4. 기초 수학 문제 2, 4, 6

* 문제 2 $T(n) = T(n-1) + n$

$T(n) = T(n-1) + n \quad \cdots (\neg)$

$T(n-1) = T(n-2) + (n-1) \quad \cdots (\llcorner)$

$T(n-2) = T(n-3) + (n-2) \quad \cdots (\sqsubset)$

$\vdots$

$T(2) = T(1) + 2$

($\neg$)에 ($\llcorner$) 대입,

$T(n) = (T(n-2) + (n-1)) + n$

$= T(n-2) + n + (n-1)$

($\sqsubset$) 대입,

$T(n) = \{T(n-3) + (n-2)\} + n + (n-1)$

$= T(n-3) + n + (n-1) + (n-2)$

$\vdots$

$T(n) = T(1) + n + (n-1) + (n-2) + \cdots + 2$

$= (n+1) \times \frac{n}{2}$

$= \frac{n^2+n}{2}$

$\therefore T(n) = O(n^2)$ ($\because$ n^2보다 작은 차수와 계수들 무시가능)

문제 4 $T(n) = T(\frac{n}{2}) + 1$

$T(n) = T(\frac{n}{2}) + 1$

$T(\frac{n}{2}) = T(\frac{n}{4}) + 1$ $\Rightarrow T(n) = (T(\frac{n}{4}) + 1) + 1$

$= T(\frac{n}{4}) + 2 \quad \cdots (\text{ㄱ})$

$T(\frac{n}{4}) = T(\frac{n}{8}) + 1 \quad \cdots (\text{ㄴ})$

ㄱ에 ㄴ 대입

$T(n) = (T(\frac{n}{8}) + 1) + 2$

$= T(\frac{n}{8}) + 3$

$\vdots$

$T(n) = T(\frac{n}{2^k}) + k \quad \cdots (k\text{번째})$

$\frac{n}{2^k} = 1 \Rightarrow n = 2^k \Rightarrow k = \log n$

$T(n) = T(1) + \log n \quad \cdots (\log n \text{ 번째})$

$\therefore T(n) = O(\log n)$ (∵ T 속에 상수가 들어가면 생략 가능)

문제 6 $T(n) = 2T\left(\frac{n}{2}\right) + n$

$$T(n) = 2T\left(\frac{n}{2}\right) + n \quad \cdots (\text{ㄱ})$$

$$T\left(\frac{n}{2}\right) = 2T\left(\frac{n}{4}\right) + \frac{n}{2} \quad \cdots (\text{ㄴ})$$

$$T\left(\frac{n}{4}\right) = 2T\left(\frac{n}{8}\right) + \frac{n}{4} \quad \cdots (\text{ㄷ})$$

(ㄱ)에 (ㄴ) 대입,

$$T(n) = 2\left(2T\left(\frac{n}{4}\right) + \frac{n}{2}\right) + n$$

$$= 4T\left(\frac{n}{4}\right) + 2n$$

(ㄷ) 대입,

$$T(n) = 4\left(2T\left(\frac{n}{8}\right) + \frac{n}{4}\right) + 2n$$

$$= 8T\left(\frac{n}{8}\right) + 3n$$

$$\vdots$$

$$T(n) = 2^k T\left(\frac{n}{2^k}\right) + k \cdot n \quad \cdots (\text{k번째})$$

$$\frac{n}{2^k} = 1 \rightarrow n = 2^k \rightarrow k = \log n$$

$$T(n) = n \cdot T(1) + (\log n) \times n \quad \cdots (\log n \text{ 번째})$$

$$= n\left(T(1) + \log n\right) \quad (\because T(상수) \Rightarrow \log n \text{보다 아주 작음})$$

$$\therefore T(n) = O(n \cdot \log n)$$