1 Perfect Square

(a) Prove by contraposition.n is even.Then, by definition $n = 2k$.Because, $n^2 = 4k^2 = 2(2k^2)$, and $k \in Z$, so n^2 is even

(b) assume that n^2 is odd, by definition for some integer l $n = 2l + 1$, then $8k + 1 = 2(4k) + 1$, since $4k$ is an integer, thus n^2 can be written in the form $8k + 1$ for some integer k

2 Numbers of Friends

3 Pebbles

Prove by contradiction.Assume that there does not exist an all-red column.Then every column has a blue pebble.So there must exist a way that can choose one pebble from each column but all pebbles are blue.This cause a contradiction.Thus original proposition is right.