Module 2
Inner Products (Part 1)

In order to measure angles, lengths and distances
we need to equip the vector space with inner
product, which will allow us to talk
about geometric properties in vector space.

An example of inner product that we may know
already is the dot product, between 2 vectors
x and y.

If x and y are two vectors in $\mathbb{R}^n$, then dot product
is defined $x^T y = \text{sum } i=1 \text{ to } N$ of $x_i y_i$,

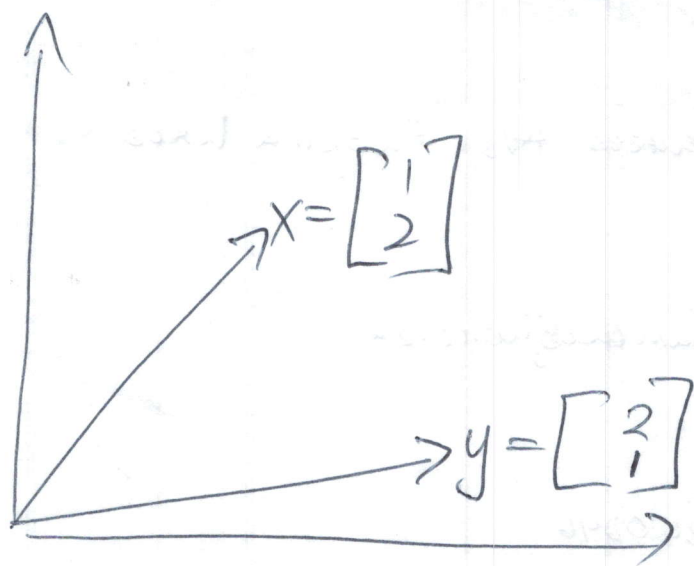where x_i and $x_i, y \in \mathbb{R}^n$

$$\therefore \; x^T y = \sum_{i=1}^{N} x_i y_i, \; x, y \in \mathbb{R}^N$$

The length of x is then defined as the
square root of dot product of x with itself.

∴ "Length of x is square root of $x^T x$, which we can also write as square root of sum of i=1 to N, of x_i squared"

$$\|x\| = \sqrt{x^T x} = \sqrt{\sum_{i=1}^{N} x_i^2}$$

Lets have a look at an example: we take vector x, as (1,2) and vector y as (2,1) in 2 Dim plane.

$$x = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

$$y = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

Then we can compute the length of vector X,

$$\|x\| = \sqrt{1^2 + 2^2} = \sqrt{5}$$

$$\|y\| = \sqrt{2^2 + 1^2} = \sqrt{5}$$

If we interested in the distance between vector x, y we simply compute the length of the different distance vector.

We generally define the distance between x, y to be length or norm of $x - y$, which is square root of $(x - y)$ transpose times $(x - y)$

$$d(x, y) = \|x - y\| = \sqrt{(x - y)^T (x - y)}$$

Lets compute the distance between our two vectors over here: (see pic)

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The distance between these two vectors are effectively the length of difference.

$$d(x, y) = \left\| \begin{bmatrix} 1 \\ 2 \end{bmatrix} - \begin{bmatrix} 2 \\ 1 \end{bmatrix} \right\| = \left\| \begin{bmatrix} -1 \\ 1 \end{bmatrix} \right\| = \sqrt{1 + 1} = \sqrt{2}$$

The last thing we still entertained in is the (length) angle
between the two vectors,
and we can compute the angle, also using
the dot product

$\therefore$ Cos of angle (α), given by x transpose y,
divided by length of x times length of y.

$$\cos \alpha = \frac{x^T y}{\|x\| \, \|y\|}$$

$$\cos \alpha = \frac{4}{5} \approx 0.64 \text{ radians}$$

In this session, we looked at dot product, a special
case of inner product, to compute length of
vectors, compute distances between vectors,
and angles between two vectors

Part 2 we will look at general inner product,
to compute exactly the same quantities