Eigen Problems

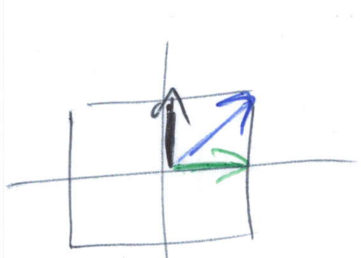
Eigen Basis : Example

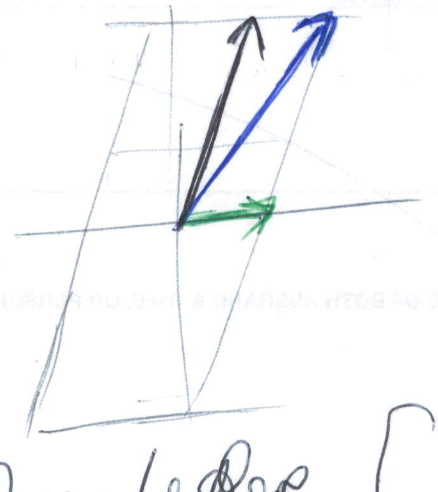
We know the theory of Eigen Basis and Diagonalization

— lets do a 2D example to see the answer graphically to verify our method.

Consider the transformation matrix:

$$T = \begin{bmatrix} 1 & 1 \\ 0 & 2 \end{bmatrix}$$





· First Column is unchanged from before $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$

But second column will be moving to point $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$

and the blue vector will go to $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$$\begin{bmatrix} 1 & 1 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$$

So what happened $\Rightarrow$ this transform can be decomposed into:

1. vertical scaling by factor of 2
2. Horizontal ~~(scaling by)~~ sheer by half a step

This is a simple transformation, so we should be able to spot the eigenvectors, and see the eigenvalues

$$\lambda = 1 \Rightarrow \overset{\text{eigenvector}}{x_1} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$\lambda = 2, \quad x = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

Now lets consider what will happen to vector $(-1,1)$ when we apply T.

$$T = \begin{bmatrix} 1 & 1 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} -1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 2 \end{bmatrix}$$

and if we apply T again we get follawing:

$$\begin{bmatrix} 1 & 1 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 0 \\ 2 \end{bmatrix} = \begin{bmatrix} 2 \\ 4 \end{bmatrix} \quad *$$

But what $T^2 \rightarrow$ to do the two steps above in single step

$$T^2 = \begin{bmatrix} 1 & 1 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 3 \\ 0 & 4 \end{bmatrix}$$

$\therefore$ we can apply the to our vector and see if we get the same result

$$\begin{bmatrix} 1 & 3 \\ 0 & 4 \end{bmatrix} \begin{bmatrix} -1 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ 4 \end{bmatrix} \quad *$$

we can now try this whole process again, But using the $\boxed{\text{eigenbasis approach}}$

So we have already build our Conversion matrix C, from our eigen vectors

$$\begin{array}{ccc} V & \xrightarrow{T^n} & T^n V \\ {\scriptstyle C^{-1}}\downarrow & & \uparrow{\scriptstyle C} \\ V_E & \xrightarrow{\Delta^n} & [T^n V]_E \end{array}$$

$$\boxed{T^n = C \Delta^n C^{-1}}$$

$$\therefore \; C = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$$

Now we need to find its inverse, Easy to find since it's a simple example

$$C^{-1} = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}$$

$\therefore$ Now we can construct our problem

$$T^2 = C\Delta^2 C^{-1} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}^2 \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 4 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ 0 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 3 \\ 0 & 4 \end{bmatrix}$$

Then apply to vector $(-1, 1)$

$$\begin{bmatrix} 1 & 3 \\ 0 & 4 \end{bmatrix} \begin{bmatrix} -1 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ 4 \end{bmatrix}$$

Same result we found before

The Concept is now clear to us, and since we can use Computer for the Calculation, we never need to do this by hand.

$\boxed{\text{VERY IMPORTANT.}}$ In order to cement the idea, will need to do a few examples on my own

Note, we did not cover

- undiagonalizable matrices
- complex eigen vectors

But, confidence in core topics is sufficient at this stage

Next lesson: real world application