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# Data Acquisition and Handling Project

## J/ψ Meson Production

### Proofs and Definitions for Modeling Invariant Mass Data

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# Normalization of Exponential Decay

$$y = A \exp(-\alpha x)$$

where

$y$  = output

$A$  = Normalization factor

$\alpha$  = characterizes curve = a

$x$  = input

Aim: Find  $A$  for data between  $x_{\min}$  and  $x_{\max}$ , where

$x_{\min}$  = minimum  $x$  value

$x_{\max}$  = maximum  $x$  value

Thus,

$$\int_{x_{\min}}^{x_{\max}} y \, dy = 1 = A \int_{x_{\min}}^{x_{\max}} \exp(-\alpha x) \, dx$$

$$\Rightarrow 1 = A \left[ -\frac{1}{\alpha} \exp(-\alpha x) \right]_{x_{\min}}^{x_{\max}}$$

$$\Rightarrow A = -\frac{\alpha}{\exp(-\alpha x_{\max}) - \exp(-\alpha x_{\min})}$$

# Maximum Likelihood fit

$L$  = Joint Likelihood

$P(x_i | p_n)$  = PDF to be used in maximum likelihood fit

$x_i$  = Input data

$p_n$  = n parameters

n = index for parameters

$$L = \prod_i P(x_i | p_n)$$

$$\text{Log}_e L = \sum_i \text{Log}_e (P(x_i | p_n))$$

$\text{Log}_e L$  works as  
Logs always  
increase.

$$-\text{Log}_e L = -\sum_i \text{Log}_e (P(x_i | p_n))$$

Minimize to find parameters  $p_n$



# Gaussian PDF Model

(Probability Density Function)

$$y = (1-F) \underbrace{\frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{1}{2} \left(\frac{x-\mu}{\sigma}\right)^2\right)}_{\text{Gaussian}} + F \cdot A \exp(-ax)$$

where  $\underbrace{\exp(-ax_{\max})}_{\text{Background}}$   $\underbrace{\exp(-ax_{\min})}_{\text{Background}}$

$$A = -\frac{a}{\exp(-ax_{\max}) - \exp(-ax_{\min})}$$

$x_{\max} = \text{maximum } x$   
 $x_{\min} = \text{minimum } x$

$a$  = Background Exponential decay parameter

$x$  = input (invariant mass data)

$y$  = Probability

$F$  = Normalization factor that indicates percent fraction of data in Background

$\sigma$  = Standard deviation

$\mu$  = Mean invariant mass

# Double Gaussian PDF Model

$$y = FA \exp(-\alpha x) + (1-F)(Q G_1 + (1-Q) G_2)$$

Where

$$A = \frac{-\alpha}{\exp(-\alpha x_{\max}) - \exp(-\alpha x_{\min})}$$

$$G_1 = \frac{1}{\sigma_1 \sqrt{2\pi}} \exp\left(-\frac{1}{2} \left(\frac{x - \mu_1}{\sigma_1}\right)^2\right)$$

$$G_2 = \frac{1}{\sigma_2 \sqrt{2\pi}} \exp\left(-\frac{1}{2} \left(\frac{x - \mu_2}{\sigma_2}\right)^2\right)$$

F=Normalization factor that indicates fraction of data in background

Q=Normalization factor that indicates fraction of Gaussian data in  $G_1$  gaussian.

$\alpha$ =Background exponential decay parameter

$x$ =input data (invariant mass)

$\mu_1$ =mean on gaussian  $G_1$  &  $G_2$

~~$\mu_2$ =mean on gaussian  $G_2$~~

$\sigma_1$ =standard deviation on gaussian  $G_1$

$\sigma_2$ =standard deviation on gaussian  $G_2$

Removed  $\mu_2$  as just having  $\mu_1$  improved results residuals and thus, accuracy

# Crystal Ball PDF Model

$$y = F \exp(-\alpha x) + N \begin{cases} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right) & \text{if } \frac{x-\mu}{\sigma} > -\alpha \\ A \cdot \left(B - \frac{x-\mu}{\sigma}\right)^{-n} & \text{if } \frac{x-\mu}{\sigma} \leq -\alpha \end{cases}$$

where

$$A = \left(\frac{n}{|\alpha|}\right)^n \exp\left(-\frac{|\alpha|^2}{2}\right)$$

$$B = \frac{n}{|\alpha|} - |\alpha|$$

$$N = \frac{(1-F)}{\sigma(C+D)}$$

$$C = \frac{n}{|\alpha|} \frac{1}{n-1} \exp\left(-\frac{|\alpha|^2}{2}\right)$$

$$D = \sqrt{\frac{\pi}{2}} \left(1 + \operatorname{erf}\left(\frac{|\alpha|}{\sqrt{2}}\right)\right)$$

$$\operatorname{erf}\left(\frac{|\alpha|}{\sqrt{2}}\right) = \frac{2}{\sqrt{\pi}} \int_0^{\frac{|\alpha|}{\sqrt{2}}} e^{-t^2} dt \quad \text{where } z = \frac{|\alpha|}{\sqrt{2}}$$

$x$  = input data (invariant mass)

$\mu$  = mean of data ("")

$\sigma$  = standard deviation

$\alpha, n$  = independent parameters that characterize the relationship

$F$  = Normalization factor that indicates fraction of data in background