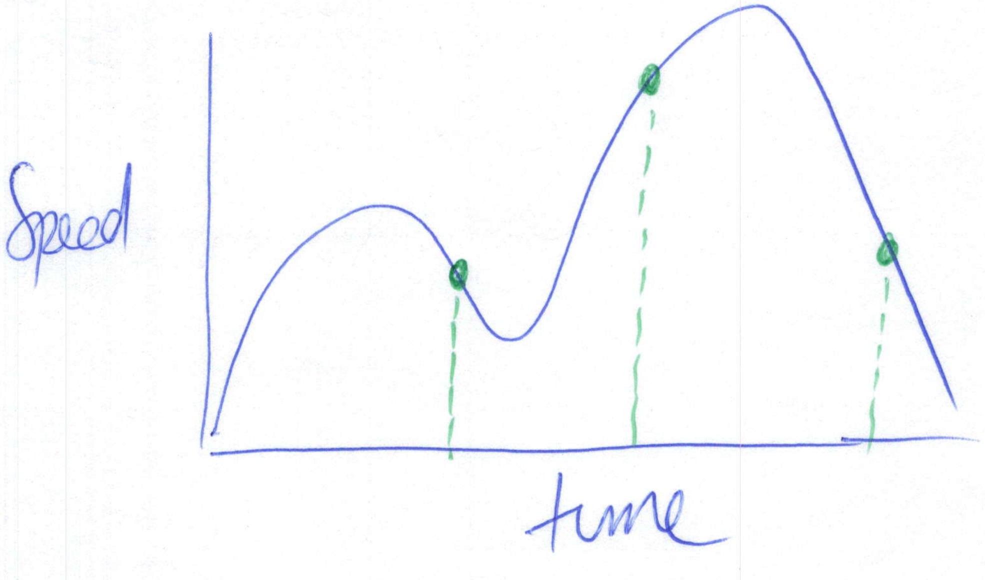
e.g. $y = f(x)$ $\quad [f(x) = y]$

But it will not make sense, to say

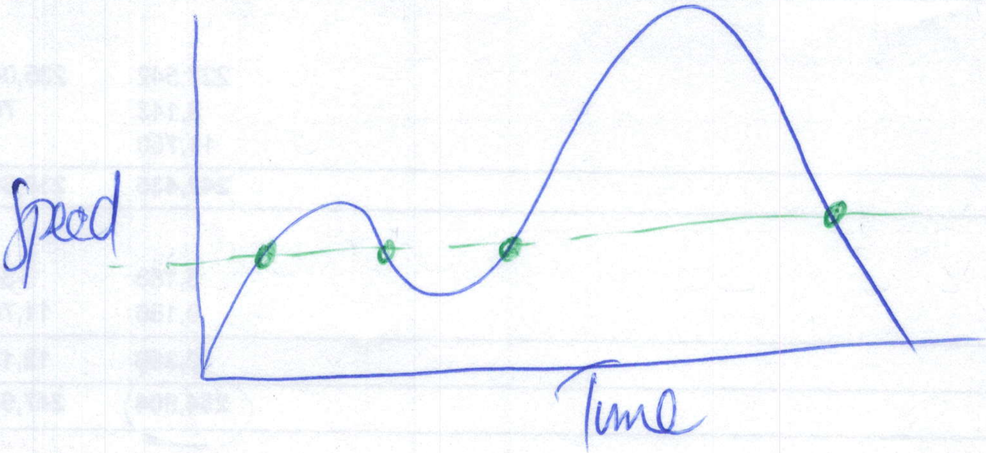
e.g. $x = g(y)$ $\quad [g(y) = x]$

Eg. a vehicle speed is clearly a function of time, as at each time, the vehicle can have only 1 speed

(~~But we cannot say the time,~~



But we cannot say the time is function of vehicles speed



As there may be multiple times at which the vehicle is traveling at any given "(same)" speed

This is why we refer to the speed as a $\boxed{\text{dependent variable}}$, as it depends on time

∴ time is an $\boxed{\text{independent variable}}$ in this particular context.

Typically, when you first learn calculus, you take functions containing variables and constants, and differentiate dependent variables, such as speed wrt independent variable, like time

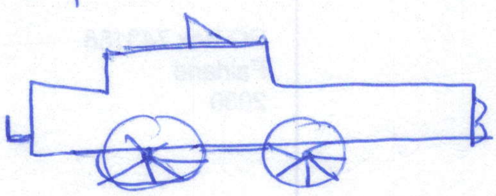
4

However, what is labelled as constant or variable, is more subtle than we may think, and will require us to understand the context, of the problem been described.

Lets continue with Car example

Consider the following highly, simplified expression:

$$F = ma + dv^2$$



Relating a Force, F generated by Cars engine, to its mass m, acceleration a, aerodynamic drag d, and velocity v, If you driving a car, you can change your speed and acceleration by pressing the accelerator pedal to adjust the force, but mass and drag are fixed features of car's design;

Force is independent variable as you control it directly

Speed and acceleration, depend on variables as they are consequences of your applied force

Your mass and drag coefficient are clearly constant in this context

However, if you are a car designer, looking to design each engine size in new fleet, so marketing will give you a specific speed / acceleration target.

But in this Context, force is still independent variable, however speed and acceleration are Constants.

And mass and drag will become variables, which you Can adjust by redesigning the Car.

We refer to these slightly confusing variable design Constants, as parameters.

We often talk of varying them as exploring a family of similar functions, rather than declaring them as variables in their own right.

But these are largely engineering distinctions not things that will worry mathematicians

As we will see later, when fitting functions to some data, it the PARAMETERS of the fitting function we are varying to find the best fit.

Which mean we will have to differentiate w.r.t. to the parameters

(There's still some logic involved But we shouldn't worry too much)

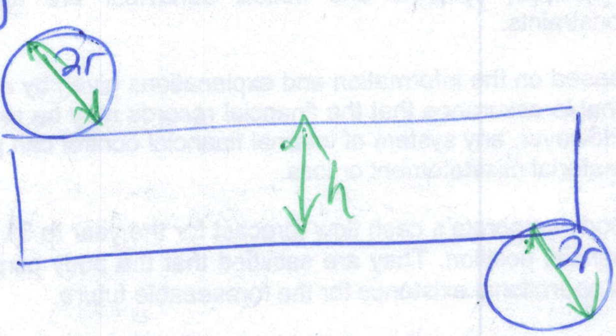
The key take away here is you can in principle differentiate any term w.r.t. to any other, so don't get caught off guard when things you thought of as constant suddenly start to vary.

Lets look at different example:

I want to build a metal Can.

I need to understand the relationship between the various key design parameters

we can start by writing a reasonable approximation on the can's empty mass m, by breaking the area down into pieces.



Circles, top and bottom, are πr^2 each.

And when we unroll the body, we get a rectangle with a width, that must be same as the circumference of circle, i.e. $2\pi r$

and we'll call the height h

So taking the area, and multiplying them by thickness t, we get volume, we get total of metal in can

Finally multiplying this, by the metal's density ρ_{metal}, we get it's mass

$$\therefore m = \underbrace{2\pi r^2}_{\text{area 2 little circles}} t\rho \;\overset{A}{}\; + \;\overset{B}{}\; \underbrace{2\pi r h}_{\text{area of rectangle}} t\rho$$

(and Both multiplied by density ρ and metal thickness t)

What should we label as a Constant or variable?

π is Constant, but not clear, we can in principle change the radius, height, wall thickness or materials density

, So Lets find the derivative of the CAN's mass w.r.t. each of it.

To calculate these, all we do, when differentiating w.r.t a certain variable, simply consider all of the others to behave as constants

Lets start with easy one:

Let $y = h$

$$\frac{dm}{dh} = 2\pi r t \rho$$

as the first term (A) did not contain h, and we treating all terms as constant, then a constant simply differentiate to 0, so we ignore it

But second term (B) does contain h, but just multiplied to some constant, so differentiating this just leaves constants

So we can see from this expression, that partial ~~diffe~~ derivative w.r.t h, no longer even contain h, which is what we expect as the mass will vary linearly with height, when all else is kept constant

∴ Instead of using the normal "d", when doing derivatives in last module, we must use the curly partial symbol "∂"

– which signifies you have differentiated a function with more than 1 variable

Lets find partial derivative w.r.t the other variable

Starting with r:

$$\frac{\partial m}{\partial r} = 4\pi r t \rho + 2\pi h t \rho$$

Next do variable t:

$$\frac{\partial m}{\partial t} = 2\pi r^2 \rho + 2\pi r h \rho$$

Next do variable ρ

$$\frac{\partial m}{\partial \rho} = 2\pi r^2 t + 2\pi r h t \Rightarrow$$

This is quite straightforward example, but that is it for partial differentiation

No more complicated than unit ~~[illegible]~~ variate Calculus (last module),

The only difference, you have to be Careful to keep track, what you Consider to held Constant when you take each derivative

⇒ Short intro to multivariate Calculus →

∴ Partial Differentiation is essentially just taking a multidimensional problem and pretending that it is just a standard one dim problem, when we Consider each variable separately.