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Data Aquisition and Handling Project

J/ψ Meson Production

Proofs and Definitions

For Modeling Invariant Mass Data of J/ψ Meson Production

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Normalization of Exponential Decay

$$y = A\exp(-\alpha x)$$

where

y = output

A = Normalization factor

α = characterizes curve = a

x = input

Aim: Find A for data between x_{min} and x_{max}, where

x_{min} = minimum x value

x_{max} = maximum x value

Thus,

$$\int_{\cancel{x_{min}}\,0}^{\cancel{x_{max}}\,\infty} y\,dy = \overset{\text{Normalized}}{\overset{\downarrow}{1}} = A\int_{x_{min}}^{x_{max}} \exp(-\alpha x)\,dx$$

$$\Rightarrow 1 = A\left[-\frac{1}{a}\exp(-ax)\right]_{x_{min}}^{x_{max}}$$

$$\Rightarrow A = -\frac{a}{\exp(-ax_{max}) - \exp(-ax_{min})}$$

Maximum Likelihood fit

L = Joint Likelihood

$P(x_i | p_n)$ = PDF to be used in maximum likellihood fit

x_i = Input data

P_n = n parameters

n = index for parameters

$$L = \prod_i P(x_i | P_n)$$

$$\log_e L = \sum_i \log_e(P(x_i | P_n))$$

$\log_e L$ works as Logs always increase.

$$-\log_e L = -\sum_i \log_e(P(x_i | P_n))$$

Minimize to find parameters p_n

Gaussian PDF Model

(Probability Density Function)

$$y = \underbrace{(1-F)\frac{1}{\sigma\sqrt{2\pi}}\exp\left(-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right)}_{\text{Gaussian}} + \underbrace{F\cdot A\exp(-ax)}_{\text{Background}}$$

where

$$A = -\frac{a}{\exp(-ax_{max}) - \exp(-ax_{min})}$$

x_{max} = maximum x

x_{min} = minimum x

a = Background Exponential decay parameter

x = input (invariant mass data)

y = Probability

F = Normalization factor that indicates ~~percent~~ fraction of data in Background

σ = Standard deviation

μ = mean invariant mass

Double Gaussian PDF Model

$$y = FA\exp(-\alpha x) + (1-F)\left(QG_1 + (1-Q)G_2\right)$$

where

$$A = \frac{-a}{\exp(-a x_{max}) - \exp(-a x_{min})}$$

$$G_1 = \frac{1}{\sigma_1\sqrt{2\pi}} \exp\left(-\frac{1}{2}\left(\frac{x-\mu_1}{\sigma_1}\right)^2\right)$$

$$G_2 = \frac{1}{\sigma_2\sqrt{2\pi}} \exp\left(-\frac{1}{2}\left(\frac{x-\mu_2}{\sigma_2}\right)^2\right)$$

F = Normalization factor that indicates fraction of data in background

Q = Normalization factor that indicates fraction of Gaussian data in G_1 gaussian.

a = Background exponential decay parameter

x = input data (invariant mass)

μ_1 = mean on gaussian G_1

μ_2 = mean on gaussian G_2

σ_1 = standard deviation on gaussian G_1

σ_2 = standard deviation on gaussian G_2

Crystal Ball PDF Model

$$y = F(\gamma)\exp(-ax) + N\begin{cases}\exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right) & \text{if } \frac{x-\mu}{\sigma} > -\alpha \\ A\cdot\left(B-\frac{x-\mu}{\sigma}\right)^{-n} & \text{if } \frac{x-\mu}{\sigma} \leq -\alpha\end{cases}$$

where

$$A = \left(\frac{n}{|\alpha|}\right)^n \exp\left(-\frac{|\alpha|^2}{2}\right)$$

$$B = \frac{n}{|\alpha|} - |\alpha|$$

$$N = \frac{(1-F)}{\sigma(C+D)}$$

$$C = \frac{n}{|\alpha|}\frac{1}{n-1}\exp\left(-\frac{|\alpha|^2}{2}\right)$$

$$D = \sqrt{\frac{\pi}{2}}\left(1+\operatorname{erf}\left(\frac{|\alpha|}{\sqrt{2}}\right)\right)$$

$$\operatorname{erf}\left(\frac{|\alpha|}{\sqrt{2}}\right) = \frac{2}{\sqrt{\pi}}\int_0^z e^{-t^2}\,dt \text{ where } z = \frac{|\alpha|}{\sqrt{2}}$$

$$\gamma = \frac{-a}{\exp(-ax_{max}) - \exp(-ax_{min})}$$

a = Background Exponential decay parameter

x = input data (invariant mass)
μ = mean of data (" ")
σ = standard deviation

α, n = independent parameters that characterize the relationship

F = Normalization factor that indicates fraction of data in background