Simulating a Disc Galaxy inside a DM halo

Técnicas de Simulación Numérica Module 2, Project 2

Objective: To model a disc galaxy rotating within a dark matter halo.

Background: We model stars within a disc, as they rotate within a dark matter halo. We will simulate a Milky Way like galaxy.

Part 1: The Sun revolving around the Centre of the Galaxy.

First model the orbit of the Sun around the Centre of the Milky Way Galaxy, assuming that the gravitation potential is provided by the dark matter halo. Model the halo using the analytic NFW profile as shown below:

Initial Conditions

Set phi=0 and use the distance of the Sun from the centre of the Galaxy (8kpc), the Mass of the Milk Way (1×10^{12} solar masses) and the velocity of the Sun around the Centre (about 200 km/s).

```
x=8      # units are kpc. We set up our axes so that the Sun is in the x-y plane
y=0.     # initially the Sun us at y=0, i.e. we chose phi=0 so sin(phi)=0
z=0
vx=0
vy=127 # km/s  Sun is moving perpendicular to its position, i.e. 180*cos(phi)
vz=0
```

Units: We can use similar units employed in Gadget earlier in the year

Unit Velocity 1e5 cm/s
This sets the velocity unit to km/sec (which we want). The specification of Unit Length in cm, Unit Mass in g, and Unit Velocity in cm per s also determines the internal unit of time: one internal time unit corresponding to 9.8×10^8 yr.
Unit Length 3.085678e21 cm
This sets the length units to 1.0 kpc
Unit Mass in g 1.989e33
This sets the mass units to 1 $M_\odot$
Gravitational Constant
G= 4.302e-6 in these units of kpc $(km/s)^2$ $M_\odot^{-1}$

NFW potential

Use the NFW profile to model the gravitational potential. So

1. determine the mass within the radius of the Sun. i.e. the mass of the halo that is at lower radii than the sun according to the NFW profile.
2. assume that mass within that radius is all at the center

So the model is **similar to the Earth orbiting the Sun**, but instead of having the mass of the sun at the centre, you have the dark matter at the centre.

Recall the NFW profile:

$$\rho(r) = \frac{\rho_0}{\frac{r}{R_s}\left(1 + \frac{r}{R_s}\right)^2}$$

You can find the mass enclosed within any radius using:

$$M = \int_0^{R_{\max}} 4\pi r^2 \rho(r)\, dr = 4\pi\rho_0 R_s^3 \left[\ln\left(\frac{R_s + R_{\max}}{R_s}\right) - \frac{R_{\max}}{R_s + R_{\max}}\right]$$

Use concentration C=12.5, Virial Radius, Rv=250, so Scale Length, Rs=20. Using rho0= 5932371 Msol/kpc^3 gives a total mass of 1x10**12 Msol out to 250 Kpc.

Task and Questions:

Run the code for 18.6×10^8 years, i.e. tfinal=2 in system units.
Use 1000 steps, and run using Euler and RK4. Plot the path of the orbits.
(Note that in the real galaxy, the sun is moving at around 200km/s rather than 127. This is because there are also stars adding mass to the central regions so the mass is greater than just the dark matter).

Part 2: A Disk revolving around the Centre of the Galaxy.

The file Disk10.txt contains data of a rotating disc with 10 “Star particles”, whilst Disk100.txt and Disk1000.txt have 100 and then 1000 star particles in a disc. Columns are m,x,y,z,vx,vy,vz. The total mass of the disk of the Milky Way is about 5x10^{10}

Task and Questions:

Simulate the disc of the Milky Way using 10, then 100 and then 1000 star particles, using the NFW dark matter potential without gravitation interaction between stars. Because each star particle is at a different radius, you have to calculate the mass of the dark matter halo enclosed within that radius **for each star**. So use the value given by the NFW profile, with a new calculation for each star. Again run the simulation for 1000 steps for 18.6×10^8 years, i.e. 2 system units, using RK4. Plot (or animate) the orbits.

Does the time taken for running this simulation scale as number of particles, i.e O(n)? Or as the square of the number of particles, O(n^2).

Part 3: A Disk revolving around the Centre of the Galaxy with “self gravity”

Although the total mass of the Milky Way is more than 80% dark matter, in the inner regions, the stars are actually gravitationally relevant. There is approximately equal mass of stars and dark matter within the radius of the sun. The dark matter is only dominant in the outer parts. Due to the importance of the stellar mass in the inner regions, we should include the mass effect of stars on the orbits of other stars.

Task and Questions:

Re-simulate the disc of the Milky Way first using 10, then 100 star particles, using the potential of the DM **plus also the effects of the stars gravitational interaction with the other stars**. Again run the simulation for 1000 steps for 18.6×10^{8} years, i.e. 2 internal units, and run using RK4. Plot (or animate) the orbits.

Do all the stars stay in the disk? If not, you may need to add a gravitational softening of e.g. 1kpc for the 10 particle simulation. Explain why this was required. Can you decrease that gravitational softening for the simulations with more particles?

Does time taken for running this simulation scale as number of particles, i.e O(n)? Or as the square of the number of particles, $O(n^2)$. Can you still run the 1000 particle simulation on your laptop?

Extra Work/Bonus

(really, you will not lose marks for not doing this… I just think you may find it interesting, at least to think about how this code may work even if you don't make the code :)

If all this was particularly easy to code, you may want to develop the model a little more. You can speed up the code by only directly calculating gravity between a star particle and its nearest neighbours. So you could start with just including the gravitational effects of the 10 nearest neighbours. This can be done efficiently using the Tree algorithm and doing a neighbor search, in a similar way that we use to speed up the Virus code. Then the remaining stars (those that are not neighbours) could be treated in a simple way: sum the mass of those remaining stars which have radius less than the star we are computing… that mass can be added to the potential. In other words, the potential would include *all* mass inside the radius of the star, rather than just the dark matter mass.

Does such a code speed up the running of 1000 particles? Does the final galaxy look ok? If not, increasing the neighbours use d from 10 to 50 may be required (which of course slows down the code…)