Gravitation
NCERT Exercises

1. How does the force of gravitation between two objects change when the distance between them is reduced to half?
2. Gravitational force acts on all objects in proportion to their masses. Why then, a heavy object does not fall faster than a light object?
3. What is the magnitude of the gravitational force between the earth and a 1 kg object on its surface? (Mass of the earth is 6×10^{24} kg and radius of the earth is 6.4×10^{6} m
4. The earth and the moon are attracted to each other by gravitational force. Does the earth attract the moon with a force that is greater or smaller or the same as the force with which the moon attracts the earth? Why?
5. If the moon attracts the earth, why does the earth not move towards the moon?
6. What happens to the force between two objects, if
   i. the mass of one object is doubled?
   ii. the distance between the objects is doubled and tripled?
   iii. the masses of both objects are doubled?
7. What is the importance of universal law of gravitation?
8. What is the acceleration of free fall?
9. What do we call the gravitational force between the earth and an object?
10. Amit buys few grams of gold at the poles as per the instruction of one of his friends. He hands over the same when he meets him at the equator. Will the friend agree with the weight of gold bought? If not, why? [Hint: The value of *g* is greater at the poles than at the equator.]
11. Why will a sheet of paper fall slower than one that is crumpled into a ball?
12. Gravitational force on the surface of the moon is only $\frac{1}{6}^{th}$ as strong as gravitational force on the earth. What is the weight in newtons of a 10 kg object on the moon and on the earth?
13. A ball is thrown vertically upwards with a velocity of 49 $m\ s^{-1}$.
    Calculate

   i. the maximum height to which it rises,
   ii. the total time it takes to return to the surface of the earth.

14. A stone is released from the top of a tower of height 19.6 m. Calculate its final velocity just before touching the ground.

15. A stone is thrown vertically upward with an initial velocity of 40 $m\,s^{-1}$. Taking g = 10 $m\,s^{-2}$, find the maximum height reached by the stone. What is the net displacement and the total distance covered by the stone?

16. Calculate the force of gravitation between the earth and the Sun, given that the mass of the earth = 6×10^{24} kg and of the Sun = 2×10^{30} kg. The average distance between the two is 1.5×10^{11} m.

17. A stone is allowed to fall from the top of a tower 100 m high and at the same time another stone is projected vertically upwards from the ground with a velocity of 25 $m\,s^{-1}$. Calculate when and where the two stones will meet.

18. A ball thrown up vertically returns to the thrower after 6 s.
    Find

    a. the velocity with which it was thrown up,
    b. the maximum height it reaches, and
    c. its position after 4 s.

19. In what direction does the buoyant force on an object immersed in a liquid act?

20. Why does a block of plastic released under water come up to the surface of water?

21. The volume of 50 g of a substance is 20 cm^3. If the density of water is 1 $g\,cm^{-3}$, will the substance float or sink?

22. The volume of a 500 g sealed packet is 350 cm^3. Will the packet float or sink in water if the density of water is 1 $g\,cm^{-3}$? What will be the mass of the water displaced by this packet?

Answers

1. The universal law of gravitation says that the force of gravitation between two objects of mass M and m respectively, separated by a distance r is:

   (a) Directly proportional to the product of their masses, and

   $$F \propto Mm$$

   (b) Inversely proportional to the square of the distance between them.

   $$F \propto \frac{1}{r^2}$$

   If the distance between them is reduced by half, keeping their masses equal, the gravitational force between them would become

   $$F' \propto \frac{1}{\left(\frac{r}{2}\right)^2} = \frac{4}{r^2}$$

   Therefore, when the distance between the objects is reduced to half, the force of gravitation increases by four times.

2. Gravitational force acts on all objects in proportion to their masses, but to move any object we need to overcome inertia which is also proportional to the mass. Hence, the mass term cancels out and we see all objects, heavy or light, fall at the same rate under the influence of gravity given that no other forces such as air drag is affecting them.

3. Given that,
   Mass of earth, $M = 6 \times 10^{24}\ kg$
   Mass of object, $m = 1\ kg$
   Distance between them, $r = 6.4 \times 10^6\ m$
   Universal gravitational constant, $G = 6.67 \times 10^{-11}\ N\,m^2\,kg^{-2}$

   $$\begin{aligned} F &= G\,\frac{Mm}{r^2} \\ &= 6.67 \times 10^{-11} \times \frac{6 \times 10^{24} \times 1}{(6.4 \times 10^6)^2} \\ &= 9.8\ N \end{aligned}$$

4. The earth attracts the moon with a force that is equal as the for with which moon attracts the earth. The only difference is that these two forces acts in opposite direction as virtue of Newton's third law of motion.

5. The universal law of gravitation says that the force of gravitation between two objects of mass M and m respectively, separated by a distance r is:

(a) Directly proportional to the product of their masses, and

$$F \propto Mm$$

(b) Inversely proportional to the square of the distance between them.

$$F \propto \frac{1}{r^2}$$

Using this fact we can answer the above questions:

a. If the mass of one object is doubled, the force of gravitation also doubles.
b. If the distance between the objects is doubled, from the second statement, the force of gravitation will decrease by 4 times. If the distance is tripled, the force of gravitation reduces nine-fold.
c. If the masses of both objects are doubled, then by the first statement, the force of gravitation becomes four times the original value.

6. Importance of universal law of gravitation:

   (a) the force that binds us to the earth;
   (b) the motion of the moon around the earth;
   (c) the motion of planets around the Sun; and,
   (d) the tides due to the moon and the Sun.

7. The earth attracts objects towards it due to the gravitational force. Whenever objects fall towards the earth under this force alone, we say that the objects are in free fall. While falling, there is no change in the direction of motion of the objects. But due to the earth's attraction, there will be a change in the magnitude of the velocity. Any change in velocity involves acceleration. Whenever an object falls towards the earth, an acceleration is involved. This acceleration is due to the earth's gravitational force. Therefore, this acceleration is called the acceleration due to the gravitational force of the earth (or acceleration due to gravity). It is denoted by g. The unit of g is the same as that of acceleration, that is, $m\ s^{-2}$ and its value is 9.8 $m\ s^{-2}$. The acceleration is also independent of the mass of the object.

8. Weight

9. Weight is just the force experienced by an object due to gravity.

$$F = mg$$

As acceleration due to gravity is not constant on earth (as earth is not a perfect sphere), the force experienced by a body is also no a constant, it changes with loacation. The distance from the centre of the earth to the surface is lowest at the poles and is highest at the equator. Therfore, when Amit buys few grams of gold at the poles, the weight of the gold will be higher and at the eqautor it will be lower. Hence, the friend will not agree with the weight of the gold bought.

10. From the day to day observation of nature, we can aggree that the air resistnace must be directly proportional to the surface area of the object. A crumpled paper has a lower surface area than a sheet of paper. Hence, the crumpled paper will face lower air resistnace that the sheet of paper. Hence, the crumpled paper will fall faster and the sheet of paper, slower.

11. As the gravitational force on the surface of the moon is only $\frac{1}{6}^{th}$ as strong as gravitational force on the earth, it implies that the acceleration due to gravity on moon will also be $\frac{1}{6}^{th}$ times the acceleration due to gravity on earth. Hence, the object's weight on moon can be calculated as,

$$\begin{aligned}\text{Weight on moon} &= \text{Mass} \times \text{Acceleration on moon}\\ &= \text{Mass} \times \frac{\text{Acceleration on earth}}{6}\\ &= 10 \times \frac{9.8}{6}\\ &= 16.3\ N\end{aligned}$$

Weight of the same object on earth will be six times as much or can be calculated as follows,

$$\begin{aligned}\text{Weight on earth} &= \text{Mass} \times \text{Acceleration on earth}\\ &= 10 \times 9.8\\ &= 98\ N\end{aligned}$$

12. Given,
Initial velocity, $u = 49\ m\ s^{-1}$
At the maximum height, the object will stop.Hence,
Final velocity, $v = 0\ m\ s^{-1}$
As acceleration due to gravity is acting opposite to the direction of motion, the acceleration due to gravity, $g = -9.8\ m\ s^{-2}$

a. Maximum height is the distance travelled by the ball. We know that,

$$\begin{aligned} v^2 &= u^2 + 2as \\ 0 &= (49)^2 + 2 \times (-9.8) \times s \\ \implies s &= \frac{-49 \times 49}{-2 \times 9.8} \\ &= 122.5\ m \end{aligned}$$

b. Total time to return to the surface is equal to twice the time it takes to reach (or drop from) the maximum height. We know that,

$$\begin{aligned} v &= u + at \\ 0 &= 49 + (-9.8)t \\ \implies t &= \frac{-49}{-9.8} \\ &= 5\ s \end{aligned}$$

Therefore, total time to it takes to return to the surface of the earth is $2 \times 5\ s = 10\ s$.

13. Given,
Initial velocity, $u = 0\ m\ s^{-1}$
As acceleration due to gravity is acting along to the direction of motion, the acceleration due to gravity, $g = 9.8\ m\ s^{-2}$
We know that,

$$\begin{aligned} v^2 &= u^2 + 2as \\ v^2 &= (0)^2 + 2 \times (9.8) \times 19.6 \\ \implies v &= \sqrt{19.6^2} \\ &= 19.6\ m\ s^{-1} \end{aligned}$$

14. Given,
Initial velocity, $u = 40\ m\ s^{-1}$
At the maximum height, the object will stop. Hence,
Final velocity, $v = 0\ m\ s^{-1}$
As acceleration due to gravity is acting opposite to the direction of motion, the acceleration due to gravity, $g = -10\ m\ s^{-2}$

Maximum height is the distance travelled by the ball. We know that,

$$\begin{aligned} v^2 &= u^2 + 2as \\ 0 &= (40)^2 + 2 \times (-10) \times s \\ \implies s &= \frac{-40 \times 40}{-2 \times 10} \\ &= 80\ m \end{aligned}$$

As the stone is vertically thrown upwards, it will come back to its original or starting position. Hence, the net displacement is 0 m.

The total distance covered is twice the maximum height. Hence, total distance covered by the stone is $2 \times 80\ m = 160\ m$.

15. Given that,
Mass of earth, $M = 6 \times 10^{24}\ kg$
Mass of sun, $m = 2 \times 10^{30}\ kg$
Distance between them, $r = 1.5 \times 10^{11}\ m$
Universal gravitational constant, $G = 6.67 \times 10^{-11}\ N\ m^2\ kg^{-2}$

The force of gravitation between two objects of mass MM and mm respectively, separated by a distance r is given by,

$$\begin{aligned} F &= G\frac{Mm}{r^2} \\ &= 6.67 \times 10^{-11} \times \frac{6 \times 10^{24} \times 2 \times 10^{30}}{(1.5 \times 10^{11})^2} \\ &= 3.57 \times 10^{22}\ N \end{aligned}$$

16. Assume that the stones meet after travelling for a time of t *seconds*.

(a) For the stone which is allowed to fall from the top of a tower
Initial velocity, $u = 0\ m\ s^{-1}$
As acceleration due to gravity is acting along to the direction of motion, the acceleration due to gravity, $g = 9.8\ m\ s^{-2}$
Distance travelled,

$$\begin{aligned} s_1 &= ut + \frac{1}{2}at^2 \\ s_1 &= 0 + \frac{1}{2} \times (9.8) \times t^2 \\ &= 4.9t^2 \end{aligned}$$

(b) For the stone that is projected vertically upwards from the ground:

Initial velocity, $u = 25\ m\ s^{-1}$
As acceleration due to gravity is acting opposite to the direction of motion, the acceleration due to gravity, $g = -9.8\ m\ s^{-2}$
Distance travelled,

$$
\begin{aligned}
s_2 &= ut + \frac{1}{2}at^2 \\
s_2 &= 25t + \frac{1}{2} \times 9.8 \times t^2 \\
&= 25t + 4.9t^2 \\
\implies s_2 &= 25t - s_1 \ (\because s_1 = 4.9t^2) \\
\implies s_1 + s_2 &= 25t
\end{aligned}
$$

As the height of the tower is 100 m,

$$s_1 + s_2 = 100$$

By comparing the above two equations, we get

$$
\begin{aligned}
25t &= 100 \\
\implies t &= \frac{100}{25} = 4\ s
\end{aligned}
$$

17. If the ball takes 6 s to travel up and down, it means it took only 3 s to cover the distance till the maximum height. Hence,
Time taken, $t = 3\ s$

At the maximum height, the object will stop. Hence,
Final velocity, $v = 0\ m\ s^{-1}$

As acceleration due to gravity is acting opposite to the direction of motion, the acceleration due to gravity, $g = -9.8\ m\ s^{-2}$

(a) We know that,

$$
\begin{aligned}
v &= u + at \\
0 &= u + (-9.8) \times 3 \\
\implies u &= 29.4\ m\ s^{-1}
\end{aligned}
$$

(b) To calculate the maximum height, we use the initial velocity calculated above.
Initial velocity, $u = 29.4\ m\ s^{-1}$

We know that,

$$
\begin{aligned}
v^2 &= u^2 + 2as \\
0 &= (29.4)^2 + 2 \times (-9.8) \times s \\
\implies s &= \frac{-29.4 \times 29.4}{-2 \times 9.8} \\
&= 44.1\ m
\end{aligned}
$$

(c) As it takes 3 s to reach the maximum height, at 4 s - the ball will be descending. We can think of this as if the ball was dropped from a height of 44.1 m and we have to calculate its position after 1 s. To calculate the position after 1 s, we are given,

Initial velocity, $u = 0\ m\ s^{-1}$
Time taken, $t = 1\ s$

As acceleration due to gravity is acting along to the direction of motion, the acceleration due to gravity, $g = 9.8\ m\ s^{-2}$

We know that,

$$
\begin{aligned}
s &= ut + \frac{1}{2}at^2 \\
s &= 0 + \frac{1}{2} \times (9.8) \times 1^2 \\
&= 4.9\ m
\end{aligned}
$$

The ball travelled 4.9 m downwards, it means that, after 4 s, the ball is at the height of

$$44.1 - 4.9 = 39.2\ m$$

above the ground.

18. The buoyant force on an object immersed in a liquid always acts opposite to the direction of the gravitational pull.

19. From the Archimedes' principle, the buoyant force experienced by an object, when partially or fully immersed in a liquid, is equal to the weight of the liquid displaced. As the weight of water displaced is more than the weight of the plastic block - mainly due to the fact that the density of plastic is very low, hence if we have same volume of plastic and water, plastic weighs less - the plastic block floats.

20. As the acceleration due to gravity on moon is six time less as that on the earth, it is reasonable to say that a person can lift six times more massive objects. If a person on earth can lift a mass of 15 kg on earth, the same person can lift $15 \times 6 = 90\ kg$ on the

moon with the same amount of force.

21. From the Archimedes' principle, the buoyant force experienced by an object, when partially or fully immersed in a liquid, is equal to the weight of the liquid displaced. The substance when immersed in water will displace 20 cm^3 of water. The mass of 20 cm^3 of water is,

$$\text{Density} = \frac{Mass}{Volume}$$
$$1 = \frac{Mass}{20}$$
$$\implies \text{Mass} = 20\ g$$

As weight is directly proportional to the acceleration due to gravity, the weight of the water displaced will be less than the weight of the substance. Hence, it will experience a lower buoyant force than its weight and will sink.

22. From the Archimedes' principle, the buoyant force experienced by an object, when partially or fully immersed in a liquid, is equal to the weight of the liquid displaced. The substance when immersed in water will displace 350 cm^3 of water. The mass of 350 cm^3 of water is,

$$\text{Density} = \frac{Mass}{Volume}$$
$$1 = \frac{Mass}{350}$$
$$\implies \text{Mass} = 350\ g$$

As weight is directly proportional to the acceleration due to gravity, the weight of the water displaced will be less than the weight of the sealed packet. Hence, it will experience a lower buoyant force than its weight and will sink.