40:00

Module 1:

Taming a beast

Here we going to work through a nasty looking function, that will require us to use all 4 (timesaving) Rules

Note: This function will not describe anything familiar

$\therefore$ we will be flying blind and have to trust the maths.

This will give us confidence to dive into next set questions.

Consider the rather nasty function:

$$\boxed{f(x) = \frac{\sin(2x^5+3x)}{e^{7x}}}$$

The essence of the Sum, Product and Chain Rules are about breaking the function down into manageable pieces

So first thing to spot, is that although it is currently expressed as a fraction we can rewrite f(x) as a PRODUCT:
- by moving the denominator (Bottom part) up
- and raising it to the power of -1

$$\boxed{f(x) = \left(\sin(2x^5+3x)\right)e^{-7x}}$$

But there is also another rule in dealing with fractions directly, called the QUOTIENT RULE, But it requires memorizing an Extra expression so we won't be covering it here.

Next, we split $f(x)$ up into 2 parts of product, and work out how to differentiate each part separately, ready to apply the product rule later on.

First Part: Called $g(x)$

$$\therefore \underbrace{\sin(2x^5+3x)}_{g(x)}$$

we have the trigonometric function sin applied to a polynomial $(2x^5+3x)$, which is a classic target for the chain rule

all we need to do is take our function and split it up into 2 parts, in order to apply chain rule

$g(u) = \sin(u)$

$u(x) = 2x^5 + 3x$

So we have 2 separate functions, and differentiate each of them, to apply chain rule.

$\therefore g(u) = \sin(u) \rightarrow g'(u) = \cos(u)$

$\therefore u(x) = 2x^5 + 3x \rightarrow u'(x) = 10x^4 + 3$

Next we'll use a mixed notation for convenience:

$$\frac{dg}{\cancel{du}} \cdot \frac{\cancel{du}}{dx} = \cos(u)(10x^4+3)$$

$$\frac{dg}{dx} = \cos(2x^5+3x)(10x^4+3)$$

we now have an expression for the derivative $g(x)$, and already we made use of

Chain Rule, Sum and Power Rule

$$\therefore g'(x) = (10x^4+3)\sin(2x^5+3x)$$

Half of an expression

For Second Part: which we will call $h(x)$

$$\underbrace{e^{-7x}}_{h(x)}$$

we can just apply the Chain rule after splitting our function up.

$$\therefore h(v) = e^v$$

$$v(x) = -7x$$

So we have our 2 functions, now find the derivatives:

$$\therefore h(v) = e^v \rightarrow h'(v) = e^v$$

$$\therefore v(x) = -7x \rightarrow v'(x) = -7$$

then combine them back together.

$$\frac{dh}{\cancel{dv}} \cdot \frac{\cancel{dv}}{dx} = -7e^{-7x} \rightarrow h'(x) = -7e^{-7x}$$

So all v's have disappeared so this is our final expression.

So now we have expression for both parts of our product — we can just apply the product rule, to generate the final answer.

$$\frac{df}{dx} = \frac{dg}{dx} h + g \frac{dh}{dx}$$

$$= (10x^4+3)\cos(2x^5+3x)e^{-7x} + \sin(2x^5+3x)(-7e^{-7x})$$

Can also then be rearranged. (and expressed in many ways)

$$= e^{-7x}\left[(10x^4+3)\cos(2x^5+3x) - 7\sin(2x^5+3x)\right]$$

$$= \frac{(10x^4+3)\cos(2x^5+3x)}{e^{7x}} - 7f(x)$$