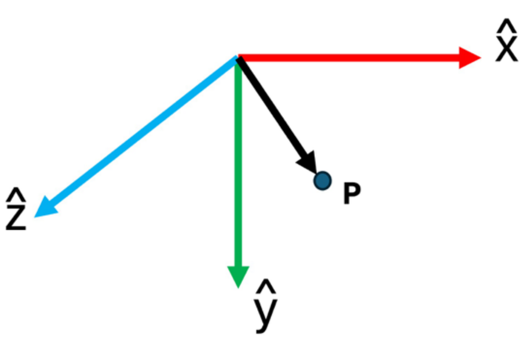
CG_LEI 2024

T_05 - camera

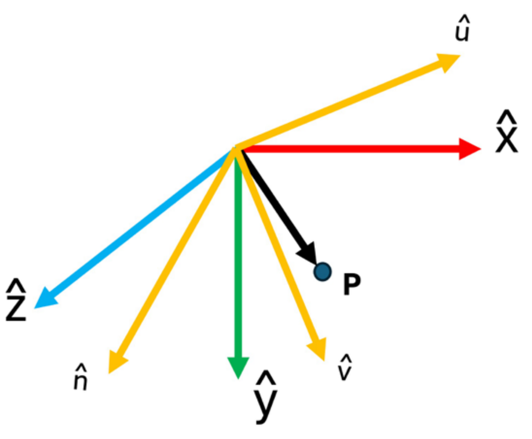
Formulação matemática para mudança de base

Considere $\{\hat{x}, \hat{y}, \hat{z}\}$ e $\{\hat{u}, \hat{v}, \hat{n}\}$ bases de $\mathbb{R}^3$

Considere um ponto $P_{\hat{x}\hat{y}\hat{z}} = (x, y, z) = x\hat{x} + y\hat{y} + z\hat{z}$



Queremos obter $P_{\hat{u}\hat{v}\hat{n}} = (u, v, n) = u\hat{u} + v\hat{v} + n\hat{n}$



Estratégia:

1. definir $\{\hat{x}, \hat{y}, \hat{z}\}$ na base $\{\hat{u}, \hat{v}, \hat{n}\}$

2. descrever o ponto $P_{\hat{x}\hat{y}\hat{z}}$ utilizando a definição 1.

$$\hat{x} = (\hat{x} \bullet \hat{u})\hat{u} + (\hat{x} \bullet \hat{v})\hat{v} + (\hat{x} \bullet \hat{n})\hat{n}$$
$$\hat{y} = (\hat{y} \bullet \hat{u})\hat{u} + (\hat{y} \bullet \hat{v})\hat{v} + (\hat{y} \bullet \hat{n})\hat{n}$$
$$\hat{z} = (\hat{z} \bullet \hat{u})\hat{u} + (\hat{z} \bullet \hat{v})\hat{v} + (\hat{z} \bullet \hat{n})\hat{n}$$

$$\begin{aligned} P_{\hat{x}\hat{y}\hat{z}} = (x, y, z) = x\hat{x} + y\hat{y} + z\hat{z} = \\ x\Big\{(\hat{x} \bullet \hat{u})\hat{u} + (\hat{x} \bullet \hat{v})\hat{v} + (\hat{x} \bullet \hat{n})\hat{n}\Big\} \\ +y\Big\{(\hat{y} \bullet \hat{u})\hat{u} + (\hat{y} \bullet \hat{v})\hat{v} + (\hat{y} \bullet \hat{n})\hat{n}\Big\} \\ +z\Big\{(\hat{z} \bullet \hat{u})\hat{u} + (\hat{z} \bullet \hat{v})\hat{v} + (\hat{z} \bullet \hat{n})\hat{n}\Big\} = \\ \hat{u}\Big\{x(\hat{x} \bullet \hat{u}) + y(\hat{y} \bullet \hat{u}) + z(\hat{z} \bullet \hat{u})\Big\} \\ +\hat{v}\Big\{x(\hat{x} \bullet \hat{v}) + y(\hat{y} \bullet \hat{v}) + z(\hat{z} \bullet \hat{v})\Big\} \\ +\hat{n}\Big\{x(\hat{x} \bullet \hat{n}) + y(\hat{y} \bullet \hat{n}) + z(\hat{z} \bullet \hat{n})\Big\} \end{aligned}$$

Lembrando que:

$P_{\hat{u}\hat{v}\hat{n}} = (u, v, n) = u\hat{u} + v\hat{v} + n\hat{n}$

posso reescrever a equação em forma de matriz:

$$\begin{bmatrix} u \\ v \\ n \end{bmatrix} = \begin{bmatrix} (\hat{x} \bullet \hat{u}) & (\hat{y} \bullet \hat{u}) & (\hat{z} \bullet \hat{u}) \\ (\hat{x} \bullet \hat{v}) & (\hat{y} \bullet \hat{v}) & (\hat{z} \bullet \hat{v}) \\ (\hat{x} \bullet \hat{n}) & (\hat{y} \bullet \hat{n}) & (\hat{z} \bullet \hat{n}) \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

Este desenvolvimento resolve a parte das rotações entre as bases $\{\hat{x}, \hat{y}, \hat{z}\}$ e $\{\hat{u}, \hat{v}, \hat{n}\}$, mas apenas nas situações onde a origem delas seja a mesma. No caso de haver uma translação entre as origens, temos de utilizar coordenadas homogêneas e adicionar uma dimensão ao problema.

Tal qual vimos anteriormente na aula sobre transformações.

Então, ficamos com a matrix final:

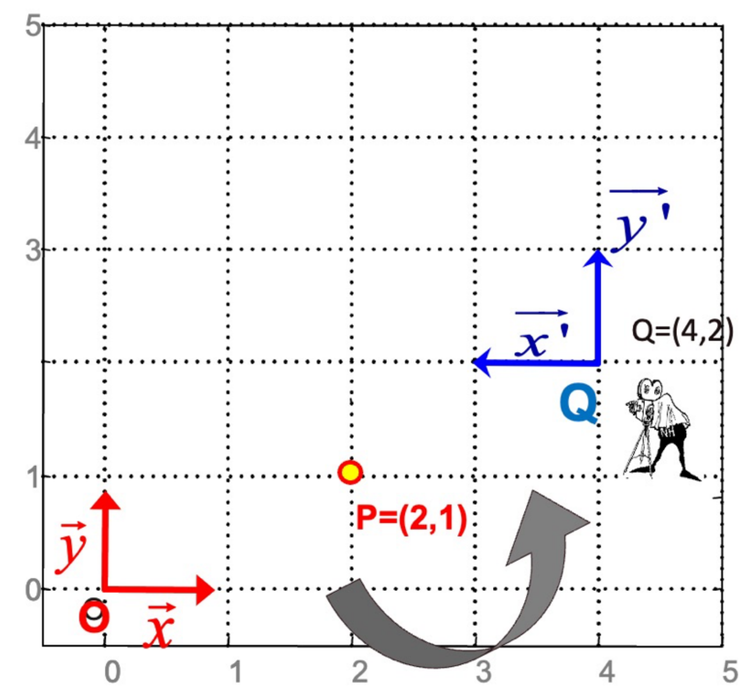
$$P_{\hat{u}\hat{v}\hat{n}} = \begin{bmatrix} u \\ v \\ n \\ 1 \end{bmatrix} = \begin{bmatrix} (\hat{x} \bullet \hat{u}) & (\hat{y} \bullet \hat{u}) & (\hat{z} \bullet \hat{u}) & T_{\hat{u}} \\ (\hat{x} \bullet \hat{v}) & (\hat{y} \bullet \hat{v}) & (\hat{z} \bullet \hat{v}) & T_{\hat{v}} \\ (\hat{x} \bullet \hat{n}) & (\hat{y} \bullet \hat{n}) & (\hat{z} \bullet \hat{n}) & T_{\hat{n}} \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

onde $T_{\hat{u}}, T_{\hat{v}}, T_{\hat{n}}$ representam as coordenadas da origem da base $\{\hat{x}, \hat{y}, \hat{z}\}$ no referencial $\{\hat{u}, \hat{v}, \hat{n}\}$

Aqui, também podemos obter T ao fazer:

$$T_{\hat{u}} = -((origem_{\hat{u}}, origem_{\hat{v}}, origem_{\hat{n}}) \bullet (\hat{u}))$$
$$T_{\hat{v}} = -((origem_{\hat{u}}, origem_{\hat{v}}, origem_{\hat{n}}) \bullet (\hat{v}))$$
$$T_{\hat{n}} = -((origem_{\hat{u}}, origem_{\hat{v}}, origem_{\hat{n}}) \bullet (\hat{n}))$$

Exemplo:



$$P_{\hat{x'}\hat{y'}\hat{z'}} = MP_{\hat{x}\hat{y}\hat{z}}$$

$$\overrightarrow{x} = (1, 0, 0)$$
$$\overrightarrow{y} = (0, 1, 0)$$
$$\overrightarrow{z} = (0, 0, 0)$$
$$\overrightarrow{x'} = (-1, 0, 0)$$
$$\overrightarrow{y'} = (0, 1, 0)$$
$$\overrightarrow{z'} = (0, 0, 0)$$

$$T_{\overrightarrow{x'}} = -((4, 2, 0) \bullet (-1, 0, 0)) = 4$$

$$T_{\overrightarrow{y'}} = -((4, 2, 0) \bullet (0, 1, 0)) = -2$$

$$T_{\overrightarrow{z'}} = 0$$

$$P_{\hat{x'}\hat{y'}\hat{z'}} = \begin{bmatrix} x_Q \\ y_Q \\ z_Q \\ 1 \end{bmatrix} = \begin{bmatrix} (\hat{x} \bullet \hat{x'}) & (\hat{y} \bullet \hat{x'}) & (\hat{z} \bullet \hat{x'}) & T_{\overrightarrow{x'}} \\ (\hat{x} \bullet \hat{y'}) & (\hat{y} \bullet \hat{y'}) & (\hat{z} \bullet \hat{y'}) & T_{\overrightarrow{y'}} \\ (\hat{x} \bullet \hat{z'}) & (\hat{y} \bullet \hat{z'}) & (\hat{z} \bullet \hat{z'}) & T_{\overrightarrow{z'}} \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 & 4 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ 0 \\ 1 \end{bmatrix}$$

$$P_{\hat{x'}\hat{y'}\hat{z'}} = (2, -1, 0)$$

In [6]:
```
! jupyter nbconvert --to html CG_LEI_2024_T05_mudaBase.ipynb
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[NbConvertApp] Converting notebook CG_LEI_2024_T05_mudaBase.ipynb to html
[NbConvertApp] WARNING | Alternative text is missing on 1 image(s).
[NbConvertApp] Writing 285529 bytes to CG_LEI_2024_T05_mudaBase.html
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In [ ]: