Module II {what we can do with vectors}

- Look at their modulus or magnitude
- Combine vectors together, called dot product
- Look basis vectors and linearly independence
- How to get scalars and vector projection
- How vectors define a space

Modulus of Dot product :-

⇒ Length of vector or size of vector

⇒ dot product of vector, also called inner, scalar or projection product. (one of most beautiful parts of vector) (and huge number of implication.

we defined r initially without any reference to a coordinate system

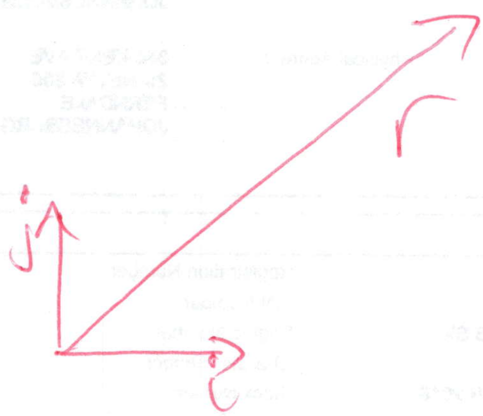
at moment it only has:

$\Rightarrow$ length

$\Rightarrow$ direction

$\Rightarrow$ So we need to know how to calculate these two properties.

So if we use a coordinate system, i, j



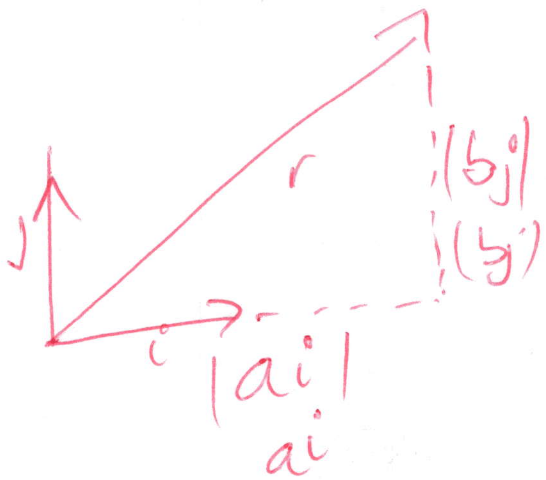
we can now say: r equals

$$ai + bj = \begin{bmatrix} a \\ b \end{bmatrix}$$

⇒ If we want to know the length

- we can draw a triangle and use pythagoras theorem.

$| \; |$ means length



So using pythagoras:

$|r| = \sqrt{a^2+b^2}$ ← is equal to length of r.

$$r = \begin{bmatrix} a \\ b \end{bmatrix}$$

$$\|r\| = \sqrt{a^2+b^2}$$

we have done this for unit vector i, j which are right angles to each other.

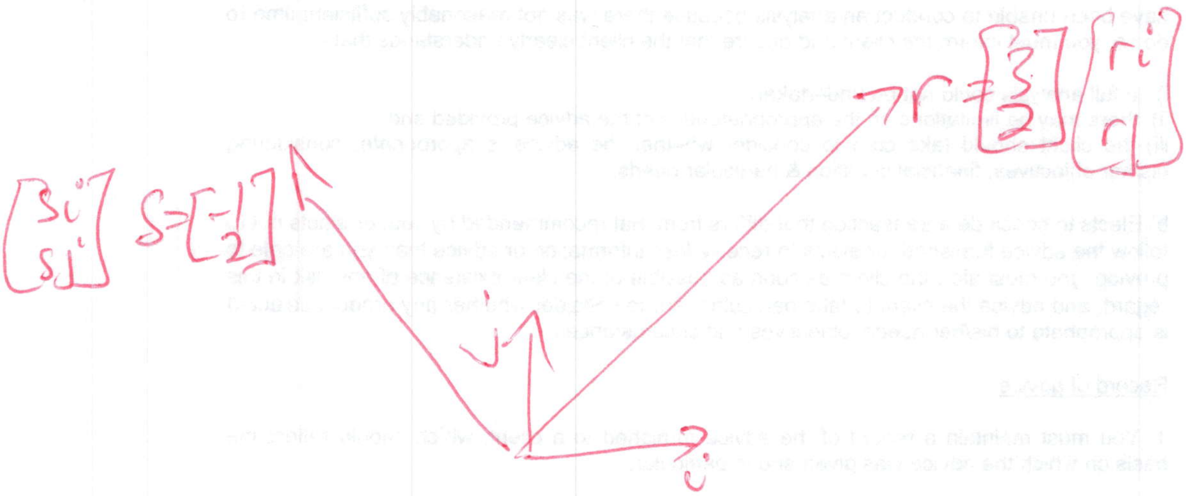
What about other cases:

we always define the size of vectors as sum of squares of each component rooted.

Next thing is to find the dot product.

Some say: multiplying two vectors together.

— with components



we define dot product as

$$r \cdot s = r_i s_i + r_j s_j$$

$$= 3 \cdot -1 + 2 \cdot 2$$

$$= -3 + 4$$

$$= \boxed{1 \Leftarrow \text{dot product}}$$

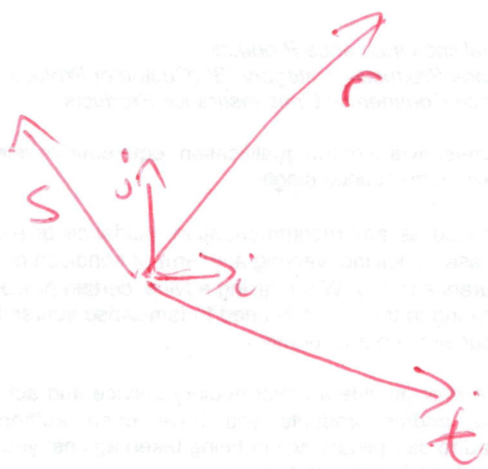
Properties of dot product:

#1 ⇒ $\boxed{\text{Commutative:}}$

∴ Same as $s \cdot r = s \cdot r$

(order does not matter)

#2 ⇒ $\boxed{\text{Distributive}}$ (over addition)



∴ have third vector (t)

$$\therefore\ r \cdot (s + t) = r \cdot s + r \cdot t$$

Also if we multiply a vector with scalar as and do dot product on that

$$r \cdot (as) = a(r \cdot s)$$

#3 $\boxed{\text{Associative}}$ over scalar multiplication

No lets draw the relationship between

- Dot product
- Size of vector.

$$r \cdot r = r_i r_i + r_j r_j$$
$$= r_i^2 + r_j^2$$

But we did say size of vector is square root of it

$\therefore$ Size vector $\Rightarrow r \cdot r = |r|^2$

So size of vector is r "dotted" with itself.