Module 3 1:10:46 1

Example: Projection onto a 1-Dimensional Subspace.

In the last session we derived the formula for projection of vectors into 1 Dim subspace

$$\pi_u(x) = \frac{x^T b}{\|b\|^2} b \quad (*)$$

In particular we (des) arrived at this equation if we chose the dot product as the inner product.

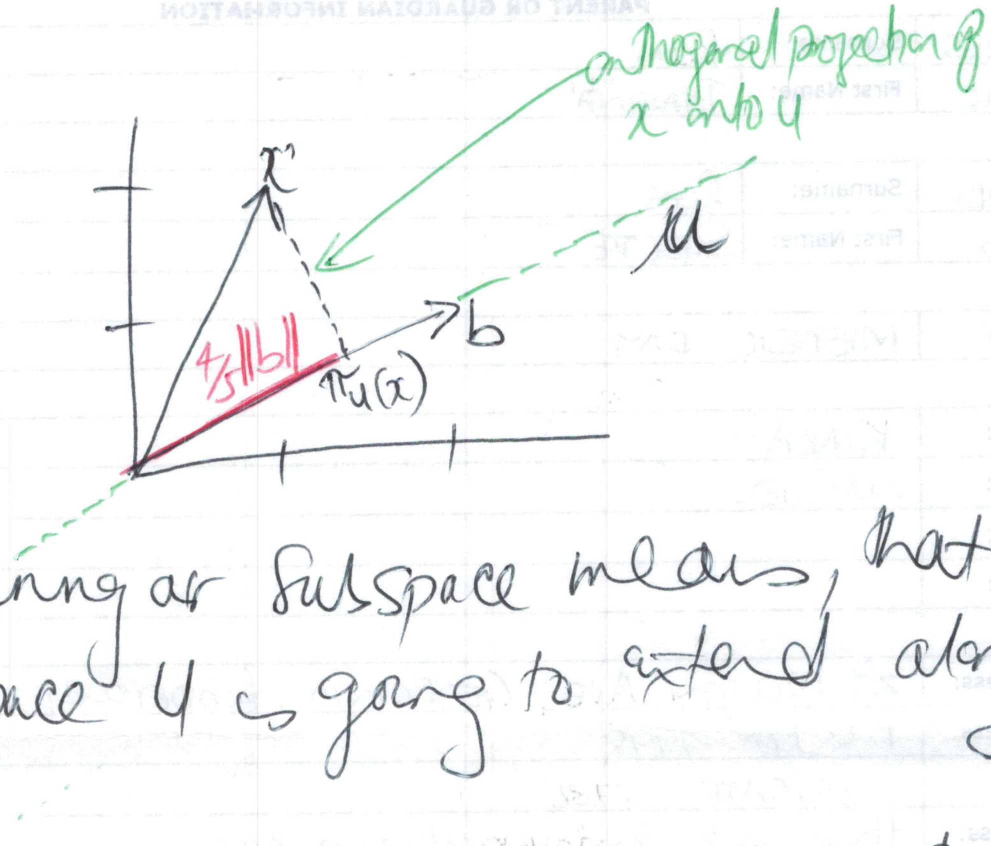
Here x a d dimensional vector, and b is the basis vector that spans the 1 Dim subspace that we want to project x onto.

In this session we're going to look at an example.

"Assume our vector b that spans our 1 Dim subspace, is the vector $\begin{bmatrix} 2 \\ 1 \end{bmatrix}$, and vector x we want to project onto that subspace, is given by $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$"

$b = \begin{bmatrix} 2 \\ 1 \end{bmatrix}, x = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

Lets quickly draw this



And b spanning our subspace means, that our subspace U is going to extend along this line

So now we interested in computing the orthogonal projection of x onto U

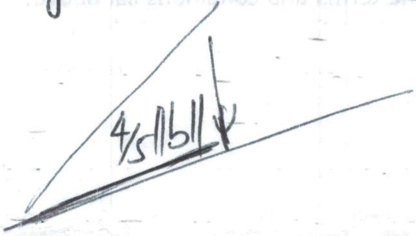
Using this Equation here (A) (page 1), we get

"X transpose times b is 2+2, divide by squared norm of b, so length of b, is $2^2+1^2=5$, times b, will give us $\begin{bmatrix} 2 \\ 1 \end{bmatrix}$, so overall $\frac{4}{5}$ times vector $\begin{bmatrix} 2 \\ 1 \end{bmatrix}$"

$$= \frac{2+2}{5}\begin{bmatrix}2\\1\end{bmatrix}$$

$$= \frac{4}{5}\begin{bmatrix}2\\1\end{bmatrix}$$

This means that our orthogonal projection is 4 over 5 times vector b, i.e. if we take 4/5 ths of this vector, then we will get to our orthogonal projection

length of 

is 4/5 times b

We've gone through orthogonal projections onto 1 dim subspaces.

In next session we'll look at orthogonal projections onto high dimensional subspaces