## 1 Perfect Square

- (a) Prove by contraposition.n is even. Then, by definition n=2k. Because,  $n^2=4k^2=2(2k^2)$ , and  $k\in \mathbb{Z}$ , so  $n^2$  is even
- (b) assume that  $n^2$  is odd, by definition for some integer 1 n = 2l + 1, then 8k + 1 = 2(4k) + 1, since 4k is an integer, thus  $n^2$  can be written in the form 8k + 1 for some integer k

## 2 Numbers of Friends

## 3 Pebbles

Prove by contradiction. Assume that there does not exist an all-red column. Then every column has a blue pebble. So there must exist a way that can choose one pebble from each column but all pebbles are blue. This cause a contradiction. Thus original proposition is right.