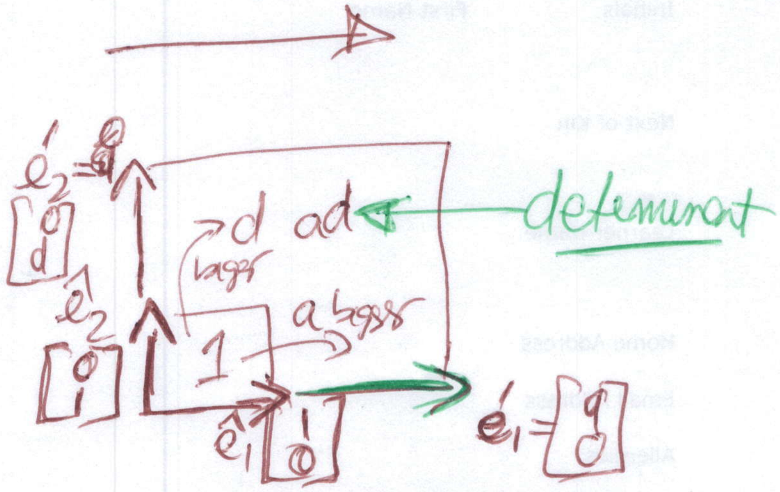
Determenant and inverse:

LOOK AT: property of matrix called the determenant

- also look at what happens when matrix does NOT have linearly independent basis vectors

Simple matrix

$$\begin{bmatrix} a & 0 \\ 0 & d \end{bmatrix}$$



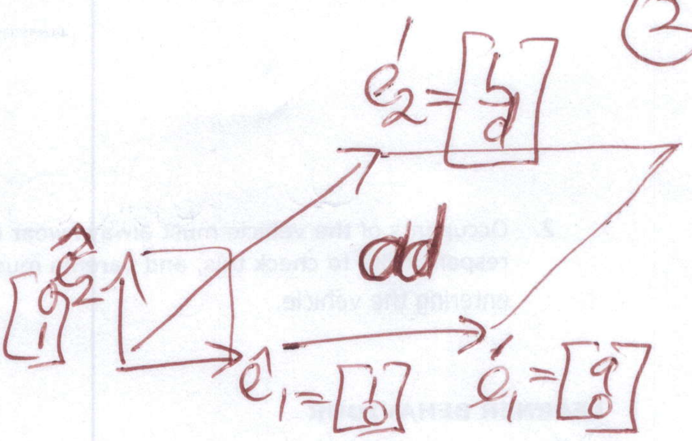
multiply matrix with $\hat{e}_1$, then get new basis vectors

changed area of space to ad (bigger) from 1 unit (Smaller Box)

ad ⇒ called the determenant

2

$A = \begin{bmatrix} a & b \\ 0 & d \end{bmatrix}$



∴ taken original grid (smaller box)

⇒ in addition to stretching it out by a,

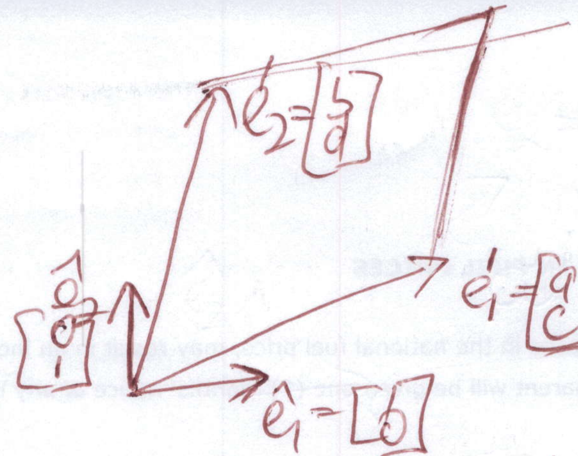
it is also stretched and skewed to $\begin{matrix} b \\ d \end{matrix}$ sheared

⇒ area is just base × perpendicular height, but still ad (determinant)

⇒ still changed size of space by factor of ad

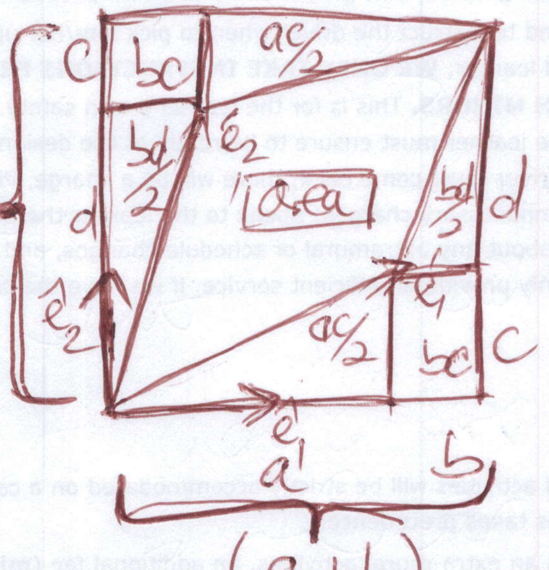
More general matrix

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}$$



But to find this area, we will have to do a lot of calculation (But complicated)

Same Maths:



$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$\text{area} = (a+c)(b+d)$$
$$= -ac - bd - 2bc$$
$$= ad - bc \qquad \therefore |A| = ad - bc$$

(Nice)

So to find the area of the parallelogram by finding the area of the whole 'Box', and subtract it.

But there is another way to find the INVERSE

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \cdot \begin{bmatrix} d & -b \\ -c & a \end{bmatrix} = \begin{bmatrix} ad-bc & 0 \\ 0 & ad-bc \end{bmatrix}$$

1. By exchanging the a and d
2. Changing the signs of b and c

divide by $\frac{1}{ad-bc}$

$$\frac{1}{ad-bc}\begin{bmatrix} \cancel{ad-bc}^{\,1} & 0 \\ 0 & \cancel{ad-bc}^{\,1} \end{bmatrix} = I$$

$$\boxed{\frac{1}{ad-bc}\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}}$$

= Inverse of 2x2 matrix.

∴ The <u>determinant</u> is the amount ~~the~~ that the original matrix "<u>stretched out</u>" space and by dividing by determinant $\frac{1}{ad-bc}$, we normalizing the space <u>BACK</u> to the original size

$$\frac{1}{ad-bc}\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

⇒ we can show how to get determinant ~~compad~~ using row reduction | Elimination | Back substitution methods, but its tricky to show and derive and POINTLESS, so knowing how to do this is not a useful skill any more, we just type

det(A) ⇐ command

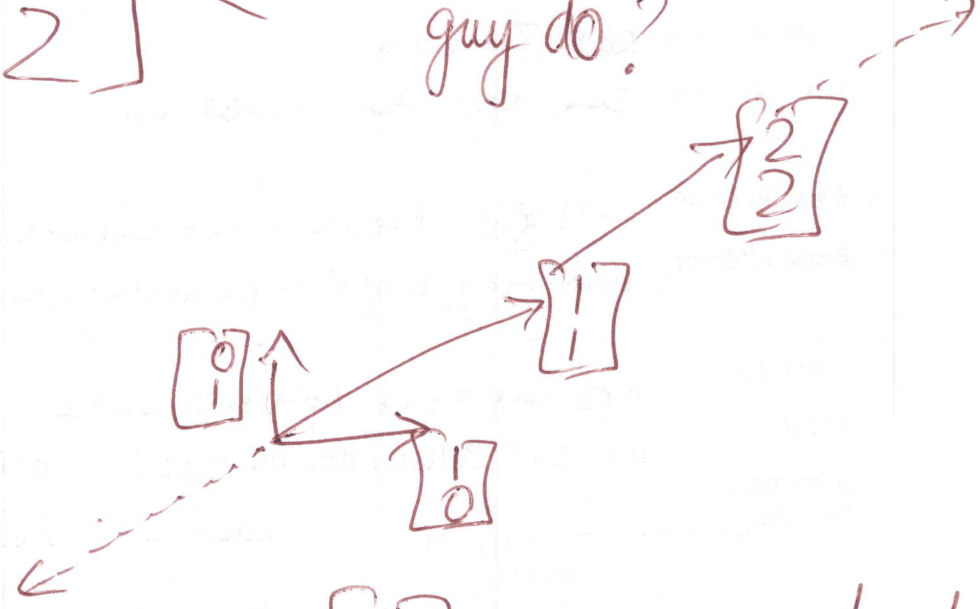
and computer gives answer.

⇒ from learning perspective it does not add much, row echelon add much, thats why we covered it

⇒ so we will not cover how to do determinants, need to read up on:

- QRD composition
- an LA text book.

$$A = \begin{bmatrix} 1 & 2 \\ 1 & 2 \end{bmatrix}$$ ← what does this guy do?



- takes first basis vector to $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$, then second vector to $\begin{bmatrix} 2 \\ 2 \end{bmatrix}$ ⇒ taken a space with area, and collapsed it onto a line, y's and x's collapse onto line.

∴ so every point in space will map onto this line

∴ The determinant of the matrix will be 0, as the area enclosed by the new Basis vectors is 0

$$\therefore A \begin{bmatrix} 1 & 2 \\ 1 & 2 \end{bmatrix} \quad (2-2)$$

$|A| = 0$ (graphically we can also see its zero) (and compute it)

Lets turn Back to our row echelon Form.

$$\begin{bmatrix} 1 & 1 & 3 \\ 1 & 2 & 4 \\ 2 & 3 & 7 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 12 \\ 17 \\ 29 \end{bmatrix}$$

Notice: 1) Sum of raw 1 and 2 = raw 3.
2) 2x col 1 + 1 col 2 = col 3

⇒ if this matrix will be the new basis vectors, we can see they cannot be, and that they are not linearly independent (i.e linearly combinations)

this problem

it will collapse my vector space of 3D to 2D (into plane)

Lets reduce to REF [raw Echelon Form]

$$\begin{bmatrix} 1 & 1 & 3 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 12 \\ 5 \\ 0 \end{bmatrix}$$

$0c = 0$?
Can't back substitute.
∴ Don't have enough information.

∴ when I went Back into shop the 3rd time
I made a mistake, I just ordered a copy
or sum of first two orders....
so I did not get any new information.

∴ 3rd order is not linearly independent
from first 2.

∴ determinant = 0 and can't solve system
that means also can't invert matrix
so stuck

∴ this matrix is SINGULAR

(Remember there may be situations where I
want to ~~collapse~~ do transformations
that collapse the dimensions in space
— may want to do this sometimes
→ But it will come at COST)

Note: The inverse matrix let me undo
my transformation, lets me get from
the new vectors, Back to the original
vectors

So if I now dump or Remove a dimension
2d to line or 3D to plane (or line)
I can undo that any more,
⇒ I don't have enough information
any more (loss some of it during transformation)

So IN GENERAL:
it's worth checking before you propose
a new basis vector set, and
use matrix to transform the basis vectors,
that this is a transformation that
you can undo.
— and you do this by checking that
new basis vectors are linearly
independent ⇒ ✓