Module 4

[Einstein] Summation Convention and Symmetry of the dot product

But there is **another** way to write matrix transformations: **Einstein Summation Convention.**

⇒ which lists what the actual operations are of the elements of matrix. (usefull when coding)

- also shows you something neat about dot product
- and allows you to deal with NON-SQUARE matrices.

we know from before:

multiplying a matrix by vector, or another matrix, taking every element in row, times every element in column (corresponding) in other matrix, and then adding up (i.e normal matrix / vector operation)

i.e

$$\underset{A}{\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & \\ \vdots & & & \\ a_{n1} & & & a_{nn} \end{bmatrix}} \underset{B}{\begin{bmatrix} b_{11} & b_{12} & \cdots & b_{1n} \\ \vdots & & & \\ b_{n1} & & & b_{nn} \end{bmatrix}} = \underset{AB}{\begin{bmatrix} & & \\ & \circ & \\ & & \end{bmatrix}}$$

row column

∴ to get AB: row A × Col B ⇒ into AB (col)

So to get element:

$(ab)_{23}$ = row 2 of a, column 3 of b

$= a_{21}b_{13} + a_{22}b_{23} + \ldots + a_{2n}b_{n3}$ ○

$$\boxed{\text{Now using Einstein's Convention}} \quad ab_{ik} = \sum_j a_{ij}b_{jk} = a_{ij}b_{jk}$$

Don't Bother with Sum.

$ab_{ik} = a_{ij} b_{jk}$

$= a_{i1} b_{1k} + a_{i2} b_{2k} + \ldots$ all j's
and do for all possible i's and k's

to get AB (right hand result)

Nice: So if you coding, run 3 loops,
i, j and k
and use accumulator on j's
to find element of Product
Matrix AB

Summation Convention gives you a quick way
of coding up these matrix operations

$\longrightarrow$

$AB = C$

$c_{ik} = a_{ij} b_{jk}$

So long as the matrices have the
same number of entries in j

this convention
will work and show you
how to get a result

∴ allows you to multiply non square matrices

$$\begin{array}{cc} & 3 \\ 2 & \begin{bmatrix} \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet \end{bmatrix} \end{array} \begin{array}{cc} & 4 \\ 3 & \begin{bmatrix} \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet \end{bmatrix} \end{array} = \begin{array}{cc} & 4 \\ 2 & \begin{bmatrix} \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet \end{bmatrix} \end{array}$$

$$[2 \times 3] \times [3 \times 4] \qquad [2 \times 4]$$

∴ we have the same number of j's in each <u>case</u> (row × column) =

multiply non square matrix, to get another non-square matrix ⇒

, But when you do this sort of things, determinants, inverses gets messy (just take note)

∃ But there are <u>times</u> when you want to do it.

<u>eg</u> Lets revisit the <u>dot product</u>

in light of the summation convention

<u>i.e</u> now having new found knowledge.

$$\begin{array}{ccc} u & & v \\ \begin{bmatrix} u_i \\ \vdots \\ \\ \\ \end{bmatrix} & \bullet & \begin{bmatrix} v_i \\ \\ \\ \end{bmatrix} \end{array}$$

← two column vectors.

Summation Convention ⇒ $u_i v_i$ ..... repeat all i's and add up ⇒

But, same as just writing:

$$\begin{bmatrix} u_1 & u_2 & \ldots & u_n \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ \\ \\ v_2 \end{bmatrix}$$

(as dot product above)

⇒ its same thing ...

So, there is some equivalence between a matrix transformation / multiplication and dot product

Lets look at that:

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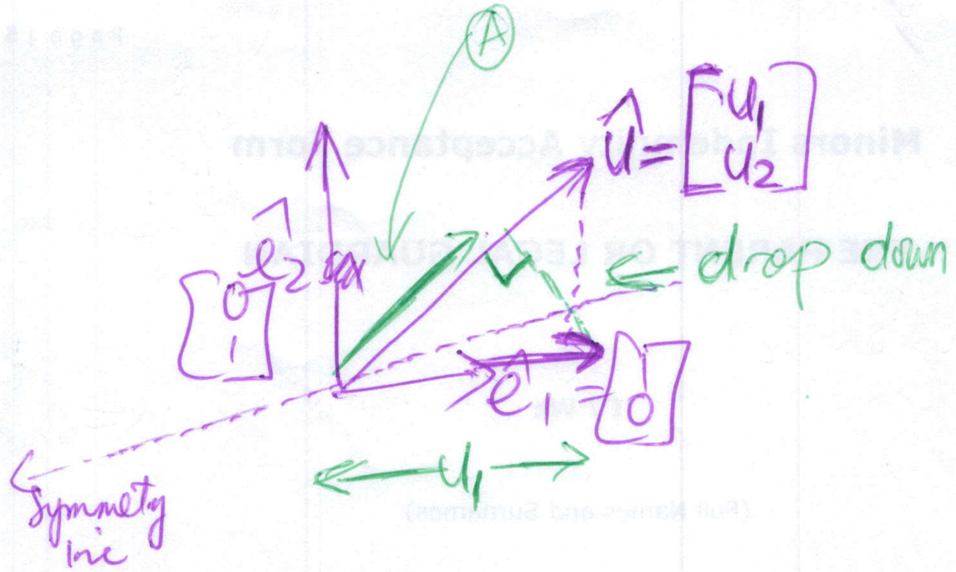

→ Lets take unit vector $\hat{u}$ $\begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$ ....

→ what will happen if we <u>dot</u> it with axis/unit vectors (e)

do the <u>project</u> of $\hat{u}_1$ onto $\hat{e}_1$

or <u>dot</u> u_1 onto e_1

, But what happens if we drop/project $\hat{e}_1$ on $\hat{u}_1$ A

the project A and u_1 is <u>same length</u>

this is implied by <u>dot</u> product

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If we repeat this for $\hat{e}_2$ — or any other axis — we will also get same result.

⇒ This is why the projection is SYMMETRIC and why the DOT PRODUCT is PROJECTION

(x) Connection between numeric multiplication and geometric projection which mind blowing and beautiful

⇒ why we talk about matrix multiplication with matrix as been projection of that vector onto the vectors comprising the matrix. (i.e columns of matrix)

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Recap

- Looked at the Summation Convention which is a compact, and computationally useful, But not very visual way to write down matrix operations
- this allows us to look at funny shaped matrices
- and then also allowed us to re-examine the dot product ✓