Translation from CCS to CSP: the m-among-n Synchronisation Approach

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Introduction - 1/2

- CCS (Calculus of Communicating Systems) and CSP (Communicating Sequential Processes) are two prominent calculi for reasoning about concurrent programs
 - CCS late Prof. R. Milner, Turing Award 1991
 - ► CSP Sir Prof. T. Hoare, Turing Award 1980
 - Hundreds of papers published based off both calculi
- ▶ What is the difference between CCS and CSP?
 - ► Ekembe et al. (CCS to CSP Translation, Dec.2021) answer this question
 - ▶ By translating CCS into CSP, excluding parallel under recursion
 - ▶ Shows that the translation function, ccs2csp, is correct up to strong bisimulation, viz., $ccs2csp(P) \sim P$
- Our main interest: Unify both CCS and CSP worlds (incl. Pi-calculus and CSPmob)

Introduction - 2/2

- ► This paper: Translate CCS into CSP, including parallel under recursion
 - Our new translation, ccs2csp3 is correct up to strong bisimulation
 - Compositional
- ► How? Extend CSP with m-among-n synchronisation, called CSPmn
 - CSPmn is a conservative extension of CSP
 - binary synchronisation can be defined through multiway or n-among-n synchronisation (the default CSP synchronisation mechanism) and renaming
 - m-among-n synchronisation can be defined through multiway synchronisation and renaming

Formalisation – Labelled Operational Correspondence between CCS and CSP

	CCS	CSP
Termination	0 →	STOP →
Prefix	$a.P \xrightarrow{a} P$	$(a \leadsto P) \xrightarrow{a} P$
Tau Prefix	$\tau.P \xrightarrow{\tau} P$	$(tau \leadsto P) \setminus_{csp} \{tau\} \xrightarrow{\tau} P$
Ext Choice	$a.P + b.Q \xrightarrow{a} P$	$a \leadsto P \square b \leadsto Q \xrightarrow{a} P$
	$a.P + b.Q \xrightarrow{b} Q$	$a \leadsto P \square b \leadsto Q \xrightarrow{b} Q$
Int Choice	$\tau . P + \tau . Q \xrightarrow{\tau} P$	$P \sqcap Q \xrightarrow{\tau} P$
	$\tau . P + \tau . Q \xrightarrow{\tau} Q$	$P \sqcap Q \xrightarrow{\tau} Q$
Mixed Choice	$a.P + \tau.Q \xrightarrow{a} P$	$(a \leadsto P \square tau \leadsto Q) \Big\backslash_{csp} \{tau\} \xrightarrow{a} P \Big\backslash_{csp} \{tau\}$
	$a.P + \tau.Q \xrightarrow{\tau} Q$	$(a \leadsto P \square tau \leadsto Q) \Big\backslash_{csp} \{tau\} \xrightarrow{\tau} Q \Big\backslash_{csp} \{tau\}$
Restriction	$(a.P) \upharpoonright \{a\} \not\rightarrow$	$(a \leadsto P) \parallel STOP \not\to$
	$(a.P) \upharpoonright \{b\} \xrightarrow{a} P \upharpoonright \{b\}$	$(a \leadsto P) \parallel STOP \xrightarrow{a} P \parallel STOP$ $ \{b\} $
	$ (a.P \mid \bar{a}.Q) \upharpoonright \{a\} \xrightarrow{\tau} P \mid Q $	_
		$(P \underset{\{a\}}{\parallel} Q) \Big\backslash_{csp} \{a\} \underset{\{a\}}{\parallel} STOP$
Recursion	$\mu X.P \xrightarrow{\alpha} P'$	$\mu X.P \xrightarrow{\alpha} P'$

Formalisation: Parallel - 1/2

CCS Parallel is complex:

$$\left. \begin{array}{l} a.0 \mid \bar{a}.0 \equiv a.\bar{a}.0 + \bar{a}.a.0 + \tau.0 \\ a.\bar{a}.0 + \bar{a}.a.0 \equiv a.0 \parallel \bar{a}.0 \\ \tau.0 \equiv (a.0 \mid \bar{a}.0) \uparrow \{a\} \end{array} \right\} \Rightarrow a.0 \mid \bar{a}.0 \equiv (a.0 \parallel \bar{a}.0) + \\ \left. \begin{array}{l} (a.0 \mid \bar{a}.0) \uparrow \{a\} \end{array} \right\}$$

► In CSP:

$$\underbrace{\left(\underbrace{(a \leadsto STOP \parallel a \leadsto STOP)}_{\text{Interleaving} \times \text{Visible!}} \Box \left(a \leadsto STOP \underset{\{a\}}{\parallel} a \leadsto STOP\right)\right) \Big\backslash_{esp} \{a\}}_{esp} \{a\}$$

 ${\sf Synchronisation} \times {\sf Invisible!}$

(Req-sep) Thus, we need to separate interleaving from synchronisation! (Req-sync) We need to make synchronisation visible!

- Solution from Ekembe et al. (CCS to CSP Translation, Dec.2021): $((a \leadsto STOP \parallel a \leadsto STOP) \square (a_{12} \leadsto STOP \parallel a_{12} \leadsto STOP)) \backslash_{csp} \{a_{12}\}$
- ► Limitation: the translation needs to generate every synchronisation index, hence cannot terminate for CCS terms with parallel under recursion

Formalisation: Parallel - 2/2

- New solution: define binary synchronisation in CSP, then translate CCS binary sync. into CSP binary sync.
 - Let \parallel denote m-among-n synchronisation on event a.
- ▶ CCS parallel case $a.0 \mid \bar{a}.0 \mid \bar{a}.0$ corresponds with the following CSPmn process:

Contrast with Ekembe et al. (CCS to CSP Translation, Dec.2021):

$$((a \leadsto STOP \parallel a \leadsto STOP \parallel a \leadsto STOP) \square$$

$$(a_{12} \square a_{13} \leadsto STOP \parallel a_{12} \leadsto STOP \parallel a_{12} \leadsto STOP) \setminus_{csp} \{a_{12}, a_{13}\}$$

CSPmn semantics

lacktriangle The rules for m/n indexed interface paralell composition are given hereafter.

$$M/N-IndxIfacePar: \frac{P_{j} \overset{a}{\rightarrow} P' \quad [a \# m \not\in B^{\checkmark} \times \{2,...,n\}, k \neq j]}{\displaystyle \parallel P_{j} \overset{a}{\rightarrow} (\prod_{B \times \{2,...,n\}} P_{k}) \quad \parallel P'} \\ \frac{P_{1} \overset{a}{\rightarrow} P'_{1} \ldots P_{n} \overset{a}{\rightarrow} P'_{n} \quad [a \# m \in B^{\checkmark} \times \{2,...,n\}, j \in J, k \neq j]}{\displaystyle \parallel P_{j} \overset{a}{\rightarrow} \prod_{\{J \subseteq I | card(J) = m\}} \left((\prod_{B \times \{2,...,n\}} P_{k}) \quad \parallel (\prod_{B \times \{2,...,n\}} P'_{j}) \right)} \\ \frac{P_{1} \overset{a}{\rightarrow} P'_{1} \ldots P_{n} \overset{a}{\rightarrow} P'_{n} \quad [a \# m \in B^{\checkmark} \times \{2,...,n\}, j \in J, k \neq j]}{\displaystyle \parallel P_{2} \stackrel{a}{\rightarrow} \prod_{B \times \{2,...,n\}} \left((\prod_{B \times \{2,...,n\}} P'_{k}) \right)} \\ \frac{P_{2} \overset{a}{\rightarrow} P'_{1} \ldots P_{n} \overset{a}{\rightarrow} P'_{n} \quad [a \# m \in B^{\checkmark} \times \{2,...,n\}, j \in J, k \neq j]}{\displaystyle \parallel P_{2} \stackrel{a}{\rightarrow} \prod_{B \times \{2,...,n\}} \left((\prod_{B \times \{2,...,n\}} P'_{k}) \right)} \\ \frac{P_{2} \overset{a}{\rightarrow} P'_{1} \ldots P_{n} \overset{a}{\rightarrow} P'_{n} \quad [a \# m \in B^{\checkmark} \times \{2,...,n\}, j \in J, k \neq j]}{\displaystyle \parallel P'_{2} \stackrel{a}{\rightarrow} \prod_{B \times \{2,...,n\}} \left((\prod_{B \times \{2,...,n\}} P'_{k}) \right)} \\ \frac{P_{3} \overset{a}{\rightarrow} P'_{1} \ldots P_{n} \overset{a}{\rightarrow} P'_{n} \quad [a \# m \in B^{\checkmark} \times \{2,...,n\}, j \in J, k \neq j]}{\displaystyle \parallel P'_{2} \stackrel{a}{\rightarrow} P'_$$

▶ We derive binary-only synchronisation by imposing that *every* event in set *B* allows 2(only)-among-n processes to synchronise.

$$2/N-IndxIfacePar: \frac{P_1 \xrightarrow{a} P_1' \dots P_n \xrightarrow{a} P_n' \quad [a \# 2 \in B^{\checkmark} \times \{2\}, j \in J, k \neq j]}{ \underset{B \times \{2\}}{||P_j \xrightarrow{a} \prod} \left(\left(\underset{B \times \{2\}}{||P_k \cap A|} \underset{B \times \{2\}}{||P_j \cap A|} \right) \right)} (1)$$

Correctness of CSPmn

- ▶ Every CSP process with a_{ij} synchronisation is equivalent to a CSPmn process obtained by mapping every a_{ij} to a single a_S (via renaming), and mapping $\|$ unto $\|$ $\{a_{ij}\}$ $a_S \# 2$
 - Conversely, binary synchronisation can be defined from renaming and mutliway synchronisation
- ► More generally, m-among-n synchronisation can be defined from renaming and multiway synchronisation
 - ► Consequence: CSPmn is a conservative extension of CSP
- ► E.g.

Translation Workflow

- 1. Make the result of synchronisation visible:
 - ▶ in CCS: $(a, \bar{a}) \mapsto \tau$
 - ▶ in CCSTau: $(a, \bar{a}) \mapsto \tau[a \mid \bar{a}]$
- 2. Separate interleaving from synchronisation:
 - ▶ Generate a unique index per complementary prefix pairs, e.g., $(a, \bar{a}) \mapsto (a_S, \bar{a}_S)$
- 3. Translate CCS operators into corresponding CSP operators,
 - lacktriangle e.g., au maps to tau, tau, tau maps to tau, tau, tau, tau
- 4. Hide a_S synchronisation names

CCSTau - Syntax and Semantics

- CCSTau extends CCS with visible synchronisation and hiding
- ► To make synchronisation observable we use the following Com rule:

$$Com: \frac{P \xrightarrow{\overline{a}} P' \quad Q \xrightarrow{a} Q'}{P\mid_{T} Q \xrightarrow{\tau[\overline{a}\mid a]} P'\mid_{T} Q'}$$

▶ We introduce the following hiding rules in the LTS which are similar to the CSP rules:

$$\mathit{Hide1}: \frac{P \xrightarrow{\beta} P' \quad \beta \not \in B}{P \backslash_{_{T}} B \xrightarrow{\beta} P' \backslash_{_{T}} B} \qquad \mathit{Hide2}: \frac{P \xrightarrow{\beta} P' \quad \beta \in B}{P \backslash_{_{T}} B \xrightarrow{\tau} P' \backslash_{_{T}} B}$$

CCSTau - Link from CCS

- ▶ We describe here a translation of CCS processes into CCSTau.
 - This encoding is concerned with hiding the now-observable synchronisation actions.

Definition 1 ($c2ccs\tau$)

Translation function c2ccs au, when applied to a CCS process, returns a CCSTau process.

$$c2ccs\tau(P\mid Q) \triangleq (c2ccs\tau(P)\mid_{T} c2ccs\tau(Q)) \Big\backslash_{T} \{\tau[a\mid \overline{a}]\mid a\in \mathcal{A}(P), \bar{a}\in \mathcal{A}(Q)\}$$

Theorem 1

Let P be a CCS process. Then: $P \sim c2ccs\tau(P)$.

Every CCS process is a CCSTau process

$$CCS \xrightarrow{c2ccs\tau} CCSTau \xrightarrow{ix} \xrightarrow{g^*} \xrightarrow{conm} CCSTau \xrightarrow{tl} CSP \xrightarrow{\left\{tau, a_{ij}\right\}} \xrightarrow{ai2a} CSP$$

Figure: ccs2csp Translation workflow

Figure: $ccs2csp_3$ Translation workflow

- ▶ Running Example: $(a.P \mid \overline{a}.Q) \mid \overline{a}.R$
- We initially translate this process into CCSTau through the $c2ccs\tau$ function (Def.1), which gives us:

$$((a.P'\mid_{\scriptscriptstyle T} \overline{a}.Q') \big\backslash_{\scriptscriptstyle T} \{\tau[a\mid \overline{a}]\}\mid_{\scriptscriptstyle T} \overline{a}.R') \big\backslash_{\scriptscriptstyle T} \{\tau[a\mid \overline{a}]\}$$

- ▶ E.g.: $(a.P \mid \overline{a}.Q) \mid \overline{a}.R$
- \blacktriangleright (ccs2csp) Then $ix: a_1.P'' \mid_T \overline{a}_2.Q''' \mid_T \overline{a}_3.R''$
- ightharpoonup (ccs2csp) Then g^* :

$$(a_1 + a_{12} + a_{13}).P''' \mid_T ((\overline{a}_2 + \overline{a}_{12}).Q''') \mid_T (\overline{a}_3 + \overline{a}_{13}).R'''$$

where (a + b).S is syntactic sugar for a.S + b.S.

▶ In contrast, for $ccs2csp_3$, apply g^2 :

$$(a + a_S).P''' \mid_T ((\overline{a} + \overline{a}_S).Q''') \mid_T (\overline{a} + \overline{a}_S).R'''$$

- ightharpoonup E.g.: $(a.P \mid \overline{a}.Q) \mid \overline{a}.R$
- ightharpoonup c2ccs au (Def.1): $((a.P' \mid_{T} \overline{a}.Q')\setminus_{\pi} \{\tau[a \mid \overline{a}]\} \mid_{T} \overline{a}.R')\setminus_{\pi} \{\tau[a \mid \overline{a}]\}$
- $ix \circ g^*$: $(a_1 + a_{12} + a_{13}).P''' \mid_{\tau} ((\overline{a}_2 + \overline{a}_{12}).Q''') \mid_{\tau} (\overline{a}_3 + \overline{a}_{13}).R'''$ $ightharpoonup q^2$: $(a+a_S).P''' \mid_{\tau} ((\overline{a}+\overline{a}_S).Q''') \mid_{\tau} (\overline{a}+\overline{a}_S).R'''$
- conm identifies co-names synchronisation events. For ccs2csp:

$$((a_1 + a_{12} + a_{13}).P''' \mid_T (\bar{a}_2 + a_{12}).Q''') \mid_T (\bar{a}_3 + a_{13}).R'''$$

ightharpoonup And for $ccs2csp_3$:

$$((a + a_S).P''' \mid_{\tau} (\bar{a} + a_S).Q''') \mid_{\tau} (\bar{a} + a_S).R'''$$

Operator Translation

 \blacktriangleright We can now present the translation tl from CCSTau to CSP.

Definition 2

Let P,Q be CCSTau processes and tau be a fresh CSP event.

$$tl_{3}(0) \stackrel{\frown}{=} STOP \qquad tl_{3}(P \mid_{T} Q) \stackrel{\frown}{=} tl_{3}(P) \underset{\{a \# 2 \mid a \in \mathcal{A}(P) \cap \mathcal{A}(Q)\}}{\parallel} tl_{3}(Q)$$

$$tl_{3}(\tau.P) \stackrel{\frown}{=} tau \leadsto tl_{3}(P) \qquad tl_{3}(P \upharpoonright B) \stackrel{\frown}{=} tl_{3}(P) \upharpoonright_{csp} B$$

$$tl_{3}(a.P) \stackrel{\frown}{=} a \leadsto tl_{3}(P) \qquad tl_{3}(P \backslash_{T} B) \stackrel{\frown}{=} tl_{3}(P) \backslash_{csp} B$$

$$tl_{3}(P + Q) \stackrel{\frown}{=} tl_{3}(P) \sqcap tl_{3}(Q) \qquad tl_{3}(\mu X.P) \stackrel{\frown}{=} \mu X.tl_{3}(P)$$

Definition 3

$$\begin{split} tl(P\mid_T Q) & \; \widehat{=} \; \; tl(P) \quad \underset{\{a\mid a\in\mathcal{A}(P)\cap\mathcal{A}(Q)\}}{\parallel} tl(Q) \\ tl(P) & = tl_3(P) \quad \text{If P is not parallel composition} \end{split}$$

- ► E.g.: $(a.P \mid \overline{a}.Q) \mid \overline{a}.R$
- ▶ $ix \circ g^*$: $(a_1 + a_{12} + a_{13}).P''' \mid_T ((\overline{a}_2 + \overline{a}_{12}).Q''') \mid_T (\overline{a}_3 + \overline{a}_{13}).R'''$ ▶ g^2 : $(a + a_S).P''' \mid_T ((\overline{a} + \overline{a}_S).Q''') \mid_T (\overline{a} + \overline{a}_S).R'''$
- ► Then *conm*:

$$((a_1 + a_{12} + a_{13}).P''' \mid_T (\bar{a}_2 + a_{12}).Q''') \mid_T (\bar{a}_3 + a_{13}).R'''$$

- And: $((a + a_S).P''' \mid_{\pi} (\bar{a} + a_S).Q''') \mid_{\pi} (\bar{a} + a_S).R'''$
- ▶ tl (Def.3):

$$\left(\left(a_{1} \square a_{12} \square a_{13}\right) \rightsquigarrow P'''' \mathop{\parallel}_{\left\{a_{12}\right\}} \left(\bar{a}_{2} \square a_{12}\right) \rightsquigarrow Q''''\right) \mathop{\parallel}_{\left\{a_{13}\right\}} \left(\bar{a}_{3} \square a_{13}\right) \rightsquigarrow R''''$$

► tl₃ (Def.2):

$$((a \square a_S) \rightsquigarrow P'''' \underset{\{a_S \# 2\}}{\parallel} (\bar{a} \square a_S) \rightsquigarrow Q'''') \underset{\{a_S \# 2\}}{\parallel} (\bar{a} \square a_S) \rightsquigarrow R''''$$

- \triangleright E.g.: $(a.P \mid \overline{a}.Q) \mid \overline{a}.R$
- $\qquad \qquad tl: \left(\left(a_1 \,\square\, a_{12} \,\square\, a_{13} \right) \stackrel{\checkmark}{\leadsto} P'''' \, \mathop{\parallel}_{\{a_{12}\}} \left(\bar{a}_2 \,\square\, a_{12} \right) \rightsquigarrow Q'''' \right) \mathop{\parallel}_{\{a_{13}\}} \left(\bar{a}_3 \,\square\, a_{13} \right) \rightsquigarrow R'''' \right.$: $((a_1 \sqcup a_{12} \sqcup a_{13}) \longrightarrow \{a_{12}\}$ tl_3 : $((a \sqcup a_S) \leadsto P'''' \parallel (\bar{a} \sqcup a_S) \leadsto Q'''') \parallel (\bar{a} \sqcup a_S) \leadsto R''''$
- The final CSP term is thus:

$$(((a \Box a_{12} \Box a_{13}) \leadsto P'''') \|_{\{a_{12}\}} (\bar{a} \Box a_{12}) \leadsto Q'''') \|_{\{a_{13}\}}$$

$$(\bar{a} \Box a_{13}) \leadsto R'''') \setminus_{csp} \{tau\} \setminus_{csp} \{a_{12}, a_{13}\}$$

The final CSPmn term is thus:

$$(((a \square a_S) \leadsto P'''') \parallel_{\{a_S \# 2\}} (\bar{a} \square a_S) \leadsto Q'''') \parallel_{\{a_S \# 2\}}$$
$$(\bar{a} \square a_S) \leadsto R'''') \backslash_{csp} \{tau\} \backslash_{csp} \{a_S\}$$

Example 2 - Recursion

Let $P \cong \mu X(a \mid \bar{a}.X)$ (or equiv. $P \cong a.0 \mid \bar{a}.P$) be a CCS process. Then, $ix(P) = a_1 \mid \bar{a}_2.ix_{\{3..\}}(P)$,

$$P = a \mid \bar{a}.(a \mid \bar{a}.P)$$

 $ix(P) = a_1 \mid \bar{a}_2.(a_3 \mid \bar{a}_4.ix_{\{5..\}}(P))$

The synchronisation pairs are thus $(a_1, \bar{a}_2), (a_1, \bar{a}_4), ..., (a_3, \bar{a}_4), ...$ We will not be able to generate all the a_{1*2k} $(k \ge 1)$ indices since recursion is unbounded. In contrast, let us define $ccs2csp_3(P)$. Then:

$$g^{2}(P) = (a + a_{S}) \mid (\bar{a} + \bar{a}_{S}).g^{2}(P)$$

= $(a + a_{S}) \mid (\bar{a} + \bar{a}_{S}).((a + a_{S}) \mid (\bar{a} + \bar{a}_{S}).g^{2}(P))$

We can unfold ${\cal P}$ multiple times, we only ever generate a single name for synchronisation. Then:

$$ccs2csp_3(P) = ((a \square a_S) \parallel (\bar{a} \square a_S) \leadsto t2csp_3 \circ c2ccs\tau(P)) \setminus_{csp} \{a_S\}$$

Correctness of the Translation - 1/2

Theorem 2 (Correctness of ccs2csp)

Let P be a CCS process. Then:

- 1. $P \xrightarrow{\tau} P'$ imply that $\forall S \mid S \cap \mathcal{A}(ix(P)) = \{\} : ccs2csp(S, P) \xrightarrow{\tau} ccs2csp(S, P')$
- 2. $\forall S \mid S \cap \mathcal{A}(ix(P)) = \{\} : ccs2csp(S, P) \xrightarrow{\tau} Q \text{ imply that } \exists !P' : P \xrightarrow{\tau} P' \text{ and } Q = ccs2csp(S, P')$
- 3. $P \xrightarrow{a} P'$ imply that $\forall S \mid S \cap \mathcal{A}(ix(P)) = \{\} : ccs2csp(S, P) \xrightarrow{a} ccs2csp(S, P')$
- 4. $\forall S \mid S \cap \mathcal{A}(ix(P)) = \{\} : ccs2csp(S, P) \xrightarrow{a} Q \text{ imply that } \exists !P' : P \xrightarrow{a} P' \text{ and } Q = ccs2csp(S, P')$

We say that ccs2csp is correct up to strong bisimulation.

Corollary 4

Let P be a CCS process. Then: $P \sim ccs2csp(P)$.

Correctness of the Translation - 2/2

Definition 5 (gstar2m/n)

Let a_{ij} be an g^* name, a_S an g^2 name. Then: $g^*2g^2 \ \widehat{=}\ \{\tau \mapsto \tau, a_{ij} \mapsto a_S\}$

Definition 6

Let P be a CSP process.

$$g^* 2g^2(\alpha \leadsto P) \stackrel{?}{=} g^* 2g^2(\alpha) \leadsto g^* 2g^2(P)$$

$$g^* 2g^2(P \parallel_{\{a_{ij}\}} Q) \stackrel{?}{=} g^* 2g^2(P) \parallel_{\{a_S \# 2\}} g^* 2g^2(Q)$$

. . .

Theorem 3

Let P be a CCS process. Then: $g^*2g^2 \circ ccs2csp(P) = ccs2csp_3(P)$.

Corollary 7

Let P be a CCS process. Then: $P \sim ccs2csp_3(P)$.

Conclusion, Future Work

- ccs2csp translation has been implemented in Haskell
 - ► GitHub Repo: https://github.com/andrewbutterfield/ccs2csp
 - ► Next: implement $ccs2csp_3$
- ▶ Next: extend FDR with m-among-n synchronisation
- Ongoing: Translate Pi-calculus into CSPmob
- Questions?

Structural Properties

- Gorla (Towards a Unified Approach to Encodability and Separation Results for Process Calculi, 2010) proposes five requirements for a translation to be valid:
 - operational correspondence
 - √ a CCS term is strong bisimilar to its translation
 - divergence reflection
 - √ if a CSPmn translation diverges then its source CCS term does;
 - success sensitiveness
 - $\checkmark\,$ a CCS term converges if, and only if, its CSPmn translation converges, and both converge to the same success final term
 - name invariance
 - \checkmark typically, $ccs2csp_3(f(P)) \sim f(ccs2csp_3(P))$
 - compositionality
 - ✓ Our translation is compositional