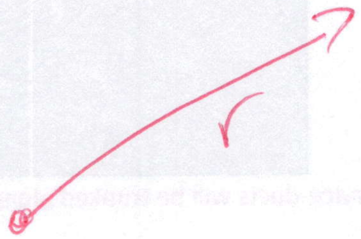
Module 2:

- Changing Basis (Co-ordinate system) Example
  Using the projection product

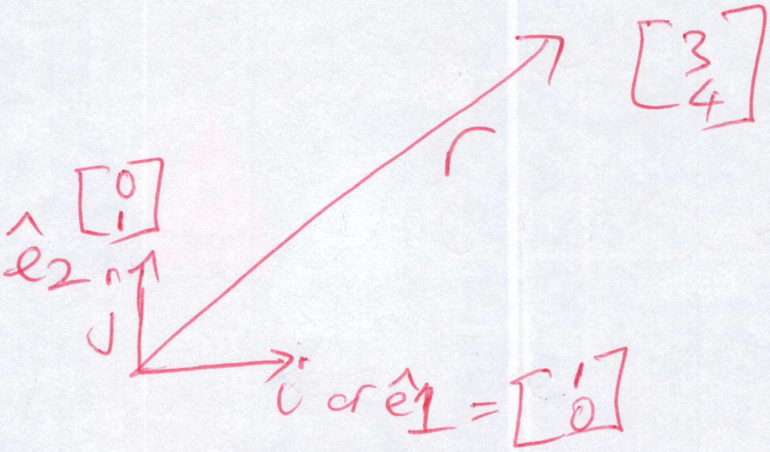
We'll look here at the coordinate system.
Then changing from one coordinate system to another.



⇒ here we have vector r from origin to some point (or space), physical or space of data (eg bedroom in house)

But what's the coordinate system to describe space.

we can use coordinate system of $\hat{i}, \hat{j}$ or $\hat{e}_1, \hat{e}_2$



and defined

$$e_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \qquad \text{[get \^{}, meaning they are of \underline{unit length}]}$$

$$e_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

(if I have more dimensions, I can have $\hat{e}_3, \ldots \hat{e}_7 \ldots \hat{e}_{1\,\text{million}}$)

~~tho~~

$\therefore$ r = some number of $\hat{e}_1$, and some number of $\hat{e}_2$

$$\therefore r = 3\hat{e}_1 + 4\hat{e}_2 = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$$

But my choice of $\hat{e}_1$ and $\hat{e}_2$ ~~to~~ is kind of arbitrary

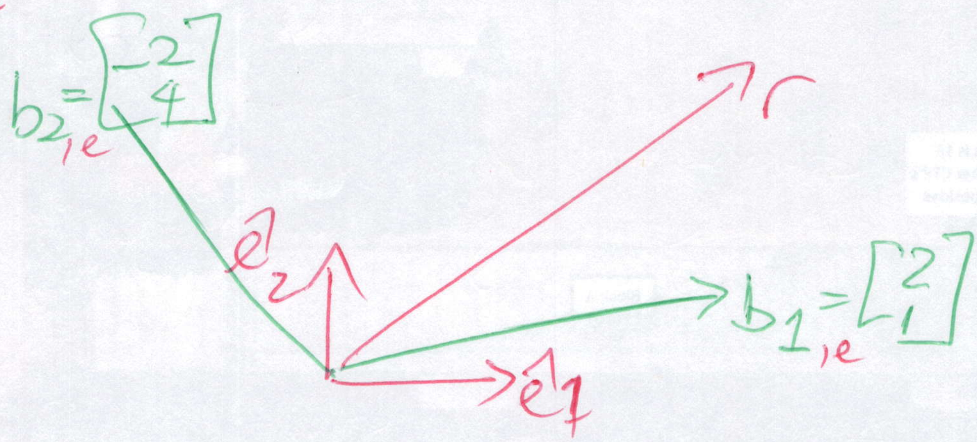
Could have set up a coordinate systems with same angle

eg

or they don't even have to be 90° to eachother

or even different lengths...

$\therefore$ Lets add more vectors b_1 and b_2 defined according to e_1 and e_2
$\downarrow$
our coordinate system.



$\Rightarrow$ But now I can describe r i.t.o. b_1 and b_2 numbers in r will now be different.

We (~~define~~) call the vectors we ~~dep~~ use to define the space e or b Basis vectors.

④

So numbers we use to define r : $\begin{bmatrix}3\\4\end{bmatrix}$, only have any meaning, when we know something about basis vectors

$\therefore r$ using basis vector $e = \begin{bmatrix}3\\4\end{bmatrix}$

But r using basis vector $b = \begin{bmatrix}?\\?\end{bmatrix} = r_b$

So if the new basis vectors b's are 90° to each other, then the projection product has nice application

$\Rightarrow$ we can use the projection or dot product to find number for r (for new basis), so long as we know what the b's are to e

$b_1 = \begin{bmatrix}2\\1\end{bmatrix} \Rightarrow \begin{matrix}2\ \hat{e}_1\\1\ \hat{e}_2\end{matrix}$

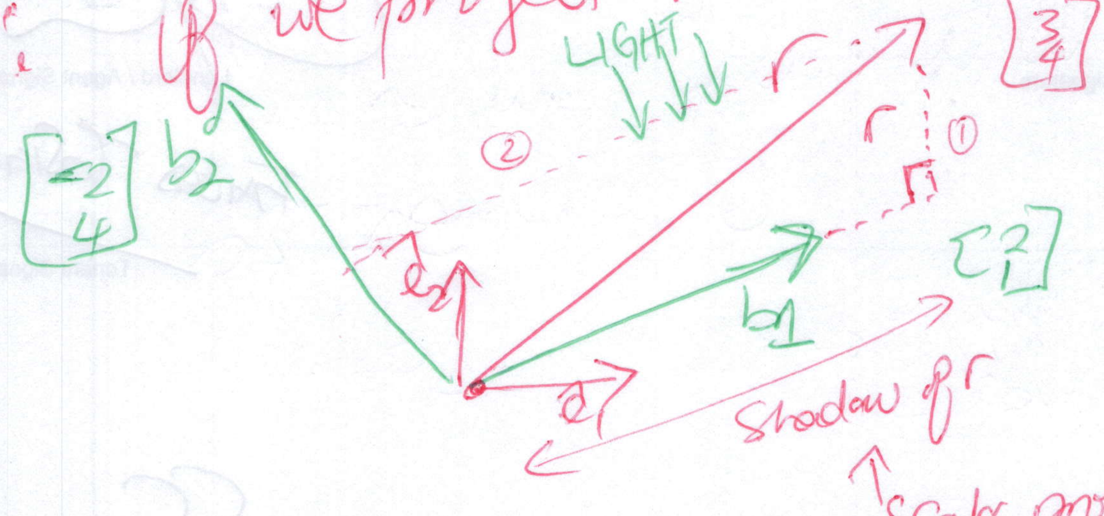
$b_2 = \begin{bmatrix}-2\\4\end{bmatrix} = \begin{matrix}-2\ \hat{e}_1\\4\ \hat{e}_2\end{matrix}$

, But $\vec{B}_1$ and $\vec{B}_2$ must be 90° to each other.
(ok, if not we will need matrices
to do a "transformation of axis"
from e to b, set of basis vectors)

[~~using dot product with the~~]

Ok, we look here first at good picture
⇒ b is (new basis vectors) are orthogonal
to each other
(using dot product)
, its computationally much faster, and easier but
less generic

⇒ If we project r onto b_1 ① projection



↑ Scalar projection
↓
number?

Next take vector project ~~and~~ of r onto b_2
② projection

that length ② + ① we will get r
$\underbrace{② + ①}_{sum}$

do that two vector projections and add it up we'll get $r_b = \begin{bmatrix} ? \\ 2 \end{bmatrix}$

found how to get r_e using e's to getting values for r_b using b's

But how do I check that the two new basis vectors (b's) are 90° to each other.

just take dot product →

[dot product divided by lengths]

$$\cos\theta = \frac{b_1 \cdot b_2}{|b_1||b_2|}$$

If b_1 and b_2 is 0 then $\cos\theta = 0$

and $\cos\theta = 0$ if they 90° to each other

$\therefore b_1 \cdot b_2 = 2 \cdot -2 + 1 \cdot 4$
$= 0$

$\therefore \cos\theta = 0$ (they are $\perp$ to each other)

$\Rightarrow$ so safe to do the projection.

Lets now do it numerically:

$e_1 = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$ $b_1 = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$

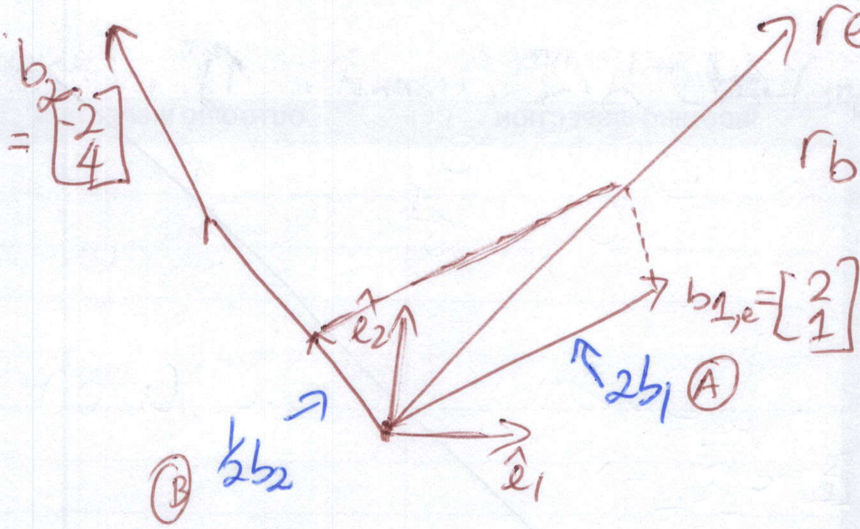
$$\therefore \frac{r_e \cdot b_1}{|b_1|^2} = \frac{3 \cdot 2 + 4 \cdot 1}{2^2 + 1^2} = \frac{10}{5} = 2 \quad (A)$$

$$\Rightarrow 2b_1$$

$$\frac{r_e \cdot b_1}{|b_1|^2} = b_1 = 2\begin{bmatrix} 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 4 \\ 2 \end{bmatrix}$$

$$\frac{r_e \cdot b_2}{|b_2|^2} = \frac{3 \cdot -2 + 4 \cdot 4}{(-2)^2 + (4)^2} = \frac{10}{20} = \frac{1}{2} \quad (B)$$

$$\therefore \frac{r_e \cdot b_2}{|b_2|^2} \, b_2 = \frac{1}{2}\begin{bmatrix} -2 \\ 4 \end{bmatrix} = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$$



$r_e = 3\hat{e}_1 + 4\hat{e}_2 = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$

$r_b = \begin{bmatrix} ? \\ ? \end{bmatrix}$

$r_b = \begin{bmatrix} 2 \\ \frac{1}{2} \end{bmatrix}$

$$\begin{bmatrix} 4 \\ 2 \end{bmatrix} + \frac{1}{2}\begin{bmatrix} -1 \\ 2 \end{bmatrix} = \begin{bmatrix} 3 \\ 4 \end{bmatrix} \rightarrow$$ original vector $r = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$ in basis e

↑ Check

Converted from e set vectors to b set of vectors. (Neat)

⇒ just using dot product.

Recap/Summary:

The vectors describing the data is NOT tied to the axis we originally used to describe it, we can "re-describe" it, using some other axis, or basis vector.

⇒ we can move the basis vectors, to another by using the dot or projection product. where new Basis vectors are ⊥ orthogonal to each other