Gravitation - Lesson 11
A Little Help from Kepler

Johannes Kepler was a very talented astronomer and a mathematician. He lived in the 17th Century and is regarded as one of the key personalities to bring the scientific revolution to its peak. In 1602, Kepler was trying to calculate the position of the earth in its orbit and in turn stumbled on the fact that **"The radius vector describes equal areas in equal times."**, which, incidentally, became his second law.

In 1605, when sifting through the data collected by Tycho Brahe (another great scientist) on the orbit of Mars, he found that the orbit would perfectly fit an ellipse. Hence, he published the first law as **"Planets move in ellipses with the Sun at one focus".**

And in May 1618, he found the third law, which ultimately led Newton to form the Universal law of gravitation, which goes like **"The squares of the periodic times are to each other as the cubes of the mean distances".**

But how did these help Newton formulate his law? Suppose we step into Newton's shoes for a moment. In that case, we can think that by Kepler's first law, **"The orbit of a planet is an ellipse with the Sun at one of the foci"**, we can assume that the orbit is circular as a circle is a special type of ellipse with both its foci coinciding at the centre. In this exceptional case, the planet of mass M is going around the Sun at velocity v, and the radius is r.

The centripetal force experienced by the planet is:

$$F_c = \frac{Mv^2}{r}$$

As mass is a constant, we can write the above as:

$$F_c \propto \frac{v^2}{r}$$

As the motion of the planet is periodic, let us assume the period to be T, which means the planet takes T amount of time to complete one revolution around the Sun.
Then the velocity of the planet is given by

$$velocity = \frac{Distance}{Time}$$

In T amount of time, the distance covered by the planet is the circumference of the circle of revolution which is $2\pi r$. Therefore,

$$v = \frac{2\pi r}{T}$$

Substituting this in F_c gives,

$$F_c \propto \frac{r}{T^2}$$

which can also be written as,

$$F_c \propto \frac{r^3}{T^2} \times \frac{1}{r^2}$$

By Kepler's III law,

$$\frac{r^3}{T^2} = constant$$

which implies,

$$F_c \propto \frac{1}{r^2} = constant$$

This is how Newton arrived at the famous inverse square relationship for the universal law of gravitation.

You should, now, be able to answer the following questions:

1. What is the shape of the orbit under the influence of the gravitational force?

2. State three Kepler's laws of planetary motion?

Conclusion

Kepler's laws of planetary motion:

1. Planets move in ellipses with the Sun at one focus.
2. The radius vector describes equal areas in equal times.
3. The squares of the periodic times are to each other as the cubes of the mean distances.

Note to Teacher

The text intends to explain the three laws of planetary motion by Kepler and its importance. It is important to realise that these laws helped Newton to discover the universal law of gravitation. The text tries to explain the route that Newton might have taken to react that conclusion. This lesson gives an insight into the great minds of Newton and Kepler.

Student Worksheet

1. State the second law of planetary motion?
2. Describe its consequnece? *Hint:* Think about speed

Answers

1. Elliptical
2. Kepler's laws of planetary motion:
   (a) Planets move in ellipses with the Sun at one focus.
   (b) The radius vector describes equal areas in equal times.
   (c) The squares of the periodic times are to each other as the cubes of the mean distances.

Student Worksheet Answers

1. The radius vector describes equal areas in equal times.
2. As the area swept by the radius vector needs to be equal. If the object that is revolving a massive planet, is not tracing a circle. This means that the path is ellipse. When the object is at the closet to the massive planet, the object will speed up to cover more area and when it is farthest, it will slow down such that the area swept is at equal time duration is equal.