1 Natural Induction on Inequality

Base case:when n = 0,then $(1+x)^0 \geq 1+0x$,$1 \geq 1$
Inductive Hypothesis:assume that $(1+x)^k \geq 1+kx$ for some $n=k$,where k $\in$ Z
Inductive Step:for $n=k+1$

$$\begin{aligned}(1+x)^{k+1} = (1+x)^k(1+x) &\geq (1+kx)(1+x)\\ &\geq 1+kx+x+kx^2\\ &\geq 1+(k+1)x+kx^2\\ &\geq 1+(k+1)x\end{aligned}$$

2 Make It Stronger

(a) Base case:when n = 1,then $a_n \leq 3^{2^n} \leq 9$

Inductive Hypothesis:assume that $a_n \leq 3^{2^n}$ for some $n \geq 1$,where n $\in$ Z

Inductive Step:for $n+1$

$$\begin{aligned}a_{n+1} = 3a_n^2 &\leq 3(3^{2^n})^2\\ &\leq 3(3^{2^{n+1}})\\ &\leq 3^{2^{n+1}+1}\end{aligned}$$

(b) Base case:when n = 1,then $a_n \leq 3^{2^n-1} \leq 3$

Inductive Hypothesis:assume that $a_n \leq 3^{2^n-1}$ for some $n \geq 1$,where n $\in$ Z

Inductive Step:for $n+1$

$$\begin{aligned}a_{n+1} = 3a_n^2 &\leq 3(3^{2^n-1})^2\\ &\leq 3(3^{2^{n+1}-2})\\ &\leq 3^{2^{n+1}-1}\end{aligned}$$

this is the induction hypothesis for n+1

(c) for every n $\geq$ 1,we have $2^n-1 \leq 2^n$,therefore $3^{2^n-1} \leq 3^{2^n}$

3 Binary Numbers

Base case:when n = 1,$n=c_02^0$,where $c_0=1$
Inductive Hypothesis:assume that the proposition is right
Inductive Step:for $n+1$,if n is an even,so the c_0 of n is 0,and the c_0 of $n+1$ is 1,and other part of $n+1$ is the same as n.
If n is an odd,so n + 1 is an even,according to the hypothesis:
$(n+1)/2 = c_k2^k + \cdots + c_12^1 + c_02^0$
$n+1 = 2(n+1)/2 = c_k2^{k+1} + \cdots + c_12^2 + c_02^1 + 0 \cdot 2^0$
therefor the statement is true.

4 Fibonacci for Home

Base case:when n = 1,$F_{3n} = F_3 = 2$,is even.
Inductive Hypothesis:for an arbitrary fixed value of k, F_{3k} is even.
Inductive Step:for $k + 1$,$F_{3k+3} = F_{3k+1} + F_{3k+2} = 2F_{3k+1} + F_{3k}$.
By the hypothesis,we know that $F_{3k} = 2q$ for some integer$q$.So $F_{3k+3} = 2(F_{3k+1} + q)$,which implies F_{3k+3} is even.
The statement is true.