CS 373 Homework 1
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1) a) Sample space for the problem will be,

H, TH, TTH,…..T^nH

The probability for the problem can be calculated as a Bernoulli Trials.

p(X = i) = $(1 - p)^{i-1} \cdot p$

p = 0.5

p(X = i) = $(\frac{1}{2})^i$

b) Set of outcomes:

E = {H, TH, TTTH,…., T^{2k+1}H}

P(X = E) = 0.5

2) S = (1,1) (1,2) (1,3) (1,4) (1,5) (1,6)
(2,1) (2,2) (2,3) (2,4) (2,5) (2,6)
(3,1) (3,2) (3,3) (3,4) (3,5) (3,6)
(4,1) (4,2) (4,3) (4,4) (4,5) (4,6)
(5,1) (5,2) (5,3) (5,4) (5,5) (5,6)
(6,1) (6,2) (6,3) (6,4) (6,5) (6,6)

E = (1,2) (1,4) (1,6)
(2,1) (2,3) (2,5)
(3,2) (3,4) (3,6)
(4,1) (4,3) (4,5)
(5,2) (5,4) (5,6)
(6,1) (6,3) (6,5)

F = (1,1) (1,2) (1,3) (1,4) (1,5) (1,6)
(2,1) (3,1) (4,1) (5,1) (6,1)

G = (2,3) (3,2) (1,4) (4,1)

a) E(E ∩ F) = (1,2) (2,1) (1,6) (6,1) (4,1) (1,4)

P(E ∩ F) = 6 / 36 = 1 / 6

b) E(E ∪ F) = (1,2) (1,4) (1,6) (1,1) (1,3) (1,5)
(2,1) (2,3) (2,5) (3,1) (5,1)
(3,2) (3,4) (3,6)
(4,1) (4,3) (4,5)
(5,2) (5,4) (5,6) (6,1) (6,3) (6,5)

P(E ∪ F) = 23 / 36

c) E(F ∪ G) = (1,1) (1,2) (1,3) (1,4) (1,5) (1,6)
(2,1) (3,1) (4,1) (5,1) (6,1) (2,3)
(3,2)

P(F ∪ G) = 13 / 36

d) P(E ∪ ¬F) = P(E) + P(¬F) - P(E ∩ ¬F)
= 1/2 + 25/36 - 1/3
= 0.8611

e) E(E ∪ F ∪ G) = E(E ∪ F) since G is a subset of E

P(E ∪ F ∪ G) = P(E U F) = 23 / 36

3) a) Let F_1, F_2, F_3 be the events when d_1, d_2, d_3 fail respectively.

Since E occurs when 2 or more disks fail,

P(E) = P($F_1 \cap F_2$) + P($F_1 \cap F_3$) + P($F_3 \cap F_2$) + P($F_1 \cap F_2 \cap F_3$)

Since F_1, F_2, F_3 are independent,

P(E) = P(F_1) . P(F_2) + P(F_1) . P(F_3) + P(F_3) . P(F_2) + P(F_1) . P(F_2) . P(F_3)

P(E) = 0.01 x 0.03 + 0.01 x 0.05 + 0.05 x 0.03 + 0.01 x 0.03 x 0.05
P(E) = 0.0008 + 0.0015 + 0.000015 = 0.002315

b) Let F_1 be event when d_1 fails, F_2 be event when d_2 fails, F_3 be event when d_3 fails.

P(F) = P(F_1) + P($F_2 \cap F_3$) + P($F_1 \cap F_2 \cap F_3$)

Since F_1, F_2, F_3 are independent,

P(F) = P(F_1) + P(F_2) . P(F_3) + P(F_1) . P(F_2) . P(F_3)
P(F) = 0.01 + 0.03 x 0.05 + 0.01 x 0.03 x 0.05
P(F) = 0.011515

c) P(F | d_3) = P(d_3 | F) . P(F) / P(d_3) = 0.5 * 0.011515 / 0.05 = 0.11515

4) a) Let C be the event that a student is studying computer science and F be the event that the student is a female,

P(F | C) = P(F ∩ C) / P(C) = 0.0055 / 0.5 = 0.011

b) Using the same events from part a. ,

P(C | F) = P(F | C) . P(C) / P(F) = 0.011 x 0.05 / 0.52 = 0.00106

c) P(C | F) = P(F | C) x P(C) / P(F) = 0.15 x 0.05 / 0.57 = 0.01316

5) a) Let ‘H’ be the number of heads and ’T’ denote number of tails in ’n’ flips

H + T = n
X = H - T = 2H - n for H = 0,…n

E(X) = 2E(H) - E(n) = 2E(H) = 2np Since H is a Bernoulli Trial of ’n’ times

b) Var(X) = 4Var(H) = 4np(1 - p) Since H is a Bernoulli Trial of ’n’ times

c) E(X_3) = 2 x 3 x p = 6p

Var(X_3) = 4 x 3 x p(1 - p) = 12p(1 - p)

CS373 Homework1 – Part B

By: Siddharth Shah

3) a)

```
> names(yelp)
 [1] "business_id"         "name"                "fullAddress"         "city"                "state"
 [6] "latitude"            "longitude"           "stars"               "reviewCount"         "checkins"
[11] "open"                "neighborhoods"       "categories"          "alcohol"             "noiseLevel"
[16] "attire"              "priceRange"          "delivery"            "ambience"            "parking"
[21] "dietaryRestrictions" "waiterService"       "smoking"             "outdoorSeating"      "caters"
[26] "recommendedFor"      "goodForGroups"       "goodForKids"
```

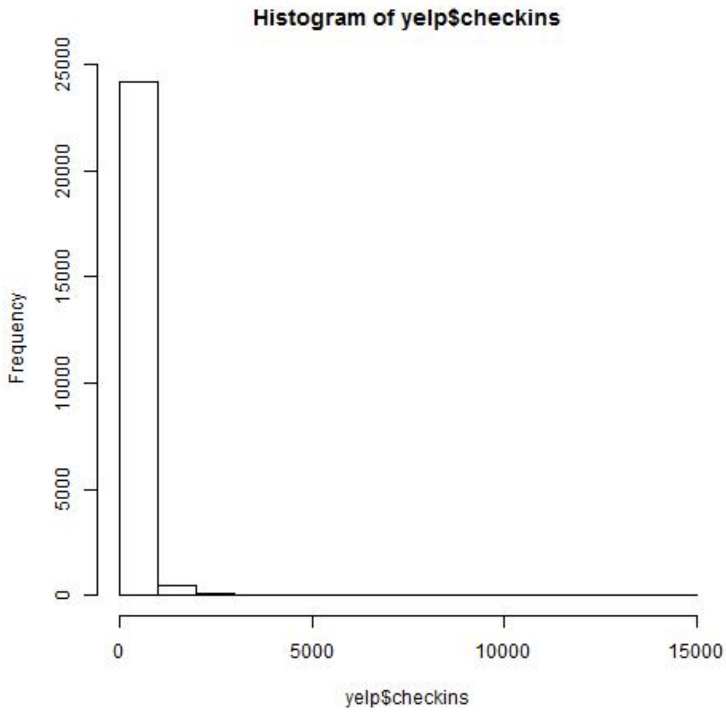
b)

```
> summary(yelp)
                  business_id                     name
 --1emggGHgoG6ipd_RMb-g:     1   Starbucks :  407
 --5jkZ3-nUPZxUvtcbr8Uw:     1   McDonald's:  275
 -024YEtnIsPQCrMSHCKLQw:     1   Subway    :  256
 -0bUDim5OGuv8R0Qqq6J4A:     1   Walgreens :  158
 -0D_CYhlD2ILkmLR0pBmnA:     1   Taco Bell :  148
 -0GkcDiIgVm0XzDZC8RFOg:     1   Wendy's   :  113
 (Other)               :24807   (Other)   :23456
                                                                          fullAddress             city
 Bellagio Las Vegas\n3600 S Las Vegas Blvd\nThe Strip\nLas Vegas, NV 89109          :   21   Las Vegas : 5256
 Las Vegas, NV                                                                        :   17   Phoenix   : 3072
 5000 S Arizona Mills Cir\nTempe, AZ 85282                                            :   14   Charlotte : 1993
 3131 Las Vegas Blvd. South\nThe Strip\nLas Vegas, NV 89109                           :   13   Pittsburgh: 1467
 Monte Carlo Hotel and Casino\n3770 Las Vegas Blvd S\nThe Strip\nLas Vegas, NV 89109:   13   Scottsdale: 1296
 2000 E Rio Salado Pkwy\nTempe, AZ 85281                                              :   12   Montral   : 1267
 (Other)                                                                              :24723   (Other)   :10462
     state          latitude        longitude              stars          reviewCount          checkins          open
 AZ     :9301   Min.   :32.88   Min.   :-115.370   Min.   :1.000   Min.   :   3.00   Min.   :    3   Mode :logical
 NV     :6296   1st Qu.:33.54   1st Qu.:-114.977   1st Qu.:3.000   1st Qu.:   8.00   1st Qu.:   16   FALSE:3580
 QC     :2389   Median :36.03   Median :-111.924   Median :3.500   Median :  18.00   Median :   48   TRUE :21233
 NC     :2370   Mean   :37.53   Mean   : -97.298   Mean   :3.544   Mean   :  49.03   Mean   :  166
 PA     :1613   3rd Qu.:40.41   3rd Qu.: -80.807   3rd Qu.:4.000   3rd Qu.:  48.00   3rd Qu.:  155
 WI     :1089   Max.   :55.99   Max.   :   8.549   Max.   :5.000   Max.   :4578.00   Max.   :14203
 (Other):1755
        neighborhoods                                   categories              alcohol              noiseLevel
 []            :15727   ['Mexican', 'Restaurants']               : 1331                :    3              : 7947
 ['The Strip']:  816   ['Food', 'Coffee & Tea']                 :  844   beer_and_wine: 2497   average  :10957
 ['Southeast']:  639   ['Pizza', 'Restaurants']                 :  831   full_bar     : 7565   loud     : 1622
 ['Downtown'] :  533   ['Chinese', 'Restaurants']               :  776   none         :14748   quiet    : 3562
 ['Westside'] :  526   ['Burgers', 'Fast Food', 'Restaurants']:  549                          very_loud:  725
 ['Eastside'] :  447   ['Restaurants', 'Italian']               :  509
 (Other)      : 6125   (Other)                                  :19973
     attire         priceRange       delivery                ambience                    parking
        : 7005   Min.   :1.000   Mode :logical   ['casual']:7878   ['lot']               :10348
 casual:17129   1st Qu.:1.000   FALSE:14471               :7875   []                    : 6675
 dressy:  640   Median :2.000   TRUE :3093      []        :6348   ['street']            : 3046
 formal:   39   Mean   :1.631   NA's :7249      ['divey'] : 716                         : 2456
                3rd Qu.:2.000                   ['trendy']: 567   ['garage']            :  907
                Max.   :4.000                   ['classy']: 320   ['street', 'lot']:  364
                NA's   :903                     (Other)   :1109   (Other)               : 1017
              dietaryRestrictions waiterService       smoking        outdoorSeating    caters
                          :24696   Mode :logical            :21862   Mode :logical   Mode :logical
 ['vegan']                :   45   FALSE:6208      no     :  904   FALSE:10989     FALSE:6503
 ['vegetarian']           :   23   TRUE :10351     outdoor: 1415   TRUE :8698      TRUE :5932
 []                       :   20   NA's :8254      yes    :  632   NA's :5126      NA's :12378
 ['dairy-free', 'vegetarian']:    7
 ['vegan', 'vegetarian']  :    5
 (Other)                  :   17
             recommendedFor goodForGroups   goodForKids
                     :7859   Mode :logical   Mode :logical
 []                  :4932   FALSE:2054      FALSE:506
 ['lunch']           :4324   TRUE :17078     TRUE :1283
 ['dinner']          :2553   NA's :5681      NA's :23024
 ['lunch', 'dinner']:1966
 ['breakfast']       :1004
 (Other)             :2175
```

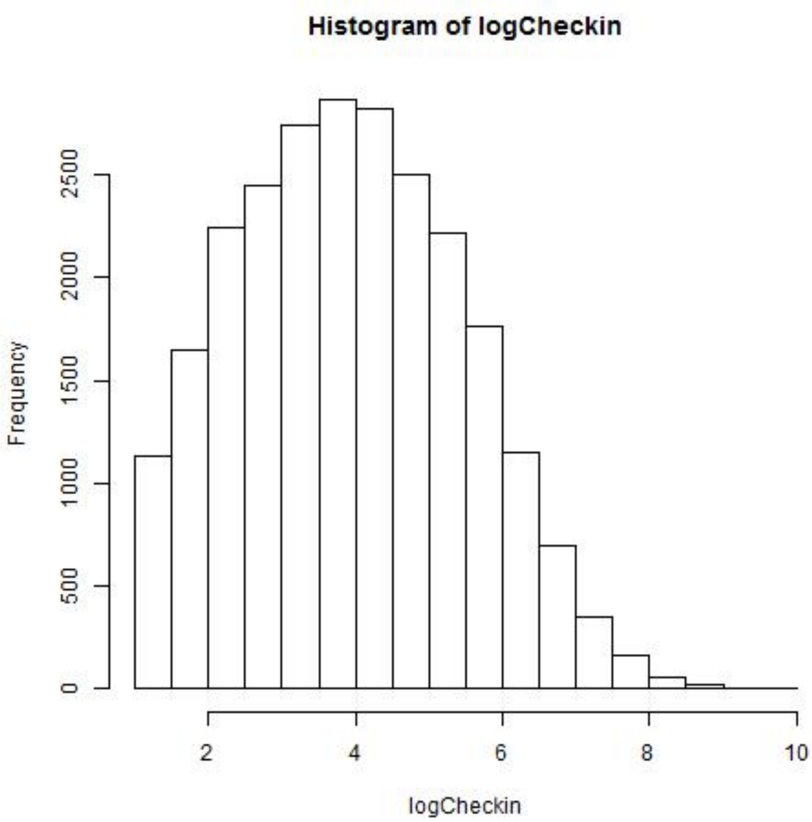
c)

```
> summary(yelp$noiseLevel)
            average      loud     quiet very_loud
     7947     10957      1622      3562       725
> summary(yelp$stars)
   Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
  1.000   3.000   3.500   3.544   4.000   5.000
```

4) a)


Histogram of yelp$checkins
Frequency
0
5000
10000
15000
20000
25000
0
5000
10000
15000
yelp$checkins

b)


Histogram of logCheckin
Frequency
0
500
1000
1500
2000
2500
2
4
6
8
10
logCheckin

c) Since the Yelp data has data about restaurants some very popular restaurants as well as new/upcoming or not-so-popular restaurants, the raw check-in histogram is skewed. This causes the frequency of higher check-ins diminish the expressivity of data by shadowing the less frequency data.

Using log-scale causes a normalization of the data generally, there by weighting all data equally, and also getting rid of broken data like negative check-ins if the exist. The shape is also more normal, there by creating ease of applying inference and statistical techniques.

5) a)

```
> summary(yelp$isAmerican)
   Mode   FALSE    TRUE
logical   21456    3357
> summary(yelp$goodForDinner)
   Mode   FALSE    TRUE
logical   19670    5143
```

b)

```
> quantile(yelp$reviewCount)
  0%  25%  50%  75% 100%
   3    8   18   48 4578
```

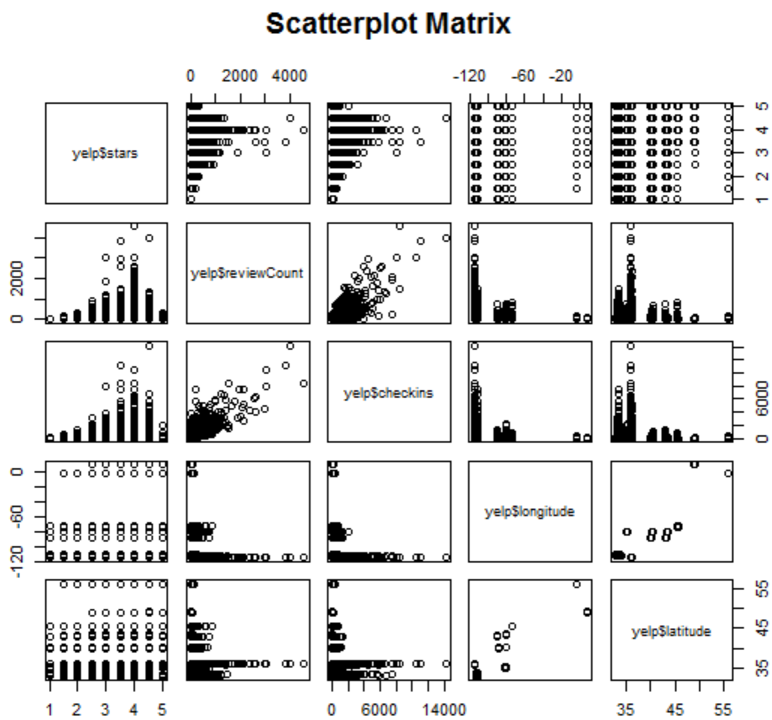
c)

```
> summary(yelp_subset$reviewCount)
   Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
  3.000   4.000   5.000   5.247   7.000   8.000
> summary(yelp_subset$stars)
   Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
  1.000   3.000   3.500   3.418   4.000   5.000
> summary(yelp_subset$attire)
        casual dressy formal
  3248    3581    107     24
> summary(yelp_subset$priceRange)
   Min. 1st Qu.  Median    Mean 3rd Qu.    Max.    NA's
  1.000   1.000   1.000   1.546   2.000   4.000     825
> summary(yelp_subset$delivery)
   Mode   FALSE    TRUE    NA's
logical    2899     693    3368
> summary(yelp_subset$goodForKids)
   Mode   FALSE    TRUE    NA's
logical      15      31    6914
```

```
> summary(yelp$reviewCount)
   Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
   3.00    8.00   18.00   49.03   48.00 4578.00
> summary(yelp$stars)
   Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
  1.000   3.000   3.500   3.544   4.000   5.000
> summary(yelp$attire)
        casual dressy formal
  7005  17129    640     39
> summary(yelp$priceRange)
   Min. 1st Qu.  Median    Mean 3rd Qu.    Max.    NA's
  1.000   1.000   2.000   1.631   2.000   4.000     903
> summary(yelp$delivery)
   Mode   FALSE    TRUE    NA's
logical   14471    3093    7249
> summary(yelp$goodForKids)
   Mode   FALSE    TRUE    NA's
logical     506    1283   23024
```

On general, since we are comparing subset to a superset, we see a decrease in means for every variable. However, for something skewed like the “reviewCount” we see a sharp decrease in the mean. We see that most priceRange data is unavailable below 1st quantile reviewCount. While the “goodForKids” data is mostly available below the reviewCount below 1st quantile. Median number of stars has less effect on the reviewCount, since it is the same for both the datasets.

6) a)



These correlations have some obviousness, like the strong co-relation between the latitude and longitude, since these 2 variables are literally at the cross-section for places available in the data. The correlation between stars and reviewCount and starts and check-ins is interesting, since this gives valuable inference to the credibility of Yelp as a platform. Why do the stars decrease after a certain number of reviewCounts? Do people see more reviews as something misleading? And why do people have less check-ins at restaurants with more stars after a certain threshold? These questions can be investigated based on the data. A striking contrast to that is the reviewCount vs. the Check-ins, that relationship seems to uniformly increasing. So that means that people who visit places more write more reviews, this means that Yelp as a platform is achieving its desired model.

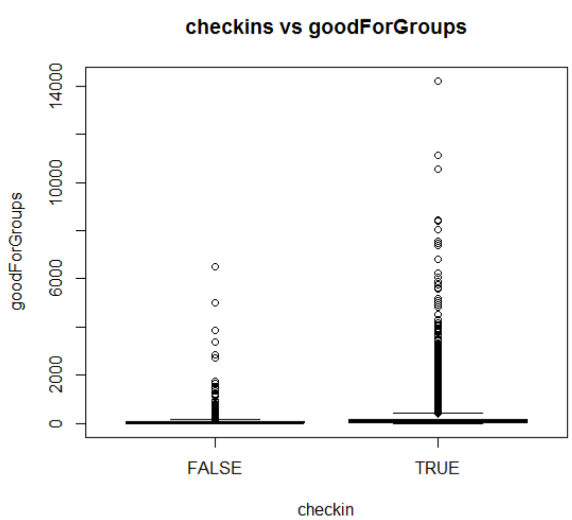
b)

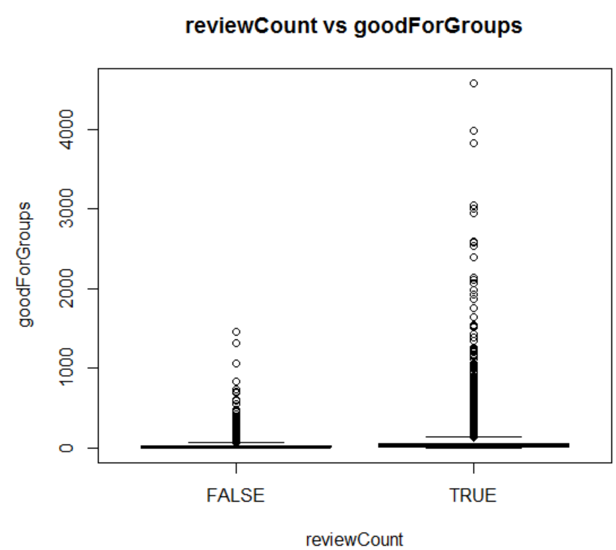
```
> cor(yelp$stars, yelp$reviewCount)
[1] 0.1070506
> cor(yelp$stars, yelp$checkins)
[1] 0.09440071
> cor(yelp$stars, yelp$longitude)
[1] 0.1174446
> cor(yelp$stars, yelp$latitude)
[1] 0.1211631
> cor(yelp$reviewCount, yelp$checkins)
[1] 0.8274936
> cor(yelp$reviewCount, yelp$longitude)
[1] -0.1294142
> cor(yelp$reviewCount, yelp$latitude)
[1] -0.09850936
> cor(yelp$checkins, yelp$longitude)
[1] -0.1789531
> cor(yelp$checkins, yelp$latitude)
[1] -0.1526046
> cor(yelp$longitude, yelp$latitude)
[1] 0.8811018
```

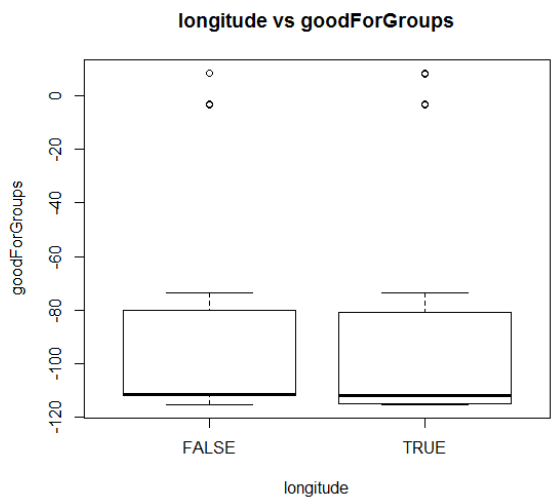
The largest pairwise positive correlation exists between longitude and latitude. While the largest pairwise negative correlation exists between check-ins and longitude.

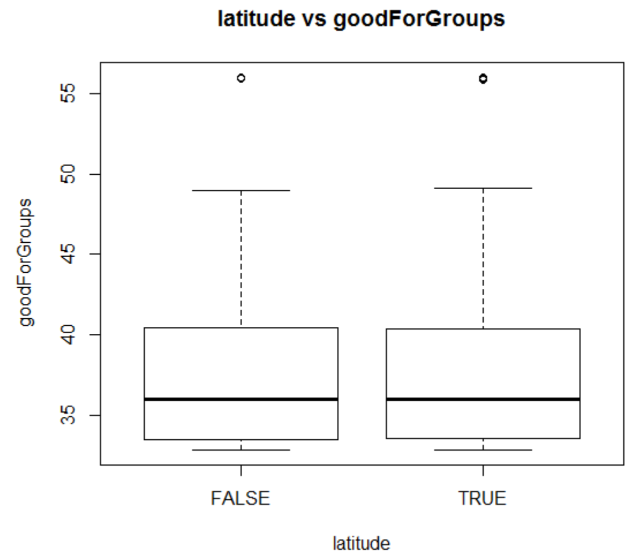
Visually, these correlations comply, since we can see a linear relationship between longitudes and latitudes both ways, while check-ins and longitudes do not seem to have any correlation at all.

c)





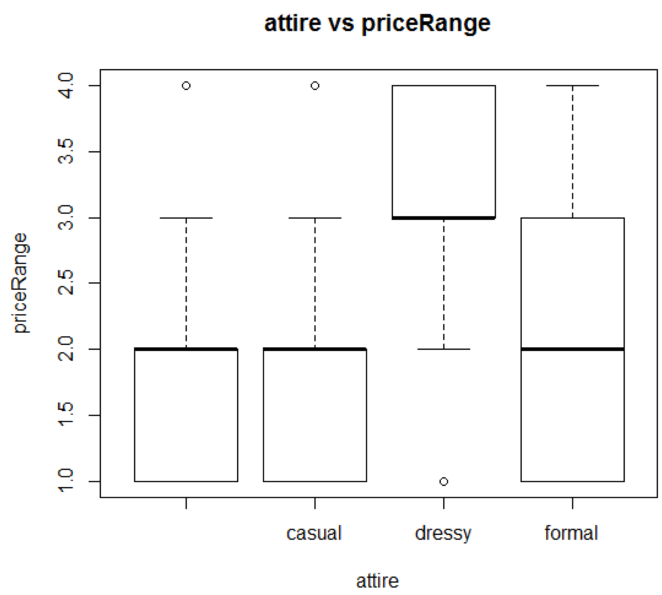


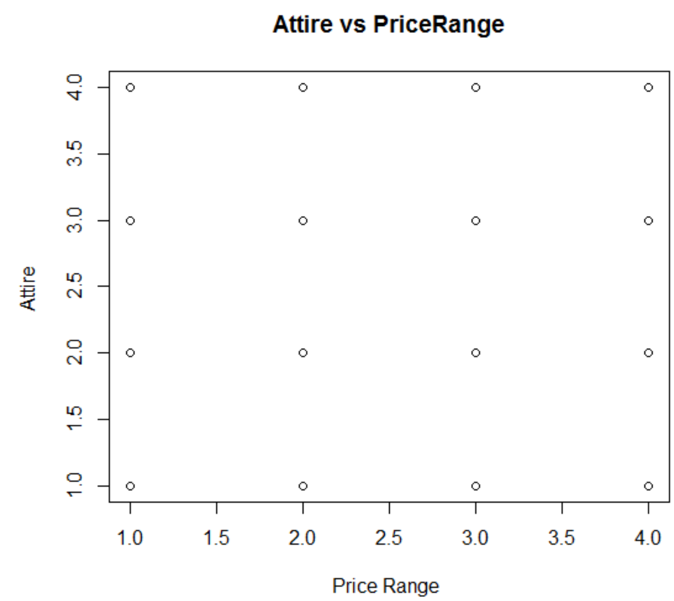



```
> tru_ch <- subset(yelp, yelp$goodForGroups == TRUE)
> quantile(tru_ch$checkins)
   0%   25%   50%   75%  100%
    3    19    59   181 14203
> quantile(tru_ch$reviewCount)
  0%  25%  50%  75% 100%
   3   10   24   61 4578
> quantile(tru_ch$longitude)
        0%        25%        50%        75%       100%
-115.36973 -115.04307 -111.92574  -80.82606    8.54856
> quantile(tru_ch$latitude)
      0%      25%      50%      75%     100%
32.87687 33.53849 36.02708 40.36092 55.99042
> false_ch <- subset(yelp, yelp$goodForGroups == FALSE)
> quantile(false_ch$checkins)
  0%  25%  50%  75% 100%
   3   11   25   66 6485
> quantile(false_ch$reviewCount)
  0%  25%  50%  75% 100%
   3    7   13   30 1453
> quantile(false_ch$longitude)
         0%         25%         50%         75%        100%
-115.328981 -112.152874 -111.840497  -80.018910    8.410954
> quantile(false_ch$latitude)
      0%      25%      50%      75%     100%
32.87918 33.51192 36.04116 40.45204 55.97743
```

The variable "checkins" exhibits most association with "goodForKids". This is interesting because one would expect more checkins creating happy vibe in the restaurant that would make place more kid-friendly. However, this is bizarre because a lot of checkins also happen at places like bars, which aren't kid-friendly.

7) a) Based on the data, I'd like to propose a relationship between PriceRange and Attire type.





b) These variables are discrete since these are discrete price ranges, and categorical variables corresponding to the attire type.

c) The function would relate “attire” (X) to “priceRange” (Y), and not the other way around. We can see there is no association the other way, but attire to price-range does have inferable association.

d) Based on the box plot, we can see that these variables have some inference property. We can see that more “dressy” attire relates to high priced places, and formal dressing is not just limited to high-prices places.

Is the fourth unknown type of dressing also “casual”? Since the data tightly corresponds to that of casual dressing.

Would inducing high prices, yield an environment of the restaurant where “dressy” clothing is preferred?

e) The hypothesis purely empirical, since the data is more or less empirical and so is the inference.