Fall 2021 Rick Ramirez

Computer Science 5300

Advanced Algorithm Design and Analysis

Assignment # 1

Problem 1

Let X(1..n) and Y(1..n) contain two lists of n integers, each sorted in nondecreasing order. Give the best (worst-case complexity) algorithm that you can think for finding

(a) the largest integer of all 2n combined elements.

(b) the second largest integer of all 2n combined elements.

(c) the median (or the nth smallest integer) of all 2n combined elements.

For instance, X = (4, 7, 8, 9, 12) and Y = (1, 2, 5, 9, 10), then median = 7, the nth smallest, in the combined list (1, 2, 4, 5, 7, 8, 9, 9, 10, 12). [Hint: use the concept similar to binary search]

Solution:

(a)

Algorithm 1 Calculate the maximum of two sorted arrays

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max(X[n-1], Y[n-1])
```

(b)

Algorithm 2 Calculate the second largest element of two sorted arrays

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if X[n-1] == Y[n-1] then
    return max(X[n-2], Y[n-2])
else if X[n-1] < Y[n-1] then
    return max(X[n-1], Y[n-2])
else
    return max(X[n-1], Y[n-2])
end if
```

Problem 2

1-to-2 PARTITION:

Instance: A finite set of positive integers Z = z1 , z2 , ... , zn .

Question: Is there a subset Z' of Z such that Sum of all numbers in Z' = 2 x Sum of all numbers in Z-Z'

(a) Obtain the dynamic programming functional equation to solve the 1-to-2 PARTITION problem.

(b) Give an algorithm to implement your functional equation.

(c) Give an example of 5 numbers with a total of 21 as an input instance for 1-to-2 PARTITION problem, and show how your algorithm works on this input instance.

(d) What is the complexity of your algorithm?

Solution

******* SOLUTION GOES HERE *******

Problem 3

Decide True or False for each of the followings. You MUST briefly justify your answer.

Satisfiability:

Instance: Set U of variables, collection C of clauses over U.

Question: Is there a satisfying truth assignment for C?

(a) If P $\neq$ NP, then no problem in NP can be solved in polynomial time deterministically.

(b) If a decision problem A is NP-complete, proving that A is reducible to B, in polynomial time, is sufficient to show that B is NP-complete.

(c) It is known that SAT (Satisfiability) is NP-complete, and 3SAT (all clauses have size 3) is NP-complete. 1SAT (all clauses have size 1) is also NP-complete.

Solution

******* SOLUTION GOES HERE *******

Problem 4

Solution

******* SOLUTION GOES HERE *******

Problem 5

Solution

******* SOLUTION GOES HERE *******

Problem 6

Solution

******* SOLUTION GOES HERE *******