Module 3

How matrices transform space

Now we going to look at different types of matrices and what they do to space

and what happens when we apply one matrix transformation, then another (term composition) →

Since we can make any vector out of vector ~~of scaled versions of the~~ vector sum of scaled versions of $\hat{e}_1$ and $\hat{e}_2$

— means that the result of the transformation is going to be same sum of the transformed vectors $\hat{e}_1'$ and $\hat{e}_2'$

— hard to see

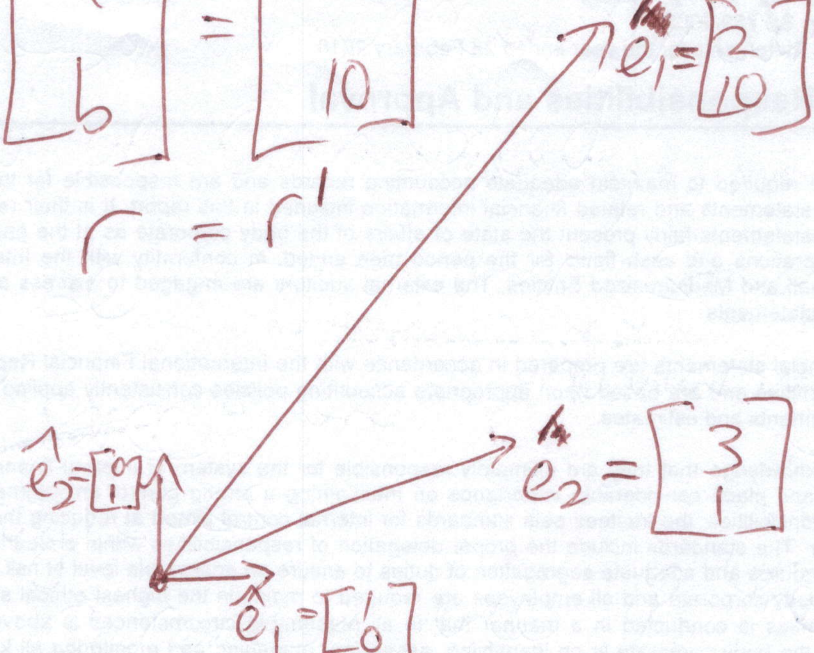
②

hard to see, but what it means that the grid lines of our space remain parallel and evenly spaced. they may be stretched or sheared, But origin space ⇒ there is no warping or curvyness.

⇒ that is a consequence of the scalar addition + multiplication rules of vectors

1.1

$$\underbrace{\begin{pmatrix}2 & 3\\ 10 & 1\end{pmatrix}}_{A}\underbrace{\begin{bmatrix}a\\ b\end{bmatrix}}_{r}=\underbrace{\begin{bmatrix}8\\ 10\end{bmatrix}}_{r'}$$



- Matrix we name as A
- vector we name r
- resultvector as r'
- and we do algebra on it

- Let multiply r by some number n

(nr)

and apply to A

$$A(nr) = nr'$$

Simelary if multiply A by $(r+s)$

$$A(r+s) = Ar + As$$

$$\therefore A(n\hat{e}_1 + m\hat{e}_2) = nA\hat{e}_1 + mA\hat{e}_2$$

$$= n\acute{e}_1 + m\acute{e}_2$$

$$= n\acute{e}_1 + m\acute{e}_2$$

(vecter sum of same multiple)

$\therefore$ all vector sum rules work →

But lets demonstrate in example: Using vecter $(3,2)$ → arbitrary picked

$$\begin{bmatrix} 2 & 3 \\ 10 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ 2 \end{bmatrix} = \begin{bmatrix} 12 \\ 32 \end{bmatrix} \quad A$$

$$\begin{bmatrix} 2 & 3 \\ 10 & 1 \end{bmatrix} \left[3\underbrace{\begin{bmatrix} 1 \\ 0 \end{bmatrix}}_{\hat{e}_1} + 2\underbrace{\begin{bmatrix} 0 \\ 1 \end{bmatrix}}_{\hat{e}_2} \right] = 3\left[\underbrace{\begin{bmatrix} 2 & 3 \\ 10 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix}}_{\acute{e}_1}\right] + 2\left(\underbrace{\begin{bmatrix} 2 & 3 \\ 10 & 1 \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix}}_{\acute{e}_2}\right)$$

$$= 3\begin{bmatrix} 2 \\ 10 \end{bmatrix} + 2\begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 12 \\ 32 \end{bmatrix} \quad B$$

(4)

The vector rules work [with matrices]

A = B

We can thus think of matrix multiplication as been multiplication of the vector sum (geometrically on graph we contract) of the transformed (e_1', & e_2') Basis vectors.

⇒ This is very deep

getting A
getting B
Showing they same thing

$\begin{pmatrix} 2 & 3 \\ 0 & 1 \end{pmatrix}$

Matrix just tells us WHERE the Basis vectors go, that's the transformation it does.