

1.

- a. $2 \cdot 3 \bmod 13 = 6 \bmod 13$
- b. $2 \cdot 8 \bmod 7 = 2 \bmod 7$
- c. $5 \cdot 1 \bmod 11 = 5 \bmod 11$
- d. $4 \cdot 4 \bmod 15 = 1 \bmod 15$

2.

- a. $1 \cdot 5^{-1} \bmod 13 = 1 \cdot 8 \bmod 13 = 8 \bmod 13$
- b. $1 \cdot 5^{-1} \bmod 7 = 1 \cdot 3 \bmod 7 = 3 \bmod 7$
- c. $3 \cdot 2 \cdot 5^{-1} \bmod 7 = 3 \cdot 2 \cdot 3 \bmod 7 = 4 \bmod 7$

3.

a.

+	0	1	2	3
0	0	1	2	3
1	1	2	3	0
2	2	3	0	1
3	3	0	1	2

b.

x	0	1	2	3
0	0	0	0	0
1	0	1	2	3
2	0	2	0	2
3	0	3	2	1

c.

+	0	1	2	3	4		x	0	1	2	3	4
0	0	1	2	3	4		0	0	0	0	0	0
1	1	2	3	4	0		1	0	1	2	3	4
2	2	3	4	0	1		2	0	2	4	1	3
3	3	4	0	1	2		3	0	3	1	4	2
4	4	0	1	2	3		4	0	4	3	2	1

d.

+	0	1	2	3	4	5		x	0	1	2	3	4	5
0	0	1	2	3	4	5		0	0	0	0	0	0	0
1	1	2	3	4	5	0		1	0	1	2	3	4	5
2	2	3	4	5	0	1		2	0	2	4	0	2	4
3	3	4	5	0	1	2		3	0	3	0	3	0	3
4	4	5	0	1	2	3		4	0	4	2	0	4	2
5	5	0	1	2	3	4		5	0	5	4	3	2	1

- e. In Z_4 , 0 and 2 did not have an inverse, while in Z_6 , 0, 2, 3 and 4 did not have one. All elements in Z_5 had an inverse because 5 is prime, so no nonzero number smaller than it can have a gcd other than 1.
4. 9 in Z_{11} , 5 in Z_{12} , 8 in Z_{13}
- 5.
- $3 \cdot 3 \bmod 13 = 9 \bmod 13$
 - $7 \cdot 7 \bmod 13 = 8 \bmod 13$
 - $3^2 \cdot 3^2 \cdot 3^2 \cdot 3^2 \bmod 13 = 9 \cdot 9 \cdot 9 \cdot 9 \bmod 13 = 3 \cdot 3 \cdot 9 \bmod 13 = 3 \bmod 13$
 - $(7^2)^{50} \bmod 13 = 3^{50} \bmod 13 = (3^{10})^5 \bmod 13 = 3^5 \bmod 13 = 81 \cdot 3 \bmod 13 = 9 \bmod 13$
6. $x=5$
7. $m=4$: (1, 3) $\varphi=2$, $m=5$: (1, 2, 3, 4) $\varphi=4$, $m=9$: (1, 2, 4, 5, 7, 8) $\varphi=6$, $m=26$: (1, 3, 5, 7, 9, 11, 15, 17, 19, 21, 23, 25) $\varphi=12$
8. $a^{-1} = 15$. $X = 15(y-22)$
- Pseudocode: convert char to int, put through decryption equation, convert back
- First the sentence and then the evidence said the queen