

1.
 - a. $n=33, \Phi(33)=20, e=7^{-1} \bmod 20=3, y=5^3 \bmod 33=26, x=26^7 \bmod 33=5$
 - b. $n=55, \Phi(55)=40, d=3^{-1} \bmod 40=27, y=9^{27} \bmod 55=4, x=8^3 \bmod 55=9$
2. $p=11, q=13, n=143, c_p=6, c_q=6$
 - a. $(1, 0): [13*6]1 + [11*6]0 \bmod 143 = 78 \bmod 143$
 - b. $(4, 5): [13*6]4 + [11*6]5 \bmod 143 = [78]4 + [66]5 = 312 + 330 \bmod 143 = 70 \bmod 143$
 - c. $(5, 4): [13*6]5 + [11*6]4 \bmod 143 = [78]5 + [66]4 = 390 + 264 \bmod 143 = 82 \bmod 143$
3. $p=7, q=11, n=77$
 - a. $x^2=1 \bmod 77$
 - b. $x^2=1 \bmod 7, -1 = 6 \bmod 7$
 - c. $x^2=1 \bmod 11, -1 = 10 \bmod 11$
 - d. $34=6+4(7)=(-1, 1), 43=10+3(11)=(1, -1)$
4. My fermats test occasionally (1 out of my 30ish tests) accidentally lets a non-carmichaels composite number out, but im accepting that rare mistake that only happens anyway due to the weakening of the function required to return every carmichael on purpose, as solving it seems computationally expensive.
 - a. 10^6 : [997633, 852841, 838201, 825265, 748657]
 - b. 10^7 : [9890881, 9613297, 9585541, 9582145, 9494101]

```
# -*- coding: utf-8 -*-  
"""
```

Created on Wed Apr 10 18:01:56 2019

```
@author: Erikson  
"""
```

```
from random import sample  
from math import sqrt, gcd, floor  
import time
```

```
def sqAndMul(base, exp, mod):  
    result = 1  
    binlist = [int(x) for x in '{:b}'.format(exp)]  
    for x in binlist:  
        result**=2  
        result %=mod  
        if x is 1:  
            result *= base  
            result %=mod  
    return result
```

```
def isPrime(p):  
    for x in range(2, floor(sqrt(p))):  
        if p % x == 0:  
            return False  
    return True
```

```
def fermat(p, s):  
    subset = sample(range(2, p-1), s)  
    passedGcd = False  
    for a in subset:  
        if gcd(a, p) is 1:  
            passedGcd = True  
            if sqAndMul(a, p-1, p) is not 1:  
                return False  
    if passedGcd:#catch cases where all s values for a were thrown out, often  
        return True#due to p being even  
    else:  
        return False
```

```
def isCarmichael(C):  
    for a in range(C):  
        if gcd(a, C)==1:
```

```
        if sqAndMul(a, C-1, C) is not 1:
            return False
    return True
```

```
def main():
    start_time = time.time()
    start = 10**7
    security = 15
    carmichaels = []
    while(True):
        if(fermat(start, security)):#likely prime
            if(not isPrime(start)):#not prime
                #if(isCarmichael(start)):#is car
                carmichaels.append(start)
                if(len(carmichaels)==5):
                    break
        start-=1
        if(start < security+3):
            break
    print(carmichaels)
    print("--- %s seconds total" % (time.time() - start_time))
```

```
if __name__ == "__main__":
    main()
```