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Inverses: Solving the apples/Banana problem and Gaussian Elimination.

⇒ Finally solving apples/bananas problem!

$$2a + 3b = 8$$
$$10a + b = 13$$

∴ write it as matrix ~~and~~ (times) vector

$$\begin{pmatrix} 2 & 3 \\ 10 & 1 \end{pmatrix}\begin{pmatrix} a \\ b \end{pmatrix} = \begin{bmatrix} 8 \\ 13 \end{bmatrix}$$

$$A \quad r \quad = s$$

∴ need to find what vector r (a,b) is to get (8,13)

But what matrix can I use to multiply with A to get Identity

$$\therefore A^{-1} A = I$$

A^{-1} is called inverse

so what A^{-1} does, reverses what A does as gives me Identity

So if take

$$Ar = S$$

then multiply on Both Sides by A^{-1}

$$\therefore \underbrace{\cancel{A^{-1}A}}_{I}\boxed{r = A^{-1}S} \Rightarrow \text{then left with}$$

[inverse matrix A times S]

will just be Identity

↓

i.e. matrix that does nothing

$\therefore$ if I can find the inverse of A

$\therefore A^{-1}$

then can solve problem and find out what a and b are

But first lets revisit inverse:

we said $\boxed{A^{-1}A = I}$

(but don't need inverse to solve problem we can just use substitution)

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By looking at a slightly more complicated problem:

$$\begin{bmatrix} 1 & 1 & 3 \\ 1 & 2 & 4 \\ 1 & 1 & 2 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 15 \\ 21 \\ 13 \end{bmatrix}$$

Idea: if I take a, b, c first row, from second row a, b, c, I have not really changed anything - I can make problem simpler by doing this. using a process of elimination.

1) Subtract row 1 from row 2 we get Ⓐ
2) Subtract row 1 from row 3, we get Ⓑ

$$\begin{array}{c} \\ A \\ B \end{array} \begin{bmatrix} 1 & 1 & 3 \\ 0 & 1 & 1 \\ 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 15 \\ 6 \\ -2 \end{bmatrix} \begin{array}{c} \\ A \\ B \end{array}$$

$$-c = -2 \Rightarrow +c = 2$$

now we have a triangular matrix:
— everything below the diagonal is zero

reduced it to echelon form
all numbers below (leading) diagonal = zero

Now we can do back substitution.

- take answer for C, and put it back into first two rows.

$$\begin{bmatrix} 1 & 1 & 3 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 15 \\ 6 \\ 2 \end{bmatrix}$$

Solve First row

Solve Second row

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 9 \\ 4 \\ 2 \end{bmatrix}$$

$2 \times 3 = 6 \; [15-6=9]$

$3 \times 1 = 3 \; [3-3=0]$

$2 \times 1 = 2 \; [6-2=4]$

$2 \times 1 = 2 \; [2-2=0]$

$4 \times 1 = 4 \; [9-4=5]$

$4 \times 1 = 4 \; [4-4=0]$

Solve third row

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 5 \\ 4 \\ 2 \end{bmatrix}$$

So we did not really need to ~~Create~~ Compute A^{-1} to get answer

But we only got answer for the specific set of result or output vector

specific(s) = $\begin{bmatrix} 15 \\ 6 \\ 2 \end{bmatrix}$

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So we did
⇒ Elimination
⇒ Back substitution (putting numbers back onto first two rows)
⇒ to solve solution, and most computationally efficient way to do it

Also notice:

Transformed A into Identity Matrix

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

and this will be the key to finding the inverse