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Physics 134

Measurement of the Lifetime of a Muon

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Abstract

By measuring light pulses produced by incoming particles through a plastic scintillator, we aimed to measure the lifetime of a muon. The device was allowed to run for 94 hours and recorded 7558 events. Analysis of the data produced a lifetime of $2.017 \pm 0.02\mu s$, which was used to calculate a Fermi coupling constant value of $1.205 \pm 0.006 \times 10^{-5} GeV^{-2}$.

1 Introduction

As cosmic rays strike the top of Earth's atmosphere, they interact with the nuclei of air molecules. This produces a range of particles, particularly pions. A pion that does not interact with the nuclei of surrounding molecules will decay via weak interactions. The outcome of this type of interaction produces both muons and neutrinos

$$\begin{aligned} \pi^+ &\longrightarrow \mu^+\nu_\mu \\ \pi^- &\longrightarrow \mu^-\bar{\nu}_\mu \end{aligned} \tag{1}$$

The muons that are produced travel toward the surface of earth at near the speed of light, however, because a muon may interact with matter via weak and electromagnetic forces, it looses kinetic energy along its trajectory. Most muons are produced at an altitude of approximately 15 km, and if traveling at the speed of light, require a transit time of 50μs. The average lifetime of a muon at rest is approximately 2μs, but due to relativistic effects, there is an average flux of 1 muon $\text{min}^{-1}\text{cm}^{-2}$ at sea-level [2].

The mean energy of muons that reach sea-level is about 4GeV and the energy loss through coulombic interactions through matter is about 2MeV $\text{g}^{-}1\ \text{cm}^2$. Because the interaction length of the atmosphere is about 1000 g cm^{-2}, the original energy of a muon detected at sea-level is roughly 6 GeV. Given the dimensions of the scintillator from Table 1, the muons that manage to stop and decay in the detector have a total energy of about 160 MeV as they enter the cylinder.

2 Experimental Setup

The detector is composed of a plastic sciltillator that is monitored by a photomultiplier tube which is in turn connected to an adjustable high voltage and a discriminator, followed by an FPGA timer, and finally a computer as shown in Fig.1. As a particle enters the cylinder, scintillation light is produced and detected by the photomultiplier tube. The signal output signal feeds a two stage amplifier that feeds the discriminator. For signals above the threshold, a transistor-transistor logic output pulse is produced, which triggers the timing circuit of the FPGA. A second signal arriving at the FPGA within a fixed time interval resets the timing circuit, and the event is recorded.

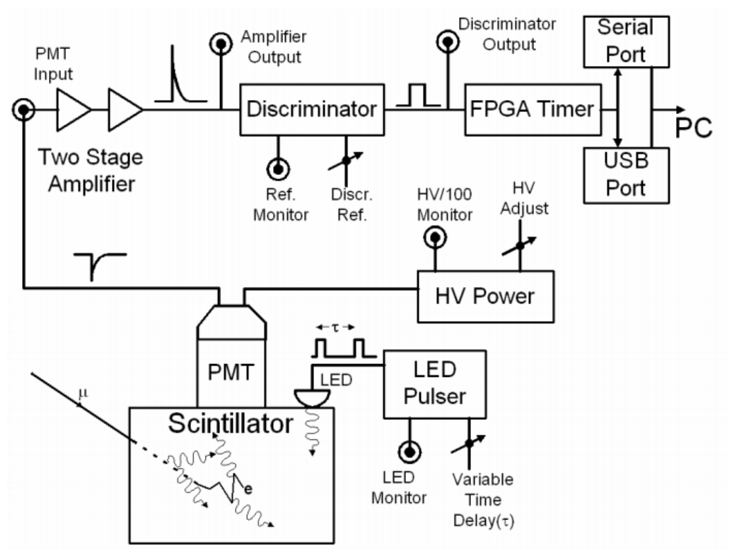

Figure 1: Block diagram of electronics. Taken from [1].

Mass density	$1.032\frac{\text{g}}{\text{cm}^3}$
Refractive index	1.58
Diameter	15 cm
Height	12.5 cm

Table 1: Scintillator dimensions and properties. Adapted from [1]

3 Procedure

The threshold of the detector controls the voltage output. Sine waves sent from a function generator to the discriminator produce a square pulse output. The timing properties of the FPGA can be verified by using the pulser located in the detector. Figure 2 show a plot of the time between successive pulses as measured by an oscilloscope and the FPGA. Altering the time between rising edges of detector pulser demonstrates the linearity of the FPGA.

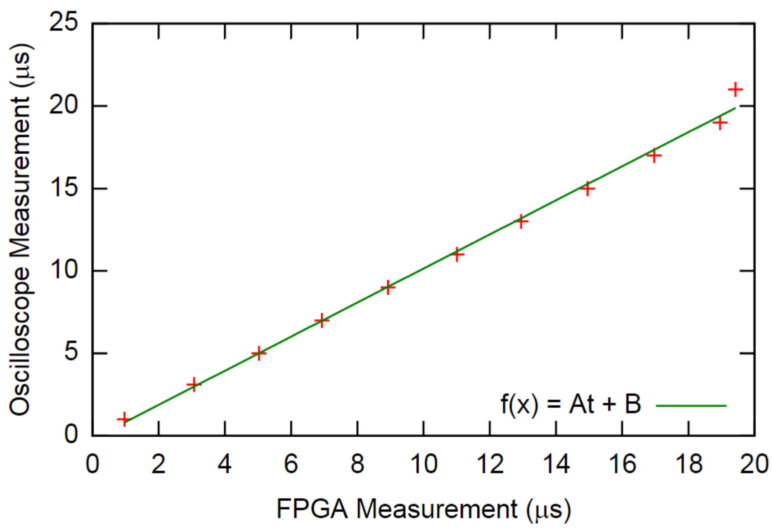

Figure 2: Measurements of the time between successive rising edges. The x-axis corresponds to measurements made by the FPGA and the y-axis corresponds to an oscilloscope. The slope between the two is approximately 1.

Figure 2 shows shat the maximum time between successive pulses registered is around 20μs, which implies that increasing the number of bins above this value should have no effect on the spread of the data. This is the result of the timing circuit being reset, during which the FPGA no longer records results. A linear fit of the data produces a slope of $1.03\pm0.02\mu$s. The result of decreasing the time interval between successive pulses indicates that the minimum internal timing bin width is about 0.02μs. This value corresponds to the resolution of the FPGA. In terms of events, increasing the threshold of the discriminator correlates to an decrease in the number of pulses with decay fluctuations ranging from $20ns$ to $1180ns$. With the information at hand, we set the discriminator to a final value of 220mV, and proceeded to examine the high voltage. By monitoring the muon count rate, we increased the voltage and attempted to determine then end of the plateau region. Figure 3 shows a sharp increase between 1050 and 1100V. A value of 1100V was used for the remainder of the experiment.

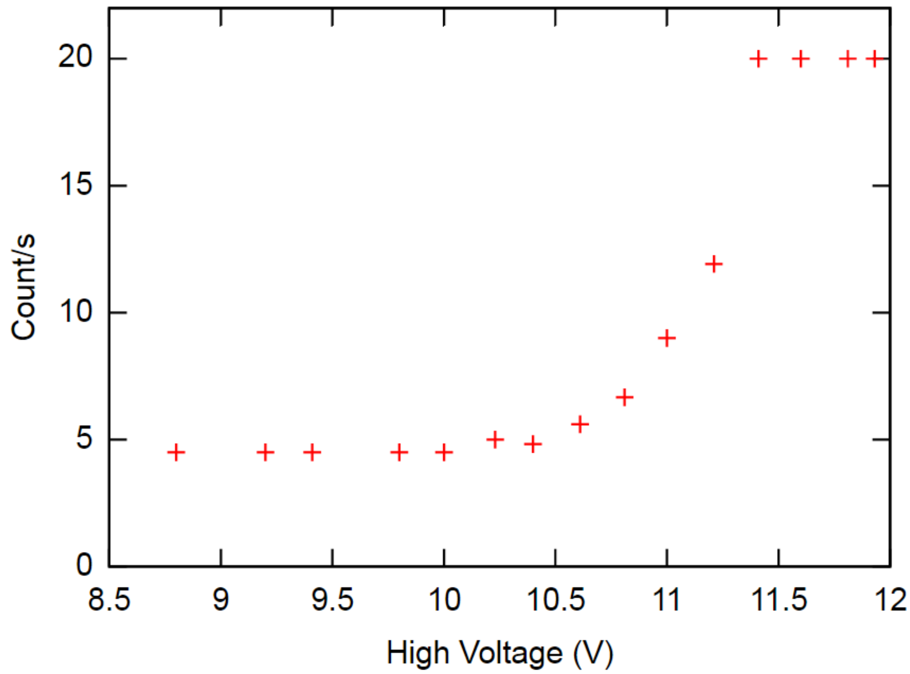

Figure 3: Muon count rate as a function of Voltage.

4 Analysis

The detector was allowed to run for a total of 94 hours during which time, 3091355 events were recorded. Timing data greater than 40000ns indicates a scenario where the time between successive signals exceeded the maximum number of clock cycles. After sifting the data, we were left with total 7558 events. As shown in Fig. 4, the data were arranged into bins of width 0.1μs for events with pulse separations between 0 and 10μs which produced 99 bins. Our next task was to fit an exponentially decaying function to the data. With the number of events in bin j denoted by d_j, with statistical variance $\sqrt{d_j}$ and a fitting function of

$$y(t_j) = A\ \exp(-t_j/\tau) + B\ , \tag{2}$$

our goal is to find values of A, B, and τ that minimized the sum of the squares of the normalized discrepancies χ^2 defined as

$$\chi^2 \equiv \sum_{j=1}^{N} \frac{(y_j - d_j)^2}{\sigma^2} = \sum_{j=1}^{N} \frac{(y_j - d_j)^2}{d_j}\ . \tag{3}$$

Inserting the trial function in Eq.2, and minimizing with respect to A and B produces

$$\frac{\partial\chi^2}{\partial A} = \sum\left[\frac{2}{d_j}[(A\exp(-t_j/\tau)+B)\exp(-t_j/\tau)] - 2\exp(-t_j/\tau)\right] = 0$$

$$\text{and} \tag{4}$$

$$\frac{\partial\chi^2}{\partial B} = \sum\left[\frac{2}{d_j}[A\exp(-t_j/\tau)+B] - 2\right] = 0$$

While holding τ constant at a value of 2μs, we are free to minimize the two equations. This is achieved by evaluating the following sums

$$N \equiv \sum 1 \qquad \alpha \equiv \sum d_j \qquad \beta \equiv \sum \exp(t_j/\tau)$$
$$\gamma \equiv \sum \frac{1}{d_j} \qquad \delta \equiv \sum \frac{\exp(t_j/\tau)}{d_j} \qquad \lambda \equiv \frac{\exp(-2t_j/\tau)}{d_j}\,, \tag{5}$$

and is summarized in table 2.

N	99
α	7119
β	19.04
γ	5.61
δ	0.23
λ	0.055

Table 2: Numerical evaluation of sums in Eq.5

Inserting the variables from Eqn.5 into Eqn.4, we can solve for A and B.

$$A = \frac{\beta\gamma - N\delta}{\lambda\gamma - \delta^2} = 361.74 \qquad \text{and} \qquad B = \frac{N\lambda - \beta\delta}{\lambda\gamma - \delta^2} = 2.86\,. \tag{6}$$

Re-writing the equation for χ^2 in terms of the variables in Eqn.5, we obtain

$$\chi^2 = \lambda A^2 + 2\delta AB + \gamma B^2 - 2\beta A - 2NB + \alpha = 86.04 \tag{7}$$

Holding A and B as they are in Eqn.6, it was found that the lowest value obtainable is $\chi^2 = 85.26$, corresponding to a lifetime value of 2.017μs. Varying τ by hand and allowing A and B to vary as well, produces a value of $\chi^2 = 82.31$ and a life time of $\tau = 2130\mu s$

With 3 free parameters and $N = 99$ bins, we are left with $\nu = 96$ degrees of freedom. For an appropriate fitting function, the goodness of fit should fall between

$$\nu - \sqrt{2\nu} \leq \chi^2 \leq \nu + \sqrt{2\nu} \quad \text{for} \quad \nu \geq 30$$
$$82.14 \leq \chi^2 \leq 109.86 \tag{8}$$

With the value of $\chi^2 = 82.31$, we are free to calculate the variance of the parameters. An estimation on the variance of the three parameters was obtained by holding two variables static, and varying the third until the goodness of fit increased a factor of $1 + \sqrt{2/\nu} = 1.144$. The optimal values of the three parameters, along with their calculated variances, are summarized in Table 3, and a plot of the data can be found in Fig.4.

A	347.731 ± 4.77
B	1.225 ± 0.26
τ	$2.125 \pm 0.02\ \mu s$

Table 3: Calculation of the values for parameters of fitting function.

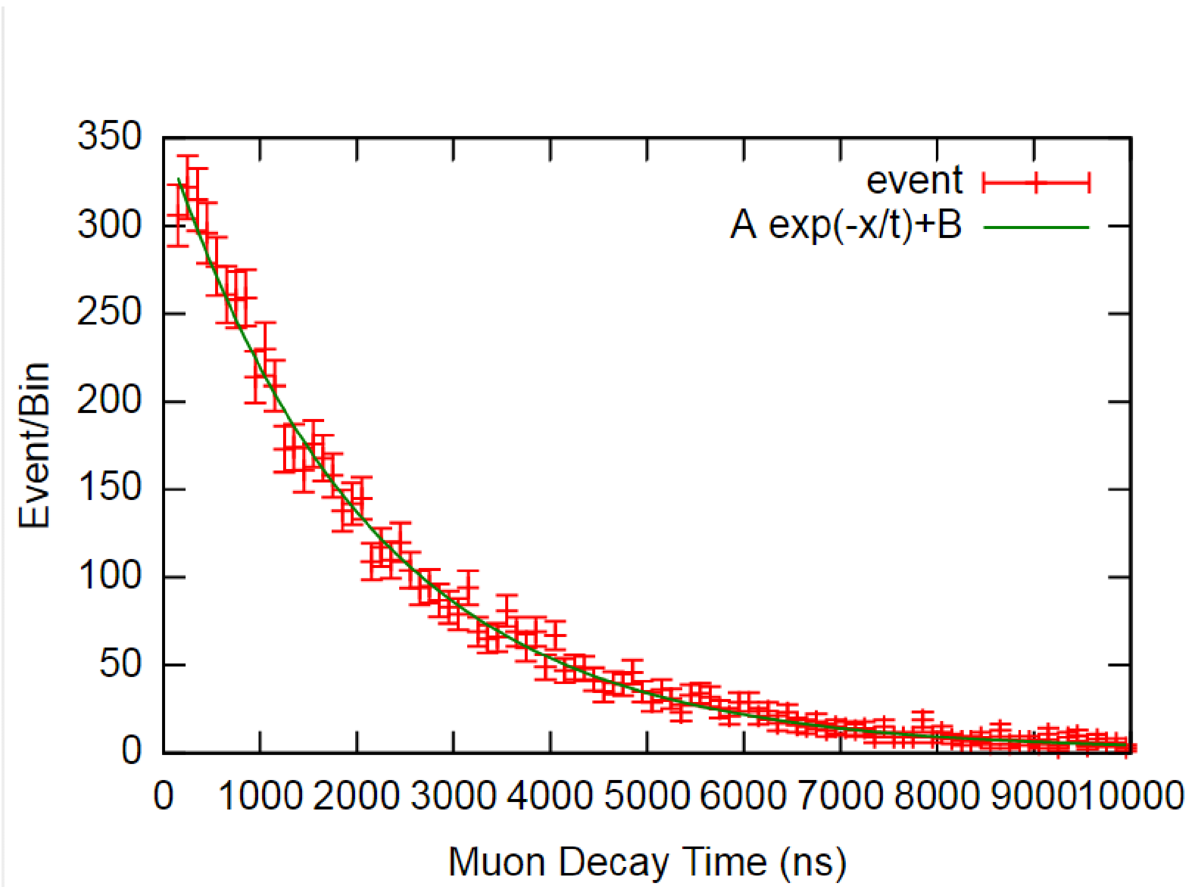

Figure 4: Decay time for 7558 events collected over 94 hours. The error on each point is $\sqrt{d_j}$, where d_j is the number of events in bin j.

To get a more precise fit to the data, a least square approach was also used. The data was binned the same as above and fit to the decaying exponential

$$N(t) = Ae^{-\frac{t}{\tau_\mu}} + B\,, \tag{9}$$

where τ_μ is the mean muon lifetime, A is a normalization constant, and B is number of background events per time bin. The values of the initial parameters were set to the values calculated above. All three parameters were allowed to vary until a convergence was reached. The results of this method can be seen in Table 4, and a plot can be seen in Fig. 4. For 96 degrees of freedom and a χ^2 value of 76.64, the confidence level is %93.81.

A	350.005 ± 6.155
B	1.471 ± 0.692
τ	$2.109 \pm 0.04\ \mu s$
χ^2	75.64

Table 4: Values for parameters of fitting function produced by gnuplot.

With the value of the muon lifetime measured, we can now calculate the value and corresponding error of the Fermi coupling constant

$$\frac{G_f}{(\hbar c)^3} = \sqrt{\frac{192\pi^3\hbar}{\tau m^5}}\,. \tag{10}$$

The current value measured values of the mass of a muon at rest and Planck's constant are $m_\mu = 105.65837(35)$MeV and $\hbar = 6.58211899(16) \times 10^{-22}$ MeV s. Compared to the level of uncertainty in the measurement of the muon lifetime above, these values can be taken to be constant to three significant digits. The speed of light in a vacuum is $c = 299792458$ m s^{-1} and is taken to be exact. [3, 4, 5].

$$\frac{G_f}{(\hbar c)^3} = \sqrt{\frac{192\pi^3(6.582 \times 10^{-25} GeV \cdot s)}{(2.109 \times 10^{-6} s)(0.106\, GeV)^5}} = 1.178 \times 10^{-5}\; GeV^{-2} \tag{11}$$

$$\sigma_G^2 = \left(\frac{\partial G}{\partial \tau}\right)^2 \sigma_\tau^2 = \left[-\frac{1}{2}\left(\frac{1}{\tau}\right)^{3/2}\sqrt{\frac{192\pi^3\hbar}{m^5}}\right]^2 \sigma_\tau^2 \tag{12}$$

$$\frac{G_f}{(\hbar c)^3} = 1.178 \pm 0.0119 \times 10^{-5} GeV^{-2} \tag{13}$$

The value of the coupling constant reported by the Particle Data Group is [4]

$$\frac{G_f}{(\hbar c)^3} = 1.16637(1) \times 10^{-5} GeV^{-2}\,. \tag{14}$$

The calculated value of $\tau = 2.109 \pm 0.004\mu$s being less than that of the lifetime of a muon in free space, $\tau_{free} = 2.19698(22)\mu$s is reasonable due to the fact that the incoming muons can have positive or negative charge. Unlike the positive muon, a negative particle that stops inside the scintillator will tend to bind to the carbon and hydrogen nuclei with the effect of producing a shorter measured lifetime.

5 Conclusion

By measuring pulses of scintillation light produced by incoming particles, we were able to measure a muon lifetime of $\tau = 2.109 \pm 0.004\ \mu$s. The observed value was lower than the lifetime of a muon in free space ($\tau_{free} = 2.19698(22)\,\mu$s) as expected. With this measurement, we were able to calculate the value of the Fermi coupling constant $G_f/(\hbar c)^3 = 1.178 \pm 0.0119 \times 10^{-5} GeV^{-2}$, which agrees with the value $1.16637(1) \times 10^{-5} GeV^{-2}$ to within three decimal places of error.

References

[1] Physics 134 Lab Manual. Spring 2016

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