Homework 5. Petri Nets

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1 Part A

Solution for a part A of the home assignment can be found in the file HW5.pnlm or on the Figure 1. In the solution the M1 means machine 1 (the same for M2), and M1 s1 means first slot in the machine M1 (the same for M1 s2, M2 s1 ans M2 s2).

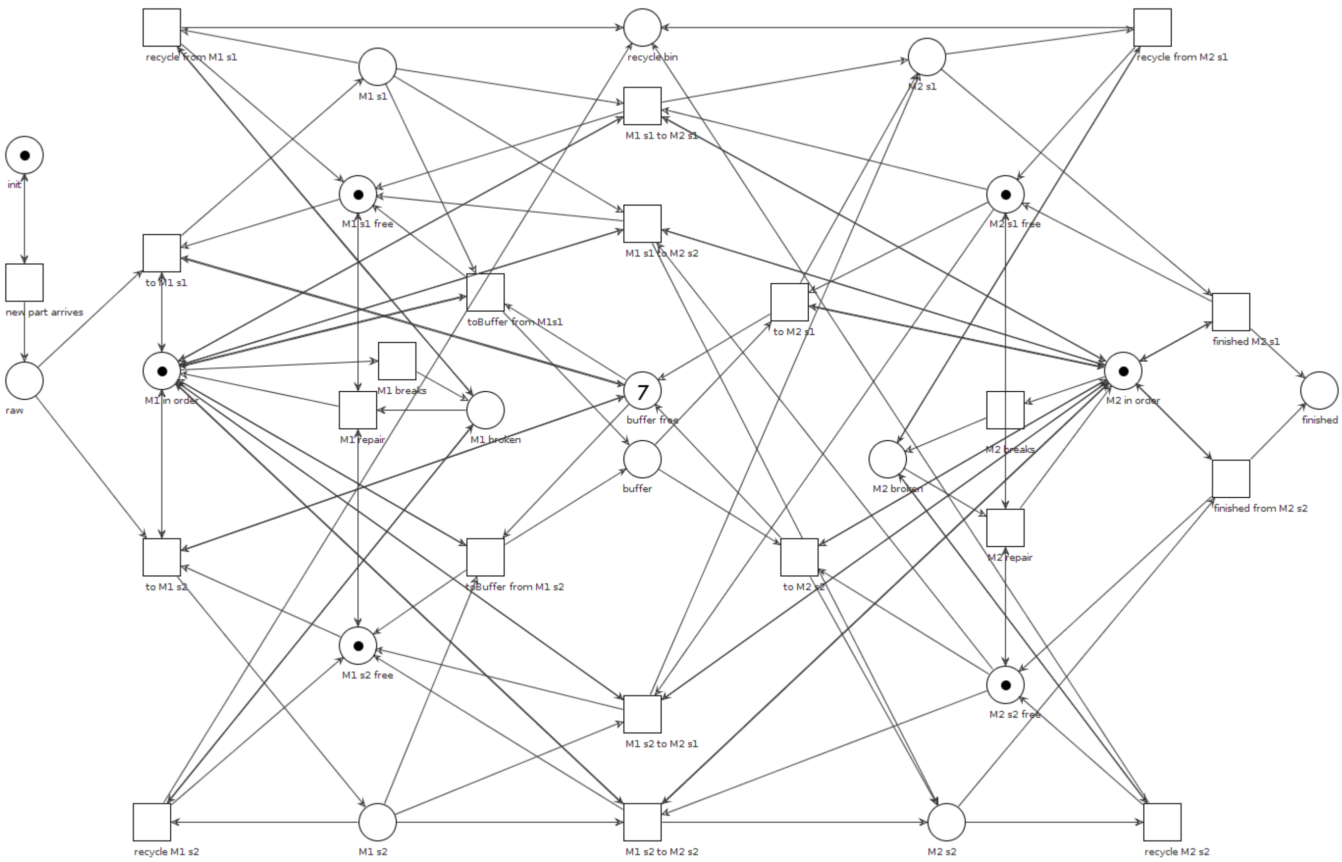

Figure 1: Petri Net. Solution to Part A

2 Part B

1. In the first net it is clear that the shortest firing sequence that leads to all odd-numbered places having a token in them (let's call this the final marking) is 2. From the initial marking only transition t2 can fire. After firing t2 in initial state, only transition t1 can fire and then we arrive in the final marking.

   Now let's note that the second petri net can be obtained from the first by adding a component identical to the first petri net. We know that the first component in the second petri net can be taken to final marking in two steps. This is required before we can take the second component into final marking, because otherwise we cannot fire t3. After taking second component to final marking we have to take the first component again to final marking. First we have to fire transition 1 which takes the first component back to initial marking and from there, as we already know, we can go to final marking in two steps. Thus in total we have 2 steps (first to final) plus 2 steps (second to final) plus 1 step (first to initial) plus 2 steps (first to final again). Total $2 + 2 + 1 + 2 = 7$.

We get third net by again adding one additional component. We know that the first two components can be taken into final marking in 7 steps. Then we can take the third component into final marking through t6 and t5. These steps change the marking of the second component, which can be taken to initial marking in 1 step, which in turn changes the marking of the first component, which also can be taken into initial marking in 1 step. Both of them have to be taken to initial marking to get them to final marking. And in 7 steps minimum, as shown in previous part, we can take the first 2 components into final marking. In total we have 7 steps (first two to final) plus 2 steps (third to final) plus 2 steps (first two to initial) plus 7 steps (first two to final). Total $7 + 2 + 2 + 7 = 18$.

2. Let's denote the number of steps needed as a function f of the number of components n (identical elements in the n^{th} member of the infinite family of nets). The formula is

$$f(n) = \begin{cases} 2, & \text{if } n = 1 \\ f(n-1) \cdot 2 + n + 1 & \text{otherwise} \end{cases}$$

Let's use induction to prove it.

From the previous part we can clearly see induction pattern. For the induction base, let's use just 1 component, that is $n = 1$. Then we have $f(1) = 2$ as shown in previous part.

Now the induction step. Let's assume that the formula holds for $n - 1$. Let's show that the formula then holds for n. To get the last component to final marking, we first take all the previous components to final marking, which takes $f(n-1)$ steps by the induction assumption. The last component can be taken to final marking in 2 steps. After that we take all but last components back to initial marking, which takes $n - 1$ steps. This is necessary to take them again to final marking. Taking them to final marking takes $f(n-1)$ steps by the induction assumption. In total we have $f(n) = f(n-1) \cdot 2 + n + 1$ steps. Since we cannot skip any steps, this is the minimum number of steps needed to take the whole net from initial to final marking. This concludes the proof.