Module 12

Linear Transformation of variances (Part 2)

we've seen what shifting and scaling does to the mean, now lets a look at the effect to the variance.

Remember the variance is spread of the data. what do we expect when a dataset is shifted? Lets have a look at dataset (see pic)

we have 3 datapoints : -(-1, 2, 3), and now we shifting the data set toward the right.

The variance of data set is indicated by Blue bar at bottom (see pic)

2.

The shift of data set is given by red dots (see pic)

and we shift every individual data point by 2.

Question now: what happens to variance?

Remember, the shift of data does not really affect the relation of the data points amongst themselves, therefore the variance does not change.

∴ variance is identical (see pic)

The variance of blue data set is identical to variance of red data set. (see pic)

Therefore a general result is that, if we have variance of D is exactly the same as variance of our dataset D + a, where a is an offset applied to every individual element of D.

$Var[D] = var[D + a]$

Lets now scale the dataset, and see what effect it does have on variance of data

we going to take exactly the same dataset as before, the variance is indicated by blue bar, and we going to scale every individual datapoint by 2

Question is what's variance of 2 times D?

Previous (Pic)

New (Pic)

New dataset is indicated by those 2 dots, Remember the variance is average square distance of datapoint from mean.

If we scale the dataset by factor of 2, the distance of every datapoint to mean is scaled by 2.

But the square distance, is scaled by 4. (see pic)

and variance is therefore, 4 times as big, as it used to be

Our next result: variance of α times D, is α squared times the variance of D, where α is a real number that scales every individual data set D.

$$Var[\alpha D] = \alpha^2 Var[D]$$

Now let's have a look at high dimensional problems:

assume we have dataset D, which is a collection of datapoints x_1 to x_N, and it live in $\mathbb{R}^p$

$$D = \{x_1 \ldots x_N\}, x \in \mathbb{R}^p$$

Remember the variance of this data set is given by co-variance matrix.

If you perform a linear transformation of every datapoint, say $Ax_i + b$, for given matrix A and offset vector b, the question is what happens to our dataset:...

$$Ax_i + b$$

If we do it to every single data point well, we get the covariance matrix of the transformed dataset, as follows:

"variance of A times D plus b is A times variance of D times A transpose"

$$Var[AD+b] = A\,var[D]A^T.$$

In, this session we say what effect the linear transformation of dataset has on the mean and variance.

In particular, we saw that shifting data has only an effect on mean, not variance where scaling the data effects both the mean and variance.