Homework 5

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1 Automated Factory Line

The petri net for automated factory line is presented as Figure 1.

- The arc from place 'M1L free' to transition 'fix M1L' has a weight of 2.
- The arc from transition 'fix M1L' to place 'M1L free' has a weight of 2.
- The arc from place 'M2L free' to transition 'fix M2L' has a weight of 2.
- The arc from transition 'fix M2L' to place 'M2L free' has a weight of 2.

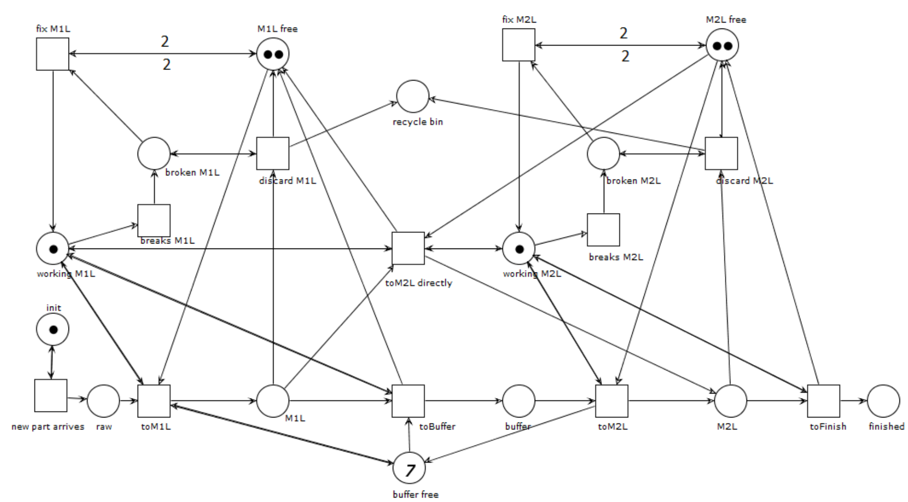

Figure 1: Petri net of the system in its initial (empty) state

2 Shortest Firings

1. The lengths of the shortest firings for petri nets 1, 2, 3 are 2, 7 and 18, respectively.

   We obtained the answer by constructing reachability graphs for the petri nets. (Sub-)graphs which were used to obtain the answers are presented as Figures 2, 3 and 4. (Note that for simplicity, we do not give descriptions of complete reachability graphs.)

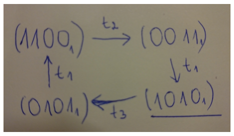

Figure 2: n=1

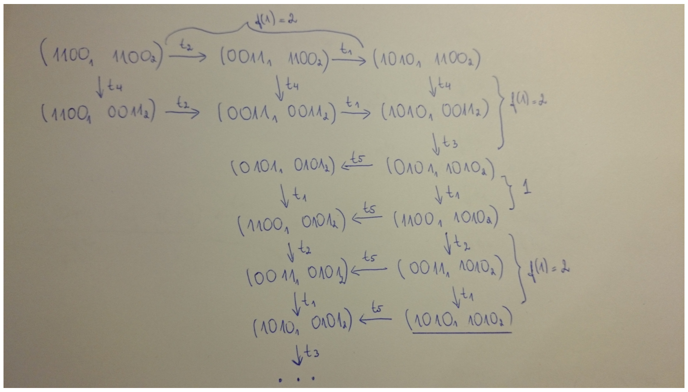

Figure 3: n=2

We denote markings by $(\mathrm{s1}\dots s4_1\dots \mathrm{sn}_m)$, i.e. $(\mathrm{s1s2s3s4}_1) = (1010_1)$ means that for $n = 1$ only odd-numbered places have a token.

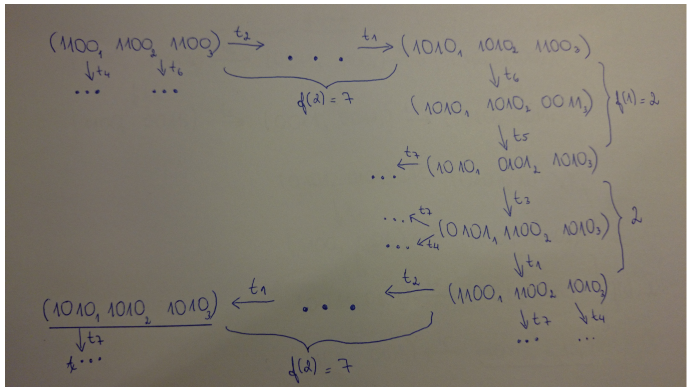

Figure 4: n=3

2. For n-th number of the family the length of the shortest path which leads to marking in which only all odd-numbered places have a token is

$$f(n) = 2f(n-1) + f(1) + n - 1 = 2f(n-1) + n + 1$$

where $f(1) = 2$.

For n the initial marking is $(1100_1 \dots 1100_n)$, i.e 1100 is repeated n times. We obtain the wanted marking with following steps.

(a) We need $f(n-1)$ steps to reach the marking $(1010_1 \dots 1010_{n-1}1100_n)$.

(b) We do $f(1)$ steps to reach the marking where n-th sub petri net has the wanted marking.

(c) Now we need $n-1$ steps to obtain the marking in which first $n-1$ sub petri nets have 'initial' marking, i.e $(1100_1 \dots 1100_{n-1}1010_n)$.

(d) To reach the marking in which first $n-1$ sub petri nets have the wanted marking we need $f(n-1)$ firings.

Thus, we have reached the marking in which only all odd-numbered places have a token. In total we need $f(n) = f(n-1) + f(1) + n - 1 + f(n-1) = 2f(n-1) + n + 1$ firings.

From Figures 2 and 3 it follows that the presented path is indeed (one of) the shortest.