

COMPUTER SCIENCE 5300

ADVANCED ALGORITHM DESIGN AND ANALYSIS

ASSIGNMENT # 1

Problem 1

Let $X(1..n)$ and $Y(1..n)$ contain two lists of n integers, each sorted in nondecreasing order. Give the best (worst-case complexity) algorithm that you can think of for finding

- (a) the largest integer of all $2n$ combined elements.
- (b) the second largest integer of all $2n$ combined elements.
- (c) the median (or the n th smallest integer) of all $2n$ combined elements.

For instance, $X = (4, 7, 8, 9, 12)$ and $Y = (1, 2, 5, 9, 10)$, then median = 7, the n th smallest, in the combined list $(1, 2, 4, 5, 7, 8, 9, 9, 10, 12)$. [Hint: use the concept similar to binary search]

Solution

***** SOLUTION GOES HERE *****

Problem 2**1-to-2 PARTITION:**

Instance: A finite set of positive integers $Z = z_1, z_2, \dots, z_n$.

Question: Is there a subset Z' of Z such that Sum of all numbers in $Z' = 2 \times$ Sum of all numbers in $Z - Z'$

- (a) Obtain the dynamic programming functional equation to solve the 1-to-2 PARTITION problem.
- (b) Give an algorithm to implement your functional equation.
- (c) Give an example of 5 numbers with a total of 21 as an input instance for 1-to-2 PARTITION problem, and show how your algorithm works on this input instance.
- (d) What is the complexity of your algorithm?

Solution

***** SOLUTION GOES HERE *****

Problem 3

Decide True or False for each of the followings. You MUST briefly justify your answer.

Satisfiability:

Instance: Set U of variables, collection C of clauses over U .

Question: Is there a satisfying truth assignment for C ?

- (a) If $P \neq NP$, then no problem in NP can be solved in polynomial time deterministically.
- (b) If a decision problem A is NP -complete, proving that A is reducible to B , in polynomial time, is sufficient to show that B is NP -complete.
- (c) It is known that SAT (Satisfiability) is NP -complete, and 3SAT (all clauses have size 3) is NP -complete. 1SAT (all clauses have size 1) is also NP -complete.

Solution

***** SOLUTION GOES HERE *****

Problem 4

Solution

***** SOLUTION GOES HERE *****

Problem 5

Solution

***** SOLUTION GOES HERE *****

Problem 6

Solution

***** SOLUTION GOES HERE *****