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Module 2.

Differentiate w.r.t. anything.

We've seen that partial differentiation as an extension of single variable method. (as derived in last module)

Now, we going to explore a slightly more complicated partial differentiation examples and also build the total derivative of function.

Lets Dive into tricky problem.

Consider Function:

$$f(x,y,z) = \sin(x)e^{yz^2}$$

we will find the derivative w.r.t. each of these 3 variables.

Lets start with x:

The exponential term does not refer to x, we can treat it as a constant and sin differentiates to cos.

$$\frac{\partial f}{\partial x} = \cos(x)e^{yz^2}$$

Next differentiate w.r.t. y

The ~~sign~~ sin term does not refer to y ~~so~~ we'll treat it ~~as~~ as a constant

But for the exponential term, we can either apply the CHAIN RULE, or remember that the result of this operation for exponential, will just be to multiply the derivative of the exponent to front.

③

and the derivative of yz^2 w.r.t y is just z^2

$$\frac{\partial f}{\partial y} = \sin(x) e^{yz^2} z^2$$

Next we differentiate w.r.t z

Once again, the only z is in the exponential term.

So similar to previous example, we simply multiply through by the derivative of the exponential w.r.t z.

$$\frac{\partial f}{\partial z} = \sin(x) e^{yz^2} 2yz$$

Now that we have these 3 partial derivatives, we going to introduce new idea called $\boxed{\text{TOTAL Derivative}}$

Imagine the variable x, y, z ~~are~~ were actually all themselves a function of a single other parameter t ④

Where:

$x = t-1; \quad y = t^2, \quad z = \frac{1}{t}$

What we looking for is the derivative of x w.r.t t

In this simple case, we can just substitute for all our 3 variable directly into t

Simplify a little bit, then differentiate directly w.r.t t, which gives us

$$f(t) = \sin(t-1)\, e^{t^2\left(\frac{1}{t}\right)^2}$$

$$f(t) = \sin(t-1)\, e$$

$$\frac{df(t)}{dt} = \cos(t-1)\, e$$

However, in a more complicated scenario, with many variables, the expression we needed to differentiate may have become unimaginably complex, and perhaps we may not have a nice analytical expression at all.

The alternative approach, is to use the logic of CHAIN RULE to solve this problem.

Where the derivative w.r.t our new variable t, is the sum of the chains of the other 3 variables.

As shown in the expression

$$\frac{f(x,y,z)}{dt} = \frac{\partial f}{\partial x}\frac{dx}{dt} + \frac{\partial f}{\partial y}\frac{dy}{dt} + \frac{\partial f}{\partial z}\frac{dz}{dt}$$

Since we already have our 3 partial derivates of f w.r.t (x,y,z), now we just need to find the derivates of the 3 variables w.r.t. t.

and we will ~~al~~ have all things we need to evaluate our expression

$$f(x,y,z) = \sin(x)e^{yz^2}$$

$$\frac{df}{dx} = \cos(x)e^{yz^2} \qquad \frac{df}{dy} = z^2\sin(x)e^{yz^2} \qquad \frac{df}{dz} = 2yz\sin(x)e^{yz^2}$$

$$\frac{dx}{dt} = 1 \qquad \frac{dy}{dt} = 2t \qquad \frac{df}{dt} = -t^{-2}$$

$$\frac{df(x,y,z)}{dt} = \frac{df}{dx}\frac{dx}{dt} + \frac{df}{dy}\frac{dy}{dt} + \frac{df}{dz}\frac{dz}{dt}$$

$$\frac{df(x,y,z)}{dt} = \cos(x)e^{yz^2} \times 1 + z^2\sin(x)e^{yz^2} \times 2t + 2yz\sin(x)e^{yz^2} \times (-t^{-2})$$

$x = t-1$; $y = t^2$, $z = \frac{1}{6}$

$$\frac{df(x,y,z)}{dt} = \cos(t-1)e + t^{-2}\sin(t-1)e \times 2t + 2t\sin(t-1)e \times (-t^{-2})$$

$$\therefore \frac{df(x,y,z)}{dt} = \cos(t-1)e + 2t^{-1}\sin(t-1)e - 2t^{-1}\sin(t-1)e$$

$\therefore$ second / third terms are same just different signs so cancel each other.

$$\frac{df(x,y,z)}{dt} = \cos(t-1)e$$

arrive at same result, at beginning of session?