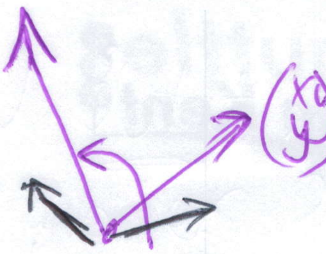
2:30 2:30 ①

Doing Transformations in a Changed Basis

$$\underbrace{\left(\begin{bmatrix}3\\1\end{bmatrix}\begin{bmatrix}1\\1\end{bmatrix}\right)}_{B}\begin{bmatrix}x\\y\end{bmatrix}$$



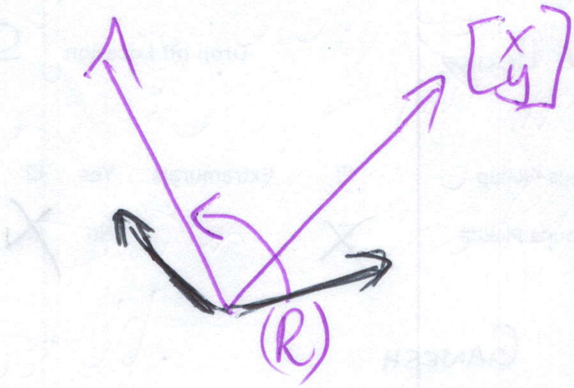
Beans basis in my world

Let's say I have a vector that I want to rotate, transform or send it somewhere, But I only know that vector in Bean's world.

?, But I do not know what that transformation matrix (rotation/reflection etc) is, in Bean's world. A

(Beans have crazy, non-orthogonal, non-unit vector to describe his axis)

2.1



Remember I only know transformation(s) in my system.

First thing going to do, is take bears vector, and put it in my world, then apply the rotation (transformation) in my world.

Let do this calculation first, to give the transformation in my world, then do the rotation.

∴ we have rotation (R) and in my world we know this rotation is

$\frac{1}{\sqrt{2}}\begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}$ ⇐ This is a rotation of 45°.

⇒ Now I have my vector $(R \times B)$, when 45° rotation, then I have vector rotated in my basis

How do I get a vector in my basis, back into bears vector basis

⇒ remember bears not interested in my basis.

③

→ So now I need to turn it back into vector in Bears basis:

$$B^{-1} = \frac{1}{2}\begin{bmatrix}3 & -1\\ -1 & 1\end{bmatrix}$$

(ad) determinant $= 2$ ∴ divide by $\frac{1}{2}$

$$\underbrace{\frac{1}{\sqrt{2}}\begin{bmatrix}1 & -1\\ 1 & 1\end{bmatrix}}_{R_{45^\circ}} \underbrace{\begin{bmatrix}3 & 1\\ 1 & 1\end{bmatrix}}_{B \text{ Bears basis in my world}} \begin{bmatrix}x\\ y\end{bmatrix} = \frac{1}{\sqrt{2}}\begin{bmatrix}2 & 0\\ 4 & 2\end{bmatrix}$$

vector rotated in my world

$$\underbrace{\frac{1}{2}\begin{bmatrix}3 & -1\\ -1 & 1\end{bmatrix}}_{B^{-1}} \underbrace{\frac{1}{\sqrt{2}}\begin{bmatrix}1 & -1\\ 1 & 1\end{bmatrix}}_{R} \underbrace{\begin{bmatrix}3 & 1\\ 1 & 1\end{bmatrix}}_{B} \begin{bmatrix}x\\ y\end{bmatrix}$$

vector Rotation in Bears world.

45° Rotation in Bears world $\Rightarrow \frac{1}{2}\begin{bmatrix}-1 & -1\\ 5 & 3\end{bmatrix}$

④

The following Equation is very useful when you want to do a transformation within some Funny Basis (abnormal Basis), use following:

$$\boxed{B^{-1} R B}$$

ReCap:

- when we transform to non-orthonormal coordinate system, then transformation matrix will also change

We see $B^{-1} \boxed{R} B$ wrapping around the transformation matrix to do ~~to the~~ the translation from my world into world of new Basis system.

(5)

Recap again:

- we've looked at how numbers in matrix to describe a vector change, when we change the basis (bit counter intuitive)
- if we have an orthonormal basis, then we can just use dot product to do projection
- also saw that if we use an arbitrary basis, the transformations themselves will change