

1. Reducible polynomials degree < 2 are  $x$ ,  $x+1$ ,  $x^2+x+1$ . Products of degree 5 MUST use one of these, along with something of degree 3 or 4.
  - a.  $x^5+x^4+1 = (x^2+x+1)(x^3+x+1)$
  - b.  $x^5+x^3+1 = \text{irreducible}$ .
  - c.  $x^5+x^4+x^2+1 = (x+1)(x^4+x+1)$  -- reducible
2. Generators are bolded
  - a.  $a \bmod 29$ 
    - order of 2 mod 29 is 28**
    - order of 3 mod 29 is 28**
    - order of 4 mod 29 is 14
    - order of 5 mod 29 is 14
    - order of 6 mod 29 is 14
    - order of 7 mod 29 is 7
    - order of 8 mod 29 is 28**
    - order of 9 mod 29 is 14
    - order of 10 mod 29 is 28**
    - order of 11 mod 29 is 28**
    - order of 12 mod 29 is 4
    - order of 13 mod 29 is 14
    - order of 14 mod 29 is 28**
    - order of 15 mod 29 is 28**
    - order of 16 mod 29 is 7
    - order of 17 mod 29 is 4
    - order of 18 mod 29 is 28**
    - order of 19 mod 29 is 28**
    - order of 20 mod 29 is 7
    - order of 21 mod 29 is 28**
    - order of 22 mod 29 is 14
    - order of 23 mod 29 is 7
    - order of 24 mod 29 is 7
    - order of 25 mod 29 is 7
    - order of 26 mod 29 is 28**
    - order of 27 mod 29 is 28**
    - order of 28 mod 29 is 2
  - b.  $a \bmod 30$  -- most never become 1.
    - order of 2 mod 30 is nonexistent
    - order of 3 mod 30 is nonexistent
    - order of 4 mod 30 is nonexistent
    - order of 5 mod 30 is nonexistent
    - order of 6 mod 30 is nonexistent
    - order of 7 mod 30 is 4

order of 8 mod 30 is nonexistent  
 order of 9 mod 30 is nonexistent  
 order of 10 mod 30 is nonexistent  
 order of 11 mod 30 is 2  
 order of 12 mod 30 is nonexistent  
 order of 13 mod 30 is 4  
 order of 14 mod 30 is nonexistent  
 order of 15 mod 30 is nonexistent  
 order of 16 mod 30 is nonexistent  
 order of 17 mod 30 is 4  
 order of 18 mod 30 is nonexistent  
 order of 19 mod 30 is 2  
 order of 20 mod 30 is nonexistent  
 order of 21 mod 30 is nonexistent  
 order of 22 mod 30 is nonexistent  
 order of 23 mod 30 is 4  
 order of 24 mod 30 is nonexistent  
 order of 25 mod 30 is nonexistent  
 order of 26 mod 30 is nonexistent  
 order of 27 mod 30 is nonexistent  
 order of 28 mod 30 is nonexistent  
 order of 29 mod 30 is 2

c.  $a \bmod 31$

order of 2 mod 31 is 5  
**order of 3 mod 31 is 30**  
 order of 4 mod 31 is 5  
 order of 5 mod 31 is 3  
 order of 6 mod 31 is 6  
 order of 7 mod 31 is 15  
 order of 8 mod 31 is 5  
 order of 9 mod 31 is 15  
 order of 10 mod 31 is 15  
**order of 11 mod 31 is 30**  
**order of 12 mod 31 is 30**  
**order of 13 mod 31 is 30**  
 order of 14 mod 31 is 15  
 order of 15 mod 31 is 10  
 order of 16 mod 31 is 5  
**order of 17 mod 31 is 30**  
 order of 18 mod 31 is 15  
 order of 19 mod 31 is 15  
 order of 20 mod 31 is 15  
**order of 21 mod 31 is 30**

**order of 22 mod 31 is 30**

order of 23 mod 31 is 10

**order of 24 mod 31 is 30**

order of 25 mod 31 is 3

order of 26 mod 31 is 6

order of 27 mod 31 is 10

order of 28 mod 31 is 15

order of 29 mod 31 is 10

order of 30 mod 31 is 2

3.  $(x^3+x+1)*(x^3+x^2+1) = x^6+x^5+x^4+3x^3+x^2+x+1 = x^6+x^5+x^4+x^3+x^2+x+1$ 
  - a.  $x^6+x^5+x^4+x^3+x^2+x+1 \bmod P(x^4+x+1)$
  - b.  $x^4 = x+1 \bmod P$ ,  $x^5 = x^2+x \bmod P$ ,  $x^6 = x^3+x^2 \bmod P$
  - c.  $x^6+x^5+x^4+x^3+x^2+x+1 \bmod P = (x^3+x^2)+(x^2+x)+(x+1)+x^3+x^2+x+1 = x^2+x$
4.  $(x^7+x^6+x^3+x^2)^{-1} = x^4+x^3+x+1$ ,  $P(x^8+x^4+x^3+x+1)$ 
  - a.  $(x^4+x+1) * (x^4+x^3+x+1) = x^8+x^7+2x^5+3x^4+x^3+x^2+2x+1 = x^8+x^7+x^4+x^3+x^2+1$
  - b.  $x^8 = x^4+x^3+x+1 \bmod P$
  - c.  $x^8+x^7+x^4+x^3+x^2+1 \bmod P = (x^4+x^3+x+1)+x^7+x^4+x^3+x^2+1 = x^7+x^2+x$
5.  $(x^7+x^6+x+1) * (x^5+x^4+x+1) = x^{12}+2x^{11}+x^{10}+x^8+2x^7+x^6+2x^5+2x^4+x^2+2x+1$ 
  - a.  $x^{12}+x^{10}+x^8+x^6+x^2+1$ ,  $P(x^8+x^4+x^3+x+1)$
  - b.  $x^8 = x^4+x^3+x+1 \bmod P$ ,  $x^{10} = x^6+x^5+x^3+x^2 \bmod P$ ,  $x^{12} = x^8+x^7+x^5+x^4 \bmod P$
  - c.  $((x^4+x^3+x+1)+x^7+x^5+x^4)+(x^6+x^5+x^3+x^2)+(x^4+x^3+x+1)+x^6+x^2+1$
  - d.  $x^7+x^4+x^3$
- 6.

i	q	r	s	t
0		$x^8+x^4+x^3+x+1$	1	0
1	$x^2+x$	$x^6+x^5+x^4+x$	0	1
2	x	$x^5+x^4+x^2+x+1$		$x^2+x$
3	x	$x^4+x^3+x^2$		$x^3+x^2+1$
4	x	$x^3+x^2+x+1$		$x^4+x^3+x^2$
5	$x^2+x+1$	x		$x^5+x^4+x^2+1$
6		1		$x^7+x^4+x^2+x+1$

Find $r_2$		$x^2+x$
		_____
$x^6+x^5+x^4+x$		$x^8+x^4+x^3+x+1$
	-	$x^8+x^7+x^6+x^3 = x^7+x^6+x^4+x$
	-	$x^7+x^6+x^5+x^2 = x^5+x^4+x^2+x+1$

Find $r_3$		$x$
$x^5+x^4+x^2+x+1$		_____
		$x^6+x^5+x^4+x$
	-	$x^6+x^5+x^3+x^2+x = x^4+x^3+x^2$

Find $r_4$		$x$
$x^4+x^3+x^2$		_____
		$x^5+x^4+x^2+x+1$
	-	$x^5+x^4+x^3 = x^3+x^2+x+1$

Find $r_5$		$x$
$x^3+x^2+x+1$		_____
		$x^4+x^3+x^2$
	-	$x^4+x^3+x^2+x = x$
	-	

Find $r_6$		$x^2+x+1$
$x$		_____
		$x^3+x^2+x+1$
	-	$x^3 = x^2+x+1$

	-	$x^2 = x+1$
	-	$x = 1$

Find $r_7$		
		_____
		x
	-	