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Module 1

Chain rule

- Forth and Final tool

: ToolBox will be significantly stacked to tackle more interesting problems.

Sometimes we use functions as inputs to other functions.

: Describing this can get a little bit confusing

So we need to give each of our function meanings, to keep track of what's going on.

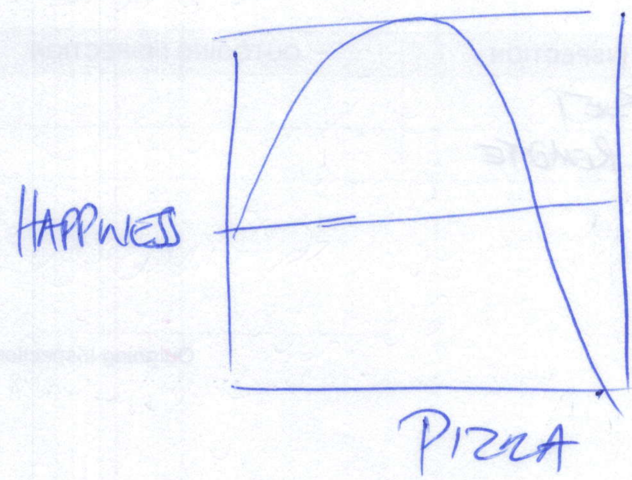
Eg nested function: $h(p(m))$: h, function how happy I am, as function of p, how many pizza I have eaten, and pizza I have eaten, is itself a function of m, of how much money I make.

→ This nested functions come up a lot in science and engineering, and you relate chains of concepts together.

So, first we going to build the function relating happines and pizza, which has the following polynomial form.

$$h(p) = -\frac{1}{3}p^2 + p + \frac{1}{5}$$

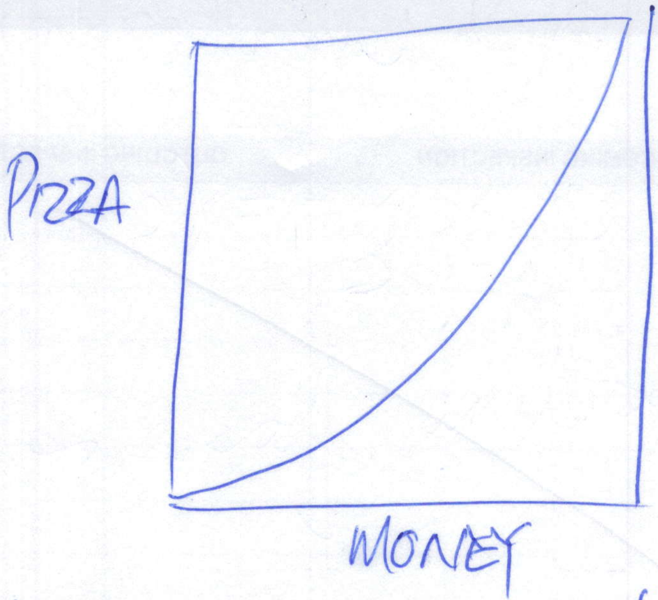
which is easily understandable from plot.



What we can see, is that although without any pizza, it is still possible to be happy in principle, my peak happines is with about 1½ pizzas, and more pizza's than two, I become less happy, and beyond about 3 pizza's, my happines become rapidly negative

Next is a function relating PIZZA and money:

$$p(m) = e^{m} - 1$$



If you have no money, you can't buy any PIZZA. But more money you have, your PIZZA purchasing power increases exponentially

- so first you take advantage of bulk discount,
- But as you get rich, you can buy own PIZZA oven
- even Build your own pizza factory.

What we would like to know: By considering how much money I have now, how much effort I need to put in now

If my aim is to be happy.

To Calculate this, we will need to ~~work out~~ ~~what~~ know what the rate of change of happiness is w.r.t. money

Which is off course just Δh dm or $\frac{dh}{dm}$

For now, using our relatively simple example, we could just directly substitute, our PRZA money, into our happiness PRZA function, which would gives this:

$$h(p(m)) = -\frac{1}{3}(e^m - 1)^2 + (e^m - 1) + \frac{1}{5}$$

Then differentiate this directly

$$\frac{dh}{dm} = -\frac{1}{3}e^m(5 - 2e^m)$$

But the chain rule provides us with a more elegant approach.

which will still work for even more complicated functions, where direct substitution like this may not be an option.

Consider derivative of h w.r.t. p, and p w.r.t m

$$\frac{dh}{dp} \qquad \frac{dp}{dm}$$

Notice: with this particular notation convention, where the derivatives are represented by quotients, the product of these quantities looks like it would give you the desired function dh by dm ($\frac{dh}{dm}$)

In actual fact this is a sensible way to think about what's going on

$$\frac{dh}{\cancel{dp}} \times \frac{\cancel{dp}}{dm} = \frac{dh}{dm}$$

This approach is called the **chain rule**

we making a chain of derivative relationships

CHAIN RULE

If $h = h(p)$ and $p = p(m)$

then $$\frac{dh}{dm} = \frac{dh}{dp} \times \frac{dp}{dm}$$

It may not be described as a formal derivation. But it is already good enough to enable you to make use the chain rule effectively.

Lets apply this rule to our Function:

Firstly, lets differentiate our two functions:

$$h(p) = -\frac{1}{3}p^2 + p + \frac{1}{5} \qquad p(m) = e^m - 1$$

$$\frac{dh}{dp} = 1 - \frac{2}{3}p \qquad \frac{dp}{dm} = e^m$$

then multiply them together:

$$\frac{dh}{dp} \times \frac{dp}{dm} = \left(1 - \frac{2}{3}p\right)e^m$$

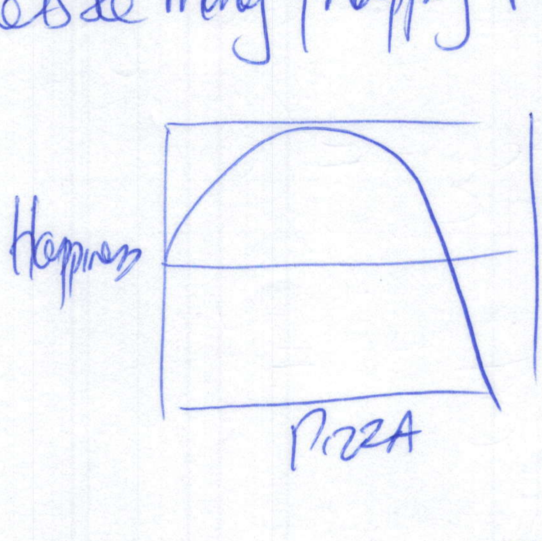
Just remember, if we don't want pizza p to appear in final expression, then we just need to sub in our expression ~~for~~ for p in terms of m

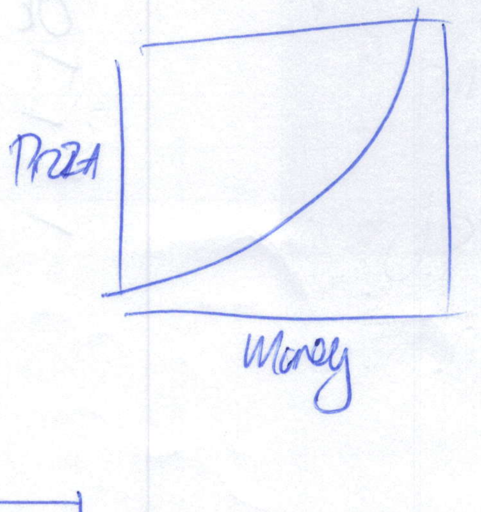
$$= \left(1 - \frac{2}{3}(e^{x}-1)\right)e^{m}$$

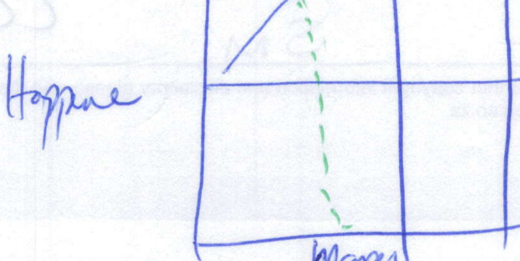
Finally, rearranging the terms, we recover the expression we saw at start of video

$$\therefore \frac{dh}{dm} = \frac{1}{3}e^{m}(5-2e^{m})$$

Lets see money / happy function and derivative on graph







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$$h(p(m)) = e^m - \frac{1}{3}(e^m - 1) - \frac{4}{5}$$

As we can see, if you broke, and its really worth while making some money;

$$\frac{dh}{dm} = \frac{1}{3}e^m(5 - 2e^m)$$

the Benefit of getting more (money), especially if you have enough pizza, decreases dramatically and quickly becomes negative ?

Next: session put them all (4 tools) to use.