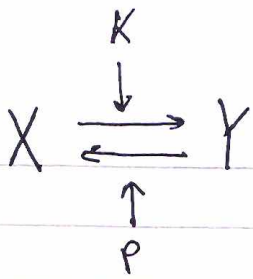
Phosphorylation



Full kinetics

$$X + K \underset{b}{\overset{a}{\rightleftarrows}} XK \xrightarrow{c} Y + K$$

$$Y + P \underset{f}{\overset{e}{\rightleftarrows}} YP \xrightarrow{g} X + P$$

ODEs

$$\frac{dX}{dt} = -a \cdot X \cdot K + b \cdot XK + g \cdot YP$$

$$\frac{dY}{dt} = c \cdot XK - e \cdot Y \cdot P + f \cdot YP$$

$$\frac{dK}{dt} = -a \cdot X \cdot K + (b+c) \cdot XK$$

$$\frac{dXK}{dt} = a \cdot X \cdot K - (b+c) \cdot XK$$

$$\frac{dP}{dt} = -e \cdot Y \cdot P + (f+g) \cdot YP$$

$$\frac{dYP}{dt} = e \cdot Y \cdot P - (f+g) \cdot YP$$

cons. eq-hs

$$\frac{dK}{dt} + \frac{dXK}{dt} = 0 \Rightarrow K + XK = K_T$$

$$\frac{dP}{dt} + \frac{dYP}{dt} = 0 \Rightarrow P + YP = P_T$$

$$\frac{dX}{dt} + \frac{dY}{dt} + \frac{dXK}{dt} + \frac{dYP}{dt} = 0$$

$$X + Y + XK + YP = X_T$$

M-M kinetics

$$\frac{d\,XK}{dt} = 0 = a \cdot X \cdot K - (b+c)\,XK$$

~~$X + K + XK + YP = X_T$~~

$$K_T = K + KX$$

$$a \cdot X \cdot (K_T - KX) - (b+c)\,XK = 0$$

$$a \cdot X \cdot K_T - a \cdot X \cdot KX - (b+c)\,XK = 0$$

$$a \cdot X \cdot K_T - XK\,(aX + b + c) = 0$$

$$XK = \frac{a \cdot X \cdot K_T}{aX + b + c} = K_T \cdot \frac{X}{\frac{b+c}{a} + X} = \frac{a}{b+c}\,X \cdot K$$

$$YP = P_T \cdot \frac{Y}{\frac{f+g}{e} + Y}$$

$$\frac{dY}{dt} = c \cdot K_T \cdot \frac{X}{\frac{b+c}{a} + X} - e \cdot Y \cdot \left(P_T - P_T \cdot \frac{Y}{\frac{f+g}{e} + Y}\right) + g \cdot P_T \cdot \frac{Y}{\frac{f+g}{e} + Y}$$

Sum of DDEs:

$$dX = -a \cdot X \cdot K + b \cdot XK + g \cdot YP$$

$$\frac{dY}{dt} = c \cdot XK - e \cdot Y \cdot P + g \cdot YP$$

~~$K + XK = K_T$~~ $\quad XK = K_T \cdot \frac{X}{\frac{b+c}{a} + X}$

~~$P + YP$~~

$$YP = P_T \cdot \frac{Y}{\frac{f+g}{e} + Y}$$

$$K = K_T - XK$$

$$P = P_T - YP$$

M-M kinetics + neglect complexes

X, Y, kX, PY, k, P

~~$\frac{dX}{dt} = -a \cdot X \cdot k + b \cdot Xk + g \cdot YP$~~

$$\frac{dY}{dt} = c \cdot Xk - e \cdot Y \cdot P + d \cdot YP$$

$$Xk = k_T \cdot \frac{X}{\frac{b+c}{a} + X}$$

$$YP = P_T \cdot \frac{Y}{\frac{f+g}{e} + Y}$$

$$X + Y + \cancel{Xk} + \cancel{YP} = X_T$$

$$k = k_T - Xk$$

$$P = P_T - YP$$

!!! $Xk, YP << X_T$

Full + neglect complexes

$$\frac{dY}{dt} = c \cdot Xk - e \cdot Y \cdot P + f \cdot YP$$

$$\frac{dXk}{dt} = a \cdot X \cdot k - (b+c) \cdot Xk$$

$$\frac{dYP}{dt} = e \cdot Y \cdot P - (f+g) \cdot YP$$

~~$k + kX$~~ $\quad k + Xk = k_T$

$$P + YP = P_T$$

$$X + Y = X_T$$

but!

if $Xk, YP < X_T$

$\Leftrightarrow \frac{dXk}{dt} = 0$

$\frac{dYP}{dt} = 0$

so if j remains