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Building approximate functions

The Taylor series is a family of approaches used for building approximations to functions

So before diving into the maths, it's worth stopping for a minute to talk when it may be useful to to have an approximation

One example that stuck in people mind is with cooking a chicken

Imagine you can write a function which describe the relationship between mass of chicken m, and the amount of time t it should be put in oven before it will be perfectly cooked.

There are alot of assumptions that need to have be put in place for such a function to even exist.

Eg. this will only apply for certain type of oven preheated to certain temp, applied to certain configuration of chicken, i.e. one whole chicken, and not chopped up.

Furthermore, the heat transfer properties of chicken will almost certainly vary in a highly non linear fashion as a function of mass.

Lets have look at the sample function, and how some of the factors we have mentioned.

Eg $t(m, T, \text{OvenFactor}, \text{ChickenShapeFactor})$

$$= 7.33m^5 - 72.3m^4 + 253m^3 - 368m^2 + 250m + 0.02 + \text{OvenFactor} + \text{ChickenShapeFactor}$$

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You can imagine that on the one hand, the chicken cooking time t will be quite sensitive to a lot of parameters that we might expect to vary case to case. However, on the other hand, if we like to sell a nice recipe book, we going to need to be pragmatic and simplify this monster.

As people tend not to want to solve complicated equations while making dinner.

So, what do we do?

Firstly we going to make 2 assumptions
- one about oven
- other about chicken

So let's assume that it's reasonable to suggest that everyone who buys my cookbook will have a sufficiently ~~simple~~ similar oven, such that we can assume that they behave basically the same

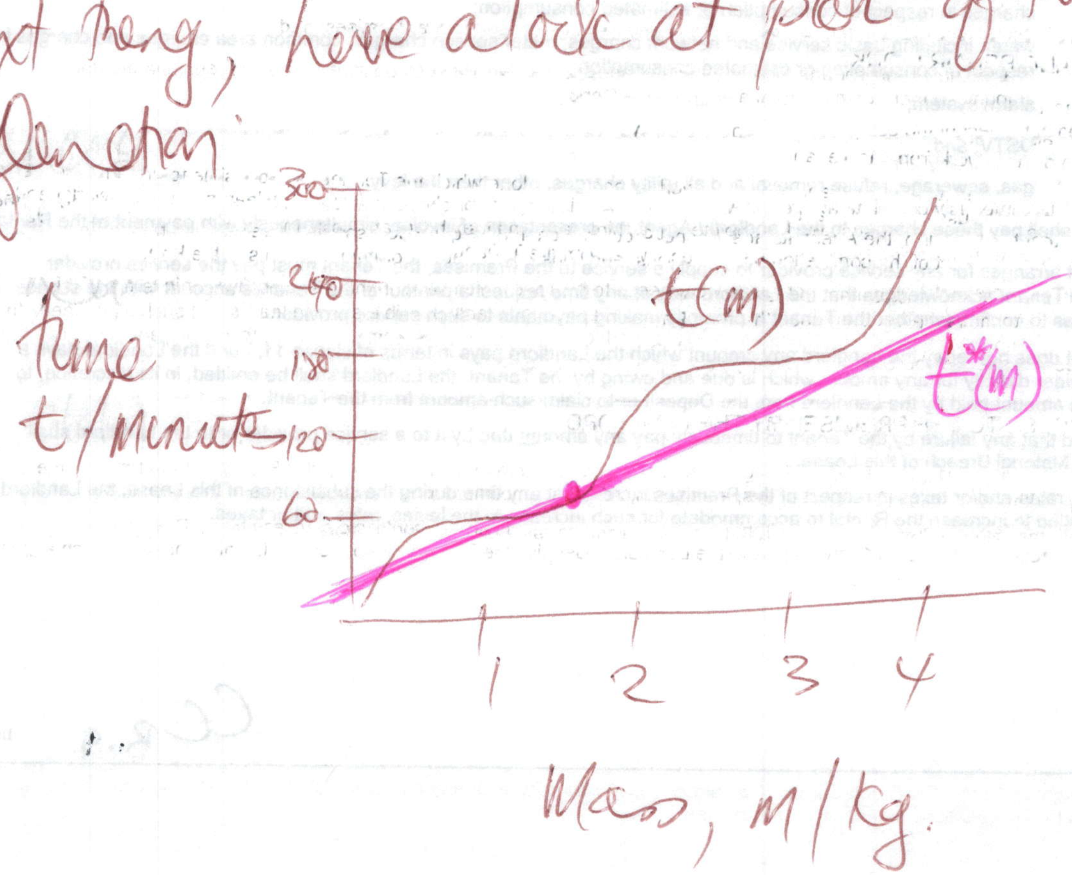
Secondly, everyone will be cooking a sufficiently similar chicken so the heat transfer prop as a function of mass then you can also assume these will the same in each case

This allows us to remove 2 potentially problematic terms that may themselves have a function of many other variable

But we still left with messy function.

$$t(m) = 7.33m^5 - 72.3m^4 + 253m^3 - 368m^2 + 250m + 0.02$$

Next thing, have a look at plot of this function:



So this is what I new as relevant information in the system

As I am not showing timings for chicken heavier the 4 kg or chicken with negative mass

In fact, take it one step further, Considering a typical chicken from supermarket is about 1.5 kg, so let focus here()

And build a nice straight line approximation of form $y=mx+c$

By using the Taylor series approach, which we will cover in next session, it is possible to derive a function with the same height and slope as one of point in graph

This line is a fairly reasonable approximation to the function in the region close to the point of interest

But as you move ~~close~~ further away the approximation becomes pretty poor.

However, the cookbook is not for people roasting giant or miniature chickens

So we end up being able to write down a expression in much more palatable format where an approximation t^* equal to 50 times mass/kg + 15

$$t^*(m) = 50\frac{m}{kg} + 15$$

So if you want to roast a 2kg bird leave it for about 115 mins

In next session, we going to go into more detail about Taylor series and also find out how to ~~arrive~~ to derive higher order terms