Gravitation - Lesson 13
Understanding Freefall

What happens when you let go of a stone from a multi-storey building? It just falls. Why does it fall? Simple, because the earth's gravity is pulling the object towards the centre. More precisely, due to the earth's gravitational force, the object is pulled. But a simple consequence of force acting on an object is to change its velocity (either magnitude or direction). And therefore, the object does accelerate, and this acceleration is called the acceleration due to gravity, g. An interesting fact about this effect is that It does not depend on mass, as the force of gravity is proportional to the mass of the object. Still, the reaction of any object under the influence of a force is also proportional to its mass. Hence, the acceleration under gravity is independent of the mass of the object. Mathematically speaking,

$$Force = m \times a$$

$$Force(Gravity) = G\frac{Mm}{r^2}$$

When an object falls, the objects experiences an accelration given by,

$$ma = G\frac{Mm}{r^2}$$

$$a = G\frac{M}{r^2} = g$$

This has units of $m\ s^{-2}$.

The above also shows that the acceleration due to gravity is independent of the mass and constant *(but not quite!).* This fact was tested by Galileo Galilei, as stated earlier in the chapter. This means that any two objects will fall at the same time when left from the same height (in the absence of air resistance). We can find the value of 'g' by using the data already available,

Universal Gravitation Constant, $G = 6.67 \times 10^{-11}\ N\ m^2\ kg^{-2}$,
Mass of the Earth, $M = 6 \times 10^{24}\ kg$, and
The radius of the earth, $r = 6.4 \times 10^6\ m$

Substituing this in the above equation,

$$ma = G\,\frac{Mm}{r^2}$$
$$g = 6.67 \times 10^{-11}\ N\ m^2\ kg^{-2}\ \frac{6 \times 10^{24}\ kg}{(6.4 \times 10^6\ m)^2} = 9.8\ m\ s^{-2}$$

Sidenote: If we closely look at the above equation ($g = .../r^2$), it is an inverse function of radius from the centre. Therefore, g reduces as altitude increases, and it is also greater at the equator than at the poles, as the earth is not a perfect sphere and instead a sphere with a flattened top and bottom. But for the most part, this deviation is minimal and often neglected. For simplicity, we will assume that the acceleration due to gravity is a constant on the earth.

If that's the case, the trajectory of any object under gravitational acceleration can be figured out by equations of motion in a straight line. These are the set of 3 equations that were derived in the earlier chapters, to refresh,

Final velocity, v, of an object starting with an initial velocity, uu, after a time, t, is

$$v = u + at$$

If the distance, s, is to found after this elapsed time, t, then

$$s = ut + \frac{1}{2}at^2$$

Final velocity, v, of an object starting with an initial velocity, u, after covering a distance, s, is

$$v^2 = u^2 + 2as$$

Note: If acceleration is opposing the trajectory of motion, we will assign a -ve sign to it to show that it decreases the velocity rather than adding to it.

Let us look at a few examples:

1. **Example 1:** A ball is thrown upwards with an initial velocity of 2 $m\ s^{-1}$. Calculate the maximum height reached. Take g = 10 $m\ s^{-2}$.

   Answer
   Initial velocity, u = 2 $m\ s^{-1}$,
   At the maximum height, the object will stop.
   Hence, Final velocity, v = 0 $m\ s^{-1}$,
   As acceleration due to gravity is acting opposite to the direction of motion.
   Hence, g = −10 $m\ s^{-2}$.

   Maximum height is the distance travelled by the ball.
   We know that,

$$\begin{aligned} v^2 &= u^2 + 2\,a\,s \\ 0 &= (2)^2 + 2 \times (-10\ m\ s^{-2}) \times s \\ -4 &= -20 \times s \\ s &= 0.2\ m \end{aligned}$$

2. **Example 2:** A ball is dropped from a height of 20 m. Calculate the velocity with which it hits the ground. Take g = 10 $m\ s^{-2}$.

   Answer
   Initial velocity, u = 0 $m\ s^{-1}$,
   Distance, s = 20 m,
   As acceleration due to gravity is acting along to the direction of motion.
   Hence, g = 10 $m\ s^{-2}$.

   We know that,

$$\begin{aligned} v^2 &= u^2 + 2\,a\,s \\ v^2 &= (0)^2 + 2 \times (10\ m\ s^{-2}) \times 20 \\ v^2 &= 20 \times 20 \\ v &= \sqrt{400}\ m \\ v &= 20\ m\ s^{-1} \end{aligned}$$

3. **Example 3:** A ball is dropped from a certain height takes 5 s to hit the ground. Calculate the velocity of the ball when it hits the surface. Take g = 10 $m\ s^{-2}$.

Answer
Initial velocity, $u = 0\ m\ s^{-1}$,
As acceleration due to gravity is acting along to the direction of motion.
Hence, $g = 10\ m\ s^{-2}$.
Time taken, $t = 5\ s$

We know that,

$$v = u + a\,t$$
$$v = 0 + (10\ m\ s^{-2}) \times 5$$
$$v = 50\ m\ s^{-1}$$

4. **Example 4:** A ball is thrown down with an initial velocity of $10\ m\ s^{-1}$ and it takes 20 seconds to hit the ground. Calculate the distance travelled by the ball. Take $g = 10\ m\ s^{-2}$.

Answer
Initial velocity, $u = 10\ m\ s^{-1}$,
As acceleration due to gravity is acting along to the direction of motion.
Hence, $g = 10\ m\ s^{-2}$.
Time taken, $t = 20\ s$

We know that,

$$s = ut + \frac{1}{2}\,a\,t^2$$
$$s = 10\ m\ s^{-1} \times 20\ s + \frac{1}{2} \times 10\ m\ s^{-2} \times (20\ s)^2$$
$$s = 2200\ m$$

5. **NCERT Example:** A car falls off a ledge and drops to the ground in $0.5\ s$. Let $g = 10\ m\ s^{-2}$ (for simplifying the calculations).

(i) What is its speed on striking the ground?
(ii) What is its average speed during the $0.5\ s$?
(iii) How high is the ledge from the ground?

Answer
Time taken, $t = \frac{1}{2}\ s$
Initial velocity, $u = 0\ m\ s^{-1}$,
As acceleration due to gravity is acting along to the direction of motion.

Hence, g = 10 $m\ s^{-2}$.
Acceleration of the car, a = +10 $m\ s^{-2}$ (downward)

(i) speed,

$$
\begin{aligned}
v &= at \\
&= 10\ m\ s^{-2} \times 0.5\ s \\
&= 5\ m\ s^{-1}
\end{aligned}
$$

(ii) average speed,

$$
\begin{aligned}
&= \frac{u+v}{2} \\
&= \frac{(0\ m\ s^{-1} + 5\ m\ s^{-1})}{2} \\
&= 2.5\ m\ s^{-1}
\end{aligned}
$$

(iii) distance travelled,

$$
\begin{aligned}
s &= \frac{1}{2}\ a\ t^2 \\
&= \frac{1}{2} \times 10\ m\ s^{-2} \times (0.5\ s)^2 \\
&= \frac{1}{2} \times 10\ m\ s^{-2} \times 0.25\ s \\
&= 1.25\ m
\end{aligned}
$$

Thus,

(i) its speed on striking the ground, = 5 $m\ s^{-1}$
(ii) its average speed during the 0.5 s = 2.5 $m\ s^{-1}$
(iii) height of the ledge from the ground, = 1.25 m

6. **NCERT Example:** An object is thrown vertically upwards and rises to a height of 10 m. Calculate (i) the velocity with which the object was thrown upwards and (ii) the time taken by the object to reach the highest point.

Answer
Distance travelled, s = 10 m
Final velocity, v = 0 $m\ s^{-1}$,
Acceleration due to gravity, g = 9.8 $m\ s^{-2}$.

Acceleration of the object, $a = -9.8\ m\ s^{-2}$ (upward motion)

(i)

$$
\begin{aligned}
v^2 &= u^2 + 2\,a\,s \\
0 &= u^2 + 2 \times (-9.8\ m\ s^{-2}) \times 10\ m \\
-u^2 &= -2 \times 9.8 \times 10\ m^2\ s^{-2} \\
u &= \sqrt{196}\ m\ s^{-1} \\
u &= 14\ m\ s^{-1}
\end{aligned}
$$

(ii)

$$
\begin{aligned}
v &= u + a\,t \\
0 &= 14\ m\ s^{-1} u^2 - 9.8\ m\ s^{-2} \times t \\
-u^2 &= -2 \times 9.8 \times 10\ m^2\ s^{-2} \\
t &= 1.43\ s
\end{aligned}
$$

Thus,

(i) initial velocity, $u = 14\ m\ s^{-1}$
(ii) Time taken, $t = 1.43\ s$

You should, now, be able to answer the following questions:

1. Does a freefalling object accelerate? If yes, what is the value of the acceleration?
2. Why is the acceleration under freefall independent of the object's mass?
3. Does the value of the acceleraion under gravity vary on the surface of the earth? If yes, at what point on the surface of the earth is it the highest and what point is it the lowest?
4. Two objects, one weighing 2 *kg* and another 200 *kg*, are dropped from a height of 10 *m*. Will they hit the ground at the same moment with the same velocity?

Conclusion

On any planet, a freefalling object (under gravity) experiences an acceleration independent of its mass.

Note to Teacher

The goal of the lesson is to introduce the important concept of freefall with all the mathematical tools reqired to solve problems related to it. It relates different consequences of freefall such as acceleration, position, time and velocity. The lesson also explains the dependency of the value of acceleration on the location on the earth. Sufficient time must be given to understand this lesson and solve many numericals to get the required familiarity for the topic.

Student Worksheet

1. Does a small object falling towards the earth makes it accelerate?
2. If yes, calculate the acceleration experienced by the earth?

   Hint: g = 10 $m\ s^{-2}$, m = 0.2 kg, r = 6371 $\times$ 10^3 m
3. Does the motion of an object depends on its mass under freefall?
4. If not, why does a nail fall faster than a feather?
5. Is the value of g constant on the earth?
6. Why is it not constant?
7. How much distance does an object of mass 10 kg fall in 1 s?
   (Take g as 10 $m\ s^{-2}$).
8. How much distance does an object of mass 100 kg fall in 2 s?
   (Take g as 10 $m\ s^{-2}$).
9. Will a ball point pen work in space? If yes, why? If no, why?
10. Take a matchbox and keep a small piece of paper on top such that no part of the paper is protruding from the sides. With this setup, drop this from a height. Which will hit the ground first?
    (a) Matchbox
    (b) Paper
    (c) Both
    (d) None of the above

Answers

1. Yes. It's value is 9.8 $m\ s^{-2}$
2. While freefalling, gravitational force acts on all objects which is directly proportional to their masses, but to move any object we need to overcome inertia which is also directly proportional to the mass. Hence, the mass term cancels out and we see all objects, heavy or light, fall at the same rate under the influence of gravity given that no other forces such as air drag is affecting them.
3. Yes. As the distance from the centre of the earth to the surface is lowest at the poles and is highest at the equator. Therefore, the value of acceleration due to gravity is highest at the poles and lowest at the equator.
4. Yes, as acceleration due to gravity is independent of the object's mass, both of them will experience the same acceleration and hence fall at the same rate.

Student Worksheet Answers

1. Yes
2. The acceleration experienced by earth,

$$Ma = G\frac{mM}{r^2}$$
$$a = G\,\frac{m}{r^2}$$
$$a = 6.67 \times 10^{-11} \times \frac{0.2}{(6371 \times 10^3)^2}$$
$$a = 3.28 \times 10^{-28}\ m\ s^{-2}$$

3. No
4. Because of air drag
5. No
6. Because the earth is not a perfect sphere and hence the distance from the center is not constant.
7. As the mass of object has no effect on an object under freefall, the distance covered in 1 second can be calculated as follows:
   Initial velocity, u = 0 $m\ s^{-1}$,
   Time taken, t = 1 s
   As acceleration due to gravity is acting along to the direction of motion.
   Hence, g = 10 $m\ s^{-2}$.

We know that,

$$s = ut + \frac{1}{2}at^2$$
$$= 0 + \frac{1}{2} \times 10 \times 1^2$$
$$s = 5\ m$$

8. As the mass of object has no effect on an object under freefall, the distance covered in 2 seconds can be calculated as follows:
Initial velocity, $u = 0\ m\ s^{-1}$,
Time taken, $t = 2\ s$
As acceleration due to gravity is acting along to the direction of motion.
Hence, $g = 10\ m\ s^{-2}$.

We know that,

$$s = ut + \frac{1}{2}at^2$$
$$= 0 + \frac{1}{2} \times 10 \times 2^2$$
$$s = 20\ m$$

9. No, as there is no gravity to pull the ink downwards.

10. (c) Both