Gravitation - Lesson 9
Mathematical Formulation of Gravity

In the summer of 1687, Newton published “Principia Mathematica”, which also described the mathematics of gravity among many other groundbreakings and astonishing mathematical works. He describes the **Universal law of gravitation** as follows:

Every object in the universe attracts every other object with a force that is:

1. Directly proportional to the product of their masses, and
2. Inversely proportional to the square of the distance between them.

This statement can be simply divided into two parts to make it easier to write the mathematical form.

Consider two bodies, A and B, of mass M and m respectively, then by the first part of the law, the force of gravity.

$$F \propto Mm \tag{1}$$

And assuming that they are separated by a distance, r, then

$$F \propto \frac{1}{r^2} \tag{2}$$

Combining both the bits gives,

$$F \propto \frac{Mm}{r^2} \tag{3}$$

A constant is introduced to remove the proportionality. It's called the **"Universal Gravitational Constant"**, G, whose SI Units is $N\,m^2\,kg^{-2}$. The value of G is $6.673 \times 10^{-11}\,N\,m^2\,kg^{-2}$.

So the final equation is,

$$F = G\,\frac{Mm}{r^2} \tag{4}$$

Let us look at a few examples:

1. **Example 1:** At some point of the year, Jupiter is about $5.8 \times 10^{11}\,m$ away from earth. Calculate the gravitational force between the two. Given that the mass of Jupiter is $1.9 \times 10^{27}\,kg$, the mass of the Earth is $6 \times 10^{24}\,kg$, and the universal gravitational constant is $6.67 \times 10^{-11}\,N\,m^2\,kg^{-2}$

   Answer
   The mass of the earth, $M = 6 \times 10^{24}\,kg$,
   The mass of Jupiter, $m = 1.9 \times 10^{27}\,kg$,
   The distance between the earth and Jupiter, $r = 5.8 \times 10^{11}\,m$
   From Eq. (4), the gravitational force between the two is,

$$\begin{aligned} F &= G\,\frac{Mm}{r^2} \\ &= (6.67 \times 10^{-11}\,N\,m^2\,kg^{-2}) \times \frac{(6 \times 10^{24}\,kg)(1.9 \times 10^{27}\,kg)}{(5.8 \times 10^{11})^2} \\ &= 2.26 \times 10^{18}\,N \end{aligned}$$

   Thus, the gravitational force between Earth and Jupiter is $2.26 \times 10^{18}\,N$.

2. **NCERT Example:** The mass of the earth is $6 \times 10^{24}\,kg$ and that of the moon is $7.4 \times 10^{22}\,kg$. If the distance between the earth and the moon is $3.84 \times 10^5\,km$, calculate the force exerted by the earth on the moon. (Take $G = 6.67 \times 10^{-11}\,N\,m^2\,kg^{-2}$)

Answer
The mass of the earth, $M = 6 \times 10^{24}\ kg$,
The mass of moon, $m = 7.4 \times 10^{22}\ kg$,
The distance between the earth and the moon,

$$\begin{aligned} d &= 3.84 \times 10^{5}\ km \\ &= 3.84 \times 10^{5} 1000\ m \\ &= 3.84 \times 10^{8}\ m \\ G &= 6.7 \times 10^{-11}\ N\, m^2\, kg^{-2} \end{aligned}$$

From Eq. (4), the force exerted by the earth on the moon is,

$$\begin{aligned} F &= G\frac{Mm}{d^2} \\ &= \frac{(6.710^{-11} N\, m^2\, kg^{-2}) \times (6 \times 10^{24}\ kg) \times (7.4 \times 10^{22}\ \ kg)}{(3.8410^{8}\ \ m)^2} \\ &= 2.0210^{20}\ \ N \end{aligned}$$

Thus, the force exerted by the earth on the moon is $2.02 \times 10^{20}\ N$.

You should, now, be able to answer the following questions:

1. What is the universal law of Gravitation?
2. Write down the formula for the force of gravitation between two objects?
3. Consider the gravitational force between two objects of the same mass to be F. If the mass of one object is reduced by half, the gravitational force between them
   (a) Halved
   (b) Doubled
   (c) Tripled
   (d) Quadrupled
4. If the distance between them is reduced by half, keeping their masses equal, the gravitational force between them is
   (a) Halved
   (b) Doubled
   (c) Tripled
   (d) Quadrupled

Conclusion

Universal Law of Gravitation states that every object in the universe attracts every other object with a force that is directly proportional to the product of their masses, and inversely proportional to the square of the distance between them.

Note to Teacher

Having introduced centripetal force in the previous lesson, the purpose of the lesson is to give the reader an understanding about the absence of centipetal force. It enforces the importance of it as well. The text relates the whirling of stone to centripetal force, an important comparision. The goal here is to understand that any object under the influence of centripetal force will travel in a straight line when the centripetal force is switched off suddenly.

Student Worksheet

1. Gravitational force always acts along the _____.
2. Gravitational force is directly proportional to _____.
3. Gravitational force is inversely proportional to _____.
4. What is the SI units of the universal gravitational constant?
5. What is the value of the universal gravitational constant?

Answers

1. Every object in the universe attracts every other object with a force that is:

   (a) Directly proportional to the product of their masses, and

   (b) Inversely proportional to the square of the distance between them.

2. Consider two bodies, A and B, of mass M and m respectively, which are separated by *r m*. Then by the Universal law of Gravitation, the force of gravity is,

$$F = G\frac{Mm}{r^2}$$

   where G is the universal gravitational constant which is equal to $6.67 \times 10^{-11}\ N\ m^2\ kg^{-2}$

3. (a) Halved

   The universal law of gravitation says that the force of gravitation between two objects of mass M and m respectively, separated by a distance r is:

   (a) Directly proportional to the product of their masses, and

$$F \propto Mm$$

   (b) Inversely proportional to the square of the distance between them.

$$F \propto \frac{1}{r^2}$$

   If the mass of one object is reduced by half without changing distance between them, then the gravitational force would become

$$F' \propto M \times \frac{m}{2} = \frac{Mm}{2}$$

4. (d) Quadrupled

   The universal law of gravitation says that the force of gravitation between two objects of mass M and m respectively, separated by a distance r is:

   (a) Directly proportional to the product of their masses, and

$$F \propto Mm$$

   (b) Inversely proportional to the square of the distance between them.

$$F \propto \frac{1}{r^2}$$

If the distance between them is reduced by half, keeping their masses equal, the gravitational force between them would become

$$F' \propto \frac{1}{\left(\frac{r}{2}\right)^2} = \frac{4}{r^2}$$

Student Worksheet Answers

1. Line joining the center of masses
2. Product of the masses
3. Square of the distance between the centre of masses
4. $N\,m^2\,kg^{-2}$
5. $6.67 \times 10^{-11}\,N\,m^2\,kg^{-2}$