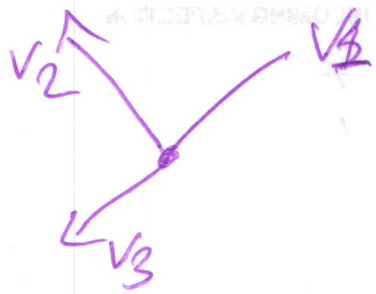
How to construct an orthonormal basis

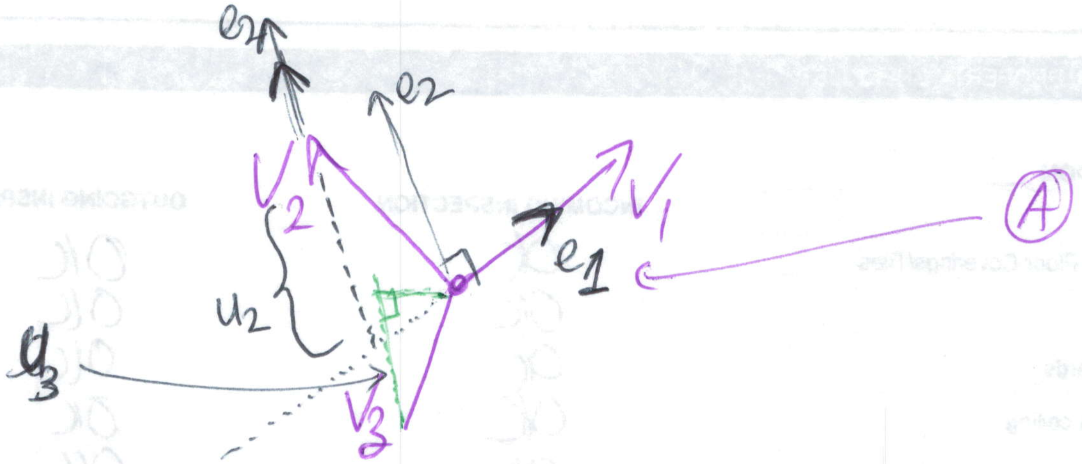
- So we already have linearly independent vectors that span our vector space...

- Lets say we have vectors V=

$V = \{v_1, v_2 \ldots v_n\} \Rightarrow$ span space.

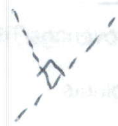


- And they linearly independent (write down their columns in matrix, and check the determinant is not 0)
  - If they linearly dependent, one of them will give you determinant = 0.



V_2 will be composed of 2 things:

- Component that is in ~~at~~ the direction of V_1 ( )
- ~~that~~ plus component that is perpendicular to e_1



(A) the component that is in the direction of V_1, we can find by taking the vector projection (dot product) of V_2 onto e_1

, But these vectors are NOT :

- orthogonal to eachother
- or of unit length ⇒ ☹ ✗

But I will be ☺, if I can construct some orthonormal basis some how...

⇒ But there is a process for doing it.

$\boxed{\text{GRAM-SCHMIDT}}$

⇒ Lets take the first vector in set, call it V_1

V_1 will get to survive...

and normalize V_1

$$e_1 = \frac{V_1}{|V_1|} \quad \text{(normalized)}$$

e_1 will be same normalized version of V_1 (see A)

A

3

$$V_2 = (V_2 \cdot e_1) \cdot \cancel{\boxed{e_1}}$$

and toget it as vector, we have toget e_1 e_1

$= 1$ so ignore

and if we ~~take~~ ~~tat~~ that off of V_2, then we have u_2

$$\therefore V_2 = (V_2 \cdot e_1) \cdot e_1 + u_2$$

(then re-arrange)

$$u_2 = V_2 - (V_2 \cdot e_1) e_1$$

then normalise:

$$\frac{u_2}{|u_2|} = e_2$$

will be another unit vector, normal to e_1

normalized version of V_2

<u>and</u> V_3 is not a linearly combination of V_1 or V_2

$\Rightarrow$ so V_3 is not in plane defined by V_1 and V_2

$\Rightarrow$ and thats not in plane of e_1 or e_2 either

So we can project V_3 onto plane e_1 and e_2, and that projection, will be same vector in plane, composed of e_1 and e_2

we can then say:

$$U_3 = V_3 - (V_3 \cdot e_1)e_1 - (V_3 \cdot e_2)e_2$$

⇒ then normalize U_3.

$$\frac{U_3}{|U_3|} = e_3$$ then we get unit vector that's normal to other two (e_1 and e_2)

we have orthonormal basis for e_1, e_2 and e_3 and keep on going to all the V's ... V_n – until we have all orthonormal basis to complete set and span space

⇒ then I can transformation matrices of (~~pattern~~) rotation, inverses, and are dot product projections